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SPECTRAL SHIFT CONTROL REACTOR
BASIC PHYSICS PROGRAM

Theoretical Analysis
Part II
BPG Computer Program Report

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FOREWORD

Report BAW-1230 has been prepared as a part of the Spectral Shift Control Reactor Basic Physics Program. There are two, separately bound parts to the report. Part I covers the analysis of nuclear reactor experiments on heterogeneous lattices moderated by mixtures of light and heavy water. Part II, presented here, describes the BPG computer code, which is used extensively in the analysis of the experiments. Included in the description of the code are derivation of the equations, physical data for the calculations, input and output format, and general operating conditions for use with the Burroughs 205 computer.

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CONTENTS

	Page
1. INTRODUCTION	1
2. DERIVATIONS	3
2.1 The B-1 Approximation	4
2.2 The P-1 Approximation	8
2.3 The Greuling-Goertzel Approximation for $L_1(u)$	10
2.4 The Multigroup Equations	14
2.5 BPG Program Equations	19
2.6 Th-232 and U-238 Absorption Cross Sections	20
2.7 2-Group and Polygroup Quantities	22
2.8 Absorption and Fission Rates	23
2.9 Polygroup Coefficients	23
2.10 Leakage and Non-Leakage Probabilities	24
2.11 Age	24
2.12 Resonance Escape Probability	25
2.13 Multiplication Constant	25
2.14 Conversion Ratio	25
2.15 Computation of Σ^T	25
3. PHYSICAL DATA	27
3.1 Cross Sections	27
3.2 Thermal Cross Sections	28
3.3 Buckling	28
3.4 Source	28
3.5 Inelastic Matrix	29
4. PROGRAM DESCRIPTION	31
4.1 Limitations	31
4.2 Features	31
4.3 Input	31
4.4 Standard Case Input	33
4.5 General Case Input	37
4.6 Output	48
4.7 Operating Instructions	51
5. APPENDICES	57
A. The Quantities T_{ik}^n	
B. Notation and Definitions Used in Derivations	
C. Coordinate System Configuration and Probability of Neutron Scatter	
D. References	
E. Input Forms	
F. Sample Output	
G. Elements Available for Option 1 Input	
H. Elements Available on Tape for Option 2 Input	

1 INTRODUCTION

The BPG Code has been written to compute the criticality of reactors using all kinds of moderators, specifically including mixtures of H_2O and D_2O . Since slowing down with hydrogen moderator is susceptible to exact calculation, and since the Fermi age equation accurately treats heavy moderators such as carbon, the important innovations in this code deal with the transition from light to heavy scatterer, and particularly with the treatment of moderation by deuterium.

The applicability of the Greuling-Goertzel equations and of an improved expansion of the scattering kernel to the analysis of the slowing down problem has been pointed out by several investigators. ^{1, 6, 7, 8, 9} In the BPG code, these equations in the B_1 expansion approximation have been formulated to obtain a multigroup, bare-reactor criticality calculation. The resulting neutron balance can in turn be used to obtain effective coefficients for few-group, many-region calculations in several spatial dimensions. For example, the four-group coefficients can be fed directly into the four-group code ¹⁰ for one-dimensional calculations or into PDQ ¹¹ for two-dimensional calculations. The code formulation of BPG is similar to that of MUFT. ^{12, 13} BPG uses the consistent expansions, and the Greuling-Goertzel equations are directly available in normal input for all elements up to mass 27, MUFT, however, uses the inconsistent expansions, and the Greuling-Goertzel equations are obtained by special identification of several of the coefficients, permitting only a single Greuling-Goertzel coefficient to be used. MUFT is available on the IBM-704 computer, while BPG is written for the Burroughs 205 computer.

The derivation of equations and coefficients used in the code is presented along with specific instructions for using the code.

2. DERIVATIONS

The BPG program utilizes either the B-1 or the P-1 approximations to the neutron transport equation at the option of the program user, with either the Greuling-Goertzel or the Fermi Age slowing-down methods as a further option. The purpose of this section of the report is to present a straightforward derivation of these methods from the neutron transport equations through the multigroup approximations used by the machine code. The material is similar to that found in Zweifel and Hurwitz¹, Amster² and Greuling, et al, ³ and has the virtue of avoiding the use of improper functions having the character of the Dirac δ -function.

Appendix A contains the rather lengthy formulae for the quantities T_{1k}^n . Appendix B may be referred to for definitions.

The transport equation for slab geometry is

$$\begin{aligned} & \mu \frac{d}{dx} \Phi(\mu, u, x) + \Sigma^T(u, x) \Phi(\mu, u, x) \\ = & \int_0^{2\pi} d\theta \int_{-1}^1 d\mu_0 \Sigma^0(\mu_0, u', x) \Phi(\mu', u', x) \\ & + \frac{\nu}{2} f(u) \int_0^\infty du' \int_{-1}^1 d\mu' \Sigma^F(u', x) \Phi(\mu', u', x) \\ & + \frac{1}{2} \int_0^\infty du' \int_{-1}^1 d\mu' \Sigma^I(u', u, x) \Phi(\mu', u', x). \end{aligned} \quad (1)$$

(See Appendix C.1.)

The primed variables, as indicated in Appendix B, refer to the values before scatter or fission. The elastic scatter integral is described in terms of a local spherical-coordinate system whose polar axis is along the final velocity vector. In this integral u' is a function of μ and u .

In the fission and the inelastic scatter integrals, the local coordinate system is again spherical, but the polar axis is parallel to the x axis. In these latter integrals, emission is assumed to be spherically symmetric in the laboratory system, requiring a factor of $1/4\pi$ for angular distribution. Since there is no dependence on the azimuthal angle, it has been integrated out. The assumption is made that all cross sections are independent of x.

2.1 The B-1 Approximation

Define:

$$\psi(\mu, u, B) = \int_{-\infty}^{\infty} \Phi(\mu, u, x) e^{iBx} dx, \quad (2)$$

that is, ψ the Fourier transform of Φ . When one applies the Fourier transform to Equation (1), the first term on the left side becomes

$$\begin{aligned} \mu \int_{-\infty}^{\infty} \frac{d\Phi(\mu, u, x)}{dx} e^{iBx} dx &= \mu \Phi(\mu, u, x) e^{iBx} \Big|_{-\infty}^{\infty} \\ &\quad - \mu iB \int_{-\infty}^{\infty} \Phi(\mu, u, x) e^{iBx} dx. \end{aligned}$$

For a solution in which Φ vanishes at $\pm \infty$, the first term on the right hand side of this equation goes to zero. This is generally the case for physical systems. For periodic solutions of Φ , in general, this term is not zero and the Fourier transform (2) does not exist. However, it may be possible to obtain the same result through a process similar to box normalization¹⁷ in quantum mechanics. With these assumptions, the term becomes

$$\mu \int_{-\infty}^{\infty} \frac{d\Phi(\mu, u, x)}{dx} e^{iBx} dx = -iB\mu\psi(\mu, u, B) \quad (3)$$

In other terms of the transformed equation (1), the transform integral with respect to x may be interchanged with the other integrals, and the result is merely to replace Φ by ψ in these terms. Thus, one gets

$$\begin{aligned}
\left[-\frac{iB\mu}{\Sigma^T(u)} + 1 \right] \Sigma^T(u) \psi(\mu, u, B) &= \int_0^{2\pi} d\theta \int_{-1}^1 d\mu_0 \Sigma^0(\mu_0, u') \psi(\mu' u', B) \\
&+ \frac{\nu}{2} f(u) \int_0^\infty du' \Sigma^F(u') \int_{-1}^1 d\mu' \psi(\mu', u', B) \\
&+ \frac{1}{2} \int_0^\infty du' \Sigma^I(u', u) \int_{-1}^1 d\mu' \psi(\mu', u', B). \quad (4)
\end{aligned}$$

The next step is to expand ψ in a series of Legendre polynomials

$$\psi(\mu, u, B) = \sum_{i=0}^{\infty} \frac{2i+1}{4\pi} \psi_i(u, B) P_i(\mu). \quad (5)$$

This expansion is introduced into Equation (4), after first dividing by the factor

$$1 - \frac{iB\mu}{\Sigma^T(u)}$$

and the equation becomes

$$\begin{aligned}
&\sum_{i=0}^{\infty} \frac{2i+1}{4\pi} \Sigma^T(u) \psi_i(u, B) P_i(\mu) \\
&= \left[1 - \frac{iB\mu}{\Sigma^T(u)} \right]^{-1} \left\{ \sum_{i=0}^{\infty} \frac{2i+1}{4\pi} \int_0^{2\pi} d\theta \int_{-1}^1 d\mu_0 \Sigma^0(\mu_0, u') \psi_i(u', B) P_i(\mu') \right. \\
&\quad + \frac{\nu}{2} f(u) \frac{1}{2\pi} \int_0^\infty du' \Sigma^F(u') \psi_0(\mu', B) \\
&\quad \left. + \frac{1}{4\pi} \int_0^\infty du' \Sigma^I(u', u) \psi_0(\mu', B) \right\} \quad (6)
\end{aligned}$$

Equation (6) can now be multiplied by $P_k(\mu)$ and integrated over all directions Ω . The integration is easily performed except for the scatter

integral term. Here one must use the additional theorem for Legendre polynomials (see e. g., Reference 4) in the form

$$P_i(\mu') = P_i(\mu)P_i(\mu_0) + 2 \sum_{m=1}^i \frac{(i-m)!}{(i+m)!} P_i^m(\mu) P_i^m(\mu_0) \cos m(\theta - \hat{\theta}). \quad (7)$$

In this equation, P_i^m are the associated Legendre functions, and θ and $\hat{\theta}$ are the azimuthal angles associated with the Ω and the (local) x-axis, respectively. (See Appendix C, 1.) When one integrates the terms of the sum, one sees from the relation

$$\int_0^{2\pi} d\theta \cos m(\theta - \hat{\theta}) = 0 \quad (8)$$

that they contribute nothing. The integration of the first term with respect to θ contributes the factor 2π , and we get

$$\begin{aligned} \Sigma^T(u) \psi_k(u, B) &= \sum_{i=0}^{\infty} \frac{2i+1}{2} \int_{-1}^1 \frac{P_k(\mu) P_i(\mu)}{1 - \frac{iB\mu}{\Sigma^T}} d\mu \int_{-1}^1 2\pi \Sigma^0(\mu_0, u') \psi_i(u', B) P_i(\mu_0) d\mu_0 \\ &+ \frac{\nu}{2} f(u) \int_{-1}^1 \frac{P_k(\mu) d\mu}{1 - \frac{iB\mu}{\Sigma^T}} \int_0^{\infty} du' \Sigma^F(u') \psi_0(u', B) \\ &+ \frac{1}{2} \int_{-1}^1 \frac{P_k(\mu) d\mu}{1 - \frac{iB\mu}{\Sigma^T}} \int_0^{\infty} du' \Sigma^I(u', u) \psi_0(u', B). \end{aligned}$$

$$k = 0, 1, 2, \dots \quad (9)$$

Now we adopt the definitions:

$$L_i(u) = 2\pi \int_{-1}^1 d\mu_0 \Sigma^0(\mu_0, u') \psi_i(u', B) P_i(\mu_0)$$

$$u' = u'(\mu_0, u), \quad (10)$$

$$S(u) = \nu f(u) \int_0^{\infty} du' \Sigma^F(u') \psi_0(u', B), \quad (11)$$

$$I(u) = \int_0^\infty du' \Sigma^I(u', u) \psi_0(u', B), \quad (12)$$

$$A_{ki}(u) = \int_{-1}^1 \frac{P_k(\mu) P_i(\mu) d\mu}{1 - \frac{iB\mu}{\Sigma^T(u)}}, \quad (13)$$

and Equation (9) becomes

$$\Sigma^T(u) \psi_k(u, B) = \sum_{i=0}^{\infty} \frac{2i+1}{2} A_{ki}(u) L_i(u) + \frac{1}{2} A_{k0}(u) [S(u) + I(u)]$$

$$k = 0, 1, 2, \dots \quad (14)$$

The B-1 approximation consists of retaining only the first two equations of (14), and in these, dropping all terms in the sum for which $i > 1$. The B-1 approximation is thus represented by the equations

$$\Sigma^T \psi_0 = \frac{1}{2} A_{00} L_0 + \frac{3}{2} A_{01} L_1 + \frac{1}{2} A_{00} (S + I)$$

$$\Sigma^T \psi_1 = \frac{1}{2} A_{10} L_0 + \frac{3}{2} A_{11} L_1 + \frac{1}{2} A_{10} (S + I). \quad (15)$$

Knowing that $P_0(\mu) = 1$ and $P_1(\mu) = \mu$, we can readily compute the relations

$$A_{00} = 2\beta,$$

$$A_{01} = A_{10} = \frac{2i\Sigma^T}{B} (1 - \beta), \text{ and}$$

$$A_{11} = 2 \left[\frac{\Sigma^T}{B} \right]^2 (1 - \beta), \quad (16)$$

$$\text{where } \beta = \frac{\Sigma^T(u)}{B} \tan^{-1} \left[\frac{B}{\Sigma^T(u)} \right]. \quad (17)$$

Using these definitions and taking two linear combinations of Equations (15), one soon derives a slightly different set of equations, in which L_0 and L_1 are separated. These are

$$\Sigma^T \psi_0 = L_0 + iB\psi_1 + S + I$$

$$\Sigma^T \psi_1 = L_1 + \frac{iB\psi_0}{3}, \quad (18)$$

$$\text{where } h = \frac{1}{3} \left[\frac{B}{\Sigma T} \right]^2 \frac{\beta}{1-\beta}. \quad (19)$$

2.2 The P-1 Approximation

The consistent P-1 approximation is derived in the same way as the B-1, except that the Fourier transform is omitted. When the expansion

$$\Phi(\mu, u, x) = \sum_{j=0}^{\infty} \frac{2j+1}{4\pi} \Phi_j(u, x) P_j(\mu) \quad (20)$$

is substituted into Equation (1), the result is

$$\begin{aligned} & \sum_{j=0}^{\infty} \frac{2j+1}{4\pi} \frac{d\Phi_j}{dx} \mu P_j(\mu) + \sum_{j=0}^{\infty} \frac{2j+1}{4\pi} \Sigma^T \Phi_j P_j(\mu) \\ &= \sum_{j=0}^{\infty} \frac{2j+1}{4\pi} \int_0^{2\pi} d\theta \int_{-1}^1 d\mu_0 \Sigma^0(\mu_0, u') \Phi_j(u', x) P_j(\mu') \\ &+ \frac{1}{2\pi} \frac{\nu}{2} f(u) \int_0^{\infty} du' \Phi_0(u', x) \Sigma^F(u') \\ &+ \frac{1}{2\pi} \frac{1}{2} \int_0^{\infty} du' \Sigma^I(u', u) \Phi_0(u', x). \end{aligned} \quad (21)$$

When (21) is multiplied by $P_k(\mu)$ and integrated over all directions Ω , the result of the computation proceeds as before with the exception of the first term. Here we make use of the relation

$$\mu P_k(\mu) = \frac{k+1}{2k+1} P_{k+1}(\mu) + \frac{k}{2k+1} P_{k-1}(\mu). \quad (22)$$

The Addition Theorem used before is needed again for the scatter integral. The final result is

$$\begin{aligned} & \frac{k+1}{2k+1} \frac{d\Phi_{k+1}}{dx} + \frac{k}{2k+1} \frac{d\Phi_{k-1}}{dx} + \Sigma^T(u) \Phi_k(u, x) \\ &= 2\pi \int_{-1}^1 d\mu_0 \Sigma^0(\mu_0, u') \Phi_k(u', x) P_k(\mu_0) \end{aligned}$$

$$\begin{aligned}
& + \nu f(u) \delta_{k_0} \int_0^\infty du' \Phi_0(u', x) \Sigma^F(u') \\
& + \delta_{k_0} \int_0^\infty du' \Phi_0(u', x) \Sigma^I(u', u),
\end{aligned} \tag{23}$$

where δ_{k_0} is the Kronecker δ . As before, we define

$$L_k(u, x) = 2\pi \int_{-1}^1 d\mu_0 \Sigma^0(\mu_0, u') \Phi_k(u', x) P_k(\mu_0), \tag{24}$$

$$S(u, x) = \nu f(u) \int_0^\infty du' \Phi_0(u', x) \Sigma^F(u'), \text{ and} \tag{25}$$

$$I(u, x) = \int_0^\infty du' \Phi_0(u', x) \Sigma^I(u', u), \tag{26}$$

and Equation (23) becomes

$$\begin{aligned}
& \frac{k+1}{2k+1} \frac{d\Phi_{k+1}}{dx} + \frac{k}{2k+1} \frac{d\Phi_{k-1}}{dx} + \Sigma^T(u) \Phi_k(u, x) \\
& = L_k(u, x) + \delta_{k_0} \left[S(u, x) + I(u, x) \right], \\
& k = 0, 1, 2, \dots
\end{aligned} \tag{27}$$

The P-1 equations are obtained by writing (27) for $k = 0$ and $k = 1$ and omitting the term involving Φ_2 . We have

$$\begin{aligned}
& \frac{d\Phi_1}{dx} + \Sigma^T \Phi_0 = L_0(u, x) + S(u, x) + I(u, x) \\
& \frac{1}{3} \frac{d\Phi_0}{dx} + \Sigma^T \Phi_1 = L_1(u, x).
\end{aligned} \tag{28}$$

Rearrangement of these equations gives

$$\begin{aligned}
& \Sigma^T \Phi_0 = L_0(u, x) - \frac{d\Phi_1}{dx} + S(u, x) + I(u, x) \\
& \Sigma^T \Phi_1 = L_1(u, x) - \frac{1}{3} \frac{d\Phi_0}{dx}.
\end{aligned} \tag{29}$$

We define the Fourier transform of $\Phi(\mu, u, x)$ as

$$\psi(\mu, u, B) = \int_{-\infty}^{\infty} \Phi(\mu, u, x) e^{iBx} dx, \quad (30)$$

and by applying this transform to Equations (29) we obtain

$$\begin{aligned} \Sigma^T \psi_0 &= L_0 + iB\psi_1 + S + I \\ \Sigma^T \psi_1 &= L_1 + \frac{iB}{3} \psi_0 \end{aligned} \quad (31)$$

which are seen to be identical to Equations (18) with h taken to be 1. Here, $L_i = L_i(u, B)$, $S = S(u, B)$, and $I = I(u, B)$ are the Fourier transforms of $L_i(u, x)$, $S(u, x)$, and $I(u, x)$.

2.3 The Greuling-Goertzel Approximation for $L_i(u)$

We turn our attention to the quantity

$$L_i(u) = 2\pi \int_{-1}^1 d\mu_0 \Sigma^0(\mu_0, u') \psi_i(u', B) P_i(\mu_0),$$

where $u' = u'(\mu_0, u)$.

Since Σ^0 is a probability distribution, one can change the variable of integration from μ_0 to μ_C (the scattering angle in the center of mass system) by using the recipe

$$\Sigma^0(\mu_0, u') d\mu_0 = \Sigma^C(\mu_C, u') d\mu_C, \quad (32)$$

to get

$$L_i(u) = 2\pi \int_{-1}^1 d\mu_C \Sigma^C(\mu_C, u') \psi_i(u', B) P_i(\mu_0),$$

$$\text{where } \mu_0 = \mu_0(\mu_C). \quad (33)$$

A third form of this quantity is obtained by introducing the lethargy change $U = u - u'$ to get

$$L_i(u) = 2\pi \int_{-1}^1 d\mu_C \Sigma^C(\mu_C, u - U) \psi_i(u - U, B) P_i(\mu_0),$$

$$\text{where } U = U(\mu_C). \quad (34)$$

Equation (34) is seen to be identical with that of Amster's Equation (1) in Reference 2. The quantity $\psi_i \Sigma^c$ can be expanded in a Taylor's series about u in powers of U to give

$$\begin{aligned} L_i(u) &= 2\pi \sum_{n=0}^{\infty} \int_{-1}^1 d\mu_c P_i(\mu_0) \frac{(-U)^n}{n!} \frac{d^n}{du^n} [\psi_i(u, B) \Sigma^c(\mu_c, u)] \\ &= \sum_{n=0}^{\infty} \frac{d^n}{du^n} [\psi_i(u, B) G_i^n(u)], \end{aligned} \quad (35)$$

$$\text{where } G_i^n(u) = \frac{2\pi}{n!} \int_{-1}^1 d\mu_c P_i(\mu_0) (-U)^n \Sigma^c(\mu_c, u). \quad (36)$$

Note: U is not dependent on u .

Now, if we expand $\Sigma^c(\mu_c, u)$ into a Legendre series,

$$\Sigma^c(\mu_c, u) = \sum_{k=0}^{\infty} \frac{2k+1}{4\pi} B_k^c(u) P_k(\mu_c), \quad (37)$$

we can write

$$\begin{aligned} G_i^n(u) &= \sum_{k=0}^{\infty} \frac{2k+1}{2n!} B_k^c(u) \int_{-1}^1 d\mu_c P_i(\mu_0) (-U)^n P_k(\mu_c) \\ &= \sum_{k=0}^{\infty} T_{ik}^n B_k^c(u), \end{aligned} \quad (38)$$

$$\text{where } T_{ik}^n = \frac{2k+1}{2n!} \int_{-1}^1 d\mu_c P_i(\mu_0) (-U)^n P_k(\mu_c). \quad (39)$$

The quantities T_{ik}^n can be calculated using the definitions of the Legendre polynomials and performing the integrations as indicated, or by changing the variables of integration to U . The pertinent formulae (see Reference 5) are

$$\mu_c = 1 - \frac{(A+1)^2}{2A} (1 - e^{-U})$$

$$\mu_0 = \frac{A+1}{2} e^{-U/2} - \frac{A-1}{2} e^{U/2} = (1+A\mu_c) (1+2A\mu_c + A^2)^{-1/2}$$

$$U = -\ln \left[1 - \frac{2A}{(A+1)^2} (1-\mu_c) \right]. \quad (40)$$

Returning to Equation (35), we can write

$$\begin{aligned} L_i(u) &= \psi_i(u, B) G_i^0(u) + \frac{d}{du} \sum_{n=1}^{\infty} \frac{d^{n-1}}{du^{n-1}} (\psi_i G_i^n) \\ &= \psi_i(u, B) G_i^0(u) - \frac{d}{du} [q_i(u)], \end{aligned} \quad (41)$$

$$\text{where } q_i(u) = - \sum_{m=0}^{\infty} \frac{d^m}{du^m} (\psi_i G_i^{m+1}). \quad (42)$$

The quantity $q_i(u)$ obviously satisfies the identity

$$\begin{aligned} q_i + \lambda_i \frac{dq_i}{du} &= -\psi_i G_i^1 - \frac{d}{du} (\psi_i G_i^2) - \lambda_i \frac{d}{du} (\psi_i G_i^1) \\ &\quad - \sum_{m=2}^{\infty} \left[\frac{d^m}{du^m} (\psi_i G_i^{m+1}) + \lambda_i \frac{d^m}{du^m} (\psi_i G_i^m) \right], \\ i &= 0, 1. \end{aligned} \quad (43)$$

The Greuling-Goertzel approximation is described in terms of Equation (43) by

1. Dropping the series term, which contains second and higher derivatives.
2. Choosing λ_i to eliminate the terms containing $\frac{d\psi_i}{du}$.
3. Adding the resulting equations for $i = 0, 1$ to the previous system with the new unknowns q_0 and q_1 .

Requirements (1) and (2) lead to the equation

$$G_i^2 \frac{d\psi_i}{du} + \lambda_i G_i^1 \frac{d\psi_i}{du} = 0, \quad (44)$$

or

$$\lambda_i = -G_i^2/G_i^1, \quad (45)$$

and (43) becomes

$$\begin{aligned} q_i + \lambda_i \frac{dq_i}{du} &= -\psi_i G_i^1 - \psi_i \frac{d}{du} (G_i^2) - \lambda_i \psi_i \frac{d}{du} (G_i^1) \\ &= -\psi_i G_i^1 \left(1 - \frac{d\lambda_i}{du}\right). \end{aligned} \quad (46)$$

The complete set of B_i equations, or P_i equations with $h = 1$, with the Greuling-Goertzel approximation is, therefore,

$$\begin{aligned} \Sigma^T \psi_0 &= \psi_0 G_0^0 - \frac{dq_0}{du} + iB\psi_1 + S + I \\ \Sigma^T h\psi_1 &= \psi_1 G_1^0 - \frac{dq_1}{du} + \frac{iB}{3} \psi_0 \\ q_0 + \lambda_0 \frac{dq_0}{du} &= -G_0^1 \psi_0 \left(1 - \frac{d\lambda_0}{du}\right) \\ q_1 + \lambda_1 \frac{dq_1}{du} &= -G_1^1 \psi_1 \left(1 - \frac{d\lambda_1}{du}\right). \end{aligned} \quad (47)$$

In these equations

a) ψ_0, ψ_1, q_0, q_1 are the unknown quantities of u to be determined by Equations (47),

b) $\Sigma^T(u)$ is known

$$c) \quad G_i^n(u) = \sum_{k=0}^{\infty} T_{ik}^n B_k^c(u) \quad (48)$$

d) $B_k^c(u)$ are the Legendre coefficients of $\Sigma^c(\mu_c, u)$

e) $\lambda_i = -G_i^2/G_i^1$.

$$f) \quad h(u, B) = \frac{B}{3\Sigma^T} \frac{\tan^{-1} \frac{B}{\Sigma^T}}{1 - \frac{\Sigma^T}{B} \tan^{-1} \frac{B}{\Sigma^T}}.$$

It should also be observed at this point that

$$\Sigma^S(u) = 2\pi \int_{-1}^1 \Sigma^0(\mu_0, u) d\mu_0 = 2\pi \int_{-1}^1 \Sigma^C(\mu_c, u) d\mu_c \quad (49)$$

from Appendix B and Equation (32). Inserting this in (37), we obtain the equality

$$\Sigma^S(u) = B_0^C(u). \quad (50)$$

The last question to be considered in this section is that of the proper point for the truncation of the series for $G_1^n(u)$ in (48), which values are needed in the computation of some of the energy dependent input to the program. Since these values consist of the sum of the products of $T_{ik}^n B_k^C(u)$, the point of truncation will depend on each of these values, and the desired number of terms may vary from element to element because of the magnitudes of the $B_k^C(u)$, and further because of the degree of accuracy to which these values are known for the various elements. In looking at the T_{ik}^n , consideration should be given to a theorem of Amster² which states: "As $1/A$ approaches zero, $T_{ik}^n(A)$ approaches zero at least as fast as the faster of $(1/A)^n$ and $(1/A)^{|i-k|}$." One approach has been to take two terms for G_0^1 to give an approximation of order $1/A$, whereas three terms were kept for G_1^0 and G_1^1 to give the same accuracy. Three terms for G_0^2 gives a series correct up to $1/A^2$. Four terms would be required for G_1^2 to give a series correct to order $1/A^2$, however, since the $B_3^C(u)$ are not known with any precision, it would not be practical to take more than three terms. Reference to Appendix A may be made for the analytical expressions for the required T_{ik}^n .

2.4 The Multigroup Equations

Equations (47) consist of a system of differential equations in the lethargy u . They are solved on the computer by using the multigroup approximation. In place of the point fluxes $\psi(u)$, we introduce a group flux ϕ_j and certain average values for the coefficients. Specifically we define

$$\phi_j = \int_{u_{j-1}}^{u_j} \psi_0(u) du, \quad (51)$$

$$J_j = -i \int_{u_{j-1}}^{u_j} \psi_1(u) du, \quad (52)$$

$$S_j = \int_{u_{j-1}}^{u_j} S(u) du, \quad (53)$$

$$I_j = \int_{u_{j-1}}^{u_j} I(u) du, \quad (54)$$

$$\Sigma_j = \frac{1}{\Delta u_j} \int_{u_{j-1}}^{u_j} \Sigma(u) du, \quad (55)$$

(where $\Delta u_j = u_j - u_{j-1}$, the above equation holds for A, S, I, F, and T)

$$q(u_j) = q_0(u_j), \quad (56)$$

$$p(u_j) = -iq_1(u_j), \quad (57)$$

$$h_j(B) = h(B, \Sigma_j^T), \quad (58)$$

$$\mu_j = \frac{1}{\Delta u_j} \int_{u_{j-1}}^{u_j} \frac{G_1^0(u)}{\Sigma^S(u)} du, \quad (59)$$

$$\xi_j = -\frac{1}{\Delta u_j} \int_{u_{j-1}}^{u_j} \frac{G_0^1}{\Sigma^S(u)} du, \quad (60)$$

$$\eta_j = -\frac{1}{\Delta u_j} \int_{u_{j-1}}^{u_j} \frac{G_1^1}{\Sigma^S(u)} du, \quad (61)$$

$$\Gamma_j = \frac{1}{\Delta u_j} \int_{u_{j-1}}^{u_j} \lambda_0(u) du, \text{ and} \quad (62)$$

$$Z_j = \frac{1}{\Delta u_j} \int_{u_{j-1}}^{u_j} \lambda_1(u) du. \quad (63)$$

From these equations, it can be seen that ϕ_j , J_j , S_j and I_j are group quantities, that are integrated over a lethargy interval. On the other hand, q and p remain as quantities defined at a specific lethargy value. The remainder of the quantities are averages over a lethargy interval, except that h_j is a function of the average value Σ_j^T .

We next integrate the Equations (47) over the interval (u_{j-1}, u_j) and use approximations which involve replacing an unknown quantity by an average value. In most cases, the group quantity can be assumed to be positive (or negative) throughout the interval. Then using the mean value theorem of calculus, one gets, for example,

$$\int_{u_{j-1}}^{u_j} \Sigma^T(u) \phi(u) du = \Sigma^T(u^*) \int_{u_{j-1}}^{u_j} \phi(u) du,$$

$$\text{where } u_{j-1} \leq u^* \leq u_j = \Sigma^T(u^*) \phi_j \approx \Sigma_j^T \phi_j.$$

The approximation is seen to consist of using the average Σ_j^T in place of Σ^T at the unknown value u^* . In similar fashion, the other terms are approximated to give the system of multigroup equations:

$$(a) \quad (\Sigma_j^T - \Sigma_j^S) \phi_j = q(u_{j-1}) - q(u_j) - BJ_j + S_j + I_j,$$

$$(b) \quad [h_j \Sigma_j^T - \Sigma_j^S \mu_j] J_j = p(u_{j-1}) - p(u_j) + \frac{B}{3} \phi_j,$$

$$(c) \quad \frac{\Delta u_j}{2} [q(u_j) + q(u_{j-1})] + \Gamma_j [q(u_j) - q(u_{j-1})]$$

$$= \xi_j \Sigma_j^S \phi_j - \frac{\xi_j \Sigma_j^S \phi_j}{\Delta u_j} [\lambda_0(u_j) - \lambda_0(u_{j-1})], \text{ and}$$

$$(d) \quad \frac{\Delta u_j}{2} [p(u_j) + p(u_{j-1})] + Z_j [p(u_j) - p(u_{j-1})]$$

$$= \eta_j \Sigma_j^S J_j - \frac{\eta_j \Sigma_j^S J_j}{\Delta u_j} [\lambda_1(u_j) - \lambda_1(u_{j-1})].$$

(64)

Equations (64) may be rewritten as follows:

$$\begin{aligned}
 (a) \quad & BJ_j + (\Sigma_j^A + \Sigma_j^I) \phi_j + q(u_j) = q(u_{j-1}) + S_j + I_j, \\
 (b) \quad & [h_j \Sigma_j^T - \Sigma_j^S \mu_j] J_j - \frac{B}{3} \phi_j + p(u_j) = p(u_{j-1}), \\
 (c) \quad & -\xi_j \Sigma_j^S \left\{ 1 - \frac{1}{\Delta u_j} [\lambda_0(u_j) - \lambda_0(u_{j-1})] \right\} \phi_j + \left(\frac{\Delta u_j}{2} + \Gamma_j \right) q(u_j) \\
 & = \left(\Gamma_j - \frac{\Delta u_j}{2} \right) q(u_{j-1}), \text{ and} \\
 (d) \quad & -\eta_j \Sigma_j^S \left\{ 1 - \frac{1}{\Delta u_j} [\lambda_1(u_j) - \lambda_1(\mu_{j-1})] \right\} J_j + \left(\frac{\Delta u_j}{2} + Z_j \right) p(u_j) \\
 & = \left(Z_j - \frac{\Delta u_j}{2} \right) p(u_{j-1}). \tag{65}
 \end{aligned}$$

The isotropic slowing-down density $q(u)$ is described by two different models. These models are:

1. The contribution due to elastic scattering of aluminum and elements lighter than aluminum may be represented by an equation of the form of Equation (c) in group (65) for each of such elements in the system. (Greuling-Goertzel slowing down model.)
2. The contribution due to elastic scattering by elements heavier than aluminum is considered to be represented by the following expression, in which the contribution of all species heavier than aluminum are lumped as indicated by the subscript L.

$$q_F(u_j) = \frac{2}{\Delta u_j} \sum_L \xi_{Lj} N_L \Sigma_{Lj}^S \phi_j - q_F(u_{j-1}) \tag{66}$$

(Fermi age model).

It is also possible to treat the lighter elements by the Fermi age model if the proper choice of input is chosen (as shown later).

The slowing-down densities for each element (by whichever model is chosen for each element) are added to obtain the total isotropic slowing down density $q(u)$.

The anisotropic slowing-down density has a contribution from only three elements: H, D, and Be. This contribution is represented by Equation (d) of group (65), one such equation representing the anisotropic slowing-down density contributed by each of the elements of this type present in the system. The total of such contributions is represented by $p(u_j)$ in the second equation of the group. We will now replace this symbol with $\sum_h p_h(u_j)$, where the subscript h refers to the elements H, D, and Be.

Inelastic scattering slows neutrons down, but because this is treated in a discontinuous manner, the neutrons being slowed down inelastically are not included in the slowing-down density. Inelastic scattering into each lethargy group is represented by the term I_j in Equation (a) of group (65), which gives the neutrons inelastically scattered into the group from all lower lethargy groups. In the same equation, the term $\sum_j^I \phi_j$ represents the neutrons removed from the group by inelastic scattering. Because the lethargy groups have finite width, not all the neutrons that undergo inelastic scattering are removed from the group. Therefore, this term must be corrected slightly, and this correction is incorporated in A_j . It is probable that the inelastically scattered neutron will lose enough energy to be removed from the group to a higher lethargy group. Thus, $\sum_j^I \phi_j$ now gives the number of neutrons inelastically scattered in a group, and $A_j \sum_j^I \phi_j$ now gives the number of neutrons removed from the group by inelastic scattering.

In order to simplify the Equations (65) we define

$$\sum_j^S \mu_j = \sum_k N_k M_{kj}, \text{ and} \quad (67)$$

$$\begin{aligned} \text{(a)} \quad \xi_{fj} \sum_{fj}^S \left\{ 1 - \frac{1}{\Delta u_j} [\lambda_{of}(u_j) - \lambda_{of}(u_{j-1})] \right\} &= N_f \Xi_{fj}^*, \\ \text{(b)} \quad \xi_{kj} \sum_{kj}^S &= N_k \Xi_{kj}, \text{ and} \end{aligned} \quad (68)$$

$$\eta_{hj} \sum_{hj}^S \left\{ 1 - \frac{1}{\Delta u_j} [\lambda_{lh}(u_j) - \lambda_{lh}(u_{j-1})] \right\} = N_h H_{hj}, \quad (69)$$

where f implies the element aluminum, or one of the elements lighter than aluminum, h refers to the elements H, D, or Be, and N is the element number density.

2.5 BPG Program Equations

We may now again rewrite Equations (65), indicating by the use of subscripts k, f, and h, the element dependent expressions, where k refers to all elements, and f and h have been defined above. These are the BPG equations.

$$\begin{aligned}
 (a) \quad & B J_j + (\Sigma_j^A + A_j \Sigma_j^I) \phi_j + \sum_f q_f(u_j) + q_F(u_j) \\
 & = S_j + I_j + \sum_f q_f(u_{j-1}) + q_F(u_{j-1}), \\
 (b) \quad & \left[h_j \Sigma_j^T - \sum_k N_k M_{kj} \right] J_j - \frac{B}{3} \phi_j + \sum_h p_h(u_j) = \sum_h p_h(u_{j-1}), \\
 (c) \quad & \left[\frac{\Delta u_j}{2} + \Gamma_{fj} \right] q_f(u_j) - N_f \Xi_{fj}^* \phi_j = \left[\Gamma_{fj} - \frac{\Delta u_j}{2} \right] q_f(u_{j-1}), \\
 (d) \quad & q_F(u_j) = \frac{2}{\Delta u_j} \sum_L N_L \Xi_{Lj} \phi_j - q_F(u_{j-1}), \text{ and} \\
 (e) \quad & \left[\frac{\Delta u_j}{2} + Z_{hj} \right] p_h(u_j) - N_h H_{hj} J_j = \left[Z_{hj} - \frac{\Delta u_j}{2} \right] p_h(u_{j-1}), \quad (70)
 \end{aligned}$$

where one equation of the type (c) and one of the type (e) appear in the system for each of the elements of types f and h respectively.

The above system of equations is solved by the BPG program for the unknowns J_j , ϕ_j , $q_f(u_j)$, $q_F(u_j)$, and $p_h(u_j)$, in the epithermal range.

The following equations are added for the thermal range to complete the spectrum

$$\begin{aligned}
 & B J_{th} + \Sigma_{th}^A \phi_{th} = \sum_f q_f(u_{th}) + q_F(u_{th}) \\
 & \left[\Sigma_{th}^T - \mu_{th} \Sigma_{th}^S - \frac{4}{5}(1 - \mu_{th}) \Sigma_{th}^A \right] J_{th} - \frac{B}{3} \phi_{th} = \sum_h p_h(u_{th}), \quad (71)
 \end{aligned}$$

where u_{th} is the thermal cutoff lethargy (upper lethargy bound of the last epithermal group).

2.6 Th-232 and U-238 Absorption Cross Sections

The absorption contribution of Th-232 and U-238 are not computed from the σ^A cross sections as for other elements. In order to allow for various degrees of self-shielding, the program normalizes the absorption cross sections of these two materials so that their resonance integrals are made to agree with values specified in the problem input. This is accomplished by having normalized distribution functions $\psi_{02,i}$ and $\psi_{28,i}$ which are normalized in the sense that

$$\sum_{j=1}^m \psi_j \Delta u_j = 1,$$

where m is the group containing the last resonance peak.

Three energy regions are considered by the code:

1. The high energy region where the absorption cross section varies slowly with energy. Groups $j = 1$ to $j = \ell - 1$ are included in this region.
2. The low energy region, between the lowest resonance and thermal, is assumed to have a cross section proportional to $1/v$. Groups $m + 1$ through the last (including thermal) are in this region.
3. Resonance region includes both the resolved and unresolved resonances. It is assumed that self-shielding and Dancoff mutual shielding of the resonances occurs uniformly in this region. Groups $j = \ell$ to $j = m$ are included in this region. ℓ is chosen as a boundary between the unresolved resonance region and the high energy region, and m includes the lowest energy resolved resonance.

The resonance integral values R_{02} and R_{28} are then used to construct the group cross sections. These values are:

1. In the lethargy interval $j = 1$ to $j = \ell - 1$,

$$\sigma_{ij}^A = R_i \psi_{ij} \quad i = 02 \text{ or } 28. \quad (72)$$

2. In the lethargy interval $j = m + 1$ to thermal,

$$\sigma_{ij}^A = \psi_{ij}. \quad (73)$$

3. In the lethargy interval $j = \ell$ to $j = m$, it is necessary to modify Equation (a) of group (70) when either, or both, of the elements Th-232 or U-238 are present in the system. In this equation, the absorption term is indicated by

$$\Sigma_j^A \phi_j.$$

The modification consists of separating the absorption into two components – smooth capture and resonance capture, as follows:

$$\text{Absorption} = \Sigma_j^A \phi_j + (1 - P_j)q(u_{j-1}),$$

in which the term for smooth capture,

$$\Sigma_j^A \phi_j$$

does not contain any contribution from Th-232 or U-238. In this equation, P_j is the resonance escape probability, and is given by the expression

$$P_j = P_{02,j} P_{28,j}$$

where $P_{02,j}$ or $P_{28,j}$ is taken as 1 if the element is not present in the system, and is defined by

$$P_{ij} = \exp \left[- \left(\frac{1}{\sum_k N_k \Xi_{k_j}} \right) (N_i R_i \psi_{i,j} \Delta u_j) \left(\frac{P_i^* + \gamma_i}{P_i^* + 1} \right) \right],$$

where P_i^* = ratio of volume part to surface part of the resonance integral,

γ_i = Dancoff factor,

N_i = number density, and

ψ_{ij} = normalized distribution function.

Rewriting Equation (a) of group (70) so as to include this expression for absorption, we obtain

$$\begin{aligned} & B J_j + (\Sigma_j^A + A_j \Sigma_j^I) \phi_j + \sum_f q_f(u_j) + q_F(u_j) \\ &= S_j + I_j + P_j \left[\sum_f q_f(u_{j-1}) + q_F(u_{j-1}) \right] \end{aligned} \quad (74)$$

This equation is substituted for Equation (a) in group (70) only in the groups in the interval including the Dancoff boundary, denoted by " ℓ ", and the last resonance peak, denoted by " m ", as indicated above.

It should be noted that due to the definition of the normalized function ψ , the resonance integral R is defined as

$$R = \int_{\text{lowest resonance peak}}^{10 \text{ Mev}} \sigma_{\text{eff}} \frac{dE}{E}, \quad (75)$$

and must not include the $1/v$ absorption.

2.7 2-Group and Polygroup Quantities

The program computes a number of quantities related to the spectrum in addition to the spectrum itself. Some of these are related to the epithermal spectrum as a whole such as age, resonance escape probability, etc. These are called two-group epithermal quantities. Others are related to portions of the epithermal spectrum. These are called polygroup quantities.

The latter class includes polygroup coefficients. These are the coefficients to be used in the multiregion, several-group diffusion calculations, characterized by the equations

$$\begin{aligned} D_1 \nabla^2 \phi_1 - w_1 \phi_1 + \frac{1}{k_{\text{eff}}} \left[f_1 \phi_1 + f_2 \phi_2 + \dots + f_N \phi_N \right] &= 0 \\ D_2 \nabla^2 \phi_2 - w_2 \phi_2 + \beta_1 \phi_1 &= 0 \\ \cdot &\cdot \\ \cdot &\cdot \\ \cdot &\cdot \\ D_N \phi_N - w_N \phi_N + \beta_{N-1} \phi_{N-1} &= 0. \end{aligned} \quad (76)$$

For defining the lethargy (or energy) boundaries of each of the groups of this polygroup representation, it is necessary to make a correspondence between each of these groups and the appropriate groups of the BPG multigroups.

<u>Polygroup Representation</u>		<u>Polygroup Number</u>	<u>Upper Boundary</u>	<u>Lower Boundary</u>
2	{	1	u_0	u_{th}
		th	u_{th}	
4	{	1	u_0	u_{16}
		2	u_{16}	u_{20}
		3	u_{20}	u_{th}
		4	u_{th}	

u_n is the upper lethargy boundary of the n^{th} multigroup of the BPG.

The polygroup boundaries, with the exception of the u_{th} , are stored on tape, but they can be changed or read from a card if that is desired. u_{th} is input for each case.

The program also provides room for storing boundaries for 3, 5, and 6 polygroup representations. In the representations below, "group N" refers to polygroup N.

2.8 Absorption and Fission Rates

$$A_N = \sum_{j \text{ in group N}} \Sigma_j^A \phi_j$$

$$\nu F_N = \sum_{j \text{ in group N.}} \nu \Sigma_j^F \phi_j$$

(77)

2.9 Polygroup Coefficients

The polygroup coefficients are

$$\phi_N = \sum_{j \text{ in group N}} \phi_j$$

$$D_N = \frac{\sum_{j \text{ in group N}} J_j}{B \phi_N}$$

$$w_N = D_N B_z^2 + \frac{A_N}{\phi_N} + \beta_N$$

$$\beta_N = \frac{1}{\phi_N} \left[\sum_k q_k(u_N) + \sum_{j \text{ in group } N} \sum_{\ell \text{ in group } N-1} a_{\ell j} \sum_{\ell}^I \phi_{\ell} \right]$$

$$f_N = \frac{1}{\phi_N} \nu F_N. \quad (78)$$

2.10 Leakage and Non-Leakage Probabilities

Leakage Probability:

$$\tau_N B^2 = \sum_{j \text{ in group } N} D_j B^2 \phi_j = B \sum_{j \text{ in group } N} J_j$$

Non-Leakage Probability:

$$P = 1 - \tau_{\text{epi}} B^2 - \tau_{\text{th}} B^2 = 1 - \sum_N \tau_N B^2. \quad (79)$$

2.11 Age

The age τ is computed from the leakage by solving the following equation for τ_N :

$$\frac{1}{1 + \tau_N B^2} = 1 - \tau_N B^2 = \text{non leakage probability} \quad (80)$$

This calculation of the age assumes a 2-group model. Therefore, the value of the age is valid only to the limit of validity of the 2-group model. Two epithermal ages are computed. τ_{epi} is based on the previous equation

$$\frac{1}{1 + \tau_{\text{epi}} B^2} = 1 - \tau_{\text{epi}} B^2.$$

τ_{epi}^* is computed according to the Fermi age treatment:

$$\tau_{\text{epi}}^* = \sum_{b=1}^{J-1} S_b \sum_{j=b}^{J-1} \frac{D_j \phi_j}{S_j + \sum_f q_f(u_{j-1}) + q_F(u_{j-1}) - B_z J_j}, \quad (81)$$

where $J-1$ is the last epithermal group.

2.12 Resonance Escape Probability

The total resonance escape probability is computed from the absorption rate by solving the following equation for p :

$$A_{\text{epi}} = (1-p) \frac{1}{1 + \tau_{\text{epi}} B^2} = \sum_{j=1}^{J-1} \Sigma_j^A \phi_j + \sum_{j=\ell}^m (1-P_j) q(u_{j-1}). \quad (82)$$

The program also computes resonance escape probability in each element present in the case. These partial resonance escape probabilities are computed by solving the following formulas for p_k :

$$A_{k \text{ epi}} = (1-p_k) \frac{1}{1 + \tau_{\text{epi}} B^2} \quad \text{if } k = \text{Th-232 or U-238}$$

$$A_{k \text{ epi}} = P_{02} P_{28} (1-p_k) \frac{1}{1 + \tau_{\text{epi}} B^2} \quad \text{if } k \neq \text{Th-232 or U-238}. \quad (83)$$

The calculation of the resonance escape probability of the individual element assumes the modified 2-group model. The p_k , therefore, are meaningful only within the framework of this model.

2.13 Multiplication Constant

With the source normalized to 1 the following formulas for the multiplication constants are valid:

$$\begin{aligned} k_{\text{res}} &= \nu F_{\text{epi}} \\ k_{\text{th}} &= \nu F_{\text{th}} \\ k_{\text{tot}} &= k_{\text{res}} + k_{\text{th}}. \end{aligned} \quad (84)$$

2.14 Conversion Ratio

$$\text{C. R.} = \frac{\sum_N (A_{02, N} + A_{28, N})}{\sum_N (A_{23, N} + A_{25, N} + A_{49, N} + A_{13, N} + A_{39, N})}. \quad (85)$$

2.15 Computation of Σ^T

As indicated in the above derivation, $\Sigma^T = \Sigma^A + \Sigma^I + \Sigma^S$. However, a provision has been made whereby Σ^I and Σ^A may be altered

by the use of a factor β in the program. This applies only in the Equation (b) of group (70), where the expression becomes

$$\Sigma_j^T = \Sigma_j^S + \beta(\Sigma_j^A + \Sigma_j^I) . \quad (86)$$

See Chapter 2, Section A.4, BAW-1230, for the theoretical justification and results of the use of this option. 14

3. PHYSICAL DATA

3.1 Cross Sections

Microscopic cross section data are stored with the program so that the coefficients of the equations of group (70) can be calculated automatically when concentrations of the elements are specified.

The stored microscopic cross sections and the constants for each species k in each group are described below, along with the number of the defining equation.

σ_{kj}^A = absorption cross section (55)

σ_{kj}^F = fission cross section (55)

σ_{kj}^I = inelastic scattering cross section (55)

σ_{kj}^S = elastic scattering cross section (55)

Σ_{kj}^* = slowing-down cross section in Greuling-Goertzel slowing-down model (68)

Σ_{kj} = slowing-down cross section in Fermi age slowing-down model (68)

M_{kj} = $\mu_{kj} \sigma_{kj}^S$ where μ_{kj} is the average cosine of the scattering angle (67)

Γ_{kj} = slowing-down constant (62)

Z_{kj} = anisotropic slowing-down constant (63)

H_{kj} = anisotropic slowing-down cross section (69)

Each of these cross sections and constants can be read from cards if that is desired.

In addition to these cross sections, there are stored with the program up to 12 sets of $\psi_{i,j}$ ($i = 02, 28$), the normalized distribution functions used in the computation of the absorption contributions of thorium-232 and uranium-238, as described above.

3.2 Thermal Cross Sections

The program calculates the thermal group absorption and fission cross sections for all elements by the formula

$$\sigma_{th} = \sigma_{2200} w g, \quad (87)$$

where σ_{2200} , w , and g are specified in the input. Here, g is the spatial disadvantage factor. w , the thermal spectrum averaging factor, is given in WAPD-185 for all water moderated reactors. Data also is given in Reference 15 (defined in 5.10.5 and tabulated in Appendix C-1) for the effect of spectrum changes, and from this data, w may be calculated by the formula

$$w = \frac{1}{1.128} \sqrt{\frac{528}{T + 460}} G(\text{non } 1/v), \quad (88)$$

where T is the temperature in degrees Fahrenheit, and $G(\text{non } 1/v)$ is the factor tabulated in the handbook.

3.3 Buckling

The equation used here describes a bare reactor. More precisely, a situation is described in which the spatial variation of the flux is characterized by

$$(\nabla^2 + B^2) \phi = 0.$$

B is constant and independent of u and r . Therefore, this approximation might not be adequate in a medium where B is a function of u or r . For many applications, zero buckling calculations are needed. In these cases, although the code does not accept $B^2 = 0$, an arbitrarily small number may be used.

3.4 Source

The program allows two alternatives for the source distribution S_j :

1. The source for a region may be taken as a Watt fission spectrum, and

2. The source for a region may be taken as the leakage from the region previously calculated:

$$S_j = \frac{\tau_j B^2}{\tau_{\text{epi}} B^2} = \frac{B J_j}{\sum_{j=1}^{\text{th}} J_j}.$$

For a region with fuel material, the fission spectrum source is appropriate. For reflector regions and most of the non-fuel regions, the leakage spectrum may be useful.

A source S_j can be read from cards if that is desired.

3.5 Inelastic Matrix

The slowing down by inelastic scattering is given by a matrix multiplication:

$$\begin{pmatrix} I_1 \\ I_2 \\ . \\ . \\ I_j \\ . \\ . \\ I_n \end{pmatrix} = \sum_k N_k \begin{pmatrix} 0 & 0 & 0 & . & . & . & . & 0 \\ a_{12} & 0 & 0 & . & . & . & . & 0 \\ a_{13} & a_{23} & 0 & . & . & . & . & 0 \\ . & . & . & . & . & . & . & . \\ a_{1j} & a_{2j} & a_{3j} & . & . & . & 0 & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ a_{1n} & a_{2n} & . & . & . & . & a_{n-1,n} & 0 \end{pmatrix} \begin{pmatrix} \sigma_{1k} \phi_1 \\ \sigma_{2k} \phi_2 \\ \sigma_{3k} \phi_3 \\ . \\ \sigma_{jk} \phi_j \\ . \\ . \\ \sigma_{nk} \phi_n \end{pmatrix}$$

I_j = number of neutrons inelastically scattered from all lower lethargy groups into the group j .

a_{ij} = proportion of neutrons scattered inelastically in group i , which scatter into group j .

All neutrons undergoing an inelastic collision are not transferred into a higher lethargy group; some remain in the same group. The coefficient A_j takes this effect into account:

A_j = inelastic proportionality factor: The probability that a neutron undergoing an inelastic collision in group j will be transferred to another group.

Therefore, the following relation must hold

$$A_j = \sum_{m=j+1}^{80} a_{jm}. \quad (90)$$

Nine different matrices can be stored on tape and any one of them can be chosen. At the present time three matrices are stored. Matrix 1 is based on the inelastic scattering by U-238. This matrix should be a fairly good approximation for inelastic scattering by thorium.¹⁴

Matrix 2 is based on the following approximation: The distribution in lethargy of a neutron which has suffered an inelastic collision, is uniform between the lethargy at which the collision took place and a cut-off lethargy $u = 4.29$ ($E = 0.137$ Mev).

Matrix 3 is a zero matrix which zeros all inelastic contributions.

A matrix and a set of inelastic proportionality factors can be read from cards if that is desired.

4. PROGRAM DESCRIPTION

4.1 Limitations

The BPG Program is designed to handle a maximum of up to eighty energy groups consisting of 79 epithermal groups and one thermal group. The energy groups may be combined into from two to six polygroups with the last polygroup consisting of the thermal group.

A maximum of 19 elements may be used in any region or in any case.

The number of regions is not restricted except in the case where the running of the BPG Program is to be followed with an entry into the 2-group or 4-group programs, in which case the restrictions applicable to these programs must be taken into account. See the appropriate report for the limitations of these programs.

4.2 Features

Most of the data used with the program that is normally not subject to change have been stored on tape. If, however, any of these data need to be changed, the program user has the option of reading in such data from cards. For a description of how this is done, see the Input Section of this report.

The program is set up to use either the Fermi age or the Greuling-Goertzel slowing-down model for the element aluminum and for the elements lighter than aluminum. The program user has the option of choosing either of these options through the use of either a positive or a negative element identification number.

4.3 Input

There are two options available to the user of the BPG Program as to cross section data:

1. If all elements can be selected from the group of 19 elements listed in Appendix G of this report,

and further provided that is is not necessary to change any of the cross sections for any of these elements, then Option 1 should be used to eliminate the running of the assembly program.

2. If some element other than an element listed in Appendix G is needed, or if it is necessary to change any of the cross sections stored on tape, then Option 2 must be used requiring the running of the assembly program.

The most efficient use of the program requires that all cases be run at one time rather than splitting up the scheduling of machine time, regardless of which option is to be used. If it is necessary to use Option 2, the user may compile his own table of EID numbers and corresponding cross sections by obtaining the cross sections either from tape or from cards, and by selecting his elements from the expanded table of EID numbers listed in Appendix H, therefore running through the assembly program. Once this table has been compiled, he may run as many cases as desired in which he can select his elements and cross sections from this new list.

The program allows the user to change any data stored on tape for his own particular case. The Input Data Sheets have been designed with this in mind. Since there are large quantities of data of several different types used by the program, it has been necessary to have 8 different types of Input Sheets to handle all possibilities. A complete set of input forms are included in Appendix E for reference. However, since in most cases it will be possible to use the tape data, the user will be able to prepare his input on only two or three different types of forms, depending on which of the above described options he may use. The minimum data required are described as follows:

For Option 1 (Standard Case)

BPG Data Sheet 3:

Fill in only that data listed under Case Data.

BPG Data Sheet 7:

Fill in data under heading Region Data.
Fill in data for as many elements as there are in the region under heading Element Data. If there are more than 4 elements, use two or more of these forms, leaving data under Region Data blank on the second and succeeding forms.

For Option 2 (General Case)

BPG Data Sheet 1:

Fill in data under heading Initial Input.
Fill in element identification list.

BPG Data Sheet 3:

Same data as described for Option 1.

BPG Data Sheet 7:

Same data as described for Option 1.

Where it is necessary to change some of the tape data, flags have been provided in the above data to indicate such changes to the program. Where any of these flags are set to indicate the read in of new data, the user should provide the data on the proper forms. The best procedure to follow, therefore, is to fill out the minimum data described above for the particular option being used, disregarding all other data descriptions in the detailed report below. Then, if additional data are indicated, look up the description of this data and fill in the forms required. Generally speaking, all input data are listed on the forms in the order in which it must be read in. Input forms have been prepared for IBM card input, 7 words, each consisting of 10 digits plus sign, per card.

4.4 Standard Case Input

For most cases, the data stored on tape (see Appendix G) are adequate, and a simplified input is possible.

The following is a description of the input for each case to be run

<u>No. of Cards</u>	<u>1st Word</u>	<u>2nd Word</u>	<u>3rd Word</u>	<u>4th Word</u>	<u>5th Word</u>	<u>6th Word</u>	<u>7th Word</u>
<u>Case Data</u> (see Math Form 68)							
2	CID	R	P	δ	ϵ	T_{co}	S
	LS	Δu	ν	IM	PG		
<u>Region Data</u> (see Math Form 72)							
1	E_R	B	B_z	0_1	0_2	0_3	0-0
<u>Element Data</u> (see Math Form 72)							
$2E_R$	EID	N	σ_{2200}^A	σ_{2200}^F	σ_{2200}^D	σ_{2200}^S	g
	ℓ	ψ	w^A	w^F	R	p	γ

The symbols under the heading Case Data are defined:

- CID - Case identification number.
- R - number of regions in case, no upper limit.
- P - number of polygroups. If a value from +2 to +6 is used here, then the polygroup boundaries for the corresponding number of polygroups as stored on tape will be used. If 0 is used, the polygroup boundaries will be read in from cards. If -1 is used, the same number of polygroups and the same boundaries as used in the previous case will be used.
- δ - determines whether P_1 or B_1 approximation will be used. Enter zero for the P_1 approximation and 1 for the B_1 approximation.
- ϵ - determines whether the consistent or inconsistent slowing-down approximation is to be used. Enter 1 for the consistent approximation and 0 for the inconsistent approximation.
- T_{co} - thermal cutoff. Enter here the number of the last energy group to be included in the epithermal range.
- S - determines the source to be used. Enter +1 if the source stored on tape is to be used. Enter 0 if it is desired to read in the source values from cards. Enter -1 if the same source as used for the previous case is desired. If -1 is used, the source will be the source used in the first region of the last case, and not the leakage spectrum.
- LS - determines if the leakage spectrum is to be used for the second and succeeding regions or if the original source is to be used in all regions. Use +1 if the leakage spectrum is desired and 0 if the original source is desired.
- Δu - determines the set of lethargy intervals to be used. If the values stored on tape are to be used, enter +1 here. If it is desired to read these values in on cards, enter 0. If the same values as used in the last case can be used, enter -1 in this location.
- ν - determines the values to be used for ν , the number of neutrons produced per fission. If +1 is entered, the set of values stored on tape will be used. If a negative integer is entered, the program will read in and replace the number of values of ν that are indicated by the size of this integer.

- IM - determines the Inelastic Matrix to be used by the program. There may be up to 9 matrices stored on tape. If any value from +1 to +9 is entered in this location, the corresponding inelastic matrix will be used. Numbers +1, +2, and +3 correspond with matrices 1, 2, and 3 as described previously. If 0 is entered, the matrix will be read in from cards. If -1 is entered, the same matrix as was used in the previous case will be used again. With the storage of each Inelastic Matrix, there is also stored a set of Inelastic Proportionality Constants and, therefore, the set of these values used depends on the input word IM. If 0 is entered here, these values must be entered on the cards preceding the Inelastic Matrix.
- PG - Enter 2 here if it is desired to follow the BPG Program with the 2-group program. Enter 4 if it is desired to follow with the 4-group program. Enter 0 if these options are not desired.

The symbols used under Region Data are:

- E_R - the number of elements in the region.
- B - total buckling in cm^{-1} . This entry must be input as B rather than B^2 .
- B_z - perpendicular buckling in cm^{-1} . This value must be input as B_z rather than as B_z^2 .
- 0_1 - printout option 1. This controls the option of printing out absorptions and fissions for each element for each polygroup, also the resonance escape probabilities for each element. Enter 1 if printout is desired, 0 if not.
- 0_2 - printout option 2. This controls the option of printing out fluxes, absorption, leakage, etc., for each multigroup (see Output). Enter 1 for printout, 0 if not desired.
- 0_3 - printout option 3. This controls the option of printing out absorptions and fissions for each element for each multigroup. Enter 1 if desired, 0 if not. Note: This is an expensive option in regard to machine time. It doubles the running time for a region.
- β - See Section 2.15 of this report for use of this quantity. Enter the value if some value other than 1 is desired. Leave blank if 1 is to be used.

The symbols used under Element Data are:

- EID - element identification number. These are given in Appendices G and H. The number should

occupy the first five digits of the input word.
Important: For aluminum and for the elements lighter than aluminum, use a minus sign with the element identification number if the Greuling-Goertzel slowing-down model is desired. Use a positive sign for the Fermi age slowing-down model.

- N - the number density of the element is atoms/cm³.
- σ_{2200}^A - the thermal absorption cross section of the element in barns.
- σ_{2200}^F - the thermal fission cross section of the element in barns.
- σ_{2200}^D - the thermal transport cross section of the element in barns.
- σ_{2200}^S - the thermal scattering cross section of the element in barns.
- g - the thermal disadvantage factor: $\frac{\phi_f}{\phi_t}, \frac{\phi_w}{\phi_t}, \frac{\phi_{cl}}{\phi_t}$,
where the subscripts f, w, cl, and t refer to fuel, water, clad and total respectively.
One of these factors should be chosen according to the region of the cell where the element is located.
- ℓ - Dancoff boundary. This coefficient is used only for Th-232 and U-238. This entry is the number of the lowest lethargy multigroup in which the resonance integral is corrected according to the Dancoff effect. The resonance integral is corrected in the same way in every following multigroup until the multigroup (m) containing the last resonance peak. The value ℓ must always be less than the m value stored with the ψ sets on tape, or less than the m read in from cards if that option is used.
- ψ - the flag for determining the set of distribution functions to be used. Enter from +1 to +12 here for the corresponding set of $\psi_{i,j}$ stored on tape. See Reference 16 for listing of data. Enter 0 here if it is desired to read these values in on cards. In which case, use BPG Input Sheet No. 8, Math Form No. 73.
- w^A - thermal spectrum averaging factor. For the water moderated reactor, the factor can be found in Reference 15, or it can be computed by

$$w^A = \frac{1}{1.128} \sqrt{\frac{528}{T + 460}} g^A.$$

g^A is given in the Physics Methods Handbook, Appendix C.

- w^F - thermal spectrum averaging factor. For the water moderated reactor, the factor can be found in Reference 15, or it can be computed by

$$w^F = \frac{1}{1.128} \sqrt{\frac{528}{T + 460}} g^F.$$

g^F is given in the Physics Methods Handbook, Appendix C.

- R - single pin resonance integral of the element in barns. See Section 5.13.1, of the Physics Methods Handbook, or Equation (75) of this report.
- P^* - ratio of the volume part of the resonance integral to the surface part in the resolved-resonance energy band according to Nordheim. (See Reference 14 for recipe.)
- γ - Dancoff coefficient.

4.5 General Case Input

The Following is a detailed description of input:

<u>No. of Cards</u>	<u>1st Word</u>	<u>2nd Word</u>	<u>3rd Word</u>	<u>4th Word</u>	<u>5th Word</u>	<u>6th Word</u>	<u>7th Word</u>
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Initial Input (see Math Form 66)

1	SID	C	I
---	-----	---	---

Table of Element Identification Numbers (see Math Form 66)

	EID ₁	EID ₂	EID ₃	EID ₄	EID ₅	EID ₆	EID ₇
I/7	EID ₈	----	----	----	----	----	----
	----	----	EID _I				

Types of Cross Section Changes (see Math Form 66)

"d" sets of 1 or 2 each	T ₁	T ₂	----	----	----	----	----
	----	T _a					

Element Cross Sections (see Math Form 67)

	XS _{t₁}	XS _{t₂}	XS _{t₃}	XS _{t₄}	XS _{t₅}	XS _{t₆}	XS _{t₇}
J/7	XS _{t₈}	----	----	----	----	----	----
	----	----	XS _{t_{J-1}}				

The subscripts used in the foregoing description of the input are:

- I - total number of distinct elements in all of the cases to be run.
- a - number of types of cross sections of a particular element for which it is desired to read in the values.
- d - number of elements for which one or more of the types of cross sections are desired to be changed.
- J - number of groups for which particular cross sections were computed.

The symbols used are:

- SID - identification number of the series of cases.
- C - number of cases. Any integral value may be used here. There is no upper limit.
- I - total number of different elements in all of the cases to be run $1 \leq I \leq 19$.
- EID - element identification numbers. These numbers are of the following form:

	+	x	x	y	y	y	0	0	0	z	z	,
Position	S	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈	A ₉	A ₁₀	

where: xxyyy in positions A₁ to A₅ is the element identification number as described in Appendix G or Appendix H. A₆, A₇, and A₈ are zero, and the number zz in A₉ and A₁₀ is the number of types of cross sections to be changed, i. e., it is desired to read in from cards the cross sections for all energy groups, except thermal, for each of the zz types of cross sections for the element xxyyy, $0 \leq zz \leq 10$. Note: It is required that all entries under the heading "Table of Element Identification Numbers" be in the same order as the elements appear in Appendix H. Also, no element may be specified which is not listed in that appendix. If an element, or isotope, is desired which is not listed in Appendix H, then the user may specify some element which is listed, but which will not be present in his system, and read in all the required cross sections and data pertaining to the desired element, using the identification number appearing in the table. Further, if it is deemed worthwhile, the element may be added to the master list on tape, and a set of cross sections and other pertinent data may be put on tape (up to a total of 58 elements). In this case, a revised copy of Appendix G that includes the new element will be issued.

- T - the types of cross sections, or values handled as

cross sections, which are to be read in from cards. These must be listed consecutively and in ascending order. Only those values pertaining to a particular element are entered on one card, or on a pair of two cards, if there are more than 7 types of cross sections to be read in. For the purpose of making changes, the types of cross sections are numbered as follows:

<u>Cross Section</u>	<u>T</u>
σ_{kj}^F	1
σ_{kj}^A	2
σ_{kj}^I	3
σ_{kj}^S	4
M_{kj}	5
H_{kj}	6
H_{kj}^*	7
Γ_{kj}	8
H_{kj}	9
Z_{kj}	10,

where: the subscript k refers to element k and the subscript j refers to the j^{th} energy group. The symbols are described in the section on cross sections.

$XS_{t,j}$ - cross sections listed consecutively, beginning with energy group 1. There are J-1 of these values. There will be one set of cards for each value of T. A set will consist of the minimum number of cards on which it is possible to enter the cross sections. For example, if 40-group cross sections are being used, there will be 6 of these cards. This number will be invariant regardless of where thermal cutoff may be for a particular case.

Order of Input for Option 2 includes:

<u>Order of Input</u>	<u>Description</u>
1	Initial input, 1 card.
2	Element identification numbers, 1 to 3 cards.

Order of Input

3

Description

Types of cross section changes. There must be a set of these cards, consisting of one or two cards, for each element for which it is desired to change any of the cross sections. The sets must be assembled in the same order as the corresponding element in the element identification list.

4

Cross sections - the cross sections for all elements to be changed. It is essential that these values be loaded in the following order:

1. By types of cross section, i. e., all changes involving σ^F , type 1, followed by all changes involving σ^A , type 2, etc.
2. By order of the elements read in.

It is absolutely necessary that the above order of read in be followed for correct results. If, as an example, the elements H, D, and O appear in the element ID list in that order, and it is desired to change the σ^A and σ^S values for each of these elements by read in from cards, then they must be in the following order:

σ^A_H

σ^A_D

σ^A_O

σ^S_H

σ^S_D

σ^S_O .

The following is a description of the input for each case to be run:

<u>No. of Cards</u>	<u>1st Word</u>	<u>2nd Word</u>	<u>3rd Word</u>	<u>4th Word</u>	<u>5th Word</u>	<u>6th Word</u>	<u>7th Word</u>
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Case Data (see Math Form 68)

2	CID	R	P	δ	ϵ	T_{co}	S
	LS	Δu	ν	IM	PG		

Polygroup Boundaries (see Math Form 68)

1	B_1	B_2	--	B_{n-2}
---	-------	-------	----	-----------

Source (see Math Form 68)

	S_1	S_2	S_3	S_4	S_5	S_6	S_7
$\frac{J-1}{7}$	S_8	S_9	--	--	--	--	--
	--	--	--	S_{J-1}			

Δu Values (see Math Form 69)

	$\frac{\Delta u_1}{2}$	$\frac{\Delta u_2}{2}$	$\frac{\Delta u_3}{2}$	$\frac{\Delta u_4}{2}$	$\frac{\Delta u_5}{2}$	$\frac{\Delta u_6}{2}$	$\frac{\Delta u_7}{2}$
$\frac{J-1}{7}$	$\frac{\Delta u_8}{2}$	$\frac{\Delta u_9}{2}$	---	---	---	---	---
	---	---	---	$\frac{\Delta u_{J-1}}{2}$			

Element ID Numbers for ν Changes (see Math Form 69)

	EID_{c_1}	EID_{c_2}	EID_{c_3}	EID_{c_4}	---	---	---
$\frac{L}{7}$	---	EID_{c_L}					

Note that subscript L refers to total number of elements for which ν is to be charged.

<u>No. of Cards</u>	<u>1st Word</u>	<u>2nd Word</u>	<u>3rd Word</u>	<u>4th Word</u>	<u>5th Word</u>	<u>6th Word</u>	<u>7th Word</u>
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Corrected ν 's (see Math Form 69)

	ν_{c_1}	ν_{c_2}	ν_{c_3}	ν_{c_4}	---	---	---
$\frac{L}{7}$	---	ν_{c_L}					

<u>No. of Cards</u>	<u>1st Word</u>	<u>2nd Word</u>	<u>3rd Word</u>	<u>4th Word</u>	<u>5th Word</u>	<u>6th Word</u>	<u>7th Word</u>
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Inelastic Proportionality Constants (see Math Form 70)

$\frac{J-1}{7}$	{	A_1	A_2	A_3	A_4	A_5	A_6	A_7
		A_8	A_9	--	--	--	--	--
		--	--	--	A_{J-1}			

Inelastic Matrix (see Math Forms 70 and 71)

M	{	J	$a_{2,1}$	$a_{3,1}$	$a_{3,2}$	$a_{4,1}$	$a_{4,2}$	$a_{4,3}$
		$a_{5,1}$	$a_{5,2}$	$a_{5,3}$	$a_{5,4}$	$a_{6,1}$	$a_{6,2}$	$a_{6,3}$
		$a_{6,4}$	$a_{6,5}$	---	---	---	---	---
		---	---	---	---	---	---	---
		---	---	$a_{j,1}$	$a_{j,2}$	$a_{j,3}$	---	---
		---	$a_{j,j-2}$	$a_{j,j-1}$	---	---	---	---
		---	---	---	---	---	$a_{J,1}$	$a_{J,2}$
		$a_{J,3}$	$a_{J,4}$	---	---	---	---	---
		---	---	---	---	$a_{J,J-2}$	$a_{J,J-1}$	---

M = 34 for 20 groups

M = 112 for 40 groups

M = 253 for 60 groups

M = 452 for 80 groups

<u>No. of Cards</u>	<u>1st Word</u>	<u>2nd Word</u>	<u>3rd Word</u>	<u>4th Word</u>	<u>5th Word</u>	<u>6th Word</u>	<u>7th Word</u>
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Region Data (see Math Form 72)

1	E_R	B	B_z	0_1	0_2	0_3	0--0
---	-------	---	-------	-------	-------	-------	------

Element Data (see Math Form 72)

$2E_R$	{	EID	N	σ_{2200}^A	σ_{2200}^F	σ_{2200}^D	σ_{2200}^S	g
		ℓ	ψ	w^A	w^F	R	P	γ

ψ_1 Values (see Math Form 73)

$\frac{J-1}{7}$	{	$\psi_{1,1}$	$\psi_{1,2}$	$\psi_{1,3}$	$\psi_{1,4}$	$\psi_{1,5}$	$\psi_{1,6}$	$\psi_{1,7}$
		$\psi_{1,8}$	$\psi_{1,9}$	---	---	---	---	---
		---	---	---	$\psi_{1,J-1}$			

<u>No. of Cards</u>	<u>1st Word</u>	<u>2nd Word</u>	<u>3rd Word</u>	<u>4th Word</u>	<u>5th Word</u>	<u>6th Word</u>	<u>7th Word</u>
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ψ_2 Values (see Math Form 73)

$\frac{J-1}{7}$	{	$\psi_{2,1}$	$\psi_{2,2}$	$\psi_{2,3}$	$\psi_{2,4}$	$\psi_{2,5}$	$\psi_{2,6}$	$\psi_{2,7}$
		$\psi_{2,8}$	$\psi_{2,9}$	---	---	---	---	---
		---	---	---	$\psi_{2,J-1}$			

The symbols under the heading Case Data are:

- CID - case identification number.
- P - number of polygroups. If a value from +2 to +6 is used here then the polygroup boundaries for the corresponding number of polygroups as stored on tape will be used. If 0 is used, the polygroup boundaries will be read in from cards. If -1 is used, the same number of polygroups and the same boundaries as used in the previous case will be used.
- δ - determines whether the P_1 or B_1 approximation will be used. Enter zero for P_1 approximation and 1 for B_1 approximation.
- ϵ - determines whether the consistent or inconsistent slowing-down approximation is to be used. Enter 1 for the consistent approximation and 0 for the inconsistent approximation.
- T_{co} - thermal cutoff. Enter here the number of the last energy group to be included in the epithermal range.
- S - determines the source to be used. Enter +1 if the source stored on tape is to be used. Enter 0 if it is desired to read in source values from cards. Enter -1 if the same source as used for the previous case is desired. If -1 is used, the source will be the source used in the first region of the last case and not the leakage spectrum.
- R - number of regions in case, no upper limit.
- LS - determines if the leakage spectrum is to be used for the second and succeeding regions, or if the original source is to be used in all regions. Use +1 if the leakage spectrum is desired and 0 if the original source is desired.
- Δu - determines the set of lethargy intervals to be used. If the values stored on tape are to be used, enter +1 here. If it is desired to read these values in on cards, enter 0. If the same values as used in the last case can be used, enter -1 in this location.

- ν - determines the values to be used for ν , the number of neutrons produced per fission. If +1 is entered, the set of values stored on tape will be used. If a negative integer is entered, the program will read in and replace those values of ν indicated by the size of this integer.
- IM - determines the inelastic matrix to be used by the program. There may be up to 9 matrices stored on tape. If any value from +1 to +9 is entered in this location, the corresponding inelastic matrix will be used. If 0 is entered, the matrix will be read in from cards. If -1 is entered, the same matrix as was used in the previous case will be again used. With the storage of each inelastic matrix, there is also stored a set of inelastic proportionality constants and, therefore, the set of these values used depends on the input word IM. If 0 is entered here, these values must be entered on the cards preceding those of the inelastic matrix.
- PG - enter 2 here if it is desired to follow the BPG Program with the 2-group program. Enter 4 if it is desired to follow with the 4-group program. Enter 0 if these options are not desired.

The symbols under Polygroup Boundaries are: (Enter only when P = 0)

- B - the polygroup boundaries. These entries are the numbers of the last multigroups which are to be included in the respective polygroups. The boundary of the last epithermal multigroup, which is entered in the case input under T_{CO} , must not be entered. This card of input is used only when P under the case input is zero. If this is the case, the program determines the number of polygroups by the number of entries on this card, i.e., if there are two entries on this card, four-group coefficients will be computed - three epithermal and one thermal group.

The symbols under Source and Δu Values are:

- S_j - source values in neutrons per cm^3 per second. The values are entered consecutively beginning with energy group 1.
- $\frac{\Delta u_j}{2}$ - Values of the lethargy intervals divided by 2. Unitless. Entered consecutively, beginning with energy group 1.

The symbols used under the headings Element ID Numbers for ν Changes and Corrected ν 's are:

- EID_{ci} - the element identification numbers of the elements

for which the tape values are not to be used, but for which it is desired to read in the ν values. These numbers should be entered consecutively, using as many cards as necessary. The subscript i refers to the i^{th} value changed.

- ν_{ci} - neutrons per fission. The values which are to be used in place of the tape values for the elements in the above list. They must be entered in the same order as the element identification numbers.

The symbols listed under the headings Proportionality Constants and Inelastic Matrix are:

- A_j - inelastic proportionality factors. These values represent the proportion of neutrons which will be scattered out of the lethargy interval Δu_j . An entry is required for each energy group except thermal, and they are listed consecutively, beginning with group 1.
- J - number of groups.
- a_{ij} - elements of the inelastic matrix, describing the scattering from lower lethargy intervals to higher lethargy intervals. The subscript i refers to the group from which the scattering takes place, and j refers to the group into which the neutrons are scattered.

The matrix has the following form:

0	0	0	0	0
$a_{2,1}$	0	0	0	0
$a_{3,1}$	$a_{3,2}$	0	0	0
.	.	.	.	.
.	.	.	.	.
$a_{j,1}$	$a_{j,2}$	$a_{j,j-1}$	0	0
.	.	.	.	.
.	.	.	.	.
$a_{J-1,1}$	$a_{J-1,2}$		$a_{J-1, J-2}$	0

The first word on the first card is the number of groups for which the matrix applies. The $a_{j,i}$'s follow this value by rows, eliminating all of the elements indicated in the above matrix by zero, i. e., all elements to the right of and including the main diagonal.

The symbols used under Region Data are:

- E_R - the number of elements in the region.
- B - total buckling in cm^{-1} . This entry must be input as B rather than as B^2 .

- B_z - perpendicular buckling in cm^{-1} . This value must be input as B_z rather than as B_z^2 .
- 0_1 - printout option 1. This controls the option of printing out absorptions and fissions for each element for each polygroup, also resonance escape probabilities for each element. Enter 1 if printout is desired, 0 if it is not.
- 0_2 - printout option 2. This controls the option of printing out fluxes, absorptions, leakage, etc., for each multigroup (see Output). Enter 1 for printout, 0 if it is not desired.
- 0_3 - printout option 3. This controls the option of printing out absorptions and fissions for each element for each multigroup. Enter 1 if it is desired, 0 if it is not. Note: This is an expensive option in regard to machine time. It approximately doubles the running time for a region.
- β - See Section 2.15 of this report for use of this quantity. Enter the value if some value other than 1 is desired. Leave blank if 1 is to be used.

The symbols used under Element Data are:

- EID - element identification number. These are given in Appendices G and H. The number should occupy the first five digits of the input word. Important: For aluminum and for the elements lighter than aluminum, use a minus sign with the element identification number if the Greuling-Goertzel slowing-down model is desired. Use a positive sign for the Fermi age slowing-down model.
- N - the number density of the element in atoms/cm^3 .
- σ_{2200}^A - the thermal absorption cross section of the element in barns.
- σ_{2200}^F - the thermal-fission cross section of the element in barns.
- σ_{2200}^D - the thermal-transport cross section of the element in barns.
- σ_{2200}^S - the thermal-scattering cross section of the element in barns.
- g - the thermal disadvantage factor: $\frac{\phi_f}{\phi_t}, \frac{\phi_w}{\phi_t}, \frac{\phi_{cl}}{\phi_t}$,
where the subscripts f, w, cl, and t refer to fuel, water, clad, and total respectively.
One of these factors should be chosen according

to the region of the cell where the element is located.

- ℓ - Dancoff boundary. This coefficient is used only for the Th-232 and U-238. This entry is the number of the lowest lethargy multigroup in which the resonance integral is corrected according to the Dancoff effect. The resonance integral is corrected in the same way in every following multigroup until the multigroup (m) containing the last resonance peak. The value ℓ must always be less than the m value stored with the ψ sets on tape, or less than the m read in from cards if that option is used.
- ψ - the flag for determining the set of distribution functions to be used. (See the $\psi_{i,j}$ listing below for a description of the functions.) Enter from +1 to +12 here for the corresponding set of $\psi_{i,j}$ stored on tape. See Reference 16 for listing of data. Enter 0 here if it is desired to read these values in on cards, in which case BPG Input Sheet No. 8, Math Form No. 73.
- w^A - the thermal-spectrum averaging factor. For water moderated reactors, the factor can be found in Reference 15, or it can be computed by
- $$w^A = \frac{1}{1.128} \sqrt{\frac{528}{T + 460}} g^A.$$
- g^A is given in the Physics Methods Handbook, Appendix C.
- w^F - the thermal-spectrum averaging factor. For water moderated reactors, the factor can be found in Reference 15, or it can be computed by
- $$w^F = \frac{1}{1.128} \sqrt{\frac{528}{T + 460}} g^F.$$
- g^F is given in the Physics Methods Handbook.
- R - the single-pin resonance integral of the element in barns. See Section 5.13.1, of the Physics Methods Handbook. See also Equation (75) of this report.
- P^* - the ratio of the volume part of the resonance integral to the surface part in the resolved-resonance energy band according to Nordheim. (See Reference 14 for recipe.)
- γ - Dancoff coefficient.

The symbols used under ψ_1 Values and ψ_2 Values are

ψ_{ij} - normalized distribution function used in computing σ_{02}^A and σ_{28}^A in the epithermal ranges (as described elsewhere in this report). The subscript "i" in this special case refers to the order of the elements Th-232 and U-238 in the region input. In case both elements are present in a region, and it is desired to read a set of ψ values in on cards for both elements rather than using the tape values, it will be necessary to read the ψ values in the same order as the elements are listed under Element Data. If the values are read in from cards, they must be read in for each region or each case. The value m must be stored in the last two digits of $\psi_{i,m}$, that is the ψ value for the first group. m is the group containing the last resonance peak.

All of the above data, beginning with case data, is in the same order as that in which it should be read. The data under the headings Polygroup Boundaries, Source, Δu Values, Element ID Numbers for ν Changes, Corrected ν 's, Proportionality Constants, Inelastic Matrix, and ψ_1 and ψ_2 Values, are optional input and their use will depend on certain words in the Case Input and Element Data as described.

4.6 Output

All input to the BPG code is printed out at seven words per line in exactly the order read in. In addition to the printout of the input, the following data is printed out.

If $0_3 = 1$, the following printout is made: (This is deleted if $0_3 = 0$)

<u>No. Lines</u>	<u>1st Word</u>	<u>2nd Word</u>	<u>3rd Word</u>	<u>4th Word</u>	<u>5th Word</u>	<u>6th Word</u>	<u>7th Word</u>
k	j	$A_{1,j}$	$\nu_1 F_{1,j}$	I_{j+1}	0---0	0---0	0---0
	0---0	$A_{2,j}$	$\nu_2 F_{2,j}$	0---0	0---0	0---0	0---0
	.	.	.	.	.	.	.
	.	.	.	.	.	.	.
	0---0	$A_{k,j}$	$\nu_k F_{k,j}$	0---0	0---0	0---0	0---0
2	-----blank-----						

The above k lines, followed by two blank lines, are repeated for each of the J-1 groups, where k is the total number of elements in the

region and J-1 is the number of epithermal groups.

The following printout is nonoptional and will be made for all cases.

<u>No. Lines</u>	<u>1st Word</u>	<u>2nd Word</u>	<u>3rd Word</u>	<u>4th Word</u>	<u>5th Word</u>	<u>6th Word</u>	<u>7th Word</u>
1	S _{epi}	$\tau_{\text{epi}} B^2$	A _{epi}	νF_{epi}	P _{epi}	τ_{epi}	τ_{epi}^*
1	k _{tot}	P	C. R.				
2	-----blank-----						
N	S ₁	$\tau_1 B^2$	A ₁	νF_1	p ₁	τ_1	0--0
	S ₂	$\tau_2 B^2$	A ₂	νF_2	p ₂	τ_2	0--0
	.	.	.	.	.	.	.
	.	.	.	.	.	.	.
	.	.	.	.	.	.	.
	S _{th}	$\tau_{\text{th}} B^2$	A _{th}	νF_{th}	0--0	0--0	0--0
2	-----blank-----						
N	ϕ_1	0--0	D ₁	W ₁	β_1	f ₁	0--0
	ϕ_2	0--0	D ₂	W ₂	β_2	f ₂	0--0
	.	.	.	.	.	.	.
	.	.	.	.	.	.	.
	.	.	.	.	.	.	.
	ϕ_{th}	0--0	D _{th}	W _{th}	0--0	f _{th}	0--0
2	-----blank-----						

N is the number of polygroups.

The following printout is optional, depending on whether or not 0₁ in the Region Input is one or zero.

<u>No. Lines</u>	<u>1st Word</u>	<u>2nd Word</u>	<u>3rd Word</u>	<u>4th Word</u>	<u>5th Word</u>	<u>6th Word</u>	<u>7th Word</u>
N + 1	EID ₁	EID ₂	----	----	----	EID _k	
	A _{1,1}	A _{2,1}	----	----	----	A _{k,1}	
	.	.	.	.	.	.	
	.	.	.	.	.	.	
	A _{1,th}	A _{2,th}	----	----	----	A _{k,th}	

<u>No. Lines</u>	<u>1st Word</u>	<u>2nd Word</u>	<u>3rd Word</u>	<u>4th Word</u>	<u>5th Word</u>	<u>6th Word</u>	<u>7th Word</u>
N + 1	$\nu F_{1,1}$	$\nu F_{2,1}$	----	----	----	$\nu F_{k,1}$	
	.	.	.	.	.	.	
	.	.	.	.	.	.	
	$\nu F_{1,th}$	$\nu F_{2,th}$	----	----	----	$\nu F_{k,th}$	
	p_1	p_2	----	----	----	p_k	
2	-----blank-----						

There will be 1, 2, or 3 groups of printout with the same format as above, depending on whether $k \leq 7$, $8 \leq k \leq 14$, or $k \geq 15$.

The following printout is optional, depending on whether or not 0_2 in the Region Input is one or zero.

<u>No. Lines</u>	<u>1st Word</u>	<u>2nd Word</u>	<u>3rd Word</u>	<u>4th Word</u>	<u>5th Word</u>	<u>6th Word</u>	<u>7th Word</u>
J-1	j	ϕ_j	$\phi_j/\Delta u_j$	$\tau_j B^2$	A_j	νF_j	$\Sigma_{q_k}(u_j)$

Note: j in the above printout is in fixed point $\times 10^{-10}$. All other pointout is in the floating point notation.

The subscript N used above refers to the N^{th} polygroup.

Symbols used in the above description of output are:

- j - energy group.
- $A_{k,j}$ - absorption of the element k in energy group j.
- νF_{kj} - production rate of the element k in energy group j.
- I_{j+1} - inelastic scattering out of groups 1 to j into group j+1.
- S_{epi} - total source over the region.
- $\tau_{\text{epi}} B^2$ - leakage rate of epithermal neutrons.
- A_{epi} - total absorption rate of epithermal neutrons.
- νF_{epi} - total production rate of epithermal neutrons.
- p_{epi} - resonance escape probability.
- τ_{epi} - age (2-group model).
- τ_{epi}^* - age (Fermi model).
- k_{tot} - effective multiplication constant.

P - total non leakage probability.
 $C.R.$ - conversion ratio.
 S_N - fraction of fission neutrons born in N^{th} polygroup.
 $\tau_N B^2$ - leakage rate from N^{th} polygroup.
 A_N - total absorption rate in N^{th} polygroup.
 νF_N - total rate of neutron production in N^{th} polygroup.
 p_N - partial-resonance escape probability in the N^{th} polygroup.
 τ_N - partial age in the N^{th} polygroup.
 ϕ_N - integrated flux in the N^{th} polygroup.
 D_N - diffusion constant in the N^{th} polygroup.
 W_N - removal coefficient in the N^{th} polygroup.
 β_N - transfer coefficient in the N^{th} polygroup.
 f_N - source coefficient in the N^{th} polygroup.
 EID_k - element identification number.
 $A_{k,N}$ - absorption rate of element k over polygroup N.
 νF_{kN} - production rate of element k over polygroup N.
 p_k - resonance escape probability of element k.
 ϕ_j - integrated flux in energy group j.
 $\phi_j / \Delta u_j$ - average flux in energy group j.
 $\tau_j B^2$ - leakage rate in energy group j.
 A_j - total absorption rate in energy group j.
 νF_j - total production rate in energy group j.
 $\Sigma_{q_k}(u_j)$ - total slowing-down density at lethargy u_j .

4.7 Operating Instructions

If a program is to be run using Option 1 as described under the input section of this report, place the appropriate BPG Program reel on unit 2 and the appropriate data reel on unit 1. Appropriate means the program and the data corresponding to the number of groups for

which the case is to be run. Place the BPG 1 assembly card on top of the deck of the input and load into the hopper.

If a program is to be run using Option 2, place the appropriate program reel on unit 2. Warning: Do not use the data reel for this option! Use the general purpose reel on tape unit 1. Place the BPG 2 assembly card on top of the deck of the input and load into the hopper.

Switch Settings:

k - in - 7 words per card

k - out - 7 words per card

B - suppress - immaterial

Breakpoint - off. The only breakpoint used in the program is
 1. If it is convenient for the desired options
 in the 2 and 4-group programs, it may be
 set on 2 or 4.

Tape Unit { - tape unit 1 - off
Not-Write {
Lights { - tape unit 2 - on

The program should run through to completion for all cases in which the cross sections are the same with no further supervision from the operator, and should stop on a read in command if Option 1 is used. It will stop with the command S 2603 in the C register if Option 2 is used. If any of the cases are followed with an entry into the 2 or 4-group programs, this entry will be automatic, but from that point, operator supervision for these programs will be required. Upon convergence, the BPG Program may be re-entered to run other cases by entering 9 at the digit add option.

The following is a list of stops used in the program:

- S 0000 - Three possible causes for this stop are:
1. a tape sum failure on the read in of the 2-group program by the 2-group buffer program.
 2. a tape sum failure on the read in of the 4-group program by the 4-group buffer program.
 3. a tape sum failure on the read in of counter storage by the second entry program.

S 0000+k - $1 \leq k \leq 19$. The k^{th} element in the element identification list is not in master element identification table. (Stop occurs in the assembly program.)

- S 0200 - a tape sum failure on the read in of the ψ set for thorium from tape. (Stop occurs in the main routine.)
- S 0280 - a tape sum failure on the read in of the ψ set for U-238 from tape. (Stop occurs in the main routine.)
- S 0360 - a tape sum failure on the read in of the source values from either tape unit one or two. (Stop occurs in the main routine.)
- S 0440 - a tape sum failure on the read in of the Δu values from tape. (Stop occurs in the main program.)
- S 0520 - a tape sum failure on the read in of the ν values. (Stop occurs in the second entry program.)
- S 0560 - a tape sum failure on the read in of part of the inelastic matrix. (Stop occurs in the main program.)
- S 0660 - a tape sum failure on the read in of the auxiliary program for storing inelastic matrix when read in from cards. (Stop occurs in the main program.)
- S 0680 - a tape sum failure on the read in of the second entry program. (Stop occurs in the second entry program.)
- S 0700 - a tape sum failure on the read in of the main BPG program. (Stop occurs in the second entry program.)
- S 0856 - occurs in the auxiliary program for storing inelastic matrix on tape after the storing of the matrix on tape unit 2.
- S 1340 - a tape sum failure after the read in of the inelastic proportionality factors by the main program.
- S 1500 - a tape sum failure on the read in of the 4-group buffer program. (Stop occurs in the second part of the main routine.)
- S 1860 - a tape sum failure on the read in of the group cross section data. (Stop occurs in the main routine.)
- S 2200 - a tape sum failure on the read in of the 2-group flux integration routine. (Stop occurs in the 2-group buffer program.)
- S 2320 - Two possible causes for this stop are:
 1. a tape sum failure on the read in of the main routine by the assembly program.

2. a tape sum failure on the read in of the 2-group buffer program by the second part of the main routine.
- S 2499 - the element ID number read in in the region data is not in the compiled ID list.
- S 2540 - Two possible causes for this stop are:
1. a tape sum failure on the read in of an additional program to the main BPG program.
 2. a tape sum failure on the read in of a required section of the program to restore the main BPG program.
- S 2603 - last of the series of the cases completed. This occurs in the main program only when Option 2 is used.
- S 2667 - a breakdown of program logic. This is generally due to machine failure.
- S 2682 - an error in value, designated by ν under case input.
- S 2725 - an element identification number read in to indicate a change in one of the ν values not in the compiled element identification list.
- S 2759 }
S 3020 }
S 3291 } - a breakdown of program logic. This is generally
S 3295 } due to machine failure.
S 3323 }
- S 3541 - a tape sum failure on the read in of the ν values from tape unit 2 by assembly program.
- S 3802 - a breakdown of the program logic. This is generally due to machine failure.
- S 9998 - a tape sum failure on the read in of the master element identification list by assembly program.
- S 9999 - a tape sum failure on the read in of the data assembly program.
- S (0692+10k) - a tape sum failure on the read in of the R^{th} region coefficients by the 2-group buffer program.
- S (0882+20k) - a tape sum failure on the read in of R^{th} region coefficients by the 4-group buffer program.
- S $\left[0039 + (T - 1) M + \frac{M}{60} (S - 1) \right]$ - a tape sum failure in the read in of the cross sections of type T of the element listed as the S^{th} element in the master element identification list. This stop is applicable only to the first 6 types of cross sections.

2. a tape sum failure on the read in of the 2-group buffer program by the second part of the main routine.
- S 2499 - the element ID number read in in the region data is not in the compiled ID list.
- S 2540 - Two possible causes for this stop are:
1. a tape sum failure on the read in of an additional program to the main BPG program.
 2. a tape sum failure on the read in of a required section of the program to restore the main BPG program.
- S 2603 - last of the series of the cases completed. This occurs in the main program only when Option 2 is used.
- S 2667 - a breakdown of program logic. This is generally due to machine failure.
- S 2682 - an error in value, designated by ν under case input.
- S 2725 - an element identification number read in to indicate a change in one of the ν values not in the compiled element identification list.
- S 2759 }
 S 3020 }
 S 3291 } - a breakdown of program logic. This is generally
 S 3295 } due to machine failure.
 S 3323 }
- S 3541 - a tape sum failure on the read in of the ν values from tape unit 2 by assembly program.
- S 3802 - a breakdown of the program logic. This is generally due to machine failure.
- S 9998 - a tape sum failure on the read in of the master element identification list by assembly program.
- S 9999 - a tape sum failure on the read in of the data assembly program.
- S (0692+10k) - a tape sum failure on the read in of the R^{th} region coefficients by the 2-group buffer program.
- S (0882+20k) - a tape sum failure on the read in of R^{th} region coefficients by the 4-group buffer program.
- S $\left[0039 + (T - 1) M + \frac{M}{60} (S - 1) \right]$ - a tape sum failure in the read in of the cross sections of type T of the element listed as the S^{th} element in the master element identification list. This stop is applicable only to the first 6 types of cross sections.

Types of cross sections are described under
Input Section.

For the 20-group program, $M = 60$.

For the 40-group program, $M = 120$.

For the 60-group program, $M = 180$.

For the 80-group program, $M = 240$.

$S [0399 + 9(T - 7) + S - 1]$	}	a tape sum failure on the read in of the element cross sections of type 7 or 8 for 20, 30, 60 and 80-group programs respectively.
$S [0759 + 18(T - 7) + 2(S - 1)]$		
$S [1119 + 27(T - 7) + 3(S - 1)]$		
$S [1479 + 36(T - 7) + 4(S - 1)]$		
$S [0417 + 5(T - 9) + S - 1]$	}	a tape sum failure on the read in of the element cross sections of type 9 or 10 for 20, 40, 60 and 80-group programs respectively.
$S [0795 + 10(T - 9) + 2(S - 1)]$		
$S [1173 + 15(T - 9) + 3(S - 1)]$		
$S [1551 + 20(T - 9) + 4(S - 1)]$

5. APPENDICES

APPENDIX A

The Quantities T_{ik}^n

According to Equation (39)

$$T_{ik}^n = \frac{2k+1}{2^n!} \int_{-1}^1 P_i(\mu_0) P_k(\mu_c) (-U)^n d\mu_c,$$

$$\text{where } -U = \ln \left[1 - \frac{2A}{(1+A)^2} (1 - \mu_c) \right]$$

$$\mu_0 = \frac{A\mu_c + 1}{\sqrt{(A^2 + 2A\mu_c + 1)}}$$

Defining $a = \left(\frac{A-1}{A+1} \right)^2$, analytical expressions for T_{ik}^n are

$$T_{00}^0 = 1 \quad T_{01}^0 = 0$$

$$T_{10}^0 = \frac{2}{3A} \quad T_{11}^0 = 1 - \frac{3}{5A^2} \quad T_{12}^0 = -\frac{2}{3A} + \frac{4}{7A^3}$$

$$T_{00}^1 = -\left[1 + \frac{(A-1)^2}{4A} \ln a \right] \quad T_{01}^1 = \frac{3(A^2+1)}{4A} + \frac{3(A^4-2A^2+1)}{16A^2} \ln a$$

$$T_{00}^2 = 1 + \frac{(A-1)^2}{4A} \ln a - \frac{(A-1)^2}{8A} [\ln a]^2$$

$$T_{01}^2 = \frac{9(A^2+1)}{8A} - \frac{3(A-1)^2(3A^2+2A+3)}{32A^2} \ln a + \frac{3(A^2-1)^2 [\ln a]^2}{32A^2}$$

APPENDIX B (Cont'd)

Definitions

$\Sigma^S(u, x)$	macroscopic cross section for scatter and equal to $2\pi \int_{-1}^1 \Sigma^0(\mu_0, u, x) d\mu_0 \ .$
$\Sigma^F(\mu', x)$	macroscopic cross section for fission = $\sum_k N_k \sigma_k^F \ .$
$\Sigma^I(u', x)$	macroscopic cross section for inelastic scattering $= \sum_k N_k \sigma_k^I \ .$
$\Sigma^A(u', x)$	macroscopic cross section for absorption $= \sum_k N_k \sigma_k^A \ .$
Σ^T	macroscopic total cross section ($\Sigma^T = \Sigma^A + \Sigma^I + \Sigma^S$).
Σ_{th}^D	macroscopic thermal transport cross section.
ν	average number of neutrons per fission.
$f(u)$	lethargy distribution of neutrons from fission (angular distribution is assumed uniform).
$q(u)$	slowing down density due to elastic scattering.
$p(u)$	anisotropic component of slowing down density.
I	inelastic slowing down.
B^2	buckling.
$\xi(u)$	average logarithmic energy decrement per collision.
$\gamma(u), \zeta(u), \eta(u)$	slowing down constants.

APPENDIX C

1. Coordinate System Configuration

x, y, z Lab (rectangular) system.

x', y', z' Local system parallel to the lab system for fission and inelastic scatter integrals.

x'', y'', z'' Local system for scatter integral.

2. Probability of Neutron Scatter

APPENDIX D

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APPENDIX E

Input Forms

DATE: _____
PREPARED BY: _____

SID	C	I					CARD NUMBER

[illegible][illegible]

B-P-G PROGRAM
INPUT FORM NO.2
MATH FORM 67

DATE: _____

PREPARED BY: _____

ELEMENT CROSS SECTIONS

[illegible][illegible]

PREPARED BY: _____

CID	R	P	δ	ϵ	T _{c.o.}	S	CARD NO.
LS	$\Delta\mu$	ν	IM	PG			

B ₁	B ₂	B ₃	B ₄				CARD NO.

[illegible]

B-P-G PROGRAM
INPUT FORM NO. 4
MATH FORM 69

DATE: _____

PREPARED BY: _____

$\Delta\mu$ VALUES

[illegible][illegible]

B-P-G PROGRAM
INPUT FORM NO. 5
MATH FORM 70

DATE: _____
PREPARED BY: _____

[illegible][illegible]

B-P-G PROGRAM
INPUT FORM NO.6
MATH FORM 71

DATE: _____

PREPARED BY: _____

[illegible]

B-P-G PROGRAM
INPUT FORM NO 7
MATH FORM 72

DATE _____

PREPARED BY: _____

REGION DATA

E	B	B _z	P O. 1	P O. 2	P O. 3		CARD NO

ELEMENT DATA

EID	N	σ_{2200}^A	σ_{2200}^F	σ_{2200}^D	σ_{2200}^S	g^A	CARD NO.
<i>l</i>	γ	W^A	W^F	R	P	γ	CARD NO
EID	N	σ_{2200}^A	σ_{2200}^F	σ_{2200}^D	σ_{2200}^S	g	CARD NO
<i>l</i>	γ	W^A	W^F	R	P	γ	CARD NO
EID	N	σ_{2200}^A	σ_{2200}^F	σ_{2200}^D	σ_{2200}^S	g	CARD NO
		W^A	W^F				CARD NO
EID	N	σ_{2200}^A	σ_{2200}^F	σ_{2200}^D	σ_{2200}^S	g	CARD NO
		W^A	W^F				CARD NO

B - P - G PROGRAM
INPUT FORM NO. 8
MATH FORM 73

DATE: _____

PREPARED BY: _____ CARD
NUMBER

Y, VALUES

[illegible]

APPENDIX F

Sample Output

The output of a typical case, with a description of the quantities, which was run on the Burroughs 205 computer follows. This case employed 40-Group Cross Sections with Thermal Cut-Off taken at the 38th group. There were six elements in the region and 4-group coefficients were found. O_3 was set equal to zero, hence the printing out of absorptions and fissions for each element for each multigroup was not made.

The above described case required 8 minutes and 57 seconds of machine time for running. However, this required from 1 to 2 minutes of setup time which would be eliminated for successive cases.

TYPICAL SAMPLE OF PRINTOUT OF CASE EMPLOYING OPTION 1,
4 GROUPS, WITH 6 ELEMENTS

	1	2	3	4	5	6	7
(1)	5475000000	5110000000	5140000000	5110000000	5110000000	5238000000	5110000000
(2)		5110000000	5110000000	5110000000			
(3)	5160000000	4991440000	4923240000	5110000000	5110000000		
(4)	- 1000000000	4932980000	5029430000		5231140000	5245560000	5111500000
(5)			5110000000	5110000000			
(6)	- 1002000000		4740780000		5139700000	5150930000	5111500000
(7)			5110000000	5110000000			
(8)	- 8000000000	4930220000	4717730000		5140240000	5142000000	5110800000
(9)			5110000000	5110000000			
(10)	-1300000000	4852950000	5020390000		5113650000	5114000000	5110800000
(11)			5110000000	5110000000			
(12)	9223500000	4742900000	5359130000	5350260000	5216000000	5216000000	5110000000
(13)			5110000000	5110000000			
(14)	9023200000	4864340000	5167020000		5212460000	5212500000	5110000000
(15)	5217000000	5160000000	5110000000	5110000000	5219300000	5111714000	5054860000
(16)	5110000000	5028483259	5027125075	5025430316	5062071750	5247633122	5242535140
(17)	5110003812	5071142580	5035972936				
(18)	5110000000	5023139687	4937633468	4940440572	5095103660	5236006663	
(19)		4924399459	4951969638	4931168341	5094673070	5129911528	
(20)		4929036065	5018164765	5018269945	5081292040	5135765327	
(21)		4837418164	5044017497	5074607307			
(22)	5217622889		5115703915	4944450703	4941467051	4822947640	
(23)	5136181107		5080654001	5019572266	5018092330	4886145350	
(24)	5147470360		5073154796	5013217511	4993514521	4938487055	
(25)	5114239519		5031427855	5030929182		5052394542	
(26)	- 1000000000	- 1002000000	- 8000000000	-1300000000	9223500000	9023200000	
(27)	4662851956		4334907901	4714296548	4913192905	4924234716	
(28)	4717987501		4399712027	4618715181	4916453154	4933317795	
(29)	4835632540		4519705618	4738490672	5011423578	4963461779	
(30)	4915894031		4582399061	4816603599	5036121060	4961401757	
(31)					4928613562	4911826610	
(32)					4931168341		
(33)					5018269945		
(34)					5074607307		
(35)	5099359430	5110000000	5099999650	5099908010	5075447120	5083078900	

TYPICAL SAMPLE OF PRINTOUT OF CASE EMPLOYING OPTION 1,
4 GROUPS, WITH 6 ELEMENTS
(Cont'd)

	1	2	3	4	5	6	7
(36)	1	4998116567	5029729320	4821801933	4728570986	4768481851	4851295763
(37)	2	5035353901	5110717427	4873416167	4771405281	4816138306	4924636536
(38)	3	5078485519	5123781256	4915127031	4812138768	4827841437	4974294585
(39)	4	5112930171	5139159196	4923353890	4820994265	4845385262	5015337632
(40)	5	5117485382	5152965622	4934189989	4826810131	4856890610	5023963906
(41)	6	5115013083	5145560737	4925208430	4821674605	4841566216	5034696306
(42)	7	5114322661	5143371117	4919571082	4818172053	4821033019	5045154455
(43)	8	5115146026	5145896353	4919658830	4820474372	4818965260	5053839774
(44)	9	5114059731	5142599178	4919019121	4820483002	4817804350	5060906706
(45)	10	5091916508	5127851714	4886950714	4814797139	4812225303	5066835205
(46)	11	5111687443	5135441087	4911094737	4822622457	4816388242	5070529114
(47)	12	5084947441	5125717723	4885175406	4817460097	4812896941	5072966527
(48)	13	5071564301	5121673279	4866839821	4816625384	4811846511	5074326477
(49)	14	5115610339	5117351723	4913538665	4848987352	4830697458	5074609573
(50)	15	5111997839	5113328281	4893864074	4849482914	4831032273	5074154526
(51)	16	5110768311	5111966518	4878303291	4855614567	4836844387	5073076924
(52)	17	5095242937	5110565271	4860278340	4873842084	4845117420	5071695746
(53)	18	5093165984	5110365672	4862606941	4910364809	4856239004	5070033198
(54)	19	5087995066	5097807636	4859004032	4914283440	4881961915	5068014816
(55)	20	5085407098	5094910716	4856105292	4919937121	4912836508	5065460054
(56)	21	5024820383	5088261705	4816474426	4883950523	4857994554	5064455810
(57)	22	5031155033	5091163360	4819882406	4867944819	4866841967	5065577541
(58)	23	5016973179	5082878995	4810745588	4910045789	4840818898	5062465507
(59)	24	5026283041	5089470831	4816916864	4837001317	4858776031	5061926324
(60)	25	5021270191	5080696032	4811763058	4910368055	4857255677	5060771391
(61)	26	5028309041	5084134708	4817045375	4912952685	4913463576	5059306167
(62)	27	5026645103	5078155023	4816660269	4895169617	4913582460	5058187168
(63)	28	5017602992	5084776701	4810656025	4842238546	4858392083	5057658420
(64)	29	5015770797	5060110438	4778674246	4930842504	4893120327	5054495498
(65)	30	5022389224	5077826104	4812949351	4890298665	4912080725	5053403012
(66)	31	5020690734	5066710947	4812985759	4892625065	4911228968	5052344904
(67)	32	5023620198	5074171529	4813721960	4911300818	4912717323	5051076166
(68)	33	5025881870	5065850054	4810219191	4873300851	4853826372	5050181065
(69)	34	5039197914	5072723901	4823594269	4852055355	4663105740	5049425189
(70)	35	5035324219	5065702755	4822343266	4856465105	4856636157	5048637084
(71)	36	5019875650	5072726428	4811900954	4831584556	4835512515	5048202227
(72)	37	5051851201	5064293568	4832723186	4918085252	4930662402	5046066469
(73)	38	5027042883	5060595238	4815910821	4915156782	4924735972	50443931680

PRINT OUT OF INPUT

CASE DATA

	1	2	3	4	5	6	7
(1)	CID	R	P	δ	ϵ	$T_{c,o}$	S
(2)	LS	Δu	ν	IM	PG	--	--

REGION DATA

(3)	E_R	B	B_z	O_2	O_2	O_3	--
-----	-------	---	-------	-------	-------	-------	----

ELEMENT DATA

(Repeated for Each of the Six Elements, i. e. , Lines 6, 7, 8, 9, 10, 11, 12, 13, 14, and 15 in addition to 4 and 5 contain element data)

(4)	EID	N	σ_{2200}^A	σ_{2200}^F	σ_{2200}^D	σ_{2200}^S	g
(5)	l	ψ	W^A	W^F	R	p	γ

PRINT OUT OF CALCULATED VALUES

NON-OPTIONAL PRINT OUT - TWO GROUP DATA

(16)	S_{epi}	$\tau_{epi} B^2$	A_{epi}	νF_{epi}	P_{epi}	τ_{epi}	τ_{epi}^*
(17)	k_{tot}	P	C. R.				

NON-OPTIONAL PRINT OUT - POLYGROUP DATA

(Subscripts Indicate Polygroup Number - Example Case has 4 Polygroups)

(18)	S_1	$\tau_1 B^2$	A_1	νF_1	P_1	τ_1	--
(19)	S_2	$\tau_2 B^2$	A_2	νF_2	P_2	τ_2	--
(20)	S_3	$\tau_3 B^2$	A_3	νF_3	P_3	τ_3	--
(21)	S_{th}	$\tau_{th} B^2$	A_{th}	νF_{th}	--	--	--

NON-OPTIONAL PRINT OUT - POLYGROUP DATA (Cont'd)

	1	2	3	4	5	6	7
(22)	ϕ_1	--	D_1	W_1	β_1	f_1	--
(23)	ϕ_2	--	D_2	W_2	β_2	f_2	--
(24)	ϕ_3	--	D_3	W_3	β_3	f_3	--
(25)	ϕ_{th}	--	D_{th}	W_{th}	--	f_{th}	--

OPTIONAL PRINT OUT - POLYGROUP DATA (ELEMENT DATA) $0_1 \neq 0$
(Example Case has 6 Elements)

(26)	EID ₁	EID ₂	EID ₃	EID ₄	EID ₅	EID ₆	--
(27)	A _{1,1}	A _{2,1}	A _{3,1}	A _{4,1}	A _{5,1}	A _{6,1}	
(28)	A _{1,2}	A _{2,2}	A _{3,2}	A _{4,2}	A _{5,2}	A _{6,2}	
(29)	A _{1,3}	A _{2,3}	A _{3,3}	A _{4,3}	A _{5,3}	A _{6,3}	
(30)	A _{1,th}	A _{2,th}	A _{3,th}	A _{4,th}	A _{5,th}	A _{6,th}	
(31)	$\nu F_{1,1}$	$\nu F_{2,1}$	$\nu F_{3,1}$	$\nu F_{4,1}$	$\nu F_{5,1}$	$\nu F_{6,1}$	
(32)	$\nu F_{1,2}$	$\nu F_{2,2}$	$\nu F_{3,2}$	$\nu F_{4,2}$	$\nu F_{5,2}$	$\nu F_{6,2}$	
(33)	$\nu F_{1,3}$	$\nu F_{2,3}$	$\nu F_{3,3}$	$\nu F_{4,3}$	$\nu F_{5,3}$	$\nu F_{6,3}$	
(34)	$\nu F_{1,th}$	$\nu F_{2,th}$	$\nu F_{3,th}$	$\nu F_{4,th}$	$\nu F_{5,th}$	$\nu F_{6,th}$	
(35)	P ₁	P ₂	P ₃	P ₄	P ₅	P ₆	

OPTIONAL PRINT OUT - MULTIGROUP DATA $0_2 \neq 0$
(T_{c,o} of Example Case = 38) (Thermal Group not Included)

(36)	$1 = j \times 10^{-10}$	ϕ_1	$\phi_1/\Delta u_1$	$\tau_1 B^2$	A ₁	νF_1	$\sum_k q_k(\mu_1)$
(37)	2	ϕ_2	$\phi_2/\Delta u_2$	$\tau_2 B^2$	A ₂	νF_2	$\sum_k q_k(\mu_2)$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(73)	38	ϕ_{38}	$\phi_{38}/\Delta u_{38}$	$\tau_{38} B^2$	A ₃₈	νF_{38}	$\sum_k q_k(\mu_{38})$

APPENDIX G

Elements Available for Option 1 Input

<u>Element</u>	<u>Element Identification No.</u>
Hydrogen	01000
Deuterium	01002
Beryllium	04000
Boron	05000
Carbon	06000
Oxygen	08000
Aluminum	13000
Thorium-232	90232
Uranium-238	92238
Uranium-233	92233
Uranium-235	92235
Plutonium-239	94239
Chromium	24000
Manganese	25000
Iron	26000
Nickel	28000
Zirconium	40000
Cadmium	48000
Stainless Steel (304)	26001

October 16, 1961

APPENDIX H

Elements Available on Tape for Option 2 Input

<u>Element</u>	<u>Element Identification No.</u>
Hydrogen	01000
Deuterium	01002
Beryllium	04000
Boron	05000
Carbon	06000
Oxygen	08000
Sodium	11000
Magnesium	12000
Aluminum	13000
Thorium-232	90232
Uranium-238	92238
Uranium-233	92233
Uranium-234	92234
Uranium-235	92235
Uranium-236	92236
Plutonium-239	94239
Plutonium-240	94240
Plutonium-241	94241
Chromium	24000
Manganese	25000
Iron	26000
Nickel	28000
Copper	29000
Zirconium	40000
Molybdenum	42000
Cadmium	48000
Bismuth	83000
Stainless Steel (304)	26001

October 16, 1961