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HYDRODYNAMIC BEHAVIOR OF A BARE ROD BUNDLE

by

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ABSTRACT

Temperature distributions within the rod bundle of a nuclear reactor is of major importance in nuclear reactor design. However temperature information presupposes knowledge of the hydrodynamic behavior of the coolant which is the most difficult part of the problem due to complexity of the turbulence phenomena. In the present work a 2-equation turbulence model - a strong candidate for analyzing actual three dimensional turbulent flows - has been used to predict fully developed flow of infinite bare rod bundle of various aspect ratios (P/D). The model has been modified to take into account anisotropic effects of eddy viscosity.

Secondary flow calculations have been also performed although the model seems to be too rough to predict the secondary flow correctly. Heat transfer calculations have been performed to confirm the importance of anisotropic viscosity in temperature predictions.

All numerical calculations for flow and heat have been performed by two computer codes developed in the present work which were based on the TEACH code [71]. Also experimental measurements of the distribution of axial velocity, turbulent axial velocity, turbulent kinetic energy and radial Reynolds stresses were performed in the developing and fully developed regions. A 2-channel Laser Doppler Anemometer working on the Reference mode with forward scattering was used to perform the measurements in a simulated interior subchannel of a triangular rod array with $P/D=1.124$.

Comparisons between the analytical results and the results of this experiment as well as other experimental data in rod bundle array available in literature are presented. The predictions are in good agreement with the results for the high Reynolds numbers.

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NOMENCLATURE

AE, AW, AN, AS, AP	PDE coefficients (Eq. 4.44)
c_L	constant (Eq. 3.58)
c_p	coolant specific heat
c_μ	constant (Eq. 3.42)
c_2	constant (Eq. 3.39)
c_1, c_2	constants (Eq. 3.47)
$c_{\epsilon 1}, c_{\epsilon 2}$	constants (Eq. 3.72)
d	scattering particle size
D	rod diameter
D_H	hydraulic diameter of the infinite array
D_H^T	hydraulic diameter of the test section
D_i	clad inner diameter
E	constant (Eq. 3.80)
f_D	Doppler frequency
g	gap length
k	turbulent kinetic energy
l_r, l_θ	length scales
L	entrance length
P	pitch
q'	linear heat generation rate
q_o''	outside clad heat flux
q_i''	inside clad heat flux
r	radius
$2r_o$	diameter of the pipe or the distance between parallel plates
T_c	coolant temperature

T_{cl}	clad temperature
T_{co}	outside clad temperature
$\bar{T}_{co}$	average outside clad temperature
u_b	bulk velocity of the infinite array
u_b^T	test section bulk velocity
u_o	maximum channel velocity at $L/DH = 113$
u_{oL}	maximum channel velocity in position L
u^*	local friction velocity
$\bar{u}^*$	average friction velocity
v_r, v_r'	mean and rms radial velocity
v_θ, v_θ'	mean and rms tangential velocity
v_z, v_z'	mean and rms axial velocity
y	radial distance from the wall
y^+	$y^+ = \frac{yu^*}{\nu}$
$\bar{y}$	radial distance between the wall and zero shear stress line

GREEK LETTERS

α	constant (Eq. 3.63)
α_H	coolant heat diffusivity
$\delta\phi$	error in quantity ϕ
ϵ	energy dissipation
ζ	general turbulent quantity
θ	angle
K	Von Karman constant
λ_c	coolant heat conductivity
λ_{cl}	clad heat conductivity

ν	viscous kinematic viscosity
$\nu_{\alpha\beta}^T$	turbulent viscosity ($\alpha, \beta = r, \theta, z$)
ρ	coolant density
$\sigma_1, \sigma_2, \sigma_3$	dimensions of light intersection volume (Fig. 2.6)
$\sigma_k, \sigma_\epsilon, \sigma_T$	diffusion constants for k , ϵ and T_c
$(\tau_w)_{avg}$	average wall shear stress
Φ	quantity $v_z, k, \epsilon, v_\theta, v_r, P/\rho$
Φ_m	eddy viscosity ratio

CHAPTER 1

INTRODUCTION

1.1 General

Heat transfer within the rod bundle of a nuclear reactor is of major importance in nuclear reactor design. The temperature field affects the mechanical and physical behavior of the reactor materials such as fuel and clad, as well as the neutron physics parameters. Thus, the temperature directly or indirectly imposes a limit on the thermal power produced by the fuel element. Especially in the LMFBR, the clad temperature is one of the significant design limits. Thus a detailed knowledge of the temperature within the fuel element and especially in the clad region is very important. However, temperature information presupposes knowledge of the hydrodynamic behavior of the coolant which is the most difficult part of the problem due to the complexity of the turbulence phenomena.

1.2 Assessment of Existing Methods for Rod Bundle Analysis

The methods which yield the detailed knowledge of the hydrodynamic (and/or thermal) behavior of the rod bundle are called Distributed Parameter (DP) Techniques. The DP techniques with respect to flow, as presently formulated, are restricted to fully developed flow and bare rod bundles which correspond

to special cases in a reactor such as nearly incompressible flow in an interior subchannel far away from disturbances.

The advantages of such a simplified case which is 2-dimensional are evident, i.e.,

1. Low computer time, since the problem is 2-dimensional and the solution is limited to a characteristic triangle (Fig. 1.1).
2. Modeling of the turbulent quantities is easier here than for 3-dimensional problems.

In addition such a simplified case is a powerful tool for understanding phenomena such as shear stress distribution and heat transfer coefficient variation along the periphery of the rod, secondary flow effects and thermal exchange coefficients between subchannels. Such information can be used for improving the solution from Lumped Parameter Techniques (LP) in which the properties are lumped over a characteristic region of the core (subchannel, bundle, etc.)

It is evident that any attempt to use a DP technique for analysis of a rod bundle of a reactor will have to cope with the following problems:

1. Modeling of the turbulent flow in 3-dimensions.
2. Very long computer time (which is the most severe difficulty).

With respect to modeling by existing methods which treat turbulent flows in 3 dimensions, the turbulence model methods

are the most attractive due to

- a) relative simplicity and flexibility to handle complicated problems since the prediction is based on adjustment of constants;
- b) use of some turbulence "information" for predicting flow characteristics in a turbulent domain.

With respect to the long computer time the high computational costs will be a persistent difficulty. Because of the computer cost a 3-D DP technique can be most beneficial in design when it is applied "locally" at regions where one is interested in detailed information (e.g., hot spot, blockage) with boundary conditions supplied by an LP technique.

So it is evident from the above discussion that there is a need for developing 3-D DP techniques which can be applied in a local region of interest. A strong candidate technique is the turbulence model methods.

The turbulence models can be distinguished with respect to Reynolds shear stress closure as follows:

- a. Models using the concept of eddy viscosity
- b. Models using transport equations for Reynolds stresses.

In this approach the models using the concept of eddy viscosity are considered because they are simple, widely used for engineering problems, and require less computer cost. The most advanced eddy viscosity approach is the two equation turbulence model and therefore, it is used in the present approach.

However, before starting on the analysis of three-dimensional flows, it is better to fully understand the two-D fully developed flows as predicted by the turbulence equation models.

Therefore, in the present work the following goals were established:

1. Obtain, through experimental and analytical work a good prediction of the hydrodynamic behavior of the channel through a two-equation turbulence model in fully developed flow. Utilize this prediction method to make some preliminary calculations of heat transfer performance.
2. Extend experiments into the developing region to provide a data base for future model development.

1.3 Summary of Present Work

1.3.1 Experimental work

The experimental task was a continuation of the experimental work initiated by P. Carajilescov [1] in a test section designed to simulate a typical interior subchannel of a triangular array of a bare rod bundle with P/D ratio equal to 1.124.

A two-channel Laser Doppler Velocimeter used by Y.B. Chen [84] with 120 mm lens has been used for the measurements instead of the one-channel approach used in [1].

Measurements of the hydrodynamic parameter (axial velocity, turbulent kinetic energy, Reynolds stresses) were made in the

fully developed region ($L/D_H = 113$) at Reynold numbers of 65,000, 26,500 and 9,000.

With respect to the developing region, the above measurements were repeated for $L/D_H = 15$ and 46 for $Re = 65,000$ and 26,000.

Attempts were made to measure the secondary flow at $Re = 65,000$, but were unsuccessful.

1.3.2 Analytical work

For prediction of the flow in the fully developed region, the two-equation turbulent model using transport equation for turbulent kinetic energy and energy dissipation have been used. A non-isotropic model of eddy viscosity has been introduced to improve the model.

Secondary flow effects have been studied using the Launder and Ying [37] model and the Bobkov et al. [2] model.

All calculations have been performed by a computer program developed in the present work based on the TEACH code [71].

For the convection terms in the finite difference equation the Up-Wind Derivative Difference Scheme Developed in the present work by the author has been used.

CHAPTER 2

EXPERIMENTAL WORK

2.1 Introduction

Experiments on test section simulating the infinite bare rod bundle have been performed by several authors. These experiments refer mainly to axial velocity measurements and to a lesser extent wall shear stress distribution. Very limited measurements exist for parameters such as turbulent kinetic energy, Reynolds stresses and secondary flow.

Table 2.1, which is based on Table II.1 of Ref. 1, summarizes the experimental work done in bare rod bundles.

The purpose of the present experimental work is the extension of P. Carajilescov's work as follows.

I. Fully Developed Region

Comparisons of the mean velocity profiles between positions $L/D_H = 46$ and 77 done in Ref. 1 did not confirm that the flow in position $L/D_H = 77$ was fully developed. Measurements are performed at $L/D_H = 113$ by extending the test section length.

To study the effect of Reynolds Number, measurements at $Re = 9,000$ and $65,000$ are performed besides the $Re = 27,000$ used in Ref. 1. The Reynolds stresses are performed by simultaneous measurements of both velocity components utilizing a two-channel Laser Doppler Anemometer (see §2.2.3.)

II. Developing Region

The above measurements are also performed for $L/D_H = 15$ and $L/D_H = 46$ to examine entrance effects.

2.2 Apparatus

2.2.1 Test section

As mentioned earlier, the test section used by P. Carajilescov [1] has been used in the present work. For corrosion protection, however, the test section has been anodized. The cross section is shown in Fig. 2.1 and the dimensions are given in Table 2.2.

The test section used in Ref. 1 is composed of three 2-foot long sections, one of which is provided with windows for the laser light to pass through the moving fluid. The interchange of the three sections permits measurements at positions $L/D_H = 15, 46, \text{ and } 77$.

In order to examine whether the data obtained at position $L/D_H = 77$ represents data in the fully developed region, the following modifications have been made in the test section.

I. A new section 28 inches long has been added which permits measurements at the position $L/D_H = 113$.

II. In order to decrease the entrance length for the flow, an entrance region has been inserted between the test section and the hydraulic loop pipe as shown in Figures 2.2 and 2.3. The entrance region has been added to suppress the undesired large-scale turbulence coming from the two-inch loop pipe. Such turbulence, the decay of which is roughly proportional to the inverse of the square of the eddy size, increases the entrance length. Thus by killing the pipe turbulence, the entrance region reduces the entrance length.

Additionally, the entrance region has been added to provide approximately the boundary conditions of uniform laminar mean flow at the entrance of the test section useful for the developing region measurements.

This is obtained by including within the entrance region a honeycomb consisting of 2mm coffee straws three inches long. The flow within each straw has a Reynolds Number below 2,000 in all cases, so that the flow at the exit of the honeycomb is approximately laminar.

The overall view of the modified test section is shown in Figure 2.2.

2.2.2 The loop

Since our original purpose was to obtain a Reynolds Number of 100,000 or even higher, the following modifications have been made in the loop used in Ref. 1.

I. Insertion of a 5 HP centrifugal pump in parallel with the already existing 1 HP pump

II. Replacement of the one-inch piping system by one and a half-inch pipe to decrease the pipe resistance and keep the cost relatively low

III. Addition of a heat exchanger connected with cold and/or hot water. The reasons for adding the heat exchanger were:

- a. to cool the water due to pump warm-up
- b. to heat the water to obtain higher Reynolds Number by decreasing the viscosity

IV. Increase the flow meter maximum capacity from 30 GPM to 74 GPM by changing the float

V. Addition of a purification loop using a fixed bed demineralizer to protect the whole loop from corrosion

The whole hydraulic loop is shown schematically in Fig. 2.4.

2.2.3 Laser Doppler Velocimeter

For the flow measurements, the two-channel Doppler Velocimeter operated in a Reference Beam Mode [58] used by Y. B. Chen [84] has been used. The details of the system as used in the present work are as follows:

2.2.3.1 Optical arrangement

The Laser light is provided by a SPECTRA-PHYSICS Laser Model 164. The beam consists of green light of wavelength 0.5145μ and a diameter of 1.5mm as shown in Table 2.2, and passes through a DISA 55L88 LDA Transducer [56].

Within the transducer, the laser beam is 50% split. One of the splitted beams is 40 MHz shifted through a Bragg cell acousto-optic modulator which is part of LDA transducer and then is again 50% splitted to form what we call reference beams. The remaining beam is the so-called scattering beam. In other words, what has been obtained through the LDA transducer is a set of three parallel beams, one relatively strong unshifted (the scattering beam) and two weak 40 MHz shifted (the reference beams.)

The relative position of the three beams is shown in Fig. 2.5, whereas the geometric distances are given in Table 2.2. In the present work the three beams pass through a 120mm lens. The use of 120mm focal length instead of 300mm used in [84] is due to the desire to obtain relatively small spatial resolution. The three beams intersect at the point under consideration. A drawing of the intersection region is shown in Fig. 2.6. The dimensions of the intersection regions are given in Table 2.2.

The scattering light generated within the intersection region is collected along the directions of the reference beams by two DISA 55L12 Photomultipliers, as shown in Fig. 2.5 and Fig. 2.7.

The beat frequency f_B of the reference and scattering light is directly related to the flow velocity component U_1 perpendicular to the symmetry line as shown in Fig. 2.6 as follows:

$$f_{PM} = 40 \text{ MHz} - f_D \quad (2.1)$$

where f_D is the Doppler frequency given by the relation [58]

$$f_D = \frac{1}{C} U_1 \quad (2.1a)$$

where C the calibration factor given by the relation

$$c = \frac{\lambda}{2n \sin \frac{\beta}{2}} \quad (2.2)$$

where λ is the light wavelength;

n is the fluid refractive index;

β is the intersection angle shown in Fig. 2.6.

2.2.3.2. Electronics

For processing the photomultiplier signal output, a number of techniques exist with associated advantages and disadvantages, given by F. Durst, A. Melling and T. H. Whitelaw in Ref. 58 Chapter 6.

In the present work the frequency tracking procedure has been used since it was available and it was used by previous investigators [1], [84.] The PM tube signal passes through a frequency mixer of a preselected frequency f_{LO} so that the frequency mixer output becomes $f_{LO} + (40 \text{ MHz} - f_D)$.

The final signal which passes through a frequency tracker is translated to a continuous voltage signal from which mean values, rms values and cross correlations are obtained by analog electronics.

The main advantages of the frequency tracker are its [57]:

- I. Capability to provide real time demodulation of the Doppler signal
- II. Relatively rapid data processing rate
- III. Relative insensitivity to variations in particle concentration.

The main disadvantages are:

I. Performance deteriorates as the signal-to-noise ratio decreases.

II. Since its output is based on scattering light for more than one particle simultaneously, a relatively large broadening of the Doppler signal occurs due to random phase fluctuation [65] from the many particle system.

2.3 The Measurements

2.3.1 General

The parameters of interest in the present work are

$$\overline{v_z}, \overline{v_r}, \overline{v_\theta}, v_z', v_r', v_\theta', \overline{v_r'v_z'}, \overline{v_\theta'v_z'}$$

The parameters that are convenient to measure are

(see Fig. 2.8)

$$v_z, v_1, v_2, v_z', v_1', v_2', \overline{v_1'v_2'}, \overline{v_2'v_z'}$$

The relations between these two groups of parameters are simple and are given in Table 2.4.

2.3.2 Experimental Grid

With respect to the experimental grid, points have been selected which lie on lines parallel to the two-axis (see Fig. 2.1) and corresponding to angle differences of

3° for axial velocity

6° for stresses

The distance between two adjacent 2-lines is taken as:

$$\begin{aligned} & 0.3\text{mm for axial velocity} \\ (\delta s)_2 &= 0.6\text{mm for stresses} \end{aligned}$$

Some stress measurements close to the wall have been performed at $(\delta s)_2 = 0.3\text{mm}$.

The detailed grid is given in Appendix C.

2.3.3 Point positioning

Due to different refraction indices between water and air, the intersection point displacement measured in the air has to be corrected to correspond to real displacement within the water. According to Figure 2.9

$$(\delta s)_{\text{air}} = \frac{\tan 7.46^\circ}{\tan 9.46^\circ} (\delta s)_{\text{water}} = 0.7474 (\delta s)_{\text{water}}$$

where

$(\delta s)_{\text{water}}$ stands for the actual displacement

$(\delta s)_{\text{air}}$ is the displacement as measured in the air

2.3.4 Scattering particles

One of the primary concerns was the scattering particle size since it affects the signal-to-noise ratio [85]. With respect to the particle size Yu. N. Dubnitshev, et al [12]

using Fourier optics has proved that for small sizes of particles and photomultiplier aperture, the optimum signal is obtained by

$$d = (2m + 1) \frac{\lambda f}{2a} \quad \text{for differential mode} \quad (2.3)$$

$$d = (2m + 1) \frac{\lambda f}{4a} \quad \text{for reference mode}$$

where m is an integer

f is the focal distance

$2a$ is the two-beam distance at the lens.

Present geometry gives for $m = 1$, an optimum particle size of $.77\mu$.

In present work, polystyrene particles of sizes 0.5, 0.79, 1.0, and 2.0 were investigated. The best signals were obtained utilizing 0.79μ and 1.00μ particles. This result supports the correctness of Eq.(2.3.)

2.3.5 Measurements

Measurements were performed at the axial position $L/D_H = 15, 46$, and 113 and Reynolds Numbers close to $9,000$, $26,500$, and $65,000$. Results are given in Appendix C and in Figures 5.1-5.18.

The measurements are characterized by good results for axial velocity and axial velocity fluctuations.

The measurements in the lateral and radial directions are less precise due to relatively low signal-to-noise ratio.

The measurements and their errors will be discussed below.

Table 2.5 gives the frequency ranges and the Tracker used to obtain the results.

As we can see from Table 2.5, DISA 55L20 tracker has been used more in the low frequencies due to greater flexibility of frequency ranges and frequency bandwidths. For the case of $Re = 65,000$ due to low signal-to-noise ratio, the only way to get lateral velocity measurements is a TSI Tracker combined with a TSI (0.6 - 2.0 MHz) bandpass filter.

2.4 Experimental Errors

2.4.1 Axial Velocity

As usual, we can distinguish between random errors and systematic errors.

2.4.1.1 Random errors

This error represented by $\delta v_z'$ is due to the various signal frequency broadening mechanisms. While these broadening mechanisms cannot be eliminated, their effect on the error can be compensated by adopting a large integration time since the error is inversely proportional to $(T)^{1/2}$ [38], i.e.,

$$\delta v_z' = (2v_z'^2 \frac{F}{T})^{1/2} \quad (2.4)$$

where v_z' is the rms value of axial velocity

F is the time scale defined as [38]

$$F = \int_0^T \frac{(v_z(t) - \bar{v}_z)(v_z(t+T) - \bar{v}_z)}{v_z'^2} dt \quad (2.4a)$$

T is the integration time

The integration time has been taken equal to 10s so that the error has been limited for all cases below 0.5%.

Achievement of this range of error has been confirmed by the spread of the data output of the digital voltmeter.

2.4.1.2 Systematic errors

The systematic errors can be mainly categorized as:

- I. Nonuniform velocity errors
- II. Position errors
- III. Frequency Tracker errors

2.4.1.2.1 Nonuniform velocity error--The velocity gradient is associated with the fact that the velocity is not constant within the control volume. Therefore, what we take as an output is not the velocity at the center position of the control volume but some average over the control volume.

An exact analytical expression for velocity gradient broadening is very difficult to obtain. Approximate expressions have been obtained by many authors [57], [59], [60], [61.] Melling [57] considers Gaussian intensity distribution within the elliptic intersection region and velocity gradient only along one of the principal axes of the elliptic control volume whose length is $2\sigma_1$ (see Fig. 2.6.)

In this case,

$$(v_z)_{\text{meas}} - v_z = \frac{\sigma_1^2}{2} \left(\frac{d^2 v_z}{dy^2} \right)_0 \quad (2.5)$$

where 0 stands for the probe volume center point.

Preliminary calculations based on Formula (2.5) show that the above error is negligible.

2.4.1,2.2 Position error--The position error is of order of the probe volume size, i.e.

$$\Delta y = 2b_0 \approx 0.053 \text{ mm}$$

Since the important velocity change is along the radial line, we can approximate

$$(v_z)_{\text{meas}} - v_z = \pm \Delta y \frac{dv_z}{dy} \quad (2.6)$$

Taking

$$\frac{dv_z}{dy} = \frac{u^*}{\kappa y} \quad (2.7)$$

we find finally

$$\frac{(v_z)_{\text{meas}} - v_z}{u_0} = \pm \frac{u^*/u_0}{\kappa \frac{y}{\Delta y}} = \pm \frac{u^*/u_0}{\kappa \frac{y}{2b_0}} \quad (2.8)$$

It is evident from above expressions that above error is significant only for the point close to the wall, whereas for the points far from the wall the error is insignificant. For example, for the point ($\theta = 0$, $\frac{y}{2b_0} = 8.96$) which is closer to the wall and $Re = 65,000$ which is the worst case, the error is calculated to be 1.2% of the maximum velocity.

2.4.1.2.3 Frequency tracker errors-- Such errors can be mainly due to dynamic response of the instrument, slew rates limitation, and finite bandwidth of the tracker.

The error is taken as 1% full-scale deflection of the range in use [1], [75], i.e. 0.1v for all cases, since the range in use is always 10 volts.

Thus for $Re = 9,000$ ($f_{max} = 500$ KHZ, 1v = 50 HZ,) the error is of order 2% of the maximum velocity. For $Re = 26,500$ (i.e. $f_{max} = 1500$ KHZ, 1v = 150 HZ) the error is of order 2% of the maximum velocity, whereas for $Re = 65,000$ (i.e. $f_{max} = 5000$ HZ, 1v = 500 HZ,) the error is of the order of 3% of the maximum velocity.

2.4.2 Turbulent axial velocity

The major errors in obtaining rms values of velocities in general are due to various frequency signal broadenings.

If we could estimate the broadening effect, the real rms value, assuming Gaussian distributions, is given by the relation

$$v_z'^2 = \left[\left(v_z'^2 \right)_{\text{MEAS}} - \left(v_z'^2 \right)_{\text{BROAD}} \right]^{1/2} \quad (2.9)$$

The broadening effects can be distinguished mainly as

1. Finite transit time broadening $(\delta v_z')_L$
2. The non-uniform mean velocity broadening $(\delta v_z')_G$
3. The turbulent velocity gradient error $(\delta v_z')_T$
4. Signal processing error $(v_z')_{\text{LAM}}$

Broadenings due to particle Gaussian motion, particle size and population fluctuation, Laser linewidth and flow rate fluctuation have been neglected as small compared to above-mentioned broadening effects. The quantity $v_z'^2$ will be given by the relation

$$\left(v_z'^2 \right)_{\text{BROAD}} = \left(\delta v_z' \right)_L^2 + \left(\delta v_z' \right)_G^2 + \left(\delta v_z' \right)_T^2 + \left(v_z' \right)_{\text{LAM}}^2 \quad (2.10)$$

2.4.2.1 Finite transit time broadening $(\delta v_z')_L$

The finite transit broadening is due to the fact that a particular particle scatters light only for a finite time—the time required to cross the intersection volume. Assuming that the aperture of photomultiplier is such that all the scattered light in the intersection region is heterodyned, the frequency broadening due to transit time effect is expected to be given by a relation [57]

$$(\delta v_z')_L = \text{CONST} \frac{v_z}{l} \quad (2.11)$$

where L is the mean particle velocity and l a length scale representing the finite size of the probe volume.

It is evident that l will depend mainly on

- a. Beam waist at the intersection region (probe volume)
- b. Velocity vector orientation
- c. Angle of intersection of the two beams

Thus a precise knowledge of $(\delta v_z')_L$ is rather difficult to obtain.

George and Lumley [62], assuming Gaussian intensity distribution within the intersection volume and a many particle system arriving at random times, obtained

$$(\delta v_z')_L = \frac{\lambda}{2\sqrt{2}\pi n_w \sin^2 \frac{\theta}{2}} \cdot \frac{v_z}{2b_0} \quad (2.12)$$

From the data of Table 2.3 we find

$$\frac{(\delta v_z')_L}{v_z} = 0.01 \quad (2.13)$$

The typical error due to finite transit time broadening based on Eq.(2.13) and a typical value $v_z'/v_z = 0.05$ and calculated from Eq.(2.9) is of order 4%.

2.4.2.2 Non-uniform mean velocity broadening $(\delta v_z)_G$

The nature of the error is described in §2.4.1.2.1.

Melling [57] gives the following approximate relation for turbulent velocity error due to non-uniform velocity

$$(\delta v_z)_G = \sigma_1 \frac{\partial v_z}{\partial y} \quad (2.14)$$

$$\text{where } \sigma_1 = b_0$$

As in the case of the mean axial velocity, the error is more important close to the wall. As that region

$$\frac{\partial v_z}{\partial y} = \frac{u^*}{ky} \quad (2.15)$$

Thus $(\delta v_z)_G = \frac{1}{2k} \frac{u^*}{y/2\sigma_1} \quad (2.16)$

The error due to non-uniform mean velocity is higher close to the wall as seen in Eq. (2.16.) Calculations based on (2.66) show an error of no more than 1%.

2.4.2.3 Turbulent velocity broadening $(\delta v_z)_T$

Due to finite scattering volume there is a fluctuation of the volumetric averaged instantaneous velocity which introduces additional ambiguity $(\delta v_z)_T$:

W. E. George and J. L. Lumley [62], considering Gaussian statistics, end up with the following approximate relation for the velocity broadening

$$(\delta v_z')^2_T = \int_{-\infty}^{+\infty} \phi_{11}(k_1, k_2, k_3) \exp \left[-(\sigma_1 k_1)^2 - (\sigma_2 k_2)^2 - (\sigma_3 k_3)^2 \right] dk_1 dk_2 dk_3 \quad (2.17)$$

where $\phi_{11}(k_1, k_2, k_3)$ is the 3-dimensional turbulence spectra.

Further, the authors [62] assuming isotropic turbulence, Pao's [66] form of energy spectra and $\sigma_3 \gg \sigma_2, \sigma_1$ (which is actually our case) obtain Fig. 7 in Ref. 62 which gives

$$\frac{(\delta v_z')^2_T}{(\nu \epsilon)} \quad \text{vs} \quad \frac{\sigma_3 \sqrt{2}}{\eta}$$

where ϵ is the energy dissipation

and $\eta = \frac{\nu^3}{\epsilon}^{1/4}$ is Kolmogorov's microscale

For the case $\frac{\sigma_3 \sqrt{2}}{\eta} < 0.3$

the authors [62] suggest the simplified relation

$$(\delta v_z')^2_T = \frac{2\sigma_3}{\sqrt{15}} \frac{\epsilon^{1/2}}{\nu} \quad (2.18)$$

In our case we can use Fig. 7 directly since the range of

$\frac{\sigma_3 \sqrt{2}}{\eta}$ of interest is in the region where $\frac{(\delta v_z)_T}{(\nu \epsilon)^{1/4}}$

can be roughly taken as a constant equal to 1.6. Specifically for our case, using the definition of Komogorov's microscale

$$\frac{\sigma_3 \sqrt{2}}{\eta} \approx \frac{\sigma_3 \sqrt{2}}{\frac{\nu}{\epsilon}} = \frac{\sigma_3 \sqrt{2}}{\frac{\nu}{0.4 k_0^{3/2} \frac{\lambda}{2}}} = 11^{1/4}$$

$$\text{and } (\delta v_z)_T / (\nu \epsilon)^{1/4} = 1.6 \quad (2.19)$$

where $2\sigma_3 = 0.6 \text{ mm}$

$$\nu = 1.0 \times 10^{-6} \text{ m}^2/\text{s}$$

k_0 = a typical turbulent kinetic energy level

$$= 0.003 v_b$$

$$\lambda = 0.2 D_H = 4 \text{ mm}$$

$$\text{and } v_b = 2 \text{ m/s (Re = 65,000)}$$

Eq. (2.19) gives a typical error of 3% for $\text{Re} = 65,000$, 5% for $\text{Re} = 26,500$, and 7% for $\text{Re} = 9,000$.

2.4.2.4 Signal processing broadening

The electronic equipment introduces some broadening due to instrument noise. The above broadening is measured in our case as follows. The rms value of the laminar flow ($Re = 1500$) has been measured by using frequency shift for each DISA frequency range and each IF bandwidth of the Tracker used in present measurements. The rms values in mV are presented in Table 2.6.

Fig. 2.10 shows the $(v_z'^2)_{LAM}$ vs ΔBW in the range of 1.5 MHz. $(v_z')_{LAM}$ varies linearly with ΔBW , which means that $(v_z')_{LAM}$ is a white noise effect in which the Tracker acts as a low pass filter.

The signal processing broadening cannot be attributed to transit time broadening since Formula (2.13) gives very low values compared with what have been measured.

More specifically, for the mean velocity measured (100mv) Eq. (2.13) gives a finite transit time broadening of 1mv which is much smaller than any value given in Table 2.6.

Since the above broadening is measured, it is not treated as an error but it has been subtracted from the measured rms values, i.e.

$$v_z'^2 = (v_z'^2)_{meas} - (v_z'^2)_{LAM}$$

2.4.2.5 Other errors

A reading error is introduced also from the DISA 55D35 RMS Voltmeter. It is taken 2% of the output [76].

2.4.3 Turbulent velocities v'_r , v'_θ

The main characteristic of these measurements is the low signal-to-noise ratio; therefore the main error comes from the relatively low performance of the tracker at low signal-to-noise ratios. The reason is that the tracker can be locked into frequencies other than the Doppler signals (e.g. the shifted frequency.)

With respect to the broadening mechanisms mentioned in §2.4.2 the only significant broadening mechanisms seem to be

- a. Turbulent velocity broadening
- b. Signal processing broadening

which are taken of the same order of magnitude as in the case of v'_z .

The overall error based on best judgment is taken of order $\pm 15\%$.

The results are tabulated in Appendix C.

2.4.4 The Reynolds stresses $\overline{v'_r v'_z}$ and $\overline{v'_\theta v'_z}$

Above stresses are obtained by measuring the quantities $\overline{v'_1 v'_z}$ and $\overline{v'_2 v'_z}$ and applying the relations shown in Table 2.4. In these measurements again the main difficulty was the low signal to noise ratio in the signals v'_1 and v'_2 , especially very close to the wall. We could estimate based on best

judgement an error 15% for $\overline{v_1 v_z}$ and 100% for $\overline{v_0 v_z}$

The results for $\overline{v_r v_z}$ are shown in APPENDIX C.

CHAPTER 3

THE ANALYTIC MODEL DEVELOPMENT

3.1 The Equations

Our ultimate purpose is to be able to solve the Reynolds equations within the characteristic triangle shown in Fig.

1.1. The Reynolds equations are the time-smoothed momentum equations obtained from the Navier-Stokes equations. In cylindrical coordinates the equations are as follows:

I. Continuity Equation

$$\nabla \cdot \underline{V} = 0 \quad (3.1)$$

II. Momentum Equations

$$r - \text{direction: } \nabla \underline{v}_r \underline{V} + \bar{\nabla} \underline{r}_r = - \frac{\partial (p/\rho)}{\partial r} + \frac{v_{\theta}^2}{r} + \tau_{\theta\theta} \quad (3.2)$$

$$\theta - \text{direction: } \nabla r v_{\theta} \underline{V} + \bar{\nabla} r \underline{r}_{\theta} = - \frac{\partial (p/\rho)}{\partial \theta} \quad (3.3)$$

$$z - \text{direction: } \nabla \underline{v}_z \underline{V} + \bar{\nabla} \underline{r}_z = - \frac{\partial (p/\rho)}{\partial z} \quad (3.4)$$

where $\underline{V}$ is the velocity vector; i.e.

$$\underline{V} = (v_r, v_{\theta}, v_z) \quad (3.5)$$

∇ is the divergence operator; i.e.

$$\nabla \cdot = \left(\frac{1}{r} \frac{\partial}{\partial r} r + \frac{1}{r} \frac{\partial}{\partial \theta} + \frac{\partial}{\partial z} \right) \quad (3.6)$$

p is the pressure

ρ is the density

$\underline{I}_r$, $\underline{I}_\theta$, $\underline{I}_z$ are stress vectors; i.e.

$$\begin{aligned}\underline{I}_r &= (\tau_{rr}, \tau_{r\theta}, \tau_{rz}) \\ \underline{I}_\theta &= (\tau_{r\theta}, \tau_{\theta\theta}, \tau_{\theta z}) \\ \underline{I}_z &= (\tau_{rz}, \tau_{\theta z}, \tau_{zz})\end{aligned}\quad (3.7)$$

consisting of a laminar part, the laminar stresses and a turbulent part, the Reynolds stresses.

$$\tau_{\alpha\beta} = \tau_{\alpha\beta}^{(l)} + \tau_{\alpha\beta}^{(t)} \quad (\alpha, \beta = r, \theta, z)$$

where

$$\left. \begin{aligned}\tau_{rr}^{(l)} &= -2\nu \frac{\partial v_r}{\partial r}, \quad \tau_{rr}^{(t)} = \overline{v_r'^2} \\ \tau_{\theta\theta}^{(l)} &= -2\nu \left(\frac{\partial v_\theta}{r \partial \theta} + \frac{v_r}{r} \right), \quad \tau_{\theta\theta}^{(t)} = \overline{v_\theta'^2} \\ \tau_{zz}^{(l)} &= -2\nu \frac{\partial v_z}{\partial z}, \quad \tau_{zz}^{(t)} = \overline{v_z'^2} \\ \tau_{r\theta}^{(l)} &= -\nu \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right], \quad \tau_{r\theta}^{(t)} = \overline{v_r' v_\theta'} \\ \tau_{\theta z}^{(l)} &= -\nu \left[\frac{\partial v_\theta}{\partial z} + \frac{\partial v_z}{r \partial \theta} \right], \quad \tau_{\theta z}^{(t)} = \overline{v_\theta' v_z'} \\ \tau_{rz}^{(l)} &= -\nu \left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right], \quad \tau_{rz}^{(t)} = \overline{v_r' v_z'}\end{aligned}\right\} \quad (3.8)$$

Obviously, in order to be able to solve Equations (3.1 - 3.4) some knowledge of the Reynolds stress is required. Accurate knowledge of Reynolds stresses does not exist due to

the complexity of the turbulence, especially in the shear flows. Thus, approximate relations, rather crude from the scientific point of view, are used, which however are very useful in solving engineering problems. The additional relations used in order to be able to solve Equations (3.1 - 3.4) consist of the Reynolds stress closure.

3.2 Reynolds Stress Closure - Previous Work

3.2.1 General

As was pointed out in the previous section, the difficulty of solving the continuity and momentum equations is strongly related to the difficulty of obtaining a Reynolds stress closure. In this section an attempt has been made of reviewing the main methods used by other investigators to obtain a Reynolds closure. The methods that will be of interest to the present work will be described in more detail.

We can distinguish three main approaches toward Reynolds stress closure.

1. Analytical expressions (i.e. semi-empirical expressions not included in the concept of eddy viscosity),
2. The eddy viscosity approach,
3. Transport equations for the Reynolds stresses.

3.2.1.1 Analytical Expressions (Algebraic relations)

The simplest way that one could think of studying the behavior of Reynolds stresses is to try to express them as a

function of the local and global characteristics of the flow that fits the experimental data. However, such an approach has disadvantages, such as:

- a. Different expressions are generally needed for different geometries,
- b. The larger the number of independent parameters, the more difficult it is to obtain such analytical expressions.

Therefore, such an approach seems more attractive for simple geometries (e.g. pipe flow, flow between parallel plates, boundary layer in a flat plate) or the wall regions of more complex geometries where parameters such as friction velocity u^* and local Reynolds number $y^+ = \frac{yu^*}{\nu}$ are enough to describe the Reynolds stresses. However, in the case of the simple geometries it is more logical to find analytical expressions directly for the velocity field rather than to go first into the Reynolds stresses.

Parenthetically it is noticed here that in the case of pipe flow and flow between parallel plates the shear stress is given by the relation (Ref. 1, p. 50):

$$\overline{v^i v^j} = -\nu \frac{dv_z}{dr} + \frac{r}{r_0} u_*^2 \quad (3.9)$$

where

ν is the kinematic viscosity

r is the distance from the center line

r_0 is the diameter of the pipe or the distance between the parallel plates.

According to (3.9) an analytical expression for $\frac{dv_z}{dr}$ gives an analytical expression for $\overline{v_r^2 v_z^2}$.

With respect to the normal stresses the only analytical expressions as far as the author knows that are valid for closed ducts of different form are those obtained by V. P. Bobkov, M. Ch. Ibragimov and G. I. Savelev [2]. We will mention below their expressions for the normal stress of a circular pipe since we will assume in Section 3.3.3 that these are also approximately valid for rod bundles.

$$\left. \begin{aligned} \overline{v_r^2} &= A_r v_z \left(1 - \frac{u_b}{u_o} \right) e^{-A_r \xi} \\ \overline{v_\theta^2} &= A_\theta v_z \left(1 - \frac{u_b}{u_o} \right) e^{-A_\theta \xi} \\ \overline{v_z^2} &= A_z v_z \left(1 - \frac{u_b}{u_o} \right) e^{-A_z \xi} \end{aligned} \right\} \quad (3.10)$$

where A's and B's are constants with values:

$$\left. \begin{aligned} A_r &= 0.4 & B_r &= 1.48 \\ A_\theta &= 0.6 & B_\theta &= 1.17 \\ A_z &= 0.95 & B_z &= 1.48 \end{aligned} \right\} \quad (3.10a)$$

$$\xi = 1 - \frac{r}{r_o} \quad (3.10b)$$

u_b is the bulk velocity

u_o is the maximum velocity in the channel

Equations (3.10) are valid for the center region of the flow up to $y^+ = 11$. The constants (3.10) fit well the experimental data in the range of $Re = 7.5 \times 10^4$ up to 2.4×10^5 .

With respect now to using algebraic expressions into the wall region, limited work has been done towards this direction. One of the reasons for this is the difficulty of obtaining reliable experimental data due to the presence of the wall. For $\overline{v_r'v_z'}$ we can use again Eq. (3.9). We will mention here the expression given by A. P. Ivanov [3] who claims that it is consistent with the experimental data of Laufer [6,7] and it is valid in the outer wall region as well. Ivanov's relation is as follows:

$$\frac{\overline{v_r'v_z'}}{u_*^2} = \frac{r}{r_0} \exp\left(-\frac{n_*^2}{1-n_*} \frac{r}{r_0-r}\right) \quad (3.11)$$

where

$$n_* = 6.6 Re^{-0.44} \quad (3.11a)$$

With respect to the normal stress — very close to the wall — as far as the author knows, analytical expressions exist only for $\overline{v_z'^2}$ very close to the wall ($y^+ < 5$).

H. Eckelman [4] gives

$$\frac{(\overline{v_z'^2})^{1/2}}{u_*} = 0.24y^+ e^{1/2\alpha y^+} \quad (y^+ < 5) \quad (3.12)$$

where

$$\alpha = 0.3 \quad (3.12a)$$

3.2.1.2 Eddy Viscosity Approach

Instead of inventing analytical expressions for each case, it seems much more logical and practical to try to understand the general behavior of the Reynolds stresses — in other words to try to build a model for the Reynolds stresses.

Boussinesq [5] was the first to attack the problem by introducing the concept of eddy viscosity assuming that the behavior of the Reynolds shear stresses are similar to the viscous stresses. The eddy viscosity concept is the most widely used in engineering problems. The eddy viscosity concept in its most general form [8] can be expressed as:

$$\tau_{\alpha\beta}^{(t)} = \frac{\nu_{\alpha\beta}^T}{\nu} \tau_{\alpha\beta}^{(l)} + \frac{2}{3} k \delta_{\alpha\beta} \quad (3.13)$$

where

- $\tau_{\alpha\beta}^{(t)}$: the turbulent Reynold stress
- $\tau_{\alpha\beta}^{(l)}$: the viscous stress
- k : the turbulent kinetic energy
- ν : the kinematic viscosity
- $\nu_{\alpha\beta}^T$: the eddy viscosity

and

$$\delta_{\alpha\beta} = \begin{cases} 0 & \text{for } \alpha \neq \beta \\ 1 & \text{for } \alpha = \beta \end{cases} \quad (3.14)$$

The first term of the right hand side of Eq. (3.13) expresses the analogy between the Reynolds and the viscous stresses whereas the second term was added to satisfy two additional requirements:

- a. The sum of the normal stresses is equal to turbulent kinetic energy.
- b. In the case of absence of viscous stresses, i.e. in isotropic turbulence, the normal Reynolds stresses are equal.

Such an approach could be completely successful if we could assume that the introduced eddy viscosity $\nu_{\alpha\beta}^T$ was constant like the viscosity ν . Indeed, in some cases, like turbulent jets and wakes propagated in an infinite space or turbulence in a free atmosphere, such an assumption is satisfactory [Ref. 9, p. 365]. However, in the case of closed ducts with their boundary layers, such an assumption completely fails.

So in general $\nu_{\alpha\beta}^T$ is position dependent and, even worse, is a tensorial quantity rather than a scalar. It is thus evident that in order for this approach to become practical some knowledge of $\nu_{\alpha\beta}^T$ is required. Since we are going to use the same approach in the present work, this topic will be examined in more detail in §3.2.2.

3.2.1.3 Transport Equations

The idea here is that the Reynolds stresses, as the other hydrodynamic quantities, obey a transport equation of the following form, which can be obtained from the Navier-Stokes equations [10]

$$\text{CONVECTION} + \text{DIFFUSION} + \text{SOURCE} + \text{SINK} = 0$$

However, such equations cannot be solved since they are func-

tions of triple velocity correlations and double pressure velocity correlations. So modeling of the various terms in such equations seems necessary. Such an effort has been made by several authors such as Rotta [11], Davydov [12], Donaldson [13], Daly and Harlow [14], Hanjalic and Launder [15], Naot et al. [16], Launder et al. [17].

In the present work we will not follow this approach. Our reason is that application of an eddy viscosity approach to our geometry should give reasonable results with far less computation time than a transport equation approach would require.

3.2.2 Eddy Viscosity Approach

As we have pointed out, the eddy viscosity is generally not constant, especially in the case of a duct flow which the rod bundle flow resembles. Therefore, detailed knowledge of eddy viscosity within the flow region is necessary. There are several approaches towards this direction. The main approaches can be classified as follows:

1. Analytical Expressions (Algebraic Expressions)
2. Prandtl Mixing Length Hypothesis
3. Van Karman Similarity Hypothesis
4. Kolmogorov-Prandtl Hypothesis
5. Transport Equations

3.2.2.1 Analytical Expressions

Efforts to express the eddy viscosity as an analytical function of the local and global characteristics of the flow

in simple terms have appeared in literature. Reichardt [18] has developed an eddy viscosity expression for the parallel plate geometry.

$$v_{rz}^T = \frac{k}{3} \left(1 - \frac{r}{r_0}\right) \left[0.5 + \left(\frac{r}{r_0}\right)^2\right] \left(1 + \frac{r}{r_0}\right) u^* \quad (3.15)$$

where

$$k \approx 0.4 \quad (3.15a)$$

Eifler [19,20] has generalized the expression for v_{rz}^T for round tubes, parallel plates and concentric annuli and the generalized expression has been applied satisfactorily for the rod bundle geometry [21]:

$$v_{rz}^T = y_0 \frac{K'}{2(1+b)} [b+x^2][1-x^2] \frac{2K - x(k-1)}{(K+1)[K-2(K-1)]} u^* \quad (3.16)$$

$$x = 1 - \frac{y}{y_0} \quad (3.16a)$$

$$K = \begin{cases} 1 & \text{for parallel plates} \\ 0 & \text{for circular tubes} \\ 1 + \frac{y_0}{R} & \text{for flows outside a rod of radius } R \end{cases} \quad (3.16b)$$

$$K' = \begin{cases} 0.407 & \text{for parallel plates or pipe flow} \\ 0.407 + 0.02 y_0/R & \text{for flows outside a circular rod} \end{cases} \quad (3.16c)$$

$$b = \begin{cases} 0.55 & \text{for parallel plates and pipe flows} \\ (1.8) - 1.3(y_0/R)[1. + 0.14(\log Re_\ell - 5)]^{-1} & \text{for flows outside a circular rod} \end{cases} \quad (3.16d)$$

y_0 is the radial distance between the wall and the zero shear stress line

and

Re_θ is the local Reynolds number at the peripheral position considered.

With respect to the lateral eddy viscosity, Nijsing and Eifler [21,22] use an average of $\nu_{\theta z}^T$ over the radial direction of the form

$$\nu_{\theta z}^T = cy_0 u^* \quad (3.17)$$

with

$$c = 0.185 \quad (3.18)$$

Rapier and Redman [24] take

$$c = 0.1 \quad (3.19)$$

3.2.2.2 Prandtl Mixing Length Hypothesis

Efforts to understand the general behavior of eddy viscosity has been carried out by several authors. The main idea behind these efforts is that the eddy viscosity can be expressed as a product of a velocity scale U times a length scale l ; i.e.

$$\nu^T = U \cdot l \quad (3.20)$$

in analogy with the approach used in modeling laminar viscosity. The velocity scale U can be associated with representative velocity of the fluid eddies in a particular position. The

length scale l can be considered as the representative length an eddy travels before losing its identity. This is the basis of Prandtl's mixing length theory [25]. In developing his hypothesis, we will refer to the simplest case of parallel flow. According to Prandtl

$$v'_z \sim v'_r \sim l' \frac{dv_z}{dz} \quad (3.21)$$

where v'_r, v'_z represent the fluctuation velocities of fluid lumps arriving at r from the position $r + l'$ and $r - l'$. A reasonable assumption is to take

$$U \sim (\overline{v_z'^2})^{1/2} = (\overline{v_r'^2})^{1/2} \\ l \sim (\overline{l'^2})^{1/2}$$

where l is Prandtl's mixing length. Hence, from Eq. (3.21) we obtain

$$U \sim l \frac{dv_z}{dz} \quad (3.22)$$

Finally, Eq. (3.20) becomes

$$v_{rz}^T = \text{const} \times l^2 \left| \frac{dv_z}{dz} \right| \quad (3.23)$$

l is not a constant but depends generally on the position under consideration and is usually given by analytical expressions based on experimental data. A more general form for the mixing length has been given by Buleev, [26]:

$$\frac{1}{l} = \text{const} \times \int_{\Omega} \frac{d\Omega}{l(\Omega)} \quad (3.24)$$

where Ω denotes the direction

and $l(\Omega)$ is the distance of the point under consideration from the wall in the direction Ω .

Prandtl's similarity hypothesis has been successfully applied to the study of turbulent motion along walls (pipe, channel, plate) and in free turbulent flows [see, for example, Ref. 44, Chapters XIX, XX, XXI, XXIV].

In rod bundles it has been applied by Bender et al. [27, 28]. For ℓ Buleev's formula (3.24) is used:

However, Prandtl's similarity hypothesis proved to be unsatisfactory in the region where $dv_z/dy = 0$ since it gives zero eddy diffusivity, which is in contrast to the experimental evidence that gives considerably higher values of eddy viscosity.

3.2.2.3 Von Karman Similarity Hypothesis

Prandtl's theory does not solve completely the problem of eddy viscosity behavior since it does not say anything about the behavior of ℓ . A general hypothesis that allows us to establish a general connection between the length ℓ and the mean velocity field was proposed by Von Karman [29]. His approach is called "the local kinematic similarity hypothesis." According to this approach the turbulent fluctuations are similar at all points of the field. They differ from point to point only by time and length scales or by equivalent length and velocity scales. Quantitatively, such an assumption leads to the following relation for the velocity scale and length scale:

$$U \sim \frac{dv_z}{dz} \quad (3.25)$$

$$l \sim \frac{dv_z/dz}{d^2v_z/dz^2}$$

Consequently the Karman similarity hypothesis gives the following relations for the shear stress, which is essentially the same as Eq. (3.23)

$$\tau_{rz}^T = l^2 \left| \frac{dv_z}{dz} \right| \left| \frac{dv_z}{dz} \right| \quad (3.26)$$

where

$$l = k \frac{dv_z/dy}{d^2v_z/dy^2} \quad (3.26a)$$

and k is called the Von Karman constant and is equal to 0.4. Von Karman's hypothesis imposes excessively rigid constraints in the velocity fluctuations that do not generally agree with experimental evidence [30, p. 107].

3.2.2.4 Kolmogorov - Prandtl Hypothesis

Prandtl's mixing length hypothesis and Von Karman's similarity hypothesis reflect the effort to find a relation between eddy viscosity and the characteristics of the mean flow. Other efforts on the same line of approach have been made, such as Taylor's vorticity transfer theory [31], improved theories of Prandtl's mixing length hypothesis [44, p. 549], Buleev's theoretical model for turbulent transfer [26] and its modification by Ramm [32] for the case of rod bundle geometry.

The above models do not directly take into consideration statistical characteristics of turbulence upon which the eddy

viscosity depends. The first attempt to express eddy viscosity as a function of the turbulent characteristics of the turbulence is due to Kolmogorov [33] who postulated a characteristic turbulent frequency f .

If we assume that the length scale l and velocity scale U (of Eq. 3.20) are functions of the turbulent kinetic energy k and the frequency f we can find immediately from dimensional analysis that

$$U \sim k^{1/2} \quad (3.27a)$$

$$l \sim \frac{k^{1/2}}{f} \quad (3.27b)$$

and consequently Eq. 3.20 becomes

$$v^T = \text{const} \times \frac{k}{f} \quad (3.28)$$

Prandtl [34] has substituted in Eq. (3.20) only the scale U given by the relation (3.27a). In such a case, equation (3.20) becomes

$$v^T = \text{const} \times k^{1/2} \times l \quad (3.29)$$

where l is the length scale equivalent to the mixing length used in (3.23).

It is evident from the above discussion that any quantity ζ which is assumed to be a function of k and l only (like f in 3.27b) can be expressed by dimensional analysis in the form of:

$$\zeta = k^\alpha l^\beta \quad (3.30)$$

Then, one can write v^T from Eq. 3.20 as a function of k and

ζ as follows:

$$\nu^T = \text{const} \times k^{1/2 - \alpha/8} \zeta^{1/8} \quad (3.31)$$

Eqn. (3.31) is a general relation of eddy viscosity as a function of the statistical characteristics k and ζ . However, the utilization of (3.31) requires the knowledge of k and ζ . This can be obtained either by semi-empirical equations or by transport equations for k and ζ . The way of evaluating k and ζ defines the type of the turbulence model as 0-equation model, 1-equation model and 2-equation model where the term "equation" refers to transport equations for k, ζ .

0-Equation Model -- In this case k and ϵ are prescribed by semi-empirical equations. This model will be applied in this work for comparison with 1- and 2-equation models.

1-Equation Model -- In this case a transport equation for k is used, whereas ϵ is described by a semi-empirical relation. This model has been widely used (see references in 35 and 36). With respect to the closed channel flows and rod bundles the 1-equation model is applied by Launder and Ying [37] and P. Carajilescov [38] in square ducts and rod bundles respectively.

The transport equation for k can be obtained from the Navier-Stokes equation. The way of modeling it is similar to that discussed in §3.2.1.3 concerning the Reynolds stresses themselves although the task is much easier, as we will see later.

2-Equation Model — In this model, transport equations are required for both k and ζ . In the literature one can find a variety of ζ 's one used by different authors, as we can see from Table 3.1 taken from [39].

The transport equation for ζ usually is found by one of two ways:

1) Derivation from Navier-Stokes equation. For example, a transport equation for the turbulent energy dissipation ϵ can be obtained [40, 41]. Such a procedure, however, is not easy for all ζ . Therefore, the second procedure is sometimes used.

2) Derivation from turbulent kinetic energy and dimensional analysis. To be more specific, the turbulent kinetic energy according to what we have said in §3.2.3 can be written:

$$\text{convection } (k) + \text{diffusion } (k) + \text{source } (k) + \text{sink } (k) = 0 \quad (3.32)$$

The transport equation for ζ can be assumed to be

$$\begin{aligned} \text{convection } (\zeta) + \text{diffusion } (\zeta) + (\text{const}) \times \frac{\zeta}{k} \text{ source } (k) \\ + (\text{const})' \times \frac{\zeta}{k} \text{ sink } (k) = 0 \end{aligned} \quad (3.33)$$

where the constants have to be determined from experimental data.

From the relations (3.31) and (3.33) it is evident that such an approach presupposes the adequacy of k and ζ for describing the "integral" behavior of turbulence at particular points. However, this cannot be true for nonisotropic turbulence where more than one velocity scale and/or length scale

exists. Monin [9] has proposed the following alternative relation for the Reynolds stresses based on the fact that the Reynolds stress is a second order symmetric tensor and the assumption that the Reynolds stress can be expressed as a linear combination of the mean strains:

$$\overline{u'_\alpha u'_\beta} = \frac{2}{3} k \delta_{\alpha\beta} - \frac{1}{2} k^{1/2} \sum_i \left(l_{\alpha i} \frac{\tau_{i\beta}^{(l)}}{\nu} + l_{\beta i} \frac{\tau_{i\alpha}^{(l)}}{\nu} \right) \quad (3.34)$$

where $\frac{\tau_{i\beta}^{(l)}}{\nu}$, $\frac{\tau_{i\alpha}^{(l)}}{\nu}$ is the mean strain and $l_{\alpha\beta}$ are the length scales to be determined. However, such a relation which includes the unknowns $l_{\alpha\beta}$ is rather difficult to utilize in solving engineering problems.

3.2.2.5 Transport Equation

A logical thought originating from Eq. (3.31) is to use for τ the eddy viscosity itself, assuming that it is isotropic. This idea has been put forward by Nee and Kovaszny [42] and is used in the code VARR-II [43]; i.e., a transport equation for ν^T is used, which deals with the solution of two-dimensional transient flows.

This approach is another alternative based on Kolmogorov-Prandtl approach, which makes more difficult the introduction of a non-isotropic eddy viscosity.

3.3 Reynolds Stress Closure — Present Approach

3.3.1 General

As we have pointed out in the previous section, the eddy

viscosity approach will be used in this work for obtaining a Reynolds stress closure. From all the eddy viscosity approaches, the Kolmogorov-Prandtl hypothesis is adopted since it relates eddy viscosity directly to the turbulence characteristics of the flow that are the "natural" parameters. The most advanced model based on Kolmogorov-Prandtl hypothesis is the two-equation model in which transport equations for turbulent kinetic energy k and another characteristic quantity of the turbulence, ϵ , are used. The selection of ϵ varies from author to author. We select for ϵ the energy dissipation ϵ which is the most widely used. Calculations with 0-equation model and one-equation model will be performed for comparison with two-equation model.

It is repeated here that authors who apply the Kolmogorov-Prandtl hypothesis for channel flow analysis, including the rod bundle and square duct flows, and consider isotropic viscosity are not consistent with the experimental evidence [23,88]. In fact, present analysis will show (see Chapter 5) that non-isotropic eddy viscosity considerably affects the prediction of flow and heat in rod bundles. The importance of anisotropic viscosity in fluid flow and heat transfer calculations in a bare rod bundle has also been confirmed by other investigators [94, 32]. In this approach, turbulent anisotropy will be introduced by using more than one length scale in modeling turbulent viscosity.

3.3.2 Shear stress relations

In the present approach, the shear stresses will be expressed as follows:

$$\overline{v'_r v'_z} = -v_{rz}^T \frac{\partial v_z}{\partial r} \quad (3.35)$$

$$\overline{v'_\theta v'_z} = -v_{\theta z}^T \frac{\partial v_z}{r \partial \theta} \quad (3.36)$$

$$\overline{v'_r v'_\theta} = -v_{r\theta}^T \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right] \quad (3.37)$$

We will further assume

$$v_{r\theta}^T = v_{rz}^T \quad (3.38)$$

for the following reasons:

1) $\overline{v'_r v'_\theta}$ does not play an important role in the creation of the secondary flow [52] so a very accurate knowledge of $\overline{v'_r v'_\theta}$ is not necessary. In fact, M. Kh. Ibragimov et al. [53] and P. Carajilescov [1] neglect $\overline{v'_r v'_\theta}$ in their calculations so a more accurate knowledge of $v_{r\theta}^T$ seems worthless.

2) $\overline{v'_r v'_\theta}$ includes v'_θ fluctuations and v'_θ is expected to have a length scale close to v'_z .

3.3.2.1 Eddy viscosity for shear stresses

In this approach the eddy viscosities v_{rz}^T and $v_{r\theta}^T$ will be expressed as follows:

0- or 1-Equation Model

$$v_{rz}^T = c_v k^{1/2} l_r \quad (3.39)$$

$$v_{\theta z}^T = c_v k^{1/2} l_\theta \quad (3.40)$$

where c_v will be a constant consistent with experimental evidence.

2-Equation Model — For the energy dissipation ϵ Eq.

(3.30) gives for $\epsilon = \epsilon$

$$\epsilon = k^{3/2}/l \quad (3.41)$$

Based on the relations (3.41), (3.39) and (3.40) the eddy viscosity relations can be put in the following form

$$v_{rz}^T = c_\mu k^2/\epsilon \quad (3.42)$$

$$v_{\theta z}^T = c_\mu \frac{l_\theta}{l_r} k^2/\epsilon \quad (3.43)$$

It is evident from the above formulations that the eddy viscosity ratio is given by the relation

$$\psi_m = \frac{v_{rz}^T}{v_{r\theta}^T} = \frac{l_r}{l_\theta} \quad (3.44)$$

Thus, additional knowledge of ψ_m is needed.

3.3.3 Normal stress relations

For the normal stresses, i.e. $\overline{v_r'^2}$ and $\overline{v_\theta'^2}$, which are mainly responsible for the secondary flow [52], two options have been introduced in the present work. The reason for selecting two options is primarily for comparison, and secondly because both options have been used for prediction of the secondary flow in square ducts.

First option — In this case the normal stress modeling used by Launder and Ying and P. Carajilescov [1] has been used. The idea behind this approach is that the $\overline{v_r'^2}$ and $\overline{v_\theta'^2}$ transport equation that can be obtained from the Navier-Stokes Equation can be simplified considerably if one realizes that:

- 1) they are important with respect to secondary flow only

close to the wall region, and

- 2) the convection and diffusion mechanisms are insignificant compared with generation and dissipation mechanisms.

Under these circumstances the $\overline{v_r'^2}$ and $\overline{v_\theta'^2}$ transport equations are reduced to [Ref. 1, p. 227]:

$$\frac{\overline{p'} \frac{\partial v_r'}{\partial r}}{\rho} \approx \frac{\epsilon}{3} \quad (3.45)$$

$$\frac{\overline{p'} \frac{\partial v_\theta'}{r \partial \theta}}{\rho} \approx \frac{\epsilon}{3} \quad (3.46)$$

What remains to be done is the modeling of pressure-velocity correlations.

This has drawn the attention of several investigators. Here we will adopt the approach discussed in Ref. [17]. The velocity-pressure correlations are expressed as functions of shear stresses and mean velocity gradients, which for our case, neglecting the secondary flow effects, are expressed as follows:

$$\begin{aligned} \frac{\overline{p'} \frac{\partial v_r'}{\partial r}}{\rho} = & -\frac{c_1}{2} \frac{\epsilon}{k} (\overline{v_r'^2} - \frac{2}{3} k) - \frac{8c_2 - 2}{11} v_{rz}^T \left[\frac{\partial v_z}{\partial r} \right]^2 \\ & + \frac{9c_2 + 6}{33} \left\{ v_{rz}^T \left[\frac{\partial v_z}{\partial r} \right]^2 + v_{\theta z}^T \left[\frac{\partial v_z}{r \partial \theta} \right]^2 \right\} \end{aligned} \quad (3.47a)$$

$$\begin{aligned} \frac{\overline{p'} \frac{\partial v_\theta'}{r \partial \theta}}{\rho} = & -\frac{c_1}{2} \frac{\epsilon}{k} (\overline{v_\theta'^2} - \frac{2}{3} k) - \frac{8c_2 - 2}{11} v_{\theta z}^T \left[\frac{\partial v_z}{r \partial \theta} \right]^2 \\ & + \frac{9c_2 + 6}{33} \left\{ v_{rz}^T \left[\frac{\partial v_z}{\partial r} \right]^2 + v_{\theta z}^T \left[\frac{\partial v_z}{r \partial \theta} \right]^2 \right\} \end{aligned} \quad (3.47b)$$

where c_1 and c_2 are constants. Combining Eqs. (3.46) and (3.47) we find finally

$$\overline{v_r'^2} = \frac{2}{3} (1 - 1./c_1)k + \frac{2(9c_2 + 6)}{33c_1} \frac{k}{\epsilon} \left[v_{rz}^T \left(\frac{\partial v_z}{\partial r} \right)^2 + v_{\theta z}^T \left(\frac{\partial v_z}{r \partial \theta} \right)^2 \right] - \frac{2(8c_2 - 2)}{11c_1} \frac{k}{\epsilon} v_{rz}^T \left(\frac{\partial v_z}{\partial r} \right)^2 \quad (3.47a)$$

$$\overline{v_\theta'^2} = \frac{2}{3} (1 - 1./c_1)k + \frac{2(9c_2 + 6)}{33c_1} \frac{k}{\epsilon} \left[v_{rz}^T \left(\frac{\partial v_z}{\partial r} \right)^2 + v_{\theta z}^T \left(\frac{\partial v_z}{r \partial \theta} \right)^2 \right] - \frac{2(8c_2 - 2)}{11c_1} \frac{k}{\epsilon} v_{\theta z}^T \left(\frac{\partial v_z}{r \partial \theta} \right)^2 \quad (3.47b)$$

The constants c_1, c_2 have to be found by experiment.

Launder et al. [17] suggest

$$\left. \begin{aligned} c_1 &= 1.5 \\ c_2 &= 0.4 \end{aligned} \right\} \quad (3.48)$$

Second option — In this case the analytical expressions (3.10) will be used.

3.3.4 The length scales

Viscosity ratios are given in the literature, especially for the fully developed flow condition in rod bundles, from authors who do not use Kolmogorov-Prandtl's approach, e.g. Nijssing and Eifler [22]. However, such a ratio is strongly related to the models for $v_{\theta z}^T$ and v_{rz}^T adopted which are different from our approach. Here we will try to estimate l_r and l_θ from empirical relations based on experimental observations.

3.3.4.1 The length ℓ_r

We will restrict our discussion to the region $y^+ \geq 30$ since it represents the region where the calculations are to be performed. It is called the outer region of the flow. The region $y^+ < 30$ which is very close to the wall is characterized by sharp velocity gradients and strongly anisotropic turbulent kinetic energy distribution. In this region a model based on a Kolmogorov-Prandtl hypothesis would not seem useful although efforts towards this direction have been made by some authors [50, 52]. Even worse, the calculation of hydrodynamic characteristics within the wall region directly from the set of equations needs relatively high computational effort due to the large number of points needed for the numerical calculations.

However, things can be simplified considerably in the case of moderate acceleration or deceleration due to the fact that the hydrodynamic characteristics are satisfactorily described by u^* and y^+ only. Returning to the outer region (i.e. $y^+ \geq 30$) let us review the length scales taken by some authors.

Launder and Ying [37] for the case of rectangular duct flow use Buleev's formulation, Eq. (3.24),

$$\frac{1}{\ell} = \int_0^{2\pi} \frac{d\phi}{s} \quad (3.49)$$

Such a relation gives ℓ proportional to the wall distance for points very close to the wall.

D. Naot et al. [54] for the case of rectangular duct flow used

$$\left. \begin{aligned} l &= y & y &\leq 0.54 D_H \\ l &= 0.135 D_H & 0.135 D_H < y \leq \frac{D_H}{2} \end{aligned} \right\} (3.50)$$

where y is the distance from the wall and D_H is the hydraulic diameter of the flow.

P. Carajilescov [1] for the case of rod bundles developed the following relation

$$\left. \begin{aligned} l &= y & \text{for } y &\leq 0.44\hat{y} \\ l &= 0.44\hat{y} + 0.066\hat{y} \sin \left| \frac{\pi}{0.38} \left(\frac{y}{\hat{y}} - 0.44 \right) \right| & & \\ & & \text{for } y &> 0.44\hat{y} \end{aligned} \right\} (3.51)$$

where $\hat{y}$ is the radial distance of the centerline from the wall at a particular angle θ .

Relations (3.46), (3.47) and (3.48) agree that close to the wall

$$l = y$$

In this approach we will distinguish three subregions within the outer region:

- A. The dynamic equilibrium subregion
- B. The central subregion
- C. The subregion in between.

3.3.4.1.1 The dynamic equilibrium subregion — The dynamic equilibrium subregion is the region closer to the wall and has the following characteristics:

a) The diffusion and convective mechanisms of turbulent kinetic energy transport are small compared with source and dissipation mechanisms. In this region turbulent kinetic energy has its highest values whereas the turbulent kinetic energy gradients are relatively small. In other words, if δk is the change of kinetic energy within this region and k_0 is a representative value of the turbulent kinetic energy level, the following relation holds:

$$\frac{\delta k}{k_0} \ll 1$$

Thus, in such a case we can approximate

$$k = k_0 \quad (3.52)$$

b) The shear stress $\overline{v_r'v_z'}$ is approximately constant. A good approximation consistent with the experimental evidence is that

$$\overline{v_r'v_z'} \text{ is } \approx u_*^2 \neq f(r) \quad (3.53)$$

If we want to put $\overline{v_r'v_z'}$ in the form of (3.37) using the relation (3.41), Eq. (3.53) leads to the following relation for l_r :

$$l_r = \frac{1}{c_v} \frac{u_*^2}{k_0^{1/2}} \frac{u_*^2}{dv_z/dr} \quad (3.54)$$

Since k_0 is mainly a function of u_*^2

$$\frac{1}{c_v} \frac{u_*^2}{k_0^{1/2}} = \text{const}$$

In order to have the same order of magnitude for the constants with other investigators who take $l_r = y$ we will define the above constant as follows.

Within dynamic equilibrium region the logarithmic law is expected to be valid, i.e.

$$\frac{dv_z}{dr} = \frac{u^*}{\kappa y} \quad (3.55)$$

when κ is the Von Karman constant, i.e.

$$\kappa \approx 0.4$$

So we take

$$\frac{1}{c_v} \frac{u^*}{k_o^{1/2}} = \frac{1}{\kappa} \quad (3.56)$$

Thus, from Eq. (3.54)

$$l_r = \frac{1}{\kappa} \frac{u^*}{dv_z/dr} \quad (3.57)$$

3.3.4.1.2 The central region — The central region is characterized by small or zero velocity energy gradients. The selection of l constant and proportional to the distance between the wall and the zero shear stress line gives satisfactory results; i.e.

$$l_r = c_L \hat{y} \quad (3.58)$$

The value of c_L has to be found from the experimental data. A quick estimation can be found, however, from the simple geometries as pipe flow

$$\text{for } c_v = 0.2 \quad (3.59a)$$

$$\left. \frac{k^{1/2}}{u^*} \right|_{y=\hat{y}} = 0.9 \quad (\text{See Ref. 2}) \quad (3.59b)$$

$$\left. \frac{v_{rz}^T}{u^*} \right|_{y=\hat{y}} = 0.07 \hat{y} \quad (\text{See Ref. 94}) \quad (3.60)$$

Substituting Equations (3.58), (3.59), and (3.60) into Eq. (3.39) gives

$$0.07 u^* \hat{y} = 0.2(0.9 u^*) c_L \hat{y} \quad (3.61)$$

or

$$c_L = 0.4$$

which is in good agreement with P. Carajilescov [1] who takes at zero shear stress line $c_L = 0.374$ and higher values for points outside the zero shear stress line.

3.3.4.1.3 The intermediate subregion -- The bigger subregion in the outer flow region is the intermediate region which has neither the properties of the dynamic equilibrium region nor the properties of the central region. Taking into consideration equations (3.57) and (3.58), it is reasonable to assume for this region that the

$$l_r = \text{function} \left(\frac{dv_z/dr}{u^*}, \hat{y} \right) \quad (3.62)$$

A logical approximation for the function can be the following:

$$\left(\frac{1}{l_r} \right)^a = \left(\frac{2}{u^*} \right)^a + \left(\frac{1}{c_L \hat{y}} \right)^a \quad (3.63)$$

where a is a real number to be determined from the experiments. The characteristic behavior of Eq. (3.63) is that in the vicinity of the central region the first term of the right hand side is negligible compared to the second term. Thus Eq. (3.63) reduces to Eq. (3.58).

On the other hand, in the vicinity of the wall the first term dominates in Eq. (3.63), which then reduces to Eq. (3.57).

With respect to the constant α , one expects α to be a function of the Reynolds number, since the higher the Reynolds number the larger the central region becomes, i.e. the larger the α becomes. However, from the nature of the relation (3.63) for large α 's (i.e. $\alpha > 4$), large changes in α have little effect. For example, as Fig. 3.2 shows, the prediction of velocity with the 1-equation model is not substantially affected by taking $\alpha = 20$ versus $\alpha = 10$. The significance of α is further diminished in the 2-equation model, as Fig. 3.3 shows. In our case, α is taken as

$$\alpha = 10$$

The advantages of introducing the length scale of Eq. (3.63) are the following:

1. A continuous function is given for the mixing length which is much more accurate than Buleev's length scale (3.49) which is also continuous.
2. As we can see from Fig. 3.2, a large α (i.e. $\alpha = 10$) gives results close to those obtained by using P. Carajilescov's mixing length (3.51).
3. Formula (3.57), which represents the first term of Eq. (3.63), identifies the length scale as not being a pure geometric characteristic of the flow. In a sense, it reflects the distortion of the turbulent kinetic energy spectrum due to the presence of the nonuniform velocity field, which is quite reasonable. Therefore, it is expected to be a better formula for length scales where the logarithmic law of velocity is not valid (e.g. for non-fully developed flows or in

the regions very close to the wall. For example, in the laminar sublayer, formula (3.57) predicts

$$l_r = \nu/u^*$$

which is the proper length scale; whereas Eq. (3.52) predicts

$$l_r = 0$$

4. Extrapolation of Eq. (3.57) in the tangential direction, as we will see below, leads to an estimate of the length scale l_θ .

The main disadvantage seems to be that Eq. (3.63) does not predict a maximum value within the intermediate region for the length scale, as the relation of P. Canajilescov predicts. However, the absence of this peaked portion in the mixing length prediction has no substantial significance in flow predictions with the turbulence model, especially in the 2-Equation model.

3.3.4.2 The length scale l_θ

In order to estimate l_θ the following assumptions will be made.

1. $v_{rz}^T = v_{\theta z}^T$ in the central region. This is a reasonable assumption, taking into consideration the fact that $\overline{v_r'^2}$, $\overline{v_\theta'^2}$, $\overline{v_z'^2}$ are of the same order of magnitude; i.e., $\overline{v_z'^2} : \overline{v_r'^2} : \overline{v_\theta'^2} = 1.16 : 1 : 1$ [2].

2. Neglect the effect of dv_z/dr on l_θ as a second order effect.

3. An expression similar to (3.63) holds for l_θ ; i.e.

$$\left(\frac{1}{\ell_\theta}\right)^\alpha = \left[\kappa \frac{dv_z/dr d\theta}{u^*}\right]^\alpha + \left(\frac{1}{c_L L_\theta^*}\right)^\alpha \quad (3.64)$$

where L^* is the "free" tangential length scale. Since $dv_z/dr d\theta$ is expected to be small, we can approximate

$$\ell_\theta \approx c_L L_\theta^* \quad (3.65)$$

where

$$c_L \approx 0.4$$

as given by (3.61)

The length scale ratio (or the eddy viscosity ratio) is given by the relation

$$\psi_m = \frac{v_{rz}}{v_{r\theta}} = \frac{\ell_r}{\ell_\theta} \left\{ \left(\frac{L^*}{L_r^*}\right)^\alpha + \left[c_L \frac{dv_z/dr}{u^*/\kappa L_\theta^*}\right]^\alpha \right\}^{1/\alpha} \quad (3.66a)$$

In Fig. 3.1 ψ_m is compared with experimental data taken by A. Quarmby and R. Quirk [68]. These experimental data are obtained by measuring the mass and heat diffusivities ($Sc \approx 0.77$, $Pr \approx 0.71$) in the fully developed region of a plain tube.

For the utilization of (3.66) in the case of plain tubes, take $L_r^* = L_\theta^* = r_0$, $dv_z/dr \approx u^*/\kappa y$, and $\alpha = 1, 2, 10$ respectively, yielding

$$\psi_m = \left\{ 1 + \left(\frac{0.4}{y/r_0}\right)^\alpha \right\}^{1/\alpha} \quad (3.66b)$$

Equation (3.66b) suggests that ψ_m should not vary significantly for $\alpha \geq 2$. This behavior is confirmed by the data of Quarmby [68] as illustrated in Fig. 3.1. Also of even more importance is the consistency of the data with the prediction

of Eq. (3.66b). Note that while the measurements are heat and mass transfer results, since the Schmidt Number and Prandtl numbers were close to unity (0.77 and 0.71, respectively), ψ_m is effectively equal to ψ_D and ψ_H .

With respect to the proper selection of ψ_m for the case of rod bundles, we have tested the following possible cases:

- a) Isotropy of the eddy viscosity along the zero stress line, i.e., $L_\theta^* = \hat{y}$
- b) Isotropy of the eddy viscosity at $y = \hat{y}_{\max}$, i.e., $L_\theta^* = \hat{y}_{\max} \equiv L_r^*$

In Figure 3.3, the quantity v_z/v_o is plotted vs. $y/\hat{y}$ at $\theta=0$. The fitting of data by taking $L_\theta^* = \hat{y}_{\max}$ considerably improves the data prediction.

It must be pointed out here that experimental data obtained by K. Rehme in a side subchannel of a rod bundle [88] indicate isotropic eddy viscosity in the central channel region together with nonisotropy in the gap region, which is consistent with case (b).

H. Ramm and K. Johannsen [32], working with an approach based on Buleev's theoretical model [26] have also tested isotropy along the zero shear stress line versus isotropy at the point $y=y_{\max}$ but with respect to length scales, not the eddy viscosity. Although the length scales used in [32] have similar physical meaning compared with the present length scales, they are defined differently due to the different

overall approach. Therefore, the length scale ratio in [32] is not equal to the eddy viscosity ratio in the present approach. The above authors have chosen a method of calculation similar to case (a) on the ground that it gives more conservative heat transfer results with their model.

From Figure 3.3 the importance of introducing the eddy viscosity anisotropy is very clear. The improvement of the match between data and turbulence model predictions using the above introduced length scales has been considerable.

3.3.5 The final form of the equations

3.3.5.1 The differential equations

3.3.5.1.1 General form — In the above discussion, it has been made clear that the equations to be solved are those of

continuity and momenta conservation and transport equations for turbulent kinetic energy and energy dissipation.

The momentum and transport equations can be put in the general form

$$\left. \begin{aligned} & \left[\begin{array}{c} \text{<---Convection-->} \\ \nabla \phi \underline{V} \end{array} + \begin{array}{c} \text{<---Diffusion-->} \\ \nabla \underline{T}_\phi \end{array} = \begin{array}{c} \text{<---Source-->} \\ \nabla S_\phi + S_\phi^* \end{array} \right] \\ & \underline{T}_\phi = (T_{\phi r}, T_{\phi \theta}) \\ & \underline{S}_\phi = (S_{\phi r}, S_{\phi \theta}) \end{aligned} \right\} \quad (3.67)$$

where ϕ stands for v_z , v_r , v_θ , k and ϵ as shown in Table 3.2a. The separation of the diffusion term into a T_ϕ term and an S_ϕ term which is treated as a source term is made on the ground that T_ϕ includes only the terms that depend explicitly on ϕ .

3.3.5.1.2 Momenta equations -- The momenta equations are obtained from the momenta Reynolds Equations given in §3.1.1 and the Reynolds stress selections given in §3.3.1. Momentum equations can be put in the general form (3.67). Each individual term is given in Table 3.2a.

3.3.5.1.3 Turbulent kinetic energy equation -- The transport equation for k can be obtained by multiplying Navier-Stokes equations for v_r , v_θ , v_z with velocity fluctuations v'_r , v'_θ , v'_z and taking the time average. The exact form of the k -equation for our particular geometry has been given by P. Carajilescov [Ref. 1, Eq. B-6, p 221-222]. Using the present shear stress relations, it can be reduced to Eq. (3.67)

which, neglecting the time and z-dependent terms, can be rewritten as follows:

$$\begin{aligned}
 & \overbrace{\left[\frac{1}{r} \frac{\partial}{\partial r} r v_r k + \frac{\partial}{r \partial \theta} v_\theta k \right]}^{\text{Convection}} - \\
 & \overbrace{\left[\frac{1}{r} \frac{\partial}{\partial r} r \left(v \frac{\partial k}{\partial r} - \overline{v' k'} - \frac{\overline{v' p'}}{\rho} \right) + \frac{\partial}{r \partial \theta} \left(v \frac{\partial k}{r \partial \theta} - \frac{\overline{v' p'}}{\rho} - \overline{v' k'} \right) \right]}^{\text{Diffusion}} \\
 & \overbrace{= S_k - \epsilon}^{\text{Source}}
 \end{aligned} \tag{3.68}$$

where

$$\begin{aligned}
 S_k &= - \left\{ \left[\overline{v'^2} \frac{\partial v_r}{\partial r} + \overline{v' v'_\theta} \frac{\partial v_\theta}{\partial r} + \overline{v' v'_z} \frac{\partial v_z}{\partial r} \right] \right. \\
 &\quad \left. + \left[\overline{v'_\theta v'_r} \frac{\partial v_r}{r \partial \theta} + \overline{v'^2} \frac{\partial v_\theta}{r \partial \theta} + \overline{v'_\theta v'_z} \frac{\partial v_z}{r \partial \theta} \right] \right\} \\
 \epsilon &= v \left\{ \left[\left(\frac{\partial v'_r}{\partial r} \right)^2 + \left(\frac{\partial v'_\theta}{\partial r} \right)^2 + \left(\frac{\partial v'_z}{\partial r} \right)^2 \right] \right. \\
 &\quad \left. + \left[\left(\frac{\partial v'_r}{r \partial \theta} \right)^2 + \left(\frac{\partial v'_\theta}{r \partial \theta} \right)^2 + \left(\frac{\partial v'_z}{r \partial \theta} \right)^2 \right] \right. \\
 &\quad \left. + \left[\left(\frac{\partial v'_r}{\partial z} \right)^2 + \left(\frac{\partial v'_\theta}{\partial z} \right)^2 + \left(\frac{\partial v'_z}{\partial z} \right)^2 \right] \right. \\
 &\quad \left. + \left[\frac{\overline{v'^2}}{r} + \frac{\overline{v'^2}}{r} + 2 \left[\overline{v'_r \frac{\partial v'_\theta}{r \partial \theta}} - \overline{v'_\theta \frac{\partial v'_r}{r \partial \theta}} \right] \right] \right\}
 \end{aligned} \tag{3.69}$$

Equation (3.68) is modelled as follows.

Diffusion term — The diffusion term is put in a typical diffusion form, in accordance with several authors:

$$\left. \begin{aligned} \overline{v_r' k'} + \frac{\overline{v_r' p'}}{\rho} &= - \frac{v_{rz}^T}{\sigma_k} \frac{\partial k}{\partial r} \\ \overline{v_\theta' k'} + \frac{\overline{v_\theta' p'}}{\rho} &= - \frac{v_{\theta z}^T}{\sigma_k} \frac{\partial k}{r \partial \theta} \end{aligned} \right\} (3.70)$$

where σ_k is a constant of order 1.0 [67].

Source term — Term S_k , representing turbulence generation, can be reduced by neglecting mean radial and tangential velocity terms as negligibly small and substituting the shear stress relations as follows:

$$S_k = + v_{rz}^T \left(\frac{\partial v_z}{\partial r} \right)^2 + v_{\theta z}^T \left(\frac{\partial v_z}{r \partial \theta} \right)^2 \quad (3.71)$$

Term ϵ is defined as the energy dissipation.

With these steps the turbulent kinetic energy equation can be put in the general form (3.67) where each term is shown in Table 3.2a.

3.3.5.1.4 Energy Dissipation — The equation for energy dissipation ϵ is obtained as suggested by Laufer [67] and based on the philosophy of Eq. (3.33), i.e.

$$\left. \begin{aligned} &\overbrace{\left[\frac{1}{r} \frac{\partial}{\partial r} r v_r \epsilon + \frac{\partial}{r \partial \theta} v_\theta \epsilon \right]}^{\text{Convection}} \\ &\underbrace{\left[- \frac{1}{r} \frac{\partial}{\partial r} r \left(v + \frac{v_{rz}^T}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial r} - \frac{\partial}{r \partial \theta} \left(v + \frac{v_{\theta z}^T}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{r \partial \theta} \right]}^{\text{Diffusion}} = \\ &\underbrace{\left[c_{\epsilon 1} \frac{\epsilon}{k} S_k - c_{\epsilon 2} \frac{\epsilon}{k} \epsilon \right]}^{\text{Source}} \end{aligned} \right\} \quad (3.72)$$

where S_k is given from Eq. (3.71) and σ_ϵ , $c_{\epsilon 1}$, $c_{\epsilon 2}$ are constants determined from experiment.

From the decay of turbulence behind a grid the value of c_2 is suggested to be 1.8 — 2.0 [67].

A relation between $c_{\epsilon 1}$, $c_{\epsilon 2}$, σ_ϵ is obtained from near to the wall turbulence [67] using Eq. (3.69) and the near wall relations

$$\left. \begin{aligned} \frac{\partial v_z}{\partial y} &= \frac{u_*}{ky} \\ \epsilon &= \frac{u_*^3}{ky} \\ k &= \frac{u_*^2}{\sqrt{c_\mu}} \end{aligned} \right\} \quad (3.73)$$

Neglecting diffusion and tangential contributions and taking into consideration that

$$\frac{y}{D/2} \ll 1 \quad \text{and} \quad \frac{v}{v_{rz}^T} \ll 1$$

Eq. (3.73) reduces to

$$\frac{\partial}{\partial y} \left(\frac{v_{rz}^T}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial r} = c_{\epsilon 1} \frac{\epsilon}{k} S_k - c_{\epsilon 2} \frac{\epsilon}{k} \epsilon \quad (3.74)$$

If further the tangential component of S_k in Eq. (3.71) is neglected, the near wall relations of Eq. (3.73) are used and v_{rz}^T is represented by Eq. (3.42), we further obtain

$$\frac{d}{dy} \left[y \frac{d}{dy} \left(\frac{1}{y} \right) \right] = \frac{(c_{\epsilon 1} - c_{\epsilon 2}) \sigma_\epsilon \sqrt{c_\mu}}{\kappa^2 y^2}$$

By performing the derivatives we end up with

$$c_{\epsilon 2} - c_{\epsilon 1} = \frac{\kappa^2}{\sigma_{\epsilon} \sqrt{c_{\mu}}} \quad (3.75)$$

Lauder [67] suggests that

$$c_{\epsilon 1} = 1.44 \quad c_{\epsilon 2} = 1.92 \quad \text{and} \quad \sigma_{\epsilon} = 1.3$$

3.3.5.2 The Solution Domain

The equations are to be solved within the characteristic triangle of the infinite rod bundle as shown in Fig. 1.1.

The secondary flow quantities (i.e., v_r , v_{θ} , p/ρ) are going to be solved in the whole flow area of the characteristic triangle, whereas the v_z momentum equation and the k , ϵ transport equations are going to be solved only in the outer region of the flow (i.e., $y_B^+ \geq 25-30$). As pointed out in Ref. 1 (p. 56) the exclusion of the wall region is made due to the following reasons:

- a) The sharp gradients of the hydrodynamic quantities within the wall region need a relatively high number of points to describe the hydrodynamic field sufficiently well.
- b) The strongly anisotropic turbulent kinetic equation makes a turbulent model based on the Kolmogorov-Prandtl hypothesis questionable. However, on the other hand, the prescription of the quantities themselves within the wall region is relatively not difficult since the main parameters of dependence are the wall shear stress, the distance from the wall and

the fluid viscosity.

The approach of excluding the wall region from the domain of solution has been used by several authors (see, e.g., [37, 54]) and in the efforts preceding this work at MIT [1].

3.3.5.2.1 Wall Layer Thickness Prescription — The wall layer thickness is defined from the relation

$$\frac{y_B \bar{u}^*}{\nu} = 30 \text{ to } 60 \quad (3.76)$$

where $\bar{u}^*$ is the average friction velocity, i.e.

$$\bar{u}^* = \left(\frac{\tau_w}{\rho} \right)^{1/2}_{\text{avg}}$$

y_B is the wall layer thickness

and ν is the fluid kinematic viscosity.

The exact value of relation (3.76) is left as problem-dependent, and its selection should take into consideration that

- 1) The higher the value of $y_B \bar{u}^* / \nu$ is, the fewer points are necessary to obtain the numerical solution; and
- 2) The closer to 30 the value of $y_B \bar{u}^* / \nu$ is, the more valid Eq. (3.52) is.

In case the bulk velocity v_B is given instead of $\bar{u}^*$, we use the friction factor relation

$$\bar{u}^* = \sqrt{\frac{f}{2}} v_B \quad (3.77)$$

For the friction factor estimation we will use Ibragimov's expression, which has also been used in Ref. [1]. This expression is as follows:

$$\frac{f}{f_o} = (0.58 + 0.42 e^{\gamma}) [1 + 0.1 (\frac{2\bar{y}}{D})^{4/3}]$$

where

$$\gamma = 0.21 \frac{P}{D} (\frac{A}{\bar{y}^2})^{1/4} (\frac{1}{\sqrt{3}} - \frac{1}{2}) \frac{D}{\bar{y}} \quad (3.78)$$

$$\bar{y} = \frac{1}{A} \iint_A y r dr d\theta = \frac{1}{2} (\frac{3}{\pi\sqrt{3}} \frac{P}{D} - 1) D$$

A is the flow area within the characteristic triangle
and

$$f_o = 0.046 Re^{-0.2} \quad (3.79)$$

is the friction factor for circular pipe.

3.3.5.3 Boundary Conditions — The boundary conditions are given in Table 3.2b. The nature of the boundary conditions at the symmetry lines is profound. We will discuss below the boundary conditions at the near wall boundary (i.e., at the position y_B).

3.3.5.3.1 Near Wall Boundary Conditions —

Axial velocity — Along the y_B circumferential line we will make the widely used assumption of logarithmic profile, i.e.

$$v_z(y_B, \theta) = \frac{u^*(\theta)}{\kappa} \ln E \frac{u^*(\theta) y_B}{v} \quad (3.80)$$

where κ and E are constants.

According to Ref. [1], $\kappa = 0.419$, $E = 9.8$.

The velocity profile in the region $0 \leq y \leq y_B$ needed for calculation of the bulk velocity is taken as follows:

$$\frac{v_z(y, \theta)}{u^*(\theta)} = \frac{u^*(\theta)y}{\nu} \quad \text{for } \frac{u^*(\theta)y}{\nu} \leq 5 \quad (3.81a)$$

$$\frac{v_z(y, \theta)}{u^*(\theta)} = \alpha + \beta \ln \frac{u^*(\theta)y}{\nu} \quad \text{for } 5 \leq \frac{u^*(\theta)y}{\nu} \leq \frac{u^*(\theta)y_B}{\nu} \quad (3.81b)$$

where

$$\beta = \frac{\frac{v_z(y_B, \theta)}{u^*(\theta)} - 5}{\ln \frac{y_B u^*(\theta)}{5\nu}} \quad (3.81c)$$

$$\alpha = \frac{1}{5} \left[\frac{v_z(y_B, \theta)}{u^*(\theta)} + 5 - \beta \ln \frac{5y_B u^*(\theta)}{\nu} \right] \quad (3.81d)$$

The expressions for α , β come from the velocity continuity condition at the points

$$\frac{yu^*(\theta)}{\nu} = 5 \quad \text{and} \quad \frac{yu^*(\theta)}{\nu} = 45 \quad (3.82)$$

3.3.5.3.2 Turbulent Kinetic Energy and Energy Dissipation

— As we have pointed out in §3.3.1.1 the near wall boundary lies within the dynamic equilibrium subregion where the importance of the convection and diffusion mechanism are minor.

Thus the transport equation for k can be reduced to

$$\left(-\frac{\overline{v_r v_z}}{r} \right)_{y=y_B} \left(\frac{\partial v_z}{\partial r} \right)_{y=y_B} = \varepsilon_B \quad (3.83)$$

where ε_B is the energy dissipation at $y=y_B$.

On the other hand the axial momentum equation neglecting the lateral gradient as small compared with the radial gradient

is reduced to

$$\frac{1}{r} \frac{\partial}{\partial r} (r \overline{v'_r v'_z}) - \nu \frac{\partial^2 v_z}{\partial r^2} = \frac{4}{D_H} (\bar{u}^*)^2 \quad (3.84)$$

where D_H is the hydraulic diameter and $\bar{u}^*$ is the average friction velocity.

Integrating Eq. (3.80) over y_B and neglecting the friction effect at y_B we obtain

$$(-\overline{v'_r v'_z})_{y=y_B} = \frac{r_1}{r_2} [u^*(\theta)]^2 - \frac{1}{2} \frac{r_2^2 - r_1^2}{r_2 D_H} (\bar{u}^*)^2 \quad (3.85)$$

where

$$r_1 = D/2$$

$$r_2 = D/2 + y_B$$

Since

$$\left(\frac{\partial v_z}{\partial r} \right)_{y=y_B} = \frac{u^*(\theta)}{\kappa y_B}$$

and

$$c_\mu \frac{k_B^2}{\epsilon_B} \left(\frac{\partial v_z}{\partial r} \right)_{y=y_B} = (-\overline{v'_r v'_z})_{y=y_B}$$

we find finally

$$\epsilon_B = (-\overline{v'_r v'_z})_{y=y_B} \frac{u^*(\theta)}{\kappa y_B} \quad (3.86)$$

$$k_B = \frac{1}{\sqrt{c_\mu}} (-\overline{v'_r v'_z})_B \quad (3.87)$$

3.3.5.4 Reynolds Number vs. Mean Wall Shear Stress —

Since we are going to solve the problems for the fully developed flow, some information on flow level is needed. This has been done by two methods:

- a) Specify the average wall shear stress $(\tau_w/\rho)_{avg}$ which appears in the source term of mean velocity equation (See Table 3.2a).
- b) Specify the Reynolds number, or equivalently the bulk velocity.

Both approaches are mathematically well defined. However, there is a difference when the problem is solved numerically.

In the second case, since there is an integral type of restriction and the problem has to be solved iteratively, some normalization of velocities is needed per iteration or per number of iterations. Since each numerical technique has a different effect per iteration, due to finite number of points, the answer will depend not only on the number of points but from the numerical method used as well.

This is illustrated in Fig. 3.4 where the shear stress distribution is plotted for a GRID 30x16, $P/D = 1.124$, $Re = 27,000$, using a 0-equation model for simplicity.

The solution has been obtained for a Reynolds number input by line iteration (See Chapter 4) excluding secondary flow and successive relaxation method with and without secondary flow. The effect of the numerical method is clear.

It is worth pointing out here that the solution of the no secondary flow problem with line iteration technique (curve 1) and the solution of the secondary flow problem with successive relation (curve 2) leads to the conclusion that the secondary flow increases the non-uniformity of wall shear stress distribution -- which is WRONG!

Therefore, in the present work the input of average wall shear stress is used.

CHAPTER 4

NUMERICAL WORK

4.1 Grid

We have tried to make the grid space as uniform as possible. The reason is that although grid nonuniformity can improve the accuracy of the problem in some cases, it usually has a negative effect on rate of convergence and stability [70]. On the other hand, a uniform grid makes the numerical equations easier to handle. So in the θ direction a uniform spacing $\Delta\theta$ is used, i.e.

$$\Delta\theta = \theta_j - \theta_{j-1} = \frac{\pi}{6*(JN - 1)} \quad (4.1)$$

where JN is the number of mesh points in the θ -direction. With respect to the r -direction, (i.e., $\theta = 0$)

$$r_1 - r_{1-1} = \left\{ \frac{P}{2} - \left(\frac{D}{2} + y_B \right) \right\}, \quad (M - 2); \quad 1 \leq M$$

$$M = IN - (JN - 1) \quad (4.2a)$$

where

M is the number of points along $\theta = 0$ line

IN is the number of points along $\theta = 30^\circ$ line, and

y_B is the wall boundary thickness.

For $1 > M$ the spacing cannot be uniform since it is defined from the zero shear stress line as shown in Fig. 4.1.

In this case

$$r_{i=M+j-1} = \frac{P}{2 \cos \theta} - \frac{P}{2} ; \quad M < i \leq IN \quad (4.2b)$$

The grid is shown in Fig. 4.1.

So by the above procedure the mesh points (i,j) have been defined. However at these points only the scalar parameters k , ϵ , and p the axial velocity v_z and auxiliary quantities (e.g., eddy viscosities, mixing length) will be calculated. The parameters $v_r(i,j)$ and $v_\theta(i,j)$ will be defined in a slightly different position as shown in Fig. 4.3. This procedure is similar to that adopted in the TEACH code [71].

4.2 The Finite Difference Equations

4.2.1 Momenta and Transport Equations

The finite difference equations are obtained by integration of Eq (3.67) over the control volume. The control volume surrounding the point under consideration is shown in its general form in Fig. 4.2.

After integration over the control volume and taking into consideration the Divergence Theorem Eq. (3.67) becomes

$$\oint_{cs} \phi \underline{v} \cdot \underline{n} \, d\ell + \oint_{cs} \underline{T} \cdot \underline{n} \, d\ell = \oint_{cs} \underline{S} \cdot \underline{n} \, d\ell + \iint_{cv} S^*_{\phi} \, dA \quad (4.3)$$

where by $\iint_{cv}$ we denote the integration over the boundary of the control volume, and $\underline{n}$ is the unit vector perpendicular to the surface boundary as shown in Fig. 4.2. Let

$$A_e = \text{length (ne, se)}$$

$$A_w = \text{length (nw, sw)}$$

$$A_n = \text{length (ne, nw)}$$

$$A_s = \text{length (se, sw)}$$

$$V_P = \text{area (ne, se, sw, nw, ne)} \quad (4.4)^*$$

The value of above quantities for the various angles ϕ are given in Table 4.2. Equation (4.3) can be written

$$\begin{aligned} A_e \phi_e v_{re} - A_w \phi_w v_{rw} + A_n \phi_n v_{\theta n} - A_s \phi_s v_{\theta s} + T_{\phi re} A_e - \\ - T_{\phi rw} A_w + T_{\phi \theta n} A_n - T_{\phi \theta s} A_s = S_{\phi re} A_e - S_{\phi rw} A_w + \\ + S_{\phi \theta n} A_n - S_{\phi \theta s} A_s + S_{\phi P}^* V_P \end{aligned} \quad (4.5)$$

In Eq. (4.5) the following approximation has been made

$$\iint_{cv} S_{\phi}^* dA \approx S_{\phi P}^* V_P \quad (4.6)$$

where

$S_{\phi P}^*$ is the value of S^* at point P

V_P is the control volume surface

The continuity equation gives after integration over the control volume

$$A_e v_{re} - A_w v_{rw} + A_n v_{\theta n} - A_s v_{\theta s} = 0 \quad (4.7)$$

In Eq. (4.5) we can distinguish between the Convection term, the Diffusion term and the Source term as follows:

$$DT = T_{\phi re} A_e - T_{\phi rw} A_w + T_{\phi \theta n} A_n - T_{\phi \theta s} A_s \quad (4.8a)$$

(*) the nomenclature used in Fig. 4.2 is almost the same as that used in Ref. 36.

Convection term:

$$CT = A_e v_{re} \phi_e - A_w v_{rw} \phi_w + A_n v_{\theta n} \phi_n - A_s v_{\theta s} \phi_s \quad (4.8b)$$

Source term:

$$ST = S_{\phi re} A_e - S_{\phi rw} A_w + S_{\phi \theta n} v_{\theta n} - S_{\phi \theta s} v_{\theta s} + S_{\phi P}^* V_P \quad (4.8c)$$

Thus Eq. (4.5) can be written

$$DT + CT = ST \quad (4.8d)$$

4.2.1.1 Diffusion Term

4.2.1.1.1 Scalar Quantities k , ϵ and the Axial Velocity V_z

From the Table 3.9a we have

$$\left. \begin{aligned} T_{\phi r} &= - \frac{v_{rz}^{eff}}{\sigma} \frac{\partial \phi}{\partial \theta} \\ T_{\phi} &= - \frac{v_{\theta z}^{eff}}{\sigma_{\phi}} \frac{\partial \phi}{r \partial \theta} \end{aligned} \right\} \quad (4.9)$$

We can make the following approximations

$$T_{\phi re} = - \frac{\left[v_{rz}^{eff} \right]_E + \left[v_{rz}^{eff} \right]_P}{2\sigma_{\phi}} \frac{\phi_E - \phi_P}{E - V_P} \quad (4.10a)$$

$$T_{\phi rw} = \frac{\left[v_{rz}^{eff} \right]_W + \left[v_{rz}^{eff} \right]_P}{2\sigma_{\phi}} \cdot \frac{\phi_P - \phi_W}{V_P - V_W}$$

$$T_{\phi \theta n} = - \frac{\left[v_{\theta z}^{eff} \right]_N + \left[v_{\theta z}^{eff} \right]_P}{2\sigma_{\phi}} \frac{\phi_N - \phi_P}{V_P \Delta \theta}$$

$$T_{\phi \theta s} = - \frac{\left[v_{\theta z}^{eff} \right]_S + \left[v_{\theta z}^{eff} \right]_P}{2\sigma_{\phi}} \frac{\phi_P - \phi_S}{V_P \Delta \theta} \quad (4.10b)$$

Thus Eq. (4.8a) becomes

$$DT = (DE+DW+DN+DS)\phi_P - DN \cdot \phi_N - DS \cdot \phi_S - DE \cdot \phi_E - DW \cdot \phi_W$$

where ϕ equals κ , ϵ or V_z

where

(4.11)

$$DE = \frac{BE}{\sigma_\phi} \left[\left(v_{rz}^{eff} \right)_E + \left(v_{rz}^{eff} \right)_P \right] \quad (4.12a)$$

$$DW = \frac{BW}{\sigma_\phi} \left[\left(v_{rz}^{eff} \right)_W + \left(v_{rz}^{eff} \right)_P \right] \quad (4.12b)$$

$$DN = \frac{BN}{\sigma_\phi} \left[\left(v_{\theta z}^{eff} \right)_N + \left(v_{\theta z}^{eff} \right)_P \right] \quad (4.12c)$$

$$DS = \frac{BS}{\sigma_\phi} \left[\left(v_{\theta z}^{eff} \right)_S + \left(v_{\theta z}^{eff} \right)_P \right] \quad (4.12d)$$

$$BE = \frac{Ae}{2(r_E - r_P)} \quad (4.13a)$$

$$BW = \frac{Aw}{2(r_P - r_W)} \quad (4.13b)$$

$$BN = \frac{An}{2r_P \Delta\theta} \quad (4.13c)$$

$$BS = \frac{As}{2r_P \Delta\theta} \quad (4.13d)$$

The values of DE, DW, DN, DS, BE, BW, BS are given in Table 4.2.

4.2.1.1.2 The Velocity v_r

From Table 3.2a we see that

$$\left. \begin{aligned} T_{\phi r} &= -2\nu \frac{\partial v_r}{\partial r} \\ T_{\phi \theta} &= -\frac{\nu_{\text{eff}}}{r\theta} \frac{\partial v_r}{\partial \theta} \end{aligned} \right\} \quad (4.14)$$

We make the following approximations

$$\left. \begin{aligned} T_{\phi re} &= -2\nu \left[v_{re} - v_{rp} \right] / \left[v_e - v_p \right] \\ T_{\phi rw} &= -2\nu \left[v_{rp} - v_{rw} \right] / \left[v_p - v_w \right] \\ T_{\phi \theta n} &= -\frac{\nu_{\text{eff}}}{r\theta n} \left[v_{rn} - v_{rp} \right] / r_p / \Delta \theta \\ T_{\phi \theta s} &= -\frac{\nu_{\text{eff}}}{r\theta s} \left[v_{rp} - v_{rs} \right] / r_p / \Delta \theta \end{aligned} \right\} \quad (4.15)$$

Thus the diffusion term (4.8a) for v_r becomes finally

$$\boxed{DT = (DE+DW+DN+DS)v_{rp} - DN \cdot v_{rn} - DS \cdot v_{rs} - DE \cdot v_{re} - DW \cdot v_{rw}} \quad (4.16)$$

where

$$DE = BE \cdot \nu \quad (4.17a)$$

$$DW = BW \cdot \nu \quad (4.17b)$$

$$DN = BN \left[\left(\frac{\nu_{\text{eff}}}{r\theta} \right)_{Ne} + \left(\frac{\nu_{\text{eff}}}{r\theta} \right)_e + \left(\frac{\nu_{\text{eff}}}{r\theta} \right)_w + \left(\frac{\nu_{\text{eff}}}{r\theta} \right)_{Nw} \right] \quad (4.17c)$$

$$DS = BS \left[\left(\frac{\nu_{\text{eff}}}{r\theta} \right)_{Se} + \left(\frac{\nu_{\text{eff}}}{r\theta} \right)_e + \left(\frac{\nu_{\text{eff}}}{r\theta} \right)_w + \left(\frac{\nu_{\text{eff}}}{r\theta} \right)_{Sw} \right] \quad (4.17d)$$

$$BE = \frac{2A_e}{r_E - r_P} \quad (4.18a)$$

$$BW = \frac{2A_W}{r_P - r_W} \quad (4.18b)$$

$$BN = BS = \frac{A_n}{4r_P \Delta \theta} \quad (4.18c)$$

The values of DE, DW, DN, DS, BE, BW, BS are given in Table 4.2.

4.2.1.1.3 The Velocity v_θ

From Table I we have

$$T_{\phi r} = - v_{r\theta}^{eff} r^2 \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) \quad (4.19a)$$

$$T_{\phi \theta} = - 2v \frac{\partial v_\theta}{\partial \theta} \quad (4.19b)$$

We make the following approximations

$$\left. \begin{aligned} T_{\phi re} &= v_{r\theta e}^{eff} r_w^2 \left(\frac{\frac{v_{\theta E}}{r_E} - \frac{v_{\theta P}}{r_P}}{r_E - r_P} \right) \\ T_{\phi rw} &= v_{r\theta w}^{eff} r_w^2 \left(\frac{\frac{v_{\theta P}}{r_P} - \frac{v_{\theta W}}{r_W}}{r_P - r_W} \right) \end{aligned} \right\} \quad (4.20)$$

$$\left. \begin{aligned} T_{\phi\theta n} &= -2v \left(\frac{v_{\theta n} - v_{\theta p}}{\Delta\theta} \right) \\ T_{\phi\theta s} &= 2v \left(\frac{v_{\theta p} - v_{\theta s}}{\Delta\theta} \right) \end{aligned} \right\} \quad (4.20)$$

Thus Eq. (4.8a) for v_θ becomes

$$DT = (DN + DS + DE \cdot \frac{r_E}{r_P} + DW \cdot \frac{r_W}{r_P}) v_P - DE \cdot v_{\theta E} - DW \cdot v_{\theta W} - DN \cdot v_{\theta n} - DS \cdot v_{\theta s} \quad (4.21)$$

where

$$\left. \begin{aligned} DE &= BE \left[\left(v_{r\theta}^{eff} \right)_{nE} + \left(v_{r\theta}^{eff} \right)_{sE} + \left(v_{r\theta}^{eff} \right)_s + \left(v_{r\theta}^{eff} \right)_n \right] \\ DW &= BW \left[\left(v_{r\theta}^{eff} \right)_{nW} + \left(v_{r\theta}^{eff} \right)_{sW} + \left(v_{r\theta}^{eff} \right)_s + \left(v_{r\theta}^{eff} \right)_n \right] \end{aligned} \right\} \quad (4.22)$$

$$DN = DW = BN \cdot v$$

$$BE = \frac{r_e^2 A_e}{4r_E(r_E - r_P)}$$

$$BW = \frac{r_W^2 A_W}{4r_W(r_P - r_W)}$$

$$BN = BS = \frac{2An}{\Delta\theta}$$

(4.23)

The values of DE, DW, DN, DS, BE, BW, BS are given in Table 4.23.

4.2.4.2 The Convection Term

Recall that the convection term is given by the Eq.

(4.8) i.e.,

$$CT = A_e v_{re} \phi_e - A_w v_{rw} \phi_w + A_n v_{\theta n} \phi_n - A_s v_{\theta s} \phi_s \quad (4.8b)$$

where

ϕ is given in Table 3.2a

From Eq. (4.8b) it is evident that the common substitution

$$\phi_e = \frac{\phi_E + \phi_P}{2}$$

etc., can produce a finite difference equation with negative coefficients which can produce nonconvergence in the solution. Since our ultimate purpose is to obtain a finite difference equation with positive coefficients and on the other hand as much accuracy as possible we use the Upwind Derivative Difference Scheme (UDDS) proposed in Appendix A.

Let

$$\left. \begin{aligned} \phi_e &= \phi_P + \left(\frac{\partial \phi}{\partial r} \right)_P (r_e - r_P) \\ \phi_n &= \phi_P + \left(\frac{\partial \phi}{\partial \theta} \right)_P \frac{\Delta \theta}{2} \\ \phi_w &= \phi_P - \left(\frac{\partial \phi}{\partial r} \right)_P (r_P - r_w) \\ \phi_s &= \phi_P - \left(\frac{\partial \phi}{\partial \theta} \right)_P \frac{\Delta \theta}{2} \end{aligned} \right\} \quad (4.24)$$

Substituting Eqs. (4.9) into (4.86) and taking into consideration continuity Eq. (4.7) we end up with

$$CT = \left[A_e v_{re} (r_e - r_P) - A_w v_{rw} (r_P - r_w) \right] \left(\frac{\partial \phi}{\partial r} \right)_P + \frac{A_n \Delta \theta}{2} (v_{\theta n} + v_{\theta s}) \left(\frac{\partial \phi}{\partial \theta} \right)_P \quad (4.25)$$

In Eq. (4.25) we have taken into consideration the fact that

$$A_n = A_s.$$

We further approximate

$$\left(\frac{\partial \phi}{\partial r} \right)_P = \begin{cases} \frac{\phi_E - \phi_P}{r_E - r_P} & \text{if } A_e v_{re} (r_e - r_P) + A_w v_{rw} (r_P - r_w) \leq 0 \\ \frac{\phi_P - \phi_w}{r_P - r_w} & \text{otherwise} \end{cases} \quad (4.26)$$

$$\left(\frac{\partial \phi}{\partial \theta} \right)_P = \begin{cases} \frac{\phi_N - \phi_P}{\Delta \theta} & \text{if } v_{\theta P} \leq 0 \\ \frac{\phi_P - \phi_S}{\Delta \theta} & \text{otherwise} \end{cases} \quad (4.27)$$

Under these circumstances the convection term can be written

$$CT = (CE + CW + CN + CS)\phi_P - CE \cdot \phi_E - CW \cdot \phi_w - CN \cdot \phi_N - CS \cdot \phi_S \quad (4.28)$$

where

$$CE = 0.5 \left\{ |A_e v_{re}(r_e - r_P) + A_w v_{rw}(r_P - r_w)| \right. \\ \left. - A_e v_{re}(r_e - r_P) - A_w v_{rw}(r_P - r_w) \right\} / (r_E - r_P) \quad (4.29a)$$

$$CW = 0.5 \left\{ |A_e v_{re}(r_e - r_P) + A_w v_{rw}(r_P - r_w)| \right. \\ \left. + A_e v_{re}(r_e - r_P) + A_w v_{rw}(r_P - r_w) \right\} / (r_P - r_w) \quad (4.29b)$$

$$CN = A_n \frac{|v_{\theta n} + v_{\theta s}| - v_{\theta n} - v_{\theta s}}{4} \quad (4.29c)$$

$$CS = A_s \frac{|v_{\theta n} + v_{\theta s}| + v_{\theta n} + v_{\theta s}}{4} \quad (4.29d)$$

The value for CE, CW, CN, CS at a particular point (i,j) and for $\phi = k$, ϵ , v_z , v_r are given in Table 4.3.

The convection term for the component v_θ itself is slightly different (*) than Eq. (4.28) as in the case of the diffusion term, i.e.,

$$CT = \left\{ CE \frac{r_P}{r_E} + CW \frac{r_P}{r_w} + CN + CS \right\} v_\theta P$$

$$= CE \cdot v_{\theta E} + CW \cdot v_{\theta W} + CN \cdot v_{\theta N} + CS \cdot v_{\theta S}$$

The values of CE, CW, CS for v_θ are given in Table 4.3.

(*) Equation (4.28) is valid for rv_θ instead of v_θ as Table 3.2a shows.

4.2.1.3 Source Term

Recall that the source term is given by Eq. (4.8c)

i.e.,

$$ST = S_{\phi re} A_e - S_{\phi rw} A_w + S_{\phi \theta n} A_n - S_{\phi \theta s} A_s + S_{\phi}^* V_P \quad (4.8c)$$

The form of the source term for each quantity is different in character so we examine ST for each quantity separately.

4.2.1.3.1 Axial Velocity v_z

From Table we see that

$$ST = S_P^* V_P = \frac{4}{DH} \left(\frac{\tau_w}{\rho} \right)_{avg} V_P \quad (4.30)$$

where

$\left(\frac{\tau_w}{\rho} \right)_{avg}$ is the average shear stress, and
DH is the hydraulic diameter.

4.2.1.3.2 Kinetic Energy k

From Table (3.2a)

$$ST = S_P^* V_P = \left[(v_{rz}^T)_P \left(\frac{\partial v_z}{\partial r} \right)_P^2 + (v_{\theta z}^T)_P \left(\frac{\partial v_z}{r \partial \theta} \right)_P^2 - \epsilon_P \right] V_P \quad (4.31)$$

For reasons of numerical stability the energy dissipation term is calculated from Eq. (3.42) i.e.,

$$\epsilon_P = \frac{c_\mu k_P^2}{(v_{rz}^T)_P} \quad (4.32)$$

Thus

$$ST = \left[(v_{rz}^T)_P \left(\frac{\partial v_z}{\partial r} \right)_P^2 + (v_{\theta z}^T)_P \left(\frac{\partial v_z}{r \partial \theta} \right)_P^2 \right] V_P - S_P \cdot k_P \quad (4.33)$$

where

$$SP = \frac{c_\mu k_P}{(v_{rz}^T)_P} \quad (4.34)$$

4.2.3.1.3.3 Energy Dissipation ϵ

From Table (3.2a)

$$ST = \left\{ c_{\epsilon 1} \frac{\epsilon}{k} \left[v_{rz}^T \left(\frac{\partial v_z}{\partial r} \right)^2 + v_{\theta z}^T \left(\frac{\partial v_z}{r \partial \theta} \right)^2 \right] - c_{\epsilon 2} \frac{\epsilon^2}{k} \right\} \quad (4.35)$$

Since

$$\frac{\epsilon}{k} = c_\mu \frac{k}{v_{vz}^T} \quad (4.36)$$

Then

$$ST = c_{\epsilon 1} \frac{c_\mu k_P}{(v_{rz}^T)_P} \left\{ (v_{rz}^T)_P \left(\frac{\partial v_z}{\partial r} \right)^2 + v_{\theta z}^T \left(\frac{\partial v_z}{r \partial \theta} \right)^2 \right\} - SP \epsilon_P \quad (4.37)$$

where

$$SP = c_{\epsilon 2} c_\mu \frac{k_P v_P}{(v_{rz}^T)_P} \quad (4.38)$$

4.2.1.3.4 Radial Velocity v_r

From Table (3.2a) we have

$$S_{\phi r} = -(\overline{v_{r\theta}^2}) + \frac{p}{\rho}$$

$$S_{\phi \theta} = v_r^{\text{eff}} r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right)$$

$$S_\phi^* = \frac{v_\theta^2 + \overline{v_\theta^2}}{r} - \frac{2v}{r} \left(\frac{\partial v_\theta}{r \partial \theta} + \frac{v_r}{r} \right) \approx \frac{\overline{v_\theta^2}}{r}$$

Thus Eq. (4.86) becomes

$$\begin{aligned}
 ST = & A_w \left[\overline{v_{rw}^2} + (p/\rho)_w \right] - A_e \left[\overline{v_{re}^2} + (p/\rho)_e \right] \\
 & + (v_r^{eff})_n r_p \left[\frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) \right]_n A_n - v_r^{eff} r_p \left[\frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) \right]_s A_s \\
 & + \frac{(\overline{v_\theta^2})_p}{r_p} V_p
 \end{aligned} \quad (4.40)$$

The ST value for the point (i,j) is given in Table 4.4.

4.2.1.3.5 Velocity v_θ

From Table (3.2a) we have

$$S_{\phi r} = -v_{r\theta}^{eff} \frac{\partial v_r}{\partial \theta} \quad (4.41)$$

$$S_{\phi \theta} = 2vv_r - r\overline{v_\theta^2} - \frac{rp}{\rho} - r(\overline{v_\theta^2} + \frac{p}{\rho}) \quad (4.42)$$

Thus Eq. (4.8c) can be approximated

$$\begin{aligned}
 ST = & v_{r\theta e}^{eff} \left(\frac{\partial v_r}{\partial \theta} \right)_e A_e - v_{r\theta w}^{eff} \left(\frac{\partial v_r}{\partial \theta} \right)_w A_w \\
 & + r_p A_n \left[(\overline{v_\theta^2})_s + (p/\rho)_s - (\overline{v_\theta^2})_n - (p/\rho)_n \right]
 \end{aligned} \quad (4.43)$$

The ST value for the point (i,j) is given in Table 4.4.

4.2.1.4 General Form of FDE

The general form of the differential equations of all the quantities under consideration, has as follows

$$(AP + SP)\phi_P = AE \cdot \phi_E + AN \cdot \phi_N + AW \cdot \phi_W + AS \cdot \phi_S + SU \quad (4.44a)$$

where

$$\left. \begin{aligned} AE &= CE + DE \\ AW &= CW + DW \\ AN &= CN + DN \\ AS &= CS + DS \\ SU &= ST + SP \cdot \phi_P \end{aligned} \right\} \quad (4.44b)$$

$$AP = \begin{cases} \frac{r_P}{r_E} CE + \frac{r_E}{r_P} DE + \frac{r_P}{r_W} CW + \frac{r_W}{r_P} DW + AN + AS & \text{for } v_\theta \\ AE + AW + AN + AS & \text{otherwise} \end{cases} \quad (4.44c)$$

where CE, CW, CN, CS, DE, DW, DN, DS, SU, SP are given in Table 4.2, 4.3 and 4.4:

4.2.1.5 The Boundary Conditions

4.2.1.5.1 Quantities k, ϵ , v_z

4.2.1.5.1.1 Boundary I and II (Fig. 4.1)

According to Table 3.2b

$$\left(\frac{\partial \phi}{\partial \theta}\right)_{I,II} = 0 \quad (4.45)$$

where

$$\phi = k, \epsilon \text{ or } v_z$$

Numerically Eq. (4.44) is translated to

$$\left. \begin{aligned} \phi_{1,j} &= \phi_{1,j+1} & \text{for Boundary I} \\ \phi_{1,j} &= \phi_{1,j-1} & \text{for Boundary II} \end{aligned} \right\} \quad (4.46)$$

4.2.1.5.1.2 Boundary III

From Table (3.2a) we have

$$\left(\frac{\partial \phi}{\partial n}\right)_{III} = 0 \quad (4.47)$$

Integrating over the triangular control volume shown in Fig. 4.4 and taking into consideration Eq. (4.47) we end up with

$$\left. \begin{aligned} (AP + SP')\phi_P &= AW \cdot \phi_W + AN \cdot \phi_N' + SU' \\ AW &= DW + CW' \\ AN &= DN + CN' \\ AP &= AW + AN \end{aligned} \right\} \quad (4.48)$$

Where DW, DN are given from Table 4.2 and 4.3 respectively

$$\left. \begin{aligned} CN' &= -AN(v_\theta)_{i,j+1}/2 \\ CW' &= A_W(v_r)_{i,j}/2 \\ SU' &= SU \frac{V_{P'}}{V_P} \\ SP' &= SP \frac{V_{P'}}{V_P} \end{aligned} \right\} \quad (4.49)$$

and A_e , A_n , V_P are given from Table 4.1. SU , SP are given from Table 4.4.

$V_{P'}$ is the area of the triangular control volume under consideration, i.e.,

$$(V_{P'}) = \left[(r_{i+1} + r_i) \sin \Delta\theta - (r_i + r_{i-1})\Delta\theta \right] \frac{r_i + r_{i+1}}{8} \quad (4.50)$$

It is evident from Eq. (4.49) that we have substituted for

$$\phi_e = \frac{\phi_E + \phi_P}{2}$$

and similarly for ϕ_w , ϕ_n , ϕ_s .

4.2.1.5.1.3 Wall Region

Axial Velocity

At the y_B line we will demand continuity of the velocity and the velocity gradient.

The velocity gradient can be calculated numerically with accuracy $O(r^2)$ from the relation

$$\begin{aligned} \left(\frac{dv_z}{dr} \right)_{2,j} &= \frac{r_4 - r_2}{(r_3 - r_2)(r_4 - r_3)} (v_z)_{3,j} - \frac{r_3 - r_2}{(r_4 - r_2)(r_4 - r_3)} (v_z)_{4,j} \\ &\quad - \frac{r_4 + r_3 - 2r_2}{(r_3 - r_2)(r_4 - r_2)} (v_z)_{2,j} \end{aligned} \quad (4.51a)$$

since

$$(v_z)_{2,j} = \frac{u_j^*}{\kappa} \ln E \frac{u_j^* y_B}{v} \quad (4.51b)$$

and

$$\left(\frac{dv_z}{dr} \right)_{2,j} = \frac{u_j^*}{\kappa y_B} \quad (4.51c)$$

we can solve Eq. (4.51) for u_j^* and $(v_z)_{2,j}$ numerically.

Turbulent Kinetic Energy and Energy Dissipation

$k_{2,j}$ and $\epsilon_{2,j}$ are calculated from the algebraic relations given in Table 3.2b.

4.2.1.5.2 V_r Velocity

Boundary I and II (Fig. 1.1)

According to Table 3.2b

$$\left(\frac{\partial v_r}{\partial \theta}\right)_{I,II} = 0 \quad (4.52a)$$

thus numerically Eq. (4.52a) is translated

$$\left. \begin{aligned} (v_r)_{i,j} &= (v_r)_{i,j+1} && \text{for Boundary I} \\ (v_r)_{i,j} &= (v_r)_{i,j-1} && \text{for Boundary II} \end{aligned} \right\} \quad (4.52b)$$

Boundary III (Fig. 1.1)

From Table 3.2b we have

$$\underline{V}_{\text{sec}} \underline{n} = 0 \quad (4.53)$$

Numerically Eq. (4.53) can be expressed as we can see from Fig. 4.4b.

$$(A_e)_{i-1} (v_r)_{i,j} = (A_n)_{i-1} (v_\theta)_{i-1,j} \quad (4.54)$$

where A_e and A_n are given in Table 4.3.

Wall

From Table 3.2b we have

$$(v_r)_{i,j} = 0 \quad (4.55)$$

from the continuity equation, since

$$\frac{\partial v_\theta}{\partial \theta} = 0 \quad \text{at the wall}$$

it is

$$\frac{\partial (rv_r)}{\partial r} = 0$$

which gives

$$(v_r)_{1,j} = 0 \quad (4.56)$$

as well

4.2.1.5.3 v_θ Velocity

Boundaries I and III

From Table 3.2b we have

$$(v_\theta)_{I,II} = 0 \quad (4.57)$$

Numerically we can approximate* (with accuracy of $O(\frac{\Delta\theta}{2})$)

$$\left. \begin{aligned} (v_\theta)_{1,2} &= 0 \\ (v_\theta)_{1,N} &= 0 \end{aligned} \right\} \quad (4.58)$$

where N is the maximum number of points for v_z along the tangential direction.

Boundary III

From Table 3.2b

$$\frac{\partial v_{\text{sec}}}{\partial n} = 0 \quad (4.59)$$

This relation leads to (see Ref. 1)

$$\frac{\partial(rv_\theta)}{\partial r} = \frac{\partial v_r}{\partial \theta} \quad (4.60)$$

Numerically Eq. (4.59) as we can see from Fig. 4.46 can be expressed

$$\frac{r_1(v_\theta)_{1,j} - r_{1-1}(v_\theta)_{1-1,j}}{r_1 - r_{1-1}} = \frac{(v_r)_{1,j} - (v_r)_{1,j-1}}{\Delta\theta} \quad (4.61)$$

*More accurate schemes for Eq. (4.57) have led to instabilities.

Wall

From Table 3.2b

$$(v_{\theta})_{1,j} = 0 \quad (4.62)$$

4.2.2 Pressure Equation

Up to now we have obtained Finite Difference Equations for the quantities v_z , v_r , v_{θ} , k and ϵ . What remains is an equation for pressure which has to come from the continuity equation.

Here we will follow the procedure followed by A.D. Gosman [67,71] and Patankar and Spalding [72].

The idea is that the previously obtained equations for v_r and v_{θ} do not necessarily satisfy the continuity equations. Thus pressure connections will lead to velocity connections so that the continuity equation is obeyed.

Thus in order to obtain a finite difference equation for p/ρ at a particular point P (see Fig. 4.2) the pressures p_P , p_E , p_W , p_N , p_S must change so that the continuity equation is obeyed. Let the new pressures and velocities be

$$\left. \begin{aligned} p_P^* &= p_P + p_P' \\ p_E^* &= p_E + p_E' & v_{re}^* &= v_{re} + v_{re}' \\ p_W^* &= p_W + p_W' & v_{rw}^* &= v_{rw} + v_{rw}' \\ p_N^* &= p_N + p_N' & v_{rn}^* &= v_{rn} + v_{rn}' \\ p_S^* &= p_S + p_S' & v_{rs}^* &= v_{rs} + v_{rs}' \end{aligned} \right\} \quad (4.63)$$

The continuity equation is

$$A_e v_{re} - A_w v_{rw} + A_n v_{\theta n} - A_s v_{\theta s} = \text{SMP} \quad (4.64a)$$

and

$$A_e v_{re}^* - A_w v_{rw}^* + A_n v_{\theta n}^* = A_s v_{\theta s} = 0 \quad (4.64b)$$

SMP is not necessarily zero.

From the general Eq. (4.44) and Eq. (4.40) we can see that the equation for v_{re} is (see Fig. 4.2).

$$\begin{aligned} (AP)_e v_{re} = & \text{length}(n,s) \cdot p_p / \rho - \text{length}(nE,sE) \cdot p_E / \rho \\ & + \text{other terms} \end{aligned} \quad (4.65)$$

By substituting v_{re} for v_{re}^* and p_E, p_p for p_E^*, p_p^* and neglecting any effect from the other terms we end up with

$$A_e (v_{re}^* - v_{re}) = DPE \frac{r_e}{r_E} p_p' / \rho - DPE \cdot p_E' / \rho \quad (4.66)$$

and

$$DPE = \frac{\text{length}(nE,sE)}{(AP)_e} \cdot A_e \quad (4.67)$$

Working similarly with the other velocities, substituting the resultant equations into Eq. (4.64a) and taking into consideration Continuity Eq. (4.64b), we end up with

$$\begin{aligned} DPE \frac{r_P}{r_E} + DPW \frac{r_P}{r_W} + DPN + DPS p_P' / \rho \\ = DPE \cdot p_E' / \rho + DPW p_W' / \rho + DPN p_N' / \rho + DPS \cdot p_S' / \rho \\ - SMP \end{aligned} \quad (4.68)$$

The expression for DPE, DPW, DPN, DPS at the particular point (i,j) are given in Table 4.5. Equation (4.68) is the FDE for pressure correction and has exactly the same form as Eq. (4.44)

which is valid for the other quantities. If we write

$$(AP+SP)p_P/\rho = AE \cdot p_E/\rho + AW \cdot p_W/\rho + AN \cdot p_N/\rho + AS \cdot p_S/\rho + SU \quad (4.69)$$

where

$$\left. \begin{aligned} AE &= DPE \\ AW &= DPW \\ AN &= DPN \\ AS &= AE \frac{r_P}{r_E} + AW \frac{r_P}{r_W} + AN + AS \\ SU &= -SMP \\ SP &= 0 \end{aligned} \right\} \quad (4.70)$$

4.3 Numerical Solution

4.3.1 General

Recall that the finite difference equations to be solved are of the form

$$\begin{aligned} (AP)_{1,j}^* \phi_{1,j} &= (AE)_{1,j}^* \phi_{1+1,j} + (AW)_{1,j} \phi_{1-1,j} + (AN)_{1,j} \phi_{1,j+1} \\ &\quad + (AS)_{1,j} \phi_{1,j+1} + SU_{1,j} \\ AP &\leq AE + AW + AN + AS \\ AP, AE, AN, AS &\geq 0 \end{aligned} \quad (4.71)$$

where ϕ stands for

- v_z the axial velocity;
- v_r the radial velocity;
- v_θ the the tangential velocity
- p'/ρ the pressure correction
- k turbulent kinetic energy, and
- ϵ energy dissipation

Equation (4.71) is a non-linear equation since its coefficients are function of the unknown quantities.

The behavior of a corresponding linear equation of form (4.71) is well known and a number of numerical techniques exist that guarantee convergence. However in the actual nonlinear problem, the applicability of such techniques is not known. In the present work our original purpose was to use the numerical technique used in TEACH [71], which here is called line iteration technique (LI). Actually LI gives a relatively good rate of convergence in the case where the secondary flow is suspended. However in the case where secondary flow is retained we have difficulties due to the very low rate of convergence. Therefore the secondary flows solutions have been worked by using the well known successive relaxation technique [74]. Both techniques as applied in our case are described below.

4.3.2 Line Iteration Technique [71]

In order to minimize the negative effect with respect to convergence that the non-linearity introduces, an under-relaxation factor is introduced as follows:

$$\phi_{1,j}^{(n)} = (1-UREL) \phi_{1,j}^{(n-1)} + UREL \phi_{1,j}^{*(n)} \quad (4.72)$$

where

n is the number of iteration

UREL the underrelaxation factor, and

$\phi_{1,j}^{*(n)}$ is given by a relation based on Eq. (4.71) as follows

$$\begin{aligned} (AP)_{1,j}^{(n-1)} \phi_{1,j}^{*(n)} &= (AE)_{1,j}^{(n-1)} \phi_{1,j+1}^{(n)} + (AW)_{1,j}^{(n-1)} \phi_{1,j-1}^{(n)} \\ &+ (AN)_{1,j}^{(n-1)} \phi_{1,j+1}^{(n-1)} + (AS)_{1,j}^{(n-1)} \phi_{1,j-1}^{(n-1)} + S_{1,j}^{(n-1)} \end{aligned} \quad (4.73)$$

Taking into consideration Eq. (4.73), Eq. (4.72) can be written in vector form as

$$[A]_j \phi_j^{(n)} + [B]_j \phi_{j+1}^{(n-1)} + [C]_j \phi_{j-1}^{(n)} = 0 \quad (4.74)$$

where

$$\phi_j = \text{vector } \{\phi_{1,j}\} \quad (4.74a)$$

$$[B]_j = \text{diagonal } \{(AN)_{1,j}\} \quad (4.74b)$$

$$[C]_j = \text{diagonal } \{(AS)_{1,j}\} \quad (4.74c)$$

$$[S] = \text{diagonal } \{S_{1,j} + \frac{1-UREL}{UREL} \cdot (AP)_{1,j}\} \quad (4.74d)$$

$$[A]_j = \begin{bmatrix} & & & 0 \\ & & & \\ & & & \\ (AW)_{1,j} & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ 0 & & & \end{bmatrix} \quad (4.74e)$$

$-(AP)_{1,j}/UREL$

From Eq. (4.74) we obtain

$$\phi_j = -[A]_j^{-1} \left\{ [B]_j \phi_{j+1}^{(n-1)} + [C]_j \phi_{j-1}^{(n)} + [S]^{n-1} \right\} \quad (4.75)$$

The inversion of matrix A is made by using the matrix factorization technique [76].

4.3.3 Successive Relaxation and Successive Under-relaxation Technique

This technique is a point iteration technique. The underrelaxation factor is again used here.

$$\phi_{1,j}^{(n)} = (1-UREL) \phi_{1,j}^{(n-1)} + UREL \cdot \phi_{1,j}^{*(n)} \quad (4.76)$$

where $\phi_{1,j}^{*(n)}$ is now given by the relation

$$\begin{aligned} (AP)_{1,j}^{(n-1)} \phi_{1,j}^{*(n)} &= (AE)_{1,j}^{(n-1)} \phi_{1+1,j}^{(n-1)} + (AW)^{n-1} \phi_{1+1,j}^{(n)} \\ &+ (AN)_{1,j}^{n-1} \phi_{1,j+1}^{(n-1)} + (AS)^{(n-1)} \phi_{1,j-1}^{(n)} + S^{(n-1)} \end{aligned} \quad (4.77)$$

4.3.4 Line Iteration vs. Point Iteration

The application of Line Iteration Technique in cases without secondary flow creates no problem with respect to convergence. Trying to solve the secondary flow with Line Iteration Technique had led to instabilities. The instability can be removed by using underrelaxation factors below 0.5. From running the first 1000 iterations, it was clear that an extremely high number of iterations is needed to obtain the solution. It was also evident that the underrelaxation factor reduces the rate of convergence.

For illustration we have taken the simple linear problem of the laminar flow with $P = 57.5\text{mm}$ $P/D = 1.124$ and $\tau_w/\rho = 2.7 \times 10^{-5}$ an underrelaxation factor of 1.0 to 0.5 has been studied. In Fig. 4.5 the CPU time required in a IBM 360 computer as a function of the underrelaxation factor is plotted. A grid of 29×16 points has been used. An accuracy of $v_z(I=5, J=5)$ up to 3 decimal points has been used as a convergence criterion. From Fig. 4.5 we can see that an $UREL = 0.9$ increases the number of iterations for solution 15 times! Figure 4.5 shows also that the Line Iteration Technique combined with an underrelaxation factor 0.7 requires more time for the solution than the point iteration technique with underrelaxation factor equal to unity.

The above example illustrates that a relatively fast technique (like Line Iteration Technique) combined with an underrelaxation factor probably is not the recommended technique to get fast solution. Some slower technique (like successive relaxation) with better underrelaxation may be preferable.

That is exactly the case with secondary flow where solution of secondary flow has been obtained faster with successive relaxation technique ($UREL = 1.0$).

Thus in the present problem recommended technique for the problems with secondary flow suppressed is the Line iteration whereas for the secondary flow solution the Successive relaxation technique is recommended.

4.3.5 The Convergence Criteria

4.3.5.1 Convergence criteria in TEACH [71]

In TEACH the following quantity related to convergence criteria is defined:

$$\begin{aligned}
 X_{\phi} = \left\{ \sum_{i,j} \left| (AE)_{i,j}^{(n)} \phi_{i+1,j}^{(n-1)} + (AW)_{i,j}^{(n)} \phi_{i-1,j}^{(n-1)} \right. \right. \\
 + (AN)_{i,j}^{(n)} \phi_{i,j+1}^{(n-1)} + (AS)_{i,j}^{(n)} \phi_{i,j-1}^{(n-1)} \\
 \left. \left. + (S_{\phi})_{i,j}^{(n)} - (AP)_{i,j}^{(n)} \phi_{i,j}^{(n-1)} \right| \right\} / \phi
 \end{aligned} \quad (4-78)$$

where

n is the number of iterations,

ϕ is a normalization constant

AE, AW, AN, AS, S are the coefficients of the finite difference equation for the quantity ϕ .

The criterion for terminating the iteration is

$$X_{\phi} \leq \epsilon \quad (4-79)$$

where ϵ is a specified sufficiently small number.

An additional criterion is related to the continuity equation and is expressed as follows

$$\sum_{i,j} \left| \frac{\int \int \nabla \cdot \underline{v} \, d(\text{vol})}{\phi_M} \right| \leq \epsilon \quad (4.80)$$

where ϕ_M is again a normalization constant

$\underline{v}$ is the velocity vector.

Although the above criteria seem to provide a correct picture of convergence, they make selection of ϵ relatively difficult, since it is determined by the coefficients of the difference equation of ϕ rather than the ϕ itself. In the present work, the convergence criteria have been selected to be easier and simpler than the TEACH approach and to take into consideration the peculiarities of the present problem.

4.3.5.2 Present convergence criteria

In the present work two criteria have been established. First,

$$\frac{\left| \phi_{1,j}^{(n)} - \phi_{1,j}^{(n-1)} \right|_{\max}}{\phi} \leq 1.0 \times 10^{-6} \quad (4.81)$$

where ϕ is also a normalization constant reflecting the size of ϕ .

ϕ stands for v_z , k , ϵ and v_θ

ϕ has been selected as follows:

Axial velocity: $\phi = u_b$

Turbulent kinetic energy: $\phi = (\tau_w/\rho)_{\text{avg}}$

Energy dissipation: $\phi = 4 (\tau_w/\rho)_{\text{avg}}^{3/2}/DH$

v_θ velocity: $\phi = 0.1 u_b$

For relatively fast converging problems, which is usually the case for problems where the secondary flow is neglected, criterion (4.81) gives sufficiently accurate

results. For some slowly convergent cases, however, which includes the secondary flow cases, the above criterion cannot be met although the solution is accurate within .1%.

For such cases an alternative criterion based on more than one iteration, the following formula is used:

$$\frac{|v_{z, is, js}^{(n+ND)} - v_{z, is, js}^{(n)}|}{u_b} \leq 1.0 \times 10^{-3} \quad (4.82)$$

where

ND = 60 for the line iteration scheme

ND = 500 for the successive relaxation scheme

(is, js) is a preselected point

Criteria (4.81) and (4.82) have been found to guarantee accurate enough solutions (i.e. within .1%) for all convergent cases.

The convergence and the accuracy is also tested by successive print outs of the quantities at a particular point. This is also done in TEACH code.

4.4 Computer Program

To obtain the solution of the Finite Difference Equations a computer program has been written. Its main structure is based on the TEACH code where subroutines calculating the grid, the wall thickness, and the average velocity are modified versions of the code used in Ref. 1. The code structure and listing are described in detail in Appendix E.

CHAPTER 5

RESULTS AND CONCLUSIONS

5.1 Experimental Results

5.1.1 Data evaluation

All the measurements performed for the estimation of the quantities v_z and v'_z were characterized by good signal to noise ratio and therefore the results were satisfactory.

The lateral measurements (i.e., measurements involving v_2 and v_1 components) are characterized by relatively low signal to noise ratio. Since the performance of the Frequency Tracker deteriorates with the decrease of signal to noise ratio, lateral measurements are characterized by relatively large errors as pointed out in Chapter 2. The reason for the low-to-signal ratio is attributed to the rather complicated geometry of the test section which for these particular directions can bring the signal below the satisfactory level. For measurement of the v_2 component in the gap region particularly, the measuring volume lies relatively close to the wall. This is because the largest dimension (0.66 mm) of the beam intersection volume lies across the gap region which is only half the thickness 3.175 mm. The close proximity of the measuring volume to the wall degrades the signal due to actual intersection and reflections of beams from the wall into the measuring volume.

Under these circumstances one can summarize the data evaluation performed in the lateral direction as follows:

Quantities v_r' , v_θ' : Although their size level seems to be reasonable, they give erroneous results for quantities like v_r'/v_θ' and $v_\theta'^2 - v_r'^2$. Specifically at some points v_r'/v_θ' is greater than one which is wrong. As a consequence of the relatively large errors, entrance effects cannot be distinguished.

Quantities $\overline{v_r'v_z'}$, $\overline{v_\theta'v_z'}$. For the quantity $\overline{v_r'v_z'}$, the results seem reasonably good, close to the gap up to a distance of 1.2 mm from the wall. The results of measurements closer to the wall give unacceptably low values for $\overline{v_r'v_z'}$. The reason is attributed to the low signal-to-noise ratio and to failure of the turbulent processor to respond to the "bursty" nature of turbulence close to the wall. Actually final measurements for points closer to the wall have not been performed.

One positive characteristic, however, of the cross-correlation measurements was that the noise carried by each signal is highly uncorrelated. To be more specific, the values for $\overline{v_r'v_z'}$ along the zero shear stress line were close to zero which means that the noise present in the signals v_1 , v_2 and v_z is almost uncorrelated. This supports the suggestion [87] that results based on continuous Doppler signal are best obtained by collecting the signal in two different directions and correlating the outputs.

With respect to the quantity $\overline{v_\theta v_z}$ due to its relatively small size and the greater dependence on v_z components, the results are characterized as rather inconclusive.

5.1.2 Fully Developed Region

One of the questions raised in [1] was whether the data taken at $L/D_H = 77$ corresponded to fully developed flow.

Figure 5.1 shows the velocity v_z normalized by the corresponding maximum channel velocity, at positions $\theta=0, 12^\circ$ and 30° as predicted by P. Carajilescov [1] at $L/D_H = 77$ and in the present work at positions $L/D_H = 77$ and 113.

Figure 5.1 shows that results taken in [1] were actually in the developing region. Also Figure 5.1 shows the positive effect of the entrance section fabricated and used only in this study versus [1] to reduce the entrance length.

With respect now to the question whether the position $L/D_H = 113$ lies in the developing region, one can distinguish in Figure 5.1 that there is a flow development from $L/D_H = 77$ to $L/D_H = 113$ which is, however, of the order of the experimental error. Therefore, we accept the data at $L/D_H = 113$ as data corresponding to fully developed flow within experimental error.

All the data taken are tabulated in APPENDIX C.

Experimental results for axial velocity v_z and turbulent axial velocity v_z' in the fully developed flow ($L/D_H = 113$)

are given in Figs. 5.2a-5.5c for $Re=26,500$ and $65,000$ and in the Figs. 5.13 and 5.14 for $Re=9,000$ at $\theta=0^\circ$, 12° and 30° .

Experimental results for radial Reynolds stress and turbulent kinetic energy are given in Figs. 5.6a-5.9c for $Re=26,500$ and $Re=65,000$ and in the Figs. 5.15 and 5.16 at $\theta=0^\circ$, 12° and 24° .

Attempts also have been made to estimate wall shear stress from the logarithmic law ($v_z = \frac{u^*}{\kappa} \ln(Ey^+)$, $\kappa=0.42$, $E=9.8$) using the measured axial velocity at points closest to the wall. The logarithmic law as applied in §3.3.5.3.1 was used to calculate the bulk velocity. Average shear stress and bulk velocity data are given in Table 5.1. Wall shear stress distribution is given in Figs. 5.10, 5.11, 5.12 for $Re=26,500$, $65,000$ and $9,000$ respectively.

5.1.3. Developing Region

The entrance effects on the axial velocity and its rms value is clear as we can see in Figs. 5.2a-5.5c where data are presented at positions $L/D_H=15$, 46 and 113 . To illustrate the combined effect of flow development and flow split, all quantities are normalized by the maximum velocity at $L/D_H=113$ for each Reynolds Number.

It is clear from the axial velocity results that

a) a flow split effect is present, i.e., higher flow rates for higher L/D_H because side subchannels experience lower flow rates than the interior subchannels. Fig. 2.1

shows that the present test section consists of twenty (20) characteristic triangles of the type of Figure 1.1, four (4) of which are side triangles.

The flow split effect also can be seen in Table 5.2 where the bulk velocities are given.

- b) Flatter velocity profiles occur for smaller L/D_H as expected
- c) The turbulent axial velocity is also developing as Figures 5.4a-5.5c show. However, the difference in turbulent axial velocity levels between $L/D_H = 46$ and 113 seems to be small.

For the other measured quantities it is rather difficult to make any definite conclusion based on the experimental results regarding development effects. One can see this from k and $\overline{v_r'v_z'}$ data plotted in Figures 5.6a-5.9c.

Comparison between all experimental data obtained in the present work at positions $L/D_H = 46$ and 113 show that the quantity mostly affected by the entrance length is the mean velocity. This suggests the mean velocity profile as the most reliable criterion of the onset of fully developed flow.

With respect to the wall shear stress distribution, we applied the logarithmic law as we did for the fully developed flow. The results are shown in Figures 5.10 and 5.11 for $S_3=26,500$ and $65,000$, respectively. Of course, use of the logarithmic law to describe the velocity distribution in the

developing region is not strictly correct. However it was done to explore the behavior of wall shear in the developing region since many experimenters may have inadvertently applied this approach to data in the developing region.

a) In the position $L/D_H=46$ the wall shear stress distribution is pretty much developed.

b) In position $L/D_H=15$ a dip is observed towards the position $\theta=30^\circ$ for $Re=26,500$. For $Re=65,000$ there is a dip tendency between 18 and 24° but reversal of the dip occurred at $\theta=30^\circ$. These results raise the question whether the shear stress distribution dips that appear in the literature, representing fully developed conditions actually are associated with developing effects.

5.1.3.1 Flow Split

As is mentioned in the previous section and we also can see from Table 5.2 and Fig. 5.17 there is a flow difference in the flow between side and central subchannels in the test section.

The values for the ratio u_o/u_t^T for the position $L/D_H=113$ were 1.12 and 1.08 for $Re=26,500$ and 65,000 respectively.

The above values are close to the values suggested by the equal pressure split which is of order 1.36 to 1.40 for turbulent flow shown in APPENDIX D.

5.1.4 Secondary Flow

An attempt was made to measure the secondary flow in $Re=65,000$ but with no success.

The first negative result came from v_θ velocity measurements along the gap ($\theta=0$) which yielded values of order 2% of the maximum velocity instead of the theoretical zero value.

Such results are attributed mainly to

- a) Frequency Tracker Error which is of order 3% of the maximum velocity.
- b) Low-signal to noise ratio.
- c) Test section inclination errors. Actually such errors could not be higher than 0.5° which introduces a maximum error of 0.9%.

All lateral velocity data used were below 3% i.e. within experimental error. It is evident from Figure 5.18 that data are inconclusive with respect to secondary flow.

The present data have been obtained with higher errors than those in [1]. (4% vs. 0.67%). The reason is that P. Carajilescov probably had a better signal to noise ratio which allowed the use of relatively small frequency range (500KHz vs. 5.0 MHz for the present case). The difference in signal to noise ratio can occur due to different Laser Beam arrangements. In this case the measurements in [1] were performed by a Dual Beam Mode instead of Reference Beam Mode used in the present approach.

5.2 Analytical Predictions

5.2.1 The Turbulence Models

Before entering into any discussion on turbulence model prediction the author wants to make clear that the turbulence models are not the tools for understanding the turbulence phenomena which are terribly complicated but are engineering tools for predicting useful hydrodynamic quantities. Therefore one has to examine their drawbacks and merits from this point of view.

The most attractive features of turbulence models as pointed out in Chapter 1 are the following:

- a) The ability to use turbulence parameters (e.g., turbulent kinetic energy, energy dissipation for this particular case as part of the model).
- b) The relative simplicity and flexibility to handle complicated problems since the predictions are based on constant adjustments.

Such an approach of course has some disadvantages.

- a) In reality for any given problem of non-homogeneous turbulence, the constants are not constants but parameters changing with position and flow characteristics. The selection of the constants is made by best fit to data. The author's experience from this work is that it is very difficult to reduce the error made in the predictions below 3% for mean velocities and 5-10% for other turbulent quantities. The author's opinion is that such error levels are due to the

change of constants with position and flow characteristics which can be affected little by more sophisticated modeling of the turbulent equations at least for eddy viscosity models (i.e., up to 2-equation models).

- b) The constants are not universal but depend in general on the problem under consideration. Such a drawback can be enhanced by errors introduced in modeling of the various terms in the transport equation of some turbulent quantity due to oversimplicity of the turbulence phenomena.
- c) More than one set of constants fit the data of any particular problem. An illustrative example is shown in Figure 5.19 where predictions are obtained for the geometry of this thesis by using the constants of Launder et al. (95) and Hoffman (96) which are as follows:

Launder et al.	Hoffman
$c_\mu = 0.09$	$c_\mu = 0.09$
$\kappa = 0.42$	$\kappa = 0.41$
$c_1 = 1.44$	$c_1 = 1.81$
$c_2 = 1.92$	$c_2 = 2.0$
$\sigma_k = 1.0$	$\sigma_k = 2.0$
$\sigma_\epsilon = 1.3$	$\sigma_\epsilon = 3.0$

Both sets of constants look reasonable and yield identical results.

However, on the positive side if for some particular experimental set you have a number of constant sets, you can single out a set that fits the experimental sets "acceptably" well. At this point two questions can be posed:

- i) Is that set of constants "universal"?
- ii) Is that set of constants physically correct for that particular family of experimental sets?

With respect to first question the answer is "not necessarily" for the reasons discussed in (a) and (b).

With respect to the second question the answer is again "not necessarily" for the simple reason that even if something is modeled wrongly or inadequately two or more wrong constants can have offsetting effects yielding the right answer.

However there is a third consequence of the relatively high number of constants. There is a danger that one can come out with a particular set of constants that fits only one particular experimental set adequately.

The immediate question which arises from the above discussion is:

How useful are the turbulence models with above advantages and disadvantages for prediction of flow in areas of Nuclear Engineering and especially in rod bundle flow prediction?

In developing the turbulent model of this thesis the following guidelines have been adopted.

a) Accepting that the constants cannot be universal there is a need to define the family of problems for which a particular set of constants can give satisfactory results.

We will consider at least the flow through a rod bundle, the duct flow and the flow along a cylinder as belonging to the same family on the grounds that they have one main direction of the flow. This decision leads us to look for experimental constants that fit simple one-dimensional flows belonging to above family (i.e., plain tube, circular pipe). At this stage we are obliged to pick something already existing in the literature. We will accept the Launder et al. constants on the ground that probably they are the most extensively tested.

Parenthetically it is noticed here that more systematic work is needed by analyzing a number of simple geometries at various Reynolds numbers and by identifying if possible all sets of constants applying to a particular case.

b) In Chapter 3 and in Figure 5.19 it has been made clear that utilization of an isotropic eddy viscosity model does not yield good predictions.

Therefore the anisotropic eddy viscosity model introduced in Chapter 3 will be used in the remainder of this chapter to predict the data.

c) The geometric peculiarities of the rod bundles will be reflected in the eddy viscosity model, e.g., the constant c_L in Eq. (3.63) can be expressed as a function of P/D ratio.

5.2.2 Secondary Flow

For the secondary flow calculations a 30×16 grid has been used. The geometrical characteristics of the experimental set has been used with an input average wall shear stress $(\tau_w/\rho)_{avg} = 4 \times 10^{-3} \text{ m}^2/\text{s}^2$ ($Re=26,000$). For normal stresses the models mentioned in §3.3.3 are used, i.e.:

a) Launder and Ying model as expressed by the Equations (3.47a) and (3.47b).

b) Bobkov et al. model as expressed by the Equation (3.10).

In Figure 5.20 the velocity v_θ at $y=y_B$ is plotted for the above two cases which shows that case (a) gives a 3 loop (!) system and case (b) a 2 loop system. The latter is in agreement with P. Carajilescov's [1] predictions. Figure 5.21 shows the prediction of v_θ velocity along the $\theta=24^\circ$ radial line for both cases.

Figures 5.20 and 5.21 show clearly the significance of the normal shear stress modeling in prediction of secondary flow independent of how well the rest of modeling has been done.

However both predictions have in common a negative loop close to the vicinity of $\theta=0$ which is smaller in strength compared with the positive one at larger θ .

However we cannot consider such predictions as conclusive for the following reasons:

a) The 2-eq. model (as well as the 1-eq. model) as pointed out in the previous section, due to oversimplification of the description of turbulence, is a relatively rough engineering tool for predicting flow characteristics (i.e., mean velocity, Reynolds stresses, turbulent kinetic energy). It is therefore very unrealistic to use such a model to predict some detailed features as secondary flow which is purely a turbulence phenomenon (there is no secondary flow in laminar flow) and is of order 1% (or below for some regions) of the mean velocity.

b) The 30×16 grid used for the present calculations is rather inadequate due to the existence of sharp v_θ gradients especially at $\theta=30^\circ$ (Fig. 5.20) and at the wall (Fig. 5.21). However increase of the grid requires more computer storage and exceedingly high computer cost. The solutions presented here already required 4,000 iterations and 90 min computer time on a IBM 360/65.

c) The small magnitude of the calculated secondary flow can be affected by computer round-off error although both cases are run with double precision.

In spite of above negative conclusions, one question still remains:

Is the secondary flow as predicted by the 2-equation turbulent model important in prediction of other hydrodynamic quantities and especially of the mean velocity which is the most important parameter? Or at least how important is secondary flow compared with anisotropic eddy viscosity as introduced in the present work? These factors are compared in Fig. 5.22 which shows v_z/u_0 along the $\theta=0$ radial line.

The anisotropy is much more significant than the secondary flow effect. Under these circumstances further investigation of secondary flow has been deferred. Such decision has also been prompted by the high computer cost due to the very low convergence rate. All subsequent data predictions are made with secondary flow suppressed.

5.2.3 Data Predictions

As mentioned before all the predictions presented below are made with the 2-equation model and the anisotropic eddy viscosity as introduced in Chapter 3.

5.2.3.1 Numerical Grid

The adequacy of the numerical grid is tested by examination of the ratio of calculated average shear stress to input average wall shear stress. The correct value of course is unity which is expected to be met with an infinite grid. However, due to use of a finite grid the ratio is always less than one.

There are two ways to improve that ratio:

- i) Grid point increase
- ii) y_B^+ increase.

How y_B^+ affects the shear stress ratio is illustrated in Figure 5.40 where the above ratio is plotted vs y_B^+ for a 30×16 grid.

In present calculations a 56×21 grid is used. The y_B^+ parameter has been selected based on the following criteria:

- a) To be as close to the wall as possible.
- b) To give a wall shear stress ratio around 0.95 (i.e., a calculation error for shear stress of order 5%). The selection of above ratio has been on the following grounds:

- 1) Quantities like v_z/u_0 are affected very little by the above ratio.

- ii) Quantities like $\overline{v_1 v_2}/u_0^2$, k/u_0^2 which are affected the most by the above ratio are not measured better than 5% in present measurements.

- iii) A finer grid than 56×21 is needed to increase the above wall shear stress ratio which means higher computer cost.

The values of y_B^+ used in the present analysis are given in Table 5.3. All solutions are obtained by a Line Iteration scheme.

5.2.3.2 Present Data Prediction

The present model has been tested against the present

fully developed data ($P/D=1.124$, $Re=65,000$, $26,500$ and $9,000$) and with mean axial velocity predictions by Eifler [19] ($P/D=1.08$, $Re=52,400$), Trupp [80] ($P/D=1.20$, 1.35 , with $Re=49,000$ and $60,000$ respectively) and Kjellstrom [82] ($P/D=1.217$, $Re=149,000$).

The above data are representative of a variety of P/D ratios (1.08 to 1.35) and Reynolds numbers ($9,000$ to $150,000$). Figures 5.28-5.34 show mean axial velocity, radial Reynolds stress, turbulent kinetic energy and wall shear stress distribution predictions compared to the present experimental results (i.e., $P/D=1.124$, $Re=65,000$, $26,500$ and $9,000$).

Figures 5.35-5.39 show mean velocity predictions of Eifler's data, Trupp's data and Kjellstrom's data.

The above results suggest that for $P/D > 1.20$ a value for c_L around 0.4 whereas for $P/D < 1.20$, c_L has to be increased. This actually is in agreement with the experimental data obtained by Rehme [88] in a side subchannel with $P/D=1.07$ which suggest an eddy viscosity along the zero shear stress line of

$$v_{rz}^T = (0.10 - 0.16) u^* \delta$$

Using this expression for ϵ in relations (3.39), (3.58) and (3.50), we find

$$c_L = 0.55 \text{ to } 0.9$$

A relation for c_L consistent with above evidence satisfactorily fitting the above data ($P/D \geq 1.08$) is the following

$$c_L = 1.62 \exp(-17(P/D-1)) + 0.4$$

All data predictions shown in Figures 5.23-5.38 were made with above relation. It is worth pointing out here that although $c_L=0.4$ gives less accurate results, it gives more conservative heat transfer results as shown in Figure 5.39.

The degree of success of the analytic predictions is a function of the Reynolds number range.

a) High Reynolds Number Predictions ($Re \geq 26,500$)

The mean axial velocity and the wall shear stress distribution predictions are satisfactory. The predictions for $\overline{v_r'v_z'}$ and k seem reasonably good although more accurate measurements are needed for more definite conclusions.

b) Low Reynolds Number Predictions ($Re=9,000$)

The predictions for v_z and k are not as satisfactory as for the High Reynolds Number. The predictions for $\overline{v_r'v_z'}$ and $\tau_w/(\tau_w)_{avg}$ are reasonably good. The present predictions for $Re=9,000$ support the approach of modifying the 2-equation turbulence model to take into account Low Reynolds Number effects already initiated by other investigators [e.g., Ref. 97]. For example Jones and Launder [97] introduce some position dependents of the constants close to the wall.

5.2.4 Heat Transfer Calculations

Two region (clad + coolant) heat transfer calculations have been performed to examine the effect of anisotropic viscosity as introduced in the present work versus isotropic viscosity working with the 2-equation turbulence model.

The calculations have been performed with the following parameters:

Coolant	Sodium
Clad	Stainless Steel
Pitch ratio	$P/D=1.10$
Rod Diameter	$D=0.25"$
Inner Clad Diameter	$D_2=0.22"$
Average Wall Shear Stress	$\frac{\tau_w}{(\tau_w)_{avg}} = 0.325 \frac{m^2}{s^2} (Re=86,000)$
Linear Heat Generation Rate	$q'=15 \frac{kw}{ft}$

All material properties were taken at 1000°F. For the numerical solution a grid of 30 x 16 points has been used. The Finite Difference Equations used are described in Appendix B. A ratio of heat diffusivity to eddy viscosity equal to one for both radial and tangential directions was used.

The numerical calculations have been performed by the Computer Code developed in the present work and described in Appendix E.

In Figure 5.40 the non-dimensional quantity $\lambda_c \frac{T_{co} - T_{co}}{q'}$ is plotted versus angle where T_{co} is the local outside clad temperature, T_{co} is the average outside clad temperature and λ_c is the coolant heat conductivity.

Figure 5.39 shows the strong effect of anisotropic

eddy viscosity where the value of $T_{co}^{max} - \overline{T}_{co}$ shows a difference almost 2 to 1. Figure 5.39 shows clearly the importance of anisotropic model of eddy viscosity in turbulence models for heat transfer calculation as demonstrated earlier by other investigators [94], [32].

5.3 Conclusions

From the present work the following conclusions can be drawn.

On the Analytical Work

1. The 2-equation turbulence model as presently formulated can be a satisfactory engineering tool for predicting useful hydrodynamic quantities (i.e., mean axial velocity, Reynolds stresses, shear stress) in a rod bundle at high Reynolds numbers (presently, it has been tested satisfactorily with Reynolds numbers above 26,500). The model does not seem to be a sufficient tool for predicting secondary flows in rod bundles.

2. The model has been used to successfully predict the data obtained in this study and published rod bundle data (principally axial velocity) utilizing one consistent set of constants for 1-dimensional flow from the literature.

3. The introduction of eddy viscosity anisotropy through length scale ratios, while keeping the model simple, also makes the model inherently extendable to 3 dimensions. (One can follow the same logic and introduce a length scale l_z similar to l_θ for axial diffusion.)

4. The strong effect of the anisotropy in eddy viscosity

on heat transfer calculations shown by other investigators is also confirmed in the present work by the present model.

On Numerical Work

1. For fully developed flows solved numerically using average velocity input instead of average shear stress input, the answer is a function of the numerical technique used due to the finite number of points. Therefore, use of an average velocity input is not recommended.

2. A relatively fast numerical technique (like the line iteration scheme used in TEACH code) combined with an under-relaxation factor is not necessarily recommended over a common point iteration technique without under-relaxation.

On Experimental Work

1. Experimental data on mean axial velocity v_z and the associated rms value v'_z in the test section used show clearly the presence of a developing flow-region.

2. The flow split model based on equal pressure drop and fully developed flow predicts higher flow rates for the interior subchannels close to those measured (within 10%).

3. The data suggest that the safest criterion of fully developed flow occurrence is the mean velocity profile.

4. Calculations of wall shear stress distribution using the experimental data of the axial velocities at the points closest to the wall and applying the logarithmic law indicate that dips observed in the vicinity of $\theta=30^\circ$ may be associated with the developing region of the flow rather than with the fully developed region.

CHAPTER 6

RECOMMENDATIONS FOR FUTURE WORK

From the experience drawn from present work the following recommendations for future work can be made.

On Turbulence Models and Flow Predictions

1. In the present work the representative constants for 1-dimensional flow are taken from Launder et al. [67] which are based on "best fits" of cases representing one and multidimensional flows. Re-examination of the 2-equation turbulence model prediction in 1-dimensional flows (plane tube, circular pipe, annulus) and identification of all reasonable constant sets that fit the data seems necessary. Reynolds number effects on constants need to be studied especially at low Reynolds numbers.

2. A new radial scale has been introduced for the wall region which seems to be reasonable for the non-logarithmic region very close to the wall. One has to examine how this can improve predictions in the hydrodynamic quantities within that region since this thesis has been focused outside this region.

3. The eddy viscosity anisotropy introduced in the present work has significant effect on flow predictions and is consistent with experimental evidence. All predictions done in 2-dimensional or 3-dimensional flows using isotropic viscosity turbulence models (like recirculating flows) could profitably be re-examined with the present

anisotropic model.

4. The present model as formulated is easily extendable to three dimensions. It can be used to predict present experimental data in the developing region of the test section used.

On Numerical Work

1. The Up-Wind-Derivative Difference Scheme (UDDS) used in present work is superior to Upwind-Difference-Scheme (UDS) since it is one-order higher in accuracy. Existing codes using UDS can be modified to use the UDDS scheme.

2. Present calculations show that in order for the wall shear stress (or equivalently the axial velocity gradients) to be calculated to better than 5% of the true value, a large number of grid points (larger than 56×21) is recommended. This poses a drawback to the finite difference schemes. Other methods like Finite Element methods need to be examined for similar flow calculations.

3. The computer program developed in the present work works with pressure and velocity (instead of vorticity and stream function) and therefore can be relatively easily extended in three dimensions.

On Heat Transfer Calculations

In the present work some heat transfer calculations were performed to study the anisotropic eddy viscosity effect in heat transfer calculations. More extensive

studies are needed to actually predict heat transfer in the fully developed flow.

On Experiments

The measurements in this thesis were performed by a 2-channel LDA working on Reference Mode using frequency Tracking Techniques.

Characteristic of the above measurements were Doppler ambiguities, instrument noise, low performance of the turbulent processor near the wall, and low performance of the Tracker at low signal-to-noise ratio.

Using the same LDA equipment the following improvements can be made.

1. Substitute for the tracking technique at least in one channel other techniques which have a better performance at low signal to noise ratio (e.g., by spectrum analysis or individual particle techniques.
2. Look for better ways of turbulence signal processing.

On Secondary Flows

1. Direct Measurement

Measurement of the secondary flow is a rather difficult task. The obvious reasons are that the mean values of secondary velocities are much smaller than their rms values. Approximate calculations of the ratio of secondary flow rms value to mean value assuming

$$v'_0 \approx u^* \approx 0.05 v_0 \quad (6.1)$$

give
$$\frac{v'_\theta}{v_\theta} = \begin{cases} 5 & \text{for } v_\theta = 0.01 v_b \\ 10 & \text{for } v_\theta = 0.005 v_b \end{cases} \quad (6.2)$$

Thus an experimentalist needs an instrument that will respond properly at instantaneous velocities at least $\pm 3v'_\theta$ (i.e., ± 15 to $30 v_\theta$) and will measure at the same time correctly the mean velocity v_θ itself.

Usually the wider the range of the instrument, the less accurate is the measurement of small quantities. Besides the above drawback the relative large magnitude of v'_θ requires high integration time which poses additional demands on test section dimensions and mean axial velocity magnitude as follows.

Recall that the error made due to finite integration time assuming a Gaussian Distribution is given by the relation (Eq. 2.4).

$$\frac{(v_\theta)_{\text{MEAS}} - v_\theta}{v_\theta} = \frac{v'_\theta}{v_\theta} \sqrt{2} \left(\frac{F}{T} \right)^{1/2} \quad (6.3)$$

where F is the time scale of the turbulence. Taking

$$\frac{v'_\theta}{v_\theta} = 10 \text{ and } \frac{(v_\theta)_{\text{MEAS}} - v_\theta}{v_\theta} = 0.2 \text{ (i.e., 20\%)}$$

Eq. (6.3) gives

$$F = T/5000$$

Taking further $T = 100$ sec,

$$F = 0.02 \text{ sec} \quad (6.4)$$

Recall that

$$F = \frac{\text{Test section dimension}}{v'_\theta}$$

For $F = 0.02 \text{ sec}$

and $v'_g = 0.05 v_b$

we find that

$$v_b \geq 1000 \text{ (Test section dimension)/sec}$$

This result represents the required v_b necessary for a given test section dimension to keep the error within 20% and the measuring time equal to or less than 100 sec.

For Test section dimension 5 mm: $v_b \geq 5\text{m/s}$.

For Test section dimension 10 mm: $v_b \geq 10\text{m/s}$.

The above preliminary calculations suggest use of a test section as small as possible and use of air as a working fluid to more easily achieve this minimum required velocity.

Future attempts to measure secondary flow in rod bundles have to take into consideration above restrictions.

2. Indirect Measurement and/or Predictions

In case direct measurements of secondary flow is not possible, a mixed scheme of experimental results and analytical calculations can be used as described below.

1. Measure mean velocities and Reynolds stresses.

11. Fit the above data with analytic expressions sensitive enough to give confidence that first and second derivations can be extracted. Note that both the data and the analytic expressions must be sensitive enough.

111. Utilize the above analytic expressions in the corresponding Reynolds equations by performing the neces-

sary differentiations to obtain the analytical expressions for the secondary flow.

The author's deep belief is that no existing turbulence model can confidently predict secondary flows which are of the bulk velocity.

On Methods for Reactor Design

1. The present model is easily extendable in three dimensions. It can be used in reactor design for flow and heat predictions in local regions of special interest (e.g., hot spot, blockage) by isolating them and applying boundary conditions from a Lumped Parameter Technique.

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Table 2.1 Summary Table of Measurements in Rod Bundles (Taken from Ref. 1)

Investigator	P/D [#]	L/D _H	Re × 10 ⁻³	Velocity		Turb. Intensity			Cross-Correlations		$\frac{\tau_w}{\tau_u}$	Techniques
				Axial	Second.	z	r	θ	$\overline{v'v'}$ r z	$\overline{v'v'}$ θ z		
Eifler, Nijssing [77, 78]	1.05	139	15, 30, 50	✓								Pitot tube
	1.10	90	15, 30, 50	✓								
	1.15	65	15, 30, 50	✓								
Eifler [19]	1.08	137	10-152	✓								Pitot tube
Subbotin, et al. [79]	1.05	154	18.8-81	✓							✓	Pitot tube Preston tube
	1.10	100	18.8-81	✓							✓	
	1.20	72	18.8-81	✓							✓	
Kjellstrom [82]	1.217	81	149-373	✓	✓	✓	✓	✓	✓		✓	Hot Wire Preston tube
Hall & Svenningsson [83]	1.217	81	270		✓						✓	Hot Wire
Trupp & Azad [80]	1.20	51	12-84	✓		✓	✓	✓	✓		✓	Hot Wire Pitot Tube Preston Tube
	1.35	30	12-84	✓		✓	✓	✓	✓		✓	
	1.50	20	12-84	✓		✓	✓	✓	✓		✓	
Rowe [81]	1.25*	85 [†]	50-200	✓		✓						LDA
	1.11*	117	50-200	✓		✓						
	1.25*	90	50-200	✓		✓						
	1.25**	105	50-200	✓		✓						
	1.25	138	50-200	✓		✓						
Carajilescov [1]	1.123	77	27	✓		✓						LDA

All data for triangular array except where mentioned otherwise

* Square Array ** Square and triangular channels in same test section

† Based on cross section hydraulic diameter instead of subchannel

Table 2.2
TEST SECTION GEOMETRICAL CHARACTERISTICS

Parameter	Symbol	Size	Units
Rod diameter	D	51.15	mm
Gap	g	6.35	mm
Pitch/Diameter	P/D	1.124	
Unit cell hydraulic diameter	D_H	20.12	mm
Cross section hydraulic diameter	D_H^T	16.375	mm
Unit cell area		6.132×10^{-5}	m^2
Total cross section area		1.346×10^{-3}	m^2

Table 2.3
LDA OPTICAL PARAMETERS

Parameter	Symbol	Value	Units
1. Refractive Indices			
Air	n_o	1.00	
Water	n_w	1.33	
Plexiglas	n_p	1.50	
2. Laser			
Wavelength	λ	0.5145	μ
Beam Diameter	D_o	1.5	mm
3. Optical Unit			
Focal Length	f	120	mm
Beam Separation	$2a$	40	mm
Half-Angle between Beams			
in air	$\theta/2$	6.64	degrees
in water	$\beta/2$	4.99	degrees
4. Beam Intersection (Fig. 1)			
Diameter of the Beam at Waist	$2b_o$	0.053	mm
Width	$2\sigma_1$	0.053	mm
Height	$2\sigma_2$	0.053	mm
Length	$2\sigma_3$	0.610	mm
5. Calibration factor			
$C = \frac{\lambda}{2n_w \sin \frac{\beta}{2}}$	C	2.224	m/s/MHz

Table 2.4 Parameter and Error Relations

Quantity	Calculation	Error ($\delta\phi = \phi_{\text{MEAS}} - \phi $)
v_z, v'_z	v_z, v'_z	$\delta v_z, \delta v'_z$
v_r	$v_1 \cos\theta + v_2 \sin\theta$	$\delta v_r = \delta v_1 \cos\theta + \delta v_2 \sin\theta$
v_θ	$-v_1 \sin\theta + v_2 \cos\theta$	$\delta v_\theta = \delta v_1 \sin\theta + \delta v_2 \cos\theta$
v'_r	$(v_1^2 \cos^2\theta + v_2^2 \sin^2\theta)^{1/2}$	$\delta v'_r = \frac{v_1}{v'_r} \cos^2\theta \delta v_1 + \frac{v_2}{v'_r} \sin^2\theta \delta v_2$
v'_θ	$(v_1^2 \sin^2\theta + v_2^2 \cos^2\theta)^{1/2}$	$\delta v'_\theta = \frac{v_1}{v'_\theta} \sin^2\theta \delta v_1 + \frac{v_2}{v'_\theta} \cos^2\theta \delta v_2$
k	$\frac{1}{2}(v_1^2 + v_2^2 + v_z^2)$	$\delta k = v_1 \delta v_1 + v_2 \delta v_2 + v_z \delta v_z$
$\overline{v'_r v'_z}$	$\overline{v'_1 v'_z} \cos\theta + \overline{v'_2 v'_z} \sin\theta$	$\delta(\overline{v'_r v'_z}) = \delta(\overline{v'_1 v'_z}) \cos\theta + \delta(\overline{v'_2 v'_z}) \sin\theta$
$\overline{v'_\theta v'_z}$	$-\overline{v'_1 v'_z} \sin\theta + \overline{v'_2 v'_z} \cos\theta$	$\delta(\overline{v'_\theta v'_z}) = \delta(\overline{v'_1 v'_z}) \sin\theta + \delta(\overline{v'_2 v'_z}) \cos\theta$

Re		v_z	v_z'	v_1	v_1'	v_2	v_2'	$v_1 v_z$		$v_2 v_z'$	
								v_1'	v_z'	v_2'	v_z'
9,000	FREQ RANGE (MHZ)	0.5	0.5	0.5	0.5	0.5	0.5	5.0	0.5	0.5	5.0
	SHIFT (MHZ)	0.05	0.05	0.25	0.25	0.25	0.25	1.0	0.05	0.25	1.0
	TRACKER	DISA	DISA	DISA	DISA	DISA	DISA	TSI	DISA	DISA	DISA
26,500	FREQ RANGE	1.5	1.5	$\frac{1.5}{5.0}$	$\frac{1.5}{5.0}$	5.0	5.0	5.0	1.5	5.0	5.0
	SHIFT	0.15	0.15	$\frac{0.7}{1.3}$	$\frac{0.7}{1.3}$	1.3	1.3	4.0	0.15	0.15	1.0
	TRACKER	DISA	DISA	DISA	DISA	TSI	TSI	TSI	DISA	DISA	TSI
65,000	FREQ RANGE	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0
	SHIFT	0.0	0.0	1.3	1.3	1.3	1.3	1.0	0.4	1.0	0.4
	TRACKER	TSI	TSI	TSI	TSI	TSI	TSI	TSI	DISA	TSI	DISA

Table 2.5 Frequency Tracking Arrangement

TABLE 2.6

 $(v'_z)_{\text{LAM}}$ (in mV) FOR VARIOUS FREQUENCY RANGES

Tracker	Frequency Range	I.F. BANDWIDTH				
DISA	v_{max} (MHZ)	18	28	48	88	TSI
	0.5	-	60	70	80	
	1.5	22	36	56	81	
	5.0	-	44	-	-	
TSI	5.0	46				

Table 3.1 Parameter ζ (Taken from Ref. 39)

Author	ζ Name	ζ Definition	Turbulent Viscosity	
Kolmogorov [3]	Frequency	$f = c \frac{k^{1/2}}{\ell}$	$\nu^T = \frac{k}{f} = k^{1/2} \ell$	
Rotta [45]	Length-energy product	$L = k\ell$		Duct Flow
Rodi & Spalding [46]	Length-energy product	$L = k\ell$	$\nu^T = k^{1/2} \ell$	Free shear flow
Ng & Spalding [67]	Length-energy product	$L = k\ell$	$\nu^T = k^{1/2} \ell$	Boundary layers
Wolfshtein [48]	Length-energy product	$L = k\ell$	$\nu^T = 0.22 k^{1/2} \ell$	One-dimensional flows
Glushko [44]	Length squared	$\lambda = \frac{1}{2} \ell^2$	$\nu^T = \alpha \ell k^{1/2}$ $0.2 < \alpha < 0.8$	High turbulence version of a low turbulence model applied to trans. flat plate boundary layer
Harlow & Nakayama [41]	Dissipation	ϵ	$\nu^T = 0.2 \frac{k^2}{\epsilon} = 1.41 k^{1/2} \ell$	
Jones & Launder [50]	Dissipation	ϵ	$\nu^T = 0.09 \frac{k^2}{\epsilon}$	High turbulence version of a low turbulence model applied to laminarizing boundary layer
Wilcox & Alber [51]	Vorticity squared	Ω^2	$\nu^T = \frac{k}{\Omega} = k^{1/2} \ell$	Boundary layers

Table 3.2a The Differential Equations

General Form: $\nabla\phi\nabla + \Delta T_\phi = \nabla S_\phi + S_\phi^*$

ϕ	T_ϕ		S_ϕ		S_ϕ^*
	$T_{\phi r}$	$T_{\phi\theta}$	$S_{\phi r}$	$S_{\phi\theta}$	
v_r	$-2v \frac{\partial v_r}{\partial r}$	$-\frac{v}{r} \frac{\partial v_r}{\partial \theta}$	$-v_r^2 - p/\rho$	$+v_{r\theta}^{\text{eff}} r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right)$	$\frac{v_\theta^2 + v_r^2}{r} - \frac{2v}{r} \left(\frac{\partial v_\theta}{r \partial \theta} + \frac{v_r}{r} \right)$
rv_θ	$= v_{r\theta}^{\text{eff}} r^2 \left(\frac{v_\theta}{r} \right)$	$-2v \frac{\partial(rv_\theta)}{r \partial \theta}$	$+v_{r\theta}^{\text{eff}} \frac{\partial v_r}{\partial \theta}$	$+2vv_r - r^2 \frac{\partial p}{\partial r}$	0
v_z	$-v_{rz}^{\text{eff}} \frac{\partial v_z}{\partial r}$	$-v_r^{\text{eff}} \frac{\partial v_z}{r \partial \theta}$	0	0	$-\frac{d(p/\rho)}{dz} = \frac{4u}{D_H}$
k	$-\frac{v_{rz}^{\text{eff}}}{\sigma_k} \frac{\partial k}{\partial r}$	$-\frac{v_r^{\text{eff}}}{\sigma_k} \frac{\partial k}{r \partial \theta}$	0	0	$v_{rz}^T \left(\frac{\partial v_z}{\partial r} \right)^2 + v_{r\theta}^T \left(\frac{\partial v_z}{r \partial \theta} \right)^2 - \epsilon$
ϵ	$-\frac{v_{rz}^{\text{eff}}}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial r}$	$\frac{v_{r\theta}^{\text{eff}}}{\sigma_\epsilon} \frac{\partial \epsilon}{r \partial \theta}$	0	0	$c_{\epsilon 1} \frac{\epsilon}{k} \left[v_{rz}^T \left(\frac{\partial v_z}{\partial r} \right)^2 + v_{r\theta}^T \left(\frac{\partial v_z}{r \partial \theta} \right)^2 \right]$ $- c_{\epsilon 2} \frac{\epsilon^2}{k}$

Table 3.2b Boundary Conditions

PARAMETER	I	II	III	NEAR WALL REGION	
v_z	$\frac{\partial v_z}{r \partial \theta} = 0$	$\frac{\partial v_z}{r \partial \theta} = 0$	$\frac{\partial v_z}{\partial r} = \frac{\partial v_z}{r \partial \theta} \tan \theta$	Logarithmic Profile	
v_r	$v_r = 0$	$v_r = 0$	$n + v_{\text{sec}} = 0$	$v_r _{\text{wall}} = 0$	
v_θ	$v_\theta = 0$	$v_\theta = 0$	$\frac{\partial v_{\text{sec}}}{\partial n} = 0$	$v_\theta _{\text{wall}} = 0$	
k	$\frac{\partial k}{r \partial \theta} = 0$	$\frac{\partial k}{r \partial \theta} = 0$	$\frac{\partial k}{\partial r} = \frac{\partial k}{r \partial \theta} \tan \theta$	$k = \frac{\overline{v_r' v_z'}}{\sqrt{c\mu}}$	
ϵ	$\frac{\partial \epsilon}{r \partial \theta} = 0$	$\frac{\partial \epsilon}{r \partial \theta} = 0$	$\frac{\partial \epsilon}{\partial r} = \frac{\partial \epsilon}{r \partial \theta} \tan \theta$	1-EQ.	2-EQ.
				$\epsilon = c_D \frac{k^{3/2}}{l}$ Algebraic	$\epsilon = \frac{u_*}{ky} \overline{(v_r' v_z')}$

ϕ	A_e	A_w	A_n	A_s	V_p
V_z k ϵ $r v_\theta$	$\frac{r_i + r_{i+1}}{2} \Delta\theta$	$\frac{r_i + r_{i-1}}{2} \Delta\theta$	$\frac{r_{i+1} - r_{i-1}}{2}$	$\frac{r_{i+1} - r_{i-1}}{2}$	$r_i \Delta\theta \frac{r_{i+1} - r_{i-1}}{2}$
v_r	r_i	r_{i-1}	$r_i - r_{i-1}$	$r_i - r_{i-1}$	$\frac{r_i + r_{i-1}}{2} (r_i - r_{i-1}) \Delta\theta$

TABLE 4.1 Control Volume Lengths and Surface

TABLE 4.2: THE DIFFUSION COEFFICIENTS

	$\phi = k, c, v_z$	v_r	v_θ
BE	$0.25 \frac{r_{i+1} + r_i}{r_{i+1} - r_i} \Delta\theta$	$\frac{4r_i}{r_{i+1} - r_{i-1}} \Delta\theta$	$\frac{(r_i + r_{i+1})^3}{32r_{i+1}(r_{i+1} - r_i)}$
BW	$0.25 \frac{r_i + r_{i-1}}{r_i - r_{i-1}}$	$\frac{4r_{i-1}}{r_i - r_{i-2}} \Delta\theta$	$\frac{(r_i + r_{i-1})^3}{32r_{i-1}(r_i - r_{i-1})}$
BN BS	$0.25 \frac{r_{i+1} - r_{i-1}}{r_i \Delta\theta}$	$\frac{r_i - r_{i-1}}{2(r_i + r_{i-1}) \Delta\theta}$	$\frac{r_{i+1} - r_{i-1}}{\Delta\theta}$
DE	$\frac{BE}{\sigma_\phi} \left[(v_{rz}^{eff})_{i+1,j} + (v_{rz}^{eff})_{i,j} \right]$	$BE \cdot v$	$BE \left[(v_{r\theta}^{eff})_{i+1,j} + (v_{r\theta}^{eff})_{i+1,j-1} \right. \\ \left. + (v_{r\theta}^{eff})_{i,j-1} + (v_{r\theta}^{eff})_{i,j} \right]$
DW	$\frac{BW}{\sigma_\phi} \left[(v_{rz}^{eff})_{i+1,j} + (v_{rz}^{eff})_{i,j} \right]$	$BW \cdot v$	$BW \left[(v_{r\theta}^{eff})_{i-1,j} + (v_{r\theta}^{eff})_{i-1,j-1} \right. \\ \left. + (v_{r\theta}^{eff})_{i,j-1} + (v_{r\theta}^{eff})_{i,j} \right]$
DN	$\frac{BN}{\sigma_\phi} \left[(v_{\theta z}^{eff})_{i,j+1} + (v_{rz}^{eff})_{i,j} \right]$	$BN \left[(v_{r\theta}^{eff})_{i,j+1} + (v_{r\theta}^{eff})_{i,j} \right. \\ \left. + (v_{r\theta}^{eff})_{i-1,j} + (v_{r\theta}^{eff})_{i-1,j+1} \right]$	$BN \cdot v$

TABLE 4.2: THE DIFFUSION COEFFICIENTS (Continued)

	$\phi = k, a, v_z$	v_r	v_θ
DS	$\frac{BS}{\sigma} \left[(v_{\theta z}^{\text{eff}})_{i,j-1} + (v_{\theta z}^{\text{eff}})_{i,j} \right]$	$BS \left[(v_{r\theta}^{\text{eff}})_{i,j-1} + (v_{r\theta}^{\text{eff}})_{i,j} \right. \\ \left. (v_{r\theta}^{\text{eff}})_{i-1,j} + (v_{r\theta}^{\text{eff}})_{i-1,j-1} \right]$	$BS \cdot v$
θ	0	0	1

$$\text{Diffusion Term } D\tilde{T} = \left[\bar{D}E \left(\frac{r_{i+1}}{r_i} \right)^\sigma + \bar{D}W \left(\frac{r_{i-1}}{r_i} \right)^\sigma + \bar{D}N + \bar{D}S \right] \phi_{i,j} - \bar{D}E \phi_{i+1,j} - \bar{D}W \phi_{i-1,j} - \bar{D}N \phi_{i,j+1} - \bar{D}S \phi_{i,j-1}$$

(Concluded)

TABLE 4.3: THE CONVECTION COEFFICIENTS

Coeff	$\phi = k, \epsilon, v_z, T_c$	$\phi = v_r$	$\phi = v_\theta$
A_1	$\left[(r_i + r_{i+1})(r_{i+1} - r_i)(v_r)_{i+1,j} \right. \\ \left. + (r_i + r_{i-1})(r_i - r_{i-1})(v_r)_{i,j} \right] \\ \cdot \frac{\Delta \theta}{8}$	$\left\{ r_i(r_i - r_{i-1}) \left[(v_r)_{i+1,j} + (v_r)_{i,j} \right] \right. \\ \left. + r_{i-1}(r_i - r_{i-1}) \left[(v_r)_{i-1,j} + (v_r)_{i,j} \right] \right\} \\ \cdot \frac{\Delta \theta}{4}$	$\left\{ (r_i + r_{i+1})(r_{i+1} - r_i) \left[(v_r)_{i+1,j} + (v_r)_{i+1,j-1} \right] \right. \\ \left. + (r_i + r_{i-1})(r_i - r_{i-1}) \left[(v_r)_{i,j} + (v_r)_{i,j-1} \right] \right\} \\ \cdot \frac{\Delta \theta}{16}$
A_2	$\frac{(r_{i+1} - r_{i-1})}{8} \left[(v_\theta)_{i,j} + (v_\theta)_{i+1,j} \right]$	$\frac{r_i - r_{i-1}}{8} \left[(v_\theta)_{i,j+1} + (v_\theta)_{i-1,j+1} \right. \\ \left. + (v_\theta)_{i,j} + (v_\theta)_{i-1,j} \right]$	$r_i \frac{r_{i+1} - r_{i-1}}{16} \left[(v_\theta)_{i,j+1} + 2(v_\theta)_{i,j} + (v_\theta)_{i,j-1} \right]$
CE	$(A_1 - A_1)/(r_{i+1} - r_i)$	$(A_1 - A_1)/(r_{i+1} - r_{i-1})$	$(A_1 - A_1) \frac{r_{i+1}}{r_{i+1} - r_i}$
CW	$(A_1 + A_1)/(r_i - r_{i-1})$	$(A_1 + A_1)/(r_i - r_{i-1})$	$(A_1 + A_1) \frac{r_{i-1}}{r_{i+1} - r_i}$
CN	$(A_2 - A_2)$	$ A_2 - A_2$	$ A_2 - A_2$
CS	$ A_2 + A_2$	$ A_2 + A_2$	$ A_2 + A_2$
o	0	0	1

$$\text{Convection term } CT = \left[CE \left(\frac{r_i}{r_{i+1}} \right)^{\sigma} + CW \left(\frac{r_i}{r_{i+1}} \right)^{\sigma} + CN + CS \right] \phi_{i,j} - CE \phi_{i+1,j} - CW \phi_{i-1,j} - CN \phi_{i,j+1} - CS \phi_{i,j-1}$$

TABLE 4.4: SOURCE TERMS

ϕ	SU	SP
v_z	$\frac{4V_p}{\Delta H} \left(\frac{T_w}{\rho} \right)_{\text{avg}}$	0
k	$v_p (v_{rz}^T)_{i,j} \left[\left(\frac{\partial v_z}{\partial r} \right)_{i,j} \right]^2 + (v_{\theta z}^T)_{i,j} \left[\left(\frac{\partial v_z}{r \partial \theta} \right)_{i,j} \right]^2$	$\frac{C_{\mu} k_{i,j} v_p}{(v_{rz}^T)_{i,j}}$
ε	$C_1 C_{\mu} \frac{k_{i,j} v_p}{(v_{rz}^T)_{i,j}} \left\{ (v_{rz}^T)_{i,j} \left[\left(\frac{\partial v_z}{\partial r} \right)_{i,j} \right]^2 + (v_{\theta z}^T)_{i,j} \left[\left(\frac{\partial v_z}{r \partial \theta} \right)_{i,j} \right]^2 \right\}$	$\frac{C_2 C_{\mu} k_{i,j} v_p}{(v_{rz}^T)_{i,j}}$
v_r	$r_{i-1} \Delta \theta \left[(v_r^2)_{i-1,j} + (p/\rho)_{i-1,j} \right] - r_i \left[(v_r^2)_{i,j} + (p/\rho)_{i,j} \right]$ $+ \frac{(v_{r\theta}^{\text{eff}})_{i,j} + (v_{r\theta}^{\text{eff}})_{i,j+1} + (v_{r\theta}^{\text{eff}})_{i-1,j+1} + (v_{r\theta}^{\text{eff}})_{i-1,j}}{8}$ $+ (r_i + r_{i-1}) \left[\frac{(v_{\theta})_{i,j+1}}{r_i} - \frac{(v_{\theta})_{i-1,j+1}}{r_{i-1}} \right]$ $- \frac{(v_{r\theta}^{\text{eff}})_{i,j} + (v_{r\theta}^{\text{eff}})_{i,j+1} + (v_{r\theta}^{\text{eff}})_{i-1,j+1} + (v_{r\theta}^{\text{eff}})_{i-1,j}}{8}$ $+ (r_i + r_{i-1}) \left[\frac{(v_{\theta})_{i,j}}{r_i} - \frac{(v_{\theta})_{i-1,j}}{r_{i-1}} \right]$ $+ \frac{2}{r_i + r_{i-1}} v_p \left[\frac{(v_{\theta}^2)_{i,j} + (v_{\theta}^2)_{i-1,j}}{2} \right]^2$	0

TABLE 4.4: SOURCE TERMS (Continued)

ϕ	SU	SP
v_θ	$\frac{(v_{r\theta}^{eff})_{i+1,j} + (v_{r\theta}^{eff})_{i+1,j-1} + (v_{r\theta}^{eff})_{i,j-1} + (v_{r\theta}^{eff})_{i,j}}{4} \frac{r_i + r_{i+1}}{2} \left[(v_r)_{i+1,j} - (v_r)_{i+1,j-1} \right]$ $- \frac{(v_{r\theta}^{eff})_{i-1,j} + (v_{r\theta}^{eff})_{i-1,j-1} + (v_{r\theta}^{eff})_{i,j-1} + (v_{r\theta}^{eff})_{i,j}}{4} \frac{r_{i+1} + r_{i-1}}{2} \left[(v_r)_{i,j} - (v_r)_{i,j-1} \right]$ $+ \frac{r_i(r_{i+1} - r_{i-1})}{2} \left[(v_\theta^2)_{i,j-1} + (p/\rho)_{i,j-1} - (v_\theta^2)_{i,j} - (p/\rho)_{i,j} \right]$	0

Source Term $ST = SU - SP \phi_p$

TABLE 4.5 Pressure Coefficients (Eq. 4.68)

QUANTITY	DEFINITION	COMMENTS
DPE	$\frac{r_{i+1} \cdot \Delta\theta}{AP_{i+1,j}} A_e$	A_e is given in Table 4.1 AP is referred to velocity v_r
DPW	$\frac{r_{i-1} \cdot \Delta\theta}{AP_{i,j}} A_w$	A_w is given in Table 4.1 AP is referred to velocity v_r
DPN	$\frac{r_{i+1} - r_{i-1}}{2 AP_{i,j+1}} A_n$	A_n is given in Table 4.1 AP is referred to velocity v_θ
DPS	$\frac{r_{i+1} - r_{i-1}}{2 AP_{i,j}} A_s$	A_s is given in Table 4.1 AP is referred to velocity v_θ

TABLE 5.1
CHARACTERISTICS OF THE FULLY DEVELOPED FLOW

Nominal Re	65,000	26,500	9,000
Actual Re	64,810	26,549	8,924
Bulk velocity $v_b^{(**)}$ (m/s)	1.975	1.103	3.67×10^{-1}
Wall shear stress $\frac{\tau_w^{(**)}}{\rho} \left(\frac{m^2}{s^2} \right)$	1.04×10^{-2}	4.12×10^{-3}	6.56×10^{-4}

(*Calculated from the logarithmic law ($k = 0.42$ $E = 98$)

(**)The wall region represented by the logarithmic law

TABLE 5.2
BULK VELOCITIES (m/s) IN THE DEVELOPING REGION

Re \ L/DH	0 (Theoretical)	15	46	113
26,000	0.984	0.997	1.087	1.103
55,000	1.829	1.830	1.95 ⁷⁵	1.975

TABLE 5.3

 y_B^+ SELECTION FOR THE NUMERICAL CALCULATIONS

Author	P/D	Re	y_B^+
Present	1.124	65,000	60
Present	1.124	26,500	45
Present	1.124	9,000	20
Eifler	1.08	52,400	60
Trupp ^T and Azad	1.20	49,000	60
Trupp ^T and Azad	1.35	60,000	60
Kjellstrom	1.217	149,000	100

Fig. 1.1 Characteristic triangle for the study of fully developed flow in an infinite triangular array of bare rods

Fig 2.1. Cross Section of the Test Section [1]

Fig 2.2 Overall View of the Test Section

Fig. 2.3 Entrance Region

Figure 2.4. The Hydraulic Loop

Fig. 2.5 LDA Optical Arrangement

$$2\sigma_1 = 2b_0$$

$$2\sigma_2 = 2 \frac{2b_0}{\sin \beta} \sin \frac{\beta}{2}$$

$$2\sigma_3 = 2 \frac{2b_0}{\sin \beta} \cos \frac{\beta}{2}$$

FIG.-2.6. Beam Intersection

Fig 2.7 Layout of Instruments (Taken from Ref.84)

Fig. 2:8. Point selection for measurements

$$\frac{\theta_1}{2} = 9.46^\circ$$

$$\frac{\theta_1}{2} = 7.10^\circ$$

$$\delta Y = (\delta s)_{\text{AIR}} \tan \frac{\theta_1}{2} = (\delta s)_{\text{WATER}} \tan \frac{\theta_1}{2}$$

Fig. 2.9. Water Displacements vs Air Displacements

Fig. 2.10. $(v_z')^2_{\text{LAM}}$ vs I.P. BANDWIDTH of the Frequency Tracker

Fig. 3.1. Comparison of ψ_m with
Quarmby's data [68]

Fig. 3.2. Mixing Lengths Comparisons (1 - Eq. Model)

Fig 3.3 Mixing Length Comparisons (2-Eq Model)

Fig 3.4. Wall Shear Stress Distribution calculated by a Reynolds Number Input

Fig 4.1. Numerical Grid

Fig. 4.2. Control Volume

Fig. 4.3 Relative positions of the control volumes at the points $P=(i,j)$

$$(A_{e,i-1}(v_r)_{i,j}) =$$

$$(A_n)_{i-1} (v_\theta)_{i-1,1}$$

$$r_i(v_{\theta})_{i,j} - r_{i-1}(v_{\theta})_{i-1,j}$$

$$r_i = r_{i-1}$$

$$(v_r)_{i,j} = (v_r)_{i,j-1}$$

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Fig.4.4. Boundary Conditions in Boundary III

Fig 4.5 Underrelaxation Factor Effect

Fig 5.1 Flow development

Fig. 5.2a. Axial velocity ($Re=26,500$, $\theta=0$)

Fig. 5.2b. Axial velocity ($Re=26,500$, $\theta=12^\circ$)

Fig. 5.2c. Axial velocity ($Re=26,500$, $\theta=30^\circ$)

Fig. 5.3a. Axial Velocity ($Re=65,000$, $u=0$)

Fig. 5.3b. Axial Velocity ($Re=65,000$, $\theta=12^\circ$)

Fig. 5.3c. Normalized Axial Velocity ($Re=65,000$, $\phi=30^\circ$)

Fig. 5.4b. Turbulent Axial Velocity ($Re=26,500$, $\theta=12^\circ$)

Fig. 5.4c. Turbulent Axial Velocity ($Re=26,500$, $\theta=30^\circ$)

Fig. 5.5a. Turbulent Axial Velocity ($Re=65,000$, $\theta=0^\circ$)

Fig. 5.5b. Turbulent Axial Velocity ($Re=65,000$, $\theta=12^\circ$)

Fig. 5.5c. Turbulent Axial Velocity ($Re=65,000$, $\theta=30^\circ$)

Fig 5.6a Radial Reynolds Stress ($Re \leq 26500, \theta = 0^\circ$)

Fig 5.6b. Radial Reynolds Stress (Re=26500, $\theta=12^\circ$)

Fig 5.6c. Radial Reynolds Stress (Re 26500, $\theta=24^\circ$)

Fig 5.7a. Radial Reynolds Stress ($Re=65,000$, $\theta=0^\circ$)

Fig 5.7b. Radial Reynolds Stress ($Re=65,000$, $\theta=12$)

Fig 5.7c. Radial Reynolds Stress ($Re=65,000$, $\theta=24$)

Fig 5.8a. Turbulent Kinetic Energy ($Re=86,500, \theta=0^\circ$)

Fig 5.8b. Turbulent Kinetic Energy ($Re=26,500$ $\theta=12$)

Fig 5.8c. Turbulent Kinetic Energy ($Re=26,500, \theta=24$)

Fig. 5.9a. Turbulent Kinetic Energy ($Re=65,000$, $\theta=0^\circ$)

Fig. 5.9b. Turbulent Kinetic Energy ($Re=65,000$, $\theta=12^\circ$)

Fig. 5.9c: Turbulent Kinetic Energy ($Re=65,000$, $\theta=24^\circ$)

Fig. 5.10. Wall Shear Stress Distribution as Predicted by the Logarithmic Law (Re=26,500)

Fig 5.11. Wall Shear Stress Distribution as Predicted by the Logarithmic Law ($Re < 65,000$)

Fig 5.12. Wall Shear Stress Distribution as predicted by the Logarithmic Law ($Re \approx 9,000$)

Fig. 5.13. Axial Velocity ($Re=9,000$)

Fig 5.14. Turbulent Axial Velocity ($Re=9,000$)

Fig. 5.15. Radial Shear Stress ($Re=9,000$)

Fig.5.16. Turbulent Kinetic Energy ($Re \approx 9,000$)

Fig 5.18. v_θ Velocity

Fig 5.19. Launder's Constants vs Hoffman's Constants

Fig. 5.20. Prediction of v_θ Velocity at $y=y_B$

Fig 5.21. Prediction of v_θ Velocity at $\theta=24^\circ$

Fig 5.22. Anisotropy vs Secondary Flow

Fig 5.23. Axial Velocity Prediction ($Re \approx 65000$).

Fig. 5.24. Reynolds Stress Prediction ($Re=65,000$)

Fig 5.25. Turbulent Kinetic Energy Predictions ($Re \approx 65000$)

Fig. 5.26. Wall Shear Stress Distribution Prediction ($Re=65,000$)

Fig 5.27. Axial Velocity Prediction ($Re \approx 26500$)

Fig 5.28, Reynolds Stress Prediction (Re 26500)

Fig 5.29. Turbulent Kinetic Energy Prediction (Re 26500)

Fig. 5.30. Wall Shear Stress Distribution Prediction ($Re=26,500$)

Fig 5.31. Axial Velocity Prediction ($Re \approx 9000$)

Fig. 5.32. Reynolds Stress Prediction ($Re \approx 9,000$)

Fig. 5.33. Turbulent Kinetic Energy ($Re \approx 9,000$)

Fig 5.34. Wall Shear Stress Distribution Prediction ($Re=9000$)

Fig 5.35. Eifler's Data Prediction

Fig 5.36 Trupp's Data Prediction

Fig 5.37. Trupp's Data Prediction

Fig 5.38. Kjellstrom's Data Prediction

Fig. 5.39. Outside clad temperature difference

Fig. 5.40. Calculated to input average wall shear stress vs. y_B^+

APPENDIX A

THE FINITE DIFFERENCE FORM OF
THE CONVECTION TERM IN FLUID FLOWA.1 Introduction

It is known that a general way of solving for any specific quantity ϕ defined within a moving fluid is to obtain a finite difference equation from the corresponding differential equation and solve it numerically. Usually the time-independent finite difference equation has the form

$$\begin{aligned} \phi_{i,j,k} = & c_{i+1,j,k} \phi_{i+1,j,k} + c_{i,j+1,k} \phi_{i,j+1,k} + \\ & c_{i,j,k+1} \phi_{i,j,k+1} + c_{i-1,j,k} \phi_{i-1,j,k} + \\ & c_{i,j-1,k} \phi_{i,j-1,k} + c_{i,j,k-1} \phi_{i,j,k-1} + \\ & S_{i,j,k}. \end{aligned} \quad (A.1)$$

The system of Eq. (A.1) is often elliptic and has to be solved iteratively. On the other hand, the system is nonlinear in most cases since the coefficients c_{ij} can change from iteration to iteration.

Of course the major concern is to use numerical techniques which unconditionally guarantee the convergence of Eq. (A.1). However, due to the complexity of the non-linear effects such numerical techniques are difficult to find.

One way to approach the problem is to try to guarantee the convergence of Eq. (A.1), assuming it is linear and then find ways to cope with the non-linearity. In such a case the first question to be asked is 'Is the

system of linear equations corresponding to Eq. (A.1) convergent?" This, of course, depends on the type of coefficients the $c_{j,i}$'s are.

One idea widely used [36] is that if the $c_{j,i}$'s obey the condition

$$|c_{i+1,i,k}| + |c_{i,i+1,k}| + |c_{i,i,v+1}| + |c_{i-1,i,k}| + |c_{i,i-1,v}| + |c_{i,i,v-1}| \leq 1 \quad (A.2)$$

then convergence is guaranteed by a number of iteration techniques [73].

The only term in equations of the type (A.1) that create difficulties within the restriction (A.2) are the convection terms.

In the remainder of this Appendix, discussion will be limited to the convective term approximations.

Consider an orthogonal system of coordinates (x_1, x_2, x_3) and a fluid with density steady with time. Without loss of generality we will limit our discussion to the particular direction, e.g., the x_1 direction.

Integrating over the control volume shown in Fig. A.1 the convection term $(CT)_1$ in x_1 direction of a physical quantity, ϕ , is given by

$$(CT)_1 = (\rho A)_e v_{1e} \phi_e - (\rho A)_w v_{1w} \phi_w \quad (A.3)$$

where A is the flow per unit velocity.

v_1 is the velocity along the direction 1.

The natural way of approximating ϕ_e and ϕ_w as we can see from Fig. 4.1 is

$$\phi_e = \frac{\phi_E + \phi_D}{2} + O(\Delta Y_e^2)$$

and

$$\phi_w = \frac{\phi_W + \phi_P}{2} + O(\Delta X_w^2)$$

(A.4)

This is the central difference scheme (CDS). It can be easily shown that such a scheme does not always obey condition (A.2). It has been found that such an approximation has led to instabilities [89], [90].

To provide for an approximation obeying condition (A.2) the "up-wind difference scheme" (UDS) has been introduced [11] and has been widely used, e.g., [36], [50], [1]. In this case

$$\phi_e = \begin{cases} \phi_D + O\left(\frac{\Delta X_e}{2}\right) & \text{when } v_{1e} > 0 \\ \phi_E + O\left(\frac{\Delta X_e}{2}\right) & \text{when } v_{1e} < 0 \end{cases} \quad (\text{A.5})$$

and similarly for ϕ_w .

Such an approximation although it obeys condition (A.2) is less accurate than the CDS by one degree of accuracy. This means that obviously a higher number of points will be required to get accurate enough results. However, in some cases the relatively low degree (1st order) of accuracy in the UDS might lead to inaccurate results [91].

One way to overcome these drawbacks of the above two schemes is to use some mixed scheme of central differencing and upwind differencing. The code SOLA [92] and VAPP-II [43] introduce an input parameter such that $0 \leq \alpha \leq 1$

when $\alpha = 0$ the central difference scheme is used
 when $\alpha = 1$ the upwind difference scheme is used.

The code TEACH [71] used the following mixed scheme. In case the diffusion is the dominant mechanism, use the central difference scheme, whereas in case the convection mechanism is dominant, use the upwind difference scheme. Even with these improvements there remain cases where UDS has to be used.

In our work we have introduced the Upwind Derivative Difference Scheme (UDDS) which is second order accurate and obeys condition (A.2).

1.2 The Upwind Derivative Difference Scheme

The second order approximation of ϕ_e and ϕ_w can be written as follows

$$\begin{aligned}\phi_e &= \phi_P + \left(\frac{\partial \phi}{\partial x_1}\right)_P \frac{\Delta x_e^2}{2} + O\left(\frac{\Delta x_e^2}{4}\right) \\ \phi_w &= \phi_P - \left(\frac{\partial \phi}{\partial x_1}\right)_P \frac{\Delta x_w^2}{2} + O\left(\frac{\Delta x_w^2}{4}\right)\end{aligned}\tag{A.6}$$

then

$$\begin{aligned}(\text{CT})_1 &= \phi_P \left((\rho A)_e v_{1e} - (\rho A)_w v_{1w} \right. \\ &\quad \left. + (\rho A)_e v_{1e} \Delta x_e + (\rho A)_w v_{1w} \Delta x_w \right) \left(\frac{\partial \phi}{\partial x_1}\right)_P\end{aligned}\tag{A.7}$$

The contribution of the first term in the right hand side of Eq. (A.7) is zero due to the continuity condition. Thus the net contribution of the convection term in the i direction is

$$(CT')_1 = (oA)_e v_{1e} \Delta v_e + (oA)_{11} v_{11} \Delta v_{11} + \left(\frac{\partial \phi}{\partial x_1}\right)_p \quad (A.8)$$

let

$$C_1 = (oA)_e v_{1e} \Delta v_e + (oA)_{11} v_{11} \Delta v_{11} \quad (A.9)$$

then

$$(CT')_1 = C_1 \left(\frac{\partial \phi}{\partial x_1}\right)_p \quad (A.10)$$

we further approximate

$$\left(\frac{\partial \phi}{\partial x_1}\right)_p = \begin{cases} \frac{\phi_p - \phi_{11}}{\Delta v_{11}} + O(\Delta v_{11}) & \text{if } C_1 > 0 \\ \frac{\phi_p - \phi_e}{\Delta v_e} + O(\Delta v_e) & \text{if } C_1 < 0 \end{cases} \quad (A.11)$$

Under approximation (A.11), Eq. (A.10) can be written as

$$(CT')_1 = (CE + CW)\phi_p - CF\phi_e - CW\phi_{11} \quad (A.12)$$

where

$$\begin{aligned} CE &= 0.5 \frac{|C_1| + \tilde{C}_1}{\Delta v_e} \\ CW &= 0.5 \frac{|C_1| + \tilde{C}_1}{\Delta v_{11}} \end{aligned} \quad (A.13)$$

and CF , CW are positive numbers.

Thus, Eqs. (A.12) and (A.13) obey condition (A.2) and are second order accurate and therefore superior to the UDS from an accuracy point of view. With respect not to the rate of convergence in the actual non-linear problems, our expedition of work in second order flows in infinite triangular meshes show that the effect in the rate of convergence seems to be minor.

Fig. A.1. Discretization in (X_1, X_j) plane

APPENDIX B

HEAT TRANSFER CALCULATIONS

B.1 General Equations

B.1.1 The Coolant Region

The time smoothed heat transfer equation [] assuming time-independent properties can also be put in the general form of Eq. (3.67) i.e.:

$$\Delta \underline{V} \cdot \underline{T}_c + \Delta \underline{\tilde{q}}_c = S^* \quad (\text{B.1})$$

where $\underline{T}_c$ is the coolant temperature

$\underline{V}$ is the velocity vector

$$\underline{\tilde{q}}_c = (\tilde{q}_{cr}, \tilde{q}_{c\theta}) \quad (\text{B.2a})$$

$$\tilde{q}_{cr} = -\alpha_H \frac{\partial T_c}{\partial r} + \frac{r}{T_c} \frac{\partial T_c}{\partial r} \quad (\text{B.2b})$$

$$\tilde{q}_{c\theta} = -\alpha_H \frac{\partial T_c}{r \partial \theta} + \frac{r}{T_c} \frac{\partial T_c}{\partial \theta} \quad (\text{B.2c})$$

α_H is the heat diffusivity

$$S^* = -\frac{1}{\rho c_p} \frac{4}{D_H} q_o'' \frac{V^2}{a_b^2} \quad (\text{B.3})$$

is the coolant density

c_p is the coolant specific heat

q_o'' is the average heat flux in the outside surface of the clad given by the relation

$$\bar{q}_0'' = \frac{q'}{\pi D} \quad (\text{B.4})$$

and q' is the Linear Heat Generation ratio (heat generation rate per unit length).

$\overline{T'_c v'_r}$ and $\overline{T'_c v'_\theta}$ are the turbulent diffusion terms.

A widely used approximation for modeling the turbulent diffusion terms is to treat them similarly to the turbulent momentum diffusion terms. Thus in our case we have

$$\overline{T'_c v'_r} = - \frac{\nu_T}{\sigma_T} \frac{\partial T_c}{\partial r} \quad (\text{B.5a})$$

$$\overline{T'_c v'_\theta} = - \frac{\nu_T}{\sigma_T} \frac{\partial T_c}{r \partial \theta} \quad (\text{B.5b})$$

where σ_T is the heat diffusivity to momentum diffusivity ratio and in general can be a function of position and flow characteristics.

B.1.2 The Clad Region

The clad heat transfer equation also can be put in the general form of Equation (3.67) with only the diffusion term as a non zero term i.e.

$$\nabla^2 \tilde{q}_{cl} = 0 \quad (\text{B.6})$$

where

$$\tilde{q}_{cl} = (\tilde{q}_{clr}, \tilde{q}_{cl\theta}) \quad (\text{B.7a})$$

$$q_{cl}^2 = -\lambda_{cl} \frac{\partial T_{cl}}{\partial r} \quad (B.7b)$$

$$q_{cl\theta}^2 = -\lambda_{cl} \frac{1}{r} \frac{\partial T_c}{\partial \theta} \quad (B.7c)$$

and λ_{cl} is the clad heat conductivity.

B.1.3 The Boundary Conditions

The boundary conditions for all the symmetry lines are written as

$$\frac{\partial T_c}{\partial n}, \frac{\partial T_{cl}}{\partial n} = 0 \text{ for all symmetry lines} \quad (B.8)$$

($\underline{n}$ is the unit vector perpendicular to a symmetry line)

The boundary conditions differ at the walls where temperature gradients are developed due to heat generation in the fuel.

B.1.3.1 Outside Clad Surface

In this case the obvious boundary conditions are temperature and heat flux continuity i.e.

$$T_c = T_{cl} \quad (B.9)$$

$$-\lambda_{cl} \frac{\partial T_{cl}}{\partial r} = -\lambda_c \frac{\partial T_c}{\partial r} \quad (B.10)$$

where λ_c is the thermal conductivity of the coolant.

B.1.3.2 Inside Clad Surface

For the inside clad region we assume heat flux uniformities i.e.

$$q_i'' = -\lambda_{cl} \frac{\partial T_c}{\partial r} = \text{const} = \frac{q_i'}{\pi D_i} \quad (\text{B.11})$$

where D_i is the inside clad diameter

B.1.3.3 Temperature Level

Since the boundary conditions for this problem are temperature gradient conditions, the resultant solution will yield the relative temperature distribution between points but not the absolute temperature levels. To obtain the absolute temperature levels, a constant temperature is algebraically subtracted from the temperature of every point in the field. This constant temperature is obtained as the difference between the calculated temperature and a prescribed temperature at a given point, the prescribed temperature being known from physical considerations.

B.2 The Finite Difference Equations

Since Equations (B.1) and (B.6) have the general form of Equation (3.67) the procedure of obtaining finite difference equations for coolant and clad temperature is quite similar to that of flow calculations.

The control volume of Figure 4.2 is also used here and it is identical with that used for the axial velocity v_z .

The equations have also the form (4.8d) i.e.

$$DT + CT = ST \quad (B.12)$$

where DT is the diffusion term given by the relation

$$DT = \tilde{q}_{re} A_e - \tilde{q}_{rw} A_w + \tilde{q}_{\theta n} A_n - \tilde{q}_{\theta s} A_s \quad (B.13)$$

CT is the convection term given by the relation

$$CT = \begin{cases} A_e V_{re} T_{ce} - A_w V_{rw} T_{cw} + A_n V_{\theta n} T_{cn} - A_s V_{\theta s} T_{cs} & \text{for } T_c \\ 0 & \text{for } T_{cl} \end{cases}$$

and ST is the source term given by the relation

$$ST = \begin{cases} - \frac{1}{\rho c_p} \frac{4}{D_H} q_o^* \frac{1}{u_b} (v_z)_p & \text{for } T_c \\ 0 & \text{for } T_{cl} \end{cases}$$

A_e, A_w, A_n, A_s are given in Table 4.1.

B.2.1 The Diffusion Term

B.2.1.1 Coolant Region

Taking equations (B.2), finite-differencing them and applying them to Equation (B.13) we end up with

$$DT = (DE + DW + DN + DS) T_{c,i,j} - DE \cdot T_{c,i+1,j} - DW \cdot T_{c,i-1,j} - DN \cdot T_{c,i,j+1} - DS \cdot T_{c,i,j-1} \quad (B.16)$$

where

$$DE = BE \left\{ 2\alpha_H + \frac{1}{\sigma_T} \left| \left(v_{rz}^T \right)_{i,j} + \left(v_{rz}^T \right)_{i+1,j} \right| \right\} \quad (B.17a)$$

$$DW = BW \left\{ 2\alpha_H + \frac{1}{\sigma_T} \left| \left(v_{rz}^T \right)_{i,j} + \left(v_{rz}^T \right)_{i-1,j} \right| \right\} \quad (B.17b)$$

$$DN = BN \left\{ 2\alpha_H + \frac{1}{\sigma_T} \left| \left(v_{\theta z}^T \right)_{i,j} + \left(v_{\theta z}^T \right)_{i+1,j} \right| \right\} \quad (B.17c)$$

$$DS = BS \left\{ 2\alpha_H + \frac{1}{\sigma_T} \left| \left(v_{\theta z}^T \right)_{i,j} + \left(v_{\theta z}^T \right)_{i,j-1} \right| \right\} \quad (B.17d)$$

BE, BW, BN, BS are given from Table 4.2.

B.2.1.2 Clad Region

Working as in the previous section and taking into consideration Equations (B.7) we find finally

$$\begin{aligned} DT = & (DE + DW + DN + DS) T_{cl,i,j} - DE \cdot T_{cl,i+1,j} \\ & - DWT_{cl,i-1,j} - DN \cdot T_{cl,i,j+1} - DST_{cl,i,j-1} \end{aligned} \quad (B.18)$$

where

$$\begin{aligned} DE &= 2BE \lambda_{cl} \\ DW &= 2 \cdot BW \lambda_{cl} \\ DN &= 2 \cdot BN \lambda_{cl} \\ DS &= 2 \cdot BS \lambda_{cl} \end{aligned} \quad (B.19)$$

where

BE, BW, BN, BS are given in Table 4.2.

B.2.2 The Convection Term

For the clad temperature there is no convection term.

For the coolant temperature the convection term is identical in form with that for axial velocity v_z , i.e.

$$CT = [CE + CW + CN + CS] T_{c,i,j} - CE \cdot T_{c,i+1,j} \\ - CW \cdot T_{c,i-1,j} - CN \cdot T_{c,i,j+1} - CS \cdot T_{c,i,j-1} \quad (B.20)$$

where

CE, CW, CN, CS are given by Table 4.3.

B.2.3 The Source Term

For the clad temperature there is no source term.

For the coolant temperature the source term is given by (B.15).

B.2.4 General Form of FDE

Equation (B.12) can be written taking into consideration Equations (B.16), (B.17), (B.18), (B.19), and (B.20) as

$$AP \cdot \phi_P = AE \cdot \phi_E + AN \phi_N + AS \phi_S + AS \phi'_S + SP \quad (B.21)$$

where

$$\begin{aligned}
 AE &= CE + DE \\
 AW &= DW + DW \\
 AN &= DN + DN \\
 AS &= CS + DS
 \end{aligned}
 \tag{B.22}$$

For clad temperature $CE = CW = CN = CS = ST = 0$

For the coolant temperature $CE, CW, CN, CS, DE, DW, DN, DS, ST$ are given from Table 4.3 and Equations (B.17), (B.19), and (B.15).

For the clad temperature DE, DW, DN, DS are given by Equation (B.19).

B.2.5 Numerical Solution

It is evident that Equation (B.22) has the same form as the general FDE for flow calculations (4.44). Thus we can apply the same numerical techniques we applied for flow calculations i.e. the Line Iterations scheme (4.32) or the successive relaxation method (§ 4.3.3). Although the computer code built in the present work has both options present calculations are performed with Line Iteration Technique.

APPENDIX C

EXPERIMENTAL DATA

In this appendix the experimental grid and the quantities measured are tabulated.

GAGE FOR AXIAL VELOCITY MEASUREMENT													TETA	
1	2	3	4	5	6	7	8	9	10	11	12			
1 0.0	4.042E-01	4.567E-01	5.001E-01	5.456E-01	5.910E-01	6.364E-01	6.819E-01	7.273E-01	7.728E-01	8.182E-01	8.637E-01	9.091E-01	30.000	
1 0.0	3.459E-01	3.962E-01	4.465E-01	4.969E-01	5.472E-01	5.975E-01	6.478E-01	6.981E-01	7.484E-01	7.987E-01	8.491E-01	8.994E-01	27.000	
4 0.0	2.759E-01	3.126E-01	3.493E-01	4.436E-01	4.507E-01	5.544E-01	6.171E-01	6.650E-01	7.215E-01	7.772E-01	8.329E-01	8.890E-01	24.000	
5 0.0	2.613E-01	3.229E-01	3.846E-01	4.460E-01	5.074E-01	5.691E-01	6.307E-01	6.922E-01	7.538E-01	8.153E-01	8.769E-01	9.385E-01	21.000	
7 0.0	2.858E-01	3.545E-01	4.232E-01	4.919E-01	5.606E-01	6.293E-01	6.980E-01	7.667E-01	8.354E-01	9.041E-01	9.728E-01	10.415E-01	18.000	
8 0.0	1.113E-01	1.845E-01	2.586E-01	3.327E-01	4.069E-01	4.810E-01	5.552E-01	6.293E-01	7.034E-01	7.776E-01	8.517E-01	9.258E-01	15.000	
9 0.0	1.167E-01	1.945E-01	2.769E-01	3.572E-01	4.376E-01	5.179E-01	5.983E-01	6.786E-01	7.590E-01	8.393E-01	9.196E-01	9.999E-01	12.000	
4 0.0	1.404E-01	2.263E-01	3.123E-01	3.983E-01	4.842E-01	5.702E-01	6.561E-01	7.421E-01	8.281E-01	9.141E-01	1.000E-01	9.000E-01	9.000	
3 0.0	7.574E-02	1.855E-01	2.763E-01	3.665E-01	4.570E-01	5.475E-01	6.380E-01	7.285E-01	8.190E-01	9.095E-01	1.000E-01	6.000E-01	6.000	
2 0.0	1.545E-01	2.570E-01	3.436E-01	4.393E-01	5.327E-01	6.262E-01	7.196E-01	8.131E-01	9.066E-01	1.000E-01	0.0	0.0	3.000	
1 0.0	1.456E-01	2.441E-01	3.286E-01	4.331E-01	5.276E-01	6.220E-01	7.165E-01	8.110E-01	9.055E-01	1.000E-01	0.0	0.0	0.0	
1 13	14	15	TETA											
11 9.091E-01	1.546E-01	1.090E-01	0.0	3.000E-01										
12 8.994E-01	1.497E-01	1.000E-01	0.0	2.700E-01										
9 8.886E-01	9.443E-01	1.000E-01	0.0	2.400E-01										
8 9.364E-01	1.000E-01	0.0	0.0	2.100E-01										
7 9.172E-01	1.000E-01	0.0	0.0	1.800E-01										
6 9.259E-01	1.000E-01	0.0	0.0	1.500E-01										
5 1.000E-01	0.0	0.0	0.0	1.200E-01										
4 0.0	0.0	0.0	0.0	9.000E-01										
3 0.0	0.0	0.0	0.0	6.000E-01										
2 0.0	0.0	0.0	0.0	3.000E-01										
1 0.0	0.0	0.0	0.0	0.0										

REYNOLDS NUMBER = 36,500 , 65,000
 LENGTH/HYDRAULIC DIAMETER = 113 , 46

GRID FOR AXIAL VELOCITY MEASUREMENT													
J=	1	2	3	4	5	6	7	8	9	10	11	12	TEVA
6	0.0	4.092E-01	4.547E-01	5.001E-01	5.455E-01	5.910E-01	6.364E-01	6.819E-01	7.273E-01	7.728E-01	8.182E-01	8.637E-01	10.000
5	0.0	2.759E-01	3.316E-01	3.873E-01	4.430E-01	4.987E-01	5.544E-01	6.101E-01	6.658E-01	7.215E-01	7.772E-01	8.329E-01	24.003
4	0.0	1.868E-01	2.545E-01	3.223E-01	3.901E-01	4.578E-01	5.256E-01	5.934E-01	6.611E-01	7.289E-01	7.967E-01	8.645E-01	18.000
3	0.0	1.162E-01	1.965E-01	2.769E-01	3.572E-01	4.376E-01	5.179E-01	5.983E-01	6.786E-01	7.590E-01	8.393E-01	9.196E-01	12.000
2	0.0	7.504E-02	1.855E-01	2.760E-01	3.665E-01	4.570E-01	5.475E-01	6.380E-01	7.285E-01	8.190E-01	9.095E-01	1.000E 00	6.000
1	0.0	1.496E-01	2.641E-01	3.386E-01	4.331E-01	5.276E-01	6.220E-01	7.165E-01	8.110E-01	9.055E-01	1.000E 00	0.0	0.0
J=	13	14	15	TEVA									
6	9.091E-01	9.546E-01	1.000E 00	1.000E 01									
5	8.886E-01	9.443E-01	1.000E 00	2.400E 01									
4	9.322E-01	1.000E 00	0.0	1.800E 01									
3	1.000E 00	0.0	0.0	1.200E 01									
2	0.0	0.0	0.0	6.000E 00									
1	0.0	0.0	0.0	0.0									

REYNOLDS NUMBER= 25,500 , 65,000
 LENGTH/HYDRAULIC DIAMETER= 15

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- - - - -
I= 1 2 3 4 5 6 7 8 9 10 11 12 IETA
J
11 0.0 4.092E-01 4.547E-01 5.001E-01 5.455E-01 5.910E-01 6.364E-01 6.819E-01 7.273E-01 7.728E-01 8.182E-01 8.637E-01 90.000
10 0.0 3.454E-01 3.962E-01 4.465E-01 4.969E-01 5.472E-01 5.975E-01 6.478E-01 6.981E-01 7.484E-01 7.987E-01 8.491E-01 27.000
9 0.0 3.316E-01 3.873E-01 4.430E-01 4.987E-01 5.544E-01 6.101E-01 6.658E-01 7.215E-01 7.772E-01 8.329E-01 8.886E-01 24.000
8 0.0 2.613E-01 3.229E-01 3.844E-01 4.460E-01 5.076E-01 5.691E-01 6.307E-01 6.922E-01 7.538E-01 8.153E-01 8.769E-01 21.000
7 0.0 1.864E-01 2.549E-01 3.223E-01 3.901E-01 4.578E-01 5.254E-01 5.934E-01 6.611E-01 7.289E-01 7.967E-01 8.645E-01 18.000
6 0.0 1.845E-01 2.586E-01 3.327E-01 4.069E-01 4.810E-01 5.552E-01 6.293E-01 7.034E-01 7.776E-01 8.517E-01 9.259E-01 15.000
5 0.0 1.162E-01 1.965E-01 2.769E-01 3.572E-01 4.376E-01 5.179E-01 5.983E-01 6.786E-01 7.590E-01 8.393E-01 9.196E-01 12.000
4 0.0 1.404E-01 2.263E-01 3.123E-01 3.983E-01 4.842E-01 5.702E-01 6.561E-01 7.421E-01 8.281E-01 9.140E-01 1.000E 00 9.000
3 0.0 9.504E-02 1.855E-01 2.760E-01 3.665E-01 4.570E-01 5.475E-01 6.380E-01 7.285E-01 8.190E-01 9.095E-01 1.000E 00 6.000
2 0.0 1.589E-01 2.523E-01 3.458E-01 4.393E-01 5.327E-01 6.262E-01 7.196E-01 8.131E-01 9.065E-01 1.000E 00 0.0 3.000
1 0.0 1.496E-01 2.441E-01 3.386E-01 4.331E-01 5.276E-01 6.220E-01 7.165E-01 8.110E-01 9.055E-01 1.000E 00 0.0 0.0
I= 13 14 15 IETA
J
11 9.091E-01 9.546E-01 1.000E 00 3.000E 01
10 8.994E-01 9.497E-01 1.000E 00 2.700E 01
9 9.443E-01 1.000E 00 0.0 2.400E 01
8 9.384E-01 1.000E 00 0.0 2.100E 01
7 9.322E-01 1.000E 00 0.0 1.800E 01
6 1.000E 00 0.0 0.0 1.500E 01
5 1.000E 00 0.0 0.0 1.200E 01
4 0.0 0.0 0.0 9.000E 00
3 0.0 0.0 0.0 6.000E 00
2 0.0 0.0 0.0 3.000E 00
1 0.0 0.0 0.0 0.0

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REYNOLDS NUMBER= 8924.
 LENGTH/HYDRAULIC DIAMETER= 993

	1	2	3	4	5	6	7	8	9	10	11	12	TETA
11 0.0	8.77E-01	9.09E-01	9.17E-01	9.40E-01	9.50E-01	9.54E-01	9.71E-01	9.78E-01	9.86E-01	9.88E-01	9.88E-01	9.88E-01	30.00
10 0.0	8.48E-01	8.69E-01	8.86E-01	9.01E-01	9.19E-01	9.30E-01	9.42E-01	9.54E-01	9.64E-01	9.71E-01	9.75E-01	9.75E-01	27.00
9 0.0	8.01E-01	8.27E-01	8.42E-01	8.63E-01	8.82E-01	8.99E-01	9.14E-01	9.24E-01	9.37E-01	9.54E-01	9.70E-01	9.70E-01	24.00
8 0.0	7.81E-01	8.03E-01	8.26E-01	8.47E-01	8.68E-01	8.84E-01	9.01E-01	9.14E-01	9.23E-01	9.32E-01	9.44E-01	9.44E-01	21.00
7 0.0	7.24E-01	7.63E-01	7.86E-01	8.10E-01	8.34E-01	8.52E-01	8.70E-01	8.87E-01	9.04E-01	9.14E-01	9.21E-01	9.21E-01	18.00
6 0.0	6.92E-01	7.37E-01	7.61E-01	7.87E-01	8.11E-01	8.31E-01	8.49E-01	8.65E-01	8.80E-01	8.92E-01	9.00E-01	9.00E-01	15.00
5 0.0	6.70E-01	7.19E-01	7.43E-01	7.69E-01	7.91E-01	8.10E-01	8.28E-01	8.44E-01	8.59E-01	8.71E-01	8.79E-01	8.79E-01	12.00
4 0.0	6.59E-01	7.09E-01	7.33E-01	7.59E-01	7.79E-01	7.98E-01	8.16E-01	8.32E-01	8.47E-01	8.59E-01	8.67E-01	8.67E-01	9.00
3 0.0	6.49E-01	6.99E-01	7.23E-01	7.49E-01	7.69E-01	7.88E-01	8.06E-01	8.22E-01	8.37E-01	8.49E-01	8.57E-01	8.57E-01	6.00
2 0.0	6.41E-01	6.91E-01	7.15E-01	7.41E-01	7.61E-01	7.80E-01	7.98E-01	8.14E-01	8.29E-01	8.41E-01	8.49E-01	8.49E-01	3.00
1 0.0	6.31E-01	6.81E-01	7.05E-01	7.31E-01	7.51E-01	7.70E-01	7.88E-01	8.04E-01	8.19E-01	8.31E-01	8.39E-01	8.39E-01	0.00
14	14	15	TETA										
11 9.46E-01	9.46E-01	1.00E-00	3.00E-01										
10 9.41E-01	9.41E-01	9.99E-01	2.70E-01										
9 9.36E-01	9.36E-01	9.78E-01	2.40E-01										
8 9.49E-01	9.51E-01	0.0	2.10E-01										
7 9.29E-01	9.13E-01	0.0	1.80E-01										
6 9.07E-01	9.12E-01	0.0	1.50E-01										
5 8.97E-01	0.0	0.0	1.20E-01										
4 0.0	0.0	0.0	9.00E-02										
3 0.0	0.0	0.0	6.00E-02										
2 0.0	0.0	0.0	3.00E-02										
1 0.0	0.0	0.0	0.0										

*REYNOLDS NUMBER 26549
 *LENGTH/HYDRAULIC DIAMETER 11.3

-----													-----												
													TURB AXIAL VELOCITY/MAX VELOCITY												
1*	1	2	3	4	5	6	7	8	9	10	11	12	-----												
J																									
11 0.0	6.579E-02	6.334E-02	6.236E-02	5.670E-02	5.350E-02	5.102E-02	4.654E-02	4.354E-02	4.001E-02	3.649E-02	3.198E-02	30.000													
10 0.0	7.008E-02	6.921E-02	6.701E-02	6.383E-02	6.084E-02	5.843E-02	5.498E-02	5.201E-02	4.804E-02	4.504E-02	4.213E-02	27.000													
9 0.0	7.559E-02	7.312E-02	7.068E-02	6.921E-02	6.579E-02	6.334E-02	5.990E-02	5.645E-02	5.473E-02	5.003E-02	4.604E-02	24.000													
8 0.0	7.677E-02	7.555E-02	7.312E-02	6.970E-02	6.726E-02	6.334E-02	5.966E-02	5.695E-02	5.300E-02	4.854E-02	4.604E-02	21.000													
7 0.0	8.205E-02	7.799E-02	7.555E-02	7.190E-02	6.770E-02	6.628E-02	6.211E-02	5.843E-02	5.473E-02	4.933E-02	4.754E-02	18.000													
6 0.0	9.962E-02	7.478E-02	7.799E-02	7.312E-02	7.190E-02	6.799E-02	6.211E-02	5.892E-02	5.666E-02	5.107E-02	4.804E-02	15.000													
5 0.0	9.740E-02	8.164E-02	7.361E-02	7.312E-02	6.872E-02	6.383E-02	6.136E-02	5.843E-02	5.231E-02	4.903E-02	4.504E-02	12.000													
4 0.0	8.528E-02	7.474E-02	7.019E-02	6.726E-02	6.579E-02	6.138E-02	5.794E-02	5.448E-02	4.854E-02	4.479E-02	4.354E-02	9.000													
3 0.0	9.498E-02	7.555E-02	7.068E-02	6.603E-02	6.457E-02	6.236E-02	5.843E-02	5.394E-02	4.754E-02	4.102E-02	4.153E-02	6.000													
2 0.0	8.205E-02	7.214E-02	6.701E-02	6.481E-02	6.138E-02	5.744E-02	5.251E-02	4.504E-02	4.077E-02	3.900E-02	0.0	1.000													
1 0.0	8.528E-02	7.312E-02	6.701E-02	6.334E-02	5.966E-02	5.597E-02	4.854E-02	4.354E-02	3.925E-02	3.798E-02	0.0	0.0													
1*	11	14	15	TEPA																					
J																									
11 3.722E-02	3.722E-02	3.722E-02	3.000E 01																						
10 3.478E-02	3.849E-02	3.798E-02	2.700E 01																						
9 4.334E-02	4.304E-02	4.153E-02	2.400E 01																						
8 4.404E-02	4.127E-02	0.0	2.100E 01																						
7 4.554E-02	4.434E-02	0.0	1.800E 01																						
6 4.554E-02	4.404E-02	0.0	1.500E 01																						
5 4.354E-02	0.0	0.0	1.200E 01																						
4 0.0	0.0	0.0	9.000E 00																						
3 0.0	0.0	0.0	6.000E 00																						
2 0.0	0.0	0.0	3.000E 00																						
1 0.0	0.0	0.0	0.0																						

REYNOLDS NUMBER= 26349.
 LENGTH/HYDRAULIC DIAMETER= 113.

AXIAL VELOCITY/MAX VELOCITY												
1	2	3	4	5	6	7	8	9	10	11	12	1ET
11 0.0	8.425E-01	8.969E-01	9.089E-01	9.235E-01	9.329E-01	9.441E-01	9.544E-01	9.619E-01	9.664E-01	9.736E-01	9.780E-01	10.00
10 0.0	8.441E-01	8.657E-01	8.873E-01	8.993E-01	9.096E-01	9.223E-01	9.329E-01	9.440E-01	9.528E-01	9.616E-01	9.650E-01	27.000
9 0.0	8.045E-01	8.285E-01	8.513E-01	8.698E-01	8.801E-01	8.969E-01	9.089E-01	9.237E-01	9.329E-01	9.468E-01	9.472E-01	26.000
8 0.0	7.734E-01	8.070E-01	8.307E-01	8.489E-01	8.657E-01	8.796E-01	8.969E-01	9.106E-01	9.168E-01	9.261E-01	9.316E-01	21.000
7 0.0	7.766E-01	7.578E-01	7.847E-01	8.082E-01	8.285E-01	8.446E-01	8.633E-01	8.813E-01	8.875E-01	8.974E-01	9.050E-01	18.00
6 0.0	6.467E-01	6.990E-01	7.240E-01	7.578E-01	7.842E-01	8.070E-01	8.261E-01	8.405E-01	8.552E-01	8.691E-01	8.850E-01	15.000
5 0.0	6.441E-01	6.811E-01	7.146E-01	7.415E-01	7.644E-01	7.840E-01	8.129E-01	8.249E-01	8.367E-01	8.465E-01	8.574E-01	12.000
4 0.0	6.411E-01	6.781E-01	7.046E-01	7.338E-01	7.594E-01	7.814E-01	7.978E-01	8.080E-01	8.191E-01	8.288E-01	8.341E-01	9.000
3 0.0	6.388E-01	6.703E-01	6.971E-01	7.050E-01	7.278E-01	7.530E-01	7.679E-01	7.841E-01	7.986E-01	8.060E-01	8.144E-01	6.000
2 0.0	6.378E-01	6.644E-01	6.882E-01	7.132E-01	7.348E-01	7.593E-01	7.805E-01	7.942E-01	8.040E-01	8.127E-01	8.200	3.000
1 0.0	6.315E-01	6.564E-01	6.835E-01	7.122E-01	7.343E-01	7.554E-01	7.707E-01	7.806E-01	7.904E-01	7.988E-01	8.0	0.0
14	15	16	17	18	19	20	21	22	23	24	25	26
11 9.794E-01	9.749E-01	9.706E-01	3.000E 01									
10 9.700E-01	9.710E-01	9.760E-01	2.700E 01									
9 9.546E-01	9.560E-01	9.578E-01	2.400E 01									
8 9.329E-01	9.353E-01	0.0	2.100E 01									
7 9.069E-01	9.113E-01	0.0	1.800E 01									
6 8.832E-01	8.841E-01	0.0	1.500E 01									
5 8.578E-01	0.0	0.0	1.200E 01									
4 0.0	0.0	0.0	1.000E 00									
3 0.0	0.0	0.0	8.000E 00									
2 0.0	0.0	0.0	3.000E 00									
1 0.0	0.0	0.0	0.0									

REYNOLDS NUMBER 20163
 LENGTH/CHARACTERISTIC DIAMETER 40

-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----												
TURN AXIAL VELOCITY/MAX VELOCITY												
1	2	3	4	5	6	7	8	9	10	11	12	REL.
J												
11 0.0	6.579E-02	6.211E-02	5.986E-02	5.740E-02	5.350E-02	4.978E-02	4.854E-02	4.404E-02	4.153E-02	3.976E-02	3.773E-02	30.00
10 0.0	7.064E-02	6.824E-02	6.579E-02	6.285E-02	5.966E-02	5.720E-02	5.473E-02	5.102E-02	4.729E-02	4.479E-02	4.228E-02	27.00
9 0.0	7.434E-02	7.312E-02	6.824E-02	6.701E-02	6.457E-02	6.089E-02	5.843E-02	5.350E-02	5.226E-02	4.679E-02	4.504E-02	24.00
8 0.0	7.555E-02	7.190E-02	7.068E-02	6.701E-02	6.285E-02	6.089E-02	5.744E-02	5.673E-02	5.102E-02	4.604E-02	4.479E-02	21.00
7 0.0	8.265E-02	7.677E-02	7.190E-02	6.944E-02	6.701E-02	6.383E-02	5.990E-02	5.843E-02	5.275E-02	4.878E-02	4.729E-02	18.00
6 0.0	8.771E-02	8.285E-02	7.677E-02	7.190E-02	6.944E-02	6.701E-02	6.408E-02	5.990E-02	5.479E-02	5.152E-02	4.729E-02	15.00
5 0.0	8.892E-02	7.921E-02	7.312E-02	7.019E-02	6.726E-02	6.383E-02	6.040E-02	5.621E-02	5.251E-02	4.854E-02	4.454E-02	12.00
4 0.0	8.285E-02	7.068E-02	6.944E-02	6.579E-02	6.211E-02	6.040E-02	5.597E-02	5.176E-02	4.804E-02	4.354E-02	4.253E-02	9.00
3 0.0	7.013E-02	7.555E-02	6.824E-02	6.579E-02	6.187E-02	5.832E-02	5.646E-02	5.152E-02	4.604E-02	4.153E-02	4.001E-02	6.00
2 0.0	5.520E-02	7.190E-02	6.457E-02	6.236E-02	5.843E-02	5.350E-02	4.854E-02	4.629E-02	4.001E-02	3.900E-02	0.0	3.00
1 0.0	6.285E-02	7.190E-02	6.457E-02	6.162E-02	5.770E-02	5.201E-02	4.604E-02	4.253E-02	3.425E-02	3.748E-02	0.0	0.0
I= 13	14	15	TEPA									
J												
11 3.544E-02	3.442E-02	3.534E-02	3.000E 01									
10 4.001E-02	3.300E-02	3.646E-02	2.700E 01									
9 4.253E-02	4.227E-02	4.102E-02	2.400E 01									
8 4.404E-02	4.327E-02	0.0	2.100E 01									
7 4.479E-02	4.354E-02	0.0	1.800E 01									
6 4.479E-02	4.404E-02	0.0	1.500E 01									
5 4.429E-02	0.0	0.0	1.200E 01									
4 0.0	0.0	0.0	9.000E 00									
3 0.0	0.0	0.0	6.000E 00									
2 0.0	0.0	0.0	3.000E 00									
1 0.0	0.0	0.0	0.0									

REYNOLDS NUMBER= 26163.
 LENGTH/HYDRAULIC DIAMETER= 46

AXIAL VELOCITY/MAX VELOCITY												
1	2	3	4	5	6	7	8	9	10	11	12	13
0.0	7.625E-01	7.722E-01	7.830E-01	7.914E-01	8.010E-01	8.093E-01	8.312E-01	8.417E-01	8.470E-01	8.565E-01	8.645E-01	30.00
0.0	7.130E-01	7.278E-01	7.458E-01	7.638E-01	8.211E-01	7.926E-01	8.165E-01	8.225E-01	8.365E-01	8.432E-01	8.484E-01	24.00
0.0	6.554E-01	6.859E-01	7.106E-01	7.386E-01	7.602E-01	7.926E-01	8.058E-01	8.168E-01	8.285E-01	8.345E-01	8.417E-01	18.00
0.0	5.935E-01	6.273E-01	6.635E-01	7.170E-01	7.361E-01	7.554E-01	7.703E-01	7.866E-01	7.980E-01	8.017E-01	8.034E-01	12.00
0.0	5.168E-01	5.411E-01	6.231E-01	6.703E-01	6.930E-01	7.163E-01	7.357E-01	7.513E-01	7.624E-01	7.676E-01	7.681E-01	6.00
0.0	4.470E-01	5.041E-01	6.137E-01	6.604E-01	6.944E-01	7.103E-01	7.285E-01	7.400E-01	7.470E-01	7.508E-01	7.510E-01	0.00
13	16	19	1512									
0.0	8.724E-01	8.777E-01	8.824E-01	8.866E-01								
0.0	8.585E-01	8.653E-01	8.710E-01	8.760E-01								
0.0	8.417E-01	8.475E-01	0.0	1.800E-01								
0.0	8.062E-01	0.0	0.0	1.200E-01								
0.0	0.0	0.0	0.0	6.000E-01								
0.0	0.0	0.0	0.0	0.0								

REYNOLDS NUMBER= 24018.
 LENGTH/HYDRAULIC DIAMETER= 15

AXIAL VELOCITY/NAE VELOCITY												
1	2	3	4	5	6	7	8	9	10	11	12	13
11 0.0	8.939E-01	9.508E-01	9.315E-01	9.402E-01	9.547E-01	9.628E-01	9.669E-01	9.723E-01	9.751E-01	9.864E-01	9.905E-01	30.00
10 0.0	8.645E-01	8.704E-01	8.865E-01	9.016E-01	9.175E-01	9.306E-01	9.456E-01	9.592E-01	9.642E-01	9.765E-01	9.823E-01	27.00
9 0.0	7.978E-01	8.205E-01	9.445E-01	8.645E-01	8.017E-01	8.494E-01	9.148E-01	9.279E-01	9.293E-01	9.479E-01	9.455E-01	24.00
8 0.0	7.743E-01	7.247E-01	8.218E-01	8.450E-01	8.676E-01	8.826E-01	9.012E-01	9.143E-01	9.225E-01	9.238E-01	9.461E-01	21.00
7 0.0	7.204E-01	7.334E-01	7.643E-01	8.024E-01	8.350E-01	8.534E-01	8.704E-01	8.867E-01	9.007E-01	9.134E-01	9.157E-01	18.00
6 0.0	6.591E-01	7.128E-01	7.316E-01	7.676E-01	7.942E-01	8.294E-01	8.519E-01	8.613E-01	8.749E-01	8.830E-01	8.867E-01	15.00
5 0.0	6.510E-01	6.406E-01	7.276E-01	7.386E-01	7.865E-01	8.087E-01	8.286E-01	8.481E-01	8.613E-01	8.767E-01	8.846E-01	12.00
4 0.0	6.500E-01	6.477E-01	7.238E-01	7.525E-01	7.741E-01	7.968E-01	8.191E-01	8.338E-01	8.499E-01	8.572E-01	8.676E-01	9.00
3 0.0	5.934E-01	6.399E-01	6.840E-01	7.176E-01	7.662E-01	7.947E-01	7.874E-01	8.033E-01	8.266E-01	8.323E-01	8.386E-01	6.00
2 0.0	6.247E-01	6.677E-01	6.941E-01	7.298E-01	7.579E-01	7.811E-01	7.978E-01	8.148E-01	8.233E-01	8.327E-01	8.4	3.00
1 0.0	6.238E-01	6.714E-01	7.032E-01	7.316E-01	7.575E-01	7.811E-01	7.983E-01	8.073E-01	8.150E-01	8.241E-01	8.3	2.0
13	14	15	16									
11 9.459E-01	9.455E-01	1.000E 00	3.600E 01									
10 9.878E-01	9.932E-01	1.000E 00	2.700E 01									
9 9.678E-01	9.746E-01	9.787E-01	2.400E 01									
8 9.547E-01	9.610E-01	0.0	2.100E 01									
7 9.325E-01	9.370E-01	0.0	1.800E 01									
6 9.066E-01	9.143E-01	0.0	1.500E 01									
5 8.921E-01	0.0	0.0	1.200E 01									
4 0.0	0.0	0.0	9.000E 00									
3 0.0	0.0	0.0	6.000E 00									
2 0.0	0.0	0.0	3.000E 00									
1 0.0	0.0	0.0	0.0									

REYNOLDS NUMBER= 54000
 LENGTH/AXIAL DIAMETER= 143

-----													AXIAL VELOCITY/MAX VELOCITY				-----				-----				TETA	
1=	1	2	3	4	5	6	7	8	9	10	11	12														
J																										
11	0.0	8.641E-01	8.866E-01	9.010E-01	9.140E-01	9.370E-01	9.428E-01	9.505E-01	9.559E-01	9.622E-01	9.689E-01	9.748E-01	30.000													
10	0.0	8.416E-01	8.605E-01	8.821E-01	8.933E-01	9.104E-01	9.280E-01	9.370E-01	9.464E-01	9.500E-01	9.599E-01	9.644E-01	27.000													
9	0.0	8.042E-01	8.177E-01	8.479E-01	8.606E-01	8.866E-01	9.032E-01	9.185E-01	9.338E-01	9.392E-01	9.442E-01	9.518E-01	24.000													
8	0.0	7.767E-01	8.038E-01	8.281E-01	8.101E-01	8.749E-01	8.888E-01	9.055E-01	9.145E-01	9.248E-01	9.334E-01	9.388E-01	21.000													
7	0.0	7.156E-01	7.444E-01	7.786E-01	8.002E-01	8.312E-01	8.533E-01	8.758E-01	8.839E-01	8.974E-01	9.073E-01	9.118E-01	18.000													
6	0.0	6.441E-01	6.872E-01	7.237E-01	7.581E-01	7.847E-01	8.043E-01	8.257E-01	8.316E-01	8.456E-01	8.740E-01	8.843E-01	15.000													
5	0.0	6.255E-01	6.789E-01	7.066E-01	7.412E-01	7.687E-01	7.888E-01	8.168E-01	8.346E-01	8.473E-01	8.542E-01	8.614E-01	12.000													
4	0.0	6.266E-01	6.829E-01	7.029E-01	7.318E-01	7.581E-01	7.797E-01	7.966E-01	8.116E-01	8.222E-01	8.344E-01	8.438E-01	9.000													
3	0.0	5.879E-01	6.283E-01	6.629E-01	6.944E-01	7.214E-01	7.426E-01	7.651E-01	7.860E-01	7.979E-01	8.101E-01	8.273E-01	6.000													
2	0.0	6.053E-01	6.672E-01	6.796E-01	7.111E-01	7.396E-01	7.552E-01	7.727E-01	7.874E-01	7.975E-01	8.074E-01	0.0	9.000													
1	0.0	6.076E-01	6.463E-01	6.787E-01	7.075E-01	7.313E-01	7.552E-01	7.722E-01	7.844E-01	7.943E-01	8.014E-01	0.0	0.0													
1=	13	14	15	TETA																						
J																										
11	9.820E-01	9.847E-01	9.860E-01	3.000E 01																						
10	9.734E-01	9.793E-01	9.829E-01	2.700E 01																						
9	9.599E-01	9.640E-01	9.658E-01	2.400E 01																						
8	9.451E-01	9.451E-01	0.0	2.100E 01																						
7	9.199E-01	9.172E-01	0.0	1.800E 01																						
6	8.893E-01	8.933E-01	0.0	1.500E 01																						
5	8.677E-01	0.0	0.0	1.200E 01																						
4	0.0	0.0	0.0	9.000E 00																						
3	0.0	0.0	0.0	6.000E 00																						
2	0.0	0.0	0.0	3.000E 00																						
1	0.0	0.0	0.0	0.0																						

REYNOLDS NUMBER= 64005
 LENGTH/HYDRAULIC DIAMETER= 46

TURN AXIAL VELOCITY/MAH VELOCITY												
1	2	3	4	5	6	7	8	9	10	11	12	TETA
11 0.0	6.595E-02	6.218E-02	5.886E-02	5.601E-02	5.024E-02	5.121E-02	4.830E-02	4.635E-02	4.537E-02	4.439E-02	4.091E-02	30.000
10 0.0	7.110E-02	6.876E-02	6.548E-02	6.218E-02	5.934E-02	5.601E-02	5.266E-02	5.024E-02	4.830E-02	4.684E-02	4.340E-02	27.000
9 0.0	7.204E-02	6.829E-02	6.736E-02	6.548E-02	6.171E-02	5.886E-02	5.505E-02	5.169E-02	4.879E-02	4.733E-02	4.488E-02	24.000
8 0.0	6.734E-02	6.829E-02	6.889E-02	6.454E-02	6.076E-02	5.601E-02	5.314E-02	4.976E-02	4.782E-02	4.537E-02	4.390E-02	21.000
7 0.0	6.736E-02	6.829E-02	6.889E-02	6.454E-02	6.265E-02	6.024E-02	5.553E-02	5.217E-02	5.162E-02	4.537E-02	4.439E-02	18.000
6 0.0	6.454E-02	6.454E-02	6.889E-02	6.548E-02	6.454E-02	6.076E-02	5.744E-02	5.410E-02	5.024E-02	4.684E-02	4.488E-02	15.000
5 0.0	6.218E-02	6.171E-02	6.265E-02	6.218E-02	6.123E-02	5.839E-02	5.505E-02	5.121E-02	4.830E-02	4.488E-02	4.390E-02	12.000
4 0.0	5.886E-02	5.744E-02	5.886E-02	5.981E-02	5.696E-02	5.601E-02	5.266E-02	4.830E-02	4.488E-02	4.340E-02	4.241E-02	9.000
3 0.0	5.791E-02	5.601E-02	5.601E-02	5.791E-02	5.646E-02	5.505E-02	5.314E-02	4.928E-02	4.537E-02	4.291E-02	4.500E-02	6.000
2 0.0	5.505E-02	6.031E-02	5.744E-02	5.744E-02	5.457E-02	5.217E-02	4.830E-02	4.488E-02	4.091E-02	3.890E-02	0.0	3.000
1 0.0	5.744E-02	5.649E-02	5.696E-02	5.505E-02	5.217E-02	4.537E-02	4.635E-02	4.241E-02	3.941E-02	3.687E-02	0.0	0.0
13	14	15	TETA									
11 3.860E-02	3.687E-02	3.687E-02	3.000E-01									
10 4.141E-02	4.091E-02	3.789E-02	2.700E-01									
9 4.340E-02	4.191E-02	4.191E-02	2.400E-01									
8 4.291E-02	4.340E-02	0.0	2.100E-01									
7 4.390E-02	4.390E-02	0.0	1.800E-01									
6 4.390E-02	4.488E-02	0.0	1.500E-01									
5 4.340E-02	0.0	0.0	1.200E-01									
4 0.0	0.0	0.0	9.000E-02									
3 0.0	0.0	0.0	6.000E-02									
2 0.0	0.0	0.0	3.000E-02									
1 0.0	0.0	0.0	0.0									

REYNOLDS NUMBER= 64005.
 LENGTH/HYDRAULIC DIAMETER= 44

RENDOS NUMBER= 60125
LENGTH/HYDRAULIC DIAMETER= 15

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00	20.00	21.00	22.00	23.00	24.00	25.00	26.00	27.00	28.00	29.00	30.00	31.00	32.00	33.00	34.00	35.00	36.00	37.00	38.00	39.00	40.00	41.00	42.00	43.00	44.00	45.00	46.00	47.00	48.00	49.00	50.00	51.00	52.00	53.00	54.00	55.00	56.00	57.00	58.00	59.00	60.00	61.00	62.00	63.00	64.00	65.00	66.00	67.00	68.00	69.00	70.00	71.00	72.00	73.00	74.00	75.00	76.00	77.00	78.00	79.00	80.00	81.00	82.00	83.00	84.00	85.00	86.00	87.00	88.00	89.00	90.00	91.00	92.00	93.00	94.00	95.00	96.00	97.00	98.00	99.00	100.00

1	2	3	4	5	6	7	8	9	10	11	12	13
6 0.0	5.505E-02	5.024E-02	5.024E-02	4.830E-02	4.635E-02	4.439E-02	4.340E-02	4.091E-02	4.782E-02	3.687E-02	3.378E-02	30.000
5 0.0	6.454E-02	6.359E-02	6.123E-02	5.781E-02	5.744E-02	5.505E-02	5.266E-02	5.121E-02	5.024E-02	4.537E-02	4.340E-02	24.000
4 0.0	6.454E-02	6.454E-02	6.218E-02	6.265E-02	5.981E-02	5.505E-02	5.410E-02	5.024E-02	4.830E-02	4.439E-02	4.231E-02	16.000
3 0.0	5.981E-02	5.791E-02	5.791E-02	5.649E-02	5.362E-02	5.121E-02	4.782E-02	4.439E-02	4.041E-02	3.789E-02	3.585E-02	12.000
2 0.0	5.744E-02	5.505E-02	5.266E-02	5.314E-02	5.217E-02	5.024E-02	4.733E-02	4.241E-02	3.789E-02	3.482E-02	3.273E-02	6.000
1 0.0	5.601E-02	5.505E-02	5.457E-02	5.217E-02	5.024E-02	4.937E-02	4.291E-02	3.789E-02	3.533E-02	3.273E-02	3.0	3.0
10 13	14	15	TETA									
6 3.482E-02	3.167E-02	3.061E-02	3.000E 01									
5 4.141E-02	3.789E-02	3.738E-02	2.460E 01									
4 3.890E-02	3.789E-02	0.0	1.800E 01									
3 3.378E-02	0.0	0.0	1.200E 01									
2 0.0	0.0	0.0	6.000E 00									
1 0.0	0.0	0.0	0.0									

REYNOLDS NUMBER= 66125.
 LENGTH/HYDRAULIC DIAMETER= 19

1	2	3	4	5	6	7	8	9	10	11	12	TETA
0.0	5.305E-02	5.024E-02	5.024E-02	4.830E-02	4.635E-02	4.439E-02	4.340E-02	4.091E-02	4.782E-02	3.687E-02	3.378E-02	30.003
0.0	6.454E-02	6.359E-02	6.123E-02	5.981E-02	5.744E-02	5.505E-02	5.266E-02	5.121E-02	5.024E-02	4.537E-02	4.340E-02	24.003
0.0	6.454E-02	6.454E-02	6.218E-02	6.263E-02	5.981E-02	5.505E-02	5.410E-02	5.024E-02	4.830E-02	4.439E-02	4.291E-02	18.531
0.0	5.981E-02	5.791E-02	5.791E-02	5.649E-02	5.362E-02	5.121E-02	4.782E-02	4.439E-02	4.041E-02	3.789E-02	3.585E-02	12.003
0.0	5.744E-02	5.505E-02	5.266E-02	5.114E-02	5.217E-02	5.024E-02	4.733E-02	4.241E-02	3.789E-02	3.482E-02	3.273E-02	6.000
0.0	5.024E-02	5.505E-02	5.457E-02	5.217E-02	5.024E-02	4.537E-02	4.291E-02	3.789E-02	3.533E-02	3.273E-02	3.0	2.53
13	14	15	TETA									
3.482E-02	3.187E-02	3.081E-02	3.006E-01									
4.141E-02	3.789E-02	3.738E-02	2.460E-01									
3.890E-02	3.789E-02	0.0	1.800E-01									
3.378E-02	0.0	0.0	1.200E-01									
0.0	0.0	0.0	6.000E-00									
0.0	0.0	0.0	0.0									

REYNOLDS NUMBER= 60125.
 LENGTH/HYDRAULIC DIAMETER= 15

				AXIAL VELOCITY/MAX VELOCITY									
I=	1	2	3	4	5	6	7	8	9	10	11	12	TETA
J													
11	0.0	9.186E-01	9.351E-01	9.458E-01	9.515E-01	9.541E-01	9.511E-01	9.479E-01	9.444E-01	9.404E-01	9.353E-01	9.276E-01	30.000
10	0.0	8.797E-01	8.939E-01	9.104E-01	9.222E-01	9.363E-01	9.434E-01	9.520E-01	9.599E-01	9.642E-01	9.741E-01	9.788E-01	27.000
9	0.0	8.561E-01	8.726E-01	8.939E-01	9.186E-01	9.127E-01	9.257E-01	9.316E-01	9.387E-01	9.514E-01	9.571E-01	9.592E-01	24.000
8	0.0	7.795E-01	8.278E-01	8.526E-01	8.615E-01	8.833E-01	8.962E-01	9.080E-01	9.127E-01	9.245E-01	9.340E-01	9.328E-01	21.000
7	0.0	7.264E-01	7.818E-01	8.137E-01	8.349E-01	8.467E-01	8.703E-01	8.774E-01	8.877E-01	8.986E-01	9.080E-01	9.127E-01	18.000
6	0.0	6.840E-01	7.571E-01	7.925E-01	8.180E-01	8.337E-01	8.476E-01	8.573E-01	8.703E-01	8.797E-01	8.844E-01	8.892E-01	15.000
5	0.0	5.495E-01	6.863E-01	7.500E-01	7.795E-01	8.014E-01	8.233E-01	8.335E-01	8.455E-01	8.538E-01	8.618E-01	8.660E-01	12.000
4	0.0	5.873E-01	6.934E-01	7.429E-01	7.783E-01	7.958E-01	8.090E-01	8.208E-01	8.318E-01	8.389E-01	8.420E-01	8.439E-01	9.000
3	0.0	4.788E-01	6.321E-01	7.075E-01	7.500E-01	7.736E-01	7.906E-01	8.050E-01	8.177E-01	8.274E-01	8.243E-01	8.278E-01	6.000
2	0.0	6.014E-01	6.840E-01	7.323E-01	7.623E-01	7.863E-01	7.972E-01	8.054E-01	8.125E-01	8.165E-01	8.196E-01	0.0	3.000
1	0.0	6.014E-01	6.816E-01	7.311E-01	7.642E-01	7.858E-01	7.948E-01	8.019E-01	8.113E-01	8.151E-01	8.137E-01	0.0	0.0
I=	13	14	15	TETA									
J													
11	9.991E-01	1.001E 00	1.000E 00	3.000E 01									
10	9.835E-01	9.858E-01	9.859E-01	2.700E 01									
9	9.658E-01	9.670E-01	0.0	2.400E 01									
8	9.387E-01	9.399E-01	0.0	2.100E 01									
7	9.175E-01	9.151E-01	0.0	1.800E 01									
6	8.915E-01	0.0	0.0	1.500E 01									
5	8.679E-01	0.0	0.0	1.200E 01									
4	0.0	0.0	0.0	9.000E 00									
3	0.0	0.0	0.0	6.000E 00									
2	0.0	0.0	0.0	3.000E 00									
1	0.0	0.0	0.0	0.0									

REYNOLDS NUMBER= 8924.
 LENGTH/HYDRAULIC DIAMETER= 113

MEASURED TURBULENT QUANTITIES

θ	$y/\hat{y}$	$v_r'/u_o (\times 10^2)$	$v_\theta'/u_o (\times 10^2)$	$k/u_o^2 (\times 10^3)$	$-\frac{\overline{v_r'v_\theta'}}{u_o^2} (\times 10^4)$
0	0.055				
	0.150				
	0.244	3.66	4.04	7.23	
	0.339	4.12	5.08	6.02	
	0.433	4.04	4.14	4.56	9.21
	0.622	3.70	4.43	3.30	10.79
	0.811	3.66	3.84	2.50	4.12
	1.000	3.53	3.74	2.30	0.95
6	0.095				
	0.186				
	0.276	3.40	4.13	6.68	
	0.367	3.92	4.33	3.37	11.70
	0.457	4.04	4.23	4.60	8.76
	0.638	3.66	4.23	3.17	5.82
	0.819	3.71	4.04	2.71	2.99
	1.000	3.72	4.23	2.62	0.63
12	0.197				
	0.277				
	0.357	4.21	5.04	5.83	
	0.438	4.48	5.56	5.62	13.61
	0.518	4.42	5.36	5.09	9.11
	0.679	3.68	4.12	3.17	6.19
	0.839	4.03	3.75	2.80	4.16
	1.000	3.96	4.94	3.22	1.41
18	0.322	4.86	5.37	6.72	
	0.390	4.46	4.85	5.25	
	0.458	4.91	4.82	5.26	
	0.526	4.86	5.37	4.99	10.73
	0.593	4.69	4.87	4.33	8.71
	0.729	4.37	5.01	3.82	5.67
	0.864	4.48	5.02	3.62	3.34
	1.000	4.26	4.58	3.21	1.86
24	0.443	4.86			
	0.499	4.47	5.35	5.05	
	0.554	4.91	4.83	4.57	
	0.610	4.86	5.48	4.62	4.13
	0.666	4.69	5.16	4.22	8.22
	0.777	4.37	4.91	3.72	4.83
	0.899	4.48	4.12	3.03	2.966
	1.000	4.25	4.08	2.95	2.039

RE = 9,000

L/DH = 113

MEASURED TURBULENT QUANTITIES

θ	y/y^*	$v_r'/u_o (\times 10^2)$	$v_\theta'/u_o (\times 10^2)$	$k/u_o^2 (\times 10^3)$	$-\frac{v_r'v_\theta'}{u_o^2} (\times 10^4)$
0	0.055				
	0.150	4.27			
	0.244	4.45			
	0.339	4.60	4.06	4.28	8.07
	0.433	4.32	3.67	3.76	7.94
	0.622	3.93	4.06	3.16	4.78
	0.811	3.57	3.67	2.30	2.69
	1.000	3.41	3.41	1.92	0.83
6	0.095				
	0.186	4.42			
	0.276	4.47			
	0.367	4.59	3.24	4.77	6.4
	0.457	4.31	3.55	3.73	3.1
	0.638	3.93	3.93	3.32	2.61
	0.819	3.75	3.67	2.56	0.93
	1.000	3.79	3.02	2.07	
12	0.197	4.91	3.86	5.43	
	0.277	4.74	3.77	4.66	
	0.357	4.65	3.47	4.48	
	0.438	4.60	3.47	4.12	8.68
	0.518	4.42	3.77	3.92	7.48
	0.679	4.30	3.95	3.49	4.41
	0.839	4.08	3.89	2.84	3.14
	1.000	3.89	3.05	2.22	1.44
18	0.322	4.78	4.37	5.08	
	0.390	4.76	3.92	4.60	
	0.458	4.57	3.89	4.34	
	0.526	4.62	3.89	4.12	7.84
	0.593	4.36	3.63	3.83	7.06
	0.729	4.83	4.14	3.59	4.38
	0.864	4.15	3.80	2.78	3.56
	1.000	3.86	3.22	2.30	1.42
24	0.443	4.63	3.98	4.36	
	0.499	4.62	3.78	4.04	
	0.554	4.36	3.62	3.70	
	0.610	4.49	4.15	3.74	7.214
	0.666	4.45	4.04	3.50	6.680
	0.777	4.54	4.06	3.16	4.638
	0.899	4.05	3.64	2.47	3.008
	1.000	3.83	3.39	2.21	1.429

Re = 26,500

L/DH = 113

MEASURED TURBULENT QUANTITIES

θ	$y/\hat{y}$	$v'_r/u_o (\times 10^2)$	$v'_\theta/u_o (\times 10^2)$	$k/u_o^2 (\times 10^3)$	$-\frac{\overline{v'_r v'_\theta}}{u_o^2} (\times 10^4)$
0	0.055				
	0.150	4.76	4.75	5.7	
	0.244	4.08	4.65	4.5	
	0.339	3.54	4.07	3.56	4.41
	0.433	3.59	3.46	3.15	4.52
	0.622	3.46	3.59	2.6	4.04
	0.811	2.92	3.33	1.88	1.61
	1.000	2.62	3.59	1.71	-0.03
6	0.095				
	0.186	4.54	4.76	5.02	
	0.276	3.84	4.19	3.95	
	0.367	3.73	4.96	4.10	3.3
	0.457	3.59	3.59	3.21	3.42
	0.638	3.07	3.95	2.85	2.55
	0.819	2.79	4.41	2.42	1.36
	1.000	3.22	4.85	2.50	-0.027
12	0.197				
	0.277	4.77	5.17	5.15	
	0.357	4.21	4.42	4.32	
	0.438	4.23	4.84	4.33	3.67
	0.518	4.20	4.31	3.85	3.34
	0.679	3.23	3.83	2.85	3.86
	0.839	3.17	4.04	2.50	2.25
	1.000	3.61	3.95	2.34	0.64
18	0.322				
	0.390	4.59	5.03	4.74	
	0.458	4.43	4.53	4.26	
	0.526	4.38	4.92	4.21	2.86
	0.593	4.35	4.72	3.86	3.08
	0.729	3.51	4.74	3.13	3.43
	0.864	3.12	4.51	2.63	2.91
	1.000	3.21	4.31	2.40	0.96
24	0.443				
	0.499	4.71	5.35	4.63	
	0.554	4.43	4.87	4.02	
	0.610	4.54	4.54	3.77	1.96
	0.666	4.30	5.11	3.66	2.46
	0.777	3.38	4.26	2.65	3.43
	0.899	3.27	4.22	2.33	2.57
	1.000	3.23	3.83	2.10	0.98

RE = 26,500

L/DH = 46

MEASURED TURBULENT QUANTITIES

θ	$y/\bar{y}$	$v'_r/u_o (\times 10^2)$	$v'_\theta/u_o (\times 10^2)$	$k/u_o^2 (\times 10^3)$	$-\frac{v'_r v'_\theta}{u_o^2} (\times 10^4)$
0	0.055				
	0.150	4.97		6.18	
	0.244	4.65	5.70	4.95	
	0.339	4.87	5.70	4.52	7.32
	0.433	4.76	5.50	4.28	5.07
	0.622	4.87	5.40	3.77	3.67
	0.811	4.54	5.08	3.06	1.55
	1.000	4.42	4.87	2.74	-0.03
6	0.095				
	0.186	5.38			
	0.276	4.67	5.18	4.31	
	0.367	4.97	4.87	4.16	5.65
	0.457	4.87	5.18	4.14	5.14
	0.638	4.76	5.08	3.53	4.87
	0.819	5.08	5.08	3.34	1.43
	1.000	4.87	5.18	3.14	0.17
12	0.197	5.47			
	0.277	5.27	4.88	4.67	
	0.357	4.98	5.18	4.44	
	0.438	5.37	4.99	4.43	6.94
	0.518	5.16	5.67	3.92	4.34
	0.679	4.67	5.06	3.32	2.5
	0.839	5.07	4.98	3.22	2.00
	1.000	5.18	5.19	3.36	0.41
18	0.322	5.42			
	0.390	5.22	6.28	5.26	
	0.458	5.11	5.36	4.45	
	0.526	5.50	5.50	4.52	4.56
	0.593	6.16	5.84	5.99	2.88
	0.729	4.94	5.53	3.70	1.69
	0.864	5.04	5.54	3.43	1.25
	1.000	5.16	5.27	3.37	0.13
24	0.443	5.11			
	0.499	5.31	5.79	4.79	
	0.554	4.89	5.38	4.21	
	0.610	5.22	5.36	4.23	2.06
	0.666	5.31	5.38	4.18	1.38
	0.777	4.81	5.03	3.48	0.71
	0.899	4.63	4.55	3.00	-0.27
	1.000	5.04	4.90	3.19	-0.97

RE = 26,500

L/DH = 15

MEASURED TURBULENT QUANTITIES

θ	y/δ	$v'_r/u_o (\times 10^2)$	$v'_\theta/u_o (\times 10^2)$	$k/u_o^2 (\times 10^3)$	$-\frac{\overline{v'_r v'_\theta}}{u_o^2} (\times 10^4)$
0	0.055				
	0.150	3.34			
	0.244	3.59			
	0.339	3.85	2.98	2.80	7.26
	0.433	3.49	3.23	2.67	7.20
	0.622	3.39	2.87	2.24	3.82
	0.811	3.02	2.92	1.78	1.84
	1.000	2.92	2.70	1.50	0.12
6	0.095				
	0.186	3.42			
	0.276	3.42			
	0.367	3.69	3.29	3.12	6.04
	0.457	3.39	3.39	2.83	5.67
	0.638	3.38	3.08	2.53	2.65
	0.819	2.92	2.76	1.88	1.95
	1.000	2.97	3.54	1.60	-0.025
12	0.197	3.16			
	0.277	3.37			
	0.357	3.27			
	0.438	3.67	3.16	3.36	7.68
	0.518	3.43	3.15	3.12	6.68
	0.679	3.47	3.10	2.70	3.06
	0.839	3.07	2.87	2.07	2.69
	1.000	3.06	2.72	1.85	0.298
18	0.322	3.32			
	0.390	3.41	2.67	3.43	
	0.458	3.45	3.03	3.40	
	0.526	3.54	3.04	3.27	7.20
	0.593	3.37	3.16	2.96	5.95
	0.729	3.51	3.17	2.63	3.33
	0.864	3.145	2.80	2.07	2.37
	1.000	2.96	2.83	1.87	0.66
24	0.443	3.33			
	0.499	3.47	2.87	3.35	
	0.554	3.42	3.11	3.16	
	0.610	3.45	3.03	2.86	6.21
	0.666	3.29	2.87	2.60	5.41
	0.777	3.37	3.06	2.24	3.08
	0.899	3.17	2.84	1.94	2.77
	1.000	3.07	2.56	1.80	0.672

RE = 65,000

L/DH = 113

MEASURED TURBULENT QUANTITIES

284

θ	y/δ	$v'_{\tau}/u_o (\times 10^2)$	$v'_{\theta}/u_o (\times 10^2)$	$k/u_o^2 (\times 10^3)$	$-\frac{\overline{v'v'}}{u_o^2} (\times 10^4)$
0	0.055				
	0.150	3.84			
	0.244	4.11			
	0.339	3.79	4.11	3.21	4.37
	0.433	4.32	4.06	3.29	3.98
	0.622	3.95	3.4	2.64	2.61
	0.811	3.23	3.57	2.07	0.80
	1.000	3.23	3.51	1.83	-0.312
6	0.095				
	0.186	3.87			
	0.276	4.00	3.79	3.11	
	0.367	4.06	4.32	3.46	4.06
	0.457	4.21	3.95	3.32	3.91
	0.638	4.10	3.74	3.08	2.70
	0.819	3.52	3.78	2.76	1.13
	1.000	3.51	3.73	3.26	-0.07
12	0.197				
	0.277	4.12			
	0.357	4.32	4.17	3.76	
	0.438	4.20	3.81	3.51	4.02
	0.518	4.00	3.82	3.43	3.63
	0.679	4.04	3.75	2.85	3.70
	0.839	3.79	3.79	2.46	2.66
	1.000	3.53	3.88	2.33	1.16
18	0.322				
	0.390	4.27	4.27	3.94	
	0.458	4.30	4.08	3.75	
	0.526	4.14	3.92	3.47	4.24
	0.593	4.37	4.10	3.39	3.61
	0.729	3.78	3.69	2.6	3.30
	0.864	4.03	3.82	2.54	3.07
	1.000	3.51	3.52	2.21	1.38
24	0.443				
	0.499	4.27	3.84	3.62	
	0.554	4.30	4.11	3.47	
	0.610	4.14	3.79	3.02	4.24
	0.666	4.44	4.10	3.11	3.62
	0.777	3.78	3.88	2.62	3.29
	0.899	4.03	3.72	2.39	3.075
	1.000	3.52	3.73	2.20	1.375

Re = 65,000

L/DH = 46

MEASURED TURBULENT QUANTITIES

θ	$y/\bar{y}$	$v'_r/u_o (x10^2)$	$v'_\theta/u_o (x10^2)$	$k/u_o^2 (x10^3)$	$-\frac{v'_r v'_\theta}{u_o^2} (x10^4)$
0	0.055				
	0.150	4.42	4.63	3.64	
	0.244	4.16	4.58	3.45	
	0.339	4.22	4.84	3.57	5.65
	0.433	4.05	4.42	3.18	6.67
	0.622	3.89	4.63	2.87	4.23
	0.811	3.78	3.84	2.21	1.33
	1.000	3.50	3.57	1.80	-0.025
6	0.095				
	0.186	4.16			
	0.276	4.06	4.58	3.28	
	0.367	3.91	4.57	3.24	4.26
	0.457	3.85	4.52	3.14	4.71
	0.638	3.52	4.31	2.68	3.43
	0.819	3.41	4.05	2.13	1.6
	1.000	3.57	3.78	1.92	0.23
12	0.197				
	0.277	4.17	4.56	3.66	
	0.357	4.06	4.50	3.36	
	0.438	3.90	3.90	2.97	4.08
	0.518	4.04	3.80	2.87	4.05
	0.679	3.70	4.19	2.56	2.96
	0.839	3.61	4.44	2.37	1.76
	1.000	3.63	3.78	1.95	0.636
18	0.322				
	0.390	3.87	4.53	3.73	
	0.458	4.06	4.82	3.66	
	0.526	4.65	4.87	3.78	3.0
	0.593	3.58	4.48	3.13	8.51
	0.729	3.61	4.35	2.78	2.31
	0.864	3.50	4.29	2.47	0.96
	1.000	3.55	4.24	2.26	-0.148
24	0.443				
	0.449	3.83			
	0.554	3.80	4.20	3.15	
	0.610	3.74	4.77	2.82	1.83
	0.666	3.84	3.06	2.89	1.84
	0.777	3.42	4.94	2.41	1.08
	0.899	3.30	3.78	2.13	-0.32
	1.000	3.19	3.67	1.90	-1.02

RE = 65,000

L/DH = 15

APPENDIX D

FLOW SPLIT-EQUAL PRESSURE DROP TEST

Let us take the well known formula of pressure drop

$$\Delta p = f \frac{L}{D_H} \frac{\rho u_b^2}{2}$$

Assuming equal pressure drops for all the subchannels and applying above relation for the central characteristic triangle and the total test section we find finally

$$\frac{f}{f^T} \frac{u_b^2}{(u_b^T)^2} = \frac{D_H^T}{D_H}$$

where D_H^T is the test section hydraulic diameter and f^T is the test section friction factor.

Assuming

$$f = Re^{-m}, f^T = (Re^T)^{-m}$$

We end up with the relation

$$\frac{u_b}{u_b^T} = \left(\frac{D_H^T}{D_H} \right)^{\frac{1+m}{2-m}}$$

From Table 2.2 $D_H = 20.12$ mm and $D_H^T = 16.375$ mm.

For $m=0.20$

$$\frac{u_b}{u_b^T} = 1.147$$

For $m=0.25$

$$\frac{u_b}{u_b^T} = 1.158$$

APPENDIX E

COMPUTER PROGRAMS

In order to solve the finite difference equations for heat and flow two computer codes were written.

The main symbols used in the above codes are explained in Table E1.

The computer program structure for flow calculation is shown in Tables D2 and D3.

The structure of the computer programs for heat transfer calculations is shown in Table D4.

Listings of both computer codes are given at the end of this appendix.

TABLE E1
SYMBOLS FOR COMPUTER CODES

<u>Symbol</u>	<u>Explanation</u>
ALHGR	Linear heat generation rate (q')
CK	Von karman constant
CMU	Constant c_μ
CA1	Constant c_1
CA2	Constant c_2
C1	Constant $c_{\epsilon 1}$
C2	Constant $c_{\epsilon 2}$
COND C	Coolant heat conductivity (λ_c)
CONCL	Clad heat conductivity (λ_{cl})
GSH	Coolant specific heat (C_p)
DENSIT	Coolant density (ρ)
DI	Clad Inner diameter (D_i)
DIA	Rod Diameter (D)
DH	Hydraulic Diameter (D_H)
ED	Energy dissipation
IN	No. of radial points in the clad
IMAX	Number of radial points in the coolant
IN	Total number of radial points in the coolant
(IMON, JMON)	Point for detailed print out (per iteration)
(IREF, JREF)	Point where pressure is preset to zero
JN	No. of tangential points
MAXIT	Maximum number of iterations
PIT	Pitch
PTR	Pitch ratio

PSI	$1/\sigma_T$
RE	Reynolds number
TE	Turbulent kinetic energy (k)
TWAVE, TALAV	Average shear stress
TT	Temperature
U	Axial velocity v_z
USTAR	Friction velocity u^*
URF____	Underrelaxation factor (UREL)
V1	Radial velocity (v_r)
V2	Tangential velocity (v_θ)
VB, VAVG	Bulk velocity
X1	Radial spacing
X2	Tangential spacing
YB	y_B^+

TABLE E2

COMPUTER PROGRAM STRUCTURE FOR FLOW CALCULATIONS

MAIN PROGRAM	SUBROUTINES	EXPLANATION
INPUT DATA	SHEAR	Define y_B
	GRID	Establish grid
GRID	INIT	Initial conditions
	KEYNU	Calculate average velocity
INITIAL CONDITIONS	TURBK	Bobkov's normal stresses and turbulent kinetic energy
	MIXLEN	Mixing Length
	GBEOM	Calculate various geom-factors
ITERATION LOOP	SECIN	Secondary flow according to Nijasing & Eifler
	PROPS	Viscosity calculations
	CALCV1	Coefficients for v_r equations
PRINT OUTPUT	CALCV2	Coefficients for v_θ equations
	CALCP	Coefficients for p' equation and p calculations
	CALCU	Coefficient for v_z equation
STRESS CALCULATION	CALCTE	Coefficients for k equation and Launder and Ying normal stresses
	CALCED	Coefficients for ϵ equation
NORMALIZATION	SOLV1	Line iteration scheme
	SOLV2	Successive relaxation
	PRINT	Print
	ANORM	Normalize

TABLE E3

ITERATION LOOP FOR FLOW CALCULATIONS

TABLE E4

COMPUTER PROGRAM STRUCTURE FOR THERMAL CALCULATIONS

C	COMPUTER PROGRAM FOR FLOW CALCULATIONS	0001
C		0002
C	I N S T R U C T I O N S	0003
C		0004
C	IPILE=0 REYNOLDS NO INPUT	0005
C	IPILE=1 AVG WALL SHEAR STRESS INPOT	0006
C		0007
C	IPIROB.EQ.1: LAMINAR FLOW	0008
C	IPIROB.GT.1: TURBULENT FLOW	0009
C	IPIROB.EQ.3: 0-EQUATION MODEL	0010
C	IPIROB.EQ.4: 1-EQUATION MODEL	0011
C	IPIROB.EQ.5: 2-EQUATION MODEL	0012
C		0013
C	ISEC=0; NO SECONDARY FLOW	0014
C	ISEC=1; NIJSING-EIFLER SECONDARY FLOW	0015
C	ISEC=2; SECONDARY FLOW	0016
C	*****	0017
C		0018
C	ITURB=1 NORMAL STRESSES FROM BOBKOV MODEL	0019
C	ITURB=2 NORMAL STRESSES FROM LAUNDER YING MODEL	0020
C		0021
C	IPR=1 ISOTROPIC VISCOCITY	0022
C	IPR=2 NON-ISOTROPIC VISCOSITY	0023
C		0024
C	ILEN=1 CARAJILESCOV RADIAL LENGTH	0025
C	ILEN=2 PRESENT RADIAL LENGTH	0026
C		0027
C	ICONV=0 UDOS SCHEME	0028
C	ICONV=1 UDS SCHEME	0029
C		0030
	IMPLICIT REAL*8 (A-B,O-Z), INTEGER (I-N)	0031
	DIMENSION V1P(30,16),V2P(30,16),V3P(30,16),T23(30,16),T13(30,16)	0032
	COMMON/HD/HEDU(9),HEDV1(9),HEDV2(9),HEDP(9),HEDK(9),HEDD(9),	0033
	1 HEDN(9),HEDL(9),HEDM(9),HEDPR(9),HEDV1P(9),HEDV2P(9),HEDV3P(9),	0034
	2 HEDT13(9),HEDT23(9)	0035
	COMMON/ALN/CL,RL,IPR,ILEN	0036

COMMON/BB/JS	0037
COMMON/BBB/NVIS	0038
COMMON/ATURB/DVS(30,16),CA,CA1,CA2,CB,ITURB	0039
COMMON/GEOMC/RM(30),ARE(30),ARN(30),ARN(30),ARP(30),DX1(30),	0040
/ ANG(16),ANGM(16),VBP(16)	0041
COMMON/GEOMD/DV1E(30),DV1W(30),DV1N(30),CV1E(30),CV1W(30)	0042
COMMON/GEOME/DV2E(30),DV2W(30),DV2N(30),CV2E(30),CV2W(30),ABPN(30)	0043
COMMON	0044
1/UVEL/URFU,RESU,NSWPU	0045
1/V1V1L/URFV1,RESV1,NSWPV1	0046
1/V2VEL/URFV2,RESV2,NSWPV2	0047
1/PCOR/RESORM,URFP,RFSP,NSWPP,IPREP,JPREP	0048
1/TEN/URFK,PESK,NSWPK	0049
1/EDIS/URFE,RFSE,NSWPD	0050
1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),	0051
2 ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)	0052
1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),	0053
2 ISEC,IPILE,JJL,ICONV,NSOLU,INUH,JL,JLP	0054
1/AGRID/R(30),X1(30),X2(16)	0055
1/STRESS/T11(30,16),T22(30,16)	0056
1/GEOM/BE(30),BN(30),BW(30),BS(30),VP(30),VP1(30)	0057
1/GEOMA/DPE(30,16),DPW(30,16),DPN(30,16),DPS(30,16)	0058
1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB	0059
1/TURB/CD,CMU,C1,C2,CK,E,CVIS,CDIS,PRTE,PRED	0060
1/WALLF/TALAV,USAV,RTAL(16),USTAR(16),PB,YB,TWAVE	0061
1/COEF/AP(30,16),AN(30,16),AS(30,16),AR(30,16),AW(30,16),SU(30,16)	0062
EQUIVALENCE (T11(1),V1P(1)),(T22(1),V2P(1)),(DPE(1),V3P(1)),	0063
1 (DPW(1),T13(1)),(DPN(1),T23(1))	0064
LOGICAL INCALU,INCALL,INCALP,INPRO,INCALK,INCALD,INCALH,	0065
1 INCLV1,INCLV2,ICLT11,ICLT22,ICLT13,ICLT23,ICLT33	0066
C	0067
C*** PROBLEM DEFINITION	0068
READ(5,700) IPILE,IPROB,ISEC,ILEN,IPR,ITURB	0069
C***GRID	0070
READ(5,700) IN,JN,IT,JT	0071
READ(5,701) YB	0072

C***	NUMERICAL PROCEDURE	0073
	READ(5,700) NSOLU, ITR, NSWPV1, NSWPV2, NSWPP, NSWPO, NSWPK, NSWPD	0074
C***	PROBLEM DATA	0075
	IF(IPILE.EQ.0) READ(5,701) PIT,DIA,VISCOS,RE	0076
	IF(IPILE.EQ.1) READ(5,701) PIT,DIA,VISCOS,TWAVE	0077
C***	NO ITERATIONS AND PRINT-OUT INFORMATION	0078
	READ(5,700) MAXIT,INDPRI,ISTRES,INORM	0079
C***	TITLES	0080
	READ(5,010) HFDU, HEDV1, HEDV2, HEDP, HEDK, HEDD, HEDL,	0081
	1 HEDN, HEDPR, HEDV1P, HEDV2P, HEDV3P, HEDT23, HEDT13	0082
	IF(IPROB.EQ.1) ISEC=0	0083
	IF(IPROB.EQ.1) ISTRES=0	0084
	IF(IPROB.EQ.1) YB=0.0	0085
	IF(IPROB.EQ.2) IPR=3	0086
	IF(IPROB.EQ.3) ITURE=1	0087
C***	PRINT INPUT DATA	0088
	WRITE(6,700) IPILE,IPROB,ISEC,ILEN,IPR,ITURB	0089
	WRITE(6,700) IN,JN,IT,JT	0090
	WRITE(6,701) YB	0091
	WRITE(6,700) NSOLU, ITR, NSWPV1, NSWPV2, NSWPP, NSWPO, NSWPK, NSWPD	0092
	IF(IPILE.EQ.0) WRITE(6,701) PIT,DIA,VISCOS,RE	0093
	IF(IPILE.EQ.1) WRITE(6,701) PIT,DIA,VISCOS,TWAVE	0094
	WRITE(6,700) MAXIT,INDPRI,ISTRES,INORM	0095
	WRITE(6,010) HEDU, HFDV1, HEDV2, HEDP, HEDK, HEDD, HEDL,	0096
	1 HEDN, HEDPR, HEDV1P, HEDV2P, HEDV3P, HEDT23, HEDT13	0097
	PITR=PIT/DIA	0098
C***	INITIAL OPERATIONS	0099
	INUM=0	0100
	IVIS=1	0101
	IF(IPROB.EQ.2) IVIS=2	0102
	ICONV=0	0103
	JJL=1	0104
	RAT=1.0	0105
	NVIS=2	0106
	INOR=1	0107
	NITER=0	0108

CDIS=0.38	0145
IF(IPROB.EQ.4) CMU=CVIS*CDIS	0146
PRSD=CK*CK / (C2-C1) / (CMU**.5)	0147
CA=2.*(2.-8.*CA2)/11./CA1	0148
CB=2.*(9.*CA2+6.)/CA1/33.	0149
IMON=5	0150
JMON=5	0151
IREF=2	0152
JREF=2	0153
C	0154
C	0155
C*** HYDRAULIC PARAMETERS OF THE FLOW	0156
C*** COMPUTE CHANNEL AREA	0157
ATOT=PIT*PIT/8./DSQRT(3.000)-DIA*DIA*3.14159/48.	0158
C*** WETTED PERIMETER	0159
PW=DIA*3.14159/12.	0160
C*** HYDRAULIC DIAMETER	0161
DH=4.*ATOT/PW	0162
C*** AVERAGE VELOCITY	0163
IF(IPILE.EQ.0) VAVG=VISCOS*RE/DH	0164
C*** INITIAL VALUES FOR SHEAR STRESS	0165
CALL SHEAR	0166
C-----GRID	0167
CALL GRID(RAT)	0168
C-----DEPENDENT VARIABLE SELECTION	0169
INCALU=.TRUE.	0170
INCLV1=.TRUE.	0171
INCLV2=.TRUE.	0172
INCALK=.TRUE.	0173
INCALD=.TRUE.	0174
INCALL=.TRUE.	0175
INCALP=.TRUE.	0176
INCALM=.TRUE.	0177
INPRO=.TRUE.	0178
ICLT1=.FALSE.	0179
ICLT2=.FALSE.	0180

ICLT33=.FALSE.	0181
ICLT13=.FALSE.	0182
ICLT23=.FALSE.	0183
IF (ISPC.EQ.0) INCLV1=.FALSE.	0184
IF (ISEC.EQ.0) INCLV2=.FALSE.	0185
IF (ISEC.LE.1) INCALP=.FALSE.	0186
IF (IPROB.EQ.1) INCALL=.FALSE.	0187
IF (IPROB.EQ.1) INPRO=.FALSE.	0188
IF (IPROB.LE.4) INCALD=.FALSE.	0189
IF (IPROB.LE.2) INCALK=.FALSE.	0190
IF (ISTRES.EQ.0) GO TO 49	0191
IF (IPROB.EQ.2) GO TO 9	0192
ICLT11=.TRUE.	0193
ICLT22=.TRUE.	0194
ICLT33=.TRUE.	0195
9 CONTINUE	0196
ICLT13=.TRUE.	0197
ICLT23=.TRUE.	0198
49 CONTINUE	0199
C	0200
C*** INITIAL OPERATIONS	0201
C	0202
C-----CALCULATE GEOMETRICAL QUANTITIES AND SET VARIABLES TO ZERO	0203
C*** SET VALUES IN STORE TO ZERO	0204
DO 30 J=1,JT	0205
DO 30 I=1,IT	0206
U (I,J)=0.0	0207
V1 (I,J)=0.0	0208
V2 (I,J)=0.0	0209
P (I,J)=0.0	0210
PP (I,J)=0.0	0211
TE (I,J)=0.0	0212
ED (I,J)=0.0	0213
AL (I,J)=0.0	0214
VIS (I,J)=VISCOS	0215
PRUT (I,J)=1.0	0216

	T11(I,J)=0.0	0217
	T22(I,J)=0.0	0218
	DPR(I,J)=0.0	0219
	DPW(I,J)=0.0	0220
	DPN(I,J)=0.0	0221
	DPS(I,J)=0.0	0222
	DVS(I,J)=0.0	0223
30	CONTINUE	0224
	IF(IPROB.GT.1) CALL INIT	0225
	IF(IPROB.GT.1) GO TO 129	0226
	DO 77 J=1,JN	0227
	IH=IMAX(J)	0228
	DO 77 I=2,IH	0229
77	U(I,J)=VB	0230
129	CONTINUE	0231
	IF(IPROB.GT.1) CALL PROPS(IVIS)	0232
	CALL BGEOM	0233
	IND=0	0234
	IF(IPROB.GT.2) CALL TURBK	0235
C-----	INITIAL OUTPUT	0236
	WRITE(6,210)	0237
	IF(IPILE.EQ.0) WRITE(6,220) VAVG	0238
	IF(IPILE.EQ.0) WRITE(6,230) RE	0239
	IF(IPILE.EQ.1) WRITE(6,270) TWAVE	0240
	WRITE(6,250) VISCOS	0241
	WRITE(6,240) PTR	0242
	IF(INCLV2) CALL PRINT(1,1,V2,HEDV2)	0243
	IF(INCLV1) CALL PRINT(1,1,V1,HEDV1)	0244
	IF(INCALP) CALL PRINT(1,1,P,HEDP)	0245
	IF(INCALU) CALL PRINT(1,1,U,HEDU)	0246
	IF(INCALK) CALL PRINT(1,1,TF,HEDK)	0247
	IF(INCALD) CALL PRINT(1,1,ED,HEDD)	0248
	IF(INCALL) CALL PRINT(1,1,AL,HEDL)	0249
	IF(IPROB.GT.1) CALL PRINT(1,1,VIS,HEDN)	0250
	IF(IPROP.GT.1) CALL PRINT(1,1,PRUT,HEDPR)	0251
	IF(IPILE.EQ.0) WRITE(6,100) PR,TALAV,DH,VAVG,VB	0252

IF(IMPRO) CALL PROPS(IVIS),	0289
C-----INTERMEDIATE OUTPUT	0290
RESU=RESU/UNORM	0291
RESK=RESK/TENORM	0292
RESE=RESE/ENORM	0293
IF(ISEC.GT.0) RESV2=RESV2/V2NORM	0294
IF(ISEC.GT.0) RESV1=RESV1/V2NORM	0295
WRITE(6,311) NITER,RESU,RESV1,RESV2,RESORN,RESK,RESE	0296
1 ,U(IMON,JMON),V1(IMON,JMON),V2(IMON,JMON),	0297
1 P(IMON,JMON),TE(IMON,JMON),ED(IMON,JMON)	0298
IF(MOD(NITER,INDPRI).NE.0) GO TO 301	0299
IF(INCLV2) CALL PRINT(1,1,V2,HEDV2)	0300
IF(INCLV1) CALL PRINT(1,1,V1,HEDV1)	0301
IF(INCALP) CALL PRINT(1,1,P,HEDP)	0302
IF(INCALU) CALL PRINT(1,1,U,HEDU)	0303
IF(INCALK) CALL PRINT(1,1,TF,HEDK)	0304
IF(INCALD) CALL PRINT(1,1,ED,HEDD)	0305
IF(IPILE.EQ.0) WRITE(6,100) PR,TALAV,DH,VAVG,VB	0306
IF(IPILE.EQ.1) WRITE(6,106) PR,VB,TWAVE,TALAV	0307
WRITE(6,200)	0308
DO 33 J=1,JN	0309
XST=X1(IL)*USTAR(J)/VISCOS	0310
WRITE(6,313) J,BTAL(J),USTAR(J),XST	0311
33 CONTINUE	0312
WRITE(6,310) IMON,JMON	0313
301 CONTINUE	0314
C-----TERMINATION TESTS	0315
IF(NITER.EQ.MAXIT) GO TO 302	0316
ERR=DMAX1(RESU,RESK,RESE,RESV2)	0317
IF(ERR.GT.CC) GO TO 300	0318
IF((NITER-N11).LT.ND) GO TO 300	0319
N11=NITER	0320
RSU=(U(IMON,JMON)-U1)/U(IMON,JMON)	0321
U1=U(IMON,JMON)	0322
IF(RSU.LT.1.0E-03) GO TO 300	0323
302 CONTINUE	0324

C		0325
C***	FINAL OUTPUT	0326
C		0327
	IF (INCLV2) CALL PRINT(1,1,V2,HEDV2)	0328
	IF (INCLV1) CALL PRINT(1,1,V1,HEDV1)	0329
	IF (INCLP) CALL PRINT(1,1,P,HEDP)	0330
	IF (INCLU) CALL PRINT(1,1,U,HEDU)	0331
	IF (INCLK) CALL PRINT(1,1,TE,HEDK)	0332
	IF (INCLD) CALL PRINT(1,1,ED,HEDD)	0333
	IF (INCLL) CALL PRINT(1,1,AL,HEDL)	0334
	IF (IPROB.GT.1) CALL PRINT(1,1,VIS,HEDN)	0335
	IF (IPROB.GT.1) CALL PRINT(1,1,PRUT,HEDPR)	0336
	IF (I-STRIP.EQ.0) GO TO 45	0337
	IF (IPROB.GT.2) CALL TORRE	0338
	DO 51 J=1,JN	0339
	IN=IMAX(J)	0340
	DO 51 I=2,IN	0341
	H1=1.0	0342
	H2=1.0	0343
	IF (I.EQ.2) H2=0.0	0344
	IF (I.EQ.IN) GO TO 63	0345
	DU1= ((U(I+1,J)-U(I,J))*H1+(U(I,J)-U(I-1,J))*H2)/((R(I+1)-R(I))*H1	0346
	/ + (R(I)-R(I-1))*H2)	0347
	GO TO 67	0348
63	DU1= (U(I,J)-U(I-1,J))/(R(I)-R(I-1))	0349
67	CONTINUE	0350
	T13(I,J)= +(VIS(I,J)-VISCOS)*DU1	0351
	VDP(I,J)=2.*TE(I,J)-T13(I,J)-T22(I,J)	0352
51	CONTINUE	0353
	DO 61 J=1,JN	0354
	IN=IMAX(J)	0355
	DO 61 I=2,IN	0356
	T23(I,J)=0.0	0357
	IF (J.EQ.1.OR.J.EQ.JN) GO TO 61	0358
	H1=1.0	0359
	H2=1.0	0360

	IF(I.EQ.IH) H2=0.0	0361
	DU2= ((U(I,J+1)-U(I,J))*H1+(U(I,J)-U(I,J-1))*H2)/X2(2)/(H2+H1)	0362
	T23(I,J)=+(VIS(I,J)-VISCOS)/PRUT(I,J)*DU2	0363
61	CONTINUE	0364
C***	RMS VALUES	0365
	DO 52 J=1,JN	0366
	IH=IMAX(J)	0367
	DO 52 I=1L,IH	0368
	V1P(I,J)=DSQRT(T11(I,J))	0369
	V2P(I,J)=DSQRT(T22(I,J))	0370
	V3P(I,J)=DSQRT(V3P(I,J))	0371
52	CONTINUE	0372
	IF(ICLT1) CALL PRINT(2,1,V1P,HEDV1P)	0373
	IF(ICLT2) CALL PRINT(2,1,V2P,HEDV2P)	0374
	IF(ICLT3) CALL PRINT(2,1,V3P,HEDV3P)	0375
	IF(ICLT13) CALL PRINT(2,1,T13,HEDT13)	0376
	IF(ICLT23) CALL PRINT(2,1,T23,HEDT23)	0377
45	CONTINUE	0378
	IF(IPILE.EQ.0) WRITE(6,100) FR,TALAV,DH,VAVG,VB	0379
	IF(IPILE.EQ.1) WRITE(6,106) FR,VB,TWAVE,TALAV	0380
	IF(IPILE.EQ.1) RE=VB*DH/VISCOS	0381
	WRITE(6,230) RE	0382
	WRITE(6,200)	0383
	DO 13 J=1,JN	0384
	XST=X1(IL)*USTAR(J)/VISCOS	0385
	WRITE(6,313) J,RTAL(J),USTAR(J),XST	0386
13	CONTINUE	0387
	IF(INORM.EQ.0) GO TO 47	0388
C***	NORMALIZE	0389
	WRITE(6,499)	0390
	DO 64 J=1,JN	0391
	IH=IMAX(J)	0392
	DO 64 I=1,IH	0393
	AL(I,J)=AL(I,J)/X1(IH)	0394
64	CONTINUE	0395
	UC=U(IH,JN)	0396

	USAV=TALAV**0.5	0397
	GO TO (71,72,73,74),INORM	0398
71	PV=1./VO	0399
	WRITE(6,504)	0400
	CALL ANORM(1,INORM,ISTRES,ICURVE,PV)	0401
	GO TO 47	0402
72	PV=1./VB	0403
	WRITE(6,500)	0404
	CALL ANORM(1,INORM,ISTRES,ICURVE,PV)	0405
	GO TO 47	0406
73	PV=1./USAV	0407
	WRITE(6,501)	0408
	CALL ANORM(2,INORM,ISTRES,ICURVE,PV)	0409
	GO TO 47	0410
74	PV=1.	0411
	WRITE(6,502)	0412
	CALL ANORM(3,INORM,ISTRES,ICURVE,PV)	0413
47	CONTINUE	0414
48	CONTINUE	0415
	C-----FORMAT STATEMENTS	0416
499	FORMAT(1H1,20X,' *****NORMALIZED MIXING LENGTH *****')	0417
504	FORMAT(1H1,20X,' *****NORMALIZED RESULTS BY MAX VEL *****')	0418
500	FORMAT(1H1,20X,' *****NORMALIZED RESULTS BY VB *****')	0419
501	FORMAT(1H1,20X,' *****NORMALIZED RESULTS BY USAV*****')	0420
502	FORMAT(1H1,20X,' *****NORMALIZED RESULTS BY USTAR(J)*****')	0421
010	FORMAT(9A4)	0422
700	FORMAT(10I5)	0423
701	FORMAT(8D10.3)	0424
210	FORMAT(1H0,47X,51HKASE T1 FULLY DEVELOPED FLOW IN A BARE ROD BUNDL	0425
	1E ,////)	0426
220	FORMAT(//1H0,15X,21H BULK FLUID VELOCITY ,T60,1H=,3X,1PD11.3)	0427
230	FORMAT(1H0,15X,16HREYNOLDS NUMBER ,T60,1H=,3X,1PD11.3)	0428
240	FORMAT(1H0,15X,15HPITCH RATIO ,T60,1H=,3X,1PD11.3)	0429
250	FORMAT(1H0,15X,18HLAMINAR VISCOSITY ,T60,1H=,3X,1PD11.3)	0430
270	FORMAT(//1H0,15X,21H AVE SHEAR STRESS/RO ,T60,1H=,3X,1PD11.3)	0431
310	FORMAT(1H0,4HITER,4X,106HI-----ABSOLUTE RESIDUAL SOURCE SUM	0432

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1S-----I          I-----FIELD VALUES AT MONITORING LOCATION,
2      1R(,I2,1H,,I2,1H),9H-----I/2X,2HNO,6X,4HUNOM,6X,4HV1MO,6X,
34HV2MO,6X,4HMASS,6X,4HTKIN,6X,4HDISP,10X,1HU,9X,2HV1,8X,2HV2,8X,1H
4P,9X,1HK,9X,1HD)
311 FORMAT(1H,I3,4X,1P6D10.3,3X,1P6D10.3)
402 FORMAT(///5X,1H1,7X,5HXU(1),6X,10HS.S.CO2PF.)
403 FORMAT(/5X,I2,3X,2(1PD11.3))
100  FORMAT(31H0FLOW INFORMATION CALCULATED IS//
      110X,50HFR,      FRICTION FACTOR.....=,1PD15.6/
      210X,50HTALAV,   AVERAGE WALL SHEAR STRESS/ROREF.....=,1PD15.6/
      310X,50HDH,      HYDRAULIC DIAMETER.....=,1PD15.6/
      410X,50HVAVG,    BULK VELOCITY.....=,1PD15.6/
      510X,50HVB,      COMPUTED BULK VELOCITY.....=,1PD15.6)
106  FORMAT(31H0FLOW INFORMATION CALCULATED IS//
      110X,50HFR,      FRICTION FACTOR.....=,1PD15.6/
      410X,50HVB,      BULK VELOCITY.....=,1PD15.6/
      210X,50HTWAVE,   AVERAGE WALL SHEAR STRESS/ROREF.....=,1PD15.6/
      510X,50HTALAV,   COMPUTED AVE SHEAR STRESS/ROREF.....=,1PD15.6)
200  FORMAT(1H130X,35HDISTRIBUTION OF WALL SHEAR STRESSES//
      /5X,1HJ,10X,9HTAL/TALAV,10X,5HUSTAR//)
313  FORMAT(4X,I2,8X,D11.4,6X,1PD11.3,6X,1PD11.3/)
      STOP
      END
      SUBROUTINE SHEAR
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)
      COMMON
      1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPOB,IGRID,IMAX(16),IND,JMIN(30),
      2 ISEC,IPILE,JJL,ICONV,NSOLU,INON,JL,JLP
      1/AGRID/R(30),X1(30),X2(16)
      1/WALLP/TALAV,USAV,RTAL(16),USTAR(16),FR,YB,TWAVE
      1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB
C*****
C  SUBROUTINE FOR COMPUTATION OF WALL FRICTION BY IBRAGIMOV METHOD
C*****
C
      IF (IPROB.EQ.1) GO TO 10

```

C*** COMPUTE YAV	0469
YAV=0.5*(3./3.1416*PIT*DLOG(3.D0)-DIA)	0470
C*** COMPUTE FRICTION FACTOR, FR	0471
X=ATOT/YAV**2	0472
XK=X**0.25*PIT*(1./DSQRT(3.D0)-0.5)/YAV	0473
XX=-0.021*XK**3	0474
RFR=(.58+.42*DEXP(XK))*(1.+.1*(2.*YAV/DIA+1.))**(.4/3.)	0475
IF(IPILE.EQ.1) GO TO 2	0476
FCIR=0.046*RE**(-.2)	0477
FR=RFR*FCIR	0478
C*** COMPUTE DISTRIBUTION OF TAL AND USTAR	0479
TALAV=0.5*FR*YAVG**2	0480
TWAVE=TALAV	0481
GO TO 3	0482
2 REE=1./1.8	0483
BAS=TWAVE*(DH**0.2)*(VISCOS**(-0.2))/REE/0.021	0484
YAVG=BAS**REE	0485
RE=YAVG*DH/VISCOS	0486
FCIR=0.046*RE**(-.2)	0487
FR=RFR*FCIR	0488
TALAV=TWAVE	0489
3 CONTINUE	0490
AC=3.85*PIT/(X**0.8*YAV)	0491
CC=(AC/2.))**0.5*3.1416/6.	0492
C=1.-6./3.1416*(3.1416/2./AC)**0.5*DEXP(-AC)*DERF(CC)	0493
C=1./C	0494
B=7.7/(X**0.8*YAV)	0495
CC=3.1416/6.*(0.25*B*PIT)**0.5	0496
CD=0.5*B*(PIT-DIA)	0497
CI=6./(3.14159*B*PIT)**0.5*DEXP(CD)*DERF(CC)	0498
C=1./(1.-CI)	0499
DTETA=3.1416/6./DFLOAT(JNM)	0500
DO 1 J=1,JN	0501
TETA=DTETA*DFLOAT(J-1)	0502
YT=0.5*(PIT/DCOS(TETA)-DIA)	0503
BY=-B*YT	0504

	RTAL(J)=C*(1.-DEXP(BY))	0505
	USTAR(J)=(TALAV*RTAL(J))**0.5	0506
1	CONTINUE	0507
	RETURN	0508
10	CONTINUE	0509
	IF(IPILE.EQ.1) GO TO 30	0510
	PR=16/RE	0511
	TALAV=0.5*PR*VAVG**2	0512
	TWAVE=TALAV	0513
	GO TO 40	0514
30	TALAV=TWAVE	0515
	RE=TWAVE*(DH/VISCOS)**2/8.	0516
	VAVG=RE*VISCOS/DH	0517
	PR=16./RE	0518
40	CONTINUE	0519
	DO 20 J=1,JN	0520
	RTAL(J)=1.0	0521
20	USTAR(J)=DSQRT(TALAV)	0522
	VB=VAVG	0523
	RETURN	0524
	END	0525
	END	0526
	SUBROUTINE GRID(RAT)	0527
	IMPLICIT REAL*8 (A-H,O-Z), INTEGER (I-N)	0528
	COMMON	0529
	1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPOB,IGRID,IMAX(16),IND,JMIN(30),	0530
	2 ISFC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP	0531
	1/AGRID/R(30),X1(30),X2(16)	0532
	1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB	0533
	1/WALLP/TALAV,USAV,RTAL(16),USTAR(16),PR,YB,TWAVE	0534
	C*****	0535
	C SUBROUTINE TO COMPUTE THE GRID COORDINATES	0536
	C*****	0537
	C	0538
	C*** CIRCUMFERENTIAL COORDINATES, X2(J)	0539
	DTETA=3.14159/6./DFLOAT(JNM)	0540

	X2(I) = 0.	0541
	DO 1 J=2, JN	0542
1	X2(J) = X2(J-1) + DTETA	0543
C***	RADIAL COORDINATES, X1(I)	0544
	X1(1) = 0.0	0545
	GAP = 0.5 * (PIT - DIA)	0546
	M = IN - JN + J1L	0547
	NN = M - 1	0548
C***	ESTIMATE WALL SUBLAYER THICKNESS	0549
	XB = YB * VISCOS / DSQRT (TALAV)	0550
	X1(IL) = XB	0551
	X1(M) = GAP	0552
	DX1 = (GAP - XB) / DFLOAT (M - IL)	0553
	IF (X1(IL) > 1.0) DX1 = (GAP - XB) * (RAT - 1.0) / (RAT * (M - IL) - 1.0)	0554
	DO 20 I = IL, M	0555
20	X1(I) = X1(I-1) + DX1 * RAT * (I - IL - 1)	0556
	JJP = J1L + 1	0557
	DO 3 J = JJP, JN	0558
	TETA = X2(J)	0559
3	X1(M + J - J1L) = PIT / 2. / DCOS (TETA) - DIA / 2.	0560
C***	RADIAL DISTANCES, R(I)	0561
	DO 4 I = 1, IN	0562
4	R(I) = 0.5 * DIA + X1(I)	0563
	DO 5 J = 1, JN	0564
	IMAX(J) = M + J - J1L	0565
	IF (J .LE. J1L) IMAX(J) = M	0566
5	CONTINUE	0567
	DO 7 I = 1, M	0568
7	JMIN(I) = 1	0569
	MP = M + 1	0570
	DO 8 I = MP, IN	0571
8	JMIN(I) = JMIN(I-1) + 1	0572
C***	PRINT OUT COORDINATES,	0573
	WRITE (6, 101) (X1(I), I = 1, IN)	0574
	WRITE (6, 102) (X2(J), J = 1, JN)	0575
	WRITE (6, 103) (IMAX(J), J = 1, JN)	0576

	WRITE(6,104) (JMIN(I),I=1,IN)	0577
101	FORMAT(25HODISTANCES IN DIRECTION-1/(1H 4D25.8))	0578
102	FORMAT(25HODISTANCES IN DIRECTION-2/(1H 4D25.8))	0579
103	FORMAT(25H0VALUES OF IMAX / (1H 4I4))	0580
104	FORMAT(25H0VALUES OF JMIN / (1H 4I4))	0581
	RETURN	0582
	END	0583
	SUBROUTINE INIT	0584
	IMPLICIT REAL*8 (A-H,O-Z), INTEGER (I-N)	0585
	COMMON/ALEN/CL,RL,IPR,IEN	0586
	COMMON	0587
	1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),	0588
	2 ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)	0589
	1/ALL/IT,JT,IN,JN,INN,JNN,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),	0590
	2 ISZC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP	0591
	1/AGRID/R(30),X1(30),X2(16)	0592
	1/PLUPR/URPVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB	0593
	1/STRESS/T11(30,16),T22(30,16)	0594
	1/WALLP/TALAV,USAV,RTAL(16),USTAR(16),PR,YB,TWAVE	0595
	1/TURB/CD,CMU,C1,C2,CK,E,CVIS,CDIS,PRTE,PRED	0596
	C*****	0597
	C SUBROUTINE FOR COMPUTATION OF INITIAL CONDITIONS	0598
	C*****	0599
	C	0600
	C*** INITIAL CONDITIONS FOR AXIAL VELOCITY	0601
	U0=14.7	0602
	DO 3 J=1,JN	0603
	IR=IMAX(J)	0604
	BK=0.387*(1.+0.05*R(IR)/R(1))	0605
	TT=DEXP(-1.26*DSQRT(R(IR)/R(1)))*10.	0606
	C=45.*(1.-0.34*45./14.7+0.039*(45./14.7)**2)-DLOG(45.0D0)/BK	0607
	DO 3 I=2,IR	0608
	XS=USTAR(J)*X1(I)/VISCOS	0609
	IF(XS.GT.45.0) GO TO 4	0610
	US=XS*(1.-0.34*XS/14.7+0.039*(XS/14.7)**2)	0611
	GO TO 3	0612

4	Y=X1(I)/X1(IH)	0613
	DS=PI*DS(X1)/PI*G*DLG(42*(1-2)/2/(1+(42-1)*1*(1-2)*2)/85	0614
3	U(I,J)=USTAR(J)*US	0615
C***	CORRECT VALUES	0616
	CALL REYNU	0617
	VB=VAVG	0618
C***	COMPUTE MIXING LENGTH	0619
	CALL MIXLEN	0620
C***	INITIAL CONDITIONS FOR THE ASSUMING BOBKOV'S CORRELATION	0621
	CALL TURBK	0622
	IF(IPROB.EQ.5) GO TO 60	0623
	GO TO 50	0624
60	DO 20 J=1,JN	0625
	IH=INAX(J)	0626
	DO 20 I=1,IH	0627
20	ED(I,J)=CDISTE(I,J)*1.5/AL(I,J)	0628
50	CONTINUE	0629
	RETURN	0630
	END	0631
	SUBROUTINE TURBK	0632
	IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)	0633
	DIMENSION PR(30,16),PT(30,16),PZ(30,16)	0634
	COMMON/ATURB/DVS(30,16),CA,CA1,CA2,CB,ITURB	0635
	COMMON/GEOMC/RH(30),ARE(30),ARW(30),ARN(30),ARP(30),DX1(30),	0636
	/ ANG(16),ANGH(16),VBE(16)	0637
	COMMON	0638
	1/VAR/D(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),	0639
	2 ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)	0640
	1/ALL/I*,J*,IN,JN,INH,INH,IL,ILP,IPROB,IGRID,INAX(16),IND,ININ(30),	0641
	2 ISEC,IFILE,JFL,IGONV,NSOLD,INOM,JL,JLP	0642
	1/AGRID/R(30),X1(30),X2(16)	0643
	1/STRESS/T11(30,16),T22(30,16)	0644
	1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB	0645
	1/WALLF/TALAV,USAV,RTAL(16),USTAR(16),PR,YB,TWAVE	0646
C		0647
C***	TURBULENT KINETIC ENERGY AND NORMAL STRESS CALCULATIONS	0648

C	PV=1.-VAVG/U (IM,JN)	0649
	IF (IND.EQ.1) GO TO 1	0650
	GO TO (15,45),ITURB	0651
1	CONTINUE	0652
	DO 10 J=1,JN	0653
	IH=IMAX(J)	0654
	DO 10 I=IL,IH	0655
	ETA=X1(I)/X1(IH)	0656
	YZ=-1.48*ETA	0657
	YT=-1.17*ETA	0658
	YR=-0.77*ETA	0659
	FP(I,J)=0.40*DEXP(YR)	0660
	FT(I,J)=0.60*DEXP(YT)	0661
	FZ(I,J)=0.95*DEXP(YZ)	0662
10	CONTINUE	0663
15	CONTINUE	0664
	DO 20 J=1,JN	0665
	IH=IMAX(J)	0666
	DO 20 I=IL,IH	0667
	T11(I,J)=(FP(I,J)*PV*U(I,J))**2	0668
	T22(I,J)=(FT(I,J)*PV*U(I,J))**2	0669
	IF (IND.EQ.0.AND.IPROB.NE.3) GO TO 20	0670
	T33=(FZ(I,J)*PV*U(I,J))**2	0671
	TE(I,J)=0.5*(T11(I,J)+T22(I,J)+T33)	0672
20	CONTINUE	0673
	RETURN	0674
45	CONTINUE	0675
	CALL MODSTB	0676
	RETURN	0677
	END	0678
	SUBROUTINE MIXLEN	0679
	IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)	0680
	COMMON/ALN/CL,RL,IPR,ILEN	0681
	COMMON	0682
	1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),	0683
		0684

2	BD(30,16),AL(30,16),WLS(30,16),GEN(30,16),PRUT(30,16)	0685
1	ALA/IT,JA,IB,IC,IN,IL,IP,IPROB,IGRID,IMAX(16),IMD,IMIN(30),	0686
2	ISEC,IPIL,JJL,ICONV,NSOLU,INUM,JL,JLP	0687
1	AGRID/R(30),X1(30),X2(16)	0688
1	WALLF/TALAV,USAV,RTAL(16),USTAR(16),PR,YB,TWAVE	0689
1	TURB/CD,CMU,C1,C2,CK,E,CVIS,CDIS,PRTE,PRED	0690
1	PLUPR/URFVIS,VISCOS,DENSIT,PIT,VIA,DH,ATOT,RE,VAVG,VB	0691
C	*****	0692
C	SUBROUTINE TO COMPUTE THE MIXING LENGTH	0693
C	*****	0694
	DO 10 J=1,JN	0695
	IH=IMAX(J)	0696
	DO 10 I=2,IH	0697
	IP=(IH+1)/2	0698
	ETA=X1(IP)/X1(IH)	0699
	IF(ETA.GT.0.44) GO TO 15	0700
	AL(I,J)=X1(IP)	0701
	GO TO 10	0702
15	CT=3.14159/.38*(ETA-0.44)	0703
	AL(I,J)=(0.44+0.066*DSIN(CT))*X1(IH)	0704
	GO TO 10	0705
20	CONTINUE	0706
	IF(I.EQ.2) GO TO 11	0707
	IF(I.EQ.IH) GO TO 12	0708
	H1=(R(I-1)-R(I))/(R(IP+1)-R(I))	0709
	H2=1./H1	0710
	DU1=((U(I+1,J)-U(I,J))*H1+(U(I,J)-U(I-1,J))*H2)/((R(I+1)-R(I))*H1+	0711
	/(R(I)-R(I-1))*H2)	0712
	GO TO 13	0713
12	DU1=(U(I,J)-U(I-1,J))/(R(I)-R(I-1))	0714
	GO TO 13	0715
11	DU1=(U(3,J)-U(2,J))/(R(3)-R(2))	0716
13	CONTINUE	0717
	IF(ILEN.EQ.2) AL(I,J)=(DU1/USTAR(J)*CK)**RL + (1./CL/X1(IH))**RL	0718
	AR=1./RL	0719
	AL(I,J)=1./AL(I,J)**AR	0720

10	CONTINUE	0721
	RETURN	0722
	END	0723
	SUBROUTINE SECIN	0724
	IMPLICIT REAL*8 (A-H,O-Z), INTEGER (I-M)	0725
	COMMON/GEOMC/RH(30),ARE(30),ARW(30),ARN(30),ARP(30),DX1(30),	0726
	/ ANG(16),ANGH(16),VBP(16)	0727
	COMMON	0728
	1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),	0729
	2 ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PROT(30,16)	0730
	1/ALL/IT,JT,IN,JN,INN,JNH,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),	0731
	2 ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP	0732
	1/AGRID/R(30),X1(30),X2(16)	0733
	1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB	0734
	1/WALLP/TALAV,USAV,RTAL(16),USTAR(16),PR,YB,TWAVE	0735
	C*****	0736
	C SUBROUTINE TO COMPUTE STREAM FUNCTION USING NIJSING APPROXIMATION	0737
	C*****	0738
	PI=3.14159	0739
C**	V2-VELOCITY	0740
	DO 100 J=3,JNH	0741
	IH=IMAX(J)-1	0742
	DU=(USTAR(J)-USTAR(J-1))/X2(2)	0743
	DO 100 I=2,IH	0744
	ANGLE=PI*2.*X1(I)/(X1(IH)+X1(IH-1))	0745
100	V2(I,J)=0.628*DU*DCOS(ANGLE)	0746
C**	V1-VELOCITY	0747
	DO 102 J=2,JNH	0748
	IH=IMAX(J)-1	0749
	DO 101 I=2,IH	0750
101	V1(I+1,J)=(ARP(I)*V1(I,J)+ARN(I)*(V2(I,J)-V2(I,J+1)))/ARE(I)	0751
102	CONTINUE	0752
	CALL MODV1	0753
	RETURN	0754
	END	0755
	SUBROUTINE REYNU	0756

IMPLICIT REAL*8 (A-H,O-Z), INTEGER (I-N)	0757
COMMON	0758
1/VAR/0 (30,16), V1 (30,16), V2 (30,16), P (30,16), PP (30,16), TE (30,16),	0759
2 ED (30,16), AL (30,16), VIS (30,16), GPN (30,16), PRUT (30,16)	0760
1/ALL/IT, JT, IN, JN, INM, JNM, IL, ILP, IPROB, IGRID, IMAX (16), IND, JMIN (30),	0761
2 ISEC, IPILE, JIL, ICONV, NSOLU, INUM, JL, JLP	0762
1/AGRID/R (30), X1 (30), X2 (16)	0763
1/FLUPR/URFVIS, VISCOS, DENSIT, PIT, DIA, DH, ATOT, RE, VAVG, VB	0764
1/WALLP/TALAV, USAV, RTAL (16), USTAR (16), PR, YB, TWAVE	0765
C*****	0766
C SUBROUTINE FOR COMPUTATION OF BULK VELOCITY, VB	0767
C*****	0768
C	0769
AV=0.0	0770
BT=TA=X2(2)	0771
DO 20 J=1, JN	0772
C*** INTERNAL POINTS	0773
TAV=0.0	0774
IH=IMAX(J)	0775
IF (IPROB.EQ.1) GO TO 40	0776
TAV=VISCOS*(R(1)*25./2.+125.*VISCOS/USTAR(J)/3.)	0777
A1=0 (IL, J)/USTAR(J)-5.	0778
A2=0 (IL, J)/USTAR(J)+5.	0779
YS=X1 (IL)*USTAR(J)/VISCOS	0780
A1=A1/DLOG (YS/5.0D0)	0781
A2=(A2-A1*DLOG (5.0D0*YS))/2.	0782
AI1= A2*(YS-5.)+A1*(YS*(DLOG (YS)-1.)-5.*(DLOG (5.0D0)-1.	0783
)))	0784
AI2= A1*(YS**2-25.)/2. + A2*(YS**2*(DLOG (YS)-0.5)/2.-25.*(DLOG (5.	0785
/DDO)-0.5)/2.)	0786
TAV=TAV+VISCOS*(AI1*R(1)+VISCOS/USTAR(J)*AI2)	0787
40 CONTINUE	0788
DO 10 I=ILP, IH	0789
DAV=(R(I)*U(I, J) +R(I-1)*U(I-1, J))*(R(I)-R(I-1))/2.	0790
10 TAV=TAV+DAV	0791
TAV=TAV*X2(2)	0792

	IF(J.EQ.1.OR.J.EQ.JN) TAV=0.5*TAV	0793
20	AV=AV+TAV	0794
C***	CALCULATE BULK VELOCITY	0795
	VB=AV/ATOT	0796
	IF(VB.EQ.0.0) VB=VAVG	0797
	IF(IPILE.EQ.1.AND.IND.EQ.0) GO TO 35	0798
C***	NORMALIZE VELOCITY DISTRIBUTION	0799
	F=VAVG/VB	0800
	DO 39 J=1,JN	0801
	IH=IMAX(J)	0802
	DO 39 I=2,IH	0803
39	U(I,J)=F*U(I,J)	0804
35	CONTINUE	0805
	RETURN	0806
	END	0807
	SUBROUTINE BGEOM	0808
	IMPLICIT REAL*8(A-H,O-Z), INTEGER(I-N)	0809
	COMMON/GEOMC/RH(30),ARE(30),ARW(30),ARN(30),ARP(30),DX1(30),	0810
	/ANG(16),ANGH(16),VBP(16)	0811
	COMMON/GEOMD/DV1E(30),DV1W(30),DV1N(30),CV1E(30),CV1W(30)	0812
	COMMON/GEOME/DV2E(30),DV2W(30),DV2N(30),CV2E(30),CV2W(30),ARPN(30)	0813
	COMMON	0814
	1/ALL/IT,JT,IN,JN,INH,JNH,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),	0815
	2 ISEC,IPILF,JJL,ICONV,NSOLU,INUM,JL,JLP	0816
	1/AGRID/R(30),X1(30),X2(16)	0817
	1/GEOM/BE(30),BN(30),BW(30),BS(30),VP(30),VP1(30)	0818
C*****		0819
C	SUBROUTINE FOR CALCULATION OF GEOM FACTORS	0820
C*****		0821
	DTETA=X2(2)	0822
C***	COMPUTE BE,BW,BN,BS AND VP	0823
	VP(IN)=DTETA*R(IN)*(R(IN)-R(INH))/2.	0824
	DX1(1)=X1(2)	0825
	RN(1)=0.5*(R(2)+R(1))	0826
	DO 21 I=2,INH	0827
	BE(I)=.25*DTETA*(R(I+1)+R(I))/(R(I+1)-R(I))	0828

	BN(I) = .25*DTETA*(R(I-1)+R(I))/(R(I)-R(I-1))	0829
	BN(I) = .25*(R(I+1)-R(I-1))/(R(I)-R(I-1))*DTETA	0830
	BS(I) = BN(I)	0831
	VP1(I) = (R(I)**2-R(I-1)**2)*DTETA/2.	0832
	VP(I) = DTETA*(R(I+1)-R(I-1))*R(I)/2.	0833
	RM(I) = 0.5*(R(I)+R(I+1))	0834
	ARR(I) = RM(I)*DTETA	0835
	ARW(I) = RM(I-1)*DTETA	0836
	ARN(I) = 0.5*(R(I+1)-R(I-1))	0837
	ARP(I) = R(I)*DTETA	0838
	DX1(I) = R(I+1)-R(I)	0839
	DV1E(I) = 2.*ARP(I)/ARN(I)	0840
	DVIN(I) = 2.*(R(I)-R(I-1))/(R(I)+R(I-1))/DTETA	0841
	CV1E(I) = DX1(I-1)/4./ARN(I)	0842
	ARP(I) = ARN(I)*R(I)	0843
	DV2E(I) = 2.*((R(I)+R(I+1))**2/4./R(I+1)*BS(I)	0844
	DV2W(I) = 2.*((R(I)+R(I+1))**2/4./R(I+1)*BW(I)	0845
	DV2N(I) = 4.*R(I)*BN(I)	0846
	CV2E(I) = R(I+1)/4./DX1(I)	0847
	CV2W(I) = R(I-1)/4./DX1(I-1)	0848
	IP(I) = EQ.2) GO TO 21	0849
	DVIN(I) = 2.*ARP(I-1)/ARN(I-1)	0850
	CV1W(I) = DX1(I-1)/4./ARN(I-1)	0851
21	CONTINUE	0852
	ANG(1) = 0.0	0853
	DO 22 J=2,JN	0854
	I = IMAX(J)	0855
	ANG(J) = DTAN(X2(J))	0856
	ANGN(J) = DTAN(X2(J))*0.500*X2(2)	0857
	VBP(J) = ((R(I+1)+R(I))*BSIN(X2(2)) - (R(I)+R(I-1))*X2(2))*R(I)*R(I-1)	0858
	/8.	0859
22	CONTINUE	0860
	VBP(1) = 0.0	0861
	VBP(JN) = RM(INN)*(R(IN)*DSIN(X2(2)/2.) - RM(INN)*X2(2)/2.)	0862
	VP(IN) = VBP(JN)	0863
	RETURN	0864

END	0865
SUBROUTINE PHOPS (IVIS)	0866
IMPLICIT REAL*8 (A-H,O-Z), INTEGER (I-N)	0867
COMMON	0868
1/VAR/U (30,16), V1 (30,16), V2 (30,16), P (30,16), PP (30,16), TE (30,16),	0869
2 ED (30,16), AL (30,16), VIS (30,16), GEN (30,16), PRUT (30,16)	0870
1/ALL/IT, JT, IN, JN, INN, JNM, IL, ILP, IPRON, IGRID, IMAX (16), IND, JMIN (30),	0871
2 ISEC, IPILE, JJL, ICONV, NSOLU, INUM, JL, JLP	0872
1/AGRID/R (30), X1 (30), X2 (16)	0873
1/PLUPR/URFVIS, VISCOS, DENSIT, PIT, DIA, DH, ATOT, RE, VAVG, VB	0874
1/THPB/CD, CMU, C1, C2, CK, E, CVIS, COIS, PRTE, PRED	0875
1/WALLF/TALAV, USAV, RTAL (16), USTAR (16), PR, YB, THAVE	0876
COMMON/ALN/CL, RL, IPR, ILEN	0877
C	0878
C*** VISCOCITY AND VISCOCITY RATIO CALCULATIONS	0879
C	0880
C*** VISCOCITY RATIO	0881
IF (IND.EQ.0) GO TO 11	0882
CTAN=0.154	0883
GO TO (13,15,17), IPR	0884
13 CONTINUE	0885
DO 14 J=1, JN	0886
IH=IMAX (J)	0887
DO 14 I=1, IH	0888
14 PRUT (I, J)=1.0	0889
GO TO 11	0890
15 CONTINUE	0891
DO 16 J=1, JN	0892
IH=IMAX (J)	0893
DO 16 I=1, IH	0894
16 PRUT (I, J)=AL (I, J)/AL (IH, JN)	0895
GO TO 11	0896
17 CONTINUE	0897
DO 10 J=1, JN	0898
IH=IMAX (J)	0899
AK=1.+X1 (IH)/R (1)	0900

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      BK=0.407+0.02*X1(IH)/R(1)
      C=30.*(1.-0.35*30./14-7*0.039*(30./14-7)**2)-DLOG(30.000)/BK
      B=(1.82-1.3*X1(IH)/R(1))/(1.+0.14*(DLOG10(RE)-5.))
      DO 10 I=IL,IH
      Y=1.-X1(I)/X1(IH)
      A2=(2.*AK-Y*(AK-1.))/(AK-Y*(AK-1.))/(AK+1.)
      PRUT(I,J)=AK*(B+Y**2)*(1.-Y**2)*A2/2./(1.+B)/CTAN
10    CONTINUE
11    CONTINUE
C
C***  RADIAL VISCOSITY
      GO TO (1,2),IVIS
1    CONTINUE
      DO 100 J=1,JM
      IH=IMAX(J)
      DO 100 I=IL,IH
      VISOLD=VIS(I,J)
      IF(IPROB.LT.5)ED(I,J)=CDIS*TE(I,J)**1.5/AL(I,J)
      IF(FC(I,J).EQ.0.)GO TO 102
      VIS(I,J)=TE(I,J)**2*CMU/ED(I,J)+VISCOS
      GO TO 101
102  VIS(I,J)=VISCOS
C-----UNDER-RELAX VISCOSITY
101  VIS(I,J)=URFVIS*VIS(I,J)+(1.-URFVIS)*VISOLD
100  CONTINUE
      RETURN
C -
2    CONTINUE
      DO 200 J=1,JM
      IH=IMAX(J)
      DO 200 I=IL,IH
200  VIS(I,J)=CTAN*PRUT(I,J)*X1(IH)*USTAR(J)
      RETURN
      END
      SUBROUTINE CALCV1
      IMPLICIT REAL*8 (A-H,O-Z), INTEGER (I-N)

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C		0937
	COMMON/GEOMC/RM(30),ARE(30),ARW(30),ARN(30),ARP(30),DX1(30),	0938
	/ ANG(16),ANGM(16),VBP(16)	0939
	COMMON/GEOMD/DV1E(30),DV1W(30),DV1N(30),CV1E(30),CV1W(30)	0940
	COMMON/ATURB/DVS(30,16),CA,CA1,CA2,CB,ITURB	0941
	COMMON	0942
	1/V1VEL/URFV1,RESV1,NSWPV1	0943
	1/PCOR/RESORM,URPP,RESPP,NSWPP,IPREF,JPREF	0944
	1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),	0945
	2 ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)	0946
	1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),	0947
	2 ISEC,IPILE,JJL,ICONV,NSOLU,INU,JL,JLP	0948
	1/AGRID/R(30),X1(30),X2(16)	0949
	1/GEOM/BE(30),BN(30),BW(30),BS(30),VP(30),VP1(30)	0950
	1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,PJA,DH,ATOT,RE,YAVG,VB	0951
	1/WALLF/TALAV,USAV,RTAL(16),NSTAB(16),FR,YB,TWAVE	0952
	1/TURB/CD,CMU,C1,C2,CK,F,CVIS,CDIS,PSTE,PRED	0953
	1/STRESS/T11(30,16),T22(30,16)	0954
	1/GEOMA/DPE(30,16),DPW(30,16),DPH(30,16),DPS(30,16)	0955
	1/COEF/AP(30,16),AN(30,16),AS(30,16),AE(30,16),AW(30,16),SO(30,16)	0956
C		0957
C***V1 VELOCITY		0958
C		0959
	DTETA=X2(2)	0960
	VISE=VISCOS	0961
	VISW=VISCOS	0962
	DO 100 J=2,JNM	0963
	IH=IMAX(J)	0964
	DO 101 I=3,IH	0965
C-----CALCULATE DIFFUSION COEFFICIENTS		0966
	IF(I.EQ.IH) VIS(I,J-1)=VIS(I,J)	0967
	VISN=(VIS(I,J)+VIS(I,J+1)+VIS(I-1,J+1)+VIS(I-1,J))*0.25	0968
	VISS=(VIS(I,J)+VIS(I,J-1)+VIS(I-1,J-1)+VIS(I-1,J))*0.25	0969
	DE=VISE*DV1E(I)	0970
	DW=VISW*DV1W(I)	0971
	DN=VISN*DV1N(I)	0972

	DS=VISS*DV1H(I)	0973
C----	CALCULATE CONVECTION COEFFICIENTS	0974
	V1E=(V1(I,J)*DX1(I) + V1(I+1,J)*DX1(I-1))/(DX1(I)+DX1(I-1))	0975
	/)	0976
	V1W=(V1(I,J)*DX1(I-2)+V1(I-1,J)*DX1(I-1))/(DX1(I-2)+DX1(I-1))	0977
	V2N=(V2(I,J+1)+V2(I-1,J+1))/2.	0978
	V2S=(V2(I,J) + V2(I-1,J))/2.	0979
	CE=ARP(I)*V1E	0980
	CW= ARP(I-1)*V1W	0981
	CN=DX1(I-1)*V2N	0982
	CS=DX1(I-1)*V2S	0983
	SMP=CN-CS+CE-CW	0984
C----	ASSEMBLE MAIN COEFFICIENTS	0985
	IF(ICONV.EQ.1) GO TO 21	0986
	A1=CZ+CW	0987
	CE=(DABS(A1)-A1)*CV1E(I)	0988
	CW=(DABS(A1)+A1)*CV1W(I)	0989
	A2P=(CN+CS)/2.	0990
	CN=(DABS(A2P)-A2P)/2.	0991
	CS=(DABS(A2P)+A2P)/2.	0992
	GO TO 22	0993
21	CE=0.5*(DABS(CE)-CE)	0994
	CW=0.5*(DABS(CW)+CW)	0995
	CN=0.5*(DABS(CN)-CN)	0996
	CS=0.5*(DABS(CS)+CS)	0997
22	CONTINUE	0998
	AE(I,J)=CE+DE	0999
	AW(I,J)=CW+DW	1000
	AN(I,J)=CN+DN	1001
	AS(I,J)=CS+DS	1002
	AP(I,J)=AE(I,J)+AW(I,J)+AN(I,J)+AS(I,J)	1003
C----	CALCULATE SOURCE TERMS	1004
	TTP=(T22(I,J)+T22(I-1,J))/2.	1005
	S1=TTP*VF1(I)/RM(I-1)	1006
	S2=ARP(I-1)*(T11(I-1,J)+P(I-1,J))-ARP(I)*(T11(I,J)+P(I,J))	1007
	S31= V1SN*RM(I-1) * (V2(I,J+1)/R(I)-V2(I-1,J+1)/R(I-1))	1008

	S32=- VISS*RN(I-1)	* (V2(I,J)/R(I)-V2(I-1,J)/R(I-1))	1009
	SU(I,J)=S1+S2+S31+S32		1010
101	CONTINUE		1011
	I=IH+1		1012
	AP(I,J)=1.0		1013
	AE(I,J)=0.0		1014
	AW(I,J)=0.0		1015
	AN(I,J)=0.0		1016
	AS(I,J)=0.0		1017
	IF(J.EQ.JNM) GO TO 104		1018
	SU(I,J)=ARN(I-1)*V2(I-1,J)/ARE(I-1)		1019
	GO TO 100		1020
104	SU(I,J)=0.0		1021
100	CONTINUE		1022
	DO 300 J=2,JNM		1023
	IH=IMAX(J)-1		1024
	DO 301 I=2,IH		1025
	DPE(I,J)=ARP(I+1)/AP(I+1,J)		1026
	DPE(I,J)=DPE(I,J)*URFV1		1027
301	CONTINUE		1028
	IH=IMAX(J)		1029
	DO 302 I=3,IH		1030
	DPW(I,J)=ARP(I-1)/AP(I,J)		1031
	DPW(I,J)=DPW(I,J)*URFV1		1032
302	CONTINUE		1033
300	CONTINUE		1034
	DO 350 J=2,JNM		1035
	IH=IMAX(J)+1		1036
	DO 350 I=3,IH		1037
	AP(I,J)=AP(I,J)/URFV1		1038
	SU(I,J)=SU(I,J)+(1.-URFV1)*AP(I,J)*V1(I,J)		1039
350	CONTINUE		1040
C			1041
C***SOLUTION OF THE DIFFERENCE EQUATION			1042
C			1043
	RESV1=0.0		1044

	DO 4000 N=1, NSWPV1	1045
	IF (NSOLU.EQ.0) CALL SOLV1(3,2,JNM,P,V1,RES)	1046
	IF (NSOLU.EQ.1) CALL SOLV2(3,2,JNM,1,V1,RES)	1047
	IF (RES.GT.RESV1) RESV1=RES	1048
	CALL MODV1	1049
400	CONTINUE	1050
	RETURN	1051
	END	1052
	SUBROUTINE CALCV2	1053
	IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)	1054
	COMMON/GEOMC/RM(30), ARE(30), ARN(30), ARN(30), ARP(30), DX1(30),	1055
	/ ANG(16), ANGN(16), VBP(16)	1056
	COMMON/GFOME/DV2E(30), DV2N(30), DV2N(30), CV2E(30), CV2N(30), ABPN(30)	1057
	COMMON/ATURB/DPS(30,16), CM, CM1, CM2, CB, CTURB	1058
	COMMON	1059
	1/V2VEL/URFV2, RESV2, NSWPV2	1060
	1/PCOR/RFSORH, URPP, RESPP, NSWPP, IPREF, JPREF	1061
	1/VAR/D(30,16), V1(30,16), V2(30,16), P(30,16), PP(30,16), TE(30,16),	1062
	2 ED(30,16), AE(30,16), VIS(30,16), GEN(30,16), PRUT(30,16)	1063
	1/ALL/IT, JT, IN, JN, INH, JNM, IL, JLP, IPROB, IGRID, IMAX(16), IND, JMIN(30),	1064
	2 ISEC, IPIL, JJJL, ICONV, NSOLU, INUM, JL, JLP	1065
	1/AGRID/R(30), XM(30), X2(16)	1066
	1/GEOM/BP(30), BM(30), BN(30), BS(30), VP(30), VP1(30)	1067
	1/GFOMA/DPE(30,16), DPN(30,16), DPN(30,16), DPS(30,16)	1068
	1/PLUPR/URFVIS, VISCOS, DFNSIT, PIT, CIA, DH, ATOT, RE, VAVG, VR	1069
	1/WALLP/TALAV, USAV, RTAL(16), USTAR(16), PR, YB, THAVE	1070
	1/TURB/CD, CMU, CP, C2, CK, E, CVIS, CDIS, PRTE, PRED	1071
	1/STRESS/T11(30,16), T22(30,16)	1072
	1/COEF/KP(30,16), RN(30,16), RS(30,16), RE(30,16), RW(30,16), ST(30,16)	1073
C		1074
C***	V2 VELOCITY	1075
C		1076
	DTETA=X2(2)	1077
	VISN=VISCOS	1078
	VISS=VISCOS	1079
	VISP=VISCOS	1080

DO 100 J=3,JNN	1081
IH=IMAX(J)-1	1082
DO 101 I=2,IH	1083
C-----CALCULATE DIFFUSION COEFFICIENTS	1084
IF (I.EQ.JH) VIS(I+1,J-1)=VIS(I+1,J)	1085
VISE=(VIS(I+1,J)+VIS(I+1,J-1)+VIS(I,J-1)+VIS(I,J))/4.	1086
VISW=(VIS(I-1,J)+VIS(I-1,J-1)+VIS(I,J-1)+VIS(I,J))/4.	1087
DE=DV2E(I)*VISE	1088
DW=DV2W(I)*VISW	1089
DN=DV2N(I)*VISN	1090
DS=DV2N(I)*VISS	1091
C-----CALCULATE CONVECTION COEFFICIENTS	1092
V1E=(V1(I+1,J-1)+V1(I+1,J))/2.	1093
V1W=(V1(I,J)+V1(I,J-1))/2.	1094
IF(J.NE.JH) V2W=(V2(I,J)+V2(I,J+1))/2.	1095
IF(J.EQ.JH) V2N=0.0	1096
V2S=(V2(I,J)+V2(I,J-1))/2.	1097
IF(J.EQ.2) V2S=0.0	1098
CE=ARF(I)*V1E	1099
CW=ARW(I)*V1W	1100
CN=ARN(I)*V2N	1101
CS=ARN(I)*V2S	1102
SNP=CN-CS+CE-CW	1103
C-----ASSEMBLE MAIN COEFFICIENTS	1104
IF(ICONV.EQ.1) GO TO 21	1105
A1=CB*DX1(I)+CW*DX1(I-1)	1106
CE=(DABS(A1)-A1)*CV2E(I)	1107
CW=(DABS(A1)+A1)*CV2W(I)	1108
A2=CN+CS	1109
CN=(DABS(A2)-A2)*R(I)/4.	1110
CS=(DABS(A2)+A2)*R(I)/4.	1111
GO TO 22	1112
21 CE=0.5*(DABS(CE)-CE)*R(I+1)	1113
CW=0.5*(DABS(CW)+CW)*R(I-1)	1114
CN=0.5*(DABS(CN)-CN)*R(I)	1115
CS=0.5*(DABS(CS)+CS)*R(I)	1116

22	CONTINUE	1117
	AE(I,J)=CE+DE	1118
	AW(I,J)=CH+DH	1119
	AN(I,J)=CH+DH	1120
	IF(J.EQ.JN) AN(I,J)=0.0	1121
	AS(I,J)=CS+DS	1122
	IF(J.EQ.2) AS(I,J)=0.0	1123
	AP(I,J)=(DE*R(I+1)+DH*R(I-1))/R(I)+R(I)*(CE/R(I+1)+CH/R(I-1))	1124
	+AN(I,J)+AS(I,J)	1125
	IF(J.EQ.2) AP(I,J)=AP(I,J)+2.*(CS+DS)	1126
	IF(J.EQ.JN) AP(I,J)=AP(I,J)+2.*(CH+DH)	1127
C----	CALCULATE SOURCE TERMS	1128
	S1=-ARP(I) * (T22(I,J)+P(I,J)-T22(I,J-1)-P(I,J-1))	1129
	S2=VISE*RM(I) * (V1(I+1,J)-V1(I+1,J-1))	1130
	- (V1(I,J)-V1(I,J-1)) * VISE*RM(I-1)	1131
	S3(I,J)=S1+S2	1132
101	CONTINUE	1133
	I=IR+1	1134
	AP(I,J)=1.0	1135
	AE(I,J)=0.0	1136
	AW(I,J)=0.0	1137
	AS(I,J)=0.0	1138
	IF(J.EQ.JNM.OR.J.LE.JJL) GO TO 104	1139
	SU(I,J)=DX1(I-1)*(V1(I,J)-V1(I,J-1))/ARP(I)	1140
	AM(I,J)=R(I-1)/R(I)	1141
	GO TO 100	1142
104	AN(I,J)=0.0	1143
	SU(I,J)=0.0	1144
100	CONTINUE	1145
C		1146
	JNM=JN-2	1147
	DO 300 J=2,JNM	1148
	IR=IMAX(J)	1149
	DO 300 I=2,IR	1150
	DPN(I,J)=ARP(I)/AP(I,J+1)	1151
	DPN(I,J)=DPN(I,J)*URPV2	1152

300	CONTINUE	1153
	DO 301 J=3,JNM	1154
	IH=IMAX(J)-1	1155
	DO 301 I=2,IH	1156
	DPS(I,J) = ARPN(I)/AP(I,J)	1157
	DPS(I,J)=DPS(I,J)*URPV2	1158
301	CONTINUE	1159
	DO 350 J=3,JNM	1160
	IH=IMAX(J)	1161
	DO 350 I=2,IH	1162
	AP(I,J)=AP(I,J)/URPV2	1163
	SU(I,J)=SU(I,J)+(1.-URPV2)*AP(I,J)*V2(I,J)	1164
350	CONTINUE	1165
C		1166
	C***SOLUTION OF THE DIFFERENCE EQUATION	1167
C		1168
	RESV2=0.0	1169
	DO 400 N=1,NSMPV2	1170
	IF(NSOLU.EQ.0) CALL SOLV1(2,3,JNM,-1,V2,RES)	1171
	IF(NSOLU.GE.1) CALL SOLV2(2,3,JNM,-1,V2,RES)	1172
	IF(RES.GT.RESV2) RESV2=RES	1173
	CALL MODV2	1174
400	CONTINUE	1175
	RETURN	1176
	END	1177
	SUBROUTINE CALCP	1178
	IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)	1179
	COMMON/GEOMC/RH(30),ABE(30),ARN(30),ARN(30),ARP(30),DX1(30),	1180
	/ ANG(16),ANGH(16),VBP(16)	1181
	COMMON	1182
	1/PCOR/RESORM,URPP,RESPP,NSWPP,IPBEP,JPREF	1183
	1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),	1184
	2 ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)	1185
	1/ALL/IT,JT,IN,JN,INN,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),	1186
	2 ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP	1187
	1/AGRID/R(30),X1(30),X2(16)	1188

1/FLUPP/DEFTIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB	1189
1/GEOM/BE(30),BN(30),BW(30),BS(30),VP(30),VP1(30)	1190
1/WALLP/TALAV,USAV,RTAL(16),USTAR(16),PR,YB,TWAVE	1191
1/TURB/CD,CHU,C1,C2,CK,E,CVIS,CDIS,PRTF,PRED	1192
1/COEF/AP(30,16),AN(30,16),AS(30,16),AE(30,16),AW(30,16),SU(30,16)	1193
1/STRESS/T11(30,16),T22(30,16)	1194
1/GEOMA/DPE(30,16),DPW(30,16),DPN(30,16),DPS(30,16)	1195
C	1196
C	1197
C*** PRESSURE	1198
	1199
DTETA=K2(2)	1200
RESORM=0.0	1201
RESPP=0.0	1202
DO 100 J=2,JHM	1203
IH=IMAX(J)	1204
DO 101 I=2,IH	1205
C--- CALCULATE COEFFICIENTS	1206
AE(I,J)=APE(I)*DPE(I,J)	1207
AW(I,J)=ARW(I)*DPW(I,J)	1208
AN(I,J)=ARN(I)*DPN(I,J)	1209
AS(I,J)=ARN(I)*DPS(I,J)	1210
AP(I,J)=AE(I,J)*R(I)/P(I+1)+AW(I,J)*R(I)/R(I-1)+AN(I,J)+AS(I,J)	1211
C*** COMPUTE SOURCE TERMS	1212
CN= V2(I,J+1)*ARN(I)	1213
CS= V2(I,J)*ARN(I)	1214
CE= V1(I+1,J)*APE(I)	1215
CW= V1(I,J)*ARW(I)	1216
SMP=CN-CS*CE-CW	1217
SU(I,J)=-SMP	1218
IF(I.EQ.IH) SU(I,J)=CW-CN	1219
C--- COMPUTE SUM OF ABSOLUTE MASS SOURCES	1220
RESORM=RESORM+DABS(SMP)	1221
101 CONTINUE	1222
100 CONTINUE	1223
C--- SOLUTION OF THE DIFFERENCE EQUATIONS	1224

	DO 400 N=1,NSWPP	1225
	IF(NSOLU.EQ.0) CALL SOLV1(2,2,JNN,0,PP,RES)	1226
	IF(NSOLU.FQ.1) CALL SOLV2(2,2,JNN,0,PP,RES)	1227
	IF(RES.GT.RESPP) RESPP=RES	1228
	CALL MODP	1229
400	CONTINUE	1230
C		1231
C---	CORRECT VELOCITIES	1232
C		1233
	DO 500 J=2,JNN	1234
	IH=IMAX(J)	1235
	DO 500 I=3,IH	1236
500	V1(I,J)=V1(I,J)+DPW(I,J)*(PP(I-1,J)-R(I)/R(I-1)*PP(I,J))	1237
	DO 501 J=2,JN	1238
	IH=IMAX(J)-1	1239
	DO 501 I=2,IH	1240
501	V2(I,J)=V2(I,J)+DPS(I,J)*(PP(I,J-1)-PP(I,J))	1241
C---	PRESSURES WITH PROVISION OF UNDER RELAXATION	1242
	PPREF=PP(IPREF,JPREF)	1243
	DO 502 J=1,JN	1244
	IH=IMAX(J)	1245
	DO 503 I=2,IH	1246
	P(I,J)=P(I,J)+URPP*(PP(I,J)-PPREF)	1247
	PP(I,J)=0.0	1248
503	CONTINUE	1249
502	CONTINUE	1250
	RETURN	1251
	END	1252
	SUBROUTINE CALCU	1253
	IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)	1254
	COMMON/GEOMC/RH(30),ARE(30),ARW(30),ARN(30),ARP(30),DX1(30),	1255
	/ ANG(16),ANGH(16),VBF(16)	1256
	COMMON	1257
	1/UVEL/URFU,RESU,NSWPU	1258
	1/PCOR/RESORM,URFP,RESPP,NSWPP,IPREF,JPREF	1259
	1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),	1260

2	END(30, 30), AN(30, 16), AS(30, 16), GEN(30, 16), PRUT(30, 16)	1261
1	MALE/IT, JT, IN, JN, INN, JNN, IL, ILP, IPROB, IGRID, IMAK(16), INB, JNIB(30),	1262
2	ISFC, IPIL, JIL, ICONV, NSOLU, INUM, JL, JLP	1263
1	AGRID/R(30), X1(30), Y2(16)	1264
1	GEOM/BE(30), BN(30), BW(30), BS(30), VP(30), VP1(30)	1265
1	FLOPR/URFVIS, VISCOS, DENSIT, PIT, PCA, DH, ATOT, RE, VAVG, VB	1266
1	WALLF/TALAV, USAV, RTAL(16), USTAR(16), FR, YB, TWAVE	1267
1	TURD/CD, CND, C1, C2, CK, E, CVIS, CDIS, PRTE, PRED	1268
1	COEF/AP(30, 16), AN(30, 16), AS(30, 16), AE(30, 16), AV(30, 16), SU(30, 16)	1269
C		1270
C***	AXIAL VELOCITY	1271
C		1272
	DDRTA=X2(2)	1273
	CP=0.0	1274
	CN=0.0	1275
	CS=0.0	1276
	CP=0.0	1277
	DO 100 J=2, JNN	1278
	IX=IMAX(J)	1279
	DO 101 I=ILP, IH	1280
C----	CALCULATE DIFFUSION COEFFICIENTS	1281
	IF(1.EQ.IH.AND.3.LE.JJL) GO TO 103	1282
	DN=(VIS(I, J)+VIS(I-1, J))*BW(I)	1283
	DN=(VIS(I, J)/PRUT(I, J)+VIS(I, J+1)/PRUT(I, J+1))*BN(I)	1284
	IF(1.EQ.IH) GO TO 104	1285
	DE=(VIS(I, J)+VIS(I+1, J))*BE(I)	1286
	DS=(VIS(I, J)/PRUT(I, J)+VIS(I, J+1)/PRUT(I, J+1))*BS(I)	1287
	GO TO 105	1288
104	DE=0.0	1289
	DS=0.0	1290
105	CONTINUE	1291
	IF(ISEC.EQ.0) GO TO 150	1292
	V1E=V1(I+1, J)	1293
	V1W=V1(I, J)	1294
	V2N=V2(I, J+1)	1295
		1296

	V2S=V2(I,J)	1297
	CE=ARE(I)*V1E	1298
	CW=ARW(I)*V1W	1299
	CN=ARN(I)*V2N	1300
	CS=ARN(I)*V2S	1301
	SHP=CN-CS+CE-CW	1302
C-----	ASSEMBLE MAIN COEFFICIENTS	1303
	IF(ICONV.EQ.1) GO TO 21	1304
	IF(I.EQ.IH) GO TO 23	1305
	A1=CE*DX1(I)+CW*DX1(I-1)	1306
	CE=(DABS(A1)-A1)/DX1(I)/4.	1307
	CW=(DABS(A1)+A1)/DX1(I-1)/4.	1308
	A2=CN+CS	1309
	CN=(DABS(A2)-A2)/4.	1310
	CS=(DABS(A2)+A2)/4.	1311
	GO TO 150	1312
21	CN=0.5*(DABS(CN)-CN)	1313
	CW=0.5*(DABS(CW)+CW)	1314
	IF(I.EQ.IH) GO TO 24	1315
	CE=0.5*(DABS(CE)-CE)	1316
	CS=0.5*(DABS(CS)+CS)	1317
	GO TO 150	1318
24	CE=0.0	1319
	CS=0.0	1320
	GO TO 150	1321
23	CE=0.0	1322
	CS=0.0	1323
	CW=CW/2.	1324
	CN=-CN/2.	1325
150	CONTINUE	1326
	AE(I,J)=CE+DE	1327
	AW(I,J)=CW+DW	1328
	AN(I,J)=CN+DN	1329
	AS(I,J)=CS+DS	1330
	AP(I,J)=AE(I,J)+AW(I,J)+AN(I,J)+AS(I,J)	1331
	AP(I,J)=(AP(I,J)+CP)/ORPU	1332

	SU(I,J)=4.*TWAVE*VP(I)/DH	1333
	IF(I.EQ.IH) SU(I,J)=4.*TWAVE*VBP(J)/DH	1334
	SU(I,J)=SU(I,J)+CP*U(I,J)+(1.-URFO)*AP(I,J)*U(I,J)	1335
101	CONTINUE	1336
	GO TO 100	1337
103	CONTINUE	1338
	TETA=X2(J)	1339
	CC=ANG(J) * (R(IH)-R(IH-1))/R(IH)/DTETA	1340
	AE(IH,J)=0.0	1341
	AN(IH,J)=CC/(1.+CC)	1342
	AW(IH,J)=1./(1.+CC)	1343
	AS(IH,J)=0.0	1344
	AP(IH,J)=1.0	1345
	SU(IH,J)=0.0	1346
100	CONTINUE	1347
C		1348
C***SOLUTION OF THE DIFFERENCE EQUATION		1349
C		1350
	RESU=0.0	1351
	DO 400 N=1,NSWPD	1352
	CALL REYNU	1353
	ISTART=1	1354
	IF(IPROB.EQ.1) ISTART=2	1355
	IF(NSOLU.EQ.0) CALL SOLV1(ISTART,2,JNN,0,0,RES)	1356
	IF(NSOLU.GE.1) CALL SOLV2(ISTART,2,JNN,0,0,RES)	1357
	IF(RES.GT.RESU) RESU=RES	1358
	CALL MODR	1359
400	CONTINUE	1360
	RETURN	1361
	END	1362
	SUBROUTINE CALCTE	1363
	IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)	1364
	COMMON/GEOMC/RM(30),ARE(30),ARN(30),ARN(30),ARP(30),DX1(30),	1365
	/ ANG(16),ANGH(16),VBP(16)	1366
	COMMON/ATURB/DVS(30,16),CA,CA1,CA2,CB,ITURB	1367
	COMMON	1368

1/TEN/URPK,RESK,NSWPK	1369
1/PCOR/RESORH,URPP,RESPP,NSWPP,IPRRP,JPREF	1370
1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),	1371
2 ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)	1372
1/ALL/IT,JT,IN,JN,INN,JNM,IL,ILP,IPROR,IGRID,IMAX(16),IND,JMIN(30),	1373
2 ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP	1374
1/AGRID/R(30),X1(30),X2(16)	1375
1/STRESS/T11(30,16),T22(30,16)	1376
1/GEOM/BE(30),BN(30),BW(30),BS(30),VP(30),VP1(30)	1377
1/FLUPP/URPVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB	1378
1/WALLP/TALAV,USAV,RTAL(16),USTAR(16),PH,YB,TWAVE	1379
1/TURB/CD,CNU,C1,C2,CK,E,CVIS,CDIS,PRTE,PRED	1380
1/CORF/AP(30,16),AN(30,16),AS(30,16),AE(30,16),AW(30,16),SU(30,16)	1381
C	1382
C*** TURBULENT KINETIC ENERGY	1383
C	1384
DTETA=X2(2)	1385
CE=0.0	1386
CN=0.0	1387
CF=0.0	1388
CS=0.0	1389
CP=0.0	1390
DO 100 J=2,JNM	1391
IH=IMAX(J)	1392
DO 101 I=ILP,IH	1393
C-----CALCULATE DIFFUSION COEFFICIENTS	1394
IF(I.EQ.IH.AND.J.LE.JJL) GO TO 103	1395
DW=(VIS(I,J)+VIS(I-1,J))*BN(I)/PRTE	1396
DN=(VIS(I,J)/PRUT(I,J)+VIS(I,J+1)/PRUT(I,J+1))*BN(I)/PRTE	1397
IF(I.EQ.IH) GO TO 104	1398
DE=(VIS(I,J)+VIS(I+1,J))*BE(I)/PRTE	1399
DS=(VIS(I,J)/PRUT(I,J)+VIS(I,J-1)/PRUT(I,J-1))*DS(I)/PRTE	1400
GO TO 105	1401
104 DE=0.0	1402
DS=0.0	1403
105 CONTINUE	1404

	IF (ISRC, ZQ, 0) GO TO 150	1405
	V1E=V1(I+1, J)	1406
	V1W=V1(I, J)	1407
	V2N=V2(I, J+1)	1408
	V2S=V2(I, J)	1409
	CE=ARE(I)*V1E	1410
	CW=ARN(I)*V1W	1411
	CN=ARN(I)*V2N	1412
	CS=ARN(I)*V2S	1413
	SMP=CN-CS+CE-CW	1414
C-----	ASSEMBLE MAIN COEFFICIENTS	1415
	IF (ICONV.EQ.1) GO TO 21	1416
	IF (I.EQ.IH) GO TO 23	1417
	A1=CE*DX1(I)+CW*DX1(I+1)	1418
	CE=(DABS(A1)-A1)/DX1(I)/4.	1419
	CW=(DABS(A1)+A1)/DX1(I+1)/4.	1420
	A2=CN+CS	1421
	CN=(DABS(A2)-A2)/4.	1422
	CS=(DABS(A2)+A2)/4.	1423
	GO TO 150	1424
21	CN=0.5*(DABS(CN)-CN)	1425
	CW=0.5*(DABS(CW)+CW)	1426
	IF (I.EQ.IH) GO TO 24	1427
	CE=0.5*(DABS(CE)-CE)	1428
	CS=0.5*(DABS(CS)+CS)	1429
	GO TO 150	1430
24	CE=0.0	1431
	CS=0.0	1432
	GO TO 150	1433
23	CE=0.0	1434
	CS=0.0	1435
	CW=CW/2.	1436
	CN=-CN/2.	1437
150	CONTINUE	1438
	AE(I, J)=CE+DE	1439
	AW(I, J)=CW+DW	1440

	AN(I,J)=DN+CN	1441
	AS(I,J)=ES+CS	1442
	SP = -CD*CHU*TE(I,J)*VP(I)/VIS(I,J)	1443
	IF(I.EQ.IH) SP = SP *VBP(J)/VP(I)	1444
	AP(I,J)=AN(I,J)+AS(I,J)+AE(I,J)+AW(I,J)-SP	1445
	AP(I,J)=(AP(I,J)+CP)/URFK	1446
C----	SOURCE TERMS	1447
	IF(I.FQ.IH) GO TO 160	1448
	H1=(R(I-1)-R(I))/(R(I+1)-R(I))	1449
	H2=1./H1	1450
	DU1=((U(I+1,J)-U(I,J))*H1+(U(I,J)-U(I-1,J))*H2)/(R(I+1)-R(I-1))	1451
	DU2=(U(I,J+1)-U(I,J-1))/R(I)/DTETA/2.	1452
	GO TO 170	1453
160	DU1=(U(I,J)-U(I-1,J))/(R(I)-R(I-1))	1454
	DU2=(U(I,J+1)-U(I,J))/R(I)/DTETA	1455
170	CONTINUE	1456
	GEN(I,J)=(VIS(I,J)*DU1**2+VIS(I,J)/PRUT(I,J)*DU2**2)	1457
	SU(I,J)=GEN(I,J)*VP(I)	1458
	IF(I.EQ.IH) SU(I,J)=SU(I,J)*VBP(J)/VP(I)	1459
	SU(I,J)=SU(I,J)+CP*TE(I,J)+(1.-URFK)*AP(I,J)*TE(I,J)	1460
	IF(ITURB.EQ.1) GO TO 101	1461
	T11(I,J)=TE(I,J)*(2.*(1.-1./CA1)/3.+(CB*GEN(I,J)+CA*VIS(I,J)*DU1**	1462
	/2)/ED(I,J))	1463
	T22(I,J)=TE(I,J)*(2.*(1.-1./CA1)/3.+(CB*GEN(I,J)+CA*VIS(I,J)*DU2**	1464
	/2/PROT(I,J))/ED(I,J))	1465
101	CONTINUE	1466
	GO TO 100	1467
103	CONTINUE	1468
	TETA=X2(J)	1469
	CC=ANG(J) *(R(IH)-R(IH-1))/R(IH)/DTETA	1470
	AE(IH,J)=0.0	1471
	AN(IH,J)=CC/(1.+CC)	1472
	AW(IH,J)=1./(1.+CC)	1473
	AS(IH,J)=0.0	1474
	SU(IH,J)=0.0	1475
	AP(IH,J)=AN(IH,J)+AW(IH,J)	1476

IF (ITURB.EQ.1) GO TO 100	1477
T11(IH,J)=AN(IH,J)*T11(IH,J+1)+AW(IH,J)*T11(IH-1,J)	1478
T22(IH,J)=AN(IH,J)*T22(IH,J+1)+AW(IH,J)*T22(IH-1,J)	1479
100 CONTINUE	1480
C	1481
C*** SOLUTION OF THE DIFFERENCE EQUATION	1482
C	1483
RESK=0.0	1484
DO 400 N=1,NSWPK	1485
IF(NSOLU.EQ.0) CALL SOLV1(3,2,JNH,0,TE,RES)	1486
IF(NSOLU.EQ.1) CALL SOLV2(3,2,JNH,0,TE,RES)	1487
IF(RES.GT.RESK) RESK=RES	1488
CALL MORTE	1489
400 CONTINUE	1490
RETURN	1491
END	1492
SUBROUTINE CALGED	1493
IMPLICIT REAL*8(A-H,O-Z), INTEGER(I-N)	1494
COMMON/GEOMC/RN(30),ARE(30),ARW(30),ARN(30),ABP(30),DX1(30),	1495
/ ANG(16),ANGH(16),VBP(16)	1496
COMMON	1497
1/TDIS/URPE,RESE,NSWPD	1498
1/PCOR/RESORH,URFP,RESPP,NSWPP,IPREF,JPREF	1499
1/VAP/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),	1500
2 ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)	1501
1/ALL/IT,JT,IN,JN,INN,JNH,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),	1502
2 ISEC,IPILE,JJL,ICCNV,NSOLD,INUM,JL,JLP	1503
1/AGRIP/R(30),X1(30),X2(16)	1504
1/GEOM/BP(30),BN(30),BB(30),BS(30),VB(30),VP1(30)	1505
1/FLUPR/URFVIS,VISCO5,DENSIT,PIT,DIA,BH,APOT,RE,VAVG,V8	1506
1/WALLP/TALAV,USAV,RTAL(16),OSTAR(16),FR,YB,TWAVE	1507
1/TURB/CD,CMU,C1,C2,CK,E,CVIS,CDIS,PRTE,PRED	1508
1/CONF/AP(30,16),AN(30,16),AS(30,16),AE(30,16),AW(30,16),SU(30,16)	1509
C	1510
C*** ENERGY DISSIPATION	1511
C	1512

DTETA=X2 (2)	1513
CP=0.0	1514
CN=0.0	1515
CW=0.0	1516
CS=0.0	1517
CP=0.0	1518
DO 100 J=2,JNM	1519
IH=IMAX(J)	1520
DO 101 I=ILP,IH	1521
C-----CALCULATE DIFFUSION COEFFICIENTS	1522
IF(I.EQ.IH.AND.J.LE.JJL) GO TO 103	1523
DW=(VIS(I,J)+VIS(I-1,J))*BW(I)/PRED	1524
DN=(VIS(I,J)/PRUT(I,J)+VIS(I,J+1)/PRUT(I,J+1))*BN(I)/PRED	1525
IF(I.EQ.IH) GO TO 104	1526
DE=(VIS(I,J)+VIS(I+1,J))*BE(I)/PRED	1527
DS=(VIS(I,J)/PRUT(I,J)+VIS(I,J-1)/PRUT(I,J-1))*BS(I)/PRED	1528
GO TO 105	1529
104 DE=0.0	1530
DS=0.0	1531
105 CONTINUE	1532
IF(ISEC.EQ.0) GO TO 150	1533
V1E=V1(I+1,J)	1534
V1W=V1(I,J)	1535
V2N=V2(I,J+1)	1536
V2S=V2(I,J)	1537
CE=ARE(I)*V1E	1538
CW=ARW(I)*V1W	1539
CN=ARN(I)*V2N	1540
CS=ARN(I)*V2S	1541
SNP=CN-CS+CP-CW	1542
C-----ASSEMBLE MAIN COEFFICIENTS	1543
IF(ICONV.EQ.1) GO TO 21	1544
IF(I.EQ.IH) GO TO 23	1545
A1=CE*DX1(I)+CW*DX1(I-1)	1546
CE=(DABS(A1)-A1)/DX1(I)/4.	1547
CW=(DABS(A1)+A1)/DX1(I-1)/4.	1548

	AN=CN+CS	1549
	CN=(DABS(A2)-A2)/4.	1550
	CS=(DABS(A2)+A2)/4.	1551
	GO TO 150	1552
21	CN=0.5*(DABS(CN)-CN)	1553
	CW=0.5*(DABS(CW)+CW)	1554
	IF(I.EQ.IH) GO TO 24	1555
	CE=0.5*(DABS(CE)-CE)	1556
	CS=0.5*(DABS(CS)+CS)	1557
	GO TO 150	1558
24	CE=0.0	1559
	CS=0.0	1560
	GO TO 150	1561
23	CE=0.0	1562
	CS=0.0	1563
	CN=CN/2.	1564
	CN=-CN/2.	1565
150	CONTINUE	1566
	AE(I,J)=CE+DE	1567
	AW(I,J)=CW+DW	1568
	AN(I,J)=DN+CN	1569
	AS(I,J)=ES+CS	1570
	SP = C2*VP(I)*CHU*TE(I,J)/VIS(I,J)	1571
	IF(I.EQ.IH) SP = SP *VBP(J)/VP(I)	1572
	AP(I,J)=AN(I,J)+AS(I,J)+AE(I,J)+AW(I,J)-SP	1573
	AP(I,J)=(AP(I,J)+CP)/URPE	1574
C----	SOURCE TERMS	1575
	SU(I,J)= C1*CHU*GEN(I,J)*VP(I) *TE(I,J)/VIS(I,J)	1576
	IF(I.EQ.IH) SU(I,J)=SU(I,J)*VBP(J)/VP(I)	1577
	SU(I,J)=SU(I,J)+CP*ED(I,J)+(1.-URPE)*AP(I,J)*ED(I,J)	1578
101	CONTINUE	1579
	GO TO 100	1580
103	CONTINUE	1581
	TETA=X2(J)	1582
	CC=ANG(J) *(R(IH)-R(IH-1))/R(IH)/DTETA	1583
	AE(IH,J)=0.0	1584

AN(IH,J)=CC/(1.+CC)	1585
AW(IH,J)=1./(1.+CC)	1586
AS(IH,J)=0.0	1587
SU(IH,J)=0.0	1588
AP(IH,J)=AN(IH,J)+AW(IH,J)	1589
100 CONTINUE	1590
C	1591
C***SOLUTION OF THE DIFFERENCE EQUATION	1592
C	1593
RESE=0.0	1594
DO 400 N=1,NSWPD	1595
IF(NSOLU.EQ.0) CALL SOLV1(3,2,JNM,0,ED,RES)	1596
IF(NSOLU.EQ.1) CALL SOLV2(3,2,JNM,0,ED,RES)	1597
IF(RES.GT.RESE) RESE=RES	1598
CALL MODED	1599
400 CONTINUE	1600
RETURN	1601
END	1602
SUBROUTINE SOLV1(ISTART,JSTART,JEND,NSOL,PHI,RES)	1603
IMPLICIT REAL*8 (A-H,O-Z), INTEGER (I-N)	1604
DIMENSION PHI(30,16),A(30),B(30),C(30),D(30)	1605
COMMON	1606
1/ALL/IT,JT,IN,JN,INM,JNM,IL,TLP,IPROB,IGRID,IMAX(16),IND,JMIN(30),	1607
2 ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP	1608
1/AGRID/R(30),X1(30),X2(16)	1609
1/COEF/AP(30,16),AN(30,16),AS(30,16),AE(30,16),AW(30,16),SU(30,16)	1610
C	1611
C***SOLUTION BY MATRIX FACTORIZATION ALONG RADIAL LINE	1612
C	1613
IS=ISTART-1	1614
A(IS)=0.0	1615
RES=0.0	1616
C-----COMMENCE W-E SWEEP	1617
DO 100 J=JSTART,JEND	1618
C(IS)=PHI(IS,J)	1619
IH=IMAX(J)	1620

	IF (NSOL.EQ.1) IH=IH+1	1621
	DO 101 I=ISTART,IH	1622
C-----	ASSEMBLE TDMA COEFFICIENTS	1623
	A(I)=AE(I,J)	1624
	B(I)=AW(I,J)	1625
	IF(J.EQ.2.AND.NSOL.GE.0) GO TO 10	1626
	IF(J.EQ.JNN.AND.NSOL.GE.0) GO TO 11	1627
	IF(J.EQ.JN) GO TO 14	1628
	IF(J.EQ.1) GO TO 15	1629
	C(I)=AN(I,J)*PHI(I,J+1)+AS(I,J)*PHI(I,J-1)+SU(I,J)	1630
	D(I)=AP(I,J)	1631
	GO TO 12	1632
10	C(I)=AN(I,J)*PHI(I,J+1)+SU(I,J)	1633
	D(I)=AP(I,J)-AS(I,J)	1634
	GO TO 12	1635
11	C(I)=AN(I,J)+AS(I,J)*PHI(I,J-1)+SU(I,J)	1636
	D(I)=AP(I,J)-AN(I,J)	1637
	GO TO 12	1638
14	C(I)=AS(I,J)*PHI(I,J-1)+SU(I,J)	1639
	D(I)=AP(I,J)	1640
	GO TO 12	1641
15	C(I)=AN(I,J)*PHI(I,J+1)+SU(I,J)	1642
	D(I)=AP(I,J)	1643
12	CONTINUE	1644
C-----	CALCULATE COEFFICIENTS OF RECURRENCE FORMULA	1645
	TERM=1./(D(I)-B(I)*A(I-1))	1646
	A(I)=A(I)*TERM	1647
	C(I)=(C(I)+B(I)*C(I-1))*TERM	1648
101	CONTINUE	1649
C-----	OBTAIN NEW PHI'S	1650
	DO 102 II=ISTART,IH	1651
	I=IH-II+ISTART	1652
	Z=PHI(I,J)	1653
	PHI(I,J)=A(I)*PHI(I+1,J)+C(I)	1654
	RS=DABS(Z-PHI(I,J))	1655
	IF(RS.GT.RES) RES=RS	1656

102	CONTINUE	1657
100	CONTINUE	1658
	RETURN	1659
	END	1660
	SUBROUTINE SOLV2 (ISTART, JSTART, JEND, NSOL, PHI, RES)	1661
	IMPLICIT REAL*8 (A-H, O-Z), INTEGER (I-N)	1662
	DIMENSION PHI (30, 16), A (30), B (30), C (30), D (30), PHO (30, 16)	1663
	COMMON	1664
	1/ALL/IT, JT, IN, JN, INN, JNN, IL, ILP, IPROB, IGRID, IMAX (16), IND, JHIN (30),	1665
	2 ISEC, IPILE, JJL, ICONV, NSOLO, INUN, JL, JLP	1666
	1/AGRID/R (30), X1 (30), X2 (16)	1667
	1/COEF/AP (30, 16), AN (30, 16), AS (30, 16), AP (30, 16), AW (30, 16), SU (30, 16)	1668
C		1669
C**SOLUTION BY POINT ITERATION		1670
C		1671
	IF (INN.EQ.0) GO TO 650	1672
	DO 651 J=1, JN	1673
	DO 651 I=1, IN	1674
	PHO (I, J)=PHI (I, J)	1675
651	CONTINUE	1676
650	CONTINUE	1677
	RES=0.0	1678
	DO 201 J=JSTART, JEND	1679
	IM=IMAX (J)	1680
	IF (NSOL.EQ.1) IM=IM+1	1681
	DO 202 I=ISTART, IM	1682
	IF (J.EQ.1) GO TO 10	1683
	DDS=PHI (I, J-1)	1684
	IF (INUN.EQ.1) DDS=PHO (I, J-1)	1685
	GO TO 11	1686
10	DDS=0.0	1687
11	CONTINUE	1688
	IF (J.EQ.JN) GO TO 14	1689
	ANUM=AE (I, J) *PHI (I+1, J)+AW (I, J) *PHI (I-1, J)+AN (I, J) *PHI (I, J+1)	1690
	/ AS (I, J) *DDS +SU (I, J)	1691
	GO TO 15	1692

14	ANUM=AE(I,J)*PHI(I+1,J)+AW(I,J)*PHI(I-1,J)+AS(I,J)*DDS+SU(I,J)	1693
15	CONTINUE	1694
	IF(AP(I,J).EQ.0.0) GO TO 202	1695
	Z=PHI(I,J)	1696
	PHI(I,J)=ANUM/AP(I,J)	1697
	RS=DABS(Z-PHI(I,J))	1698
	IF(RS.GT.RES) RES=RS	1699
202	CONTINUE	1700
201	CONTINUE	1701
	RETURN	1702
	END	1703
	SUBROUTINE PROMOD	1704
	IMPLICIT REAL*8(A-H,O-Z), INTEGER(I-N)	1705
	COMMON/GEOMC/EN(30),ARE(30),ARH(30),ABH(30),ABP(30),DTN(30),	1706
	ANG(16),ANGN(16),VBP(16)	1707
	COMMON/ATURB/DVS(30,16),CA,CA1,CA2,CB,ITURB	1708
	COMMON	1709
	1/VAR/U(30,16),V0(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),	1710
	2 ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)	1711
	1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPOB,IGRID,INAX(16),IND,JHIN(30),	1712
	2 ISEC,IPILE,JJL,ECCNV,NSQLO,INUM,JL,JLP	1713
	1/AGRID/R(30),X1(30),X2(16)	1714
	1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,V8	1715
	1/STRESS/T11(30,16),T22(30,16)	1716
	1/TURB/CD,CNU,C1,C2,CK,E,CVIS,CDIS,PRTE,PRED	1717
	1/WALLP/TALAV,USAV,RTAL(16),OSTAR(16),PR,Y8,TWAVE	1718
C		1719
C***	BOUNDARY CONDITIONS	1720
C		1721
C		1722
C	ENTRY MODU	1723
C		1724
C***	WALL	1725
C***	COMPUTE USTAR	1726
C***	IF(IPROB.EQ.1) GO TO 114	1727
		1728

	H1=(R(IL+2)-R(IL))/(R(IL+1)-R(IL))/(R(IL+2)-R(IL+1))	1729
	H2=(R(IL+1)-R(IL))/(R(IL+2)-R(IL))/(R(IL+2)-R(IL+1))	1730
	H=(R(IL+2)+R(IL+1)-2.*R(IL))/(R(IL+1)-R(IL))/(R(IL+2)-R(IL))	1731
	DO 52 J=2,JNH	1732
	DO 53 I=1,100	1733
	B3=1./CK/X1(IL)	1734
	YS=USTAR(J)*X1(IL)/VISCOS	1735
	A3=DLOG(F*YS)/CK	1736
	C4=(H1*U(IL+1,J)-H2*U(IL+2,J))/(B3+H*A3)	1737
	IF(DABS(USTAR(J)-C4)/C4).LT.1.0D-5) GO TO 54	1738
	USTAR(J)=C4	1739
	YS=USTAR(J)*X1(IL)/VISCOS	1740
53	CONTINUE	1741
54	CONTINUE	1742
	U(IL,J)=USTAR(J)*DLOG(F*YS)/CK	1743
C***	NEAR WALL REYNOLDS STRESS	1744
	UV(J)=R(1)/R(IL)*USTAR(J)**2-2.*(R(IL)**2-R(1)**2)/R(IL)/DH*TWAVE	1745
52	CONTINUE	1746
	GO TO 106	1747
114	CONTINUE	1748
	DO 49 J=2,JNH	1749
49	USTAR(J)=DSORT(VISCOS*U(2,J)/K1(2))	1750
106	CONTINUE	1751
	USTAR(1)=USTAR(2)	1752
	USTAR(JN)=USTAR(JNH)	1753
C***	COMPUTE NEW FRICTION FACTOR	1754
	S=0.5*(USTAR(1)**2+USTAR(JN)**2)	1755
	DO 20 J=2,JNH	1756
20	S=S+USTAR(J)**2	1757
	TALAV=S/DFLOAT(JNH)	1758
	FR=2.*TALAV/VB**2	1759
	IF(IPILE.EQ.0) TWAVE=TALAV	1760
C***	WALL SHEAR STRESS DISTRIBUTION	1761
	DO 30 J=1,JN	1762
	RTAL(J)=USTAR(J)**2/TALAV	1763
30	CONTINUE	1764

C***	SOUTH BOUNDARY	1765
	IH=IMAX(1)	1766
	DO 11 I=IL,IH	1767
11	U(I,1)=U(I,2)	1768
C***	NORTH BOUNDARY	1769
	DO 12 I=2,INH	1770
12	U(I,JN)=U(I,JNH)	1771
	U(IN,JN)=U(INH,JN)	1772
	RETURN	1773
C		1774
	ENTRY MODTE	1775
C		1776
C***	WALL	1777
	IF (IPROB.EQ.3) GO TO 72	1778
	DO 71 J=2,JNH	1779
71	UE(IL,J)=UV(J)/CNU**0.5	1780
72	CONTINUE	1781
C***	SOUTH BOUNDARY	1782
	IH=IMAX(1)	1783
	DO 41 I=IL,IH	1784
41	TE(I,1)=TE(I,2)	1785
C***	NORTH BOUNDARY	1786
	DO 42 I=2,INH	1787
42	TE(I,JN)=TE(I,JNH)	1788
	TE(IN,JN)=TE(INH,JN)	1789
	RETURN	1790
C		1791
	ENTRY MODEB	1792
C		1793
C***	WALL	1794
	DO 83 J=2,JNH	1795
83	ED(IL,J)=UV(J)*USTAR(J)/CK/X1(IL)	1796
C***	SOUTH BOUNDARY	1797
	IH=IMAX(1)	1798
	DO 81 I=IL,IH	1799
81	ED(I,1)=FD(I,2)	1800

C***	NORTH BOUNDARY	1801
	DO 82 I=2, INM	1802
82	ED(I, JN) = PD(I, JNM)	1803
	ED(IN, JN) = ED(INM, JN)	1804
	RETURN	1805
C		1806
	ENTRY MODV1	1807
C		1808
C***	SOUTH BOUNDARY	1809
	IH=IMAX(1)	1810
	DO 121 I=2, IH	1811
121	V1(I, 1) = V1(I, 2)	1812
	DO 122 I=1, INM	1813
122	V1(I, JN) = V1(I, JNM)	1814
	V1(IN, JN) = V1(INM, JN)	1815
	RETURN	1816
C		1817
	ENTRY MODV2	1818
	RETURN	1819
C		1820
	ENTRY MODP	1821
C		1822
	IH=IMAX(1)	1823
	DO 93 I=2, IH	1824
93	PP(I, 1) = PP(I, 2)	1825
	DO 92 I=2, INM	1826
92	PP(I, JN) = PP(I, JNM)	1827
	PP(IN, JN) = PP(INM, JN)	1828
	RETURN	1829
C		1830
	ENTRY MODSTR	1831
C		1832
	IU=IMAX(1)	1833
	DO 301 I=2, IU	1834
	T22(I, 1) = T22(I, 2)	1835
301	T11(I, 1) = T11(I, 2)	1836

	DO 302 I=2,INH	1837
	T11(I,JN)=T11(I,JNM)	1838
302	T22(I,JN)=T22(I,JNM)	1839
	T11(IN,JN)=T11(IN,JNM)	1840
	T22(IN,JN)=T22(IN,JNM)	1841
	RETURN	1842
	END	1843
	SUBROUTINE PRINT(ISTART,JSTART,PHI,HEAD)	1844
	IMPLICIT REAL*8(A-H,O-Z), INTEGER(I-N)	1845
	DIMENSION PHI(30,16),HEAD(9),STORE(50)	1846
	DIMENSION F(7),R4(11)	1847
	COMMON	1848
	1/ALL/IT,JT,IN,JN,INH,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),	1849
	2 ISEC,IPILE,ISL,ICONV,USOLV,INUM,OL,OLP	1850
	1/AGRID/R(30),I1(30),I2(16)	1851
	DATA P/4H11H,4H,4H,4H13,4H11I,4H10,4H7X,	1852
	14H46/	1853
	DATA P4/4H 1I,4H 2I,4H 3I,4H 4I,4H 5I,4H 6I,	1854
	1 4H 7I,4H 8I,4H 9I,4H10I,4H11I /	1855
	DATA HI,HY/4H I=,4H Y =/	1856
C		1857
C***	PRINT OUT SUBROUTINE	1858
C		1859
	ISKIP=1	1860
	JSKIP=1	1861
	WRITE(6,110) HEAD	1862
	ISTA=ISTART-12	1863
100	CONTINUE	1864
	ISTA=ISTA+12	1865
	IEND=ISTA+11	1866
	IEND=MINO(IN,IEND)	1867
	F(4)=F4(IEND-ISTA)	1868
	WRITE(6,F) HI, (I,I=ISTA,IEND,ISKIP), HY	1869
	WRITE(6,112)	1870
	DO 101 JJ=JSTART,JN,JSKIP	1871
	J=JSTART+JN-JJ	1872

	III=IMAX(J)	1873
	DO 120 I=ISTA,IEND	1874
	A=PHI(I,J)	1875
	IF(DABS(A).LT.1.D-20) A=0.0	1876
120	STORE(I)=A	1877
	WRITE(6,113) J,(STORE(I),I=ISTA,IEND,ISKIP),X2(J)	1878
101	CONTINUE	1879
C-----		1880
	IF(IEND.LT.IN)GO TO 100	1881
	RETURN	1882
110	FORMAT(1H0,20(2H+-),7X,9A4,7X,20(2H-*))	1883
112	FORMAT(3H J)	1884
113	FORMAT(1H ,I3,1P12D10.2,0PF7.3)	1885
	END	1886
	SUBROUTINE ANORM(L,INORM,ISTRES,ICURVE,FV)	1887
	IMPLICIT REAL*8(A-H,O-Z), INTEGER(I-N)	1888
	DIMENSION VIP(30,16),V2P(30,16),V3P(30,16),T23(30,16),T13(30,16)	1889
	COMMON/HD/HEDU(9),HEDV1(9),HEDV2(9),HEDP(9),HEDK(9),HEDD(9),	1890
1	HEDM(9),HEDL(9),HEDN(9),HEDPR(9),HRDV1P(9),HEDV2P(9),HEDV3P(9),	1891
2	HEDT13(9),HEDT23(9)	1892
	COMMON	1893
1	VAP/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),	1894
2	ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)	1895
1	ALL/IT,JT,IN,JN,INN,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMTN(30),	1896
2	ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP	1897
1	AGRID/R(30),X1(30),X2(16)	1898
1	STRESS/T11(30,16),T22(30,16)	1899
1	WALLP/TALAV,USAV,RTAL(16),USTAR(16),FR,YB,TWAVE	1900
1	GEOMA/DPE(30,16),DPW(30,16),DPN(30,16),DPS(30,16)	1901
	EQUIVALENCE (T11(1),V1P(1)),(T22(1),V2P(1)),(DPE(1),V3P(1)),	1902
1	(DPW(1),T13(1)),(DPN(1),T23(1))	1903
	FF=FV	1904
	DO 54 J=1,JN	1905
	III=IMAX(J)	1906
	IF(L.EQ.3) FF=FV/USTAR(J)	1907
	FVV=FF**L	1908

	DO 54 I=1,10	1909
	U(I,J)=U(I,J)*PP	1910
	IF(IPROB.GT.2) TE(I,J)=TE(I,J)*PVV	1911
	IF(ISTRES.EQ.0) GO TO 54	1912
	V1P(I,J)=V1P(I,J)*PP	1913
	V2P(I,J)=V2P(I,J)*PP	1914
	V3P(I,J)=V3P(I,J)*PP	1915
	T13(I,J)=T13(I,J)*PVV	1916
	T23(I,J)=T23(I,J)*PVV	1917
54	CONTINUE	1918
	CALL PRINT(1,1,U,HEDU)	1919
	IF(IPROB.GT.2) CALL PRINT(1,1,TE,HEDK)	1920
	IF(ISTRES.EQ.0) GO TO 15	1921
	CALL PRINT(2,1,V1P,HEDV1P)	1922
	CALL PRINT(2,1,V2P,HEDV2P)	1923
	CALL PRINT(2,1,V3P,HEDV3P)	1924
	CALL PRINT(2,1,T13,HEDT13)	1925
	CALL PRINT(2,1,T23,HEDT23)	1926
15	CONTINUE	1927
	RETURN	1928
	END	1929

C	I N S T R U C T I O N S	0001
C		0002
C	IWAL=0 TEMPERATURE BOUNDARY CONDITIONS	0003
C	IWAL=1 FLUX BOUNDARY CONDITIONS	0004
C		0005
C	IREG=1 COOLANT ONLY	0006
C	IREG=2 COOLANT+CLAD	0007
C		0008
C	IPSI=1 HEAT TO MOMENTUM RATIO EQUAL TO ONE	0009
C		0010
	IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)	0011
	DIMENSION RTW(16),ANU(16),TT(41,16)	0012
	COMMON/HO/HEDU(9),HEDV1(9),HEDV2(9),HEDP(9),HEDK(9),HEDD(9),	0013
	1 HEDM(9),HEDL(9),HEDN(9),HEDPR(9),HEDV1P(9),HEDV2P(9),HEDV3P(9),	0014
	3 HEDTC(9),HEDPSI(9),HEOTCL(9)	0015
	COMMON/BB/JS	0016
	COMMON/GEOMC/RM(30),ARE(30),ARW(30),ARN(30),ARP(30),DX1(30),	0017
	/ ANG(16),ANGM(16),VBP(16)	0018
	COMMON/CCLAD/CONCL,DI,FLUX1,FLUCL(16),TCL(10,16),IM,IMN	0019
	COMMON	0020
	1/TEMPC/URFTC,RESTC,NSWTC	0021
	1/VAR/U(30,16),V1(30,16),V2(30,16),VIS(30,16),PRUT(30,16),	0022
	2 TC(30,16),PST(30,16)	0023
	1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPOB,IGRID,IMAX(16),IND,JMIN(30),	0024
	2 ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP	0025
	1/AGRID/R(30),X1(30),X2(16)	0026
	1/GEOM/BE(30),BN(30),BW(30),BS(30),VP(30)	0027
	1/FLUPR/CONOC,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VB,TREF,PRAND	0028
	1/WALLF/TALAV,USAV,RTAL(16),USTAR(16),FR,YB,TNAVE,FLUC(16)	0029
	1/COEF/AP(41,16),AN(41,16),AS(41,16),AE(41,16),AW(41,16),SU(41,16)	0030
	LOGICAL INCALU,INCLV1,INCLV2,INPRO	0031
	INCALU=.TRUE.	0032
	INCLV1=.TRUE.	0033
	INCLV2=.TRUE.	0034
	INPRO=.TRUE.	0035
	N1=41	0036

N2=30	0037
N3=16	0038
READ (15) U, V1, V2, VIS, PRUT, USTAR, RTAL, THAVE, X1, X2, R, RH, ARE, ARW, ARN	0039
READ (15) ARP, DX1, ANG, VBP, BE, BW, BN, BS, DH, ATOT, RE, V8, PIT, DIA, PTR	0040
READ (15) IN, JN, JNL, IPROB, ISEC, INH, JNH, IL, ILP, IPIL, IMAX	0041
READ (15) VISCOS	0042
VAVG=V8	0043
IF (ISEC.EQ.0) INCLV1=.FALSE.	0044
IF (ISEC.EQ.0) INCLV2=.FALSE.	0045
NITER=0	0046
PJ=3.14159	0047
IND=1	0048
RESTC=0.0	0049
IL=2	0050
OTETA=X2/P2	0051
CE=0.0	0052
CN=0.0	0053
CW=0.0	0054
CS=0.0	0055
C*** NUMERICAL PROCEDURE	0056
READ(5,700) NSOLU, INUM, NSMTC, INOPR1, IMON, JMON, INORM, IWAL, IREF, JREF	0057
READ(5,700) IPSE	0058
READ(5,700) IREG, IM	0059
READ(5,701) CC, URFTC, FACTOR	0060
IF (IREG.GT.1) READ(5,701) CONCL, DI	0061
C*** PROBLEM DATA	0062
READ(5,701) ALHGR, CONDC, DENSIT, CSM, TREF	0063
C*** PLOT CURVES	0064
READ(5,700) ICURVE, ICURVS, JS	0065
C*** TITLES	0066
READ(5,010) HEDU, HEDV1, HEDV2, HEDP, HEDK, HEDD, HEOL,	0067
I HEDN, HEDPR, HEDVIP, HEDV2P, HEDV3P, HEDTC, HEDPSI, HEDTCL	0068
C*** PRINT INPUT DATA	0069
WRITE(6,700) NSOLU, INUM, NSMTC, INOPR1, IMON, JMON, INORM, IWAL, IREF, JREF	0070
WRITE(6,700) IPSE	0071
WRITE(6,700) IREG, IM	0072

WRITE(6,701)CC,URFTC,FACTOR	0073
IF(IREG.GT.1) WRITE(6,701)CONCL,DI,DIA	0074
WRITE(6,701)ALHGR,CONDC,DENSIT,CSH,TREF	0075
WRITE(6,700)ICURVE,ICURVS,JS	0076
WRITE(6,010) HEDU,HEDV1,HEDV2,HEDP,HEDK,HEDD,HEDL,	0077
1 HEDN,HEDPR,HEDVIP,HEDV2P,HEDV3P,HEDTC,HEDPSI,HEDTCL	0078
WRITE(6,210)	0079
WRITE(6,240) P1TR	0080
WRITE(6,270) TWAVE	0081
WRITE(6,220) VB	0082
WRITE(6,230) RE	0083
IF(INCALU) CALL PRINT(1,1,0,HEDU,(N,N2,N3)	0084
IF(INCLV1) CALL PRINT(1,1,V1,HEDV1,(N,N2,N3)	0085
IF(INCLV2) CALL PRINT(1,1,V2,HEDV2,(N,N2,N3)	0086
CALL PRINT(1,1,VIS,HEDN,(N,N2,N3)	0087
IF(IPROB.GT.1) CALL PRINT(1,1,PRUT,HEDPR,(N,N2,N3)	0088
WRITE(6,200)	0089
1TR=1	0090
IT=IM+IN	0091
IMP=IM+1	0092
IMN=IM-1	0093
DO 29 J=1,JN	0094
XST=X1(1L)*USTAR(J)/VISCOS	0095
WRITE(6,313) J,RTAL(J),USTAR(J),XST	0096
29 CONTINUE	0097
C	0098
C*** PARAMETERS AND CONTROL INDICES	0099
C	0100
C*** INITIAL OPERATIONS	0101
DIFFC=CONDC/DENSIT/CSH	0102
PRAND=VISCOS/DIFFC	0103
PE=RE*PRAND	0104
C	0105
C-----CALCULATE GEOMETRICAL QUANTITIES AND SET VARIABLES TO ZERO	0106
IF(IREG.GT.1) FLUX1=ALHGR/PI/DI	0107
FLUX=ALHGR/PI/DIA	0108

	CALL APSI(PSI,SPSI)	0109
	DO 30 J=1,JN	0110
	FLUC(J)=FLUX	0111
	IF(IREG.GT.1) FLUC(J)=FLUX	0112
	DO 32 I=1,IT	0113
32	TT(I,J)=0.0	0114
30	CONTINUE	0115
	CALL PRINT(IM,1,TT,HEDTC,IT,N1,N3)	0116
	CALL PRINT(1,1,TT,HEDTCL,IM,N1,N3)	0117
	CALL PRINT(1,1,PSI,HEOPSI,IN,N2,N3)	0118
	TH=TREF	0119
	TH=TC(1,1)	0120
	DO 257 J=1,JN	0121
	IH=IMAX(J)	0122
	DO 257 I=1,IH	0123
	VIS(I,J)=(VIS(I,J)-VISCOSI)*PSI(I,J)	0124
257	CONTINUE	0125
	DO 6 I=2,INH	0126
6	VP(I)=DTETA*(RI(I+1)-RI(I-1))*R(I)/2.	0127
	IND=0	0128
C----	INITIAL OUTPUT	0129
	CALL PRINT(1,1,VIS,HEDN,IN,N2,N3)	0130
	TENORM=TREF	0131
C		0132
	DO 100 J=2,JNM	0133
	IH=IMAX(J)	0134
	DO 101 I=2,IH	0135
C----	CALCULATE DIFFUSION COEFFICIENTS	0136
	IF(I.EQ.IH.AND.J.LE.JJL) GO TO 103	0137
	DN=(2.*DIFFC+VIS(I,J)+VIS(I-1,J))*BN(I)	0138
	DN=(2.*DIFFC+VIS(I,J)/PRUT(I,J)+VIS(I,J+1)/PRUT(I,J+1))*BN(I)	0139
	IF(I.EQ.IH) GO TO 104	0140
	DE=(2.*DIFFC+VIS(I,J)+VIS(I+1,J))*DE(I)	0141
	DS=(2.*DIFFC+VIS(I,J)/PRUT(I,J)+VIS(I,J-1)/PRUT(I,J-1))*BS(I)	0142
	GO TO 105	0143
104	DE=0.0	0144

	DS=0.0	0145
105	CONTINUE	0146
	IF(I)SEC.EQ.0) GO TO 150	0147
	V1E=V1(I+1,J)	0148
	V1W=V1(I,J)	0149
	V2N=V2(I,J+1)	0150
	V2S=V2(I,J)	0151
	CE=ARE(I)*V1E	0152
	CW=ARW(I)*V1W	0153
	CN=ARN(I)*V2N	0154
	CS=ARN(I)*V2S	0155
C----	ASSEMBLE MAIN COEFFICIENTS	0156
	IF(I.EQ.IH) GO TO 23	0157
	A1=CE*DX1(I)+CW*DX1(I-1)	0158
	CE=(DABS(A1)-A1)/DX1(I)/4.	0159
	CW=(DABS(A1)+A1)/DX1(I-1)/4.	0160
	A2=CN+CS	0161
	CN=(DABS(A2)-A2)/4.	0162
	CS=(DABS(A2)+A2)/4.	0163
	GO TO 150	0164
23	CE=0.0	0165
	CS=0.0	0166
	CW=CW/2.	0167
	CN=-CN/2.	0168
150	CONTINUE	0169
	AE(I,J)=CE+DE	0170
	AW(I,J)=CW+DW	0171
	AN(I,J)=CN+DN	0172
	AS(I,J)=CS+DS	0173
	AP(I,J)=AE(I,J)+AN(I,J)+AN(I,J)+AS(I,J)	0174
	SU(I,J)=ALHGR*U(I,J)/VB/DENSIT/ATOT/CSH/12.*VP(I)	0175
	IF(I.EQ.IH) SU(I,J)=SU(I,J)*VBP(J)/VP(I)	0176
	SU(I,J)=-SU(I,J)	0177
101	CONTINUE	0178
	GO TO 100	0179
103	CONTINUE	0180

TTETA=X21.0)	0181
CC=ANG(J) * (R(IH)-R(IH-1))/R(IH)/DTETA	0182
AE(IH,J)=0.0	0183
AN(IH,J)=CC/(1.+CC)	0184
AW(IH,J)=1./(1.+CC)	0185
AS(IH,J)=0.0	0186
API(IH,J)=1.0	0187
SUI(IH,J)=0.0	0188
100 CONTINUE	0189
IF(IHAL.EQ.0) GO TO 261	0190
DO 262 J=2,JNM	0191
API(J,J)=1.0	0192
AE(J,J)=1.0	0193
AW(J,J)=0.0	0194
AN(J,J)=0.0	0195
AS(J,J)=0.0	0196
SUI(J,J)=FLUX*X142)/CONQC	0197
262 CONTINUE	0198
261 CONTINUE	0199
C	0200
DX=X142)	0201
IF(IREG.GT.1) CALL CLAD(X2,CONDC,DIA,DX)	0202
C***SOLUTION OF THE DIFFERENCE EQUATION	0203
C	0204
ISTART=1	0205
IF(IHAL.EQ.0) ISTART=2	0206
C*** ITERATION LOOP	0207
300 NITER=NITER+1	0208
DO 837 I=1,ITR	0209
IF(NSOLU.EQ.0) CALL SOLV1(ISTART,2,JNM,TT,RES)	0210
IF(NSOLU.EQ.1) CALL SOLV2(ISTART,2,JNM,TT,RES,URFIC)	0211
IM=IMAX(1)	0212
DO 273 I=1,IH	0213
273 TT(I,1)=TT(I,2)	0214
DO 274 I=1,JNM	0215
274 TT(I,JN)=TT(I,JNM)	0216

	TT(IN,JN)=TT(INM,JN)	0217
837	CONTINUE	0218
C-----	INTERMEDIATE OUTPUT	0219
	RESTC=RES/TCNORM	0220
	DELTA=TT(IREF,JREF)-TREF	0221
	TTT=TT(IMGN,JMON)-DELTA	0222
	WRITE(6,313) NITER,RESTC,TTT	0223
	IF(MOD(NITER,INDPRI).NE.0) GO TO 301	0224
	IF(IWAL.EQ.0) GO TO 381	0225
	DELTA=TT(IREF,JREF)-TREF	0226
	DO 382 J=1,JN	0227
	IH=IMAX(J)	0228
	DO 382 I=1,IH	0229
	TT(I,J)=TT(I,J)-DELTA	0230
382	CONTINUE	0231
381	CONTINUE	0232
	CALL PRINT(IM,1,TT,HEOTC,IT,N1,N3)	0233
	CALL PRINT(1,1,TT,HEDTCL,IM,N1,N3)	0234
301	CONTINUE	0235
C-----	TERMINATION TESTS	0236
	IF(NITER.EQ.NSWTC) GO TO 302	0237
	IF(RESTC.GT.CC) GO TO 300	0238
302	CONTINUE	0239
C***	BOUNDARY VALUES	0240
	IF(IWAL.EQ.0) GO TO 281	0241
	DELTA=TT(IREF,JREF)-TREF	0242
	DO 282 J=1,JN	0243
	IH=IMAX(J)	0244
	DO 282 I=1,IH	0245
	TT(I,J)=TT(I,J)-DELTA	0246
282	CONTINUE	0247
281	CONTINUE	0248
	CALL PRINT(IM,1,TT,HEOTC,IT,N1,N3)	0249
	CALL PRINT(1,1,TT,HEDTCL,IM,N1,N3)	0250
	DO 610 J=1,JN	0251
	IF(IREG.GT.1) FLUCL(J)=CONCL*(TT(1,J)-TT(2,J))	0252

610	FLUC(J)=CONDC*(TT(IM,J)-TT(IMP,J))	0253
	CALL AVG2(FLUX,FLUC,JN,JNM)	0254
	IF(TREG.GT.1) CALL AVG2(FLUX1,FLUCL,JN,JNM)	0255
611	CONTINUE	0256
	CALL AVG1ATOT,TB,TT,U,VB,IN,INM,JN,JNM,IMAX,IM)	0257
	CALL AVG1(TW,TT,JN,JNM,IM)	0258
	IF(TREG.GT.1) CALL AVG1(TW1,TT,JN,JNM,1)	0259
	DO 612 J=1,JN	0260
	RTW(J)=CONDC*(TT(IM,J)-TW)/ALHGR	0261
	ANU(J)=FLUC(J)*DH/CONDC/(TT(IM,J)-TB)	0262
612	CONTINUE	0263
	CALL AVG2(ANUA,ANU,JN,JNM)	0264
	DO 613 J=1,JN	0265
	ANU(J)=ANU(J)/ANUA	0266
	IF(TREG.GT.1) FLUCL(J)=FLUCL(J)/FLUX1	0267
613	FLUC(J)=FLUC(J)/FLUX	0268
	WRITE(6,800) ALHGR,FLUX,TB,JN,KE,PRAND,PE,ANUA,TW1,FLUX1	0269
	WRITE(6,820)	0270
	DO 815 J=1,JN	0271
	IF(TREG.EQ.1) WRITE(6,313) J,RTW(J),FLUC(J),ANU(J)	0272
	IF(TREG.EQ.2) WRITE(6,319) J,RTW(J),FLUC(J),ANU(J),FLUCL(J)	0273
815	CONTINUE	0274
	IF(INORM.EQ.0) STOP	0275
	WRITE(6,615)	0276
	DO 614 J=1,JN	0277
	TH=IMAX(J)	0278
	DO 614 I=IM,TH	0279
	TT(I,J)=TT(I,J)/TT(I,1)	0280
614	CONTINUE	0281
	CALL PRINT(IM,1,TT,HEOTC,IT,N1,N3)	0282
	CALL PRINT(1,1,TT,HEOTCL,IN,N1,N3)	0283
010	FORMAT(9A4)	0284
700	FORMAT(10I5)	0285
701	FORMAT(8D10.3)	0286
210	FORMAT(1H0,47X,51HHEAT TRANSFER CALCULATIONS IN A BARE ROD BUNDLE	0287
1	//////	0288

220	FORMAT(//1H0,15X,21H BULK FLUID VELOCITY ,T60,1H=,3X,1PD11.3)	0289
230	FORMAT(1H0,15X,16HREYNOLDS NUMBER ,T60,1H=,3X,1PD11.3)	0290
240	FORMAT(1H0,15X,15HPITCH RATIO ,T60,1H=,3X,1PD11.3)	0291
250	FORMAT(1H0,15X,18HLAMINAR VISCOSITY ,T60,1H=,3X,1PD11.3)	0292
270	FORMAT(//1H0,15X,21H AVE SHEAR STRESS/RO ,T60,1H=,3X,1PD11.3)	0293
310	FORMAT(1H0,4HITER,4X,106HI-----ABSOLUTE RESIDUAL SOURCE SUM	0294
	1S-----I I-----FIELD VALUES AT MONITORING LOCATION,	0295
	2 1H(,12,1H,,12,1H),9H-----1/2X,2HNO,6X,4HUMOM,6X,4HVIMO,6X,	0296
	34HV2MO,6X,4HMASS,6X,4HTKIN,6X,4HDISP,10X,1HU,9X,2HV1,8X,2HV2,8X,1H	0297
	4P,9X,1HK,9X,1HO)	0298
311	FORMAT(1H ,13,4X,1P6D10.3,3X,1P6D10.3)	0299
402	FORMAT(///5X,1HI,7X,5HXU(1),6X,10HS.S.COEFF.)	0300
403	FORMAT(5X,12,3X,2(1PD11.3))	0301
200	FORMAT(1H130X,35HDISTRIBUTION OF WALL SHEAR STRESSES//	0302
	/5X,1HJ,10X,9HTAL/FALAV,10X,5HUSTAR//)	0303
313	FORMAT(4X,14,8X,011.4,6X,1PD11.3,6X,1PD11.3,6X,1PD11.3//)	0304
615	FORMAT(1H1,' NORMALIZATION OF THE RESULTS')	0305
800	FORMAT(31HOHEAT INFORMATION CALCULATED IS//	0306
	110X,50HALHGR, LINEAR HEAT GENERATION RATIO.....=,1PD15.6/	0307
	210X,50HFLUX, OUTSIDE CLAD HEAT FLUX.....=,1PD15.6/	0308
	310X,50HTB, COOLANT BULK TEMPERATURE.....=,1PD15.6/	0309
	410X,50HTW, AVERAGE CLAD TEMPERATURE.....=,1PD15.6/	0310
	510X,50HRE, REYNOLDS NUMBER.....=,1PD15.6/	0311
	610X,50HPRAND, PRANDTL NUMBER.....=,1PD15.6/	0312
	710X,50HPE, PECLET NUMBER.....=,1PD15.6/	0313
	810X,50HANUA, AVERAGE NUSSELT NUMBER.....=,1PD15.6/	0314
	810X,50HTWI AVERAGE INNER CLAD TEMPERATURE.....=,1PD15.6/	0315
	810X,50HFLUX1 AVERAGE INNER CLAD FLUX.....=,1PD15.6)	0316
820	FORMAT(1H130X,31HOISTRIBUTION OF HEAT PARAMETERS//	0317
	/5X,1HJ,10X,15HWALL TEMP RATIO,5X,14HOUT FLUX RATIO,5X,8HLOCAL NU,	0318
	15X,10HINNER FLUX/)	0319
	STOP	0320
	END	0321
	SUBROUTINE APSI(PSI,IPSI)	0322
	IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)	0323
	DIMENSION PSI(30,16)	0324

	COMMON	0325
	1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPOB,IGRID,IMAX(16),IND,JMIN(30),	0326
	2 ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP	0327
	1/AGRID/R(30),X1(30),X2(16)	0328
	1/FLUPR/CONDC,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,V8,TREF,PRAND	0329
C		0330
C	SUBROUTINE FOR CALCULATION OF HEAT TO MOMENTUM EDDY DIFFUSIVITY RATIO	0331
C		0332
	GO TO (1,2),IPST	0333
1	CONTINUE	0334
	DO 10 J=1,JN	0335
	DO 10 I=1,IN	0336
10	IF(I,J)=1,0	0337
	RETURN	0338
6		0339
2	CONTINUE	0340
	RETURN	0341
	END	0342
	SUBROUTINE CLAD(X2,CONDC,DIA,DX)	0343
	IMPLICIT REAL*8(A-H,O-Z); INTEGER (I-N)	0344
	DIMENSION X2(16),R(10)	0345
	COMMON/CCLAO/CONCL,DI,FLUXI,FLUCL(16),TCL(10,16),IM,INM	0346
	COMMON	0347
	1/TEMPC/URFTC,RESTC,NSWTC	0348
	1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPOB,IGRID,IMAX(16),IND,JMIN(30),	0349
	2 ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP	0350
	1/COEF/AP(41,16),AN(41,16),AS(41,16),AE(41,16),AW(41,16),SU(41,16)	0351
C		0352
	IN=IN+IM-1	0353
	INM=IN-1	0354
	INM=IM-1	0355
	DTETA=X2(2)	0356
	DO 1 J=1,JN	0357
	IMAX(J)=IMAX(J)+IM-1	0358
	IH=IMAX(J)	0359
	MP=IH-IM	0360

	DO 1 I=1,MP	0361
	I=IM-I+1	0362
	AE(I,J)=AE(I-IMN,J)	0363
	AW(I,J)=AW(I-IMN,J)	0364
	AN(I,J)=AN(I-IMN,J)	0365
	AS(I,J)=AS(I-IMN,J)	0366
	AP(I,J)=AP(I-IMN,J)	0367
	SU(I,J)=SU(I-IMN,J)	0368
1	CONTINUE	0369
	R(1)=D1/2.	0370
	R(IM)=DJA/2.	0371
	DR=(DJA-D1)/2./DFLOAT(IMN)	0372
	DO 10 I=2,IMN	0373
10	R(I)=R(I-1)+DR	0374
	DO 20 J=2,JNM	0375
	AP(I,J)=1.0	0376
	AE(I,J)=1.0	0377
	AW(I,J)=0.0	0378
	AN(I,J)=0.0	0379
	AS(I,J)=0.0	0380
	SU(I,J)=FLUXI*DR/CONCL	0381
	DO 21 I=2,IMN	0382
	BE =.25*DTETA*(R(I+1)+R(I))/(R(I+1)-R(I))	0383
	BW =.25*DTETA*(R(I-1)+R(I))/(R(I)-R(I-1))	0384
	BN =.25*(R(I+1)-R(I-1))/R(I)/DTETA	0385
	BS=BN	0386
	AE(I,J)=2.*CONCL*BE	0387
	AW(I,J)=2.*CONCL*BW	0388
	AN(I,J)=2.*CONCL*BN	0389
	AS(I,J)=2.*CONCL*BS	0390
	SU(I,J)=0.0	0391
	AP(I,J)=AE(I,J)+AW(I,J)+AN(I,J)+AS(I,J)	0392
21	CONTINUE	0393
	AE(IM,J)=CONDC/CONCL*DR/DX	0394
	AW(IM,J)=1.0	0395
	AN(IM,J)=0.0	0396

	AS(IH,J)=0.0	0397
	SU(IH,J)=0.0	0398
	AP(IH,J)=AE(IH,J)+AW(IH,J)	0399
20	CONTINUE	0400
	RETURN	0401
	END	0402
	SUBROUTINE AVG(ATOT,PB,PH,U,VB,IN,INN,JN,JNM,IMAX,IN)	0403
	IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)	0404
	DIMENSION PH(41,16),U(30,16),IMAX(16)	0405
	COMMON	0406
	1/AGRID/R(30),X1(30),X2(16)	0407
	C*****	0408
	C SUBROUTINE FOR COMPUTATION OF BULK TEMPERATURE	0409
	C*****	0410
	IMN=IM-1	0411
	IMP=IM+1	0412
	AV=0.0	0413
	DO 22 J=1,JN	0414
	TAV=0.0	0415
	IH=IMAX(J)	0416
	DO 12 I=IMP,IH	0417
	DAV=(R(I-IMN)*U(I-IMN,J)+R(I-IM)*U(I-IM,J)+R(I-IMP)*U(I-IMP,J)+R(I-1)*U(I-1,J))/4.	0418
	IMN=I-IMN-1	0419
12	TAV=TAV+DAV	0420
	TAV=TAV*X2(2)	0421
	IF(J.EQ.1.OR.J.EQ.JN) TAV=0.5*TAV	0422
22	AV=AV+TAV	0423
	VB=AV/ATOT	0424
	C	0425
	AV=0.0	0426
	DIETA=X2(2)	0427
	DO 20 J=1,JN	0428
	TAV=0.0	0429
	IH=IMAX(J)	0430
	DO 10 I=IMP,IH	0431
	DAV=(R(I-IMN)*PH(I,J)*U(I-IMN,J)+R(I-IM)*PH(I,J)*U(I-IM,J)+R(I-IMP)*PH(I,J)*U(I-IMP,J)+R(I-1)*PH(I,J)*U(I-1,J))/4.	0432

1MN)-R(I-1MN-1))/2.	0433
10 TAV=TAV+DAV	0434
TAV=TAV*X2(2)	0435
IF(J.EQ.1.OR.J.EQ.JN) TAV=0.5*TAV	0436
20 AV=AV+TAV	0437
C*** CALCULATE BULK TEMPERATURE	0438
PB=AV/ATOT/VB	0439
RETURN	0440
END	0441
SUBROUTINE AVG1(PB,PH,JN,JNM,I)	0442
IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)	0443
DIMENSION PH(41,16)	0444
S=0.5*(PH(I,1)+PH(I,JN))	0445
DO 1 J=2,JNM	0446
1 S=S+PH(I,J)	0447
PB=S/DFLOAT(JNM)	0448
RETURN	0449
END	0450
SUBROUTINE AVG2(PB,PH,JN,JNM)	0451
IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)	0452
DIMENSION PH(16)	0453
S=0.5*(PH(1)+PH(JN))	0454
DO 1 J=2,JNM	0455
1 S=S+PH(J)	0456
PB=S/DFLOAT(JNM)	0457
RETURN	0458
DEBUG SUBCHK	0459
END	0460
SUBROUTINE SOLV2(ISTART,JSTART,JEND,PHI,RES,UREL)	0461
IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)	0462
DIMENSION PH(41,16),A(41),B(41),C(41),D(41),PHO(41,16)	0463
COMMON	0464
1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROR,IGRID,IMAX(16),IND,JM(30),	0465
2 ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP	0466
1/AGRID/R(30),X1(30),X2(16)	0467
1/COEF/API(41,16),AN(41,16),AS(41,16),AF(41,16),AW(41,16),SU(41,16)	0468

E		0469
C**SOLUTION BY POINT ITERATION		0470
C		0471
	IF(INUM.EQ.0) GO TO 650	0472
	DO 651 J=1,JN	0473
	DO 651 I=1,IN	0474
	PH0(I,J)=PHI(I,J)	0475
651	CONTINUE	0476
650	CONTINUE	0477
	RES=0.0	0478
	DO 201 J=JSTART,JEND	0479
	IH=IMAX(J)	0480
	DO 202 I=I\$TART,IH	0481
	DDS=PHI(I,J=1)	0482
	IF(INUM.EQ.1) DDS=PH0(I,J-1)	0483
	IF(I.EQ.1) GO TO 10	0484
	ANUM=AE(I,J)*PHI(I+1,J)+AM(I,J)*PHI(I-1,J)+AN(I,J)*PHI(I,J+1) +	0485
	/ AS(I,J)*DDS +SU(I,J)	0486
	GO TO 12	0487
10	ANUM=AE(I,J)*PHI(I+1,J)+SU(I,J)	0488
12	CONTINUE	0489
	IF(AP(I,J).EQ.0.0) GO TO 202	0490
	Z=PHI(I,J)	0491
	PHI(I,J)=ANUM/AP(I,J)	0492
	RS=DABS(Z-PHI(I,J))	0493
	IF(RS.GT.RES) RES=RS	0494
202	CONTINUE	0495
201	CONTINUE	0496
	RETURN	0497
		0498
	END	0499
	SUBROUTINE SOLV1(I\$TART,J\$TART,J\$END,PHI,RES)	0500
	IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)	0501
	DIMENSION PHI(41,16),A(41),B(41),C(41),D(41)	0502
	COMMON	0503
	1/ALL/IT,IT,JN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),	0504

2	ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP	0505
	1/COEF/AP(41,16),AN(41,16),AS(41,16),AE(41,16),AM(41,16),SU(41,16)	0506
C		0507
C	**SOLUTION BY MATRIX FACTORIZATION ALONG RADIAL LINE	0508
C		0509
	A(1)=0.0	0510
	C(1)=0.0	0511
	RES=0.0	0512
	DO 100 J=JSTART,JEND	0513
	IH=IMAX(J)	0514
	DO 101 I=ISTART,IH	0515
	A(I)=AE(I,J)	0516
	B(I)=AW(I,J)	0517
	IF(J.EQ.2) GO TO 10	0518
	IF(J.EQ.JNM) GO TO 11	0519
	C(I)=AN(I,J)*PHI(I,J+1)+AS(I,J)*PHI(I,J-1)+SU(I,J)	0520
	D(I)=AP(I,J)	0521
	GO TO 12	0522
10	C(I)=AN(I,J)*PHI(I,J+1) +SU(I,J)	0523
	D(I)=AP(I,J)-AS(I,J)	0524
	GO TO 12	0525
11	C(I)= +AS(I,J)*PHI(I,J-1)+SU(I,J)	0526
	D(I)=AP(I,J)-AN(I,J)	0527
12	CONTINUE	0528
	IF(I.EQ.1) GO TO 101	0529
	TERM=1./ID(I)-B(I)*A(I-1)	0530
	A(I)=A(I)*TERM	0531
	C(I)=(C(I)+B(I)*C(I-1))*TERM	0532
101	CONTINUE	0533
	DO 102 II=ISTART,IH	0534
	I=IH-II+ISTART	0535
	Z=PHI(I,J)	0536
	PHI(I,J)=A(I)*PHI(I+1,J)+C(I)	0537
	RS=DABS(Z-PHI(I,J))	0538
	IF(RS.GT.RES) RES=RS	0539
102	CONTINUE	0540

100 CONTINUE	0541
RETURN	0542
END	0543
SUBROUTINE PRINT(ISTART,JSTART,PHI,HEAD,IE,IK,JK)	0544
IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)	0545
DIMENSION PHI(IK,JK),HEAD(9),STORE(50)	0546
DIMENSION F(7),F4(11)	0547
COMMON	0548
1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,9MIN(30),	0549
2 ISEC,IPILE,JAL,ICONV,NSOLU,INUM,JL,JLP	0550
1/AGRID/R(30),X1(30),X2(16)	0551
DATA F/4H1H,4H,A4,,4H13,,4H11,4H10,,4H7X,,	0552
DATA F4/4H1H,4H,A4,,4H13,,4H11,4H10,,4H7X,,	0553
DATA F4/4H1H,4H,A4,,4H13,,4H11,4H10,,4H7X,,	0554
1 DATA H1,H774H H1,4H W =7	0555
DATA H1,H774H H1,4H W =7	0556
C	0557
C*** PRINT OUT SUBROUTINE	0558
C	0559
ISKIP=1	0560
JSKIP=1	0561
WRITE(6,110)HEAD	0562
ISTA=ISTART-12	0563
100 CONTINUE	0564
ISTA=ISTA+12	0565
IEND=ISTA+11	0566
IEND=MIN(0,IEND)	0567
F(6)=F4(IEND-IESTA)	0568
WRITE(6,F) H1, '1,1=ISTA,IEND,ISKIP', H1	0569
WRITE(6,112)	0570
DO 101 JJ=JSTART,JN,JSKIP	0571
J=JSTART+JN-JJ	0572
IH=IMAX(J)	0573
DO 120 I=ISTA,IEND	0574
A=PHI(I,J)	0575
IF(DABS(A).LT.1.D-20) A=0.0	0576

120	STORE(I)=A	0577
	WRITE(6,113) J,(STORE(I),I=ISTA,IEND,ISKIP),X2(J)	0578
101	CONTINUE	0579
C	-----	0580
	IF(IEND.LT.IE)GO TO 100	0581
	RETURN	0582
110	FORMAT(1H0,20(2H*-),7X,9A4,7X,20(2H*-))	0583
112	FORMAT(3H J)	0584
113	FORMAT(1H ,13,1P12D10.2,0PF7.3)	0585
	END	0586