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SYSTEM MODEL FOR ANALYSIS OF THE MIRROR FUSION-FISSION REACTOR

D. J. Bender
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October 12, 1977

MASTER

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SYSTEM MODEL FOR ANALYSIS OF THE MIRROR FUSION-FISSION REACTOR

ABSTRACT

This report describes a system model for the mirror fusion-fission reactor. In this model we include a reactor description as well as analyses of capital cost and blanket fuel management. In addition, we provide an economic analysis evaluating the cost of producing the two hybrid products, fissile fuel and electricity. We also furnish the results of a limited parametric analysis of the modeled reactor, illustrating the technological and economic implications of varying some important reactor design parameters.

1. INTRODUCTION

As part of the Mirror Hybrid Program for FY 1976, we developed a model for the mirror fusion-fission (hybrid) reactor to assess the economic implications of variations in the reactor design and in the blanket fuel management scheme. Results from this modeling effort have been presented at several meetings.¹⁻⁴ In this document, we describe the model (Sec. 2) and the results from a parametric study (Sec. 3) that were used to design an optimized mirror hybrid reactor.

Our objective in this parametric analysis was to minimize the cost of producing the fissile material (^{239}Pu and ^{233}U) used in fission reactors that generate electricity. (Our rationale is discussed in Refs. 4 and 5.) To evaluate the cost of producing fissile fuel, we developed a system model with the following features:

- A reactor model to size reactor components and evaluate power flows, blanket geometry, and first-wall neutron loading as a function of a set of independent reactor parameters.
- A capital cost analysis for the reactor.
- A fuel-management package to evaluate the thermal power of the blanket and fissile breeding as a function of blanket exposure.
- An economic analysis to evaluate the cost of producing the two hybrid products, fissile fuel and electricity.

Two points concerning our analysis should be made at this juncture. First, although we describe a specific model in this report, the working model

is dynamic in that we continue to update and improve it. Our purpose here is to illustrate the *methodology* of fusion reactor system analysis (Sec. 2) and to provide broad and useful insight into the reactor design that can be developed with this type of model (Sec. 3).

Second, the relationships used to describe the various reactor components and their costs frequently are relatively crude scaling laws. Thus, the values of dependent variables generated by the model are expected to be less accurate than those obtained from an actual conceptual design study based on the same set of input parameters. However, the value of the model lies in its ability to show the *tradeoff* among the various input design parameters. Thus, it enables us to develop optimum design parameters *before* a point conceptual design is undertaken.

2. SYSTEM MODEL

2.1 REACTOR MODEL

2.1.1 Magnet Design

We assume the magnetic field is supplied by a superconducting Yin-Yang magnet that creates a minimum-B, MHD-stable, magnetic well. The geometrical features of this magnet are shown in Fig. 2.1. For application to reactors, the magnet design has been modified so that each half of the coil consists of two separate conductor bundles (see Figs. 2.2 and 2.3). Note that this design exhibits the attractive feature that the mirror points are very near the inside diameter of the conductor. The geometry of the magnet is characterized by the major radii of the inner and outer bundles, R_1 and R_2 , the minor radii of the inner and outer bundles, r_1 and r_2 , and the conductor dimensions, w and h .

As input for the coil calculation, we chose the following set of parameters:

$$R_1, R_2, \alpha_M, R_{v1}, R_{v2}, B_c, n_M, \Omega_{m1}, \Omega_{m2}. \quad (2.1)$$

Here, α_M is the ratio of the minor radii of the inner and outer bundles,

$$\alpha_M = r_1/r_2, \quad (2.2)$$

R_1 and R_2 are the major radii of the inner and outer bundles, and R_{v1} and R_{v2} are the mirror ratios at the inner and outer bundles,

$$R_{v1} = \frac{B_{M1}}{B_0}, \quad (2.3)$$

$$R_{v2} = \frac{B_{M2}}{B_0}. \quad (2.4)$$

The quantity B_c is the maximum field at the conductor, which is assumed to occur at the locations shown in Fig. 2.3 and η_M is the magnet efficiency defined as

$$\eta_M = \frac{B_{M1}}{B_c}. \quad (2.5)$$

The quantities Ω_{m1} and Ω_{m2} are the aspect ratios of the two conductor bundles,

$$\Omega_{m1} = \frac{w_1}{h_1}, \quad (2.6)$$

$$\Omega_{m2} = \frac{w_2}{h_2}.$$

With these parameters, we can determine the complete magnet geometry, the number of ampere-turns, and the ampere-metres of conductor (see Appendix A). Thus, the solution variables are

Fig. 2.1 (left). Yin-Yang coil configuration.

Fig. 2.2 (below). Magnetic field on-axis.

Fig. 2.3. Reactor coil design.

$$r_1, r_2, w_1, h_1, w_2, h_2, (NI)_1, (NI)_2, (AM)_1, (AM)_2, \quad (2.7)$$

where (NI) is ampere-turns and (AM) is ampere-metres.

Given the field design, we also can specify certain plasma parameters. The mirroring plasma is contained between the two mirror points (Fig. 2.4) so that the plasma length is

$$L_p = 2R_1. \quad (2.8)$$

Also, the radial extent of the plasma is determined by the last closed contour in the field. We have used single-filament models in the MAFCO⁶ code to determine the outer field line that defines the radial plasma boundary (see Fig. 2.5). The plasma radius at the midplane r_p can be described by

$$\frac{r_p}{R_1} = 0.3 + 0.626 \exp(-0.79 R_{v1}) \quad (2.9)$$

Also, the fan half-height t_F (i.e., the outer field line intercept at the mirror point, $z = R_1$) is

$$\frac{t_F}{r_1} = 0.2389 - 6.768 \times 10^{-2} R_{v1} + 6.367 \times 10^{-3} R_{v1}^2 \quad (2.10)$$

2.1.2 Plasma Physics

In this section, we evaluate the fusion power and plasma Q as a function of the input parameters: neutral atom (D^0, T^0) injection energy, W_{inj} ; vacuum mirror ratio, R_{v1} ; and the angle of injection relative to the field line, θ_{inj} ($\theta_{inj} = 90$ deg implies that injection is perpendicular to the field line). In our plasma model, we assume classical mirror confinement. That is, a neutral atom is injected into the plasma, trapped by an ionization collision, and held in the confinement volume until it is scattered into the loss-cone by ion-ion and ion-electron collisions. The kinetics of this velocity-space

Fig. 2.4. Geometric relationship of plasma and magnet.

Fig. 2.5. Field line defining the radial plasma boundary (x, z plane).

Fig. 2.6. Confinement parameter at $W_p = 100$ keV.

diffusion process are evaluated separately with a Fokker-Planck model⁷ and the results specified parametrically to the reactor code.

Values for the ratio of the perpendicular plasma pressure to the vacuum magnetic field pressure,

$$\beta = \frac{P_{\perp}}{B^2/2\mu_0} \quad (2.11)$$

and the confinement parameter ($n\tau$), are shown in Figs. 2.6 and 2.7.⁸ The ($n\tau$) product appearing in Fig. 2.6 is for an average plasma energy, W_p , of 100 keV; this quantity scales with plasma energy to the 3/2 power,

$$n\tau = (n\tau)_{100 \text{ keV}} \left(\frac{W_p}{100} \right)^{3/2} \quad (2.12)$$

The average plasma energy is related to the injection energy by a multiplicative constant,

Fig. 2.7. Stability limited beta.

Fig. 2.8. Fusion rate coefficient for mirror plasmas.³

$$C_w = W_p / W_{inj} \quad (2.13)$$

which is evaluated in the Fokker-Planck calculation.

In Eq. (2.11), $p_\perp$ is the total perpendicular pressure due to fuel ions (D^+ and T^+), alpha particles, and electrons. The values of β appearing in Fig. 2.7 are the maximum values permitted by stability considerations; we use these values for the reactor plasma. To evaluate the fuel ion density, n_0 , in the center of the machine, Eq. (2.11) is rearranged in the form

$$n_0 = \frac{\beta(B_0^2/2\mu_0)}{(kT_\perp)_f (1.6 \times 10^{-16} \text{ J/keV})} \quad (2.14)$$

Here, B_0 is the central vacuum field (see Fig. 2.3). The characteristic fuel ion energy $(kT_\perp)_f$ is related to the injection energy by a multiplicative constant,

$$C_\perp = \frac{(kT_\perp)_f}{W_{inj}} \quad (2.15)$$

that is evaluated from the Fokker-Planck results.

If we assume equal deuterium and tritium densities in the plasma,

$$n_{D,0} = n_{T,0} = 1/2 n_0 \quad (2.16)$$

the fusion power density at the center of the machine is

$$q_{f,0}'''' = \frac{n_0^2}{4} \langle \sigma v \rangle_f E_f \quad (2.17)$$

where E_f is the energy release per fusion event (17.6 MeV) and $\langle \sigma v \rangle_f$ is the fusion rate coefficient (see Fig. 2.8).⁹ The total fusion power from the plasma, P_1 , is obtained by integrating the power density over the plasma volume,

$$P_1 = \int q_f'''' dV \quad (2.18)$$

This calculation has been performed in an appropriate manner by Moir and Taylor,¹⁰ assuming the axial magnetic field variation generated by a Yin-Yang magnet. The results of this calculation may be expressed in the form

$$P_1 = q_{f,0}'' \cdot \left(\pi r_{p,p}^2 \right) \cdot C_{RA}^j(R) . \quad (2.19)$$

The coefficient $C_A(R)$ accounts for axial variations in the plasma density as a function of the plasma mirror ratio, R , where

$$R = R_{v1} / \sqrt{1-\beta} , \quad (2.20)$$

(see Fig. 2.9). The coefficient C_R^j accounts for the radial density dependence

$$C_R^j = \frac{j^2}{(j+1)(j+2)} ,$$

where the radial density distribution at the midplane is assumed to be of the form

$$\frac{n}{n_0} = 1 - \left(\frac{r}{r_p} \right)^j . \quad (2.21)$$

Detailed plasma physics considerations for reactor-size plasmas predict very flat radial profiles, which we approximate with $j = 20$.

The plasma Q is the ratio of fusion power to injection power. Because the rate of particle injection is equal to the rate of particle loss in steady state operation,

$$\begin{aligned} Q &= \frac{n^2/4 \langle \sigma v \rangle_f E_f}{n w_{inj} / \tau} , \\ &= \frac{1}{4} (n\tau) \langle \sigma v \rangle_f \frac{E_f}{w_{inj}} . \end{aligned} \quad (2.22)$$

2.1.3 Blanket Geometry

In Fig. 2.1, note that a spherical volume is available inside the magnet conductor bundles. Within this space resides the blanket, which has the form of a spherical shell. In Fig. 2.10, the two halves of the coil have been separated to expose the blanket. The component layout inside the inner conductor bundle is illustrated in Fig. 2.11. Our next task is to size these components, establish the first-wall radius, and determine the fraction of spherical area that can be covered with the blanket (while leaving the required holes for plasma leakage fans and for neutral-beam injection).

Fig. 2.9 (above). Axial power coefficient.

Fig. 2.10 (right). Blanket geometry.

The determination of the width of the inner conductor bundle, w_1 , was described in Sec. 2.1.1. On the basis of previous experience with coil design,¹¹ we assume a coil-case thickness of $t_{cc} = 0.25$ m.

The required attenuation of the fusion neutron energy by the shield is determined by specifying (arbitrarily) that the neutron heating in the coil equal the thermal heat leak into the (cryogenic) coil. Because the heat leak is surface dependent, the thermal heat leak (in megawatts) is scaled on the square of the magnet dimension obtained from a previous hybrid point design,¹²

$$q_{CHL}^o = 20 \times 10^{-3} \left(\frac{L_M}{25} \right)^2, \quad (2.23)$$

where L_M is a characteristic magnet dimension (metres) defined as

$$L_M = 2 \left(\frac{R_1 + R_2}{2} \right). \quad (2.24)$$

The required number of decades of fusion neutron power attenuation is then

$$a_t = \log_{10} \left(\frac{0.8 P_1}{q_{CHL}^o} \right), \quad (2.25)$$

where P_1 is the fusion power in megawatts. If we assume two decades of attenuation in the blanket¹² and one decade of attenuation per 20 cm of shield,^{11,12} the required shield thickness (in metres) is

$$t_{SH} = 0.2(a_t - 2) . \quad (2.26)$$

The inside radius of the shield (metres) is

$$R_{SH} = R_1 - w_1 - t_{cc} - t_{SH} . \quad (2.27)$$

We assume that helium will be used as the coolant for the blanket. The plenum dimension indicated in Fig. 2.11 includes the area for both the inlet

Fig. 2.11. Component layout inside the conductor bundles.

and outlet flow. On the basis of thermal-hydraulic considerations, the plenum dimension is scaled from a previous point design for the mirror hybrid blanket,

$$t_{PL} = 0.58 \left[\frac{\hat{P}_5}{R_{SH} (\rho_{HE} \Delta p_{PL})^{1/2} C_P \Delta T_{HE}} \right] . \quad (2.28)$$

Here, $\hat{P}_5$ is the maximum blanket thermal power, ρ_{HE} is the helium density, Δp_{PL} is the helium pressure drop in the plenum, C_P is the helium specific heat, and ΔT_{HE} is the helium temperature rise in the blanket. The blanket thermal power, P_5 , is the product of the fusion neutron power, P_3 ,

$$P_3 \approx 0.8 P_1 , \quad (2.29)$$

and the blanket energy multiplication, M ,

$$P_5 = M P_3 . \quad (2.30)$$

The blanket multiplication varies over the life of the blanket as discussed in Sec. 2.1.4. In Eq. (2.28), the plenum dimension is based on the maximum thermal power (and thus the maximum blanket multiplication). We evaluate this quantity in Sec. 2.2.

The blanket thickness for the region containing the fertile material and tritium breeding material, t_{BL} , is determined from detailed neutronics considerations and is specific to the model. The first-wall radius thus becomes

$$R_{FW} = R_{SH} - t_{PL} - t_{BL} , \quad (2.31)$$

and the first-wall 14-MeV neutron current (first-wall load in megawatts per square metre) is

$$\Gamma = \frac{P_3}{4\pi R_{FW}^2} . \quad (2.32)$$

The intersection of the outer plasma field line (radial plasma boundary) with the blanket is shown in Fig. 2.12. The hole in the blanket required to admit the plasma fan has the shape and dimensions shown in Fig. 2.13. The area of the hole in the blanket is

Fig. 2.12 (above). Blanket-plasma geometry.

Fig. 2.13 (right). Geometry of blanket hole for the plasma fan.

$$A_{FH} = 4 \theta_{FW} R_{FW} t_{FW} . \quad (2.33)$$

To calculate θ_{FW} and t_{FW} (defined in Fig. 2.12), we must find the two intercepts of the first-wall radius and the radial plasma boundary, z_{FX} and z_{FY} . The plasma boundary is curve-fit with quadratics,

$$C_x = C_{x1} + C_{x2}z + C_{x3}z^2 , \quad (z > 0) , \quad (2.34a)$$

$$C_y = C_{y1} + C_{y2}z + C_{y3}z^2 , \quad (z < 0) , \quad (2.34b)$$

as described in Appendix B. The intercepts are given by the solution to the two equations,

$$R_{FW}^2 = [C_x(z_{FX})]^2 + z_{FX}^2 , \quad (2.35a)$$

$$R_{FW}^2 = [C_y(z_{FY})]^2 + z_{FY}^2 , \quad (2.36b)$$

which are fourth-order equations in z_{FX} and z_{FY} . Then,

$$\theta_{FW} = \cos^{-1}(z_{FY}/R_{FW}) , \quad (2.37a)$$

$$t_{FW} = (R_{FW}^2 - z_{FK}^2)^{1/2} . \quad (2.37b)$$

Holes also must be provided in the blanket for neutral-beam injection. The required beam power (megawatts) is

$$P_{10} = P_1/Q ; \quad (2.38)$$

if we assume equal D^0 and T^0 injection energies, the required neutral-beam current (amperes) is

$$I_{inj} = \frac{P_{10}}{W_{inj} \times 10^{-3}} , \quad (2.39)$$

where the injection energy, W_{inj} , is in kiloelectron volts. On the basis of a previous point design,¹² we assume a beam current density at the blanket of 500 A/m^2 . The blanket hole area (square metres) for injection is thus

$$A_{IH} = I_{inj}/500 . \quad (2.40)$$

The blanket area may now be evaluated from the expression

$$A_{BL} = 4\pi R_{FW}^2 - 2A_{FH} - A_{IH} , \quad (2.41)$$

where the area of the hole for the plasma fan has been multiplied by two to account for the leakage from each end of the plasma. The fractional blanket coverage is thus

$$\Omega_{BL} = \frac{A_{BL}}{4\pi R_{FW}^2} . \quad (2.42)$$

The arrangement of the blanket internals is shown in Fig. 2.14; the fuel region immediately behind the first wall experiences the 14-MeV fusion-neutron flux. We use the term "fuel" to denote the material that contains the fertile isotope (e.g., UO_2 or UC). Behind the fuel region is the tritium breeding region, with neutrons leaking out of the fuel and into the tritium breeding material.

The initial mass of fertile material in the blanket is

$$M_{FM} = A_{BL} t_{FR} (VF)_{FR} \rho_{FM} , \quad (2.43)$$

where t_{FR} is the (radial) thickness of the fuel region, $(VF)_{FR}$ is the volume fraction of fuel in the fuel region, and ρ_{FM} is the density of the fertile material. These three quantities are determined from detailed neutronic, thermal-hydraulic, and structural design of the blanket (as described in Refs. 12 and 13) and are specified to the model. Similarly, the initial mass of tritium breeding material in the blanket is

$$M_{TBM} = A_{BL} t_{TR} (VF)_{TR} \rho_{TBM} , \quad (2.44)$$

where t_{TR} is the (radial) thickness of the tritium breeding region, $(VF)_{TR}$ is the volume fraction of tritium breeding material in the tritium breeding region, and ρ_{TBM} is the density of the tritium breeding material.

2.1.4 Blanket Neutronics

The nuclear performance calculations that evaluate the interaction of the 14-MeV fusion neutrons with the blanket are performed with neutron transport codes.¹⁴ The results of these calculations are used as input to the reactor model.

Fig. 2.14. Blanket submodule.

Fig. 2.15. Performance vs exposure for a UC blanket.

To date, we have examined three blanket fuels: uranium carbide (UC), uranium-7 wt% molybdenum (U/Mo), and thorium metal. The nuclear performances of these blankets are shown in Figs. 2.15-2.17 where we plot the pertinent nuclear parameters as a function of blanket exposure. These parameters are:

- M , the blanket energy multiplication or the ratio of the thermal energy release in the blanket per incident fusion neutron to the energy (14.1 MeV) of the incident fusion neutron.
- Pu/n or U/n , the net fissile breeding ratio or the net number of fissile atoms produced per incident fusion neutron.
- T/n , the tritium breeding ratio or the number of tritium atoms bred per incident fusion neutron.
- BU , the fuel burnup or the fraction of initial fertile atoms that has been fissioned.
- Pu or U , the fraction of initial fertile atoms that has been converted to fissile atoms.

A significant time-dependent effect is involved. As fissile material builds up in the fuel, some of these bred atoms are fissioned, providing increased energy multiplication and increased neutron yield which thus increases tritium breeding.

In the three ratios listed above, M , Pu/n (or U/n), and T/n , the number of neutrons entering the blanket is the denominator of the ratio. In the reactor, some of the neutron yield from the plasma is lost through the leakage to ports and beam ports. To first order, the plasma is approximated as an

Fig. 2.16. Performance vs exposure for a U/Mo blanket.

Fig. 2.17. Performance vs exposure for a thorium blanket.

isotropic point neutron source at the center of the magnet; thus the fraction of fusion neutrons entering the blanket is the fractional blanket coverage, Ω_{BL} . If we recall that P_3 is the fusion neutron power, the blanket thermal power is

$$P_5 = \Omega_{BL} M_B P_3, \quad (2.45)$$

and the breeding ratios are

$$Pu/n = \Omega_{BL} (Pu/n)_l, \quad (2.46)$$

$$T/n = \Omega_{BL} (T/n)_l, \quad (2.47)$$

where the subscript l denotes the "local" values shown in Fig. 2.15. Again we emphasize that the above quantities are time-dependent.

2.1.5 Power Flow

In the block diagram of the reactor power flow (Fig. 2.18), we summarize the entire reactor power flow. The starting point is the fusion power, which is a function of magnet and injector parameters; the evaluation of this quantity is discussed in Sec. 2.1.2. The neutral-beam power required to sustain the plasma is

$$P_{10} = P_1/Q, \quad (2.48)$$

where Q also is evaluated in Sec. 2.1.2. This relation follows from the definition of Q .

The fusion neutron power is 80% of the total fusion power (14.1 MeV/17.6 MeV):

$$P_3 = 0.8P_1. \quad (2.49)$$

Of these neutrons, some escape through the injection and plasma fan ports. We assume that these neutrons are lost and that their energy appears as unusable, low-grade heat. The remaining neutrons intercept the blanket, yielding a thermal energy of

$$\bar{P}_5 = \Omega_{BL} \bar{M}_B P_3. \quad (2.50)$$

Fig. 2.18. Diagram of the reactor power flow.

Here, we evaluate an average thermal power $\bar{P}_5$ that is based on a time-averaged blanket multiplication, $\bar{M}_B$ (discussed in Sec. 2.3). This thermal output is converted to electrical energy,

$$P_{7A} = \eta_{t1} \bar{P}_5 \quad (2.51)$$

where the efficiency of the thermal conversion system, η_{t1} , is specified to the model.

The charged-particle input to the direct converter consists of the fuel ion flow (D,T), the alpha particles, and the electrons. Because a mirror plasma has a negligible power loss in the confinement region as a result of radiation¹⁵ and cross-field diffusion,¹⁶ the charged-particle power is equal to the input injected power plus 20% of the fusion power (3.5 MeV/17.6 MeV) carried by the alpha-particle reaction product. Thus,

$$P_4 = 0.2P_1 + P_{10} \quad (2.52)$$

During the magnetic expansion process in the direct converter, the kinetic energy of the electrons is transferred almost completely to the ions. Thus, when the plasma has been expanded and is ready for direct conversion, all of P_4 is resident in the ions. To account for the possibility that the alpha particles are not confined, we use two direct converter efficiencies, one for the alpha particles and one for the fuel ions. Thus, the electrical output from the direct converter is

$$P_8 = 0.2P_1 \eta_{DC\alpha} + P_{10} \eta_{DCF} \quad (2.53)$$

where the alpha-particle direct conversion efficiency, $\eta_{DC\alpha}$, and the fuel-ion direct conversion efficiency, η_{DCF} , are specified to the model.¹⁷ The fraction of the input plasma power that is not directly converted to electricity appears as thermal energy in the direct converter structure:

$$P_6 = P_4 - P_8 \quad (2.54)$$

If we allow for the possibility that this energy may be recovered as high-grade heat and converted to electrical energy in a bottoming cycle, the electrical output from this source is

$$P_{7B} = \eta_{t2} P_6, \quad (2.55)$$

where the thermal cycle efficiency, η_{t2} , is specified to the model.

The gross electrical output, P_{78} , then is composed of the power output from the two thermal conversion systems,

$$P_7 = P_{7A} + P_{7B}, \quad (2.56)$$

and the electrical output from the direct converter,

$$P_{78} = P_7 + P_8. \quad (2.57)$$

The recirculating power requirements consist of the power to operate the helium and nitrogen refrigeration systems, P_{13} and P_{14} , and the electrical input to the neutral beam injectors, P_9 . Thus, the recirculating power is

$$P_{15} = P_9 + P_{13} + P_{14}. \quad (2.58)$$

The evaluation of P_{14} and P_{15} is discussed in Appendix C. The injector input power is

$$P_9 = P_{10}/\eta_{inj}, \quad (2.59)$$

where we assume a positive-ion injection system for the hybrid¹⁸; the injector efficiency as a function of W_{inj} is shown in Fig. 2.19.¹² Thus, the net electrical output is the difference between the gross electrical output and the recirculating power,

$$P_{11} = P_{78} - P_{15}, \quad (2.60)$$

where the recirculating power fraction is

$$f_{RC} = P_{15}/P_{78}. \quad (2.61)$$

The system net thermal efficiency, η_{sys} , is the ratio of the (nuclear) power generated,

$$P_2 = 0.2P_1 + \bar{P}_5, \quad (2.62)$$

to the net electrical output:

$$\eta_{sys} = P_{11}/P_2. \quad (2.63)$$

2.2 CAPITAL COST MODEL

To delineate the cost of the various reactor components, we have adopted the cost schedule proposed in Ref. 19, suitably modified for a mirror fusion reactor. The form of the cost schedule is given in Table 2.1.

Accounts 21-25 are for direct capital costs; i.e., equipment, labor, and materials. The relations derived to evaluate these individual accounts are described in Appendix D. In general, our costs are based on three sources: (1) light-water reactors (LWR), (2) cost estimates for the next major mirror fusion experiment, the Mirror Fusion Test Facility (MFTF), and (3) cost estimates from Bechtel Corporation for conceptual fusion reactor designs.

As described in Sec. 2.1.4, the thermal power generated in the blanket varies with time (i.e., exposure) due to buildup of fissile material in the fuel. Many of the reactor component costs are based on the plant thermal output; we have evaluated these costs using the maximum thermal output of the facility. The variation in the thermal power is a function of the fuel management scheme and is described in Sec. 2.3.

We denote the total direct costs (i.e., the sum of accounts 21-25) as C_D , and the cost of the indirect cost accounts as $C(91)$, $C(92)$, etc. From this, the cost of account $C(91)$ is some fraction of C_D ,

$$C(91) = i_{91} C_D, \quad (2.64)$$

where i_{91} is the fractional multiplier. Successive indirect accounts are then compounded,

$$C(92) = i_{92} (1 + i_{91}) C_D, \quad (2.65)$$

$$C(93) = i_{93} [i_{92} (1 + i_{91}) + 1] C_D, \quad (2.66)$$

$$C(94) = i_{94} \{i_{93} [i_{92} (1 + i_{91}) + 1] + 1\} C_D. \quad (2.67)$$

The total capital costs, direct and indirect, then can be written as

$$C_{cc} = (1 + i_{ID}) C_D, \quad (2.68)$$

Table 2.1. Capital cost account summary.

21	Structures and Site Facilities
211	Site improvement
212	Reactor building
213	Turbine building
215	Auxiliary building
216	Hot maintenance area
218	A. Control room building
	B. Diesel generator building
	C. Administration building
	D. Service building
22	Reactor Plant Equipment
221	Reactor equipment
	.1 Magnet
	.11 Windings
	.12 Structure
	.13 Thermal shields
	.2 Injectors
	.21 Sources
	.22 Power supplies
	.23 Accessories
	.3 Direct converter
	.31 Vacuum tank
	.32 Grids and collectors
	.33 Electrical equipment
	.34 Cooling
	.4 Shielding
	.41 Coil shield
	.42 Injectors
	.43 Direct converter
	.5 Vacuum system
	.51 Cryopanel
	.52 Vacuum envelope

Table 2.1. Continued.

222	Main Heat Transfer Loop
.1	Circulators
.2	Piping
.3	Steam generators
223	Auxiliary Cooling
.1	Refrigeration
.11	Liquid helium
.12	Liquid nitrogen
.2	ECCS
.3	Injectors
.4	Coil shield
224	Radioactive Waste Treatment and Disposal
225	Fuel Handling and Storage
.1	Refueling equip
.2	Tritium reprocessing and storage
.3	Tritium clean-up
226	Other Equipment
.1	Plasma start-up
227	Instrumentation and Control
23	Turbine Plant Equipment
24	Electrical Plant Equipment
25	Miscellaneous Plant Equipment
251	Transport and Lifting
252	Air, Water, and Steam Service Equipment
91	Indirect Field Costs
92	Professional Services
93	Other Costs
94	Interest During Construction

where i_{ID} is the indirect cost multiplier,

$$i_{ID} = (1 + i_{91}) (1 + i_{92}) (1 + i_{93}) (1 + i_{94}) - 1 \quad (2.69)$$

As described in Appendix D, we optimistically have used $i_{ID} = 0.91$; more pessimistic estimates range up to $i_{ID} \approx 1.5$. However, we have not included any

provision for escalation; the calculated capital costs are based on 1976 dollars.

2.3 BLANKET MODEL

2.3.1 Blanket Management

The underlying strategy in the blanket management scheme is to load unenriched fertile material into the reactor, expose the fuel to a specified maximum burnup, remove the fuel from the reactor, and then reprocess it to obtain the fissile material that has been bred. As fissile material is created in the blanket, some of it is fissioned *in situ*, the result being an increased blanket energy multiplication and therefore an increased thermal output. We use a blanket management scheme to decrease the power swing that results from the enrichment.

The blanket is made up of modules as shown in Fig. 2.20. The blanket modules are grouped into a number of fuel management regions denoted as N_B . For example, with 16 modules and $N_B = 4$, each fuel management region consists of 4 modules. If τ_{BL} is the blanket life (i.e., the length of time the fuel

Fig. 2.19. Positive ion injector efficiency, including creation, acceleration, neutralization, and trapping.¹²

Fig. 2.20. Top view of the blanket.

stays in the reactor), then at time intervals of τ_{BL}/N_B , the reactor is shut down and $1/N_B$ of the blanket is removed and replaced with new, unexposed blanket modules. By using the startup procedure described below, each blanket region is at a different exposure and the swing in the overall blanket multiplication can be maintained below that occurring in an individual module.

The blanket life is determined by specifying a maximum burnup for the fuel, BU_{MX} . From the relations shown in Figs. 2.15-2.17, we can determine the first-wall exposure (megawatt-years per square metre) corresponding to this burnup,

$$\phi_{MX} = \phi_{MX}^{(BU_{MX})} \quad (2.70)$$

Then, the blanket life is the maximum exposure divided by the product of the first-wall neutron flux, Γ (megawatts per square metre), and the plant capacity factor, δ_c ,

$$\tau_{BL} = \frac{\phi_{MX}}{\Gamma \delta_c} \quad (2.71)$$

where δ_c is the fraction of the time that the reactor operates at full capacity. For the period τ_{BL} , we define a plant availability factor, δ_a , as

$$\delta_a = \frac{\tau_{op}}{\tau_{op} + \tau_{sm} + \tau_{um} + N_B \tau_{BC}} \quad (2.72)$$

where τ_{op} is the time during which the reactor is operating, τ_{sm} is the time for scheduled reactor maintenance (excluding blanket change operations), τ_{um} is the time for unscheduled maintenance, and τ_{BC} is the time required to perform a blanket change operation. Now, we assume

$$\frac{\tau_{op}}{\tau_{op} + \tau_{sm} + \tau_{um}} = 0.8 \quad (2.73)$$

which yields

$$\delta_a = \frac{1}{1.25 + N_B \tau_{BC} / \tau_{op}}$$

Because $\tau_{op} = \phi_{MX} / \Gamma$, and assuming $\delta_c = \delta_a$, the plant capacity factor is

$$\delta_c = \frac{1}{1.25 + \frac{N_B \tau_{BC} \Gamma}{\phi_{mx}}} \quad (2.74)$$

This quantity has important economic ramifications because it quantifies the tradeoff between high Γ (and thus high fissile fuel production rates) and the resulting need to frequently shut down the reactor to exchange blanket modules when ϕ_{mx} is reached. At this time, it is difficult to specify realistic values of τ_{BC} because detailed studies of the blanket change operation have not been performed. However, we have included the parameter τ_{BC} in our formalism at this time and demonstrate the sensitivity of the plant capacity to this quantity.

The reactor operating procedure then is to initially load all N_B regions of the blanket with unexposed fuel and, at time intervals of

$$\tau_{BLN} = \tau_{BL} / N_B, \quad (2.75)$$

to shut down the reactor and remove one region of the blanket and replace it with modules containing fresh, unexposed fuel. The region removed at the end of the first interval will have exposure ϕ_{mx} / N_B , the region removed at the end of the second interval has exposure $2\phi_{mx} / N_B$ and so on as shown in Fig. 2.21 for $N_B = 3$. After $(N_B - 1) \phi_{mx} / N_B$ periods, we reach an equilibrium fuel cycle where, at the beginning of the time period, the exposures for the various regions are

$$\phi_i = (i - 1) \phi_{mx} / N_B, \quad i = 1, 2, \dots, N_B, \quad (2.76)$$

Fig. 2.21. Blanket exposure as a function of time for $N_B = 3$.

and, at the end of the period, the exposures are

$$\hat{\phi}_i = i\phi_{\text{mx}}/N_B, \quad i = 1, 2, \dots, N_B. \quad (2.77)$$

The subscript i is the fuel management region index.

To reach the equilibrium cycle, we have to pay a penalty; the first $(N_B - 1)$ regions removed have not reached maximum exposure. For the first nonequilibrium $(N_B - 1)$ fuel cycle periods, the initial and final exposure are

$$\tilde{\phi}_i \left\{ \begin{array}{ll} = \frac{\phi_{\text{mx}}}{N_B} (i - 1), & i = 1, 2, \dots, (n - 1), \end{array} \right. \quad (2.78a)$$

$$\left\{ \begin{array}{ll} = \frac{\phi_{\text{mx}}}{N_B} (n - 1), & i = n, \dots, N_B, \end{array} \right. \quad (2.78b)$$

$$\hat{\phi}_i \left\{ \begin{array}{ll} = \frac{\phi_{\text{mx}}}{N_B} i, & i = 1, 2, \dots, (n - 1), \end{array} \right. \quad (2.79a)$$

$$\left\{ \begin{array}{ll} = \frac{\phi_{\text{mx}}}{N_B} n, & i = n, \dots, N_B, \end{array} \right. \quad (2.79b)$$

where n is the index defining the fuel cycle period under consideration. To evaluate average blanket quantities for each period, we define the average exposure for region i in each period n as

$$\bar{\phi}_{i,n} = \left(i - \frac{1}{2}\right) \frac{\phi_{\text{mx}}}{N_B} \quad i = 1, 2, \dots, N_B, \quad n \geq N_B, \quad (2.80a)$$

$$\bar{\phi}_{i,n} = \left(i - \frac{1}{2}\right) \frac{\phi_{\text{mx}}}{N_B} \quad i = 1, 2, \dots, (n - 1), \quad n < N_B, \quad (2.80b)$$

$$\bar{\phi}_{i,n} = \left(n - \frac{1}{2}\right) \frac{\phi_{\text{mx}}}{N_B} \quad i = n, \dots, N_B, \quad n < N_B. \quad (2.80c)$$

2.3.2 Plant Output

We now have established the blanket exposure as a function of time. The dependent neutronic quantities of interest are shown in Fig. 2.15, and we denote the relations in the form

$$\begin{aligned}M_{\ell} &= M_{\ell}(\Phi) \quad , \\F_{\ell} &= F_{\ell}(\Phi) \quad , \\T_{\ell} &= T_{\ell}(\Phi) \quad .\end{aligned}$$

Here, M_{ℓ} is the "local" blanket multiplication, F_{ℓ} is the fraction of initial fertile atoms that have been converted to fissile atoms (shown as Pu or U) and T_{ℓ} is the tritium breeding ratio (shown as T/n). For a discussion of "local" values, refer to Sec. 2.1.4.

For each fuel management period, denoted by index n, the average value of the blanket multiplication is

$$\bar{M}_{B,n} = \frac{\Omega_{BL}}{N_B} \sum_{i=1}^{N_B} M_{\ell}(\bar{\Phi}_{i,n}) \quad , \quad (2.81)$$

where we have accounted for the fractional blanket coverage. Over the reactor lifetime, the average blanket multiplication is

$$\bar{M}_B = \frac{1}{N_L} \sum_{n=1}^{N_L} \bar{M}_{B,n} \quad , \quad (2.82)$$

where N_L is the number of fuel management periods in the reactor life. To evaluate the required thermal capacity of the plant, we require the maximum blanket multiplication, which occurs in the equilibrium fuel cycle at the end of the period,

$$\hat{M}_B = \frac{\Omega_{BL}}{N_B} \sum_{i=1}^{N_B} M_{\ell} \left(i \frac{\Phi_{mx}}{N_B} \right) \quad . \quad (2.83)$$

Then, the average blanket thermal power for period n is

$$\bar{P}_{5,n} = P_3 \bar{M}_{B,n} \quad , \quad (2.84)$$

and the net electrical output for period n is

$$Q_{e,n} = P_{78} (\bar{P}_{5,n}) \tau_{BLN} \delta_c, \quad (2.85)$$

where the power quantities are as defined in Sec. 2.1.5. For the average reactor power flow, the blanket output is based on $\bar{M}_B$,

$$\bar{P}_5 = P_3 \bar{M}_B. \quad (2.86)$$

The peak blanket output, used to size the plant thermal capacity, is

$$\hat{P}_5 = P_3 \hat{M}_B. \quad (2.87)$$

The tritium breeding ratio for each period is

$$\bar{T}_n = \frac{\Omega_{BL}}{N_B} \sum_{i=1}^{N_B} T_{\ell}(\bar{\Phi}_{i,n}), \quad (2.88)$$

and the corresponding quantity for the reactor life is

$$\bar{T} = \frac{1}{N_L} \sum_{i=1}^{N_L} \bar{T}_n. \quad (2.89)$$

The mass of fissile material produced in period n is

$$Q_{f,n} \left\{ \begin{array}{l} = \left(\frac{239}{238} \right) \frac{M_{FM}}{N_B} F_{\ell} \left(n \frac{\phi_{mx}}{N_B} \right), \quad n < N_B, \end{array} \right. \quad (2.90a)$$

$$= \left(\frac{239}{238} \right) \frac{M_{FM}}{N_B} F_{\ell}(\phi_{mx}), \quad n \geq N_B, \quad (2.90b)$$

where M_{FM} is the initial mass of fertile material in the blanket (see Eq. 2.43); the ratio (239/238) is replaced by (233/232) when thorium is used. In Eq. 2.90, we have accounted for the fact that in equilibrium, $1/N_B$ of the blanket is removed at maximum exposure, and for startup, the exposure is less than maximum.

2.4 ECONOMIC MODEL

In contrast to the economic analysis of fission and pure fusion reactors, the corresponding analysis for the hybrid is more complex as a result of the following considerations:

- A primary product of the hybrid, fissile material, does not produce revenues on a continuous basis but rather only when the blanket is removed from the reactor and the fuel is reprocessed. Thus, the economic model must reflect properly the "inventory charge" for producing a product over a period of time but not realizing income from the product until the end of the period. In contrast, for example, the electricity production from a fission reactor is sold as it is produced and revenues are realized (almost) simultaneously.
- The blanket thermal output, and thus the plant net electrical output, varies with time.
- The hybrid reactor produces two products, fissile material and electricity. To fix the unit price of each product, the economic analysis must be constrained in some way; alternately, the price of one product may be treated as a parameter.

2.4.1 Hybrid Economics

As derived in Appendix F, the *cash-flow relation* which is used to evaluate the hybrid economics has the form

$$\begin{aligned}
 (1 - k_f) p_e \sum_{m=1}^N (1+x)^{-m} Q_{e,m} + (1 - k_f) p_f \sum_{m=1}^N (1+x)^{-m} Q_{f,m} \\
 = \sum_{m=1}^N (1+x)^{-m} z_m - k_f \frac{z}{N_A} + (1 - k_f) \sum_{m=1}^N (1+x)^{-m} Q_m \\
 + z_0 \left[1 - \frac{k_f}{N_A} \sum_{m=1}^N (1+x)^{-m} \right]. \quad (2.91)
 \end{aligned}$$

In this equation, the symbols are defined as follows:

- m = accounting period.
- N_A = number of accounting periods in the reactor life.
- x = discount factor (effective interest rate).
- $Q_{e,m}$ = quantity of electrical energy generated in period m .
- p_e = unit price of electricity.
- k_f = federal tax rate.
- $Q_{f,m}$ = quantity of fissile material generated in period m .
- p_f = unit price of fissile material.
- z_m = fuel cycle cost for period m ($m = 1, 2, \dots, N_A$).
- $Z = \sum_{m=1}^{N_A} z_m$; total fuel cycle costs.
- O_m = operating and maintenance (O & M) costs for period m .
- z_0 = initial plant capital cost.

Equation 2.91 is a balance between revenues and expenses. On the left of the equation are the after-tax revenues from the sale of electricity and fissile material; this is balanced on the right by depreciated fuel cycle costs, after-tax O & M costs, and depreciated plant capital costs. From the analysis in the previous sections, all quantities in the equation are known

except p_e and p_f . Thus Eq. 2.91 forms the basis for evaluating these two unit costs.

An important aspect of this equation is that each term is weighted with the present-worth factor, $(1+x)^{-m}$, which quantifies the "time value" of money. The significance of the present-worth factor is that it represents the initial value of an investment plus the effective interest on the money up to period m .

In the context of the hybrid economics (if we neglect the second-order fuel cycle and O & M costs), a business organization borrows money at time $t = 0$ to construct a facility and produces products for the plant life. The revenues from the products are used to repay the initial investment *plus* interest on the money. The later in the plant life the product is produced, the more interest that must be paid. For example, a quantity of fissile material, Q_f , generated in some period m' is worth more than the same quantity of fissile material generated in a later period m'' because there will be a smaller interest payment associated with production in period m' compared to period m'' .

It is the purpose of this analysis to evaluate the cost of producing fissile material with the hybrid reactor. Thus at present, we treat p_e as a parameter and solve Eq. 2.91 for p_f . First, defining a weighted average per period,

$$\bar{y} = \frac{1}{X} \sum_{m=1}^{N_A} (1+x)^{-m} y_m, \quad (2.92)$$

where

$$X = \sum_{m=1}^{N_A} (1+x)^{-m}, \quad (2.93)$$

we obtain

$$p_f = \frac{1}{(1-k_f)\bar{Q}_f} \left[\bar{z} - k_f \frac{Z}{N_A} \right] + \frac{\bar{Q}}{\bar{Q}_f} + \frac{z_0}{(1-k_f) X \bar{Q}_f} \left[1 - k_f \frac{X}{N_A} \right] - p_e \frac{\bar{Q}_e}{\bar{Q}_f}. \quad (2.94)$$

This equation expresses the cost of producing fissile fuel as the sum of fuel cycle, O & M, and capital costs, minus the revenues from electricity sales.

To examine the important features of this equation, we focus on the fissile production costs that are attributed to initial plant costs,

$$(p_f)_{cc} = \frac{z_0}{(1 - k_f) \times \bar{Q}_f} \left[1 - k_f \frac{x}{N_A} \right]. \quad (2.95)$$

We note first that the later in the plant life the fissile material is produced, the smaller its weighted value (see Eq. 2.92) and thus the smaller the value of $\bar{Q}_f$ and the larger the value of $(p_f)_{cc}$. The practical result from this relationship is that hybrid fuel cycles which do not produce fissile material on an approximately yearly basis incur an economic penalty in the form of higher interest payments.

Now, rewriting Eq. 2.95 as

$$(p_f)_{cc} \bar{Q}_f = \frac{z_0}{x} \left[1 + \frac{k_f}{(1 - k_f)} \left(1 - \frac{x}{N_A} \right) \right], \quad (2.96)$$

the left-hand side is the fissile revenue per period that must be generated to cover the costs for the period associated with the initial plant cost.

The quantity

$$\frac{1}{\bar{x}} = \frac{x}{1 - (1 + x)^{-N_A}}, \quad (2.97)$$

is termed the *capital recovery factor*; thus, z_0/x is the payment per period that must be made to repay an initial debt of z_0 in N_A periods at a compound interest rate of x per period. The factor in brackets in Eq. (2.96) accounts for the taxes on revenues generated by fissile material minus the depreciation on the initial plant cost.

At this point, we now can relate the cash-flow formulation to the more common economic evaluation known as the *fixed charge rate*. In this case, the relation analogous to Eq. (2.96) is

$$(p_f)_{cc} \sum_{m=1}^{N_A} \frac{Q_{f,m}}{N_A} = k_c z_0, \quad (2.98)$$

where k_c is the fixed charge rate per period. Thus,

$$k_c = \frac{1}{x} \left[1 - \frac{k_f}{(1 - k_f)} \left(1 - \frac{x}{N_A} \right) \right]. \quad (2.99)$$

However, we note that Eq. (2.98) does not properly weight the fissile production except in the special case where $Q_{f,m}$ is equal for all periods. The fixed-charged-rate technique is used frequently to evaluate the economics of facilities that produce just electrical energy because the product, $Q_{e,m}$, generally is constant. Thus, the cash-flow formulation only is required for the hybrid facility because of the time-variation in its products.

2.4.2 Electricity Cost: The Hybrid/Burner Economic Model

In the previous section, the unit cost of electricity, p_e , was treated as a parameter. For a given utility, this is indeed the case, as the utilities are a highly regulated industry and electricity costs are determined by many noneconomic factors. However, as a means of comparing the hybrid reactor to other energy sources, we consider the hybrid plus fission burner reactor which is supported by the hybrid fissile output as a single economic entity producing just electricity. This nuclear complex then is directly comparable to the fast breeder reactor. The difference between these two reactor systems is that, in the hybrid/burner complex, the process of fertile-to-fissile conversion takes place in one facility and electricity production primarily in a second facility, whereas in the fast breeder reactor, both functions are performed in a single facility.

The cost model for the fission burner reactor (assuming a uniform energy output in time) is

$$(1 - k_f)p_e Q'_e = \frac{1}{X} \sum_{m=1}^N (1+x)^{-m} \left[z'_m - k_f \frac{z'_1}{N'_A} + (1 - k_f)p_f Q'_{f,m} \right] + \frac{(1 - k_f)}{X} \sum_{m=1}^N (1+x)^{-m} O'_m + \frac{z'_0}{X} \left[1 - \frac{k_f X}{N_A} \right] \quad (2.100)$$

Here, the symbols have the same definition as in the previous section, and the prime denotes the fission burner reactor component of the nuclear complex. The significant difference between this equation and Eq. (2.91) is that fissile fuel appears on the right-hand side of the equation as a fuel cycle cost and $Q'_{f,m}$ is now the fissile consumption per period. The quantity z'_1 denotes all fuel cycle costs normally associated with fission reactors¹⁹ (i.e., fertile fuel, fabrication, reprocessing, shipping spent fuel) but excepting any of the costs associated with fissile fuel.

We now form our equation for the cost of electricity based on the assumption that the fissile fuel used by the burner reactors is that produced by the hybrid. Thus, eliminating p_f in Eq. (2.100) by substitution from Eq. (2.94), we obtain

$$p_e [Q'_e + q_f \bar{Q}_e] = \frac{1}{(1 - k_f)} \left(\bar{z}' - k_f \frac{z'_0}{N_A} \right) + \frac{q_f}{(1 - k_f)} \left(\bar{z} - k_f \frac{z}{N_A} \right) + \bar{O}' + q_f \bar{O} + \frac{z'_0}{(1 - k_f)X} \left(1 - k_f \frac{X}{N_A} \right) + \frac{z_0 q_f}{(1 - k_f)X} \left(1 - k_f \frac{X}{N_A} \right), \quad (2.101)$$

where

$$q_f = \frac{\bar{Q}'_f}{Q_f}. \quad (2.102)$$

The left-hand side of Eq. (2.101) is the average revenue per period from electricity sales. The three components of the right-hand side are, respectively, fuel cycle (without fissile material) costs, O & M costs, and capital costs for the burner reactors plus the hybrid that supplies fuel. This equation illustrates the strong economic leverage of the hybrid because the quantity q_f (the ratio of fissile consumption per burner reactor to fissile production from the hybrid) typically is small (approximately 0.1 and varying with the burner and hybrid fuel cycles). Thus, the electricity cost is determined primarily by the burner reactor economics and is comparatively insensitive to the hybrid cost components. We also note that the presence of $\bar{Q}_e$ in the left-hand side of the equation reflects the economic advantage of net energy production with the hybrid, although here too, p_e is rather insensitive to this quantity.

Having thus established p_e , we return to Eq. (2.94) to calculate an equivalent value for the cost of producing fissile fuel with the hybrid. The cost of the fissile fuel to the burner reactor (typically in mills/kWh) can then be obtained from the appropriate term in Eq. (2.100).

3. PARAMETRIC STUDY

The model described in Sec. 2 does not incorporate an optimization procedure. Rather, it evaluates the reactor performance for a given set of input

parameters. Therefore, we selected a set of parameters, the base case, and then consecutively varied different parameters to find the optimum reactor configuration. In this section, we illustrate the variation of some of the more important parameters. We are able to draw general conclusions, some of which are applicable to all hybrid reactors regardless of their confinement technique.

3.1 U/Mo BLANKET

The blanket fuel which we have studied most extensively is uranium-7 wt% molybdenum (U/Mo) because of its excellent fissile breeding and energy multiplication performance. The base case parameters for this reactor are listed in Table 3.1. The selection of the reactor parameters were based on a previous point design¹² and a previous optimization study.²⁰ Fuel cycle costs were obtained from Ref. 21. For the economic evaluation, the hybrid was coupled with light-water reactors (LWR); the LWR fuel cycle parameters are described in Ref. 3 and illustrated in Fig. 3.1.

In Fig. 3.2, we present the results of varying the injection energy and mirror ratio. Of significance here is that for any mirror ratio between 2 and 3, the injection energy can be varied to give near optimum performance (within about 10%). Thus we find that the mirror ratio may be selected on the basis of some criterion other than economics (i.e., mechanical design) and the injection energy then set by economic optimization. This rather broad minimum in the (W_{inj}, R_{vl}) parameter space is used because we found that optimum economics always occurred with a first-wall neutron flux of 1 to 2 MW/m^2 , and a large combination of R_{vl} and W_{inj} yields this value. The optimum performance occurs here because the first-wall load yields the optimum combination of plant capacity factor, blanket coverage, and fissile inventory costs.

Results of varying the conductor field are shown in Fig. 3.3. The injection energy and mirror ratio were varied to obtain optimum conditions at each value of the conductor field. Increasing the field above 8 T (the maximum for NbTi superconductor) yields only a very modest improvement in the cost of producing plutonium. This situation develops because we can obtain the optimum first-wall load at 8 T. The optimum first-wall load also may be obtained at higher values of the magnetic field and the ion energy with a resultant (modest) increase in Q . However, the advantage of a higher Q is almost negated by the cost of a more expensive magnet (larger restraining

Table 3.1. Base case parameters for U/Mo blanket.

Variable	Description	Value
Reactor Parameters		
R_{v1}	Vacuum mirror ratio of mirror coil	2.5
R_{v2}	Vacuum mirror ratio of main coil	$R_{v1} - 0.25$
B_c	Maximum field at the conductor	8T
η_M	Coil efficiency	0.77
R_1	Major radius of mirror coil	7.5 m
R_2	Major radius of main coil	12.3 m
α_M	Ratio of minor radius of main coil to minor radius of mirror coil	2.2
Ω_{m1}	Aspect ratio for mirror coil cross section	0.5
Ω_{m2}	Aspect ratio for main coil cross section	2.0
W_{inj}	Injection energy	100 keV
θ_{inj}	Injection angle relative to field line	70 deg
C_w	Ratio of average plasma energy to injection energy	1.06
C	Ratio of perpendicular plasma energy to injection energy	0.92
η_{t1}	Efficiency of blanket thermal conversion system	0.35
η_{t2}	Efficiency of direct converter thermal bottoming cycle	0.35
η_{DCF}	Electrostatic conversion efficiency of direct converter (fuel ions)	0.5
$\eta_{DC\alpha}$	Electrostatic conversion efficiency of direct converter (alpha particles)	0.0
ρ_{HE}	Helium density	4.3 kg/m^3
Δp_{PL}	Pressure drop in the blanket plenum	3 psi
ΔT_{HE}	Helium temperature rise in blanket	250°C

Table 3.1. Continued.

Variable	Description	Value
<u>Blanket Parameters</u>		
t_{BL}	Blanket thickness	1.0 m
t_{FR}	Thickness of the fuel region	0.25 m
$(VF)_{FR}$	Volume fraction of fertile material in fuel region	0.46
ρ_{FM}	Fertile material density	16,500 kg/m ³
t_{TR}	Thickness of the tritium breeding region	0.75 m
$(VF)_{TR}$	Volume fraction of tritium breeding material in tritium breeding region	0.55
ρ_{TBM}	Density of tritium breeding material	1770 kg/m ³
<u>Blanket Neutronics</u>		
See Fig. 2.18		
<u>Blanket Management</u>		
N_B	Number of fuel management regions	4
τ_{RL}	Reactor life	30 y
τ_{BC}	Blanket change time	0.075 y
BU_{MX}	Maximum fuel burnup (atoms fissioned per initial fertile atoms)	0.005
<u>Accounting Information</u>		
τ_A	Length of an accounting period	0.25 y
k_c	Fixed charge rate (fraction per year)	0.15
k_f	Income tax rate	0.50
<u>Hybrid Fuel Cycle Unit Costs</u>		
P_U	Price of fertile material	\$35/kg
P_{D2}	Price of deuterium	\$60/kg
P_{T2}	Price of tritium	1.2×10^6 \$/kg
P_{TB}	Price of tritium breeding material	\$200/kg
P_{FU}	Price of fabricating fertile fuel	\$70/kg heavy metal
P_{FTB}	Price of fabricating tritium breeding material	\$50/kg
P_{RU}	Price of reprocessing fertile fuel	\$160/kg heavy metal

Table 3.1. Continued.

Variable	Description	Value
P_{RTB}	Price of reprocessing tritium breeding material	\$100/kg
P_{SU}	Price of spent fuel shipping, fertile fuel	\$37/kg heavy metal
P_{STB}	Price of spent fuel shipping, tritium breeding material	\$25/kg
Fission Burner Reactor		
z'_0	Capital cost	\$750/kWe
Q'_{HM}	Fertile material required/y/MWe	25.9 kg
Q'_{FM}	Make-up fissile material required/y/MWe	0.33 kg
τ'_c	Core life-time	3 y
δ'_c	Capacity factor	0.8
P'_{FU}	Price of fabricating mixed-oxide fuel	\$245/kg heavy metals
P'_{RU}	Price of reprocessing	\$160/kg heavy metals
P'_{SU}	Price of spent fuel shipping	\$37/kg heavy metals
p'_U	Price of natural uranium	\$35/kg
$0'$	Operating & maintenance costs/y/MWe	\$5000

Fig. 3.1. Fuel cycle for a 1000-MW LWR (0.8 capacity factor, 32,000-MWd/tonne burnup); the mass flow is given in tonnes per year with the fissile mass shown in parentheses.

Fig. 3.2. Variation of mirror ratio and injection energy.

structure). Thus we conclude that the use of the present NbTi superconductor technology does not penalize the mirror hybrid reactor.

In Fig. 3.4, we illustrate the effect of varying the two primary fuel management parameters, the fuel burnup and the number of blanket management regions, N_B (Sec. 2.3.1). The fuel burnup was varied up to the allowable maximum for U/Mo fuel, about 1%. At low burnup, excessive fabrication and reprocessing costs are incurred whereas at high burnup (>1%), fissile fuel inventory costs are excessive. We are able to obtain optimum conditions within the maximum allowable burnup of U/Mo fuel. For large N_B , frequent plant outages are required for blanket change operations resulting in a low capacity factor. For small N_B , large power swings occur resulting in excessive capital costs for thermal conversion equipment that is not fully utilized (the thermal conversion equipment must be sized to the maximum blanket power) and in increased fissile fuel inventory charges. However, at the optimum burnup, the economics are insensitive to N_B . Thus we select $N_B = 4$ because this will ameliorate the load management problems associated with large power swings.

These results illustrate a general principle of hybrid economics. Reactor parameters must be determined so that the fissile fuel is extracted from the blanket on an approximately yearly basis at burnups of about 1% or greater. Failure to meet these conditions will result in penalties, either in the form of fissile fuel inventory costs or fabrication-reprocessing costs. Thus for

Fig. 3.3. Variation of maximum field at the conductor (U/Mo blanket).

Fig. 3.4. Variation of fertile fuel burnup and number of blanket management regions (U/Mo blanket).

a fast-fission uranium blanket, we require first-wall fusion neutron fluxes of 1 to 2 MW/m², which yields average fuel power densities of 100 to 200 W/cm³.

In Fig. 3.5 we show the effect of (hypothetically) varying classical mirror confinement. This is accomplished simply by multiplying the classical $n\tau$ (Eq. 2.12) by an input parameter ($Q/Q_{\text{classical}}$). We see that the mirror hybrid benefits from improved confinement because less electrical energy must be recirculated to the injectors. Thus, more revenue is available from electricity sales and less revenue must be recovered by fissile fuel sales. However, we note that at the classical value of confinement ($Q/Q_{\text{classical}} = 1$), the cost of fissile fuel constitutes only about one-third of the total cost of electricity, certainly an acceptable economic situation. For comparison, present fissile fuel costs (3% enriched uranium) are about 3 mills/kWh, whereas with the hybrid, this cost is about 9 mills/kWh.

A general conclusion about hybrid reactors also can be drawn from the results shown in Fig. 3.5; there are significant economic advantages associated with the sale of electricity from the hybrid reactor that are not available with off-line operation (no electricity sales). For example, the hybrid reactor at $Q/Q_{\text{classical}} = 0.5$ has a recirculation power fraction of approximately 1 (i.e., no net electricity produced) whereas the reactor at $Q/Q_{\text{classical}} = 10$ has a recirculating power fraction of 8% (almost all thermal power is used to generate net electrical output). The difference in the cost of electricity for these two extremes is very significant, about 40%.

The results of varying the time required to change the blanket are illustrated in Fig. 3.6. From this, the need to develop mechanical designs for the reactor enabling relatively fast blanket change operations (on the order of 1 mo) is evident because, based on the previous discussion, economy dictates that we obtain fissile fuel from the blanket on an approximately yearly basis.

In Fig. 3.7, we show the effect of varying the LWR capital cost, a key parameter when determining the cost of electricity from the hybrid/LWR complex. As the LWR capital cost increases, electricity costs increase requiring a lower revenue from fissile material. However, within this rather broad range of LWR capital costs, the fissile fuel costs do not show a large variation.

We investigated two additional features. First, the direct converters were deleted from the reactor and replaced with simple "plasma dumps," significantly simplifying the design. Because this component handles only a small amount of power relative to the blanket, the reactor net electrical output

Fig. 3.5. Variation of confinement (U/Mo blanket).

Fig. 3.6. Variation of time required for blanket change operation (U/Mo blanket).

decreased only slightly while providing a savings on the cost of the direct convertor. The fissile fuel cost without the direct convertor increased by only about 10%, relative to that for a reactor with the direct convertor.

Second, the fissile fuel requirement of 0.33 kg/MWe/y for the LWR (see Fig. 3.1) assumes that the LWR is fed with natural uranium and that the natural uranium supplies about 0.19 kg/MWe/y of fissile fuel (^{235}U) to the reactor. We also examined the case where the LWR uranium is recycled, such that the uranium supplied to the LWR is just ^{238}U (in equilibrium). The plutonium makeup required from the hybrid is then about 0.55 kg/MWe/y. In this scenario, the cost of electricity increased from 29.8 mills/kWh to 35.9 mills/kWh, a 20% increase.

3.2 UC BLANKET

We also examined the consequences of incorporating the uranium into the blanket as uranium carbide (UC). This fuel is capable of a higher burnup (maximum burnup of approximately 10%) than U/Mo although its neutronic performance is somewhat poorer (compare Figs. 2.15 and 2.16). The input to the reactor was the same as that in Table 3.1, with the exception that the blanket neutronics are taken from Fig. 2.16 and the blanket material characteristics are somewhat different.

The results of varying the fuel management parameters for the UC blanket are shown in Fig. 3.8. We see that the high burnup capability of UC offers no advantages because the minimum fuel cost occurs at low burnup, about 1 to 1.5%. This results from the fact that at higher burnup, fissile fuel inventory charges become large. In a comparison of Figs. 3.4 and 3.8, the poorer neutronic performance of UC is reflected in an approximate 20% greater fissile cost than that for the U/Mo fuel.

One possible way to fully exploit the high burnup of UC and still extract fuel from the blanket on an approximately yearly basis is to increase the wall loading by increasing the magnetic field. We have examined this configuration and found that the penalties for a more expensive magnet, for providing cooling for very high-blanket power densities, and for providing significantly increased neutral beam currents do not lower the fissile cost significantly below those shown in Fig. 3.8.

Fig. 3.7. Variation of the capital cost of the LWR (U/Mo blanket).

Fig. 3.8. Variation of fertile fuel burnup and number of fuel management regions (UC blanket).

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APPENDIX A. MAGNET MODEL

To perform the calculations of the magnet parameters described in Sec. 2.1.1, we develop empirical scaling laws derived from detailed calculations with the magnet codes EFFI^{A1} and MAFCO.⁶ To describe the model, we first develop the equations for a coil with a single conductor bundle (see Fig. 2.1), then superimpose the equations for a two-bundle coil (Fig. 2.3).

A.1 SINGLE BUNDLE COIL

The coil modeled here is illustrated in Fig. 2.2 when the outer conductor bundle is deleted. From the input, Eq. (2.1), the mirror (B_{m1}) and the central (B_{01}) fields are calculated to be

$$\begin{aligned} B_{m1} &= \eta_M B_c, \\ B_{01} &= \frac{B_{m1}}{R_{v1}}, \end{aligned} \quad (A.1)$$

where the symbols are as defined in Sec. 2.1.1. These two fields may be expressed in terms of geometric and current variables by the empirical equations,^{A2}

$$B_{01} = \frac{5}{3} \times 10^{-6} (NI)_1 \frac{r_1}{\rho_1^2} \left(1 - \frac{r_1}{3\rho_1} \right), \quad (A.2)$$

$$\rho_1^2 = R_1^2 + r_1^2, \quad (A.3)$$

$$B_{m1} = \frac{2}{5} \times 10^{-6} \frac{(NI)_1}{r_1}, \quad (A.4)$$

where all units are expressed in the metre-kilogram-second system.

Forming the ratio of Eqs. (A.2) and (A.4),

$$\frac{1}{R_{v1}} = \frac{25}{6} \frac{\gamma_1^2}{1 + \gamma_1^2} \left(1 - \frac{\gamma_1}{3\sqrt{1 + \gamma_1^2}} \right), \quad (A.5)$$

$$\gamma_1 \equiv \frac{r_1}{R_1} . \quad (\text{A.6})$$

Because R_{v1} and R_1 are known quantities, we wish to solve for γ_1 . An approximate solution to Eq. (A.5) is

$$\gamma_1 = \sqrt{\frac{\Delta_1}{1 - \Delta_1}} , \quad (\text{A.7})$$

$$\Delta_1 \equiv \frac{\frac{6}{(25 R_{v1})}}{1 - \frac{1}{3} \sqrt{\frac{6}{25 R_{v1}}}} . \quad (\text{A.8})$$

Once we have determined r_1 , Eq. (A.2) or (A.4) can be used to evaluate $(NI)_1$. The number of ampere-metres for *both* halves of the coil is then

$$(\text{AM})_1 = 2(NI)_1 \ell_1 , \quad (\text{A.9})$$

where ℓ_1 is the length per turn,

$$\ell_1 = 2\pi (R_1 + r_1) . \quad (\text{A.10})$$

We now can determine the size of the conductor bundle from the specified coil efficiency. We assume the maximum field is located at the position indicated in Fig. 2.3 and we model the coil at this location as two straight conductors of infinite length in the x dimension (Fig. A.1). Using equations developed for the EFFI^{A1} code, the mirror field can be expressed as

$$B_{m1} = \frac{\mu_0 (NI)_1}{\pi r_1} F_{M1} , \quad (\text{A.11})$$

where

$$F_{M1} = \frac{1}{4\theta_1} \ln \left[\frac{\omega_1^2 + (1 + \theta_1)^2}{\omega_1^2 + (1 - \theta_1)^2} \right] - \frac{1}{2\omega_1 \theta_1} (1 - \theta_1) \tan^{-1} \left(\frac{\omega_1}{1 - \theta_1} \right) \\ + \frac{1}{2\omega_1 \theta_1} (1 + \theta_1) \tan^{-1} \left(\frac{\omega_1}{1 + \theta_1} \right) , \quad (\text{A.12})$$

$$\omega_1 = w_1/r_1, \quad (A.13a)$$

$$\theta_1 = h_1/r_1. \quad (A.13b)$$

For the field at the conductor,

$$B_c = \frac{\mu_o (NI)_1}{4\pi w_1} F_{Cl}, \quad (A.14)$$

where

$$F_{Cl} = \frac{\omega_1}{2\theta_1} \ln \left[\frac{\theta_1^2 + 4}{\theta_1^2} \right] + 2 \tan^{-1} \left(\frac{\omega_1}{2\theta_1} \right) + \frac{\omega_1}{2\theta_1} \ln \left[\frac{\omega_1^2}{\omega_1^2 + 4(1 - \theta_1)^2} \right] \\ - 2(\omega_1/\theta_1 - 1) \tan^{-1} \left[\frac{\omega_1}{2(1 - \theta_1)} \right] + \frac{2}{\theta_1} \tan^{-1} \left(\frac{\omega_1}{2} \right). \quad (A.15)$$

Forming the ratio of Eqs. (A.11) and A.14), we obtain

$$\eta_{m1} = 4 \frac{F'_{M1}}{F_{Cl}}, \quad (A.16)$$

where

$$F'_{M1} = \omega_1 F_{M1}. \quad (A.17)$$

Calculated results for Eq. (A.16) appear in Fig. A.2. Given Ω_{m1} and η_{m1} , we can evaluate iteratively ω_1 and θ_1 , the (nondimensional) conductor dimensions. The current density in the conductor bundle is then

$$J_1 = \frac{(NI)_1}{4w_1 h_1}. \quad (A.18)$$

A model that is not needed at this point but will be required in the two-bundle calculation, is the axial field variation. The field along the z-axis, resulting from a single-filament MAFCO calculation, is shown in Fig. A.3. The field can be represented by the following equations,

$$B_1 = B_{01} + \Delta B_1 F_{z1}(\xi_1), \quad \xi_1 < 1, \quad (A.19a)$$

Fig. A.1. Geometry for the magnet efficiency.

Fig. A.2. Coil efficiency.

Fig. A.3. Axial field variation.

$$B_1 = B_{m1} F_{z1}(\xi_1), \quad \xi_1 > 1, \quad (\text{A.19b})$$

where

$$\xi_1 = z/R_1, \quad (\text{A.20})$$

$$F_{z1}(\xi_1) = \exp \left[-\frac{(\xi_1 - 1)^2}{\sigma_1^2} \right], \quad (\text{A.21})$$

$$\sigma_1^2 = 0.0842 \exp(-0.3R_{v1}) + \frac{0.0386}{(1 - 1/R_{v1})^2}, \quad (\text{A.22})$$

$$\Delta B_1 = B_{m1} - B_{01}. \quad (\text{A.23})$$

A.2 TWO-BUNDLE COIL

Using the equations described in Sec. A.1, we can write the following relations for the field at the center of the coil and the field on-axis at the two conductor bundles,

$$B_0 = \frac{5}{3} \times 10^{-6} (NI)_1 \frac{r_1}{\rho_1} \left(1 - \frac{r_1}{3\rho_1} \right) + \frac{5}{3} \times 10^{-6} (NI)_2 \frac{r_2}{\rho_2} \left(1 - \frac{r_2}{3\rho_2} \right), \quad (\text{A.24})$$

$$B_{M1} = \frac{2}{5} \times 10^{-6} \frac{(NI)_1}{r_1} + \frac{2}{5} \times 10^{-6} \frac{(NI)_2}{r_2} F_{z2}(\beta_m) + \frac{5}{3} \times 10^{-6} (NI)_2 \frac{r_2}{\rho_2} \left(1 - \frac{r_2}{3\rho_2} \right) \left[1 - F_{z2}(\beta_m) \right], \quad (\text{A.25})$$

$$B_{M2} = \frac{2}{5} \times 10^{-6} \frac{(NI)_1}{r_1} F_{z1}(1/\beta_m) + \frac{2}{5} \times 10^{-6} \frac{(NI)_2}{r_2}, \quad (\text{A.26})$$

where

$$\beta_m \equiv R_2/R_1. \quad (\text{A.27})$$

These three fields are evaluated from the input parameters, Eq. (2.1), by the relations

$$B_{M1} = \eta_{M1} B_c, \quad (A.28)$$

$$B_0 = B_{M1} / R_{v1}, \quad (A.29)$$

$$B_{M2} = R_{v2} B_0. \quad (A.30)$$

Eliminating r_2 through Eq. (2.2), Eqs. (A.24)-(A.26) may be solved for r_1 , $(NI)_1$, and $(NI)_2$ as functions of known quantities. Because they are quite lengthy, the results of this manipulation are not described here. The quantity r_2 then is obtained from Eq. (2.2) and the ampere-metres of conductor for each bundle is found using Eq. (A.9).

The dimensions of the conductor bundles now may be determined from the coil efficiency model described in Sec. A.1. Considering first the inner conductor bundle, we assume that the field due to bundle 1 varies from the on-axis point to the inside conductor surface and that the field contribution due to bundle 2 does not vary significantly over this dimension. The field at the inside surface of bundle 1 can be written as

$$B_c = \frac{B_{m1}}{\eta_{m1}} + B_{02} + \Delta B_{2z2}(\beta_m), \quad (A.31)$$

where B_{m1} is the on-axis field due to conductor 1,

$$B_{m1} = \frac{2}{5} \times 10^{-6} \frac{(NI)_1}{r_1},$$

and η_{m1} describes the variation in this field at the bundle surface. Solving for η_{m1} , we have

$$\eta_{m1} = \frac{B_{m1}}{B_c - [B_{02} + \Delta B_{2z2}(\beta_m)]}, \quad (A.32)$$

This value is used with the input value of Ω_{m1} to evaluate θ_1 and ω_1 . Similarly for conductor bundle 2, we find

$$\eta_{m2} = \frac{B_{m2}}{B_c - B_{m1} F_{z1}(1/\beta_m)}. \quad (A.33)$$

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APPENDIX B. EQUATIONS FOR THE RADIAL PLASMA BOUNDARY

The radial plasma boundary is a function of the mirror ratio, R_{v1} (see Fig. 2.5). We seek to curve-fit this line with two quadratic equations, one for positive z and one for negative z ; the geometric parameters are defined in Fig. B.1. Expressions for r_p and t_F are given by Eqs. (2.9) and (2.10). The fan sweep angle θ_F (degrees) is a function of mirror ratio and is given by

$$\theta_F = 90.65 - 11.78 R_{v1} + 1.075 R_{v1}^2 . \quad (B.1)$$

The angle θ_{FT} , which defines the tangent to the field line at the mirror point in the negative z plane, is given by

$$\theta_{FT} \approx \sin^{-1} \left(\frac{\sin \theta_F}{\tilde{a}} \right) , \quad (B.2)$$

$$\tilde{a} = [1.01 + 0.2 \cos \theta_F]^{1/2} . \quad (B.3)$$

In the positive z plane, the curve-fitted line is

$$C_x = C_{x1} + C_{x2} \xi_1 + C_{x3} \xi_1^2 , \quad (B.4)$$

where $\xi_1 \equiv z/R_1$. Using the conditions

$$C_x = r_p/R_1 , \quad \xi_1 = 0 ,$$

$$C_x = r_p/3R_1 , \quad \xi_1 = 0.5 ,$$

$$C_x = t_F/R_1 , \quad \xi_1 = 1 ,$$

we obtain the coefficients

$$C_{x1} = r_p/R_1 , \quad (B.5a)$$

$$C_{x2} = - \left(\frac{t_F}{R_1} + \frac{5}{3} \frac{r_p}{R_1} \right) , \quad (B.5b)$$

which give the solutions

$$C_{y1} = r_p / R_1 , \quad (B.7a)$$

$$C_{y2} = 2 \frac{(\sin \theta_F - r_p / R_1)}{\cos \theta_F} - \tan \theta_{FT} , \quad (B.7b)$$

$$C_{y3} = \frac{r_p / R_1 + \cos \theta_F \tan \theta_{FT} - \sin \theta_F}{\cos^2 \theta_F} . \quad (B.7c)$$

APPENDIX C. POWER REQUIREMENTS FOR THE REFRIGERATION SYSTEM

The major source of gas in the reactor vacuum system is the gas that is fed to the neutral beam sources. Of this gas feed, a fraction is used to form the required neutral current that sustains the plasma and the remainder must be pumped from the injector tanks so it does not enter the confinement volume. The gas flow represented by the neutral beams exits from the confinement region as plasma leakage and is neutralized in the direct converters. Thus, pumping surfaces also must be provided in the direct converters to handle the gas load resulting from the plasma leakage.

Presently, we are proposing to use cryocondensation pumping^{C1} in the mirror hybrid reactor. A diagram of this type of vacuum pump is given in Fig. C.1. The fuel atoms (D and T) condense on the liquid helium (LHe) panel. Liquid nitrogen (LN) surfaces are provided on either side of the LHe panel surfaces to maintain an acceptable radiative heat load on the LHe panel. In addition, room temperature panels are provided on either side of the LN surfaces to maintain an acceptable radiative heat load on the LN panels. This radiative shield for the LN panels is required because much of the interior of the direct converter will operate at high temperatures. These high temperatures result because the direct converter cooling is designed to recover the

Fig. C.1. Diagram of the cryopump ($\epsilon = \text{emissivity}$).

residual thermal energy from the direct conversion process for the thermal bottoming cycle.

In addition to the cryogenic pumping surfaces, the other large cryogenic reactor component is the magnet. Both the superconductor and the magnet structure operate at 4.2 K, and thus must be shielded from all radiation by LN liners.

In relation to reactor power flow, we need to determine the heat load on the cryogenic panels (LHe and LN) and the required input power to the refrigeration system. The following analysis follows the calculations performed for the design of the MX experiment^{C2} by T. A. Kozman of LLL.

We consider first the LHe system. For the 77 K LN panels radiating to the 4.2 K LHe panel with the emissivities shown in Fig. C.1, the heat transferred into the LHe panel is 0.35 W/m^2 ; we increase this amount by 25% to 0.44 W/m^2 to account for hanger and lead losses. The sensible heat of the gas load, from 77 K to 4.2 K is 0.0146 W/A ($1 \text{ A} = 0.176 \text{ Torr} \cdot \text{litre/s}$). If we let A_{HE} be the pumping surface area of the LHe panels and assume a source gas efficiency of one-third, the heat load (watts) on the LHe panels is

$$q_{\text{HE}}^{\circ} = 0.44 A_{\text{HE}} + 0.0146 \nu 3 I_{\text{inj}} \quad , \quad (\text{C.1})$$

where I_{inj} is the injected current (see Eq. 2.39). Scaling from a previous design,¹² a total of 0.6 m^2 of cryopanel per ampere of injected current is required in the injectors and direct converter; i.e., $A_{\text{HE}}/I_{\text{inj}} = 0.6 \text{ m}^2/\text{A}$. Thus,

$$q_{\text{HE}}^{\circ} = 0.31 I_{\text{inj}} \quad . \quad (\text{C.2})$$

Now, adding the magnet contribution (as evaluated in Sec. 2.1.3),

$$q_{\text{HE}}^{\circ} = 0.31 I_{\text{inj}} + 20 \times 10^3 \left(\frac{I_{\text{M}}}{25} \right)^2 \quad . \quad (\text{C.3})$$

The input power to the LHe refrigeration system, which operates between ambient temperature (290 K) and 4.2 K, is

$$P_{13} = \frac{1}{\eta_{\text{HR}}} \left(\frac{290 - 4.2}{4.2} \right) q_{\text{HE}}^{\circ} \quad , \quad (\text{C.4})$$

where η_{HR} is the isentropic efficiency of the helium refrigerator and the second term is the Carnot efficiency. From Ref. C3, for $\dot{q}_{HE} \approx 10$ kW, we find that $\eta_{HR} \approx 0.25$. Thus the ratio of input power to heat load (watts per watt) is

$$P_{13}/\dot{q}_{HE} \approx 300 \quad , \quad (C.5)$$

which yields an input power (kilowatts) to the LHe refrigeration system of

$$P_{13} = 0.093 I_{inj} + 6 \times 10^3 \left(\frac{L_M}{25} \right)^2 \quad . \quad (C.6)$$

Now, considering the LN system, the heat load on the chevrons is 260 W/m^2 , including a 25% hanger and lead loss. The heat load on the liner is 66 W/m^2 and the sensible heat of the pumped gas is 0.044 W/A . Thus, if we let A_{N2} equal the of the LN liner, which is also the effective area of the chevrons, the heat load (watts) on the LN pumping surfaces is

$$\dot{q}_{N2} = 326 A_{N2} + 0.044 \times 3 I_{inj} \quad . \quad (C.7)$$

Because $A_{N2} = A_{HE}$, and $A_{HE}/I_{inj} = 0.6 \text{ m}^2/\text{A}$,

$$\dot{q}_{N2} = 196 I_{inj} \quad . \quad (C.8)$$

The heat load on the LN surfaces in the magnet structure is scaled, using an area ratio, from the MX coil design.^{C2} Thus the total heat load (watts) on the LN system is

$$\dot{q}_{N2} = 196 I_{inj} + 30.9 \times 10^3 \left(\frac{L_M}{5} \right)^2 \quad . \quad (C.9)$$

The input power to the LN system is

$$P_{14} = \frac{1}{\eta_{NR}} \left(\frac{290-77}{77} \right) \dot{q}_{N2} \quad . \quad (C.10)$$

From Ref C3, for $\dot{q}_{N2} \approx 1$ MW, $\eta_{NR} \approx 0.35$ which yields the ratio (watts per watt)

$$P_{14}/\dot{q}_{N2} \approx 10 \quad . \quad (C.11)$$

Thus, the input power (kilowatts) to the LN refrigeration system is

$$P_{14} = 1.96 I_{inj} + 309 \left(\frac{L_M}{5} \right)^2 . \quad (C.12)$$

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APPENDIX D. EVALUATION OF CAPITAL COST ACCOUNTS

The following description of the cost model does not detail the derivation of each individual cost. For many accounts, several sources were available to estimate the cost and these sources often exhibited a dismaying degree of discrepancy. We do not attempt to defend our particular choice in each case (not necessarily the lowest!) but merely quote the source.

A large number of the reactor component costs are based on the thermal capacity of the plant and, as described in Secs. 2.1.4 and 2.2, the blanket thermal output varies with time. Here, to determine the cost of the plant, we use the maximum thermal output from the blanket, $\hat{P}_5$, plus the thermal power from the direct converter,

$$P_{th} = \hat{P}_5 + P_6 . \quad (D.1)$$

The maximum thermal output from the blanket is

$$\hat{P}_5 = \Omega_{BL} \hat{M}_B P_3 , \quad (D.2)$$

where $\hat{M}_B$ is the maximum local blanket energy multiplication (see Sec. 2.3).

In the following discussion, we use the notation $C(XXX)$ to denote the cost of component XXX.

D.1 DIRECT COSTS

211. Site improvement^{D1}:

$$C(211) = \$1.5/kW_t . \quad (D.3)$$

212. Reactor Building^{D1-D3}:

The cost is scaled on the volume at $\$750/m^3$, assuming a cylindrical building of radius $3R_2$ and height $3R_2$ (R_2 is as defined in Sec. 2.1.1). Thus,

$$C(212) = \$750 \times \pi(3R_2)^3 . \quad (D.4)$$

213. Steam Turbine Building^{D2,D3};

$$C(213) = \$5/kW_t, \quad (D.5)$$

where this value is based on an LWR value and is escalated approximately 50% to account for the (assumed) more-complex fusion reactor.

214. Intake and Discharge Structures:

Because we assume that wet cooling towers will be used, the cost of these structures was neglected.

215. Auxiliary Building^{D2}:

For the hybrid reactor, this structure will house such components as the blanket emergency cooling equipment, helium coolant make-up equipment, tritium clean-up equipment, and refrigeration equipment:

$$C(215) = \$4/kW_t, \quad (D.6)$$

216. Hot Maintenance Area:

This facility is the hot cell for disassembly of exposed blanket modules, injector maintenance, and direct converter component maintenance:

$$C(216) = \$750 \times \frac{\pi}{3} (3R_2)^3, \quad (D.7)$$

217. Fuel Storage Building:

This facility is assumed to be part of the hot maintenance area.

218A. Control Room Building^{D1,D2}:

$$C(218A) = \$1.5/kW_t, \quad (D.8)$$

218B. Diesel Generator Building^{D2}:

$$C(218B) = \$0.4/kW_t, \quad (D.9)$$

218C. Administration Building^{D2}:

$$C(218c) = \$0.6/kW_t \quad . \quad (D.10)$$

218D. Service Building^{D2}:

$$C(218D) = \$0.7/kW_t \quad . \quad (D.11)$$

221.1 Magnet:

The magnet windings cost estimate includes the superconductor (NbTi), winding, assembly, and installation,^{D4}

$$C(221.11) = \$5.7 \times 10^{-3}/A \cdot m \quad . \quad (D.12)$$

The magnet structure includes the coil clamps, which are assumed to be stainless steel beams, and the stainless steel coil casing. These components are scaled from a previous reactor design,¹² where

$$C(221.12) = \$85 \times 10^6 \left(\frac{B_c}{8.1} \right)^2 \left(\frac{L_M}{25} \right)^3 + \$55 \times 10^6 \left(\frac{L_M}{25} \right)^2 \quad . \quad (D.13)$$

The thermal shields are the LN liners that form the interface between the ambient temperature and the 4.2 K magnet structure. Pricing the liners^{D4} at \$250/ft² and scaling the surface area from the point design,¹²

$$C(221.13) = \$250 \times (4 \times 10^4) \left(\frac{L_M}{25} \right)^2 \quad . \quad (D.14)$$

221.2 Injectors:

The cost of the source scales as the ion current extracted from the plasma source. The extracted current, I_E , is related to the neutral current by the neutralization efficiency, η_N ,

$$I_E = I_{inj} / \eta_N \quad . \quad (D.15)$$

Using a fit to existing sources and sources under development,

$$\eta_N = 0.7 e^{-0.006 W_{inj}}, \quad (D.16)$$

where W_{inj} is the injection energy in kiloelectron volts. Then, using a cost of $\$75 \times 10^3$ per 80-A (extracted) source,^{D4} and including 25% spares,

$$C(221.21) = \$75 \times 10^3 \times 1.25 \left(\frac{I_E}{80} \right). \quad (D.17)$$

We assume the power for the injectors is pulled directly off the line to facilitate reactor start-up. The injector power supply cost estimate includes filament and discharge power supplies, the accel-decel supplies, and the substation. Scaling from Ref. D5,

$$C(221.22) = \$85/kW_e, \quad (D.18)$$

where the total cost is based on the input power to the injectors, P_g .

The injector accessories cost estimate includes isolation valves, neutralizers (iron, ducting, and plumbing), and mounting and alignment fixtures,^{D4}

$$C(221.23) = \$35 \times 10^3 \left(\frac{I_E}{80} \right). \quad (D.19)$$

221.3 Direct Converter

The cost of the direct converter vacuum tank is scaled on the characteristic dimensions of the tank, as derived in Ref. D6. Each end of the reactor has a direct converter, and for each we use the equation

$$C(221.31) = \$1.2 \times 10^6 \left(\frac{R_{DC}}{5 \text{ m}} \right)^3 \left[0.75 \left(\frac{H_{DC}}{2R_{DC}} \right)^{1.4} \times \left(\frac{\theta_{DC}}{360 \text{ deg}} + 0.32 \right) + 0.060 \left(\frac{\theta_{DC}}{360 \text{ deg}} \right) \right]. \quad (20)$$

The quantities R_{DC} , H_{DC} , and θ_{DC} are characteristic direct converter dimensions (see Appendix E).

The collector structure, including coolant headers, for a three-stage unit^{D,6} is estimated at

$$C(221.32) = \$3.85 \times 10^3 / m^2 \left[\frac{P_{DC}(W)}{q_c''(W/m^2)} \right], \quad (D.21)$$

where P_{DC} is the power entering the direct converter and q_c'' is the power flux at the collector (see Appendix E). For a less complex single stage unit, we use

$$C(221.32) = \$2 \times 10^3 / m^2 \left[\frac{P_{DC}(W)}{q_c''(W/m^2)} \right] \quad (D.21)$$

The power conditioning equipment is estimated at ^{D5}

$$C(221.33) = \$45/kW_E, \quad (D.22)$$

where the total cost is based on the electrical output of the direct converters, P_8 .

The sides of the direct converter also are cooled to remove heat radiated from the collectors and heat deposited by some energetic particles. We base this cost on $\$2 \times 10^3$ per square metre of wall area, ^{D6}

$$C(221.34) = \$2 \times 10^3 / m^2 [A_{DCS} (m^2)] \quad (D.23)$$

for each direct converter, where A_{DCS} is the converter side wall area, evaluated in Appendix E.

221.4 Shielding:

Shielding is required between the blanket and the magnet to protect the cryogenic magnet from excessive heating and neutron damage, in the direct converters and injectors to protect the cryopanel from excessive heating, and as biological shielding. We assume the shield is a mixture of iron, lead, and carbon at a cost of $\$14 \times 10^3 / m^3$. ^{D5}

For the magnet shield,

$$C(221.41) = \$14 \times 10^3 / m^3 [4\pi R_{SH} \Omega_{BL} t_{SH} (m^3)] \quad (D.24)$$

where the shield geometry is as discussed in Sec. 2.1.3.

The cost of shielding in the injectors is scaled from a point reactor design ^{D5} on the injected current and the characteristic neutron flux,

$$C(221.42) = \$6.4 \times 10^6 \left(\frac{I_{inj}}{6250 \text{ A}} \right) \times \ln \left[\left(\frac{P_1}{1270 \text{ MW}} \right) \left(\frac{20.2 \text{ m}}{L_M} \right)^2 \right] \quad (D.25)$$

In the direct converter, we again scale from Ref. D5 on area and characteristic neutron flux,

$$C(221.43) = \$14 \times 10^6 \left(\frac{R_c}{48 \text{ m}} \right)^2 \times \ln \left[\left(\frac{P_1}{1270 \text{ MW}} \right) \left(\frac{48 \text{ m}}{R_c} \right)^2 \right] \quad (D.26)$$

221.5 Vacuum System:

The vacuum system equipment includes cryopanels, the vacuum envelope, and the external vacuum system.

Cryopanels are located in the direct converters to pump the gas resulting from the injected neutral current and in the injector tanks to pump the fraction of the input gas which is not converted to energetic neutrals. From Ref. D7, the direct converter requires 6 ft^2 of cryopanel per injected ampere, and the injector tanks requires 0.3 ft^2 of panel per extracted ampere. The cost of the cryopanels, including the LN shield, LHe panel, chevrons, plumbing, Dewar, and inspection is estimated to cost $\$500/\text{ft}^2$.^{D4} Thus the cryopanel cost is

$$C(221.51) = \$500 (6 I_{inj} + 0.3 I_E) \quad (D.27)$$

The vacuum envelope includes the main spherical tank surrounding the magnet, the injector tanks, and the direct converter tank. The later item already has been accounted for in C(221.31). For the main tank, we scale from Ref. D7 on the square of the characteristic magnet dimension, and for the injector tanks we scale from MX on the extracted current^{D4}:

$$C(221.52) = \$2.8 \times 10^6 \left(\frac{L_M}{25} \right)^2 + \$320 \times I_E \quad (D.28)$$

The external vacuum system is also scaled from MX and is proportional to the cryopanel costs:

$$C(221.53) = \$5 \times 10^5 \left[\frac{(6 I_{inj} + 0.3 I_E)}{3500} \right] \quad (D.29)$$

222. Main Heat Transfer Loop:

The helium circulators and steam generators are assumed to be similar to the components developed by General Atomic Company for the high-temperature gas reactor and the gas-cooled fast reactor. The price of these two components are, respectively,^{D8}

$$C(222.1) = \$7/\text{kW}_t, \quad (D.30)$$

$$C(222.3) = \$23.5/\text{kW}_t. \quad (D.31)$$

The main helium ducting cost was estimated from a working stress of 13,000 psi (stainless steel, 550°C, 60 atm) and a fluid dynamic head of 0.5 psi. The result was $\$0.95/\text{kW}_t \cdot \text{m}$. Estimating 100 m of ducting for a machine of characteristic 15-m dimension, the resulting ducting cost is

$$C(22.2) = \$5/\text{kW}_t \left[\frac{L_M}{15 \text{ (m)}} \right]. \quad (D.32)$$

223. Auxiliary Cooling:

Using the input power requirements for the cryogenic systems developed in Appendix C, the costs of the LHe and LN refrigeration systems are,^{D9}

$$C(223.11) = \$4 \times 10^4 [P_{13} \text{ (kW)}]^{0.7}, \quad (D.33)$$

$$C(223.12) = \$4 \times 10^4 [P_{14} \text{ (kW)}]^{0.7}. \quad (D.34)$$

In the above equations, the cost estimates have been increased threefold from Ref. D9 to account for transfer lines, valves, Dewars, storage, purification, and installation.

The ECCS (emergency core cooling system) is a secondary coolant loop provided to ensure cooling to the blanket in the event of an accident. The cost estimate of this component is based on a conceptual design for a helium-cooled mirror hybrid reactor,^{D10}

$$C(223.2) = \$7.3/\text{kW}_t. \quad (D.35)$$

The injectors, because they are not 100% efficient, suffer heat deposition which must be removed. Here, assuming a simple low-temperature cooling water circuit at $\$5/\text{kW}_t$, we obtain

$$C(223.3) = \$5 \times 10^3 \left[P_9 \text{ (kW)} \right] \left(\frac{1}{\eta_{inj}} - 1 \right) . \quad (D.36)$$

To cool the magnet shield, we assume one-hundred fold attenuation of the fusion neutron flux by the blanket and, again, the use of a simple low-temperature cooling water circuit to remove the resulting heat,

$$C(223.4) = \$5 \times 10^3 \left[10^{-2} \times P_3 \text{ (kW)} \right] . \quad (D.37)$$

224. Radioactive Waste Treatment and Disposal^{D1,D3}:

$$C(224) = \$3/\text{kW}_t . \quad (D.38)$$

225. Fuel Handling and Storage:

The cost of the blanket replacement machine is assumed to be proportional to the mass of the blanket, which we scale as the square of the first wall radius. Using the cost estimate from Ref. D10,

$$C(225.1) = \$30 \times 10^6 \left[\frac{R_{FW}}{6 \text{ (m)}} \right]^2 . \quad (D.39)$$

The tritium reprocessing and storage equipment includes facilities for purification, isotopic separation, recovery, and storage. Scaling from a previous point design,^{D7}

$$C(225.2) = \$13 \times 10^6 \left[\frac{P_1 \text{ (MW)}}{210} \right] . \quad (D.40)$$

The bulk of the cost of the tritium clean-up system is for air-handling equipment for the containment building. At a cost of \$100/m³ of containment volume^{D11} and referring to Eq. D.4,

$$C(225.3) = \$100 \times \pi (3R_2)^3 . \quad (D.41)$$

226. Other Equipment:

We include plasma start-up equipment in this cost estimate, but have not yet addressed the design of this component in enough detail to estimate the equipment cost.

227. Instrumentation and Control^{D1}:

$$C(227) = \$7/\text{kW}_t . \quad (\text{D.42})$$

23. Turbine Plant Equipment:

The thermal conversion system is assumed to be a steam cycle. From Refs. D3 and D12, we obtain

$$C(23) = \$35/\text{kW}_t . \quad (\text{D.43})$$

This cost includes equipment, materials and installation for the turbine-generators, mechanical equipment, condensers, circulating water system, and cooling towers and piping; it does not include the steam generators.

24. Electrical Plant Equipment:

We compared the costs for the equipment for a conceptual fusion reactor^{D1} to similar equipment for an LWR.^{D2,D3} There appears to be a strong dependence on volume of the buildings and cost per unit volume. We therefore defined the following factor,

$$VF = \frac{C(21)/\text{kW}_t}{\$22/\text{kW}_t} , \quad (\text{D.44})$$

to account for the relative volume and relative cost per unit volume as compared to that for an LWR. In Eq. (D.44), the numerator is the sum of accounts 211 through 218D per kilowatt; the denominator is the same sum for an LWR.^{D3}

The switchgear cost estimate is^{D2}

$$C(241) = VF \times \$0.6/\text{kW}_t , \quad (\text{D.45})$$

which is almost entirely for station service. The station service equipment includes station service transformers, low-voltage substation, and auxiliary power sources (batteries, diesel generator). Because the cost of this equipment should be independent of building volume, we use LWR costs^{D2}:

$$C(242) = \$2.5/\text{kW}_t . \quad (\text{D.46})$$

The switchboards equipment includes the main and auxiliary control panels and their instruments. These also should be independent of building volume and we use^{D2}

$$C(243) = \$0.5/kW_t . \quad (D.47)$$

The protective equipment costs are estimated at^{D1,D2}

$$C(244) = \$0.2/kW_t . \quad (D.48)$$

The electrical structures and wiring containers cost estimate includes underground ducts, cable trays, and conduit^{D1-D3}:

$$C(245) = \$3.5/kW_t . \quad (D.49)$$

The power and control wiring estimate covers the generator circuits, station service power wiring, control and instrument wiring, and containment penetrations, at a cost of^{D1-D3}

$$C(246) = VF \times \$5/kW_t . \quad (D.50)$$

25. Miscellaneous Plant Equipment:

The transport and lifting equipment (cranes and hoists) is estimated to cost^{D1-D3}

$$C(251) = \$0.5/kW_t . \quad (D.51)$$

The cost of the air, water, and steam service (auxiliary heating boilers) is estimated to be^{D2,D3}

$$C(252) = \$1.5/kW_t . \quad (D.52)$$

D.2. INDIRECT COSTS

A review of the available literature for indirect cost factors for nuclear power plants yielded the values given in Table D1. Based on these estimates, we adopted the following (optimistic) values:

$$\begin{aligned}i_{91} &= 0.20 , \\i_{92} &= 0.15 , \\i_{93} &= 0.05 , \\i_{94} &= 0.20 , \\ \text{contingency} &= 0.10 .\end{aligned}$$

This results in a total indirect cost factor of

$$i_{ID} = 0.91 . \quad (D.53)$$

Table D1. Indirect cost factors.

Reference	i_{91}	i_{92}	i_{93}	i_{94}	Contingency	i_{ID}
D1	0.23	0.16	0.08	0.24	0.30	1.48
D2	0.072	0.163	0.052	—	—	—
D3	0.079	—	—	—	—	—
D12	0.21	0.15	—	—	—	1.24
D13	0.246	—	—	0.208	—	0.77

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APPENDIX E. SCALING OF THE PLASMA DIRECT CONVERTER

The geometry of the plasma direct converter is shown in Fig. E.1. To determine the size of the vacuum tank, the plasma must be expanded to the point where the power flux is low enough to permit proper collector operation. An overall review of plasma direct conversion is presented in Ref. 18. Here, our objective is to determine R_{DC} , H_{DC} , and θ_{DC} .

The plasma power into the direct converter which is to be magnetically expanded is denoted as P_{DC} . The total plasma power leaving the confinement volume is P_4 (see Eq. 2.52). The fraction that leaks out of each end is a function of the magnet design and is specific to the model. Also, the alpha particles, if they are not adiabatically confined, cannot be magnetically expanded and their energy directly converted. Therefore, the quantity P_{DC} is given by

$$P_{DC} = \delta_F P_{10} + \delta_\alpha 0.2 P_1, \quad (E.1)$$

where δ_F is the fraction of the fission leaking into the direct converter under consideration. If $\delta_\alpha = \delta_F$, the alphas are thermalized in the plasma otherwise, $\delta_\alpha = 0$. The power flux at the collectors is then

$$q_c'' = \frac{P_{DC}}{A_C}, \quad (E.2)$$

Fig. E.1. Direct converter geometry.

where A_c is the collector area,

$$A_c = \theta_{DC} R_c H_{DC} \quad (E.3)$$

The maximum allowable heat flux at the collector is a function of the type of direct converter and is specific to the model. Typically, for a three-stage converter ($\eta_{DCF} \approx 0.65$), $q_c'' = 50$ to 100 W/cm^2 . For a single-stage model ($\eta_{DCF} \approx 0.5$), $q_c'' \approx 250 \text{ W/cm}^2$. If the plasma energy simply is dumped thermally ($\eta_{DCF} = 0$), we might use $q_c'' > 250 \text{ W/cm}^2$.

By conservation of the magnetic flux between the midplane of the magnet and the collectors,

$$\pi B_0 r_p^2 = B_c A_c \quad (E.4)$$

where B_0 is the central field (with plasma), r_p is the plasma radius, and B_c is the field at the collector. We have found from detailed magnetic field calculations that the field intensity in the region of the direct converter decreases with increasing distance from the center of the magnet as

$$B = B_{m1} \left(\frac{R_2}{z} \right)^{3.9} \quad (E.5)$$

where R_2 is the major radius of the outer conductor bundle (see Appendix A) and B_{m1} is the mirror field. Thus, using Eqs. (E.4) and (E.5),

$$A_c = \frac{\pi r_p^2}{R} \left(\frac{R_c}{R_2} \right)^{3.9} \quad (E.6)$$

where R is the plasma mirror ratio. Substituting Eq. (E.6) into Eq. (E.2), we have

$$R_c = R_2 \left[\frac{P_{DC} R}{\pi r_p^2 q_c''} \right]^{0.257}.$$

Then, we match this result to a specific design^{E1} and adjust it by a multiplicative constant to obtain

$$R_c = 0.94 R_2 \left[\frac{P_{DC} R}{\pi r_p^2 q_{c,DC}''} \right]^{0.257} \quad (E.7)$$

The radius (metres) to the outside of the vacuum tank is then approximately

$$R_{DC} = R_C + 5 \quad . \quad (E.8)$$

Using Eqs. (E.3) and (E.6), we solve for the tank height,

$$H_{DC} = 1.3 \times \frac{\pi r_p^2}{R_{DC}^2 R_C} \left(\frac{R_C}{R_2} \right)^{3.9} \quad , \quad (E.9)$$

where typically, $\theta_{DC} \approx 120$ deg and the factor 1.3 adjusts the scaling to a specific design.

The area of the tank side wall in front of the collectors is approximately

$$A_{DCS} = 2\theta_{DC} (R_C^2 - R_2^2) + H_{DC} R_C \left(1 - \frac{R_2^2}{R_C^2} \right) \quad . \quad (E.10)$$

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APPENDIX F. HYBRID CASH-FLOW FORMULATION

To properly account for the time-dependent effects in the hybrid economics, we use a present-worth cash-flow analysis patterned after the analyses described in Refs. F1-F3. Our objective is to determine levelized prices (i.e., constant over the reactor life) for the hybrid products so that all of the plant expenditures are recovered by the end of the reactor life.

For the cash-flow formulation, we divide the reactor lifetime into equal-length accounting periods and evaluate the income and expenses in each period, treating the product unit cost as an unknown. The product costs then are determined by requiring that the total indebtedness be retired by the end of the reactor life.

Defining Y_m to be the indebtedness at the start of accounting period m , the indebtedness at the start of period $m+1$ is

$$Y_{m+1} = Y_m + z_m - R_m, \quad (F.1)$$

where z_m is the net capital investment during period m and R_m is the net income.

The capital investment includes the initial plant capital cost, denoted as z_0 , plus capital investments z_m during reactor operation (these will later be identified as fuel cycle costs). Thus, these quantities are evaluated from the results of the analyses in Secs. 2.2 and 2.3. At this point, we may make the identification,

$$Y_1 = z_0; \quad (F.2)$$

Eq. (F.1) then is a recursive relation in Y_m .

The net income in period m is expressed as

$$R_m = S_m - I_m - O_m - T_m, \quad (F.3)$$

where S_m is gross revenue, I_m is investor return on the indebtedness, O_m represents O & M expenses, and T_m is tax.

The gross income from product sales is

$$S_m = p_e Q_{e,m} + p_f Q_{f,m}, \quad (F.4)$$

where p_e and p_f are the levelized unit costs of electricity (mills/kWh) and fissile material (\$/g), respectively, $Q_{e,m}$ is the quantity of electricity produced in period m , and $Q_{f,m}$ is the quantity of fissile material produced in period m . The later two quantities ($Q_{e,m}$ and $Q_{f,m}$) are evaluated from the analysis in Sec. 2.3.2 and the unit prices (p_e and p_f) are the unknowns which we will determine.

Large power projects are typically financed by selling bonds (debt financing) and, in the case of a private corporation, by selling stock (equity financing). Thus, the return on indebtedness is

$$I_m = [i_b f_b + i_e (1 - f_b)] Y_m, \quad (F.5)$$

where i_b is the bond interest rate (fraction per period), f_b is the fraction of indebtedness in bonds, and i_e is the stock return rate or dividend (fraction per period).

In the present study, we assume that the O & M costs are proportional to the gross thermal output of the plant (see Sec. 2.1.5),

$$O_m \sim \bar{P}_5 + P_6, \quad (F.6)$$

where the proportionality constant is based on fission reactor costs and is input to the model.

A detailed tax accounting would include federal and state income taxes, gross revenue (sales) tax, and property tax. However, here we consider only federal income tax because it is by far the largest tax; inclusion of the other taxes adds a great deal of complexity without significantly changing the results of the economic analysis.

The federal income tax is based on the fraction, k_f , of taxable income, where

$$\text{taxable income} = \text{gross income} - \text{deductions}, \quad (F.7)$$

$$\begin{aligned} \text{deductions} = & \text{operation and maintenance} \\ & + \text{depreciation} + \text{bond interest payments}. \end{aligned} \quad (F.8)$$

Note that bond interest payments are deductible but stock dividends are not. Thus we express the income tax for period m as

$$T_m = k_f (S_m - O_m - D_m - i_b f_b Y_m) \quad , \quad (F.9)$$

where D_m is the depreciation for period m . We have used simple straight-line depreciation assuming all capital investments are depreciable (i.e., they have no recovery value). Thus,

$$D_m = \frac{1}{N_A} \left(z_0 + \sum_{j=1}^{N_A} z_j \right) \quad , \quad (F.10)$$

where N_A is the number of accounting periods in the reactor life. Defining the capital investment for periods 1 through N_A as

$$Z = \sum_{j=1}^{N_A} z_m \quad , \quad (F.11)$$

the tax is written in terms of primary variables:

$$T_m = k_f \left[p_e Q_{e,m} + p_f Q_{f,m} - O_m - \frac{(z_0 + Z)}{N_A} - i_b f_b Y_m \right] \quad . \quad (F.12)$$

Substituting this into Eq. (F.3) and rearranging the terms, the net income for period m is

$$R_m = (1 - k_f) (p_e Q_{e,m} + p_f Q_{f,m}) - \left\{ [i_b f_b (1 - k_f) + i_e (1 - f_b)] Y_m + (1 - k_f) O_m - k_f \frac{(z_0 + Z)}{N_A} \right\} \quad . \quad (F.12)$$

In turn, substituting into the cash-flow relation, Eq. (F.1), we obtain

$$Y_{m+1} = (1 + x) Y_m + z_m - (1 - k_f) (p_e Q_{e,m} + p_f Q_{f,m}) - (1 - k_f) O_m - k_f \frac{(z_0 + Z)}{N_A} \quad , \quad (F.13)$$

where x is the discount factor,

$$x = i_b r_b (1 - k_f) + i_e (1 - f_b) \quad . \quad (F.14)$$

Equation (F.13) is a recursion relation in Y_m . Using the conditions Eq. (F.2) and

$$Y_{N_A+1} = 0 \quad , \quad (F.15)$$

which specifies that the indebtedness be reduced to zero by the end of the reactor life, the solution to Eq. (F.13) can be expressed in the form^{F1}

$$\begin{aligned} (1 - k_f) \left[p_e \sum_{m=1}^{N_A} (1+x)^{-m} Q_{e,m} + p_f \sum_{m=1}^{N_A} (1+x)^{-m} Q_{f,m} \right] \\ = \sum_{m=1}^{N_A} (1+x)^{-m} \left[z_m - k_f \frac{z}{N_A} \right] + (1 - k_f) \sum_{m=1}^{N_A} (1+x)^{-m} Q_m \\ + z_0 \left[1 - \frac{k_f}{N_A} \sum_{m=1}^{N_A} (1+x)^{-m} \right] \quad . \quad (F.16) \end{aligned}$$

This equation balances the revenues and expenses. On the left of the equation are the after-tax revenues from the sale of electricity and fissile material; this is balanced on the right by depreciated fuel cycle costs, after-tax operation and maintenance costs, and depreciated plant capital cost, respectively. We assume that all capital costs after the initial plant investment, z_0 , are due to fuel cycle costs (see Appendix G). All quantities in Eq. (F.16) are known except for p_e and p_f and this forms the basis for evaluating these two leveled costs.

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- F1. *Guide for Economic Evaluation of Nuclear Reactor Plant Designs*, NUS Corporation, Rockville, Maryland, Rept. NUS-531 (1969).
- F2. R. Salmon, *A Revision of Computer Code POWERCO (Cost of Electricity Produced by Nuclear Power Stations) to Include Breakdowns of Power Cost and Fixed Charge Rates*, Oak Ridge National Laboratory, Rept. ORNL-4116 (1969).
- F3. T. H. Pigford, *The Analysis of the Cost of Electrical Energy from Nuclear Power Plants*, Nuclear Engineering Department, University of California, Berkeley, CA, Rept. NE-3008, Rev. 2 (1976).

APPENDIX G. HYBRID FUEL CYCLE

Following the fuel cycle accounting method described in Ref. G1, the fuel cycle costs are broken down into components,

$$z_m = \sum_{i=1}^8 z_{i,m} , \quad m = 1, 2, \dots, N_A , \quad (G.1)$$

where the subscript i denotes the various components. For the hybrid, we consider the following fuel cycle costs.

$i = 1$: cost of fertile materials (uranium or thorium and tritium-breeding material).

$i = 2$: value of recovered fertile materials.

$i = 3$: cost of initial fusile materials (tritium and deuterium).

$i = 4$: value of recovered fusile materials.

$i = 5$: fabrication costs (uranium or thorium and tritium breeding-material).

$i = 6$: reprocessing costs (uranium or thorium and tritium breeding-material).

$i = 7$: spent fuel shipping costs (uranium or thorium and tritium breeding-material).

$i = 8$: blanket structure replacement costs.

In our model, we specify unit costs for all these components. Then, based on the blanket inventories (Sec. 2.1.3) and the blanket change schedule (Sec. 2.3.1), these costs are evaluated during the reactor life ($m = 1, \dots, N_A$). We also include all initial ($m = 0$) blanket costs including spare modules.

REFERENCE

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