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Development Report

GENERALIZED SNAP SAFETY ANALYSIS MODELS

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GENERALIZED SNAP SAFETY ANALYSIS MODELS

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ABSTRACT

This report presents a general mathematical approach for evaluating the risk of using radioactive materials in space missions. Two basic safety models, which encompass the general safety criteria philosophies concerning the number of people receiving a specified amount of radiation and the total radiation received by the population, are derived. Each model is applicable to presently conceived hazard situations that may arise from the use of radioactive materials.

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LIST OF SYMBOLS

$*A_{ijk}$	= region of radiation magnitude criteria validity
$a_{ijk}(x, y)$	= absorption function
$a_{ij}(x, y)$	= equivalent absorption function
$f(\cdot)$	= a function of the variables in parentheses
$g(\cdot)$	= a function of the variables in parentheses
$h(\cdot)$	= a function of the variables in parentheses
$\rho_{ij}(x, y)$	= radiation density function
i	= index representing i^{th} hazard
j	= index representing the j^{th} hazard
k	= index representing the k^{th} body part
$\overline{L}_k$	= expected number of lethal doses *
$L_{k \text{ max}}$	= maximum allowable number of lethal doses
L_{max}	= maximum allowable equivalent number of lethal doses
m	= total number of body parts
n	= total number of hazards
n_i	= total number of locations for the i^{th} hazard
N_{ijk}	= number of people who receive a specified radiation *
$\overline{N}_{ik}$	= expected number of people who receive a specified radiation from the i^{th} hazard *
$\overline{N}_k$	= expected number of people who receive a specified radiation *
$N_{k \text{ max}}$	= maximum allowable number of people who receive a specified radiation

LIST OF SYMBOLS (cont)

$N_{\max}$	= maximum allowable equivalent number of people who receive a specified radiation
P_1	= probability of the 1 th hazard occurring
$P_{j/1}$	= probability of the j th hazard location occurring given the 1 th hazard has occurred
$p(x, y, t)$	= population density function
$p(x, y)$	= time independent population density function
$r_{ij}(x, y, t)$	= radiation-rate density function
$r_{ij}(x, y)$	= total radiation density function
R_{ijk}	= total radiation
$\overline{R}_{ik}$	= expected total radiation from the i th hazard
$\overline{R}_k$	= expected total radiation from the mission
$R_{k \max}$	= maximum allowable radiation for the mission
$R_{\max}$	= maximum allowable equivalent radiation for the mission
t	= time
(x, y)	= x longitude and y latitude coordinate points
X_k	= radiation magnitude limit for the k th body part
X	= equivalent radiation magnitude limit

* k subscript is dropped when equivalent system simplification is used.

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GENERALIZED SNAP SAFETY ANALYSIS MODELS

Introduction

The purpose of a safety model is to determine whether or not a certain space mission is "safe" according to some specified criterion. An ideal safety model would be one that defines the required variables and provides mathematical equations for relating the variables in such a way that the "true" appraisal of safety is realized. This report presents two models which, it is felt, approach the ideal model more closely than existing models. The first is based on a criterion specifying the maximum amount of radiation, and the second is based on the maximum expected total radiation received by the entire world population. Each model is applicable to all presently conceived hazards that may arise from the use of radioactive materials. These models, which are general in nature and based upon fundamental variables, are thought to give a clear understanding of the radiation hazard to the population. The models are designed in such a way that it is possible to know where approximations are used and what assumptions are made when applying the models.

Safety Model Development

The safety models are developed in such a way that the total radiation hazard may be visualized in a logical manner. This approach considers the world population, the hazards with all their possible locations, the resultant radioactive debris at each hazard location, and the relationship between the population and the radioactive debris.

The safety model development is briefly illustrated as follows.

Consider a world map with population densities superimposed on it. In theory, it would be possible to define the exact location distribution of the world population and to know the environment and characteristics of the population.

Now consider one hazard where the fuel capsule is ruptured and the fuel is released. This hazard may occur at different locations. At any one location, a density function from the radioactive debris can in theory be defined.

Visualize this radiation density function superimposed on the world population density function. The overlap of these two density functions illustrates the amount of interrelation between the population and the radiation as obtained from the hazard occurring at a given hazard location. The desired relation between the two density functions is determined in accordance with the safety criterion used. While this relation may be extremely complicated, the basic approach as outlined here remains the same. The desired relation between the population and the radiation density function can be calculated for all hazard locations; this desired relation can be associated with probabilities of occurrence of hazard locations. Comparison of these data with the safety criterion establishes whether the space mission is or is not safe.

Safety Variables

Mathematical functions are used in the safety models to describe the portion of the population involved in each hazard, the hazards with their associated locations and radioactive debris, and the population/radiation relation. The population density function is used to describe the population. Associated with each hazard are the hazard locations, hazard and hazard location probabilities, and debris and radiation rate density functions. The population/radiation relation is given by the absorption functions.

Population

Every safety hazard will involve from at least one person to the total world population. A function that describes the location of the total world population can be generated. With this function, which will be called the population density function, that particular part of the population which may be involved in any hazard situation can be defined if the relationship between the world population and the hazard can be established. The population density function is defined as follows.

$p(x, y, t)$ = the population density function

= a real-valued function defined on the surface of the earth
which gives the population density (people per unit area)
at the x longitude and the y latitude, and at a specified
time t .

Hazards

Hazards are defined as any sequence of events that results in exposure of the world population or any part of that population to radioactive materials. Hazards occur from normal or abnormal events in the radioactive material use sequence (manufacture, transportation, launch, orbit, and reentry). Hazard occurrence depends primarily on the environments and the radioactive materials' reaction to the environments. Typical examples of hazards are release of radioactive fuel following a launch-pad explosion and deep-ocean fuel release after an intact fuel-capsule reentry. Knowledge of all of the possible hazards as well as of the probability of their occurrence is required for the safety analysis. The definitions given below are used for all hazards.

$i, \quad (i = 1, \dots, n)$ Let i be an index representing the i^{th} hazard
which runs from 1 to n , the total number of
hazards possible.

P_i Let P_i be the probability of the i^{th} hazard
occurring.

Hazard Locations

Given that a hazard occurs, the possible locations of the hazard must be ascertained. Hazards such as a launch-pad explosion can occur only at one geographic location. Other hazards may occur at a finite or infinite number of locations. All the locations for each hazard must be known in order to evaluate the total effect of radiation exposure on the world population.

The following definitions describe the hazard locations and their probability of occurrence:

- $j; (j = 1, \dots, n_i)$ Let j be an index representing the j^{th} location of the i^{th} hazard where a total of n_i locations are possible. n_i may be finite or infinite.
- $P_{j/i}$ Let $P_{j/i}$ be the probability of the j^{th} hazard location occurring, given that the i^{th} hazard has occurred.

Radiation

Radioactive debris is associated with every hazard location. After the occurrence of the hazard, the position of the debris can, in theory, be determined for each hazard location as a function of time. From these data, the debris density in the "earth-zone" can be determined as a function of time. The earth-zone is defined as that region which contains radioactive debris below, on, or above the earth's surface whose radioactive particles would impinge upon a person on the earth. This earth-zone would not include either deeply buried radioactive materials such as from deep ocean burial or debris in the ionosphere from a hazard such as high-altitude fuel-capsule burnup, except when the debris enters the earth-zone. The debris density function is primarily dependent upon the effect of the hazard on the radioactive material and upon meteorological variables such as rainfall, temperature, wind, and water currents.

If the type of radioactive debris and the debris density at any point in time is known, the radiation rate on the earth at this same time can also be determined. The radiation rate is obtained from the radiation-rate density function which is given as follows:

$r_{ij}(x, y, t)$ = the radiation-rate density function

= a real-valued function which gives the radiation rate in the earth-zone due to the occurrence of the i^{th} hazard at the j^{th} location at time t and at the x longitude and y latitude coordinates. The radiation-rate density function is a function of the debris density.

In general there will be different radiation-rate density functions for each hazard and each hazard location.

Population/Radiation Relation

Functions associated with the population and hazards are defined in the previous sections. If a relation between the radiation (for each i^{th} hazard at each j^{th} location) and the population density were known, the basic safety problem would be completely defined. All that remains is to select the safety criterion desired and to generate the safety model equations.

The desired relation between the population and the radiation should give the amount or fraction of the available radiation that the population will receive. The amount of radiation received by the people will generally be less than the total radiation in the earth-zone. Many variables directly or indirectly influence the population/radiation relation. Examples of these are (1) population variables such as health, eating habits, hygiene, clothing, type work, etc., (2) shielding variables such as from terrain, vegetation, buildings, etc., (3) meteorological variables such as rainfall, temperature, and wind, and (4) radioactive-material variables such as particle size, chemical composition, and type of emission. Not only is the population/radiation relation a function of at least the above-mentioned variables, but many of the variables are functions of each other. In summary, it may be said the population/radiation relation is a function of those variables which influence the amount of available radiation the population may receive. Because of this multivariable dependency, the true function which describes the relation between the population and radiation may be very complicated, in many practical applications, only approximations of the true function will be possible.

In addition to the many variables that influence the fraction of available radiation the population will receive, the portion of the body that is irradiated is also important. The population receives radiation by different means, i.e., from external sources, inhalation, ingestion, absorption, penetration through wounds, etc. Thus, various parts of the body may receive different radiation doses. Additionally, various parts of the body may demonstrate different sensitivities to radiation. These differences in sensitivity and in amount of radiation to the body part must be accounted for in the safety evaluation. If an equivalence between radiation to different body parts is determined or assumed, only one function is

necessary to describe the population/radiation relation. If this equivalence is not known or cannot be assumed, a separate function for each body part is necessary. For example, separate functions would be required for radiation to the lungs and thyroid.

Absorption Function -- The desired population/radiation relation that gives the fraction of available radiation the population receives will be called the absorption function. The absorption function, which depends on the previously mentioned variables, is defined for each body part as follows:

$a_{ijk}(x,y)$ = the absorption function for the k^{th} body part, ($k = 1, \dots, m$), and for the i^{th} hazard at the j^{th} location defined at the x longitude and y latitude coordinates. It is a real-valued function which gives the fraction of available radiation that the k^{th} body part of a person will receive if that person were at the x and y coordinates.

As indicated by the definition, it is assumed that the absorption function is time independent. Time dependency is ignored for the many variables that determine the function, and only average or expected values of the variables are used. In general this is a reasonable assumption. The potential for error occurs when, at some time t , the actual value of the absorption function at an x and y coordinate is much larger than the average value. If the radiation function is large at the same coordinate and time, the actual amount of radiation received by those people at the coordinate will be higher than the calculated value. The probability of this error occurring is thought to be low.

General Safety Models

At present there are two types of safety analysis criteria. The first determines the expected number of people who receive at least a specified amount of radiation. If the expected number of people receiving this radiation were below some predetermined value, the mission would be considered safe, and if not, it would be unsafe. The safety model for this criterion will be called the radiation magnitude safety model. The second criterion considers the total amount of radiation the world population could receive. The total radiation received is compared to a

predetermined allowable limit, and a safe or unsafe mission is again established. The safety model for this criterion will be called the total radiation safety model.

Radiation Magnitude Safety Model

The radiation magnitude safety model is summarized in Figure 1. This model requires the determination of the expected number of people who receive radiation for each k^{th} body part greater than a specified level. This specified radiation level will be called X_k . The expected number of people who receive X_k or more radiation is compared to the predetermined maximum expected number of people, and a safe or unsafe system is established.

The population density function $p(x, y)$ is used for the radiation magnitude safety model. This function is identical to the previously defined population density function except that it is assumed not to be a function of time. The assumption of a time-independent population density function is made so that the safety model can be presented in a simpler manner. In reality, some population density changes would be expected to occur during the physical life of the radioactive material. However, for a radioactive material of short half-life or for a small time interval, the assumption is reasonably valid.

The radiation-rate density function as previously defined by $r_{ij}(x, y, t)$ is the only remaining time-dependent function used in the safety model. The total radiation available to a person at any coordinate point on the earth can be obtained by integrating the radiation-rate density function with respect to time. The resultant radiation density function is defined as follows:

$$\begin{aligned} \rho_{ij}(x, y) &= \text{the (total) radiation density function} \\ &= \int_{t=0}^{t=\infty} r_{ij}(x, y, t) dt \end{aligned} \tag{1}$$

= a real-valued function of the radiation density due to the occurrence of the i^{th} hazard at the j^{th} location and is defined in the earth-zone at the x longitude and y latitude coordinates.

*Alternate solution: If the body-part radiations have a functional equivalence, Equations 2 through 6 may be solved for equivalent criteria rather than m criteria.

Figure 1. Radiation Magnitude Safety Model Summary

Because the population location is assumed to be time independent, the total radiation received by the k^{th} body part of each person in the total world population is defined by the product of the absorption function and the radiation density function. The region on the earth where this product term exceeds X_k is determined. This region, A_{ijk} , can be written as follows:

$$A_{ijk} = \left\{ (x, y) : a_{ijk}(x, y) \rho_{ij}(x, y) \in X_k \right\} . \quad (2)$$

In words, A_{ijk} is the region where the radiation magnitude limit X_k is exceeded for the i^{th} hazard at the j^{th} location. This region contains the set of all points (x, y) such that the product of the absorption function for the k^{th} body part and the radiation density function satisfies the radiation magnitude limit conditions for that k^{th} body part.

The number of people, N_{ijk} , who receive radiation for the k^{th} body part of magnitude greater than X_k is obtained directly by integration of the population density function over the region A_{ijk} as follows:

$$N_{ijk} = \int_{A_{ijk}} p(x, y) dA . \quad (3)$$

By using Equations 1 through 3, the number, of people, that satisfies the radiation magnitude limit values can be obtained for all body parts of the population for all hazards at all possible locations for each hazard. By using all of these N_{ijk} values, it is possible to obtain the expected number of people, $\bar{N}_{ik}$, involved in the i^{th} hazard, who receive a radiation dose specified by the radiation magnitude limit X_k for the k^{th} body part. This is given by

$$\bar{N}_{ik} = \sum_{j=1}^{n_1} N_{ijk} P_{j/1} . \quad (4)$$

The expected number of people, $\bar{N}_k$, who receive radiation greater than X_k to the k^{th} body part for the complete mission is given by

$$\bar{N}_k = \sum_{i=1}^n \bar{N}_{ik} P_i . \quad (5)$$

The safety criterion for this model establishes a maximum expected number of people, $N_{k \text{ max}}$, who receive radiation greater than the magnitude, X_k , for each k^{th} body part. If the maximum expected number of people is not exceeded by any $\bar{N}_k$, the safety criterion is satisfied and a safe mission is established. Conversely, if any one of $\bar{N}_k$'s exceeds $N_{k \text{ max}}$, the mission is unsafe. A safe mission is thus established if

$$\bar{N}_k \leq N_{k \text{ max}} \text{ for all } k, \text{ where } k = 1, \dots, m . \quad (6)$$

The radiation magnitude safety model may be modified in various ways to extend or to simplify its application. For example, the radiation magnitude limit, X_k , could be modified to specify the radiation level between lower and upper limit values, or it could be defined as a minimum lethal dose.

The radiation magnitude safety model can be simplified if the equivalence between radiation of different body parts were known or assumed. With this equivalence, the equivalent absorption function, radiation magnitude limit, and maximum allowable number of people can be formally expressed as follows:

$$a_{ij}(x, y) = f[a_{ij1}(x, y), \dots, a_{ijk}(x, y), \dots, a_{ijm}(x, y)] \quad (7)$$

$$X = g(X_1, \dots, X_k, \dots, X_m) \quad (8)$$

$$N_{\text{max}} = h(N_{1 \text{ max}}, \dots, N_{k \text{ max}}, \dots, N_{m \text{ max}}) \quad (9)$$

where f , g , and h define the functional relationships of the indicated variables.

Substitution of $a_{1j}(x, y)$, X , and $N_{\max}$ into Equations 2 and 6 for their k subscripted counterparts gives a solution for the new equivalent-radiation magnitude safety model, this substitution reduces the m safety analysis calculations to one.

Total Radiation Safety Model

The total radiation safety model is summarized in Figure 2. Comparison with the radiation magnitude model as summarized in Figure 1 indicates many similarities between the two models. The total radiation safety model determines the amount of radiation that the total population would be expected to receive for each k^{th} body part. This model is more general than the radiation magnitude model because the population density function may be time dependent.

The population density function, $p(x, y, t)$, and the radiation-rate density function $r_{1j}(x, y, t)$ as previously defined are both required for each i^{th} hazard at each j^{th} location. From these functions a density function can be calculated that gives the total amount of radiation the population could receive. This total radiation density function $r_{1j}(x, y)$ is obtained by multiplying the radiation-rate density function by the population density function and integrating with respect to time as follows

$$r_{1j}(x, y) = \int_{t=0}^{t=\infty} p(x, y, t) r_{1j}(x, y, t) dt \quad (10)$$

The radiation magnitude safety model required a calculation of a region on the earth where the radiation magnitude was greater than X_k . Integration of the population density function over the region gives the number of people who received more radiation than X_k . For the total radiation safety model, the region becomes the total surface of the earth, and integration over the earth of the total radiation density function times the absorption function gives the radiation which the total population receives. This resultant total radiation, R_{1jk} , for the k^{th} body part is given by

$$R_{1jk} = \int_A R_{1j}(x, y) a_{1jk}(x, y) dA \quad (11)$$

*Alternate solution: If the body-part radiations have a functional equivalence, Equations 11 through 14 may be solved for equivalent criteria rather than m criteria.

Figure 2. Total Radiation Safety Model Summary

The expected total radiation, $\bar{R}_{ik}$, received by the k^{th} body part of the world population from the i^{th} hazard is given by

$$\bar{R}_{ik} = \sum_{j=1}^{n_i} R_{ijk} P_{j/1} . \quad (12)$$

The expected total radiation $\bar{R}_k$ received by the k^{th} body part of the world population from the complete mission is given by the summation of the hazard probabilities times the expected total radiation for each hazard as follows

$$\bar{R}_k = \sum_{i=1}^n \bar{R}_{ik} P_i . \quad (13)$$

The safety-model criterion establishes the maximum amount of radiation, $R_{k \text{ max}}$, that the population will be allowed to receive for each k^{th} body part. This maximum is compared to $\bar{R}_k$ to establish a safe or unsafe mission. A safe mission is established by

$$\bar{R}_k \leq R_{k \text{ max}} \text{ for all } k, \text{ where } k = 1, \dots, m . \quad (14)$$

The total radiation safety model may be simplified if an equivalence between radiation of different body parts is known or assumed. The equivalent absorption function, $a_{ij}(x, y)$, and the maximum allowable equivalent radiation R_{max} are formally expressed as follows

$$a_{ij}(x, y) = f[a_{ij1}(x, y), \dots, a_{ijk}(x, y), \dots, a_{ijm}(x, y)] \quad (15)$$

$$R_{\text{max}} = g(R_{1 \text{ max}}, \dots, R_{k \text{ max}}, \dots, R_{m \text{ max}}) \quad (16)$$

where f and g give the functional relationship of the indicated variables. Substitution of Equations 15 and 16 for $a_{ijk}(x, y)$ and $R_{k \text{ max}}$ in Equations 11 and 14 reduces the m safety analysis calculations to one.

The total radiation safety model may be modified to consider the expected number of eventual lethalties if the appropriate functional relationships are known or assumed. The expected number of lethalties, $\bar{L}_k$, is a function of the population density function, the radiation-rate density function, and the absorption function, i.e.

$$\bar{L}_k = f[p(x, y, t), r_{ij}(x, y, t), a_{ijk}(x, y)] \quad (17)$$

For some hazards, such as those that produce a radiation distribution of low magnitude, the expected number of lethalties can be approximated by a functional relationship of the expected total radiation as follows:

$$\bar{L}_k \cong g(\bar{R}_k) \quad (18)$$

The expected number of lethal doses can then be compared to an established maximum allowable number of lethal doses, $L_{k \max}$, and a safe mission can be established as follows:

$$\bar{L}_k \leq L_{k \max} \text{ for all } k, \text{ where } k = 1, \dots, m. \quad (19)$$

A simplification of the modified total radiation safety model may be made if the equivalent absorption function of Equation 15 is known and if a maximum allowable equivalent number of lethal doses can be expressed as a function of $L_{i \max}$ ($i = 1, \dots, m$) or

$$L_{\max} = f(L_{1 \max}, \dots, L_{k \max}, \dots, L_{m \max}) \quad (20)$$

where f gives the functional relationship of the indicated variable. Substitution of Equation 20 and Equation 17 or 18 (which are calculated by using the equivalent absorption function) into Equation 19 reduces the m calculations to one.

APPENDIX A
SAFETY ANALYSIS EXAMPLE

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Introduction

This appendix is intended to illustrate the two safety models. Although the hypothetical example is an extensive simplification of a real-life situation, an attempt has been made to represent the mission, hazards, and sequence of events with their associated probabilities in a realistic manner that is representative of present state-of-the-art SNAP systems. The radiation density functions and absorption coefficients used in the example may or may not be realistic for a particular radioisotopic material.

Consider a space mission that uses a short half-life radioisotopic fuel. The safety objective is to have the fuel capsule deposited by means of a controlled deorbit in deep ocean water without any fuel release from the capsule until the radioactivity has decreased to a negligible level. If this design objective were achieved, the mission would be considered safe.

Safety Criteria

At the present time, official safety criteria have not been established. The criteria stated for this example were chosen for illustrative purposes only. The safety analysis criteria shall be twofold. First, the maximum expected immediate lethalties will be set at 0.01 people; the radiation magnitude safety model will be used for this criterion. (A lethality will be assumed to occur when a person receives 400 or more rem.) Second, the maximum expected eventual lethalties will also be set at 0.01 people. The total radiation safety model as modified by Equation 18 to consider resultant lethalties will be used for this criterion. The total radiation safety model will not include lethal radiation dose levels; these are considered in the radiation magnitude model. The assumption of an equivalence of radiation effect between different body parts will be made for both models. Hence, Equations 7, 8, and 9 (page 16) apply to the radiation magnitude model and Equations 15 and 16 (page 19) apply to the total radiation model.

In order to solve the safety problem, data must be obtained for the mission events, event sequence and probabilities, hazards, hazard locations, radiation density function for each hazard location, and the population density function

Event Sequence and Probabilities

Figure A-1 gives a simplified hazard tree diagram of the mission. Each block in Figure A-1 gives the event, event number, and event conditional probability of occurrence. The event probabilities are assumed values thought to be reasonable for this system. Event sequence probabilities are also given in Figure A-1 immediately beneath the event block that terminates the event sequence.

Event 2 in the figure indicates that the probability of the system operating properly up to reentry is 0.90. This means there is a resultant 0.10 probability associated with Event 3. Event 3 includes all launch vehicle, payload, and support equipment malfunctions that affect the mission safety before nominal reentry occurs.

A completely nominal mission is represented by Event Sequence 9 (Events 1, 2, 4, 6, and 9). Event Sequence 9 has a 0.778 probability of occurrence. Figure A-1 also shows a probability of 0.032 for all of the remaining impact event sequences (Event Sequences 10, 11, 12, 13, and 14). The remaining 0.19 mission probability is associated with Event Sequences 3 and 5 which result in nonnominal missions before reentry and in uncontrolled reentry.

Figure A-2 shows a tree diagram expanded to include the event sequences resulting from Events 3 and 5. Note the additional impact event sequences. The probability of deep ocean impact without fuel release is now 0.832, and the probability of all of the remaining impact event sequence is now 0.08. The remaining mission probability concerns hazards other than those associated with land or ocean impact as represented by Event Sequences 18 and 22. The tree diagram of Figure A-2 will be used for the safety analysis.

Hazards

Event Sequences 12, 13, 14, 18, and 22, as given in Figure A-2, are considered to be safety hazards. Event Sequences 9, 10, and 11 are not considered as safety hazards. Event Sequence 9 does not present a hazard because the radioactive material is deposited in deep ocean water without fuel release until the material has decayed to a negligible radioactivity. As such, the radioactivity never gets into the earth zone of the population. Event Sequence 11 (no fuel release in shallow ocean

* Event sequence probability.

Figure A-1. System Hazard Tree Diagram

Figure A-2. System Hazard Tree Diagram

water) does not present a safety hazard when the fuel capsule is in water below the population earth zone. For those very few cases when the fuel capsule is in the earth zone, the low population density coupled with the excellent shielding ability of the water still make the hazard negligible. For Event Sequence 10 (fuel release, deep ocean impact), much of the fuel will be released or will diffuse below the earth zone, and the small amount of fuel in the earth zone will produce a negligible hazard because of the low population density and the shielding of the water.

Fuel released in deep ocean water may become a safety hazard if people catch and eat marine life which has consumed some of the radioactive materials. In that little fishing is done in deep ocean water (defined as water greater than a few thousand feet), this does not present a significant safety hazard.

The five hazards that present a significant safety problem (numbered in Figure A-2) are defined as follows:

1. Event Sequences 7, 16, and 20 that end with fuel release from shallow ocean impact
2. Event Sequences 8, 17, and 21 that end with land impact without fuel release
3. Event Sequences 8, 17, and 21 that end with fuel release from impact onto land masses
4. Event Sequence 18 that ends with other than impact hazards from a mission that is nonnominal before the reentry command is given
5. Event Sequence 22 that ends with other than impact hazards from a mission that has a controlled reentry malfunction.

The number of hazards and hazard definitions could vary considerably. For example, Hazards 4 and 5 could be redefined into many more specific hazards, and all five hazards could be redefined and expanded for each particular event sequence. The essential point is to define all possible hazards and to include their effects in the safety model in a mutually exclusive manner.

Hazard Locations

Figure A-3 shows the hazard location regions for the three impact hazards. Each hazard location region defines the region of all possible hazard locations for the stated hazard event sequence terminating with the stated hazard. Hazard Location Regions A and C are from the controlled reentry sequence (Event 4). As such these regions are on the expected trajectory impact region as shown in the figure. Hazard Location Regions B and D result from nonnominal mission sequences (Events 3 and 5) which allow the hazard location to occur any place on the total possible region for the indicated hazard. The probability of the hazard occurring in each region (Partial Hazard Probability) is obtained from the contribution of the event sequence to the hazard probability as given in Figure A-2.

Figure A-3. Impact Hazard Locations and Probabilities

It is assumed that all of the possible hazard locations for each hazard location region may be represented by a few resultant hazard locations. Hazard Location Regions A and B for Hazard 1 each have one resultant hazard location as given in Figure A-5. Hazard Location Regions C and D for Hazard 2 or 3 have two and four resultant hazard locations respectively as given in Figure A-6. The resultant hazard locations for each of these regions is located on a different population density

region. The population density regions are shown in Figure A-4 and are discussed in the next section. The conditional probabilities of occurrence of the resultant hazard locations are given in the Safety Model section of this appendix.

Population Density Function

Figure A-4 describes the population density in the earth region of interest. It is assumed that the average population densities and population areas of Figure A-4 completely define the population density functions for this safety analysis example problem.

Figure A-4. Population Density Function

Radiation Density Function

Figure A-5 gives the radiation density function for the first location of Hazard 1 which is fuel release in shallow ocean water. The value of the radiation density function is negligible in shallow ocean water because most of the radioactive materials will not be in the population earth zone and the small amount of radioactive material in the earth zone will diffuse through the water which provides natural shielding.

Figure A-5. Radiation Density Function of Hazard 1

It is assumed that the sea life will absorb a fraction of the radioactive materials in the shallow ocean water and that a portion of the sea life will be caught. The land radiation region is assumed to occur through distribution of the radioactive sea life to the population. The radiation density function is assumed to be equal to 0.005 rem on this region for Hazard Location 1. The radiation density function for Hazard Location 2 of Hazard 1 is also assumed to be equal to 0.005 rem on the same region.

Figure A-6 defines the radiation density functions for the six hazard locations of Hazards 2 and 3. The 0.01-square-mile region of a lethal dose and the 0.5-square-mile region of an average dose of 25 rem define the radiation density function of each hazard location of Hazard 2 (no fuel release from land impact). The 0.5-square-mile lethal region and the 10.0-square-mile region of an average 25-rem dose define the radiation density function of each hazard location of Hazard 3 (fuel release from land impact). Each radiation density function is assumed to be zero outside the population region indicated in Figure A-6.

LEGEND

Lethal dose region
Each j^{th*} area = 0.01 square mile (hazard 2)
= 0.5 square mile (hazard 3)

Average dose region of 25 rem
Each j^{th*} area = 0.5 square mile (hazard 2)
= 10.0 square miles (hazard 3)

* j^{th} hazard locations are locations 1 to 6 as given in figure.

Figure A-6. Radiation Density Function of Hazards 2 and 3

Safety Models

For the sake of simplicity, the number of expected lethalties and expected total radiation are assumed known for Hazards 4 and 5. The radiation magnitude safety model only needs to be solved for Hazards 2 and 3 because Hazard 1 has no lethal dose region defined in the radiation density function of Figure A-5. The total radiation safety model must be solved for hazards 1, 2, and 3.

Radiation Magnitude Safety Model

The following is assumed for the radiation magnitude safety model:

1. Hazard numbers i ; $i = 1, 2, \dots, 5$.
2. Equivalent radiation to body parts.

$$a_{ij}(x, y) = a_{ijl}(x, y) + \dots + a_{ijm}(x, y)$$

X = lethal dose

$$N_{\max} = 0.01 \text{ people}$$

3. Expected number of lethalties for Hazards 1, 4, and 5.

$$\bar{N}_1 = 0.0 \text{ people}$$

$$\bar{N}_4 = 0.05 \text{ people}$$

$$\bar{N}_5 = 0.01 \text{ people}$$

Table A-I gives the data caluculations for the expected number of people that receive a lethal dose from Hazards 2 and 3. Hazard 2 is no fuel release on land impact, and Hazard 3 is fuel release on land impact. The number of lethalties at each hazard location is equal to the region where the people will receive a lethal dose (area of criterion validity) times the average population on the region. The expected number of lethalties for each hazard is equal to the summation of the expected number of lethalties for each hazard location times the probability of the hazard location occurring. Expected lethalties are 0.045 people for Hazard 2 and 2.2 people for Hazard 3.

The total expected lethalties, $\bar{N}$, from all possible hazard conditions are calculated from Equation 5 (page 16) to be

$$\bar{N} = \sum_{i=1}^5 \bar{N}_i P_i$$

$$\bar{N} \cong 0.0145 \text{ people}$$

The probability of the i^{th} hazard occurring is obtained from Figure A-2, and the expected number of lethalties from each hazard is obtained from Table A-I for Hazards 2 and 3 and from the given values in this section for Hazards 1, 4, and 5. The expected lethalties from all possible hazard conditions exceed the maximum expected lethalties of 0.01 people. Changes will have to be made to bring the system within the desired safety limits. The hazards of impact on populated land regions are the major contributors to the expected lethalties. Relocation and improved control of the reentry trajectory, to insure a lower impact probability into populated areas, coupled with other design improvements would probably be sufficient to achieve the maximum lethality safety objective.

Total Radiation Safety Model

The following is assumed for the total radiation safety model:

1. Hazard numbers, i ; $i = 1, \dots, 5$.
2. Equivalent absorption function and lethal dose criterion.

$$a_{ij}(x, y) = a_{ijl}(x, y) + \dots + a_{ijm}(x, y)$$

$$L_{\max} = 0.01 \text{ people}$$

3. Total nonlethal radiation to eventual lethality conversion.

$$\bar{L} = (4 \times 10^{-5} \text{ lethality/man rem}) \cdot \bar{R}$$

4. Population density function time-independent assumption.

$$p(x, y, t) = p(x, y)$$

5. Expected total nonlethal radiation for Hazards 4 and 5.

$$\bar{R}_4 = 1000.0 \text{ man rem}$$

$$\bar{R}_5 = 500.0 \text{ man rem}$$

Table A-II gives the data calculations for the expected total nonlethal radiation received by the population for Hazards 1, 2, and 3. The calculations for the expected number of eventual lethality for Hazards 1 and 2 are given in detail. Many similarities between the two safety models can be seen in the calculations. The essential difference is that the radiation magnitude model gave the number of people who receive a lethal dose, whereas the total radiation model gives the total nonlethal radiation the population receives. The total expected nonlethal radiation received by the population is calculated from Equation 13 as follows:

$$\bar{R} = \sum_{i=1}^5 \bar{R}_i P_i$$

$$\bar{R} \cong 97.5 \text{ man rem}$$

The probability of the i^{th} hazard is obtained from Figure A-2, and the expected i^{th} hazard total nonlethal radiation is obtained from this section.

The expected eventual lethalities are calculated by using the given relationship as follows:

$$\bar{L} = (4 \times 10^{-5} \text{ lethalities/man rem}) \cdot \bar{R}$$

$$\bar{L} = 0.0039 \text{ lethalities}$$

The number of expected eventual lethalities meets the safety criterion of 0.01 or fewer eventual lethalities. In that the maximum expected immediate lethality criterion (as obtained from the radiation magnitude model) is exceeded, the system is not considered safe until both criteria are met.

TABLE A-I

Calculations for the Expected Number of People to Receive a Lethal Dose from Hazards 2 and 3

Hazard 2 - No Fuel Release from Land Impact

- 1 Hazard locations $j, j = 1, \dots, 6$
- 2 Population density function, $p(x, y)$
- $p(x, y) \cong \bar{p}$ as given in Figure A-4
 $\bar{p} = 10$ people/sq mi, for $j = 1$ and 5
 $\bar{p} = 1$ person/sq mi, for $j = 2$ and 6
 $\bar{p} = 500$ people/sq mi, for $j = 3$
 $\bar{p} = 100$ people/sq mi, for $j = 4$
- 3 Absorption function, $a_{1j}(x, y)$, assumed value
- $a_{2j}(x, y) \cong 0.9$ for $j = 1, \dots, 6$, where $a_{2j}(x, y)$ is a function of radiation shielding
- 4 Hazard location probabilities

1	2	3
Hazard Location	Partial Hazard Probability (Fig A-3)	Hazard Location Probability (Column 2 $\times$ 0.036)
1	0.003	0.084
2	0.004	0.111
3	0.00014	0.004
4	0.00027	0.008
5	0.0014	0.040
6	0.0272	0.755
	~ 0.036	~ 1.0

- 5 Radiation density function
- $G_{1j}(x, y) = \int_{t=0}^{t=\infty} r_{1j}(x, y, t) dt$
- $G_{2j}(x, y) =$ the lethal dose for each j^{th} location of Hazard 2 as given in Figure A-6
- 6 Area of criterion validity
- $A_{1j} = \{(x, y) | a_{1j}(x, y)G_{1j}(x, y) \in \text{lethal dose region}\}$
- $A_{2j} \cong (0.9)(0.01) \cong 0.009$ sq mi, for $j = 1, \dots, 6$

- 7 Number of people for j^{th} location
- $N_{1j} = \int_{A_{1j}} p(x, y) dA$
- $N_{2,1} \cong (0.009 \text{ sq mi})(10 \text{ people/sq mi}) = 0.09$ people
 $N_{2,2} \cong (0.009 \text{ sq mi})(1 \text{ people/sq mi}) = 0.009$ people
 $N_{2,3} \cong (0.009 \text{ sq mi})(500 \text{ people/sq mi}) = 4.5$ people
 $N_{2,4} \cong (0.009 \text{ sq mi})(100 \text{ people/sq mi}) = 0.9$ people
 $N_{2,5} \cong (0.009 \text{ sq mi})(10 \text{ people/sq mi}) = 0.09$ people
 $N_{2,6} \cong (0.009 \text{ sq mi})(1 \text{ people/sq mi}) = 0.009$ people

- 8 Expected number of lethalties for Hazard 2
- $\bar{N}_1 = \sum_{j=1}^2 \bar{N}_{1j} P_{j/1}$
- $\bar{N}_2 \cong (0.09)(0.084) + (0.009)(0.111) + (4.5)(0.004) + (0.9)(0.008) + (0.09)(0.04) + (0.009)(0.755)$
- $\bar{N}_2 \cong 0.045$ people

Hazard 3 - Fuel Release from Land Impact

- 1 The hazard locations, population density function, absorption function, and hazard location probabilities are assumed identical to those given for Hazard 2.
2. Radiation density function, $G_{1j}(x, y) = G_{2j}(x, y) =$ the lethal dose for each j^{th} location of Hazard 3 as given in Figure A-6
3. Area of criterion validity, $A_{1j} = A_{2j} \cong (0.9)(0.5) = 0.45$ sq mi, for $j = 1, \dots, 6$
4. Number of people for j^{th} location, N_{1j}
- $N_{3,1} \cong (0.45 \text{ sq mi})(10 \text{ people/sq mi}) = 4.5$ people
 $N_{3,2} \cong (0.45 \text{ sq mi})(1 \text{ people/sq mi}) = 0.45$ people
 $N_{3,3} \cong (0.45 \text{ sq mi})(500 \text{ people/sq mi}) = 225.0$ people
 $N_{3,4} \cong (0.45 \text{ sq mi})(100 \text{ people/sq mi}) = 45.0$ people
 $N_{3,5} \cong (0.45 \text{ sq mi})(10 \text{ people/sq mi}) = 4.5$ people
 $N_{3,6} \cong (0.45 \text{ sq mi})(1 \text{ people/sq mi}) = 0.45$ people
- 5 Expected number of lethalties for Hazard 3
- $\bar{N}_1 = \sum_{j=1}^6 N_{3j} P_{j/3}$
- $\bar{N}_1 \cong 2.2$ people

The subscript k has been dropped from the safety model equations because of the equivalent absorption function assumption. The 0.029 partial hazard probability of Locations 3, 4, 5, and 6 is proportioned according to the population areas of Figure A-4. The 0.007 partial hazard probability of Locations 1 and 2 is assumed proportioned as given.

T A B L E A-II

Calculations of the Expected Total Nonlethal Radiation from Hazards 1, 2, and 3

Hazard 1 - Fuel Release in Shallow Ocean Water

1. Population density function, $p(x, y)$.

$p(x, y) \cong \bar{p}$ as given in Figure A-4.
 $\bar{p} = 500$ people/sq mi for 5 sq mi
 $\bar{p} = 100$ people/sq mi for 10 sq mi
 $\bar{p} = 10$ people/sq mi for 50 sq mi

2. Absorption function, $a_{ij}(x, y) \cong 0.02^*$

3. Radiation density function, $G_{ij}(x, y)$.

$$G_{1,1}(x, y) = 0.005 \text{ rem}$$

4. Total radiation density function.

$$R_{ij}(x, y) = \int_{t=0}^{t=\infty} p(x, y, t) r_{ij}(x, y, t) dt$$
$$= p(x, y) G_{ij}(x, y)$$

$$R_{1,1}(x, y)^{**} = (500 \text{ people/sq mi}) (0.005 \text{ rem}) = 2.5 \text{ man rem/sq mi}$$
$$= (100 \text{ people/sq mi}) (0.005 \text{ rem}) = 0.5 \text{ man rem/sq mi}$$
$$= (10 \text{ people/sq mi}) (0.005 \text{ rem}) = 0.05 \text{ man rem/sq mi}$$

5. Total radiation for Location 1.

$$R_{ij} = \int_A R_{ij}(x, y) a_{ij}(x, y) dA$$
$$R_{1,1} = (2.5 \text{ man rem/sq mi}) (0.02) (5 \text{ sq mi})$$
$$+ (0.5 \text{ man rem/sq mi}) (0.02) (10 \text{ sq mi})$$
$$+ (0.05 \text{ man rem/sq mi}) (0.02) (50 \text{ sq mi})$$
$$= 0.40 \text{ man rem}$$

6. Expected total radiation for Hazard 1.

$$\bar{R}_1 = \sum_{j=1}^{n_1} R_{ij} P_{j/i}$$

$$R_1 = 0.40 \text{ man rem}$$

Hazard 2 - No Fuel Release from Land Impact

1. The hazard locations, population density function, absorption function, and hazard location probabilities are identical to those given for Hazard 2 in Table A-I.

2. Radiation density function, $G_{ij}(x, y)$.

$$G_{2j}(x, y) = 25 \text{ rem.}$$

3. Total radiation density function, $R_{ij}(x, y)$.

$$R_{2,j}(x, y) = (10 \text{ people/sq mi}) (25 \text{ rem}) = 250 \text{ man rem/sq mi; } j = 1 \text{ and } 5$$
$$R_{2,j}(x, y) = (1 \text{ people/sq mi}) (25 \text{ rem}) = 25 \text{ man rem/sq mi; } j = 2 \text{ and } 6$$
$$R_{2,3}(x, y) = (500 \text{ people/sq mi}) (25 \text{ rem}) = 12,500 \text{ man rem/sq mi}$$
$$R_{2,4}(x, y) = (100 \text{ people/sq mi}) (25 \text{ rem}) = 2,500 \text{ man rem/sq mi}$$

4. Total radiation for j^{th} location R_{ij} .

$$R_{2,j} = (250 \text{ man rem/sq mi}) (0.9) (0.5 \text{ sq mi}) = 112.5 \text{ man rem; } j = 1, 5$$
$$R_{2,j} = (25 \text{ man rem/sq mi}) (0.9) (0.5 \text{ sq mi}) = 11.25 \text{ man rem; } j = 2, 6$$
$$R_{2,3} = (12500 \text{ man rem/sq mi}) (0.9) (0.5 \text{ sq mi}) = 5625 \text{ man rem}$$
$$R_{2,4} = (2500 \text{ man rem/sq mi}) (0.9) (0.5 \text{ sq mi}) = 1125 \text{ man rem}$$

5. Expected total radiation for Hazard 2.

$$\bar{R}_1 = \sum_{j=1}^{n_1} R_{ij} P_{j/i}$$

$$\bar{R}_2 = \sum_{j=1}^6 R_{2j} P_{j/2}$$

$$\bar{R}_2 = 55.20 \text{ man rem}$$

Hazard 3 - Fuel Release from Land Impact

An analysis identical to that of Hazard 3 yields the expected total radiation for Hazard 3 as:

$$\bar{R}_3 = 2760 \text{ man rem}$$

*It is assumed that 0.02 of the available radiation dose of the sea life will be received by the population.
**Valid in the 5-, 10-, and 50-square-mile regions, respectively, of Figure A-4.

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