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Westinghouse Astronuclear Laboratory

APPLICATION OF FRACTURE MECHANICS TO
THERMALLY STRESSED CYLINDERS USING A
FINITE ELEMENT METHOD

UNCLASSIFIED NERVA RESEARCH
AND DEVELOPMENT REPORT.

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APPLICATION OF FRACTURE MECHANICS TO THERMALLY STRESSED CYLINDERS USING A FINITE ELEMENT METHOD

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AND DEVELOPMENT REPORT.

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ABSTRACT

This report describes a method for obtaining the thermal stress intensity factor for a hollow cylinder with internal heat generation. The model is chosen to represent a section around a cooling hole in the reflector of the NERVA reactor.

The stress intensity factor is determined as a function of radial crack length for various heat generation rates, material properties and geometries. The stress intensity factor is related to the equivalent thermal stress at the inner wall. A comparison between this allowable stress and the stresses obtained from approximate mechanical models shows close agreement only for small crack lengths. In all cases the mechanical models give conservative estimates.

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1. INTRODUCTION

The final objective of this study is to develop a method that can be used to predict the reliability of beryllium reflectors when the stress gradient around coolant holes is considered. A fracture mechanics approach can be used if the stress intensity factor at the coolant hole and the fracture toughness of the material are known. This report is concerned with predicting the stress intensity factor for various length cracks and thermal conditions at the coolant holes.

2. THE BASIC MODEL

2.1 DESCRIPTION OF MODEL

A section of the reflector is shown in Figure 2-1. The cylindrical segment, shown dashed in this figure, and in more detail in Figure 2-2, is chosen as the basic model to represent the position of the reflector around a cooling hole. A radial crack is assumed to exist at the inside surface. The model is designed to accept various thermal and mechanical loadings to approximate the actual reflector conditions.

2.2 THE FINITE ELEMENT MESH

In order to determine the stress intensity factor at the crack tip it is necessary to know either the amount of the crack opening or the stress distribution behind the crack. This information can be determined from a finite element analysis.

A large portion of the time involved in using existing finite element codes is required in developing a suitable mesh. A mesh generating program has been developed to provide a suitable grid for the basic model^{1,2}. Since the area around the crack tip is highly stressed, it necessarily requires a fine mesh. This portion of the mesh is labeled Zone I and is shown in Figure 2-3. The mesh in the area away from the crack may be more coarse and is contained in Zone II. Zone III represents a transition area between the coarse and fine grids.

The variables required to define a mesh are:

- 1) the inside radius of the cylinder,
- 2) the outside radius,
- 3) the location of the crack,
- 4) the crack length,
- 5) the size of Zone I,
- 6) the width of Zone III,
- 7) the number of elements in each zone, and
- 8) the size of the elements in each zone.

Figure 2-4 shows the computer plot of the mesh for a crack depth equal to 50% of the thickness. The origin of the axis system has been transferred to the crack tip to aid in interpreting the stresses and displacements. Figure 2-5 is a blowup of the mesh in Zones I and III. Figure 2-6 is yet a larger blowup of the mesh in Zone I and gives an indication of the fine mesh size placed around the crack tip.

Section of Beryllium Reflector

FIGURE 2-1

Basic Model of Reflector Cooling Hole

FIGURE 2-2

Zone Description for Finite Element Mesh

FIGURE 2-3

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Finite Element Mesh for Cylinder with Radial Crack

FIGURE 2-4

Blowup of Mesh in Zones I and III

FIGURE 2-5

Blowup of Mesh Around Crack Tip (Zone I)

FIGURE 2-6

3. STRESS DISTRIBUTIONS AND DISPLACEMENTS

Having developed a general mesh, it is now possible to calculate the stresses and displacements associated with several loading conditions. Assume that the cylinder is subjected to a constant internal heat generation rate of 4529 BTU/hr-in^3 with the inside surface maintained at zero degrees and the outside wall adiabatic. The geometry, material properties, and loads are based on the NRX-A6 reflector conditions. The outside cylinder radius of 0.25" represents the mean distance between cooling holes near the inner diameter of the reflector. The material properties used are $E = 42 \times 10^6 \text{ psi}$, $\alpha = 6.7 \times 10^{-6}$, $\nu = 0.03$ and $k = 7.92 \text{ BTU/hr-in-}^\circ\text{R}$. The plane strain solution to this problem, when no crack is present, is shown in Figure 3-1. A comparison between the stresses and temperatures obtained from the finite element code³ and those obtained from an exact analytical solution indicates very little difference.

Figure 3-2 gives the stress distribution along a radial line whenever a crack equal in length to 50% of the wall thickness is present. The value of the stresses in the immediate vicinity of the crack tip are not exact even though small elements were used. The stress values obtained at the crack tip nodal point are listed on the figure as $\left(\begin{smallmatrix} \sigma \\ y \end{smallmatrix}\right)_{\text{C.T.}}$ and $\left(\begin{smallmatrix} \sigma \\ x \end{smallmatrix}\right)_{\text{C.T.}}$. This error, however, does not greatly affect the stress intensity calculation since an extrapolation process, as described in the next section, is used. Figure 3-3 shows the lines of constant principal stress for the in-plane model. The temperature distribution is not affected by the presence of the crack and remains the same as that shown in Figure 3-1. Figure 3-4 shows the stress distribution along a radius located 180° away from the crack. This plot shows that the presence of the crack tends only to slightly increase the stresses in this area over that obtained for the uncracked cylinder of Figure 3-1.

Figures 3-5 and 3-6 show the stress distributions along a radial line for crack depths equal to 25% and 12 1/2% of the wall thickness, respectively.

FIGURE 3-1

Temperature and Stress Distributions in a Hollow Cylinder with Heat Generation

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FIGURE 3-2

Stress Distribution Along a Radial Crack (50% Depth)

612792-208

Lines of Constant Principal Stress

FIGURE 3-3

FIGURE 3-4

612792-15B

Temperature and Stress Distributions 180° Away from Crack

PLANE STRAIN
 INSULATED OUTER SURFACE
 MATERIAL I
 MESH 2

(CRACK DEPTH
 = 25% OF
 THICKNESS)

$(\sigma_y)_{C.T.} = 7151 \text{ PSI}$
 $(\sigma_x)_{C.T.} = 4890 \text{ PSI}$

FIGURE 3-5

Stress Distribution Along a Radial Crack (25% Depth)

612792-148

FIGURE 3-6

Stress Distribution Along a Radial Crack (12 1/2% Depth)

4. DETERMINING THE STRESS INTENSITY FACTOR

4.1 GENERAL DESCRIPTION

Once the numerical stress and displacement solutions have been obtained for a particular finite element representation, crack tip stress intensity factors can be estimated by the use of established crack tip relations. The two methods considered here are 1) the displacement method and 2) the stress method⁴. The displacement method involves a correlation of the finite element nodal point displacements with the plane strain displacement equations. For each nodal point displacement v^* at some distance from the crack tip r the quantity K_I^* can be calculated as follows:

$$K_I^* = \frac{E (2\pi)^{1/2} v^*}{4 (1-\nu^2)^{1/2} r^{1/2}} \quad (4-1)$$

From plots of K_I^* as a function of r for fixed values of θ and a particular displacement component, estimates of K_I can be made. If the substituted displacements were the exact theoretical values then the value of K_I^* obtained as r approaches zero would be the exact value of K_I . Since the finite element displacements are rather inaccurate at an infinitesimal distance from the crack tip, this limiting process is not usable. Instead, a tangent extrapolation of the K_I^* curve is used to estimate K_I . The intercept of the tangent to the constant slope portion of the curve with the K_I^* axis is used as the K_I estimate.

The stress method for determining crack tip stress intensity factors is similar to the displacement method. Now the nodal point stresses are correlated with the known crack tip stress equations. For each nodal point stress σ in the vicinity of the crack tip the quantity K_I^* can be calculated as follows:

$$K_I^* = \frac{(2\pi r)^{1/2} \sigma(r, \theta)}{f(\theta)} \quad (4-2)$$

From plots of K_I^* as a function of r for a fixed θ and a particular stress component, estimates of K_I can be made. Again the tangent extrapolation value of K_I^* at $r = 0$ is the estimated K_I . It has been shown in Reference 4 that both methods give values of K_I approximately 5% lower than the exact (collocation) value.

4.2 DETERMINING K_I FOR THERMALLY STRESSED CYLINDER

Figures 4-1 and 4-2 show the curves used in estimating K_I for a 50% crack depth by the displacement and stress methods, respectively. In a similar manner estimates of K_I for a 25% crack depth are shown in Figures 4-3 and 4-4 and for a 12 1/2% crack depth in Figures 4-5 and 4-6.

The dimensionless stress intensity factor, $K_I/\sigma_w\sqrt{a}$, is plotted vs a/h for the three crack lengths in Figure 4-7. The arbitrarily chosen normalizing stress, σ_w , is taken to be the inside wall tangential stress whenever no crack is present. The limiting value of the dimensionless stress intensity factor is obtained from the fracture mechanics model shown in Figure 4-8. For this case

$$K_I = \sigma \sqrt{\pi a} f(a/b)$$

As $a/b \Rightarrow 0$, $f(a/b) \Rightarrow 1.12$, therefore

$$K_I/\sigma \sqrt{a} \approx 1.98$$

Figure 4-9, which shows the value of K_I vs a/h for a/h from zero to 0.5, is readily obtained from Figure 4-7.

The low values of the stress intensity factor shown in Figure 4-9 compared to the material fracture toughness ($\approx 9,000 \text{ psi}\sqrt{\text{in}}$) suggest that cracking the cylinder by means of this amount of thermal heat generation is highly unlikely. It should also be noted that boundary displacements, non-uniform heat generation, and thermal conduction, as are actually present in the reflector, were not included.

Figure 4-9 can be generalized to show the thermal stress intensity factor vs the dimensionless crack depth for various material properties, internal heat generation rates and cylinder geometries. The only requirement is that the inside radius, r_i , of the cylinder be equal to one-half of the outside radius, r_o , to maintain dimensional similarity. Figure 4-10 represents this generalized curve. A family of curves could be generated for other values of r_i/r_o , but at present the need is not sufficient to justify the additional time required. It can be seen from the ordinate of Figure 4-10 that if the heat generation rate is doubled over that given in Figure 4-9 and a cylinder three times as large is chosen, the stress intensity factor will be over 30 times as large.

FIGURE 4-1

Determining the Stress Intensity Factor at the Crack Tip from Displacements (50% Depth)

FIGURE 4-2

Determining the Stress Intensity Factor at the Crack Tip from the Stresses (50% Depth)

FIGURE 4-3

Determining K_I from Displacements (25% Depth)

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FIGURE 4-4

Determining K_I from Stresses (25% Depth)

FIGURE 4-5

Determining K_I from Displacements (12 1/2% Depth)

FIGURE 4-6

Determining K_I from Stresses (12 1/2% Depth)

FIGURE 4-7

Dimensionless Stress Intensity Factor for Cylinder Versus Crack Length

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FIGURE 4-8

Fracture Mechanics Model Used in Determining Limiting Value of K_I

FIGURE 4-9

Thermal Stress Intensity Factor Versus Crack Length

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FIGURE 4-10

Generalized Thermal Stress Intensity Factor Versus Crack Length

4.3 EQUIVALENT THERMAL STRESS AT INNER WALL

The tangential stress σ_w existing at the inner wall of the uncracked thermally stressed cylinder, for the particular case of $r_i = r_o/2$, is given by⁶

$$\sigma_w = .48 q''' r_o^2 B \quad (4-3)$$

where $B = E \alpha / 4k(1 - \nu)$. The parameter dimensions are E (psi), α (in/in-°F), r_o (in), k (BTU/hr-in-°R), and q''' (BTU/in³-hr). Figure 4-10 can then be replotted to give σ_w versus the relative crack depth a/h for various values of K_I and cylinder size. Figure 4-11 represents this plot for several values of K_I for each of two cylinder sizes ($r_o = .25$ and $.50$ in). The values of K_I were chosen so as to be in the vicinity of the fracture toughness of beryllium. This form of plot is similar to those obtained for mechanical stress intensity factors and will be used for comparison in the following section.

Suppose we wish to determine the mean allowable wall stress in a component that can be represented by the fracture mechanics model of Figure 4-11. Let the outer radius of the hole be 0.25 in. and assume a crack length of 20 mils at the inner wall. For a mean fracture toughness of 9,300 psi-√in. (as for irradiated beryllium) a mean allowable wall stress of 51,000 psi is obtained from Figure 4-11.

Suppose further that the fracture toughness data is normally distributed with a standard deviation of 710 psi-√in. obtained from six tests. The design minimum fracture toughness, based on a 99% reliability at a 95% confidence level, should then be reduced from 9,300 to 5,700 psi-√in. Converting this value to stress and including an 80% factor of safety results in an allowable design stress limit of 25,000 psi.

FIGURE 4-11

Stress to Propagate Crack in Cylinder with Uniform Heat Generation

5. COMPARISON OF THERMAL AND MECHANICAL STRESS INTENSITY FACTORS

5.1 COMPARISON WITH APPROXIMATE MECHANICAL MODELS

It is also of interest to compare the thermal stress intensity factor for the cracked cylinder with approximate mechanical stress intensity factors. It has been shown⁵ that in classical thermoelastic problems where the temperature distribution is stationary, the stress-field equations are identical to those for isothermal problems with applied forces. Since the presence of heat flow produces no additional singularities, the local character of the thermal stress at the crack tip is expected to be of the same nature as problems with mechanical stresses.

5.1.1 Comparison with Tensile Models

Several tensile fracture mechanics models can be used to approximate the thermally stressed cylinder. These are shown in Figure 5-6. The stress associated with each model vs crack length for a stress intensity factor of 100 psi-√in. is also shown. It should be noted that in each case the stress is directly proportional to the stress intensity factor so that the curves may be used for any value of K_I provided the stress axis is modified accordingly. For example, if it is desired to know the stress required to propagate a 0.007 inch crack in a material with a fracture toughness of 9000 psi-√in. for geometry (b) we would read a value of 600 psi from Figure 5-6 and multiply it by 90 to obtain 54,000 psi.

Curve (a) is obtained from Figure 4-11 for $r_o = .25$ in. (i.e., $h = .125$ in.) and $K_I = 100$ psi-√in. The stress indicated corresponds to the tangential stress at the inner wall that would be calculated for a cylinder with no crack. Curve (b) gives the uniform tensile stress σ_w corresponding to $K_I = 100$ psi-√in. in a tensile model of width $h = .125$ in. Curve (c) represents the uniform stress in a similar model, but of infinite width. Curve (b) approaches Curve (c) as the crack length decreases. This is to be expected since $a/h \Rightarrow 0$ can be represented mathematically by $a \Rightarrow 0$ or $h \Rightarrow \infty$.

Curve (d) represents the stress at the hole wall for $K_I = 100$ psi-√in. and the geometry shown in inset (d). The model has two diametrically opposed cracks on a hole of

FIGURE 5-1

Comparison of Tangential Wall Stress Versus
Crack Length for Several Fracture Mechanics Models

radius .125 in. and is subjected to a biaxial stress state. Under these conditions the tangential wall stress (assuming no crack present) is given by twice the value of the nominal stress.

The stress curve corresponding to a uniaxial loading or for a single crack would be slightly higher than that given by Curve (d) but yet lower than Curve (a).

6. CONCLUSIONS

The thermal stress intensity factor was obtained for a cylinder with internal heat generation. Values of K_I were obtained for a particular set of parameters that were representative of actual conditions existing in the reflector of the A6 NERVA reactor.

The low values of K_I compared to the fracture toughness of beryllium suggest that this mode of cracking for the cylinder is highly unlikely. The stress intensity factor is a function of the heat generated and the cylinder size. Increasing these parameters somewhat can raise the value of K_I to within the critical range, but such an increase is not realistic. It should also be noted that boundary displacements, non-uniform heat generation, and thermal conduction, as are actually present to some degree in the reflector, were not considered. The added effect of these conditions are to be included in subsequent investigations.

It has also been shown that simple tensile fracture mechanics models give a reasonable approximation to the thermal stress intensity factor. The models investigated all give lower estimates than the finite element-extrapolation method. In addition, the latter method is known to yield values of K_I approximately 5% lower than the exact (collocation) values insuring conservative approximations throughout.

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