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THE USE OF LINEAR PROGRAMMING TO MODEL BUSINESS AND CONSUMER
DECISION MAKING RELATED TO ENERGY CONSUMPTION

D. E. Deonigi
Manager, Economic Analysis Section
Pacific Northwest Laboratories
Richland, Washington 99352

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Introduction

In recent years there has been an intense effort to develop models that will predict the response of business and consumers to changes in energy cost. Most of these models have applied econometric principles to a history of small price fluctuations which may have been either directly or indirectly under regulation. These models thus produce uncertain results when applied to the present market situation and its rapidly changing price structure. In the past, energy prices have gone through fairly slow adjustments and fuel supply has not been a major factor in consumers' decisions. However, today, prices are rising at rates with which we have no prior experience. The availability and environmental controls also influence our choice of fuels as well as price considerations. Given this situation, it is postulated that models which simulate the decision (buying) process of business and consumer may be superior to those relying on past trends.

Battelle-Northwest and the AEC developed a model to predict the impact of a new technology, namely, the nuclear reactor, being considered by utilities to supply electrical energy. Because the nuclear reactor energy source had not previously been available, little data on how the sale of reactors would react to changes in oil or coal prices were available. Thus, instead of modeling apparent trend relationships, which are of limited value when dealing with new technology or major new constraints such as environmental controls, the utility decision process was modeled. The decision process was determined to be the minimization of present worth cost with some decision limiting constraints. Since franchised utilities are required to prove prudent management to some form of state public utility commission, their decision process is open to public inspections. Therefore, modeling of this process for utilities is easier than for other areas of business or consumer purchases.

For such a model to be truly useful, it must be understood by a group of policy makers with diverse backgrounds. Consequently, linear programming was selected over dynamic programming and heuristic techniques. The linear programming model was designed to determine the electrical generation supply for the next 50 years. The equation development was reviewed

by a task force group representing a cross section of utilities, fuel suppliers, plant vendors and policy makers. Many compromises were struck in developing the equations to represent the views of the task force membership and the availability of data. The model was extensively used for the Liquid Metal Fast Breeder Reactor cost-benefit analysis in 1969⁽¹⁾ and again in 1971.⁽²⁾

Figure 1 shows the flow of information from basic data through a linear programming (LP) equation generator, called DAEDALUS II, to a standardized LP solution system⁽³⁾ to resulting reports. External models involving non-linear equations which determine costs of nuclear fuel service are also used as sources of input.

The next section of this paper is devoted to modeling technique for utility applications. A discussion of application to consumer and other business applications is contained in the last section. Details on other modules shown in Figure 1 can be found in other reports.^(4,5,6)

Model Description

The model is formulated to simulate a schedule of building power plants that will supply projected energy requirements while also satisfying other constraints. The other constraints represent material balances, technological introduction rate constraints, fuel price step functions, committed plant construction, and fuel processing capacity. Various groups of constraints are optional and will only be generated when specified. These constraints will be discussed in more detail later. The model is displayed in the block diagram in Figure 2, where the blocks represent the groups of constraints, the lines connecting the blocks exhibit the interrelationships between groups, and the variables named on the lines are the means of these interrelationships.

The total power economy may be divided into several sub-sections, each with its own projected energy requirement constraints. These sections have been used to represent geographic regions in which fossil power costs are nearly equal.

Figure 1
ENERGY MODEL FLOW

Figure 2

In reality, the time variable is continuous over a range of a simulation. However, to make the problem fit a linear programming format, the time span is broken into a finite number of discrete time intervals. The time interval sizes, as well as the beginning and ending times are variable and specified by the user. The interval size may even be changed in the range of time under study; for instance, time intervals of one year could be specified for the first 10 years, then relaxed to two-year intervals for, say, 20 years, and then further relaxed to five-year intervals. This allows for increased detail in the early part of the simulation. Up to three different time schedules may be specified, with different groups of constraints using different time schedules. The most common time schedule used has been 35 two-year intervals, from 1970 to 2040.

In this formulation, each variable represents an activity during a specific time interval, such as the amount of a fuel type used in year 1980 or the number of plants of a certain type built in 1978. The variables in the model may be subdivided into three general classes for discussion, independent variables, dependent variables and slack variables. The slack variables are actually generated by the LP system rather than by DAEDALUS, but are included here for discussion. The independent variables represent the plants to be built, and the dependent variables represent the resulting effects from those plants, such as material balances, fuel usage, etc.

Each variable is represented in the LP matrix by a vector or column of coefficients. Each coefficient of an independent variable represents its effect on the constraint, while the dependent variables usually measure the status of a constraint and are usually represented by a coefficient of ± 1 .

Independent Variables. Each independent variable represents the construction of a particular type of power plant in a particular discrete time interval. A type of plant is identified by the plant design, fueling scheme, and a predetermined lifetime load history. Currently, two load histories are allowed, but a third could easily be added. The model assumes that all plants constructed will start power production at the midpoint of the associated time period and will shutdown only at the end of its lifetime. To date, all plants have had an assumed 30-year power producing lifetime.

The user must specify all the possible plant options available to the model. A set of mass balance data and certain economic data must be provided to DAEDALUS on magnetic tape from the computer code PACTOLUS for each plant type available to the model.⁽⁷⁾ DAEDALUS then uses that one set of data to generate a vector of coefficients for each time interval that the plant type is available. A non-zero coefficient is calculated for every constraint a plant will affect, with zero coefficients signifying no affect. Most coefficients are basically accumulations of some characteristics of the reactor history, but the cost coefficient is considerably more detailed and is discussed later. These coefficients are a measure of a plant's installed capacity, fuel usage, isotopic production or usage (optional), fuel processing (optional), and residual consumption (optional).

Dependent Variables. The dependent variables measure some characteristic of the system of plants built. In addition to measuring the characteristic, the non-negativity requirement effectively constrains the model. For instance, a variable measuring the plutonium stockpile requires the stockpile to not become negative; thus the system of reactors built must produce plutonium before the plutonium may be used. The vector of coefficients representing a dependent variable is generally simpler than the vector for an independent variable and will usually have only one or two coefficients that are ± 1.0 . That is, the stockpile at the end of one time period is used for calculating the stockpile at the end of the next time period.

The following general classes of dependent variables can be generated under user control:

1. The amount of fuel consumed at each price level.
2. Material balance and amounts of material purchased or sold outside the system.
3. Fuel processing requirements.
4. Fuel processing residuals.
5. Total nuclear plants with a given load curve required in each sub-region.
6. A technology's construction capacity.

Slack Variables. Slack variables are generated by the LP system at load time rather than by the DAEDALUS matrix generator. Slack variables, as discussed later, are non-negative variables added to inequality constraints to convert them to equality constraints. The Bonner and Moore LP system generates a slack variable for all constraints, even the equality constraints. The slacks for the equality constraints are then logically deleted (forced to zero) internally during solution. The logical deletion is performed automatically by the Bonner and Moore system. These slack variables may appear in the solution of the model and are a measure of the inequality (or slack) in the constraint, thus the term slack variable.

Constraints

Mathematically, constraints are used to define the region of feasibility of the model and the solution algorithm will select the optimum solution from the feasible region. Thus, the constraints impose whatever physical conditions the modeler wishes to impose on the system being modeled. In this model, a constraint usually will impose a physical condition during a single time interval, thus each type of constraint is normally repeated for each time interval. The discussion of the constraints will be sectioned by purpose, since sometimes more than one constraint is required to perform the desired function. The LP formulation places all variables on the left side and only a constant term on the right hand side of the constraint. However, in this discussion, the terms are sometimes transposed to clarify the intent of the constraint.

The following notation was employed in the following discussion.

$\sum_{r \in R} r$ means the sum of r , for all r that are members of the set R .

Power Demand Constraints. In the early development of this model, total installed capacity and total required energy were specified as well as a maximum installed capacity for each subregion, and the LP algorithm decided the most effective mixture of load histories within each geographic subregion. It was soon learned that this cost considerable computer time and resulted in nearly the same load mixture for each case. Thus the formulation was changed

to make the user specify the capacity of each load history that is installed in each region. This capacity must allow for both growth in energy requirements and in plants closing down and being removed from the system.

For each time period and for each load history, the present formulation requires the following constraints: one equation that equates the sum of the nuclear plants required from all regions to the particular type of nuclear plants available, and one constraint for each subregion to partition the required capacity between nuclear and fossil.

That is, for each load history i and each period j ,

$$(a) \quad \sum_{k=1}^K N_{ijk} = \sum_{r_{\ell} \in R_{ij}} c_{\ell} r_{\ell}$$

and

$$(b) \quad N_{ijk} + \sum_{f_{\ell} \in F_{ijk}} c_{\ell} f_{\ell} = T_{ijk} \quad (\text{for each subregion } k)$$

where

N_{ijk} = total capacity of nuclear plants with load history i installed in region k during time period j (gigawatts)

K = number of regions

R_{ij} = the set of all nuclear plants with load history i that may be installed in time period j

c_{ℓ} = capacity (gigawatts) of a power plant (used for both fossil or nuclear above). This coefficient is obtained from the PACTOLUS tape.

F_{ijk} = the set of fossil plants with load history i available in region k during time period j .

T_{ijk} = an input constant representing the total capacity (gigawatts) of new plants with load history i installed in region k during time period j .

Fuel Usage Constraints. Fuel can be priced as a step function dependent on the amount of fuel used. That is, there is only a fixed amount available at a certain price. After that amount is used, the next higher price must be paid. The model assumes the lower priced ore will be used first. This formulation requires one constraint for each time period plus one constraint for each price step except the last one. Ore at the last price step is assumed unlimited, thus it is not constrained. The constraint for each time period stipulates that the amount of fuel purchased at all price steps during the period must equal the total fuel required by all plants in that time period. The constraint for each price step stipulates that the total fuel purchased at a certain price in all periods in the model must not exceed the total amount available at that price. The present worthing effectively causes the model to use the lowest priced fuel first.

That is, for each time interval j

$$\sum_{i=1}^M U_{ij} = D_j + 1.34W_j - 1.34 W_{j-1} + \sum_{r_\ell \in R_j} u_\ell r_\ell$$

and for each price step i from 1 to $(M-1)$,

$$\sum_{j=1}^P U_{ij} \leq A_i$$

where

M = number of price steps

U_{ij} = the amount of fuel purchased in time interval j at price i

D_j (optional) = the deficit in the fuel processing residual stockpile. This formulation can represent any residual stockpile, where use of the actual fuel can be substituted for the use of the residual, such as depleted uranium in the uranium enrichment process. These stockpile constraints are optional, and the D_j term will appear in this constraint only when the residual stockpile constraints are generated.

W_j (optional) = the preprocessed fuel stockpile at the end of period j . If $(W_j - W_{j-1})$ is plus, it represents the amount of processing during this time interval for use in an earlier time interval and used in this time interval. Here, the coefficient of 1.34 is the amount of feed (kg) per unit of separative work (kg) at an assumed four percent enrichment. The processing constraints are optional, and the W_j terms will appear in this constraint only when the processing constraints are generated.

R_j = the set of all reactors that affect requirements in time interval j . For nuclear plants, this may be plants starting in period $j + 1$ or plants closing down in period $j-1$ or anything between.

μ_k = the amount of fuel required by r_k in period j

P = number of time intervals

A_i = the total amount of fuel available at price i

Material Balance Constraints. One constraint is required for each isotope stockpile in each time interval. The formulation is merely that the stockpile at the end of a time period is equal to the stockpile from the previous time period plus the net effect (discharged less charged) of all plants affecting the stockpile during the time interval concerned, plus any purchased and less any sold outside the system. For nuclear plants, certain isotopes required as fuel have been constrained. Up to three isotopes may be combined in a single stockpile, with an assumed mixture used for calculating radioactive decay. For example, Pu^{239} and Pu^{241} have commonly been combined as a fissile plutonium stockpile, with the assumption that the combined stockpile is 10% Pu^{241} . This constraint not only evaluates the stockpile, but the non-negativity condition of linear programming constrains the model to maintain a non-negative stockpile at the end of each interval.

The stockpile constraint for each time interval j is written

$$I_j = dI_{j-1} + b_j - s_j + \sum_{r_\ell \in R_j} a_\ell r_\ell$$

where

I_j = material balance at the end of time interval j

d = decay constant for the length of the time period

R_j = the set of all reactors that charge or discharge the material in time period j

b_j (optional) = the amount of material purchased (10^3 kg) from outside the system at the beginning of time interval j . This variable is optional and will be generated only when requested by the user. The purchase price is specified by the user and enters the objective function.

s_j (optional) = the amount of material sold (10^3 kg) outside the system at the end of time interval j . This variable is also optional and will be generated only when requested by the user. The selling price must be specified by the user and may differ from the buying price.

a_ℓ = the net amount of the material discharged by reactor r_ℓ in period j . This coefficient is the accumulation of the annual kilograms reprocessed minus kilograms fabricated from PACTOLUS.

Fuel Processing Residual Stockpile Constraint. Stockpile constraints will be generated only when requested by the user. They should be used only when a by-product of a fuel processing plant can be used as fuel in certain plants, such as depleted uranium or perhaps a heavy residue from a refinery. One constraint is required for each time period. This formulation differs slightly from the material balances since a deficit of residual material may be satisfied by the substitution of natural fuel. In other words, these equations merely calculate the amount of residual stockpile or the amount of fuel required to replace the residual requirements, rather than constrain the tails stockpile to a non-negative value.

Due to the non-negativity condition LP applies to all variables, the stockpile is represented as the difference of two non-negative variables, say $(x-y)$. Then if the stockpile is positive, x will attain a positive value; but if a deficit occurs, the variable y attains a positive value. A positive stockpile enters the constraint for the next stockpile and a deficit enters the fuel constraint as a fuel requirement, simulating purchase of fuel and a zero residual stockpile at the end of the year.

The tail stockpile for each time interval j is thus written as

$$T_j - D_j = T_{j-1} + \sum_{r_\ell \in R_j} t_\ell r_\ell$$

where

- T_j = residual stockpile at the end of the j^{th} time interval
- D_j = deficit in residual stockpile at the end of the j^{th} time interval. This variable enters the fuel usage constraint for the j^{th} time interval as a fuel requirement
- R_j = the set of reactors that affect the residual stockpile in the j^{th} time interval
- t_ℓ = the amount of residual discharged by reactor r_ℓ in period j . This coefficient is an accumulation of the residual discharged annually minus residual charged from the PACTOLUS data.

Fuel Processing Constraints. Two constraints per time interval are required for the formulation of the fuel processing constraints. To allow preprocessing of fuel, a stockpile of preprocessed fuel is maintained. One constraint stipulates that the change in the preprocessed fuel stockpile in a time interval is equal to the difference between the processing performed in the time interval and the amount required by all plants operating in the time interval. The other constraint limits the amount of processing that can be performed during a time interval. The limit must be specified by the user and simulates the projected capacity of all processing plants. The limit may be expressed as a function of time to allow for plant expansion and/or addition of new plants. These constraints are optional and are generated only when specified by the user.

The two constraints for time period j are written

$$W_j - W_{j-1} = S_j - \sum_{r_\ell \in R_j} s_\ell r_\ell$$

and

$$S_j \leq L_j$$

where

W_j = the stockpile of preprocessed fuel (10^6 kg) at the end of the j^{th} time interval. These variables enter the fuel usage constraints.

S_j = the amount of fuel processing (10^6 kg) performed during the j^{th} time interval.

R_j = the set of all reactors requiring or getting a credit for fuel processing during the j^{th} time interval. A plant gets a credit if the plant fuel is not totally consumed, such as nuclear fuel.

s_ℓ = the net fuel processing required by reactor r_j . This coefficient is an accumulation of the annual processing purchased less processing credited, which is obtained from the PACTOLUS data.

L_j = the total amount of separative work that can be performed during the j^{th} time interval. This value is a constant for a single constraint but may be input as a function of time for the total model.

Technology Introduction Constraints. The purpose of the technology introduction constraints is to limit the introduction rate of classes of plants; for instance, to simulate the learning phase of constructing plants of a new technology. These constraints are formulated such that the number of plants of a class built in a time period must not exceed the number built in previous periods plus some fixed number. This formulation effectively doubles the number built each period if the maximum allowed number is built in each time interval. For instance, if the fixed number is two, only two plants may be built in the first period constrained. Then in the next period, two plus the number built in the previous period, two, or a total of four may be built. Then in the third period, two plus the number built in the preceding period, four, plus the number built in the first period, two, or a total of eight may be built. These introduction constraints are optional and completely flexible. The user must specify which reactors are included in a class, which time intervals the class is constrained, and the fixed amount of increase. A typical set of constraints might constrain the introduction of LMFB reactors to 4 in 1980-81, 8 in 1982-83, 16 in 1984-85, 32 in 1986-87, and unconstrained from 1988.

These constraints are written for period j

$$\sum_{r_{\ell} \in R_j} r_{\ell} \leq \sum_{r_{\ell} \in P_j} r_{\ell} + k$$

where

R_j = the set of reactors in the class that can be built in the j^{th} time interval.

P_j = the set of reactors in the class that can be built prior to the j^{th} time interval.

k = the fixed growth factor.

Present Commitment Constraints. This set of constraints fixes the number of nuclear plants built in the early periods of the model to those plants that are presently committed to construction. These constraints are actually revised into the model using the Bonner & Moore REVISE micro rather than generated by DAEDALUS, but are included here as part of the model. These constraints stipulate that the total number of nuclear plants built in a region in a period is fixed.

That is, for time interval j and region i

$$\sum_{k=1}^K N_{kji} = B_{ij}$$

where

K = number of load histories.

N_{kji} = the number of nuclear plants with the k^{th} load history built in period j in region i .

B_{ij} = the total number of nuclear plants committed in region i in period j .

Technology Phaseout Constraint. One constraint is generated for each time period for each technology constrained. A variable is introduced that represents the new construction capacity for a technology. The basic assumption is that new construction capacity has a 14-year lifetime. This constraint has a smoothing effect on the number of plants of a given technology that are built. This constraint will simulate an existing technology's unwillingness to stop production, and also the caution that a corporation displays when entering a new technology. The new plant construction capability will not enter the solution unless it is economical to stay in for the 14-year lifetime.

This constraint is represented by the following equation:

$$C_i = \frac{1}{t_i} \cdot (R_i - R_{i-1} + \sum_{j=(S_i-14)}^{(f_i-14)} C_j)$$

where

C_i = technology annual new construction capacity.

t_i = years in time period i .

R_i = number of plants of the constrained class built in period i .

S_i = period i start year.

f_i = period i final year.

This constraint simply states that the annual new construction capacity equals the annual construction capacity growth of the technology plus the retired construction capacity. The constraint not only defines the new capacity, but the non-negativity condition of all LP variables effectively constrains the new construction capacity to be used for the full 14-year lifetime, since if C_i is zero, then $R_i = R_{i-1} - \sum C_j$. Thus the number of reactors built in succeeding time periods can only decrease as fast as construction capacity expires.

Objective Function

The objective function is the measure of the optimization and is used to determine the best basic solution to the set of constraints defined previously in this report. During the Simplex solution algorithm, the objective function is modified in such a way that it indicates which non-basic variable should be brought into the basic solution. In this model, the objective function is the total present worth cost of building and operating all the plants from some specified beginning time to a specified ending time, using the conditions specified by the user. Since the objective is to minimize cost, costs are positive and income is negative. The starting time, ending time, and discount factor are specified by the user. Up to 40 objective functions may be generated, but only one at a time may be optimized. The alternate objective functions are then merely evaluative, but may be used for optimization by employing controls built into the Bonner & Moore LP system.

If P = the number of time intervals
 J = the number of uranium price steps
 K = the number of isotope stockpiles,

then the objective function Z may be written

$$Z = \sum_{i=1}^P \left\{ \sum_{k=1}^K (p_{ki}^b b_{ki} - p_{ki}^s s_{ki} + p_{ki} I_{ki}) + \sum_{j=1}^J p_{ji}^u u_{ji} + c_i^s w_i + \sum_{r_\ell \in R_i} c_\ell^r r_\ell \right\}$$

where

b_{ki} = the amount of material k bought externally at beginning of period i.

s_{ki} = the amount of material k sold externally at end of period i.

I_{ki} = the balance of material k at the end of period i.

$p_{ki}^s, p_{ki}^b, p_{ki}$ = the selling, buying, and storage coefficients, respectively, for material k in period i. The derivation of these coefficients is discussed later.

u_{ji} = the amount of fuel purchased in period i at the j^{th} price step.

p_{ji}^u = the j^{th} price step for fuel, to be discussed later.

w_i = the amount of fuel processing in period i.

c_i^s = the unit variable cost of fuel processing.

R_i = the set of all plants built in time interval i.

c_ℓ^r = the total present worth cost coefficient of plant r_ℓ .

The derivation of this coefficient is discussed later.

Uranium Price Steps Coefficients (p_{ji}^u). The cost coefficients for fuel purchased is the cost at the midpoint of the time interval, for each price step, discounted to the initial time (t_0). If t is the midpoint of the i^{th} time interval and r is the discount factor, then the cost is

$$p_{ji}^u = \left(\frac{\$}{lb} \right) \left(\frac{lb}{kg} \right) \left(\frac{1}{1+r} \right)^{t-t_0}$$

Fuel Processing Coefficients (c_i^s). The cost for fuel processing may be divided into two components when the fuel processing equations are generated. One component is applied when the work is processed and the other component is applied when the fuel is actually required by a reactor. The two times will vary only when fuel processing is preprocessed and stockpiled for later use. The fuel processing cost coefficient represents the first components, that which is applied when the fuel is processed. The second cost component is included as part of the single cost coefficient of each plant. If the fuel processing constraints are not generated, both components will be included as part of the reactor coefficient.

If t is the midpoint of the i^{th} time interval and r is the discount factor

$$c_i^s = \left(\frac{\$}{kg} \right) \left(\frac{1}{1+r} \right)^{t-t_0}$$

Material Selling, Buying, and Storage Cost Coefficients ($p_{ki}^s, p_{ki}^b, p_{ki}$).

Since material will be bought in deficit periods and sold in excess years, the assumption that material is bought at the beginning of a time period interval and sold at the end of a time interval should keep the stockpile non-negative throughout the time interval. If we assume the normal systems effect on stockpile is a continuous function of time, then Figure 3 exhibits the nature of a stockpile constraint,

where

x_i = the material stockpile at the end of time interval i .

b_i = the amount bought at start of time interval i .

s_i = the amount sold at end of time interval i .

t_i = the time at the end of i^{th} time interval.

Non-zero b_i and s_i will then appear as discontinuities in the function.

Figure 3 is drawn for demonstration purposes rather than as a realistic example.

Figure 3. Isotope Stockpile as Function of Time

Assuming that unit storage cost and the stockpile each are linear over a single time period, the time integrated stockpile storage cost very nearly equals the average stockpile for the period multiplied by the average unit storage charge for the period. If c_i is the unit storage charge at time t_i , then H_i , the storage cost for period i , may be written as

$$H_i = (.5)(c_{i-1} + c_i) (.5)(x_{i-1} + b_i + x_i + s_i) \quad .$$

The total storage cost may then be written

$$\begin{aligned} \sum_{i=1}^n H_i = & \left\{ \sum_{i=1}^n \left[1/4(c_{i-1} + c_{i+1}) + 1/2 c_i \right] x_i \right\} + 1/4(c_{n-1} + c_n)x_n \\ & + \sum_{i=1}^n 1/4 (c_{i-1} + c_i)b_i + \sum_{i=1}^n 1/4 (c_{i-1} + c_i)s_i \end{aligned}$$

Collecting terms, the total coefficient:

for x_i , where $i < n$, is $1/4 (c_{i-1} + c_{i+1}) + 1/2 c_i$,

for x_n is $1/4(c_{n-1} + c_n)$,

for s_i is $1/4 (c_{i-1} + c_i) - c_i^s$,

for b_i is $1/4 (c_{i-1} + c_i) + c_i^b$

where

c_i^s is the isotope selling price at the end of the i^{th} period, and

c_i^b is the isotope purchase price at the beginning of the i^{th} period.

All of the time related cost terms are then discounted to t_0 .

Plant Cost Coefficients (C_{ℓ}^r). The single cost coefficient for a plant represents the total cost to build a plant and operate it for its entire lifetime or until the end of the problem, whichever occurs first, minus the costs that are charged in other parts of the model. The costs charged in other parts of the model are those isotopes stockpiled and those items already discussed in this report. Stockpiled material required is charged indirectly by forcing the system being modeled to either produce its own material or purchase it from outside the system. For those plants which would operate after the end of the model, the total cost is determined to fix the discounted cost to discounted energy ratio. That is, if a plant with expected lifetime of L years will operate only N years, then

$$\frac{\sum_{i=1}^N PWC_i}{\sum_{i=1}^N PWE_i} = \frac{\sum_{i=1}^L PWC_i}{\sum_{i=1}^L PWE_i}$$

where

PWC_i = present worth cost in year i .

PWE_i = present worth energy generated in year i .

Present worthing is based on the midpoint of the year the item is incurred.

It is assumed that the total revenue for a plant is equal to the total expense. The total expense is calculated so that total expense, less gross revenue tax and state and local income taxes, is equal to the initial capital, plus the sum of the yearly expenses and revenue on equity. The yearly expenses for a nuclear reactor include:

1. Fuel purchases (U_3O_8 , thorium, U^{233} , and plutonium)
2. U_3O_8 to UF_6 conversion
3. Fabrication
4. Reprocessing
5. Fuel enrichment
6. Fuel shipping
7. Fixed operating and maintenance expenses (includes nuclear liability insurance)
8. Variable operating and maintenance (includes property tax and insurance)
9. Interim capital replacements

Initial capital may be obtained from computer code PACTOLUS⁽⁷⁾ but may also be input to DAEDALUS as a function of time by the user. Yearly summaries of quantities of fuel bought and sold (material balances) and amounts of services required (fabrication, reprocessing, shipping, and fuel enrichment) are also obtained from PACTOLUS. Unit costs of fuel and services are specified by the user as a function of time. Fabrication and reprocessing unit costs have, in practice, been the result of computer code FUELCO as described in Reference 2, but may be from any source desired.

The coefficient, C_{ℓ}^r , is then the total expense less those costs charged in some other part of the model. The reason to add all costs and then back out certain costs is to get the correct tax calculation. The cost details are discounted to the startup of the plant, then the total coefficient is discounted to the initial time t_0 . When a plant is built in several time periods, only one set of PACTOLUS data is required, but the cost must be calculated for each startup period because the unit costs may vary with time.

Interpretation of the Linear Programming Solution

The objective function value at the optimal solution is a single number representing the total cost of the system. It is probably the most useful information for performing a benefit-cost analysis. In other words, how much does it change the total system cost to perturb the model in some manner.

The primal solution of a linear programming problem is the list of all variables in the optimal basis and the activity levels of those variables. All variables not in the basis have activity of zero. The reactor mix, i.e., the number of each type of plant built in each time interval, is available from the primal solution. The amount of material in each stockpile and the amount of fuel purchased at each price step in each time period is also obtained from the primal solution. Any non-zero slack variables in the primal solution represent the measure of inequality (slack) in the associated constraint.

In addition to the primal solution, other sets of very useful information are routinely available from the linear programming system. This additional information is known as sensitivity analysis, the effect on the solution caused by varying model parameters. Of particular interest are the dual solution and the reduced costs.

The dual solution is alternately known as P_i values, marginal values, or shadow prices. It is the effect on the objective function value caused by changing the constant term (right-hand side) of a constraint for small changes in the constant term. In other words, the effect of tightening or loosening a constraint. Of particular interest in this model are the shadow prices on plutonium stockpile and on the separative duty constraints, when generated. The plutonium shadow price represents the value of plutonium to the system being modeled. The separative duty shadow price represents the value per unit increase if additional diffusion cascade capacity were made available. This cost reduction could then be compared with the cost of adding additional capacity in a benefit-cost analysis.

The reduced costs are alternately known as D_j 's. There is one reduced cost for each variable not in the primal solution. The reduced cost for a variable is the reduction in its cost coefficient required to make it eligible

for entry into the optimal primal solution; i.e., how much cheaper are the variables in the solution. In this model, the reduced costs on the plant variables represent how far out of concentration a particular plant is. The reduced cost is much more than just a comparison of cost coefficients, it reflects the total condition of the model at its optimum position.

Results

Results discussed here are based on the energy demand of eight trillion kW-hr in the year 2000, the first year that the entire central-plant generating system was represented by the simulation. The capital costs for coal-fired plants were \$50/kW higher than for light water reactors. Fossil fuel costs were specified by region, which has a range of 25 to 82¢/MBtu. The costs of fuel cycle services were determined for each fuel type, as an example of the trends that were developed for this system, fabrication costs for light water reactors in 1970 were calculated at \$80/kg, declining to \$36/kg by the year 2000. Reactor introduction constraints did not apply because the light water reactor was available in 1970. The final input was the cost of nuclear materials, namely, U_3O_8 and thorium. Thorium was held at a fixed price of \$5/lb of thorium oxide, whereas the U_3O_8 price was varied as a function of the amount consumed, with 416,000 tons available at \$8/lb or less and about 1,000,000 tons available at \$15/lb or less. The computer simulation was also used where the U_3O_8 cost simply increased with time.

As can be seen in Figure 4, the light water reactor supplies a large portion of the energy during the period prior to year 2000; but after that, due to increasing cost of uranium, the light water reactor is driven out and is reduced to operation only in the high-cost fossil areas and in plutonium recycle applications.

Figure 5 illustrates how supplying the nuclear capacity shown in Figure 4 affects the major fuel-cycle parameters. The U_3O_8 price rises from \$7/lb in 1970 to \$40/lb by 2020, which forces the light water reactor out of the optimal selection. Separative work requirements rise continually to a peak of 26,000 tons per year and then falls off as the number of light water reactors diminishes. The value of plutonium starts off at approximately \$8/g fissile and rises as the U_3O_8 price rises; however, because a large fraction of the cost of the alternative uranium fuel is the result of

Figure 4

TOTAL ELECTRICAL GENERATING CAPACITY WITH LWR'S AND FOSSIL POWER PLANTS

Figure 5

MAJOR FUEL CYCLE PARAMETERS WITH LWR'S AND FOSSIL POWER PLANTS

separative work cost, the plutonium value does not go up as rapidly as the uranium cost. The value of plutonium in this system is determined by what is called in linear programming jargon the "shadow price," which is the value that a gram of plutonium has at reducing the total cost. This method is completely compatible with the indifference pricing philosophies used in the past for determining plutonium value. The simulation can also include the possibility of plutonium imports and exports at any price as a function of time, which places a maximum or minimum price for fissile material at a given time.

Figure 6 considers the situation where an LMFBFR is made available in 1984, selling for \$160/kW more than coal-fired plants. Note that the LMFBFR has now taken over a major share of the future generation capacity, with the light water reactor participation being increased substantially in the later years owing both to the reduced demand placed on uranium reserves, which keeps the price down, and to the increase in plutonium value caused by the existence of high performance LMFBFR's. Figure 7 shows the effect of added LMFBFR's on U_3O_8 price. Instead of rising to \$40/lb as in the case of light water reactors only, the uranium price reaches a maximum of \$14/lb, whereas the separative work requirements goes to a peak of nearly the same magnitude, as in the light-water-only case but somewhat earlier in time. The plutonium value begins at about \$9/g and rises to \$18/g during peak demand period at which time the rate of production of plutonium by fast breeders begins to exceed the demand. The value then drops to about \$4/g by 2020 when the excess plutonium is used as light-water reactor fuel.

Application to Other Sectors of the Energy Consumption

The modeling exercise for the AEC demonstrates an approach to modeling the decision process. In Figure 8, a comparison of the principal characteristics describing the decision process for electrical generation and three other sectors of energy consumption, an example dealing with the purchase of a device which consumes the energy is shown in each case. The second column deals with the relationship between first cost or capital cost and the annual operating cost. Based on publications by the DOT and HUD, it

Figure 6

TOTAL ELECTRICAL GENERATING CAPACITY WITH LMFBR'S ADDED

Figure 7

MAJOR FUEL CYCLE PARAMETERS
WITH LMFBR'S ADDED

Figure 8

COMPARISON OF ELECTRICAL GENERATION WITH OTHER
ENERGY USE SECTORS

	<u>Example</u>	<u>Capital Cost/Annual Operating Cost</u>	<u>Load Factor</u>	<u>Life Time of Using System</u>	<u>Ability To Change Fuel Use</u>
Electrical Generation	Fossil Plant	7	0.50	30	10%
Transportation	Auto	8	0.10	10	1%
Residential Commercial	Space Heat	6	0.15	15	1%
Industry	Electric Motor	3	0.20	15	0

appears that the capital-to-operating ratio is very similar to that for a fossil power plant for space heating and autos. The load factor on an automobile, however, is quite low as compared with a power plant. However, transportation devices other than the automobile find load factors which are equal to those for electric generation. The lifetime of the various systems vary from 10 years for the automobile to 30 years for a power plant.

The final characteristic that applies to nearly all of these is a limited ability to change fuels. Thus, once a decision to buy a particular system is made, it is likely to be used throughout its useful life without fuel changes. However, with rapidly changing fuel prices, the incremental cost of changing fuels may become a good investment. Although each of these parameters differ from the utility example, the changes are small enough that the technique used for the utility system should apply.

Applying the model just described over these additional energy use sectors, some critical assumptions are being made. One is that the choice between alternates is made on a purely utilitarian basis with the objective to minimize cost. This assumption might work well for space heat but represents greater hazards in the case of the automobile. Another assumption that must be made is that the cost of money to the buyer group can be specified in the form of a present-worth rate. Because the capital cost variation in space heat is fairly small, we expect fuel prices to show great influence; whereas the automobile has few choices as to fuel. However, the consumption rate can be altered by going to more light-weight vehicles which generally have a lowered capital cost. Battelle is presently in the process of establishing the discount rates which best fit historical patterns for space heat and automobiles.

References

1. USAEC Division of Reactor Development and Technology. Cost-Benefit Analysis of the U. S. Breeder Reactor Program. WASH-1126 (April 1969).
2. USAEC Division of Reactor Development and Technology. Cost-Benefit Analysis of the U. S. Breeder Reactor Program (Updated 1970). WASH-1184 (January 1972).
3. Bonner and Moore Associates, Inc. 1107 LP-2 Users Manual (1965).
4. Deonigi, D. E., "A Simulation of the United States Power Economy". Proceedings of the American Power Conference, Vol. 32 (1970).
5. USAEC Division of Reactor Development and Technology. Potential Nuclear Power Growth Patterns. WASH 1098 (in publication).
6. USAEC Division of Reactor Development and Technology. Reactor Fuel Cycle Costs for Nuclear Power Evaluations. WASH 1099 (in publication).
7. Bloomster, C. H., Nail, J. H., and Haffner, D. R., PACTOLUS: A Code for Computing Nuclear Power Costs. BNWL-1169, Pacific Northwest Laboratory, Richland, Washington (January 1970).