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ELASTIC WAVE SCATTERING METHODS: ASSESSMENTS AND SUGGESTIONS

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INTRODUCTION

I was asked by the meeting organizers to review and assess the developments over the past ten or so years in elastic wave scattering methods and to suggest areas of future research opportunities. I will highlight the developments, focusing on what I feel were distinct steps forward in our theoretical understanding of how elastic waves interact with flaws. For references and illustrative figures, I decided to use as my principal source the proceedings of the various annual Reviews of Progress in Quantitative Nondestructive Evaluation (NDE). These meetings have been the main forum not only for presenting results of theoretical research but also for demonstrating the relevance of the theoretical research for the design and interpretation of experiment. In my opinion a quantitative NDE is possible only if this relevance exists, and my major objective is to discuss and illustrate the degree to which relevance has developed. I apologize if any one feels slighted by my not mentioning a particular work. To keep the size of "review" manageable, I had to be brief and to the point.

BEGINNINGS

To start I would like to convey my recollections of where the state of elastic wave scattering theory stood about eleven years ago by recounting my introduction to the field. At that time I was a postdoc of Jim Krumhansl at Cornell. Jim came to me one day and told me about an interdisciplinary research program in which he was going to participate. His participation was motivated in part by a concern for strengthening the relationship between university research capabilities and industrial technological needs. Shortly afterwards he asked me to represent him at a "kick-off" meeting for the program because he had a scheduling conflict.

The meeting was the immediate predecessor to the annual Reviews of Progress in Quantitative Nondestructive Evaluation. Emphasized at the meeting was the need to convert something called Nondestructive Testing into something called Nondestructive Evaluation. The change in phrasing was to indicate a technology more firmly based in scientific understanding and quantitative interpretation of measurement.

In terms of how ultrasonic NDE was being done an incredible variety of calibration standards, like those in Fig. 1, were shown at the meeting. The problem was interpreting the measurements not only from these standards but more importantly from real flaws in fabricated parts, which these standards were suppose to represent. Emphasis was on identifying features in the frequency spectrum of measured signals that were signatures, or indicia, of the flaw. There was belief that more information simply meant going to higher frequencies. Scalar wave theories were being used to describe a vector wave phenomena.

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Fig. 1 Examples of calibration standards [1].

I learned that as part of the program I was expected to calculate the scattering from a "fuzzy tennis ball". What had been fabricated was a hemispherical flaw of known size in a test specimen (Fig. 2). Since the surface was not smooth and since a spherical flaw was in the making, the "fuzzy" ball was to become the first challenge to theory and its relation to experiment. Using controlled flaws in experiments instead of real ones was one of the steps being developed to establish the scientific base for NDE. However there was skepticism raised not only about the relevance of such a flaw to "real world" problems but also about the likelihood of theories based on smooth surfaces and homogeneous materials being successful.

Fig. 2 Hemispherical flaw test specimen [2].

The only relevant elastic wave scattering theory at that time was the exact solution for the scattering of a plane wave from a sphere. Since Dick Cohen was already studying it, I returned to Cornell and with Jim Krumhansl and Eytan Domany started to develop a statement and a general approach to scattering theory that could be applied to nonspherical shapes and from it we derived and studied the Born Approximation [3] as a first step.

With the exact scattering results and the Born Approximation several developments rapidly occurred: The inadequacy of scalar theories and the importance of mode conversion were clearly demonstrated; the angular dependence of the scattering was established as a partner to frequency dependence for information content; and successful comparisons between theory and experiments on the spherical shape occurred. In Fig. 3 is a representation of the partnership of angular dependence and "indicia". The fact that even a sphere had significant angular variation in its direct and mode converted scattering sounds trivial, but for many seeing was believing. The degree to which theory and experiment on the spherical flaws could agree is illustrated in Fig. 4. The "fuzz" was unimportant.

The significance of these developments is manifold: Theory became accepted as an important aid to the design and interpretation of experiment. Modeling became an accepted practice in the way experiment was done. No longer were only pulse-echo experiments done, as great lengths were taken to measure angular dependence (Fig. 5). In short, the way in which ultrasonic scattering experiments were done radically changed. This change was very important for the calibration of the theoretical methods that were forthcoming.

FREQUENCY OR ANGULAR DEPENDENCE

WAVELENGTH ~ DIMENSIONS ,

FLAT OBJECT WITH SCATTERING DOMINATED BY EDGES

INDICIA

EASILY SET-UP WITH PRESENT INDUSTRIAL APPARATUS

TRANSLATE

OTHERS - TO EVOLVE FROM ANALYSIS AND EXPERIMENTAL EXPERIENCE

Fig. 3 An early view of scattering analysis [4].

THE MAJOR DEVELOPMENT PHASE

Shortly after these beginnings, several different scattering methods became available and comparisons between theory and experiment using fabricated flaws became standard practice. More importantly the physics of the problem began to emerge. Instead of interpreting experiment as a function of frequency, one began to quote " ka " values, where k is the wavenumber of the probing ultrasound and a is a length characteristic of the flaw's "length". The theories developed were found to have certain ka ranges over which they could wisely be applied.

When $ka < 1$ perturbation theory provided several useful approximations. The most significant one is something I stumbled upon. I called it the quasi-static approximation (Rayleigh scattering) [7], and its importance lies in the subsequent work it inspired. This approximation becomes an exact theory when $ka \ll 1$. In practice it works well up to ka around 0.5. Very small ka values correspond to the ultrasound wavelength becoming much larger than the flaw size. The flaw has shrunk to a point. What was surprising about the elastic wave problem was this "point" did not scatter isotropically: At low frequencies there was information about the flaw in the angular variation of the scattering. Although difficult to measure because the scattering is weak, Elsely [8] with the theoretical support of Richardson measured the scattering and by fitting the data to the analytic expressions was able to extract the proper size and orientation parameters (Table I) of an oblate spheroid, that is to say, an inverse scattering problem was done. Also in a subsequent effort Budiansky and Rice [9] showed the scattering from elliptical cracks should give a measure of stress intensity factors. Their predictions were verified by measurements of Kino et al. [10].

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Fig. 4 Comparison between theory and experiment for the scattering from a sphere [5].

Fig. 5 A Goniometer to measure the angular dependence of the scattering [6].

Table I. Estimated parameters from oblate spheroidal void [8].

Quantity	True Value	Estimate	Standard Deviation Estimate
Size:			
a,b	0.04 CM	0.03947	0.0168
c	0.02 CM	0.01999	0.0608
Orientation			
γ_X	0	-1.24×10^{-5}	0.1120
Stress Intensity:			
k_1	0.364	0.362	0.008

In the intermediate region where $1 < ka < 10$ eigenfunction expansions methods are the dominant theories. Here the physical fields are expanded in terms of vector partial waves. The approximation is made good by increasing the number of terms in the expansion. Large ka requires a large number of terms. This number and the increasing difficulty in maintaining computational precision determines the upper limit of applicability. Of course, they also work well at $ka < 1$.

Two such methods are used. One, generally called the T-matrix method, was an extension of the ideas of Waterman to the elastic wave scattering problem made by Pao and Varadan [11]. The other, generally called the Optimal Truncation Method, was a rediscovery by Visscher [12] of the boundary residual method found in electrodynamic theory. For axially-symmetric shapes these methods had only modest computer requirements, and the Varadans, Visscher, and Opsal (using Visscher's approach) were able to compute the scattering from a variety of complicated shapes and their predictions generally compared satisfactorily with experiment (Figs. 6 and 7). Considerable information on how nonspherical features affect the scattering was obtained.

For $ka > 10$ Achenbach and coworkers [15] extended the geometrical diffraction theory ideas of Keller to elastic wave scattering from cracks. This theory constructs a geometrical interpretation of the scattering process that greatly aids interpretation. As the quasi-static approximation, it is an asymptotic theory. In this case it becomes exact as ka approaches infinity. Also as the quasi-static approximation, its range of validity generally extends beyond the asymptotic region. It also agrees well with experiment for simple shapes, and such a comparison is shown in Fig. 8. At first sight the comparison seems poor; however, it is actually quite good! On close observation one notes that along the frequency axis the positions of the peaks and valleys agree. Since the theory predicts a relation between these positions and the lengths of the major and minor axes of the ellipse, an accurate estimation of the lengths was determined, as seen from Table II. This determination was another example of how scattering could be used to study the inverse problem. Confidence in theory was so high that the experiment was reexamined to understand why the agreement in magnitude was not better. An effect was found missing in the interpretation of the experimental data that when included made the agreement more precise.

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Fig. 6 Theoretical and experimental scattering amplitudes from oblate voids [13].

OTHER DEVELOPMENTS

The next things I will discuss are results that plugged most of the gaps in our ability to compute the scattering of elastic waves from simple shapes. For flaws occupying a finite volume, the eigenfunction expansion methods covered ka from zero up to the asymptotic region, but there was no asymptotic theory. For cracks geometrical diffraction theory covered ka from infinity down to the mid-intermediate region, then one had the quasi-static approximation in the small ka region. A gap existed.

Visscher [17] was able to extend his method to describe the scattering from a circular crack. For this shape the scattering could now be described over the entire ka range by two quite different, but overlapping, theories that agreed in the overlap region.

For the axially symmetric flaws and circular cracks Poe and Opsal [18] developed a method that took the scattering from a conveniently computed

Fig. 7 Theoretical and experimental scattering amplitudes from several complex shapes [14].

Table II. Result of Size Determination from the Spectral Components of the Diffracted Wave [16].

Scattering θ_L' Angle	$\Delta f_{\max}$	Measured Radius of the Crack in μ
35	2.18	2530
40	1.87	2630
45	1.83	2450
50	1.68	2460
55	1.60	1410
60	1.47	2500
65	1.39	2510

ka range and extrapolated it to a range extremely difficult to compute. This is not an asymptotic theory but did broaden the range over which existing methods were applicable.

For the axially symmetric flows the asymptotic theory, the geometrical diffraction theory, was attempted by Brind, Gubernatis, and Achenbach [19]. They found the ka where it work well did not extend down into the intermediate ka region. I am unaware of any other application of geometrical diffraction theory where the range of applicability is so restricted. I

Fig. 8 Theoretical and experimental scattering amplitudes from an elliptical crack [16].

mention this because while it is true that most of the methods applied to the elastic wave problems are extensions of methods applied to scattering problems in other areas of science the extensions were in most cases non-trivial and in more than one case that something worked well elsewhere did not guarantee success here. The vector nature of the fields, the tensor nature of the scattering "potential", and mode conversion make the elastic wave scattering problem more challenging than others.

Since experiment so far can only probe up to the asymptotic region, even for axially symmetric flaws one can say that elastic waves scattering methods can give a useful description of the scattering for certain flaw shapes. In many cases the understanding developed with these methods lead to modest successes with the inverse problem.

RECENT DEVELOPMENTS

Theories are being developed for more "realistic" flaw types. Flaws on or near a surface have received considerable attention. The principal computational methods appear to be those of Visscher [20] and of Achenbach and coworkers [21]. Visscher has developed a new type of localized function expansion method for the problem that permits the full three-dimensional problem to be addressed. Achenbach and coworkers have continued their development of diffraction theory. As was the case for the other flaw problems, this type of theory, roughly speaking, builds a three-dimensional theory from lower dimensional results. The important point is the two approaches are consistent (Fig. 9): There are no gaps. Comparisons with experiment apparent, are still being developed. One such comparison is shown in [22].

Other work is under way on bifurcated crack tips, partially closed cracks, rough cracks, etc. Again Achenbach and coworkers [23] are at the leading edge.

Fig. 9 Comparison of forward scattering Rayleigh amplitude for a surface breaking crack as computer by Visscher (solid line) and Achenbach (dashed line) [20].

ASSESSMENTS

The ability to calculate increasingly more complex and more "realistic" flaws grew rapidly over a relatively short period of time. For a broad and relevant class of flaws, the frequency and angular dependence of the scattering can be accurately calculated. I feel, however, the growth is slowing: The remaining interesting problems are more difficult than those tackled to date; the relationship between theory and experiment appears less synergistic; and funding levels appear reduced.

In spite of the success some very simple and important shapes are beyond the reach of present methods or else await attention. For example, the ellipsoid has only been done in the small ka limit [7]. In the intermediate ka range a major computational effort would be required to implement the eigenfunction expansion method. It is unclear whether computer memories are sufficiently large to handle the problem at the high ka side of this range. One will recall that the successes of these methods were for axially symmetric shapes. The symmetry made possible significant reductions in the amount of computer memory needed. Surprisingly, the "flat-bottom hole", an axially symmetric shape, awaits calculation. I believe it probably can be done using Visscher's methods [20]. Since this shape still provides the basis for many calibration procedures and standards, it is hard to understand the lack of a quantitative analysis for it if such procedures and standards are to represent a quantitative NDE.

RECOMMENDATIONS

Most of the current research is progressing forward and treating the next levels of complexity. As with most research, progress will be incremental. Instead of noting what is being done or will be done in the next several years, I will direct my recommendations towards what I feel needs to be done to achieve step-wise advances. One of my premises is that in a few years existing methods are likely to run their course as the problems

become even more difficult. Theory will become more approximate, and the approximations, unfortunately, more likely to become uncontrolled.

Notwithstanding the inherently difficult nature of the elastic wave scattering problem, the basic fact is that the analytic methods and computational resources used to date are in general less sophisticated and more meager than used in most areas of physics and several other areas of engineering science. The physicists have developed quite advanced approaches to perturbation theory that may well be necessary to handle more realistic problems. Although I had mixed success in trying some of them [24], it was probably because I lacked the research time to sustain the large scale computational effort required. On the other hand some of my colleagues [25] were more successful.

The telling difference I believe will come when greater computer power is applied to the problem. Ten years ago there were no CRAYs. The computationally most intensive efforts, the calculations performed by the eigenfunctions methods, can today be performed on "scientific workstations". I do not believe I am exaggerating if I say that for several hours on one of the current "supercomputers" one would be able to redo all of the calculations published over the past ten years. The exploitation of these new "theoretical tools" should be actively pursued. One should not wait until something useful develops in some other field and then try to adapt. An emphasis on developing approaches tailored from the start for the elastic wave scattering problem is needed.

A considerable body of knowledge has been amassed over the past ten years. While it is important to increase this body, it is also important to use it properly. The theoretical and experimental research on elastic wave scattering problems of and in themselves will not directly influence NDE technology. The problem ultimately more difficult than doing the research is defining how to use the results of the research to improve the technology. At this point I feel it is just as timely to emphasize exhausting the existing knowledge base as it is to emphasize incrementally adding to it.

LEGACIES AND LESSONS

The development of the scatterings method has had several other important implications for NDE and other technological areas. One is the increased experience and confidence in the design and interpretation of experiments that has occurred. Another is improved modeling of the effective elastic behavior of composite and inhomogeneous materials. It also has provided a clear statement of scattering theory and a considerable body of knowledge so more sophisticated approaches to the inverse problem are now possible. In this context the richness of the elastic wave scattering problem is now more clear.

Besides scattering theory there are several other things I have learned over the past ten years. I feel an understanding of the needs and the problems associated with university (and national laboratory) attention on problems affecting industry. More importantly I have confidence that basic research performed at a university (or a national laboratory) can construct the knowledge base to impact industrially-related problems. Many of these problems are probably like the NDE problems, and what will be important in addressing them will be an interdisciplinary approach like the one that occurred for the elastic wave problem.

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REFERENCES

1. L. R. Gulley, Jr., Interdisciplinary Workshop for Quantitative Flaw Definition, D. O. Thompson, Ed., Air Force Materials Laboratory Report AFML-TR-238 (1974), p. 5.
2. B. R. Tittmann and E. R. Cohen, Interdisciplinary Workshop for Quantitative Flaw Definition, D. O. Thompson, Ed., Air Force Materials Laboratory Report AFML-TR-238 (1974), p. 173.
3. J. E. Gubernatis, E. Domany, J. A. Krumhansl, and M. Huberman, J. Appl. Phys. 48, 2812 (1977).
4. R. B. Thompson, Proceedings of the ARPA/AFML Review of Quantitative NDE, D. O. Thompson, Ed., Air Force Materials Laboratory Report AFML-TR-75-212 (1975), p. 33.
5. B. R. Tittmann, Proceedings of the ARPA/AFML Review of Progress in Quantitative NDE, D. O. Thompson, Ed., Air Force Materials Laboratory Report, AFML-TR-77-44 (1977), p. 173.
6. B. R. Tittmann and R. K. Elsley, Proceedings of the ARPA/AFML Review of Progress in Quantitative NDE, D. O. Thompson, Ed., Air Force Materials Laboratory Report AFML-TR-78-55 (1978), p. 26.
7. J. E. Gubernatis, J. Appl. Phys. 50, 4046 (1979).
8. R. K. Elsley, Proceedings of the ARPA/AFML Review of Progress in Quantitative NDE, D. O. Thompson, Ed., Air Force Materials Laboratory Report AFML-TR-78-205 (1978), p. 400.
9. B. Budiansky and J. R. Rice, J. Appl. Mech. 45, 453 (1978).
10. J. Tien, B. Khuri-Yakub, and G. S. Kino, Proceedings of the DARPA/AFML Review of Progress in Quantitative NDE, D. O. Thompson, Ed., Air Force Wright Aeronautical Laboratories Report AFWAL-TR-80-4078 (1980), p. 671.
11. V. Varadarajulu (Varadan) and Y.-H. Pao, J. Acoust. Soc. Am. 60, 556 (1976).
12. W. M. Visscher, J. Appl. Phys. 51, 825 (1980); *ibid.* 835 (1980).
13. F. Cohen-Tenoudji, B. R. Tittmann, and L. A. Ahlberg, and G. Quentin, Review of Progress in Quantitative Nondestructive Evaluation 1, D. O. Thompson and D. E. Chimenti, Eds., (Plenum, New York, 1982), p. 173.
14. V. V. Varadan, V. K. Varadan, and D. J. N. Wall, Proceedings of the DARPA/AFML Review of Progress in Quantitative NDE, D. O. Thompson, Ed., Air Force Wright Aeronautical Laboratories Report AFWAL-TR-81-4080 (1981), p. 276.
15. A. K. Gautesen, J. D. Achenbach, and H. McMaken, J. Acoust. Soc. Am. 63, 1824 (1978).
16. L. Adler and D. K. Lewis, Proceedings of the DARPA/AFML Review of Progress in Quantitative NDE, D. O. Thompson, Ed., Air Force Wright Aeronautical Laboratories Report AFMLA-TR-80-4078 (1980), p. 377.
17. W. M. Visscher, Wave Motion 3, 49 (1981).
18. G. D. Poe, "Frequency Response Extrapolation in the Theory of Scattering," Ph.D. Thesis, Univ. of Calif., Davis.
19. R. J. Brind, J. D. Achenbach, and J. E. Gubernatis, Wave Motion 6, 41 (1984).
20. W. M. Visscher, J. Appl. Phys. 56, 713 (1984); *ibid.* 57, 1538 (1985).
21. D. A. Mendelson, J. D. Achenbach, L. M. Keer, Wave Motion 2, 227 (1980).
22. Vu and Kindra, preprint.

23. J. D. Achenbach, K.-Y. Hu, A. M. Norris, T. A. Gray, and R. B. Thompson, Review of Progress in Quantitative Nondestructive Evaluation 4, D. O. Thompson, D. E. Chimenti, Eds., (Plenum, New York, 1985), p. 91.
24. J. E. Gubernatis and G. A. Baker, Jr., Review of Progress in Quantitative Nondestructive Evaluation, D. O. Thompson, D. E. Chimenti, Eds., (Plenum, New York, 1982), p. 111.
25. K. E. Newnan, S. Teitel, and E. Domany, J. Appl. Phys. 52, 4363 (1981).