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THE USE OF THE CONDUCTING MASK IN PULSED ELECTROMAGNETIC TESTING

by

C. W. Cox and C. J. Renken

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Metallurgy Division
Program 12.1.13

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THE USE OF THE CONDUCTING MASK IN PULSED ELECTROMAGNETIC TESTING

by

C. W. Cox and C. J. Renken

ABSTRACT

The application of conducting masks to the problem of nondestructive testing by means of pulsed electromagnetic fields is examined. The diffusion of a pulsed plane wave into a semi-infinite conductor is calculated, and formulas are derived for pulse attenuation and delay, as well as for field strength inside and outside the conductor. The plane-wave assumptions cause serious discrepancies between calculated and experimentally measured fields through conducting sheets. A qualitative analysis of the symmetrical mask offers a satisfactory explanation for most of the experimentally observed features of its operation, and provides an insight into the operation of other types of masks.

I. INTRODUCTION

Although electromagnetic test methods have been in use for over 30 years, their applicability as a reliable method of quality control is widely distrusted. In some critical applications, electromagnetic methods have been discarded in favor of ultrasonic methods.¹

One reason that electromagnetic (or eddy-current) methods are still used is the economic advantages of electromagnetic inspection over other methods, particularly in the inspection of relatively low-priced items produced at a high rate.² Lately, however, a new and important reason has developed for reemphasizing electromagnetic testing: the increasing importance of very thin layers of nonferrous metals in the form of sheets, tubing, or cladding.³

The important advances in electromagnetic methods over the years, most notably, phase discrimination, filtering, and saturation,⁴ have so improved the effectiveness of electromagnetic testing that their proper application by qualified personnel can provide a peerless inspection method in many applications. Uncertainties concerning the reliability of the electromagnetic methods of inspection, at least in the USA, are due mostly to the misapplication and the improper operation of basically well-designed commercial equipment.

II. BASIS AND CHARACTERISTICS OF THE METHOD

Since the basic principles of electromagnetic nondestructive testing are well known, they will not be treated in detail, except to introduce some terms and concepts that may make this report simpler to follow.

If any useful information is to result from an electromagnetic test, the condition to be detected must somehow correlate with the electrical resistivity or permeability, or both, of the test object. The influence of these quantities on the time-varying current field flowing in the test object provides the basis of the method. Since the time variation of this field has been sinusoidal in nature, it has been convenient to speak of the difference in phase between the current flowing at the surface of the test object and the current flowing within the object. If the current field in the metal is established by a current-carrying coil outside the metal, the current in the metal will affect the current in the coil. The normalized plots⁵ of such impedance changes under various spacings between coil and metal and various conditions in the metal are known as "impedance plane plots,"⁵ which are familiar to everyone acquainted with electromagnetic testing. If however, the amplitude of the current field is not a time-varying sinusoid, the impedance-plane concept loses its usefulness. If a current field is established on the metal surface by a short pulse of electromagnetic energy occurring in the space outside, the current field undergoes an apparent "delay" as it travels deeper and deeper into the metal.⁶ The diffusion of the current into the metal is analyzed in the next section.

The test method described in this report differs from the conventional eddy-current test in several important respects. One difference, of course, is in the time variation of the field amplitude, which is pulsed instead of sinusoidal. Another is that there is no dependence upon changes in test-coil impedance to reveal conditions in the metal. And, third, there has been a conscious attempt to restrict the size of the current system flowing in the metal to as small a volume as possible, not by restricting the size of a test coil (which is, however, a useful approach), but by blocking off a large portion of the exciting field. Thus, the test object is exposed to a field with a small cross-sectional area. This technique is possible with the pulsed method because of the very high peak powers compared with a continuous-field method; even though a very high percentage of the total field power is wasted, an effective test is still possible.

Reducing the field size has long been the obvious way to increase test resolution and discrimination in both electromagnetic and ultrasonic testing. The reduction of beam size in recent years has sharply increased the utility and versatility of the ultrasonic method as applied to tube testing. If an ultrasonic test system were forced to operate with an ultrasonic field volume in the metal the size of the current field produced by most electromagnetic

test equipment, the same complaints about lack of discrimination, spurious signals, insensitivity, etc., would probably be registered as are now voiced about many of the present-day electromagnetic test systems. Unfortunately, because of a vast difference in wavelength, electromagnetic energy cannot be focused into narrow beams, as has been possible with ultrasonic energy, and most electromagnetic systems have operated with current-field volumes in the tube thousands of times as great as the volumes of the abnormalities that were to be detected. Electromagnetic systems have been too sensitive to random metallurgical and geometrical changes that occupy volumes much greater than many defects. Phase discrimination and filtering have not always completely remedied this problem.

The test system described in this report attempts to reduce the volume of the current fields in the test specimen by the use of devices called mask-aperture assemblies. In the assembly shown in Fig. 1, only the narrow field passing through the annular slit below the field coil impinges upon the tube. The system of currents established in the tube is much smaller in volume than with the usual type of eddy-current encircling coil. A pickup coil or coils detect the field established outside the surface of the metal as a result of the current flowing in the metal. There is relatively little coupling of the pickup coil with the primary field set up around the field coil. As a result, changes in the signal from the pickup coil can be detected without a bridge or any bucking arrangement to remove the massive effects of the bulk of the exciting coil field. The influence of the tube on the impedance of the field and pickup coils is not significant in the operation of these devices, since masking of the field coil from the tube inhibits inductive coupling between the two. In addition, amplifier and impedance changes in the pickups are insignificant.

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Fig. 1. Symmetrical Mask-aperture Assembly

III. THEORETICAL CONSIDERATIONS

The analysis of eddy-current methods, in which the driving currents are pulses rather than steady-state sinusoids, is extremely complicated. Even for the simple test geometries, one can rarely expect closed solutions, but must be prepared to deal with series solutions. Often two separate series solutions are required, one valid for small values of time, the other for large values of time. The problem of analysis of the mask-aperture assembly is compounded by the fact that the geometry is far from simple. Any success in application of the mask-aperture probe is due to its extreme departure from the classic boundary conditions.

The computation time for the solution of the relatively simple problem (for example, a constant magnetic-field intensity suddenly applied at $t = 0$ to the outside of a tube of infinite length⁷) can easily exceed the man-hours needed to empirically design and construct a working mask-aperture probe. A precise quantitative design formula is simply not realistic. However, analyses of a few simple models will help describe the manner in which the method overcomes the difficulties of the conventional eddy-current test, and establish rules of thumb governing the important parameters of the method.

A. Plane-wave Analysis

In any linear, homogeneous, isotropic medium, the Maxwell equations hold. In an uncharged medium of permeability μ , permittivity ϵ , and conductivity σ , the equations may take the form

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}; \quad (1)$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}; \quad (2)$$

$$\nabla \cdot \vec{E} = 0; \quad (3)$$

and

$$\nabla \cdot \vec{H} = 0; \quad (4)$$

where $\vec{E}$ is the electric field intensity and $\vec{H}$ is the magnetic field intensity.

As a first approach to the problem of pulsed eddy currents, consider the simplified model consisting of an infinite sheet of conductor, of thickness w , excited by a current sheet carrying $i(t)$ amperes per meter of distance normal to the current flow, as shown in Fig. 2. The electric and magnetic field-intensity vectors everywhere are, in Gibbs' notation,

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Fig. 2. Conducting Slab Parallel to an Exciting-current Sheet

$$\vec{E} = \vec{i}0 + \vec{j}E(x,t) + \vec{k}0 \quad (5)$$

and

$$\vec{H} = \vec{i}0 + \vec{j}0 + \vec{k}H(x,t). \quad (6)$$

With the fields described in this manner, Eqs. 1-6 may be manipulated to yield

$$\frac{\partial E(x,t)}{\partial x} = -\mu \frac{\partial H(x,t)}{\partial t}; \quad (7)$$

$$\frac{\partial H(x,t)}{\partial x} = -\sigma E(x,t) - \epsilon \frac{\partial E(x,t)}{\partial t}; \quad (8)$$

$$\frac{\partial^2 E(x,t)}{\partial x^2} = \mu\sigma \frac{\partial E(x,t)}{\partial t} + \mu\epsilon \frac{\partial^2 E(x,t)}{\partial t^2}; \quad (9)$$

and

$$\frac{\partial^2 H(x,t)}{\partial x^2} = \mu\sigma \frac{\partial H(x,t)}{\partial t} + \mu\epsilon \frac{\partial^2 H(x,t)}{\partial t^2}. \quad (10)$$

If an initially unexcited system is assumed, Eqs. 7-10 possess the following Laplace transforms with respect to t :

$$\frac{\partial \bar{E}(x,s)}{\partial x} = -\mu s \bar{H}(x,s), \quad (11)$$

$$\frac{\partial \bar{H}(x,s)}{\partial x} = -(\sigma + \epsilon s) \bar{E}(x,s), \quad (12)$$

$$\frac{\partial^2 \bar{E}(x,s)}{\partial x^2} = \mu s(\sigma + \epsilon s) \bar{E}(x,s), \quad (13)$$

and

$$\frac{\partial^2 \bar{H}(x,s)}{\partial x^2} = \mu s(\sigma + \epsilon s) \bar{H}(x,s). \quad (14)$$

This study will be confined to nonmagnetic conductors and air dielectrics, so that no magnetization or polarization need be considered. Therefore, if μ_0 and ϵ_0 are the permeability and permittivity, respectively, of free space, then

$$\left. \begin{aligned}
 \sigma &= 0, & -\infty < x < 0, & \quad w < x < \infty, \\
 \sigma &\neq 0, & 0 < x < w, \\
 \epsilon &= \epsilon_0, & -\infty < x < \infty, \\
 \mu &= \mu_0, & -\infty < x < \infty.
 \end{aligned} \right\} \quad (15)$$

and

Also, in good conductors, displacement currents may be neglected, and Eqs. 13 and 14 reduce to

$$\frac{\partial^2 \bar{E}(x, s)}{\partial x^2} = \gamma^2(s) \bar{E}(x, s) \quad (16)$$

and

$$\frac{\partial^2 \bar{H}(x, s)}{\partial x^2} = \gamma^2(s) \bar{H}(x, s). \quad (17)$$

The space-independent parameter in these equations is the intrinsic propagation constant

$$\gamma(s) = \begin{cases} \gamma_0(s) = \sqrt{\mu_0 \epsilon_0} s, & -\infty < x < 0, \quad w < x < \infty, \\ \gamma_1(s) = \sqrt{\mu_0 \sigma s}, & 0 < x < w, \end{cases} \quad (18)$$

in nepers per meter (mks units).

The solution of Eq. 16 is of the form

$$\begin{aligned}
 \bar{E}(x, s) &= \bar{E}_f(0, s) \exp[-\gamma(s)x] + \bar{E}_b(0, s) \exp[\gamma(s)x] \\
 &= \bar{E}_f(x, s) + \bar{E}_b(x, s).
 \end{aligned} \quad (19)$$

Substitution of Eq. 19 into Eq. 11 results in

$$\begin{aligned}
 \bar{H}(x, s) &= \frac{1}{Z_0(s)} \bar{E}_f(0, s) \exp[-\gamma(s)x] - \bar{E}_b(0, s) \exp[\gamma(s)x] \\
 &= \bar{H}_f(0, s) \exp[-\gamma(s)x] + \bar{H}_b(0, s) \exp[\gamma(s)x],
 \end{aligned} \quad (20)$$

where

$$Z_0(s) = \begin{cases} \eta = \frac{\mu_0 s}{\gamma_0(s)} = \sqrt{\frac{\mu_0}{\epsilon_0}}, & -\infty < x < 0, \quad w < x < \infty, \\ Z_{01}(s) = \frac{\mu_0 s}{\gamma_1(s)} = \sqrt{\frac{\mu_0 s}{\sigma}}, & 0 < x < w. \end{cases} \quad (21)$$

Clearly,

$$\frac{\bar{E}_f(x, s)}{\bar{H}_f(x, s)} = - \frac{\bar{E}_b(x, s)}{\bar{H}_b(x, s)} = Z_0(s), \quad (22)$$

where $Z_0(s)$ is the intrinsic impedance of the medium in ohms. In free space, the subscripts f and b identify components that are in reality forward and backward traveling waves. When the conductivity is nonzero and displacement currents are negligible, subscripts f and b identify components that are analogous to the traveling waves in a lossless medium insofar as the mathematical description in the s-domain is concerned; but the process is actually one of diffusion, as will be seen.

In the list of the boundary conditions, the suffixed plus sign (+) on the boundary locator indicates a point just to the right of the boundary, and the suffixed minus sign (-) indicates a point just to the left of the boundary. The boundary conditions are as follows:

(A) From the boundedness requirement for physical systems,

$$\bar{E}(\pm\infty, s) \text{ and } \bar{H}(\pm\infty, s) \text{ are finite.} \quad (23)$$

(B) Applying the circuital laws at $x = -d$ results in

$$\bar{H}(-d^+, s) - \bar{H}(-d^-, s) = I(s) \quad (24)$$

and

$$\bar{E}(-d^+, s) - \bar{E}(-d^-, s) = 0. \quad (25)$$

(C) Applying the circuital laws at $x = 0$ results in

$$\bar{H}(0^+, s) - \bar{H}(0^-, s) = 0 \quad (26)$$

and

$$\bar{E}(0^+, s) - \bar{E}(0^-, s) = 0. \quad (27)$$

(D) Applying the circuital laws at $x = w$ yields

$$\bar{H}(w^+, s) = \bar{H}(w^-, s) \quad (28)$$

and

$$\bar{E}(w^+, s) = \bar{E}(w^-, s). \quad (29)$$

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Fig. 3. Transmission-line Analog to the Geometry of Fig. 2

These boundary conditions will also fit the transmission-line model of the system in Fig. 3, where magnetic and electric field intensities are represented, respectively, by currents and voltages.

From boundary condition A and Eqs. 19 and 20,

$$\bar{E}_f(-d^-, s) = \bar{E}_b(w^+, s) = \bar{H}_f(-d^-, s) = \bar{H}_b(w^+, s) = 0,$$

and

$$\bar{E}(x, s) = \eta \bar{H}(x, s) = \bar{E}_b(-d^-, s) \exp[-\gamma_0(s)(x + d)], \quad x < -d; \quad (30)$$

$$\bar{E}(x, s) = \eta \bar{H}(x, s) = \bar{E}_f(w^+, s) \exp[-\gamma_0(s)(x - w)], \quad x > w. \quad (31)$$

Boundary condition D and Eqs. 19-22 yield

$$\frac{\bar{E}_b(w^-, s)}{\bar{E}_f(w^-, s)} = - \frac{\bar{H}_b(w^-, s)}{\bar{H}_f(w^-, s)} = \frac{\eta - Z_{01}(s)}{\eta + Z_{01}(s)} = \Gamma(s). \quad (32)$$

This quantity is called the metal-to-air reflection coefficients.

Similarly, boundary condition C produces

$$\frac{\bar{E}_b(0^-, s)}{\bar{E}_f(0^-, s)} = - \frac{\bar{H}_b(0^-, s)}{\bar{H}_f(0^-, s)} = \frac{Z(0, s) - \eta}{Z(0, s) + \eta}, \quad (33)$$

where

$$Z(0, s) = \frac{\bar{E}(0, s)}{\bar{H}(0, s)}. \quad (34)$$

It can be shown by manipulating Eqs. 19-22 that

$$Z(0, s) = Z_{01}(s) \frac{1 + \Gamma(s) \exp[-2\gamma_1(s)w]}{1 - \Gamma(s) \exp[-2\gamma_1(s)w]}. \quad (35)$$

By Eqs. 20 and 32,

$$\bar{H}_f(w, s) = \frac{\bar{H}(0, s) \exp[-\gamma_1(s)w]}{1 - \Gamma(s) \exp[-2\gamma_1(s)w]}. \quad (36)$$

Boundary conditions A and B with Eq. 22 show that

$$\bar{H}_f(-d^+, s) = \frac{I(s)}{2}, \quad (37)$$

and Eq. 37, along with Eqs. 22 and 33, yields

$$\bar{H}(0, s) = \frac{\eta}{\eta + Z(0, s)} I(s) \exp[-\gamma_0(s)d]. \quad (38)$$

Equations 22, 32, 36, and 38 now yield

$$\bar{H}(x, s) = \frac{\eta I(s)}{\eta + Z_{01}(s)} \exp[-\gamma_0(s)d - \gamma_1(s)x] \frac{1 - \Gamma(s) \exp[-2\gamma_1(s)(w - x)]}{1 - \Gamma^2(s) \exp[-2\gamma_1(s)w]}, \quad (39)$$

$0 < x < w.$

Equation 39 is the Laplace transform of the magnetic field intensity inside the metal slab. Unfortunately, inversion of Eq. 39 to give the complete time variation of the field inside the metal is at best extremely complex and would yield very complicated series expressions of little practical value. However, the bandwidth limitations of the measuring equipment indicate approximations that show simpler but still fairly precise descriptions of behavior.

B. Effects of Bandwidth Limitations

To provide a convenient but sufficiently high value for the upper limit of useful frequencies, assume that the system bandwidth is less than 159 Mc. Also assume that the metal slab consists of Type 304 stainless steel, a typical material for tubing tested by these methods, so that

$$\sigma = 0.139 \times 10^7 \text{ mhos/m.}$$

Then, in the metal the intrinsic impedance of Eq. 21 becomes

$$Z_{01}(s) = \sqrt{\frac{\mu_0 s}{\sigma}} = 9.51 \times 10^{-7} \sqrt{s} \text{ ohms,}$$

and the intrinsic propagation constant becomes

$$\gamma_1(s) = \sqrt{\mu_0 \sigma s} = 1.32 \sqrt{s} \text{ Np/m.}$$

The parameters of free space are

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = .377 \text{ ohms}$$

and

$$\gamma_0(s) = \sqrt{\mu_0 \epsilon_0} s = \frac{s}{c} = \frac{s}{3 \times 10^8} \text{ Np/m},$$

where c is the velocity of light (3×10^8 m/sec).

At the upper frequency limit, $s = j\omega = j2\pi f = j10^9$ rad/sec and

$$\gamma_0(j\omega) = j3.33 \text{ Np/m},$$

$$\gamma_1(j\omega) = 4.10 \exp\left(j\frac{\pi}{4}\right) \times 10^4 \text{ Np/m},$$

and

$$Z_{01}(j\omega) = 0.0305 \exp\left(j\frac{\pi}{4}\right) \text{ ohms}.$$

Clearly, within the assumed frequency range,

$$\eta \gg |Z_{01}(s)|$$

and

$$|\gamma_0(s)| \ll |\gamma_1(s)|. \quad (40)$$

This leads to the following approximations:

$$\Gamma(s) \cong 1; \quad (41a)$$

$$\Gamma^2(s) \cong 1; \quad (41b)$$

$$1 - \Gamma(s) \cong 2\sqrt{\frac{\epsilon_0 s}{\sigma}}; \quad (41c)$$

$$\exp[-\gamma_0(s)d] \cong 1; \quad (41d)$$

$$\frac{\eta}{\eta + Z_{01}(s)} \cong 1. \quad (41e)$$

C. The Very Thick Slab

If we apply the approximations of Eqs. 41a-e to Eq. 39 and let $w \rightarrow \infty$, the equation for the semi-infinite conductor, or very thick slab, results in

$$\bar{H}(x,s) = I(s) \exp[-\gamma_1(s)x] = I(s) \exp(-\sqrt{\mu_0 \sigma s} x). \quad (42)$$

Let

$$H_u(x,t) = H(x,t) \Big|_{i(t)=u(t)} \quad (43)$$

where $u(t)$ is the unit step function. Then

$$\bar{H}_u(x,s) = \frac{1}{s} \exp(-\sqrt{\mu_0 \sigma s} x), \quad (44a)$$

which has the inverse transform

$$H_u(x,t) = \operatorname{erfc}\left(\frac{1}{2}\sqrt{\mu_0 \sigma} \frac{x}{\sqrt{t}}\right) u(t). \quad (44b)$$

(The symbol "erfc" signifies the complementary error function.) If Eq. 44b is normalized with respect to time base T_0 and distance base X_0 , chosen so that

$$X_0 = \sqrt{\frac{2T_0}{\mu_0 \sigma}}, \quad (45)$$

then

$$\underline{H}_u(\underline{x}, \underline{t}) = H_u(X_0 \underline{x}, T_0 \underline{t}) = \operatorname{erfc} \frac{\underline{x}}{\sqrt{2\underline{t}}} u(\underline{t}). \quad (46)$$

D. Pulse Penetration

For simplicity, assume that the density of the current in the current sheet has the time variation

$$i(t) = I[u(t) - u(t - T_w)]. \quad (47)$$

Then, by superposition,

$$H(x,t) = H_u(x,t) - H_u(x,t - T_w). \quad (48)$$

To normalize with respect to T_w , set

$$T_0 = T_w \quad (49a)$$

and

$$X_0 = \sqrt{\frac{2T_w}{\mu_0\sigma}} = \delta = \text{depth of penetration.}^8 \quad (49b)$$

Now the normalized field intensity within the metal slab becomes

$$\begin{aligned} \underline{H}(\underline{x}, \underline{t}) &= \frac{1}{I} H(\delta \underline{x}, T_w \underline{t}) \\ &= \operatorname{erfc} \frac{\underline{x}}{\sqrt{2\underline{t}}} u(\underline{t}) - \operatorname{erfc} \frac{\underline{x}}{\sqrt{2(\underline{t}-1)}} u(\underline{t}-1). \end{aligned} \quad (50)$$

Equation 50 is plotted in Fig. 4. The "center of gravity" of the response occurs at a time that is an increasing function of distance, thus

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Fig. 4. Diffusion of a Magnetic Field into a Semi-infinite Solid due to Rectangular Current Pulse

giving the appearance of a traveling wave undergoing distortion as it travels. The analogy is rather crude because the response theoretically exists for all $x > 0$ at any $t > 0$, and any velocity defined on the basis of this concept would be a function of x , not a constant. The concept may be useful, however, with regard to the time required for the surviving portion of the main bundle of energy contained in a pulse to arrive at a particular point in the metal.⁹ The apparent "signal velocity" is quite low in comparison with that of wave propagating in air, and is lowest for the best conductors.

Figure 4 also shows that, at the depth of penetration, δ ,

the peak value of the pulse has decayed to approximately 34% of the initial value at the surface.

E. Through Transmission

At the back surface of the metal slab, Eq. 39 becomes

$$\bar{H}(w, s) = \frac{\eta I(s)}{\eta + Z_{01}(s)} \exp[-\gamma_0(s)d - \gamma_1(s)w] \frac{1 - \Gamma(s)}{1 - \Gamma^2(s) \exp[-2\gamma_1(s)w]}.$$

Applying the approximations of Eqs. 41a-d to this expression yields

$$\begin{aligned}\bar{H}(w, s) &= I(s) \frac{2\sqrt{\frac{\epsilon_0 s}{\sigma}} \exp(-\sqrt{\mu_0 \sigma s} \cdot w)}{1 - \exp(-2\sqrt{\mu_0 \sigma s} \cdot w)} \\ &= 2I(s) \sqrt{\frac{\mu_0 s}{\sigma}} \sum_{n=1}^{\infty} \exp[-(2n-1)\sqrt{w^2 \mu_0 \sigma} \sqrt{s}].\end{aligned}\quad (51)$$

As in Section D above, when $i(t) = u(t)$, Eq. 51 becomes

$$\bar{H}_u(w, s) = 2 \frac{\sqrt{\epsilon_0}}{\sigma \sqrt{s}} \sum_{n=1}^{\infty} \exp[-(2n-1)\sqrt{w^2 \mu_0 \sigma} \sqrt{s}], \quad (52)$$

which has the inverse transform

$$H_u(w, t) = 2\sqrt{\frac{\epsilon_0}{\sigma}} \frac{1}{\sqrt{\pi t}} \sum_{n=1}^{\infty} \exp\left[-(2n-1)^2 \frac{w^2 \mu_0 \sigma}{4t}\right] u(t). \quad (53a)$$

Equation 53a may be written as

$$H_u(w, t) = \frac{2}{\sigma \eta X_0} \sqrt{\frac{2}{\pi}} \frac{1}{\sqrt{t/T_0}} \sum_{n=1}^{\infty} \exp\left[-\frac{(2n-1)^2}{2} \frac{(w/X_0)^2}{t/T_0}\right] u(t/T_0), \quad (53b)$$

where T_0 and X_0 are again related as in Eq. 45.

Again, the response to the rectangular pulse $i(t) = I[u(t) - u(t - T_w)]$ will be normalized by setting $T_0 = T_w$ so that $X_0 = \delta$. Then, as in Eq. 50 for the semi-infinite solid,

$$\underline{H}(\underline{w}, \underline{t}) - \frac{1}{I} H(\delta \underline{x}, T_w \underline{t}) = \underline{H}_u(\underline{w}, \underline{t}) - \underline{H}_u(\underline{w}, \underline{t} - 1), \quad (54a)$$

where

$$\underline{H}_u(\underline{w}, \underline{t}) = \frac{1}{\Psi} \frac{1}{\sqrt{\underline{t}}} \sum_{n=1}^{\infty} \exp\left[-\frac{(2n-1)^2}{2} \frac{\underline{w}^2}{\underline{t}}\right] u(\underline{t}). \quad (54b)$$

In this expression, the relationship

$$\Psi = \sqrt{\frac{\pi}{8}} \sigma \eta \delta \quad (55)$$

is a dimensionless constant that depends upon the material of the slab and the depth of penetration for the pulse width used. Equation 54a is plotted

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Fig. 5. Pulse Transmission through a Conducting Slab

plane waves and rectangular pulses is worthy of note. The experimental curve was obtained by exciting a ferrite-cored coil and then recording the output of a similar coil on the opposite side of the metal sheet and on the axis of the exciting coil.⁹

Figures 7 and 8 show experimental pulse transmissions through stainless steel; field strength was monitored by integrating the voltage induced in a thin, straight wire positioned in the field for maximum signal strength. Although the pictures are somewhat clouded by secondary instrument effects, comparison with Fig. 5 shows clearly that the plane-wave analysis correlates better with the large-diameter probe than the analysis with the small-diameter probe. The concentrated field generated by the small-diameter probe is transmitted through a given thickness with considerably less attenuation, but with about the same time delay as the larger-diameter probe. These observations indicate that the plane wave provides a pessimistic view of what might be accomplished with pulse-transmission techniques.

in Fig. 5, where W is the thickness of the material. Time delay and signal attenuation apparently increase with distance.

In Fig. 6, the response calculated by Eq. 54a overlays an actual measured response through 5/64-in.-thick Type 304 stainless steel. This material has a depth of penetration of 0.0595 in. at a 2 μ sec pulse width. Although the field source for the experimental results was definitely non-planar, the correlation along the time axis with the results calculated on the basis of

(A) Magnetic intensity, no metal present. (Vertical scale not determined)
(B) Magnetic intensity at back surface of $\frac{5}{64}$ in. Type 304 stainless steel plate intercepting pulse.

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Fig. 6. Pulse Penetration of a Metal Plate

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Fig. 7. Pulse Transmission from a 1/4-in.-diam Ferrite-cored Solenoid through Varying Thicknesses of Type 304 Stainless Steel

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Fig. 8. Pulse Transmission from a 0.057-in.-diam Ferrite-cored Solenoid through Varying Thicknesses of Type 304 Stainless Steel

Figure 9 represents an attempt to remove the secondary effects in Fig. 7; an apparent long-time transient was subtracted to reduce each response to its single initial pulse. Normalization was based on $T_W = 0.5 \mu\text{sec}$ and $\delta = 0.0298 \text{ in.}$ The result correlates fairly well with the plane-wave analyses for nonzero thickness as far as relative amplitude variations and time delays are concerned. However, the attenuation of the initial wave is much less than that indicated by plane-wave analysis. Although the factors involved in these discrepancies need more study, two aspects of the problem seem most important: (1) "perfect" coupling between probe and sensor in the plane-wave analysis, as compared with the very low coefficient of coupling between probe and pickup in the experi-

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Fig. 9. Normalized Version of Fig. 7 after Approximate Correction for Secondary Effects

ment, and (2) the fact that the depth of penetration from a field of high curvature is greater than that from a plane wave.

The plots in Fig. 10 of the variations in indicated field strength with increasing spacing between the probe and the pickup provide further indication that the more concentrated the field, the more the results deviate from those of plane-wave excitation. (The mask aperture, discussed in Section V, provides a much more concentrated field than the smallest solenoids.)

Fig. 10

Experimental Variation in Field Strength as a Function of Distance of the Pickup Wire from the Field Source. Zero distance is measured from the end of the solenoids and to the plane of the aperture. The pulse lengths are the same as in the previous figures.

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F. Surface Reflections

Because of the negligible travel times involved in surface reflections in the region $-d < x < 0$ in Fig. 2, as indicated by Eqs. 41a-e, the same condition may be assumed to hold throughout this region at a given time. That is,

$$H(x,t) = H(0,t), \quad d < x < 0.$$

Equation 39 may be written, for $x = 0$, as

$$\bar{H}(0,s) = \frac{\eta I(s)}{\eta + Z_{01}(s)} \exp[-\gamma_0(s)d] \left\{ 1 - \frac{\Gamma(s)[1 - \Gamma(s)] \exp[-2\gamma_1(s)w]}{1 - \Gamma^2(s) \exp[-2\gamma_1(s)w]} \right\}.$$

If the approximations of Eq. 41 are applied, this expression becomes

$$\bar{H}(0,s) = I(s) \left[1 - \frac{2\sqrt{\frac{\epsilon_0 s}{\sigma}} \exp(-2\sqrt{\mu_0 \sigma s} w)}{1 - \exp(-2\sqrt{\mu_0 \sigma s} w)} \right]. \quad (56)$$

When $i(t) = u(t)$,

$$\bar{H}_u(0,s) = \frac{1}{s} - 2\sqrt{\frac{\epsilon_0}{\sigma}} \frac{1}{\sqrt{s}} \frac{\exp(-2\sqrt{\mu_0 \sigma s} w)}{1 - \exp(-2\sqrt{\mu_0 \sigma s} w)} = \frac{1}{s} - \bar{H}_{0u}(w,s), \quad (57)$$

where

$$\bar{H}_{0u}(w,s) = 2\sqrt{\frac{\epsilon_0}{\sigma}} \sum_{n=1}^{\infty} \frac{1}{\sqrt{s}} \exp(-2n\sqrt{\mu_0 \sigma} w \sqrt{s}). \quad (58)$$

The time variation corresponding to Eq. 57 is

$$H_u(0,t) = [u(t) - H_{0u}(w,t)], \quad (59a)$$

where

$$\begin{aligned} H_{0u}(w, t) &= 2 \sqrt{\frac{\epsilon_0}{\sigma}} \frac{1}{\sqrt{\pi t}} \sum_{n=1}^{\infty} \exp\left(-4n^2 \frac{w^2 \mu_0}{4t}\right) u(t), \\ &= \frac{2}{\sigma \eta X_0} \sqrt{\frac{2}{\pi}} \frac{1}{\sqrt{t/T_0}} \sum_{n=1}^{\infty} \exp\left[-2n^2 \frac{(w/X_0)^2}{t/T_0}\right] u(t), \end{aligned} \quad (59b)$$

and T_0 and X_0 are defined as before.

For rectangular pulse $i(t) = I[u(t - T_w)]$, normalized as before with respect to pulse width, Eq. 59 becomes

$$\underline{H}(0, \underline{t}) = \frac{1}{I} H(0, T_w \underline{t}) = u(\underline{t}) - u(\underline{t} - 1) - \underline{H}_0(\underline{w}, \underline{t}), \quad (60)$$

where

$$\underline{H}_0(\underline{w}, \underline{t}) = \frac{1}{I} H_0(\delta \underline{w}, T_w \underline{t}) = \underline{H}_{0u}(\underline{w}, \underline{t}) - \underline{H}_{0u}(\underline{w}, \underline{t} - 1), \quad (61a)$$

and

$$\underline{H}_{0u}(w, t) = \frac{1}{\Psi} \frac{1}{\sqrt{t}} \sum_{n=1}^{\infty} \exp\left(-2n^2 \frac{w^2}{t}\right) U(\underline{t}). \quad (61b)$$

These results are displayed in Fig. 11, where Eq. 61a is plotted for several values of $\underline{w}$. Note the additional apparent time delay, over that of the through-transmission case, before the effects of the back surface are noticeable.

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Fig. 11. Surface-field Deviations of an Infinite Slab from That of a Perfect Conductor

G. Injected Current

From the circuital law of magnetism,

$$LH(\underline{w}^+, t) - LH(0^-, t) = Li_c(w, t),$$

where L is the length of a rectangular contour whose sides are just outside the metal surfaces, and i_c is the total current per meter flowing in the slab in the same direction as i . The functional notation recognizes that the injected current, i_c , depends

on slab thickness. Thus, normalized,

$$\underline{i}_c(\underline{w}, \underline{t}) = \frac{1}{I} i_c(\delta \underline{w}, T \underline{w} \underline{t}) = \underline{H}(\underline{w}, \underline{t}) - \underline{H}(0, \underline{t}). \quad (62)$$

For the rectangular pulse, substitution of previously established expressions for magnetic field intensities results in

$$\underline{i}_c(\underline{w}, \underline{t}) = -[\underline{u}(\underline{t}) - \underline{u}(\underline{t} - 1)] + i_{co}(\underline{w}, \underline{t}), \quad (63)$$

where, in a manner similar to that for Eq. 61a,

$$i_{co}(\underline{w}, \underline{t}) = i_{cou}(\underline{w}, \underline{t}) - i_{cou}(\underline{w}, \underline{t} - 1), \quad (64a)$$

and

$$i_{cou}(\underline{w}, \underline{t}) = \frac{1}{\sqrt{t}} \sum_{n=1}^{\infty} \exp\left(-\frac{n^2 \underline{w}^2}{2 \underline{t}}\right) \underline{u}(\underline{t}) \text{ Amp/m.} \quad (64b)$$

The current i_{co} is the deviation from the current that would occur in a perfectly conducting slab, in which $i_c(t) = -i(t)$. Although Eq. 64b was not plotted, point-by-point additions have been made to yield the curves of Figs. 5 and 11 for a given $\underline{w}$.

IV. THE TESTING DILEMMA

The basic problem in eddy-current testing, whether by steady-state sinusoidal excitations or pulsed currents, lies in the magnitude of the constant Ψ for normal conductors and pulse widths. For example, in Type 304 stainless steel (a common tubing material), Ψ is approximately 5×10^5 . A glance at the plane wave in Fig. 7 shows that variations to be expected, because of changes in the metal slab, fall within a range of the order of 10^{-6} times the variation expected for a perfect conductor. Detection of such a small variation in gross signal presents a difficult instrumentation problem.

Actually, the large Ψ presented by the plane-wave analysis presents a very pessimistic view of the testing problem because of the essentially perfect "coupling" between the planar current source and the metal slab. When the distribution of the exciting current in space is quantized by being constrained to conducting filaments or coil sides, the coupling between exciting current and test specimen is reduced, and the fields are distorted so that the H-vectors due to exciting coil and response current are no longer colinear everywhere. Pickup coils could then be located in regions where the effects of test-piece changes are more evident than is indicated by a plane-wave analysis. If Ψ were defined for such conditions, the value would

be considerably less than the 5×10^5 cited above. Although a reduction in Ψ by a factor of as large as 1000 might reduce the problem, it will not eliminate the difficulty.

In the application of single-frequency procedures, the problem of detecting small variations in test response has been accomplished by balancing the processed signal against a standard signal, usually by means of a bridge circuit. Similar techniques have been applied to pulsed systems, but balancing signals whose frequency content spans the entire frequency spectrum present a much more formidable practical problem than balancing

single-frequency signals. However, a single pickup coil can be made relatively insensitive to the massive signal that is to be expected from any conductor and still maintain sensitivity to the small changes due to variations in the properties of the test specimen.

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Fig. 12. Change in Intensity (H_C) That Affects Exciting Coil when Probe Is near a Metal Plate

that links the pickup coil and thereby causes a signal output. The exact nature of the flux change depends on the thickness and electrical properties of the metal plate, since the injected current and the manner in which the current diffuses away from the region of initial concentration are functions of these quantities.

Figure 12 shows one experiment that illustrates how this may be accomplished. With no metal beneath the exciting coil, the pickup coil is positioned so that its axis is normal to the direction of the H -vector. When a metal sheet is placed beneath the coil, a rapidly changing current in the exciting coil produces in the metal sheet a similarly distributed current nearly equal in magnitude but opposite in sign to the exciting current, as has been demonstrated in Section III.A for the plane-wave case. The currents in the sheets tend to prevent flux penetration so that the flux distribution is distorted in a manner similar to that shown in Fig. 12b, which indicates the conditions after some penetration has occurred. A component of flux is now present

Although the probe illustrated in Fig. 12 indicates gross variations in the parameters of the plate or tubing, the technique suffers from poor resolution. Efforts to improve resolution have led to the development of the mask-aperture assembly.

V. MASK-APERTURE ASSEMBLIES

The advantages that accrue from the use of highly conducting masks become more evident if the plane-wave model of Fig. 13 is considered.

Fig. 13

Effects of Masking a Plane-wave Source. Pickup coil is indicated by (p).

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Assume that the exciting current in Fig. 13 is rapidly increasing in the direction shown, and the flux density produced at point (p) is monitored by a small pickup coil. In Fig. 13a, the exciting current is assumed to be in a region containing no conducting masses, and a uniform field is produced of opposite sign on opposite sides of the exciting current.

When a conducting metal plate is introduced, as in Fig. 13b, the field is essentially doubled in the region between the plate and exciting current and nearly cancelled everywhere else. The only deviation from this doubling and cancelling is that described by Eqs. 61 and 54, respectively. The deviation at location (p) of the exciting coil from the massive doubling of exciting flux by a perfect conductor is that of Eq. 61. As was noted previously, the small deviation is difficult to separate from the gross signal.

The metal plate is now removed, and a mask of high conductivity is inserted between the pickup coil and the exciting current, as shown in

Fig. 13c. The flux density at (p) is consequently greatly reduced from the value in Fig. 13b, although the flux density is considerably greater than that which would result from transmission through a solid-metal barrier. If the metal plate is returned to the position occupied in Fig. 13b, the plate intercepts the field penetrating the aperture in the mask, and the currents induced in the plate increase the field in the aperture, but reduce the field that penetrates the aperture and invades the area beneath the mask, as shown in Fig. 13d. The amount of such blocking depends upon the condition of the metal plate in that region intercepting the aperture. For example, a crack in the plane of the figure and completely through the plate results in a field pattern similar to Fig. 13c, even though the length of the crack is only slightly larger than the aperture dimension.

Another effect of importance on the response of the pickup coils results from the fact that currents injected into the plate tend to diffuse away from the aperture along the plate. As Eq. 64b shows, a component of the injected current generally persists after the exciting pulse has passed, and the manner in which this current diffuses away from the aperture apparently has an important bearing on that part of the response occurring after passage of the exciting pulse. The manner of diffusion, in turn, depends upon plate conditions around the aperture.

VI. QUALITATIVE CIRCUIT ANALYSIS OF MASK-APERTURE ASSEMBLY

The penetration depths and diffusion rates given by the plane-wave analysis suggest a method to obtain additional insight into the mask-aperture assembly. The method resembles the separation of "transient" and "sub-transient" effects in synchronous machines.

Figure 14 shows cross-sectional views of a prototype encircling mask-aperture tubing tester of a type that has proven useful in tests of thin-wall metal tubing. In use, the exciting coil is driven by a pulse whose width depends upon the material and dimensions of the tubing under test. The flux penetrating the aperture and the tubing is monitored by the pickup coil.

If a step function of current is passed through the exciting coil, opposing currents appear in the adjacent metal in the manner described by Eq. 64b for the plane wave. These initial currents are shown in the left column of the sketches in Fig. 14. The initial concentration of currents in the metal nearest the exciting coil depends upon the depth of penetration for the metal: a dense concentration near the surface for good conductors with small depths of penetration, and a more diffuse concentration for poorer conductors. For example, diffusion of a 1.0- μ sec pulse into copper, which has a depth penetration of 0.167 mm for a 1.0- μ sec pulse, is on the order

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Fig. 14. Current Distributions in an Encircling Mask-aperture Probe following a Step Function of Exciting Current

of one-sixth the distance that the pulse diffuses in Type 304 stainless steel, which has a depth of penetration of 1.07 mm.

For the empty probe, the mask has a relatively high resistance under a rapidly changing excitation because of the concentration of the current into an area of small cross section near the exciting coil. However, under more slowly varying excitation, the current is distributed essentially uniformly across the mask, resulting in a much lower resistance. The inductances are somewhat different also, but the difference should be much less significant than that of the resistances. The high-resistance portion of the mask probably extends into the mask less than 1 mm for most tests, since the depth of penetration for a 1- μ sec pulse in copper is only 0.167 mm.

Pulses employed are normally in the range of 1-4 μ sec in width, and fast-changing effects die out before slow flux changes can penetrate the massive part of the mask away from the aperture. In short, rapidly changing effects concentrated near the aperture may be considered to be the results of a short-circuited coil of relatively high resistance concentrated right adjacent to the aperture, as shown on the left in Fig. 14. Conversely, slowly changing effects may be considered to result from a short-circuited coil of relatively low resistance formed by the entire cross section of the mask, as indicated on the right in Fig. 14. Analysis on this basis may give an insight into a problem whose complicated boundary conditions make a straightforward field analysis very difficult.

In this analysis of the empty probe in Fig. 14a, each flux-linkage component is tagged with two subscripts, the first locating the flux component and the second the coil whose magnetomotive force creates the flux. Total flux linkage in a coil, due to all coils, bears the single subscript of the coil in which linkage occurs. The following coil tags are adopted:

E identifies the exciting coil,

S identifies the coil formed by short-term portion of mask,

L identifies the coil formed by long-term portion of mask,

C identifies the pickup coil,

and

T identifies the tube.

We also define "constants" specifying the fraction of the flux produced by one coil, say coil N, that links another coil, say coil M. Thus,

$$K_{MN} = \frac{\lambda_{MN}}{\lambda_{NN}} = \frac{L_{MN}i_N}{L_{NN}i_N} = \frac{L_{MN}}{L_{NN}}, \quad (65)$$

where λ denotes flux linkages, L is inductance, and i is current. The coefficient of coupling between coils M and N is

$$k_{MN} = \sqrt{\frac{L_{MN}^2}{L_{MM}L_{NN}}} = \sqrt{\frac{L_{MN}}{L_{NN}} \frac{L_{NM}}{L_{MM}}} = \sqrt{\bar{K}_{MN}\bar{K}_{NM}}. \quad (66)$$

The circuit equations will be written in terms of flux linkages, recognizing that first-order terms in inductive loops are of the form

$$(R_M + L_{MM}p) i_M = (p + p_M) \lambda \text{ or } L_{MN}p i_N = p \lambda_{MN}, \quad (67a)$$

where

$$p = \frac{d}{dt} \text{ sec}^{-1} \quad (67b)$$

and

$$p_M = \frac{R_M}{L_{MM}} \text{ sec}^{-1}. \quad (67c)$$

To save clutter in this qualitative analysis, simplifying assumptions will be made when justified.

The differential equations for coils S and L, with all currents positive in the same direction, are

$$0 = (p + p_S) \lambda_{SS}(t) + p \lambda_{SL}(t) + p \lambda_{SE}(t) \quad (68a)$$

and

$$0 = p \lambda_{LS}(t) + (p + p_L) \lambda_{LL}(t) + p \lambda_{LE}(t). \quad (68b)$$

Since initial flux linkages were zero in all coils, the Laplace transforms of these equations are

$$0 = (s + p_S) \Lambda_{SS}(s) + s\Lambda_{SL}(s) + s\Lambda_{SE}(s) \quad (69a)$$

and

$$0 = s\Lambda_{LS}(s) + (s + p_L) \Lambda_{LL}(s) + s\Lambda_{LE}(s). \quad (69b)$$

By using Eq. 65, we can collect these into the matrix form

$$-\begin{bmatrix} K_{SE} \\ K_{LE} \end{bmatrix} s\Lambda_{EE}(s) = \begin{bmatrix} s + p_S & K_{SL}s \\ K_{LS}s & s + p_L \end{bmatrix} \begin{bmatrix} \Lambda_{SS}(s) \\ \Lambda_{LL}(s) \end{bmatrix}. \quad (70)$$

The determinant of the coefficient matrix of Eq. 70 is

$$\begin{aligned} |D| &= (s + p_S)(s + p_L) - K_{SL}K_{LS}s^2 \\ &= (s + p_S)(s + p_L) - k_{LS}^2 s^2. \end{aligned}$$

Since the coefficient of coupling, k_{LS} , between the long-term and short-term coils is considerably less than unity,

$$|D| \approx (s + p_S)(s + p_L).$$

The S-coil linkages are now obtained by solving Eq. 70:

$$\Lambda_{SS}(s) = - \frac{(K_{SE} - K_{SL}K_{LE})s + p_L}{(s + p_S)(s + p_L)} s\Lambda_{EE}(s).$$

However, K_{SE} is so much larger than either K_{SL} or K_{LE} that $K_{SE} - K_{SL}K_{LE} \approx K_{SE}$, and approximately,

$$\Lambda_{SS}(s) = - \frac{K_{SE}s\Lambda_{EE}(s)}{s + p_S}. \quad (71)$$

Now assume that the exciting flux is the rapidly decaying exponential

$$\lambda_{EE}(t) = \lambda_0[\exp(-p_E t)] u(t), \quad (72a)$$

so that

$$\Lambda_{EE}(s) = \frac{\lambda_0}{s + p_E}. \quad (72b)$$

Equation 71 takes the form

$$\begin{aligned}\Lambda_{SS}(s) &= -K_{SE}\lambda_0 \frac{s}{(s+p_E)(s+p_S)} \\ &= -\frac{K_{SE}\lambda_0}{p_E - p_S} \left(\frac{p_E}{s+p_E} - \frac{p_S}{s+p_S} \right).\end{aligned}\quad (73)$$

In this equation, two effects that slow up become relatively independent for narrow exciting pulses, for which $p_E \gg p_S$, and take the form

$$\Lambda_{SS}(s) = -K_{SE}\lambda_0 \left(\frac{1}{s+p_E} - \frac{p_S/p_E}{s+p_S} \right). \quad (74)$$

Substitution of Eq. 74 into Eq. 69b results in

$$0 = (s+p_L) \Lambda_{LL}(s) + s \left(\frac{K_{LE} - K_{LS}K_{SE}}{s+p_E} + \frac{K_{LS}K_{SE}p_S/p_E}{s+p_S} \right) \lambda_0.$$

Since, as shown in Fig. 14, K_{LS} is not much different from K_{LE} and K_{SE} is nearly unity, the coefficient of $1/(s+p_E)$ will be quite small. Neglecting this term and solving for $\Lambda_{LL}(s)$, we have

$$\Lambda_{LL}(s) = -\frac{K_{LS}K_{SE}p_S\lambda_0}{(p_S - p_L)p_E} \left(\frac{p_S}{s+p_S} - \frac{p_L}{s+p_L} \right). \quad (75a)$$

Since p_S is considerably larger than p_L , Eq. 75a becomes

$$\Lambda_{LL}(s) = -\frac{K_{LS}K_{SE}p_S\lambda_0}{p_E} \left(\frac{1}{s+p_S} - \frac{p_L/p_S}{s+p_L} \right). \quad (75b)$$

The pickup-coil flux linkages are

$$\begin{aligned}\Lambda_C(s) &= \Lambda_{CE}(s) + \Lambda_{CS}(s) + \Lambda_{CL}(s) \\ &= K_{CE}\Lambda_{EE}(s) + K_{CS}\Lambda_{SS}(s) + K_{CL}\Lambda_{LL}(s).\end{aligned}$$

Substituting Eqs. 72b, 74, and 75b into the above expression, and then taking the inverse Laplace transform, we obtain

$$\lambda_C(t) = K_{SE}\lambda_0 [K_E \exp(-p_E t) - K_S \exp(-p_S t) + K_L \exp(-p_L t)] u(t), \quad (76)$$

where

$$K_E = \frac{K_{CE}}{K_{SE}} - K_{CS}, \quad (76a)$$

$$K_S = \frac{P_S}{P_E} (K_{CL}K_{LS} - K_{CS}), \quad (76b)$$

and

$$K_L = \frac{P_L}{P_E} K_{CL}K_{LS}. \quad (76c)$$

The following alternative form of Eq. 76 follows from Eq. 72:

$$\lambda_c(t) = K_{SE} \{ K_E \lambda_{EE}(t) + [K_L \exp(-p_L t) - K_S \exp(-p_S t)] u(t) \}. \quad (77)$$

In spite of constants that are not really constant and a good number of approximations, useful conclusions can be obtained from these results. Some are evident from a discussion of the constants in Eqs. 76 and 77.

It is evident from Eq. 77 that the coefficient K_E is a measure of the amount of exciting-coil flux that penetrates the S-coil and causes an indication from the pickup coil. Because of the close coupling between the E-coil and the S-coil, $K_{SE} \approx 1$, and, from Eq. 76a,

$$K_E \approx K_{CE} - K_{CS}. \quad (78)$$

Since, as shown in Fig. 14, K_{CS} and K_{CE} do not differ significantly, K_E is normally a quite small number. However, $\lambda_{EE}(t)$ is changing so rapidly that it causes an appreciable signal in the pickup coil. The quantity $\lambda_{EE}(t)$ is an important component of flux, because metallic test specimens that partially close the aperture will primarily have their effects on it, the effect depending on the characteristic of the test-piece metal.

The coefficient K_S may be either positive or negative, depending on the position of the pickup coil. A positive value results when the pickup is moved a small distance away from the aperture. The required distance should not be great, since K_{CS} is much more sensitive to small changes in coil position than K_{CL} , and since K_{LS} is independent of pickup. The component of amplitude K_S can probably be eliminated when the probe is empty.

The coefficient K_L is very dependent on the amount of copper in the mask, but it is relatively insensitive to small changes in the position of the pickup coil. Increasing the cross-sectional area of the mask should reduce K_L , for p_L then will be reduced.

If we disregard impulses at the origin, the voltage in the pickup coil is

$$\begin{aligned} e_C(t) &= p \lambda_C(t) \\ &= -K_{SE} \lambda_0 [P_E K_E \exp(-P_E t) - P_S K_S \exp(-P_S t) + P_L K_L \exp(-P_L t)] u(t). \end{aligned} \quad (79)$$

Figure 15 shows that the coupled-circuit analysis leading to Eq. 79 can indeed approximate the form of the actual behavior of the probe. The function

$$e_C(t) = \left[29.4 \exp\left(-\frac{t}{3.4}\right) - 12.0 \exp\left(-\frac{t}{7}\right) + 2.73 \exp\left(-\frac{t}{20}\right) \right] u(t)$$

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Fig. 15. Experimental Curve for an Empty Probe Compared with a Mathematical Curve from Eq. 79

is plotted on the same coordinates as the actual oscillogram of the response of the empty probe. No attempt was made in the figure to determine the optimum fit of the calculated to the experimental curve; the intent was simply to show the similarities in form.

If one observes the experimental response of a mask constructed with a much greater mass of copper on one side of the aperture than on the other, an additional apparent time constant can be observed. This time constant represents a transient term of long duration. Presumably the additional term is caused by the mas-

sive part of the mask in the same manner that the third term of Eq. 79 is created. Thus the approximation of Eq. 79 appears justifiable.

To reiterate, Eq. 79 is based on a lumped-parameter model of a nonuniformly distributed system. The actual transient diffusion process in the real system yields equations that are much more complicated in form than the equations above or the equations resulting from plane-wave analysis.

A. Effects due to a Test Specimen

The first term in Eq. 77 is the fraction of the feed-through portion of exciting flux that links the pickup coil. This component is intercepted by tubing inserted into the probe. Figure 14 demonstrates that the effect of tubing is quite concentrated for a good conductor, such as copper, but less so for a poorer conductor, such as Type 304 stainless steel.

After the insertion of the tubing, the feed-through portion of the flux linkages of the pickup coil is modified by the flux linkages introduced by the tubing. If long-term effects and mutual coupling with other coils are neglected, these modified linkages are

$$\lambda_F(t) = K_{SE} K_E \lambda_{EE}(t) + K_{CT} \lambda_{TT}(t). \quad (80)$$

The voltage equation for the short-circuited coil assumed to represent the tubing is

$$0 = K_{TE} p \lambda_{EE}(t) + (p + p_T) \lambda_{TT}(t),$$

from which results

$$\lambda_{TT}(s) = - \frac{K_{TE} s \lambda_{EE}(s)}{s + p_T}.$$

Substituting Eq. 72 into the above expression leads to

$$\lambda_{TT}(s) = - \frac{K_{TE} \lambda_0}{p_E - p_T} \left(\frac{p_E}{s + p_E} - \frac{p_T}{s + p_T} \right),$$

which has the inverse Laplace transform

$$\lambda_{TT}(t) = - \frac{K_{TE} \lambda_0}{p_E - p_T} [p_E \exp(-p_E t) - p_T \exp(-p_T t)] u(t).$$

Equation 80 may now be written as

$$\begin{aligned} \lambda_F(t) &= \left(K_{SE} K_E - \frac{K_{CT} K_{TE} p_E}{p_E - p_T} \right) \lambda_0 \exp(-p_E t) + \frac{K_{TE} p_T K_{CT} \lambda_0}{p_E - p_T} \exp(-p_T t) u(t) \\ &= \left(K_{SE} K_E - \frac{K_{CT} K_{TE} p_E}{p_E - p_T} \right) \lambda_{EE}(t) + \frac{K_{TE} p_T K_{CT} \lambda_0}{p_E - p_T} \exp(-p_T t) u(t). \end{aligned} \quad (81)$$

Without the tubing, the feed-through flux is the first term in Eq. 77:

$$\lambda_{F0}(t) = K_{SE} K_E \lambda_{EE}(t). \quad (82)$$

A comparison of this equation with Eq. 81 shows that the insertion of the tubing has two effects: (1) It reduces the amount of exciting-pulse feed-through to the pickup coil, and (2) it introduces a new time constant, $1/p_T$, which depends on the conductivity of the tubing. In estimating relative magnitudes of these effects, note that K_{TE} will be largest when the tubing is a good conductor and the current is concentrated near the aperture, as in Fig. 14b.

Figure 16 shows experimental results that give qualitative support to the above statements; the higher the conductance of the current-carrying region of the tube, the more the exciting-pulse feed-through is reduced. When the time scale is expanded, evidence of time-constant changes also appears in the early time results.

Fig. 16
Probe Responses to 0.174-in.-OD
Tubing with Base-metal Wall
Thickness of 9.0 mils

106-9552

B. The Point-type Mask-aperture Assembly

Although the point-type mask aperture used for surface scanning of tubing and plate stock was developed before the encircling probe discussed above, it is interesting to observe how the former may be "evolved" from the latter. This allows the qualitative analysis of the one to carry over to the other as follows: Starting with the encircling probe as shown in Fig. 17a, one "unwraps" the cylindrical configuration to form the planar figure in Fig. 17b. Adding returns for the resulting exciting and pickup conductors to form them into coils, then replacing the strip probe by a similar cylindrical configuration of Fig. 17c completes the evolution.

The mask-aperture point probe provides a highly concentrated field of high curvature in the neighborhood of the aperture. The greatly increased pulse penetration of this field is readily seen by comparing transmissions in Fig. 18 with those of Figs. 7 and 8. There is much less pulse attenuation with the mask-aperture probe, but, again, the time delays are comparable.

106-9556 Rev. 1

Fig. 17. Evolution of the Surface Probe

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Fig. 18. Pulse Transmission from a 0.080-in.-diam Mask-aperture Assembly through Varying Thicknesses of Type 304 Stainless Steel

VII. CONCLUSION

The analytical treatment of the problem of pulsed electromagnetic fields in the presence of highly conducting shields of a complicated shape is difficult at best, and the results presented in this report can only be considered as a preliminary venture toward a usable solution. This analysis has, however, provided considerable insight as to how the mask-aperture assemblies work, and what direction attempts at a more realistic analysis of the problem of pulsed current flow in good conductors should take in order to be useful for application to nondestructive-test problems. Further analysis is presently being undertaken.

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