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DESIGN, ANALYSIS, AND TEST VERIFICATION OF ADVANCED
ENCAPSULATION SYSTEM

MASTER

Triannual Report for Period Ending July 30, 1980

By
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August 1980

Work Performed Under Contract No. NAS-7-100-955567

Spectrolab, Inc.
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TRIANNUAL REPORT
on the
DESIGN, ANALYSIS, AND TEST VERIFICATION
OF ADVANCED ENCAPSULATION SYSTEM

For Period Ending
30 July 1980

Contract 955567

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1.0 SUMMARY STATEMENT

An optical model has been developed which will predict the amount of incident sunlight reaching the cell surface as well as determining the heat absorbed in encapsulation layers above the cells in a given module design. These results can then be factored into a thermal model which will predict the NOCT of a given design.

An electrical analysis has been performed which will aid in the selection of properly sized (for thickness) layers in a given module design. Differing allowable voltages may be integrated into the model.

Work has begun on the structural model using the MSC NASTRAN finite element model computer program.

An algorithm has been developed for determining the total system life-cycle energy cost for ranking module systems.

2.0 INTRODUCTION

The objective of this program is to develop analytical methodology for advanced encapsulation systems which will aid in the determination of optimum systems for meeting the Low Cost Solar Array Project goals. The program consists of three phases. In Phase I, analytical models will be developed to perform optical, thermal, electrical, and structural analyses on candidate encapsulation systems. From these analyses an optimum candidate system will be selected for verification in Phase II, based on cost per watt over the life of the module.

During Phase II, modules of this verification system will be constructed and tested to determine the validity of the analysis methodology developed in Phase I.

In Phase III, a finalized optimum design based on any iterations in Phase II will be developed and delivered to JPL.

3.0 TECHNICAL DISCUSSION

3.1 OPTICAL MODEL

The final version of the report on the optical model is attached as Appendix A. This document shows the methodology supporting the algorithms used in the computer program.

The essential goals of the optical analysis are to (1) predict the amount of incident sunlight reaching the cell surface, and (2) predict the heat absorbed in the encapsulant and cover layers above the cell. Data input to the model include: (1) relative response curve of the cell, (2) spectrum of the incident radiant energy, (3) index of refraction for the encapsulant and cover layers, and (4) spectral transmittance for the encapsulant and cover layers. The optical computer model is currently programmed for an AM 1.5 solar spectrum, but this can be easily changed to accommodate any light source.

At JPL's request Spectrolab investigated the optimization of an anti-reflection coating. A discussion of this optimization follows.

The reflectivity from one thin film for unpolarized light is given by

$$R = \frac{r_1^2 + r_2^2 + 2r_1r_2\cos\theta}{1 + r_1^2r_2^2 + 2r_1r_2\cos\theta} \quad (1)$$

where $\theta = \frac{4\pi N_1 t_1 \cos I_1}{\lambda}$

$$r_1 = \frac{-\sin(I_0 - I_1)}{\sin(I_0 + I_1)} \quad \text{or} \quad \frac{\tan(I_0 - I_1)}{\tan(I_0 + I_1)}$$

$$r_2 = \frac{-\sin(I_1 - I_2)}{\sin(I_1 + I_2)} \quad \text{or} \quad \frac{\tan(I_1 - I_2)}{\tan(I_1 + I_2)}$$

λ = wavelength of incident light

t = thickness of film

N_0, N_1, N_2 = refractive indices

I_0, I_1, I_2 = angles of incidence and refraction

The nomenclature is shown in Figure 1.

Figure 1

Depending on the polarization of the incident light, the sine or tangent expressions are chosen for r_1 and r_2 . For unpolarized light, R is computed for each polarization and the two values are averaged.

At normal incidence, $I_0 = I_1 = I_2 = 0$, and r_1 and r_2 reduce to

$$r_1 = \frac{N_0 - N_1}{N_0 + N_1} \quad (2)$$

$$r_2 = \frac{N_1 - N_2}{N_1 + N_2} \quad (3)$$

Using these expressions for r_1 and r_2 , the thickness that yields the minimum reflectance is found to be

$$t_1 = \frac{\lambda}{4N_1} \quad (4)$$

that is, the optical thickness of the film is equal to one-quarter wavelength of the incident light. For a quarter-wave film, the reflectivity is zero when the index of refraction of the film is

$$N_1 = \sqrt{N_0 N_2} \quad (5)$$

3.2 ELECTRICAL ANALYSIS

The derivation of the electric field distribution in a multi-layer system of dielectric materials is attached in Appendix B. The expression for the electric field in the k th layer, E_k , is given below

$$E_k = \frac{V_0}{t_k + \gamma_k \sum_{i=k} \frac{t_i}{\gamma_i}} \quad (6)$$

where E_k = electric field strength in layer k , volt/cm
 γ_i, γ_k = dielectric constants of layers i and k , respectively
 t_i, t_k = thicknesses of layer i and k , respectively
 V_0 = voltage across entire stack of layers
 = 3000 Vdc (LSA requirement)

The electrical safety design criteria is that the electric field strength in each layer does not exceed the dielectric strength, S_k , of that layer. In other words

$$E_k < S_k \text{ for all layers when } V_0 = 3000 \text{ Vdc}$$

It should be pointed out that eq. (6) is for an ideal system in which there are no leakage currents, and no defects such as cracks or bubbles. The expression also does not account for the crowding of electric field lines near the corner of a solar cell or near an interconnect.

For the glass superstrate design family, the only layer of interest from an electrical safety standpoint is the encapsulant layer between the cell and the aluminum foil backing. It is assumed the Mylar backing on the foil can be cracked or nicked while inserting the module in an array frame. Thus, it must be assumed that the encapsulant layer below the cell must withstand the full voltage difference. The superstrate design family and the electrical properties of candidate pottants are shown in Figure 2.

To determine the minimum thickness of pottant below the cell to withstand a given potential drop V_0 , we make use of the following expression

$$t = \frac{V_0}{S} \quad (7)$$

where S = dielectric strength of the pottant

As seen in Figure 2, there is a wide variation in values of S for each pottant in bulk (the values for EVA/craneglass will be added as the information becomes available). In addition, the values of S for each pottant cast as a thin film vary with film thickness.

Figure 2

SUPERSTRATE DESIGN FAMILY

MATERIAL	DIELECTRIC CONSTANT	DIELECTRIC STRENGTH	
		Volt/cm	Volt/mil
EVA	2.7 - 2.9 (1 MHz) 3.16 (60 Hz, bulk)	2.4×10^5 (1 mil)	620
EVA/Craneglass			
White EVA/Craneglass			
EPR	2.25 - 3.0 (60 Hz, bulk) 2.2 - 2.95 (1 MHz, bulk)	1.97×10^5 - 3.94×10^5 (bulk)	500 - 1000
Polyurethane	4.7 - 9.53 (60 Hz, bulk) 5.5 - 9.50 (1 MHz, film)	1.30×10^5 - 2.76×10^5 (bulk)	330 - 701
PMMA	3.5 - 4.5 (60 Hz, bulk) 2.7 - 3.2 (1 MHz, bulk)	3.15×10^5 (bulk)	800
PVC Plastisol	4 - 8	1.18×10^5	300
Silicone Acrylic Copolymer	5*	5×10^5 (10 mil)*	1270

*Engineering Approximation

The minimum thicknesses of pottant required to withstand 3000 volts for the glass superstrate design family are summarized in Table 1.

The substrate family of module designs is illustrated in Figure 3, which also lists the relevant properties for each layer. From eq. (6), the appropriate expressions for the electric field in the top cover and the pottant above cell, E_1 and E_2 , respectively, are

$$E_1 = \frac{V_o}{t_1 + \frac{\gamma_1}{\gamma_2} t_2} \quad (\text{top cover}) \quad (8)$$

$$E_2 = \frac{V_o}{\frac{\gamma_2}{\gamma_1} t_1 + t_2} \quad (\text{pottant above cell}) \quad (9)$$

JPL has suggested that plotting the pottant above cell thickness against the top cover thickness might be convenient for module designers. Solving eqs. (8) and (9) for t_2 and substituting S_1 for E_1 and S_2 for E_2 (because we want to calculate the minimum thickness at which breakdown can be avoided) yields

$$t_2 = \frac{\frac{V_o}{S_1} - t_1}{\gamma_1/\gamma_2} \quad (10)$$

$$t_2 = \frac{V_o}{S_2} - \frac{t_1}{\gamma_1/\gamma_2} \quad (11)$$

TABLE 1

**MINIMUM THICKNESS OF POTTANT UNDER CELL TO WITHSTAND
3000 VOLTS FOR GLASS SUPERSTRATE MODULES**

Pottant	Thickness, mil
EVA	4.8
EVA/Craneglass	
White EVA/Craneglass	
EPR	3.0 - 6.0
Polyurethane	4.3 - 9.1
PNBA	3.8
PVC Plastisol	10
Silicon Acrylic Copolymer	2.4

Figure 3
SUBSTRATE DESIGN FAMILY

*Engineering Approximation

$$V_1/V_2 \text{ (min)} = 3.5/9.53 = .367$$

$$V_1/V_2 \text{ (max)} = 9.9/2.2 = 4.5$$

The thickness t_2 calculated by means of eq. (11) is the minimum allowable thickness of pottant over cell to prevent voltage breakdown in the pottant. For conservative design, the values of t_2 calculated by means of eq. (10) should be greater than those by means of eq. (11).

A sensitivity study was performed by calculating t_2 for different values of S_1 (top cover dielectric strength), γ_1/γ_2 , and t_1 for the LSA 3000 volt requirement. The results were calculated by means of eq. (13) and are shown in Figures 4-7. For any pottant/front cover material combination, the acceptable combinations of t_1 and t_2 lie above and to the right of the line for the value of γ_1/γ_2 corresponding to that material combination. As expected, when $t_1 = V_0/S_1$, the front cover can withstand the entire voltage, and as a consequence, the required pottant thickness becomes negligible.

3.3 THERMAL ANALYSIS

The thermal analysis portion of this program is progressing along three avenues. The first involves an investigation of the cell temperature with different module backside emissivities. The second involves comparison of radiative and convective heat loss mechanisms on a percentage basis for different designs and different emissivities on the back (earth-facing) side of a module. The last is a sensitivity study of the convective heat loss mechanism.

Table 2, as well as Figure 8, show values of cell temperature versus back side emissivity for the glass superstrate design (Design No. 1) for NOCT conditions. The temperature decreases with increasing emissivity; this reflects the lower potential

$V_0 = 3000$ Volt DC.

TOP COVER DIELECTRIC STRENGTH = 500 volt/mil.

Figure 4 - Thickness of Potant Above Cell vs Top Cover Thickness for $V_0 = 3000$ volt dc and Top Cover Dielectric Strength = 500 volt/mil.

$V = 3000$ Volts DC

TOP COVER DIELECTRIC STRENGTH = 1000 Volt/mil

Figure 5 - Thickness of Potant Above Cell vs Top Cover Thickness for $V = 3000$ volt dc and Top Cover Dielectric Strength = 1000 volt/mil.

Figure 6 - Thickness of Potant Above Cell vs Top Cover Thickness for $V = 3000$ volt dc and Top Cover Dielectric Strength = 3000 volt/mil.

Figure 7 - Thickness of Pottant Above Cell vs Top Cover Thickness for $V_0 = 3000$ volt dc and Top Cover Dielectric Strength = 5000 volt/mil.

Table 2

VARIATION OF NOCT WITH EMISSIVITY, GLASS SUPERSTRATE DESIGN

<u>ϵ</u>	<u>$T_{cell} (^{\circ}C)$</u>
.2	56.0
.6	52.1
.9	49.5

Figure 8
NOCT VS. EMISSIVITY

Table 3 provides a detailed breakdown of the energy loss components for the emissivity values previously noted (Table 2). Radiative and convective losses are tabulated with a further breakdown to show losses from both top and bottom sides of the module. The fact that the relative radiative loss increases with increasing emissivity and decreasing cell temperature (also indicative of module surface temperature changes) illustrates the nonlinear nature of the radiative loss and the shift to reradiation as the dominant heat loss mechanism at higher values of emissivity.

Table 4 shows a comparison of the superstrate and substrate module designs (Design No. 1 and 2, respectively)*. This table presents a detailed breakdown of the radiative and convective heat loss mechanisms. Both designs lose more heat by radiation than by convection with Design #2 (wood product substrate) being the more efficient radiator for the assumed emissivity values.

The surface heat transfer coefficient used to calculate convective heat loss is calculated by the following equation:

$$h = A (\Delta T_{\text{cos} X})^{1/3} + BV \quad (12)$$

where h = surface heat transfer coefficient, $\text{w}(\text{m}^2\text{C})^{-1}$
 A = coefficient for natural convection component of h
 B = coefficient for forced convection component of h
 V = wind velocity, meter/sec

*See the Triannual Report for period ending 31 March 1980.

Table 3

COMPARISON OF CONVECTIVE/RADIATIVE LOSSES WITH
VARIABLE EMISSIVITY-OPTICALLY HOT DESIGN NO. 1

<u>ϵ</u>		<u>Radiation</u>			<u>Convection</u>
		<u>Sky</u>	<u>Ground</u>	<u>Total</u>	
.2	Top	-3.00	- .26	= -3.26 watt	-2.29 watt
	Bottom	- .09	- .46	= - .55 watt	-2.28 watt
	Module Percent		45.5%		54.5%
.6	Top	-2.75	-		-1.99 watt
	Bottom	- .25		.6 watt	-1.98 watt
	Module Percent				47.2%
.9	Top	-2.59	- .20	= -2.79 watt	-1.80 watt
	Bottom	- .36	-1.64	= -2.00 watt	-1.79 watt
	Module Percent		57.2%		42.8%

Table 4

COMPARISON OF CONVECTIVE/RADIATIVE LOSSES
DESIGN NOS. 1 AND 2

<u>Case</u>		<u>Radiation</u>			<u>Convection</u>
		<u>Sky</u>	<u>Ground</u>	<u>Total</u>	
Design No. 1	Top	-2.75	- .22	= -2.97 watt	-1.99 watt
Superstrate	Bottom	- .25	-1.21	= -1.46 watt	-1.98 watt
	Module Percent		52.8%		47.2%
Design No. 2	Top	-2.78	- .23	= -3.01 watt	-2.02 watt
Substrate	Bottom	- .33	-1.41	= -1.74 watt	-1.53 watt
	Module Percent		57.2%		42.8%

Design No. 1 - Glass Superstrate Design Bottom Emissivity = .6
Design No. 2 - Substrate Design Bottom Emissivity = .9

To test the sensitivity of the NOCT to this parameter, calculations were made using a value of h both higher and lower than the reference case. The input parameters and results are shown in Table 5. The choice of coefficients A and B is based on discussions with JPL personnel. As can be seen, the temperature varies only 4.4°C from the two extremes showing little sensitivity to h .

The thermal model was rerun using white EVA for both the superstrate and substrate design families, and the results are presented in Figures 9 and 10. For the "optically-cold" case (i.e., assuming that 10% of the incident sunlight is absorbed in the encapsulant cover layers above the cell), the cell temperature remains at 52°C for the superstrate design, and increases to 51°C for the substrate design. The temperature distributions for the current superstrate design configuration, namely that a layer of white mylar (emissivity = .9) is now interposed between the aluminum foil and the external environment, are shown in Figures 11 and 12.

The optical and thermal models have been combined to permit a faster method of evaluating the thermal and optical performance of a candidate design than by running the two models separately. The thermal model was also improved by using an interactive data input procedure which permits fast program turnaround at a time-share terminal.

The optical model includes absorption, but not emission along the optical path. In effect, this decouples the optical and thermal models and allows the optical model to be used as a pre-processor in the temperature calculations. To estimate the resultant error in ignoring reradiation from the cell through the encapsulant and cover layers, an analysis was performed in which these layers are assumed to be completely transparent to

Table 5

SENSITIVITY STUDY - CONVECTIVE LOSS +15%
SUPERSTRATE DESIGN

Convective Heat Transfer Coefficient, $W(m^2 \text{ } ^\circ K)^{-1}$

Top Surface: $h_t = 1.247 (\Delta T \cos \gamma)^{1/3} + 2.685 \text{ V}$

Bottom Surface: $h_b = 1.079 (\Delta T \cos \gamma)^{1/3} + 2.685 \text{ V}$

Emissivity-Top:	.9 (Design #1)	.9 (Design #2)
-Bottom :	.6 (Design #1)	.9 (Design #2)
Velocity (Wind)-V:	1 m/s	
Tilt:	40° from horizontal	
Air Temperature:	20°C	
Ground Temperature:	20°C	
Sky Temperature:	-5.2°C	
Solar Flux, q''	.08 W/cm ²	

RESULTS

<u>Case</u>	<u>Cell Temperature (°C)</u>
Increased Heat Transfer Coefficient	50.1
Decreased Heat Transfer Coefficient	54.5
Reference Case	52.1

SUPERSTRATE DESIGN: PRELIMINARY NOCT RESULTS

HUGHES

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	$k \left(\frac{w}{cm^{\circ}C} \right)$	$t \text{ (mm)}$
TEMPERED GLASS	7.8×10^{-3}	3.175
EVA	3.3×10^{-3}	.127
CELL	.835	.254
	3.3×10^{-3}	.127
ALUMINUM FOIL	2.06	.0254

$\epsilon = .8$

Figure 9 -- Temperature Distribution in Glass Superstrate Module with Aluminum Foil on Back Side and White EVA as Pottant Below Cell - Optically Cold Condition

SUBSTRATE DESIGN: PRELIMINARY NOCT RESULTS

HUGHES

Figure 10 - Temperature Distribution in Substrate Module Design with White EVA as Pottant Below Cell - Optically Cold Condition

SUPERSTRATE DESIGN: PRELIMINARY NOCT RESULTS

HUGHES

Figure 11 - Temperature Distribution in Glass Superstrate Module with White Mylar on Back Side and White EVA as Pottant Below Cell - Optically Cold Condition

SUPERSTRATE DESIGN: PRELIMINARY NOCT RESULTS

HUGHES

$\epsilon = .9$

	$k \left(\frac{W}{cm^{\circ}C} \right)$	$t \text{ (mm)}$
TEMPERED GLASS	7.8×10^{-3}	3.175
EVA	3.3×10^{-3}	.127
CELL	.835	.254
	3.3×10^{-3}	.127
ALUMINUM	2.06	.0254
MYLAR	18×10^{-3}	.0254

$\epsilon = .9$

Figure 12 - Temperature Distribution in Glass Superstrate Module with White Mylar on Back Side and White EVA as Pottant Below Cell - Optically Hot Condition

the radiant energy emitted from the cell. In other words, it was assumed that the front surface of the cell radiated directly to the ground and to the sky, and that there was no thermal or optical interaction with the pottant and cover layers. The results are shown in Figures 13 and 14 for the "optically hot" condition. A comparison between Figures 13 and 14 of this report and earlier results indicates that inclusion of cell reradiation to the sky and ground does not yield temperature distributions significantly different from those calculated on the assumption that cell reradiation is negligible.

The sensitivity of the computed temperature distribution in a module can be viewed as consisting of two parts: (a) sensitivity to values of material properties such as thermal conductivity, and (b) sensitivity to modelling techniques and assumptions. The results for sensitivity resulting from variations in thermal conductivity for both the superstrate and substrate classes of module designs will be discussed in the next report.

Sensitivity to modelling techniques and assumptions has been partially examined in previous reports. The influence of back side emissivity on NOCT for the superstrate design was found to be significant. Also, a variation of $\pm 15\%$ in the convective heat transfer coefficient gave rise to above a $\pm 2^{\circ}\text{C}$ variation in NOCT for the superstrate design. In Figure 15 it is shown that a $\pm 10^{\circ}\text{C}$ uncertainty in sky temperature gives rise to about a $\pm 1.4^{\circ}\text{C}$ uncertainty in NOCT, and that $\pm 10^{\circ}\text{C}$ uncertainty in ground temperature gives rise to about a $\pm 1.5^{\circ}\text{C}$ uncertainty in NOCT. The results shown in Figure 15 are for the substrate design.

SUPERSTRATE DESIGN: PRELIMINARY NOCT RESULTS

HUGHES

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	$k \left(\frac{w}{cm^2 C} \right)$	$t \text{ (mm)}$
TEMPERED GLASS	7.8×10^{-3}	3.175
EVA	3.3×10^{-3}	.127
CELL	.835	.254
	3.3×10^{-3}	.127
ALUMINUM FOIL	2.08	.0254

$\epsilon = .6$

Figure 13 - Temperature Distribution for Direct Cell Reradiation to Sky and Ground: Glass Superstrate Module - Optically Hot Condition

SUBSTRATE DESIGN: PRELIMINARY NOCT RESULTS

HUGHES

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	$k \left(\frac{w}{cm^{\circ}C} \right)$	t (mm)
KORAD	1.88×10^{-3}	.127
EVA	3.3×10^{-3}	.0762
CELL	.835	.254
	3.3×10^{-3}	.1778
WOOD PRODUCT	4.0×10^{-3}	6.35
EVA	3.3×10^{-3}	.127

$\epsilon = .9$

.21

50.6	50.5
50.7	50.6
50.8	50.7
50.8	50.8
50.7	50.6
48.2	48.1
45.6	45.6
45.6	45.5

ABSORBED HEAT (w) TEMPERATURE (°C)

Figure 14 - Temperature Distribution for Direct Cell Reradiation to Sky and Ground: Substrate Module - Optically Hot Condition.

Figure 15 - Sensitivity of Cell Temperature to Changes in the Ground and Sky Temperature - Substrate Design

3.4 STRUCTURAL RESULTS

The MSC NASTRAN finite-element model for both the superstrate and substrate designs are now operational. These models include a thermal as well as structural analysis capability. Temperature distributions calculated by means of MSC/NASTRAN and CINDA 3G show are in excellent agreement. In the MSC/NASTRAN analysis, the calculated temperatures are automatically stored on a tape or disc file for subsequent use in the finite-element thermal-stress analysis. A typical stress distribution for a superstrate module design is shown in Figure 16.

Another MSC/NASTRAN model has been developed to investigate the ability of candidate modules to resist deflection when subjected to uniform pressure loading such as wind, dead-weight, and snow. This model has the capability of handling the expected nonlinear (i.e., deflection vs. load is not a linear relationship) nature of uniform loading on the module, which can be considered as a thin flat plate. At present, the module is being treated as a flat plate simply supported around periphery of the plate. The model can be easily modified to take into account stiffening ribs and different edge clamping conditions. The model will be validated initially by comparing computed deflections with results of previous JPL analyses reported by Donald M. Moore, "Low Cost Solar Array Project: Proposed Method for Determining the Thickness of Glass in Solar Collector Panels", JPL Report No. 5101-148, 1 March 1980.

3.5 LIFE COST OPTIMIZATION

The following algorithm which is a modification that was presented by Ron Ross at the 13th IEEE Photovoltaic Specialist Conference will be used for rating the cost effectiveness of the panels.

MAXIMUM PRINCIPAL STRESS, $\Delta T = 100^{\circ}\text{C}$ UNIFORM

BOUNDARY CONDITIONS

- PLANE SECTIONS
REMAIN PLANE
- FREE TO GROW
IN PLANE (X)
AND THROUGH
THICKNESS (Y)

MAT'L	MATERIAL PROPERTIES			
	E	G	ν	α IN/IN/ $^{\circ}\text{C}$
GLASS	10×10^6		.22	9.2×10^{-6}
ENC	7×10^2		.40	100×10^{-6}
CELL	17×10^6		.29	4.68×10^{-6}
ALUM	10×10^6		.33	23.4×10^{-6}

Figure 16 - Superstrate Design Stress Distribution at NOCT Conditions

$$\left(\frac{\text{BOPC} + \left(\frac{\text{IAC} + \sum_{n=1}^L C_n (1+k)^{-n}}{\text{PE} \times \frac{1\text{kW}}{\text{m}^2}} \right)}{(\text{AI}) \times \left(\sum_{n=1}^L P_n (1+k)^{-n} \right) \times \left(\frac{1\text{kW}}{\text{m}^2} \right)} \right) = \text{PC}_L$$

BOPC = balance of plant costs \$/kW

IAC = initial array cost \$/m²

L = plant life years

C_n = operating cost in year n \$/m²

k = discount rate

PE = plant efficiency

AI = annual insulation Kwh/m²/year

P_n = fraction of initial power in year n

PC_L = power cost over life of plant in \$/Kwh

A given module design's cost will be integrated into the initial array cost and any replacement costs (C_n). The module's efficiency will be factored into the plant efficiency number.

4.0 CONCLUSIONS AND RECOMMENDATIONS

There are no conclusions and recommendations for this period.

5.0 PLANNED ACTIVITIES

During the next four month period Phase I of the program will be completed. The optical, electrical, thermal, and structural analyses will be completed and an optimum design selected. A test plan for Phase II will also be submitted to JPL for critique. Phase II will begin. Experimental modules and coupons will be built to begin verifying the predicted ability of the analysis methods.

APPENDIX A

Optical Model Report

Prepared by:

C. M. Perrygo

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I. SUMMARY

This report details the development of a procedure and equations for performing the optical analysis of proposed encapsulation systems. The purpose of this analysis is to determine the effective transmittance of the encapsulation system, and the fraction of incident solar energy absorbed in the various encapsulant layers. The solar spectrum, solar cells, some basic optical principles, the radiosity-irradiation network, and the internal enhancement phenomenon are reviewed in the first part of this report. These principles and results are then combined into a general procedure for performing the optical analysis.

This procedure consist of dividing the solar spectrum into a number of spectral bands of equal solar irradiation. For each spectral band, the radiosity-irradiation network is used to calculate the fraction of solar irradiation absorbed into each layer of the encapsulation system. These values are then adjusted to account for enhancement effects due to internal reflections between intercell areas and the solar cells. These corrected values for each spectral bandwidth are then combined with the relative spectral response of the solar cell to obtain the effective transmittance, and the fraction of energy absorbed in the layers of the encapsulant system. The analysis is relatively simple and is easily carried out using a calculator, or digital computer for repeated use.

II. NOMENCLATURE

A_a	Total area of solar array
A_i	Area of intercell areas in solar array
a	Absorption coefficient
$C(\lambda)$	Solar cell spectral power conversion efficiency
CI	Correction factor for index of refraction applied to internal enhancement effect
$E(\theta)$	Intensity per unit area of surface of emitted radiation at an angle θ from the normal to the surface
E_{CIR}	Fractional increase in solar irradiation on circular solar cells due to internal enhancement
E_o	Intensity per unit area of surface of emitted radiation in the direction normal to the surface
E_{SQ}	Fractional increase in solar irradiation on square solar cells due to internal enhancement
$F_{0-\lambda}$	Percentage of solar irradiation in spectral band 0 to λ
$FQ_{1S}, FQ_{1L}, FQ_{2S}, FQ_{2L}, FQ_c$	Total fraction of solar irradiation absorbed in encapsulant surface 1, layer 1, surface 2, layer 2, and solar cells, respectively
$I(\lambda)$	Spectral solar irradiation
M	Number of spectral bands that the solar spectrum is divided into
N	Index of refraction
$P(\lambda)$	Solar cell spectral power output
P_{TOT}	Total power output of solar cell
$PE(\lambda)$	Spectral power output of encapsulated solar cell
PE_{TOT}	Total power output of encapsulated solar cell
Q^+	Radiosity - the radiant flux leaving a surface
Q^-	Irradiation - the radiant flux impinging on a surface

$Q_{1S}, Q_{2S}, Q_1, Q_2, Q_c$	Fraction of solar irradiation absorbed in encapsulant surface 1, layer 1, surface 2, layer 2, and solar cells, respectively
R	Radius of circular solar cells
S	Length of side of square solar cells
T	Transmittance of a layer
$T(\lambda)$	Spectral transmittance of an encapsulation system
T_{EFF}	Effective transmittance of an encapsulation system
t	Thickness of a layer
W	Distance between edges of square solar cells
α	Turbidity factor; or absorptivity of a surface
β	Turbidity factor
θ	Angle of incidence or refraction
θ_c	Critical angle for total internal reflection
λ	Wavelength
λ_i	Subscript indicating i^{th} spectral band
τ	Transmissivity of a surface
ρ	Reflectivity of a surface
ρ_i	Reflectivity of the intercell areas

III. INTRODUCTION

This study deals with the task of performing an optical analysis of proposed solar cell encapsulation systems. By optical analysis, I mean the study of the radiant energy transfer that occurs between the various layers of an encapsulation system when exposed to sunlight. An encapsulation system is the combination of substrates, pottants, top covers, etc., in which solar cells are sealed. The encapsulation system provides structural support and protection from the environment. A solar array consists of many encapsulated solar cells. Figure 1 shows cross sections through three proposed solar arrays detailing the various layers making up the different encapsulation systems. Plan views of two solar arrays showing the two different solar cell shapes, square and circular, and their arrangement in the arrays are shown in Figure 2.

The primary purpose of the optical analysis is to predict the effective transmittance of light to the solar cells in an encapsulated array. For light of normal incidence, an effective transmittance of at least 90 percent is required. Effective transmittance is the ratio of the power output of an encapsulated solar cell to the power output of a bare solar cell. To determine the effective transmittance requires calculating a weighted average transmittance of the encapsulation system where the weighting corresponds to the spectral response of the solar cell.

Also required of the optical analysis is the fraction of the total solar energy incident on an array absorbed in each layer of the encapsulation system. This information is used in the thermal analysis of the encapsulation system.

The next section of this report reviews the solar spectrum, solar cells, some basic optical principles, the radiosity-irradiation network analysis, and the internal enhancement phenomenon. Following that, Section V combines these principles and results into a general procedure for performing the optical analysis.

Figure 1. Cross sectional views of three proposed encapsulation systems. Not to scale.

SQUARE SOLAR CELLS

CIRCULAR SOLAR CELLS

Figure 2. Plan views of solar arrays. Arrays are 1.2 meters square.

IV. REVIEW OF BASIC PRINCIPLES AND BACKGROUND INFORMATION

IV.A. SOLAR IRRADIATION

The total energy which irradiates a unit surface normal to the sun at the average Sun-Earth distance is called the solar constant. The NASA/ASTM standard value of the solar constant is 1353 Wm^{-2} ($429.2 \text{ BTU ft}^{-2} \text{ hr}^{-1}$).¹ The distribution of this energy as a function of wavelength is the extraterrestrial solar spectrum. Of concern in this study is the solar spectrum at ground level, since the efficiency of solar cells is wavelength dependent. Figure 3 shows the extraterrestrial and ground level spectral solar irradiance for different air mass values.

The absorption of solar energy by the atmosphere follows the log decrement law for most of the solar spectrum. In the infrared region, there is also selective absorption by the polyatomic gases carbon dioxide and water vapor, and absorption and scattering by particulates and water droplets. As the air mass or turbidity increases, the total solar irradiance decreases. There is also an increase in the relative amount of energy in the infrared region and a decrease in the visible and ultraviolet. Table I points these effects out.

In addition to direct solar irradiation, there is diffuse irradiation from the sky due to the scattering mentioned above. On a horizontal surface, the total irradiation from the sky is about 20 percent of that due to direct solar irradiation. The ratio of diffuse to direct irradiation is high in the ultraviolet region (accounting for why the sky is blue), but drops rapidly in the visible and infrared regions.

IV.B. SOLAR CELLS

The efficiency of solar cells is very dependent upon the wavelength of the incident radiation. An exact analysis of solar cell efficiency requires knowledge of the incident spectral irradiation, spectral power conversion efficiency, current-voltage characteristics, and series resistance of the solar cell.² However, an approximate method can be used based on the cell spectral power conversion efficiency and incident spectral irradiation.

The spectral power output $P(\lambda)$ of a solar cell is equal to the spectral power conversion efficiency of the cell, $C(\lambda)$, and the spectral solar irradiation, $I(\lambda)$,

$$P(\lambda) = C(\lambda) \cdot I(\lambda) \quad (1)$$

Figure 3. Solar spectral irradiance for different air mass values, assuming U.S. standard atmosphere, 20 mm of precipitable water vapor, 3.4 mm of ozone, very clear air ($\alpha = 1.3$, $\gamma = 0.02$).

TABLE I. SOLAR IRRADIANCE FOR DIFFERENT AIR MASS VALUES AND TURBIDITY FACTORS, U.S. STANDARD ATMOSPHERE, 20mm OF PRECIPITABLE WATER VAPOR, 3.4mm OF OZONE

AIR MASS	SOLAR ZENITH ANGLE (DEGREES)	TURBIDITY FACTORS		TOTAL IRRADIANCE W/m^2	RATIO OF TOTAL IRRADIANCE TO SOLAR CONSTANT %	FRAC. OF THE TOTAL ENERGY IN THE		
		α	β			UV. $\lambda < 0.4 \mu m$	VIS. $0.4 \mu m < \lambda < 0.72 \mu m$	INF. $\lambda > 0.72 \mu m$
0	0			1353.0	100.0	8.7	40.1	51.1
1	0	1.30	0.02	896.2	70.7	4.8	46.9	48.3
4	75.5	1.30	0.02	595.2	44.0	1.23	44.2	54.5
7	81.8	1.30	0.02	413.8	30.6	0.35	39.4	60.3
10	84.3	1.30	0.02	302.5	22.4	0.102	34.7	65.2
1	0	1.30	0.04	924.9	68.4	4.6	46.4	49.0
4	75.5	1.30	0.04	628.9	39.1	1.04	42.1	56.9
7	81.8	1.30	0.04	342.0	25.3	0.26	35.9	63.8
10	84.3	1.30	0.04	234.5	17.3	0.065	30.2	69.6
1	0	0.66	0.085	889.2	66.7	4.7	46.4	48.9
4	75.5	0.66	0.085	448.7	33.2	1.14	42.4	56.5
7	81.8	0.66	0.085	285.2	18.9	0.30	36.3	63.4
10	84.3	0.66	0.085	183.8	11.4	0.08	30.7	69.2
1	0	0.66	0.17	800.2	59.1	4.5	45.4	50.1
4	75.5	0.66	0.17	303.1	27.4	0.88	38.3	60.8
7	81.8	0.66	0.17	133.3	9.85	0.14	30.0	69.8
10	84.3	0.66	0.17	63.4	4.67	0.039	22.9	77.1

Typical spectral power conversion efficiencies of silicon and gallium arsenide cells are shown in Figure 4. These curves are for total efficiencies of 19 percent for the silicon cell and 24 percent for the gallium arsenide cell. Figure 5 shows the spectral power output of these cells for a solar spectrum of air mass one and turbidity factors $\alpha = 0.66$, $\beta = 0.085$. The total power output of the cell is the integral of Equation 1 over all wavelengths.

$$P_{TOT} = \int_0^{\infty} C(\lambda) \cdot I(\lambda) d\lambda \quad (2)$$

Consider now an encapsulated solar cell. The encapsulation system has a spectral transmittance $T(\lambda)$. The spectral power output of the encapsulated cell $PE(\lambda)$ is given by the equation

$$PE(\lambda) = C(\lambda) \cdot I(\lambda) \cdot T(\lambda) \quad (3)$$

Figure 4. Spectral power conversion efficiency for silicon and gallium arsenide solar cells

Figure 5. Spectral power outputs for silicon and gallium arsenide solar cells for air mass one, U.S. standard atmosphere, 20MM H₂O, 3.4MM O₃, $\alpha = 0.66$, $\beta = 0.085$.

As before, the total power output of the solar cell is the integral over all wavelengths

$$PE_{TOT} = \int_0^{\infty} C(\lambda) \cdot I(\lambda) \cdot T(\lambda) d\lambda \quad (4)$$

We can now determine the effective transmittance, T_{EFF} , of the encapsulation system from Equations 2 and 4

$$T_{EFF} = \frac{PE_{TOT}}{P_{TOT}} = \frac{\int_0^{\infty} C(\lambda) \cdot I(\lambda) \cdot T(\lambda) d\lambda}{\int_0^{\infty} C(\lambda) \cdot I(\lambda) d\lambda} \quad (5)$$

Note that in Equation 5 the relative spectral response of the cell may be used in place of spectral power conversion efficiency since this term appears in both integrals. Also, the spectral solar irradiation term may be replaced with a relative spectral irradiation term. Consequently, we need not know the actual power conversion efficiency or solar irradiation as a function of wavelength, but merely how these quantities vary with wavelength.

IV.C. REFLECTION AND REFRACTION

The reflection and refraction of light at the interface of two different dielectric materials with different indices of refraction is shown in Figure 6. The portion of light reflected from the interface is given by Fresnel's equation³

$$\rho(\theta_1) = \frac{1}{2} \left[\frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)} + \frac{\tan^2(\theta_1 - \theta_2)}{\tan^2(\theta_1 + \theta_2)} \right] \quad (6)$$

where θ_1 is the angle of incidence, and θ_2 is the angle of refraction. Snell's law gives the relationship between the angles of reflection and refraction

$$N_1 \sin \theta_1 = N_2 \sin \theta_2$$

Figure 5. Reflection and refraction at the interface of two dielectric materials with different indices of refraction.

where the N 's are the indices of refraction for the two materials. If N_2 is less than N_1 , then all of the incident light is reflected for angles of incidence greater than or equal to the critical angle θ_c , where θ_c is given by the relationship

$$\theta_c = \sin^{-1}\left(\frac{N_2}{N_1}\right) \quad (7)$$

This is known as total internal reflection. If N_2 is greater than N_1 , then some portion of the incident light is always refracted through the interface. Figure 7 shows how the reflectivity of the interface varies with the angle of incidence for N_2 both less than and greater than N_1 . For small angles of incidence, Equation 6 can be approximated by the relationship

$$\rho = \left(\frac{N_1 - N_2}{N_1 + N_2}\right)^2 \quad (8)$$

The reflected light considered above is of a specular quality. The light reflected from a gray surface is diffuse and quite often can be approximated by a relationship similar to Lambert's law

$$E(\theta) = E_0 \cos \theta \quad (9)$$

Figure 7. Reflectivity from the interface of two materials with different indices of refraction.

where $E(\theta)$ is the intensity per unit area of the surface in a direction at an angle θ from the normal to the surface, and E_0 is the intensity in the direction normal to the surface.

IV.D. RADIOSITY-IRRADIATION NETWORK ANALYSIS

Consider a gray non-opaque surface as depicted in Figure 8. This surface could be that of a sheet of glass. Q_1^- and Q_2^- are incoming radiant fluxes, and Q_1^+ and Q_2^+ are outgoing radiant fluxes. The subscripts -1 and -2 denote which side of the surface the fluxes are referenced to, while the superscripts - and + denote incoming and outgoing fluxes respectively. The incoming flux is called the irradiation and the outgoing flux the radiosity.

Figure 8. Radiant fluxes for a surface

Let α , ρ , and τ stand for the absorptivity, reflectivity, and transmissivity of the surface. We can now define the radiosities in terms of the irradiations. Ignoring any radiation emission from the surface itself, these relationships are

$$Q_1^+ = \rho Q_1^- + \tau Q_2^- \quad (10)$$

$$Q_2^+ = \tau Q_1^- + \rho Q_2^- \quad (11)$$

The radiosity-irradiation equations are easily applied to radiation exchange involving several surfaces and absorbing medium. Consider a sheet of glass of thickness t as shown in Figure 9. Surface A has reflectivity ρ_A , transmissivity τ_A and zero absorptivity α_A ($\rho_A + \tau_A = 1$). Surface B is opaque, with reflectivity ρ_B , and absorptivity α_B ($\rho_B + \alpha_B = 1$). The glass material has an absorption coefficient a . The transmittance T of the glass layer is obtained from the equation

$$T = e^{-at} \quad (12)$$

Figure 9. Radiant fluxes in a sheet of glass with one opaque surface.

To solve for the radiation exchange in the layer of glass, define the radiosities and irradiances for each side of each surface involved in the radiation exchange. Figure 9 shows these six radiant fluxes. We can now write six simultaneous equations relating these six radiant fluxes. These equations are, with irradiation Q_1^- normalized to a value of one

$$Q_1^- = 1$$

$$Q_1^+ = \rho_A Q_1^- + \tau_A Q_2^-$$

$$Q_2^- = \tau Q_3^+$$

(13)

$$Q_2^+ = \tau_A Q_1^- + \rho_A Q_2^-$$

$$Q_3^- = \tau Q_2^+$$

$$Q_3^+ = \rho_B Q_3^-$$

This system of equations is given below in matrix form with the normalized value of Q_1^- substituted in.

$$\begin{bmatrix} 1 & -\tau_A & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -T \\ 0 & -\rho_A & 1 & 0 & 0 \\ 0 & 0 & -T & 1 & 0 \\ 0 & 0 & 0 & -\rho_B & 1 \end{bmatrix} \begin{bmatrix} Q_1^+ \\ Q_2^- \\ Q_2^+ \\ Q_3^- \\ Q_3^+ \end{bmatrix} = \begin{bmatrix} \rho_A \\ 0 \\ \tau_A \\ 0 \\ 0 \end{bmatrix} \quad (14)$$

$$[M] \cdot \bar{Q} = \bar{C} \quad (15)$$

The unknown radiant fluxes are easily solved for after inverting matrix M.

$$\bar{Q} = [M]^{-1} \cdot \bar{C} \quad (16)$$

Once the radiant fluxes are known, the amount of radiation absorbed in the glass and in layer B is readily obtained by considering an energy balance.

$$\text{Energy absorbed in glass} = Q_2^+ + Q_3^+ - Q_2^- - Q_3^- \quad (17)$$

$$\text{Energy absorbed in layer B} = Q_3^- - Q_3^+ = \alpha_B Q_3^- \quad (18)$$

This method of defining radiosities and irradiances for each side of each surface involved in a radiation exchange is easily expanded to many surfaces and absorbing mediums. The resulting system of simultaneous equations is readily solved by digital computer, or for smaller problems, by programmable calculator. Using this method of radiosity-irradiation equations to account for the multiple internal reflections is far easier than more traditional method of developing a set of infinite series, and determining their sums.

IV. E. THE INTERNAL ENHANCEMENT PHENOMENON

The basic solar cell array consists of solar cells embedded in some type of encapsulant system as shown in Figures 1 and 2. Due to internal reflections, described in the previous section, some of the light reflected from the intercell areas strikes the cells. This enhancement of solar cell irradiation is referred to as the zero depth concentrator phenomenon by Mark and Volk,³ who have investigated this enhancement effect for several different cell patterns. Their findings that are applicable to this analysis are discussed in this section.

The assumptions made by Mark and Volk in their investigation are:

1. The incident light on the solar cell array is assumed normal to the array.
2. The light reflected from the intercell areas is of a Lambertian form, with a hemispherical reflectivity of one.
3. The solar cell array has a single homogeneous, non-absorbing top cover. No consideration is made of any pottants or spacer materials between the top cover and the solar cell or the intercell areas.
4. Fresnal's equation, Equation 6, is approximated by its small angle simplification, Equation 8, for angles less than the critical angle. The error introduced by this approximation is relatively small.

Mark and Volk found in their analysis that the enhancement effect in the solar cells occurs primarily along the edges of the solar cells. To show this, consider two semi-infinite sheets, one a white reflector and the other a solar cell, under a top cover of thickness t as shown in Figure 10. The incident light is of unit intensity. Due to internal reflections (and the assumption of a non-absorbing top cover), the light intensity in the array will be greater than one. Figure 11 shows the increase of light intensity in the array as a function of normalized distance. We see that the enhancement effect is limited almost entirely to a depth of about $6t$ units. Figure 11 also shows that the enhancement effect is greater for higher index of refraction top covers. This is as we would suspect, since the critical angle decreases for an increase in the index of refraction. A smaller critical angle results in more of diffusely reflected light from the intercell areas being internally reflected.

Figure 10. Slice through infinite array consisting of semi-infinite cell and semi-infinite white reflecting region.

Figure 11. Enhancement effect due to multiple internal reflections in the solar array in Figure 10.

Now consider the array pictured in Figure 12. This array consists of two semi-infinite solar cells with an infinitely long white reflecting stripe in between. The importance of considering this configuration can be seen by looking at the plan views of the actual solar cell arrays shown in Figure 2. In the case of rectangular cells, the width of the intercell spaces is much smaller than the dimensions of the solar cells themselves. This allows us to treat these intercell areas as infinitely long stripes, making the results obtained from the configuration shown in Figure 12 directly applicable to the array containing rectangular cells.

Figure 12. Slice through infinite area consisting of two semi-infinite solar cells separated by an infinitely long white reflecting stripe.

Figure 13 shows the enhancement as a function of normalized distance for the infinite stripe array pictured in Figure 12. The vertical bars represent the width of the white reflecting area. These results are for a top cover index of refraction of 1.5. As shown before, the enhancement effect in the solar cells is concentrated in the first few normalized units. Figure 13 also shows that the enhancement effect initially increases for wider reflecting

CELL ENHANCEMENT IN INFINITE STRIPE GEOMETRY
MULTI REFLECTION

Figure 13. Enhancement effect due to multiple internal reflections in the solar array in Figure 12.

regions but then levels off as a saturation point is reached. As the reflecting stripe becomes very wide, the limiting case shown in Figure 11 is reached.

Mark and Volk also analyzed the case of a circular solar cell surrounded by an infinite white reflecting surface as shown in Figure 14. Their results for this geometry are shown in Figure 15, where the integrated enhancement in the solar cell is plotted as a function of solar cell radius, in normalized units. The index of refraction of the top cover is 1.5. The integrated enhancement in the solar cell decreases as the radius of the solar cell increases. This is as we would suspect, since the enhancement effect occurs only along the edge of the solar cell. As the radius of the solar cell increases, its total area goes up as the radius squared, but the enhanced area only goes up in proportion to the radius. Consequently, the integrated enhancement falls off as the inverse of the radius.

Figure 14. Circular solar cell surrounded by infinite white reflecting region under top cover of thickness t .

Figure 15. Average enhancement over entire cell for circular cell surrounded by infinite reflecting region.

V. ANALYTICAL PROCEDURE AND EQUATIONS

The task at hand now is to combine the spectral characteristics of the solar irradiation and solar cell power output with the radiosity-irradiation equations, optical principles, and solar cell enhancement phenomenon discussed in Section IV.

V.A. ACCOUNTING FOR SPECTRAL VARIATIONS

As discussed in Section III, the required results from this analysis are the effective transmittance to the solar cell, and the percentage of light absorbed in the various layers making up the encapsulation system.

The effective transmittance is given by Equation 5:

$$T_{EFF} = \frac{P_{E_{TOT}}}{P_{TOT}} = \frac{\int_0^{\infty} C(\lambda) \cdot I(\lambda) \cdot T(\lambda) d\lambda}{\int_0^{\infty} C(\lambda) \cdot I(\lambda) d\lambda} \quad (5)$$

The spectral power conversion efficiency $C(\lambda)$ of a particular solar cell and the spectral solar irradiation $I(\lambda)$ are independent of the encapsulation system. Only the spectral transmittance $T(\lambda)$ depends on the encapsulation system under study. Variation in the transmittance with wavelength is due to the absorption characteristics of the encapsulants and the spectral characteristics of any antireflection coating. The spectral transmittance will therefore be a fairly smooth function of wavelength. This allows us to break the integrals in Equation 5 into a number of spectral bands and use values of $C(\lambda)$, $I(\lambda)$ and $T(\lambda)$ that are averaged over each of the spectral bands. Since the spectral power conversion efficiency (or relative spectral response) and the spectral solar irradiation are independent of the encapsulation system, averaged values over each of the spectral bands for these two terms, once calculated, can be saved and used over and over. Only the averaged values of transmittance need be calculated for each encapsulation system. The integrals in Equation 5 can thus be approximated as

$$\int_0^{\infty} C(\lambda) \cdot I(\lambda) \cdot T(\lambda) d\lambda \approx C_{\lambda 1} I_{\lambda 1} T_{\lambda 1} + C_{\lambda 2} I_{\lambda 2} T_{\lambda 2} + \dots + C_{\lambda M} I_{\lambda M} T_{\lambda M} \quad (19)$$

$$\int_0^{\infty} C(\lambda) \cdot I(\lambda) d\lambda \approx C_{\lambda 1} I_{\lambda 1} + C_{\lambda 2} I_{\lambda 2} + \dots + C_{\lambda M} I_{\lambda M} \quad (20)$$

where the subscripts $\lambda_1, \lambda_2, \dots, \lambda_M$ refer to M spectral bands covering the solar irradiation spectrum, and $C_{\lambda i}$, $I_{\lambda i}$ and $T_{\lambda i}$ are the solar cell spectral response, solar irradiation, and encapsulation transmittance averaged over spectral band λ_i .

If we choose the spectral bands such that the amount of solar irradiation in each band is the same, then the solar irradiation quantities in each term in Equations 19 and 20 cancel out when substituted into Equation 5. Setting $I_{\lambda_1} = I_{\lambda_2} = \dots = I_{\lambda_M} = I_{\lambda}$, Equations 19 and 20 become

$$\int_0^{\infty} C(\lambda) * I(\lambda) * T(\lambda) d\lambda \approx I_{\lambda} (C_{\lambda_1} T_{\lambda_1} + C_{\lambda_2} T_{\lambda_2} + \dots + C_{\lambda_M} T_{\lambda_M}) \quad (21)$$

$$\int_0^{\infty} C(\lambda) * I(\lambda) d\lambda \approx I_{\lambda} (C_{\lambda_1} + C_{\lambda_2} + \dots + C_{\lambda_M}) \quad (22)$$

Substituting these results into Equation 5 and cancelling out the I_{λ} 's gives the following approximate equation for calculating the effective transmittance of an encapsulation system

$$T_{EFF} = \frac{PE_{TOT}}{P_{TOT}} \approx \frac{C_{\lambda_1} T_{\lambda_1} + C_{\lambda_2} T_{\lambda_2} + \dots + C_{\lambda_N} T_{\lambda_N}}{C_{\lambda_1} + C_{\lambda_2} + \dots + C_{\lambda_N}} \quad (23)$$

To determine the spectral bands such that each has the same solar irradiation, I have numerically integrated a tabulated set of solar irradiation versus wavelength data obtained from Reference 1. The results of this are presented in Table II, which gives the spectral solar irradiation data, the integrated solar irradiation, and the percentage of total solar irradiation in the spectral band 0 to λ as a function of wavelength λ . This data is for air mass one and clear, but polluted air, as might be found near large cities or industrial areas. Figure 16 is a graph of the data for the fraction of solar irradiation in wavelengths 0 to λ . Called out on Table II and listed in Table III are the wavelengths at five percent total solar irradiation intervals. This allows choosing 1, 2, 4, 5, 10, or 20 spectral bands depending on the smoothness of the $T(\lambda)$ function, or the availability of data on the optical properties of the encapsulant materials.

TABLE II. SOLAR SPECTRAL IRRADIANCE DATA. AIR MASS ONE, U.S. STANDARD
ATMOSPHERE, 20mm H₂O, 3.4 mm O₃, α 0.66, β 0.085

λ	I_A	I_{0-A}	F_{0-A}
0.280	0.0	0.0	0.00
0.295	0.0	0.0	0.00
0.300	4.1	0.0	0.00
0.305	11.4	0.0	0.01
0.310	30.5	0.2	0.02
0.315	79.4	0.4	0.05
0.320	207.8	1.1	0.13
0.325	269.5	2.3	0.26
0.330	331.6	3.8	0.43
0.335	383.4	5.6	0.63
0.340	431.3	7.8	0.88
0.345	449.2	9.8	1.11
0.350	480.5	12.2	1.37
0.355	498.0	14.8	1.84
0.360	513.7	17.1	1.93
0.365	561.3	19.8	2.23
0.370	603.5	22.7	2.56
0.375	609.4	25.8	2.90
0.380	608.0	28.8	3.24
0.385	609.8	31.9	3.58
0.390	623.9	34.9	3.93
0.395	691.2	38.2	4.30
0.400	849.9	42.1	4.73
0.405	992.8	46.7	5.26
0.410	1073.7	51.9	5.83
0.415	1104.5	57.3	6.44
0.420	1104.3	62.8	7.06
0.425	1086.5	68.3	7.57
0.430	1067.9	73.7	8.28
0.435	1100.1	79.1	8.89

5% $\oplus$
 $\lambda = 0.403$

λ	I_A	I_{0-A}	F_{0-A}
0.440	1215.5	84.9	9.54
0.445	1310.4	91.2	10.25
0.450	1388.4	98.0	11.01
0.455	1434.8	105.0	11.80
0.460	1452.2	112.2	12.61
0.465	1450.7	119.5	13.43
0.470	1451.2	126.7	14.24
0.475	1470.3	134.1	15.06
0.480	1503.4	141.5	15.90
0.485	1443.3	148.9	16.73
0.490	1435.2	156.0	17.53
0.495	1453.6	163.3	18.35
0.500	1451.2	170.5	19.16
0.505	1440.1	177.8	19.97
0.510	1416.8	184.9	20.78
0.515	1384.9	191.9	21.56
0.520	1390.0	198.8	22.34
0.525	1409.5	205.8	23.13
0.530	1406.9	212.9	23.92
0.535	1393.6	219.9	24.71
0.540	1371.7	226.8	25.48
0.545	1354.2	233.6	26.25
0.550	1336.8	240.3	27.00
0.555	1335.7	247.0	27.76
0.560	1319.2	253.7	28.50
0.565	1330.0	260.3	29.25
0.570	1338.4	267.0	29.99
0.575	1346.9	273.7	30.75
0.580	1346.7	280.4	31.51
0.585	1347.3	287.1	32.26

10% $\oplus$
 $\lambda = 0.443$

15% $\oplus$
 $\lambda = 0.475$

20% $\oplus$
 $\lambda = 0.505$

25% $\oplus$
 $\lambda = 0.537$

30% $\oplus$
 $\lambda = 0.570$

λ	I_A	I_{0-A}	F_{0-A}
0.590	1340.7	293.9	33.02
0.595	1329.4	300.5	33.77
0.600	1319.6	307.2	34.51
0.605	1311.0	313.7	35.25
0.610	1307.9	320.3	35.99
0.620	1294.2	333.3	37.45
0.630	1280.9	346.2	38.89
0.640	1272.1	358.9	40.33
0.650	1257.1	371.6	41.75
0.660	1244.2	384.1	43.16
0.670	1226.8	396.4	44.54
0.680	1209.9	408.6	45.91
0.690	1196.2	420.6	47.26
0.698	1010.3	429.5	48.26
0.700	1175.3	431.7	48.50
0.710	1157.4	443.3	49.81
0.720	1135.1	454.8	51.10
0.728	1003.1	463.3	52.06
0.730	1117.8	465.5	52.30
0.740	1095.1	476.5	53.54
0.750	1076.6	487.4	54.76
0.762	794.0	498.6	56.02
0.770	1039.2	505.9	56.85
0.780	1019.4	516.2	58.00
0.790	1000.3	526.3	59.14
0.800	981.2	536.2	60.25
0.808	874.4	541.8	60.88
0.825	931.6	559.0	62.80
0.830	921.8	563.6	63.33
0.835	912.4	568.2	63.84

35% $\oplus$
 $\lambda = 0.603$

40% $\oplus$
 $\lambda = 0.638$

45% $\oplus$
 $\lambda = 0.673$

50% $\oplus$
 $\lambda = 0.711$

55% $\oplus$
 $\lambda = 0.752$

60% $\oplus$
 $\lambda = 0.798$

(Continued next page)

(Table II, concluded)

λ	I_{λ}	$I_{0-\lambda}$	$F_{0-\lambda}$
0.846	476.2	575.8	64.70
0.860	506.4	582.7	65.47
0.870	453.8	587.5	66.01
0.875	449.2	589.8	66.26
0.887	448.9	595.1	66.87
0.900	448.9	601.0	67.53
0.907	455.2	604.1	67.88
0.915	461.6	607.8	68.29
0.925	279.0	611.5	68.71
0.930	221.8	612.8	68.85
0.940	313.4	615.4	69.16
0.950	296.5	618.5	69.49
0.955	321.1	620.0	69.87
0.965	344.4	623.3	70.04
0.975	576.9	628.0	70.56
0.985	544.8	633.8	71.19
1.018	617.5	652.7	73.34
1.082	512.8	688.9	77.41
1.094	464.1	694.8	78.07
1.098	503.7	696.7	78.28
1.101	504.8	698.2	78.45
1.128	135.1	706.9	79.42
1.131	152.2	707.3	79.47
1.137	143.1	708.2	79.57
1.144	191.2	709.4	79.70
1.147	174.5	709.9	79.76
1.178	399.3	718.8	80.76
1.189	402.2	723.2	81.26
1.193	424.0	724.9	81.44
1.222	391.8	736.7	82.77

65% @
 $\lambda = 0.851$

70% @
 $\lambda = 0.964$

75% @
 $\lambda = 1.044$

80% @
 $\lambda = 1.154$

λ	I_{λ}	$I_{0-\lambda}$	$F_{0-\lambda}$
1.236	390.8	742.2	83.39
1.264	329.2	752.2	84.52
1.276	342.6	756.3	84.98
1.288	347.3	760.4	85.44
1.314	298.3	768.8	86.38
1.335	190.6	773.9	86.96
1.384	5.7	778.8	87.50
1.432	44.6	780.0	87.64
1.457	85.4	781.6	87.82
1.472	77.4	782.8	87.96
1.542	239.3	793.9	89.20
1.572	222.6	800.8	89.98
1.599	216.0	806.7	90.64
1.608	208.5	808.7	90.86
1.628	206.7	812.4	91.28
1.644	197.9	816.0	91.69
1.650	195.7	817.2	91.82
1.678	181.9	822.1	92.37
1.732	161.5	831.7	93.46
1.782	136.7	839.2	94.29
1.862	4.0	844.8	94.92
1.955	42.7	847.0	95.17
2.008	69.4	850.0	95.50
2.014	74.7	850.4	95.55
2.057	69.5	853.6	95.90
2.124	70.0	858.2	96.42
2.156	66.0	860.3	96.67
2.201	66.1	863.3	97.00
2.266	61.6	867.5	97.47
2.320	57.2	870.7	97.83

85% @
 $\lambda = 1.277$

90% @
 $\lambda = 1.573$

95% @
 $\lambda = 1.892$

λ	I_{λ}	$I_{0-\lambda}$	$F_{0-\lambda}$
2.338	54.7	871.7	97.94
2.356	52.0	872.6	98.04
2.388	36.0	874.0	98.20
2.415	32.5	875.0	98.31
2.453	29.6	876.2	98.45
2.494	20.3	877.2	98.56
2.537	4.6	877.7	98.62
2.900	2.9	879.1	98.78
2.941	6.0	879.3	98.80
2.954	5.7	879.3	98.80
2.973	8.7	879.5	98.82
3.005	7.8	879.7	98.84
3.045	4.7	880.0	98.88
3.056	4.9	880.0	98.88
3.097	3.2	880.2	98.90
3.132	6.8	880.4	98.97
3.156	18.7	880.7	98.96
3.204	2.1	881.2	99.01
3.214	3.4	881.2	99.01
3.215	3.8	881.3	99.02
3.260	3.7	881.4	99.03
3.285	14.2	881.6	99.06
3.317	12.9	882.0	99.10
3.344	4.2	882.3	99.13
3.403	12.3	882.8	99.19
3.450	12.6	883.3	99.25
3.507	12.5	884.0	99.33
3.538	11.8	884.4	99.37
3.573	10.9	884.8	99.42
3.633	10.8	885.5	99.49

λ	I_{λ}	$I_{0-\lambda}$	$F_{0-\lambda}$
3.673	9.1	886.9	99.54
3.696	10.4	886.1	99.56
3.712	10.9	886.3	99.58
3.765	9.5	886.8	99.64
3.812	8.9	887.2	99.69
3.888	8.1	887.9	99.76
3.923	8.0	888.2	99.80
3.948	7.8	888.4	99.82
4.045	6.7	889.1	99.90
4.320	0.0	890.0	100.0

Figure 16. Percentage of solar irradiation in spectral 0 to λ . Air mass one, U.S. standard atmosphere, 20 MM precipitable water vapor, 3.4MM ozone, $\alpha = 0.66$, $\beta = 0.085$.

TABLE III. WAVELENGTHS AT 5 PERCENT INTERVALS FOR TOTAL SOLAR IRRADIATION IN SPECTRAL REGION 0 TO λ . AIR MASS ONE, U.S. STANDARD ATMOSPHERE, 20mm PRECIPITABLE WATER VAPOR, 3.4mm OZONE, $\alpha = 0.66$, $\beta = 0.085$

λ - Wavelength, μm

$F_{0-\lambda}$ - Percentage of solar irradiation in spectral band 0- λ

Total solar irradiation = 890 W/m^2

λ	$F_{0-\lambda}$	λ	$F_{0-\lambda}$
0.403	5	0.752	55
0.443	10	0.798	60
0.475	15	0.851	65
0.505	20	0.964	70
0.537	25	1.044	75
0.570	30	1.154	80
0.603	35	1.277	85
0.638	40	1.573	90
0.673	45	1.892	95
0.711	50	4.32	100

Once the spectral bands have been chosen, averaged values for $C(\lambda)$ and the optical properties of the encapsulant materials need to be determined. This information is not included here because the spectral variation of these quantities is not presently available.

The next step is to use the radiosity-irradiation network analysis to determine the percentage of incident light absorbed in each layer of the encapsulant system for each of the chosen spectral bands.

V.B. RADIOSITY-IRRADIATION EQUATIONS

The radiosity-irradiation network analysis, introduced in Section IV.D and developed in detail below, is straightforward and accurate in analyzing the specular reflections within an encapsulation system. Specular reflections typically account for most of the energy transfer in the array. To analyze the diffuse reflections and scattering within an encapsulation system, requires a significant increase in complexity in the radiation-irradiation network. It also requires knowledge of the scattering and diffuse reflectivity properties of the encapsulant materials. This includes the effects of weathering of the top cover. Since this information is not generally available at this time and may be difficult to determine, I have chosen to assume that all reflections are specular in the radiosity-irradiation network analysis. However, once this network is analyzed, internal reflections between the intercell areas and the solar cells are accounted for by applying enhancement correction factors based on the work of Mark and Volk. The amount of energy absorbed in the other layers making up the encapsulation system, as determined by the radiation network, are adjusted in proportion to the correction factors used for the solar cells. For the solar arrays using rectangular solar cells, the enhancement due to internal reflections is only a few percent. The enhancement in circular solar cells will be at most twenty percent. Consequently, an error of ten percent in determining the enhancement results in an error for the total energy absorbed by the solar cells of a fraction of a percent for rectangular cells, and only a few percent for circular cells. Therefore, I feel that it is not necessary to determine the enhancement effects to a high degree of accuracy. Because of this, I have decided to account for the enhancement phenomenon by developing general equations based on the existing work of Mark and Volk. This is in lieu of developing integral equations, requiring iterative numerical analysis to solve, to more accurately predict the enhancement effects for each of the encapsulation systems under consideration.

Based on the discussion in Section IV.D, I will now develop the radiosity-irradiation network equations for solving for the specular energy exchange in an encapsulation system. Figure 17 shows the radiosity-irradiation network model for a general cross section through a solar array, and indicates the radiative fluxes involved. The general array consists of a solar cell with two differing encapsulant layers covering it. This network model takes into account the reflectivity and transmissivity of each surface (ρ_1 , ρ_2 , τ_1 , and τ_2), the absorptance in each layer based on the thickness and absorption coefficient of each layer (t_1 , t_2 , a_1 , a_2), and the reflectivity of the solar cells (ρ_c). These quantities are to be average values over the spectral bands chosen. The energy absorbed in the two cover layers and surfaces and solar cell are Q_{1s} , Q_1 , Q_{2s} , Q_2 , and Q_c as indicated in Figure 17. The complete set of radiosity-irradiation equations to be solved simultaneously are as follows,

Figure 17. Radiosity-irradiation network for solar cell with two top cover encapsulant layers.

$$Q_1^- = 1 \text{ (Solar irradiation normalized to one)}$$

$$Q_1^+ = \rho_1 Q_1^- + \tau_1 (1 - \rho_1) Q_2^-$$

$$Q_2^- = T_1 Q_3^+$$

(23)

$$Q_2^+ = \tau_1 (1 - \rho_1) Q_1^- + \rho_1 Q_2^-$$

$$Q_3^- = T_1 Q_2^+$$

$$Q_3^+ = \rho_2 Q_3^- + \tau_2 (1 - \rho_2) Q_4^-$$

$$Q_4^- = T_2 Q_5^+$$

$$Q_4^+ = \tau_2(1 - \rho_2)Q_3^- + \rho_2 Q_4^-$$

$$Q_5^- = T_2 Q_4^+$$

$$Q_5^+ = \rho_c Q_5^-$$

where,

$$T_1 = e^{-a_1 t_1}$$

(24)

$$T_2 = e^{-a_2 t_2}$$

The simultaneous set of Equations 23 is easily reduced to the following set of solution equations

$$A = \rho_c \tau_2^2 T_2^2 (1 - \rho_2)^2$$

$$B = 1 - \rho_c \rho_2 T_2^2$$

$$Q_2^+ = \frac{\tau_1(1 - \rho_1)}{1 - \rho_1 T_1^2 \left(\rho_2 + \frac{A}{B} \right)} \quad (25)$$

$$Q_3^+ = Q_2^+ T_1 \left(\rho_2 + \frac{A}{B} \right)$$

$$Q_1^+ = \rho_1 + \tau_1 T_1 (1 - \rho_1) Q_3^+$$

$$Q_4^+ = Q_2^+ \frac{\tau_2 T_1 (1 - \rho_2)}{B}$$

$$Q_5^+ = \rho_c T_2 Q_4^+$$

$$Q_1^- = 1$$

$$Q_2^- = T_1 Q_3^+$$

$$Q_3^- = T_1 Q_2^+$$

$$Q_4^- = T_2 Q_5^+$$

$$Q_5^- = T_2 Q_4^+$$

The fraction of energy absorbed in each layer is obtained from the following energy balance for each layer.

$$\begin{aligned} Q_{1s} &= Q_1^- + Q_2^- - Q_1^+ - Q_2^+ \\ &= 1 - Q_1^+ - Q_2^+ + T_1 Q_3^+ \end{aligned}$$

$$\begin{aligned} Q_1 &= Q_2^+ + Q_3^+ - Q_2^- - Q_3^- \\ &= (Q_2^+ + Q_3^+) (1 - T_1) \end{aligned}$$

(26)

$$\begin{aligned} Q_{2s} &= Q_3^- + Q_4^- - Q_3^+ - Q_4^+ \\ &= T_1 Q_2^+ - Q_3^+ - Q_4^+ + T_2 Q_5^+ \end{aligned}$$

$$\begin{aligned}
Q_2 &= Q_4^+ + Q_5^+ - Q_4^- - Q_5^- \\
&= Q_4^+ (1 - T_2) (1 + \rho_c T_2) \\
Q_c &= Q_5^- - Q_5^+ \\
&= Q_4^+ T_2 (1 - \rho_c)
\end{aligned}$$

Equation sets 24, 25 and 26 allow us to solve for the specular radiation exchange within the encapsulation system. This analysis is carried out for each of the spectral bands that the solar spectrum is divided into.

In the absence of antireflection coatings transmissivities τ_1 and τ_2 are equal to one, and reflectivities ρ_1 , ρ_2 , and ρ_c can be calculated from Equation 27, which are small angle approximations of Fresnel's equation (Equation 6).

$$\begin{aligned}
\rho_1 &= \left(\frac{N_1 - 1}{N_1 + 1} \right)^2 \\
\rho_2 &= \left(\frac{N_2 - N_1}{N_2 + N_1} \right)^2 \\
\rho_c &= \left(\frac{N_c - N_2}{N_c + N_2} \right)^2
\end{aligned} \tag{27}$$

If surface 1 or the surface of the solar cell have antireflection coatings or special surface preparations, then their actual reflectivity and transmissivity values should be used in lieu of Equation 27.

V.C. CORRECTING FOR INTERNAL ENHANCEMENT EFFECTS

After the radiosity-irradiation network is analyzed, the results obtained need to be adjusted to account for internal reflections between the intercell areas and the solar cells. This enhancement effect is discussed in Section IV.E. Based on the information presented in that section, I will now develop equations to account for the enhancement effect. The assumptions listed on page 17 still apply, but with the following exceptions. Assumption (2) assumes a hemispherical reflectivity of one for the intercell areas. In this section I will approximate the effect of non-white intercell areas by multiplying the enhancement effect obtained assuming a hemispherical reflectance of one for the intercell areas by the square of the actual hemispherical reflectance ρ_i . Using a power of two is an approximation based on reasoning that the power must be greater than one since the enhancement effect due to multiple internal reflections is about twice that due to a single internal reflection. Therefore, much of the light is reflecting off of the intercell areas more than once.

Assumption (3) assumes a single homogeneous, non-absorbing top cover. However, an allowance can be made for more than one encapsulant layer covering the cells. This is done by using a top cover thickness equal to the total thickness of the encapsulant layers comprising the top cover. And, the index of refraction of the top cover is set equal to the highest of the indices of the layers. Figure 18 shows why this is a reasonable approximation if there is not an antireflection coating on the top cover. The effect of an antireflection coating would be to greatly reduce the internal reflections that give rise to the enhancement effect. Consequently, if an antireflection coating is present, then one can conservatively assume that there is no enhancement effect and use the results from the radiosity-irradiation analysis as they stand.

Absorptance of internally reflected light by the encapsulant layers will reduce the enhancement effect. This can be accounted for in an approximate manner. Once the average solar cell enhancement E has been calculated assuming a non-absorbing medium, multiply the energy absorbed in the cell Q_c by the term $(1 + E)$, rather than just adding E to Q_c . This has the effect of applying the absorptance effects calculated in the radiosity-irradiation network analysis, and inherent in Q_c , to the enhancement effect term.

I will develop two equations to account for the enhancement phenomenon — one for arrays using rectangular solar cells, and one for circular solar cells. One term common to both equations is a correction factor for the index of refraction of the cover material. Both equations will be derived from results based on an index of refraction of the top cover of 1.5 to account for other indices of refraction, the following correction factor CI , based on Figure 11, is used

$$CI = 2(N - 1) \quad (28)$$

where N is the index of refraction of the cover material.

$$\underline{N_1 > N_2}$$

$$\underline{N_2 > N_1}$$

$$N_2 \sin \theta_2 = N_1 \sin \theta_1 = \theta_0$$

FOR TOTAL INTERNAL REFLECTION AT SURFACE 1

$$\theta_{1C} = \sin^{-1} \left(\frac{1}{N_1} \right)$$

THERE IS NO INTERNAL REFLECTION AT SURFACE 2. THEREFORE N_1 DETERMINES WHEN TOTAL INTERNAL REFLECTION OCCURS

FOR TOTAL INTERNAL REFLECTION AT SURFACE 1,

$$\theta_{1C} = \sin^{-1} \left(\frac{1}{N_1} \right)$$

THE ANGLE θ_2 TO GIVE $\theta_1 = \theta_{1C}$ IS

$$\theta_{2|\theta_{1C}} = \sin^{-1} \left(\frac{N_1}{N_2} \sin \theta_{1C} \right)$$

$$= \sin^{-1} \left(\frac{N_1}{N_2} \frac{1}{N_1} \right)$$

$$\theta_{2|\theta_{1C}} = \sin^{-1} \left(\frac{1}{N_2} \right)$$

FOR TOTAL INTERNAL REFLECTION AT SURFACE 2

$$\theta_{2C} = \sin^{-1} \left(\frac{N_1}{N_2} \right)$$

SINCE $\theta_{2|\theta_{1C}} < \theta_{2C}$, N_2 ALONE DETERMINES

WHEN TOTAL INTERNAL REFLECTION OCCURS

Figure 18. Total internal reflection for two layer encapsulant.

A section of an array using rectangular cells indicating the principle dimensions is shown in Figure 19. This array is characterized by intercell areas that are relatively long and narrow. Consequently, the results given in Figure 13 are applicable to rectangular arrays, as discussed in Section IV. E. To put the results of Figure 13 into a usable form, I have integrated the areas under the curves in the cell regions for each of the reflecting region widths. These areas are plotted as a function of the reflecting stripe width in Figure 20. The upper limit on the enhancement effect as the reflecting strip becomes very wide is the case shown in Figure 11 for a semi-infinite reflecting area. The following equation has been derived to predict the irradiation enhancement IE as a function of reflecting strip width and top cover thickness:

$$IE = 1.4t(1 - e^{-0.22 W/t}) \quad (29)$$

Figure 19. Section of array using square solar cells.

Figure 20. Increase in irradiation as a function of reflecting stripe width. Based on results shown in Figure 10.

To determine the average increase in solar irradiation for the entire solar cell, E_{SQ} , it is necessary to multiply the above fractional increase by the ratio of solar cell perimeter to cell area. The average enhancement for the whole cell is then

$$E_{SQ} = IE \frac{4S}{S^2}$$

$$E_{SQ} = 5.6 \frac{t}{S} \left(1 - e^{-0.22 W/t} \right) \quad (30)$$

Including the correction factor C for a top cover index of refraction N and the square of the reflectivity of the intercell area, the fractional increase in cell irradiation in an array of square solar cells is

$$E_{SQ} = 11.2 \rho_i^2 \frac{t}{S} (N - 1) (1 - e^{-0.22 W/t}) \quad (31)$$

To obtain the total fraction of solar irradiation absorbed in the solar cell, the enhancement term E_{SQ} must be added to the result obtained from the irradiation-radiosity analysis for Q_c

$$Q'_c = Q_c (1 + E_{SQ}) \quad (32)$$

where the prime on Q'_c indicates that it has been corrected for internal enhancement effects.

For a single circular solar cell embedded in an infinite reflecting region, Figure 15 gives the average enhancement in the cell. As discussed in Section IV.E, this curve falls off as $1/R$. In an actual array of circular cells, the enhancement is less than shown in Figure 15 because there is less reflecting area, much of it taken up by the cells themselves. I suggest that the enhancement predicted by Figure 15 be multiplied by the ratio of inter-cell area to total array area to account for this reduction in enhancement. Based on Figure 15, the following equation relates the average increase in irradiation for the entire cell, E_{CIR} , to the cell radius, top cover thickness, and array geometry

$$E_{CIR} = 3.2 \frac{A_i}{A_a} \frac{t}{R} \quad \frac{t}{R} > 2 \quad (33)$$

where A_i is the area of the intercell spaces and A_a is the area of the whole array. Adding now the correction factor for the index of refraction N of the top cover and intercell area reflectivity ρ_i , the fractional increase in cell irradiation in an array of circular solar cells becomes

$$E_{CIR} = 6.4 \rho_i^2 (N - 1) \frac{A_i}{A_a} \frac{t}{R} \quad \frac{R}{t} > 2 \quad (34)$$

To obtain the fraction of the total incident energy absorbed in the solar cell, Equation 32 is used with E_{CIR} in place of E_{SQ} .

To obtain the fraction of the total incident energy absorbed in the solar cell, Equation 32 is used with E_{CIR} in place of E_{SQ} .

One final adjustment to be made to the results of the radiosity-irradiation network analysis is to account for increased energy absorption in the encapsulant layers above the cell due to internal reflections from the intercell areas. This can be done in a manner similar to Equation 30 for correcting the energy absorbed in the solar cell

$$Q'_{1S} = Q_{1S}(1 + E_{SQ})$$

$$Q'_1 = Q_1(1 + E_{SQ})$$

(35)

$$Q'_{2S} = Q_{2S}(1 + E_{SQ})$$

$$Q'_2 = Q_2(1 + E_{SQ})$$

This completes the process of accounting for internal reflections. Equations 31, 32, 34, and 35 are used for this purpose.

V. D. COMPLETING THE OPTICAL ANALYSIS

Once we have calculated Q'_c , Q'_{1S} , Q'_1 , Q'_{2S} and Q'_2 for each spectral band, we are ready to determine the effective transmittance of the encapsulation system and the fraction of incident energy absorbed in each layer. The spectral transmittance of the encapsulation system averaged over spectral band λ_i is Q'_c calculated for spectral band λ_i , or $Q'_{c\lambda_i}$. Therefore the effective transmittance can be determined from Equation 23 with the T_{λ_i} 's replaced with $Q'_{c\lambda_i}$'s

$$T_{EFF} = \frac{C_{\lambda 1} Q'_{c\lambda 1} + C_{\lambda 2} Q'_{c\lambda 2} + \dots + C_{\lambda M} Q'_{c\lambda M}}{C_{\lambda 1} + C_{\lambda 2} + \dots + C_{\lambda M}} \quad (36)$$

The fraction of incident solar irradiation absorbed into each layer is determined by averaging the energy absorbed into each layer for each spectral band.

$$\begin{aligned}
 FQ_c &= \frac{1}{M} \left(Q'_{c\lambda_1} + Q'_{c\lambda_2} + \dots + Q'_{c\lambda_M} \right) \\
 FQ_{1S} &= \frac{1}{M} \left(Q'_{1S\lambda_1} + Q'_{1S\lambda_2} + \dots + Q'_{1S\lambda_M} \right) \\
 FQ_1 &= \frac{1}{M} \left(Q'_{1\lambda_1} + Q'_{1\lambda_2} + \dots + Q'_{1\lambda_M} \right) \\
 FQ_{2S} &= \frac{1}{M} \left(Q'_{2S\lambda_1} + Q'_{2S\lambda_2} + \dots + Q'_{2S\lambda_M} \right) \\
 FQ_2 &= \frac{1}{M} \left(Q'_{2\lambda_1} + Q'_{2\lambda_2} + \dots + Q'_{2\lambda_M} \right)
 \end{aligned} \tag{37}$$

where FQ_c , FQ_{1S} , FQ_1 , FQ_{2S} , and FQ_2 are the fraction of incident solar irradiation absorbed in the solar cell, surface 1, layer 1, surface 2, and layer 2 respectively.

V. E. SUMMARY OF ANALYTICAL PROCEDURE

1. Required information

- a. Solar cell spectral power conversion efficiency or relative spectral response, $C(\lambda)$.
- b. Thicknesses of encapsulant layers, t_1 , and t_2 .
- c. Absorption coefficients, $a_1(\lambda)$, and $a_2(\lambda)$.
- d. Reflectivity of intercell areas, $\rho_i(\lambda)$.
- e. Reflectivities of outer encapsulant layer $\rho_1(\lambda)$ and solar cell $\rho_c(\lambda)$ if they have been specially treated to reduce reflections. Otherwise, they can be calculated from indices of refraction N_1 , N_2 , and N_c .
- f. Dimensions S and W for square solar cells, or dimension R and areas A_1 and A_2 for circular solar cells.

2. Based upon the wavelength dependence of the above quantities, decide on the number M of spectral bands that the solar spectrum should be divided into. Determine average values of the above quantities for each of these spectral bands.
3. Solve the radiosity-irradiation network for each of the M spectral bands.
 - a. Calculate encapsulant layer transmittances T_1 and T_2 from Equations 24.
 - b. Calculate reflectivities ρ_1 , ρ_2 , and ρ_c , if not specified, from Equations 27.
 - c. Solve for radiosities Q_2^+ , Q_3^+ , and Q_4^+ from Equations 25.
 - d. Solve for fraction of incident energy absorbed into each layer and surface, Q_{1S} , Q_1 , Q_{2S} , Q_2 , and Q_c from Equations 26.
4. Calculate the internal enhancement effects for each of the M spectral bands.
 - a. Set $t = t_1 + t_2$
 - b. Set N equal to the highest index of refraction of the encapsulant cover layers.
 - c. For square solar cells, calculate the average enhancement in the cells, E_{SQ} , from Equation 31.
 - d. For circular solar cells, calculate the average enhancement in the cells, E_{CIR} , from Equation 34.
 - e. Calculate corrected values for the fraction of incident energy absorbed in each layer and surface, Q'_{1S} , Q'_1 , Q'_{2S} , Q'_2 , and Q'_c .
5. Calculate the effective transmittance T_{EFF} of the encapsulation system from Equation 36.
6. Calculate the fraction of solar irradiation absorbed in each layer and surface FQ_{1S} , FQ_1 , FQ_{2S} , FQ_2 , and FQ_c for input to the thermal analysis.

VI. REFERENCES

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Appendix B

PRELIMINARY ANALYSIS OF BREAKDOWN VOLTAGE CRITERIA

Reference: Halliday, David and Robert Resnik, Physics, Parts I and II, New York, John Wiley and Sons, 1966, pp 741-763 (Chapt. 30).

Assumptions:

- (1) The system is ideal; i.e., there are no leakage currents.
- (2) There are no material imperfections such as bubbles and cracks.
- (3) Crowding of field lines near cell corners and interconnects is neglected.

Consider the system of dielectric layers shown below:

Where E_1 = electric field strength in layer 1, volt/cm

ϵ_1 = dielectric constant (DC) for layer 1

S_1 = dielectric strength for layer 1, volts/cm

t_1 = thickness of layer 1, cm

etc.

From the fundamental principles of electrostatics, the electric potential, V , is defined as

$$V = \int E \cdot dx \quad (1)$$

Assuming the electrical properties are constant within each layer.

$$V = E_1 t_1 + E_2 t_2 + \dots + E_n t_n \quad (2)$$

If we wish to solve for E_1 , eq. (2) can be rewritten as follows:

$$V = \left[E_1 t_1 + \frac{E_2 t_2}{E_1} + \dots + \frac{E_n t_n}{E_1} \right] \quad (3)$$

Another fundamental principle of electrostatics is that the electric displacement vector remains constant across any interface between dielectrics*. This condition may be expressed as follows:

$$\epsilon_1 E_1 = \epsilon_2 E_2 = \dots = \epsilon_n E_n \quad (4)$$

Substituting eq. (4) into eq. (3) then yields

$$\begin{aligned} V &= E_1 t_1 + E_1 \left(\frac{\epsilon_1}{\epsilon_2} t_2 + \dots + \frac{\epsilon_1}{\epsilon_n} t_n \right) \\ &= E_1 \left[t_1 + \sum_{i=2}^n \frac{\epsilon_1}{\epsilon_i} t_i \right] \end{aligned} \quad (5)$$

Solving for E_1 , we obtain

$$E_1 = \frac{V}{t_1 + \sum_{i=2}^n \frac{\epsilon_1}{\epsilon_i} t_i} \quad (6)$$

*See reference, pp. 750-756 for a more complete discussion

Equation (6) can be serialized for any layer, k

$$E_k = \frac{V}{t_k + \frac{a_k}{a_i} \frac{t_i}{\epsilon_i}} \quad (7)$$

For a two layer system, such as the front cover and pottant above cell combination found in the substrate design family, we obtain the following expressions for E_1 (front cover) and E_2 (pottant)

$$E_1 = \frac{V}{t_1 + \frac{a_1}{a_2} t_2} \quad (8)$$

$$E_2 = \frac{V}{\frac{a_2}{a_1} t_1 + t_2} \quad (9)$$

The criteria to be used in this analysis is that the electric field strength in each layer cannot exceed the dielectric strength of the material used in the layer. In other words

$$E_1 < S_1 \quad (10a)$$

$$E_2 < S_2 \quad (10b)$$

$$\begin{aligned} &\vdots \\ &\vdots \\ E_n &< S_n \end{aligned} \quad (10c)$$

For the substrate design family, the top cover is taken as layer 1 and the encapsulant is taken as layer 2. For 3000 volt DC impressed across this two layer system, the electric field strength in the encapsulant is shown as a function of encapsulant thickness, in Figure 1. Here, it can be seen that for layer a_1/a_2 (such as that for Tedlar/EPR), the electric field is a strongly decreasing function of encapsulant thickness. Also shown in Figure 1 are the

ranges of values of the dielectric strength for EPR and polyurethane. A similar plot of field strength in the front cover vs. encapsulant thickness is shown in Figure 2.

Figure 1. Electric Field Strength in Encapsulant vs. Encapsulant Thickness

Figure 2. Electric Field Strength in Front Cover vs. Encapsulant Thickness