IDO—19022

CEND-108

ANNUAL PROGRESS REPORT

REACTOR THEORY DEVELOPMENT PROGRAM

FOR

ABWR

October 1, 1959 to September 30, 1960

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REACTOR THEORY DEVELOPMENT PROGRAM FOR ABWR

October 1, 1959 to September 30, 1960

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REACTOR THEORY DEVELOPMENT PROGRAM FOR ABWR

I. SUMMARY

The objective of this program is to provide analytical techniques for predicting the static and dynamic behavior of boiling water reactors. The validity of these techniques will be tested by application to experimental boiling reactors. This will help determine the complexity of model and accuracy of parameters required. By distinguishing the important physical processes and providing a reliable description of them, the designer of boiling water reactors will be able to obtain a higher degree of performance in his designs. Although a large amount of work has been done in this field, the degree of confidence which has been obtained is not high, at least in a quantitative sense.

During the past year, the work has fallen into three main phases aimed at better determination of the stability, static behavior, and transient behavior of boiling water reactors.

We have approached the problem of static reactor behavior by first reviewing the available literature on theory and experiments concerning steam void distributions which play a large part through their reactivity effects. Here, the uncertainties in steam-water velocity ratios, subcooled boiling and two-phase pressure drops present the main problems. The best of the current methods were then incorporated into a boiling water reactor thermal and hydraulic code STREAC which yields the void distribution for each channel of a boiling reactor when the power distribution is known. Initial attempts were made in studying the characteristics of bubble coalescence and its role in improving the basic physical theory of fluid flow and heat transfer in a boiling channel.

Since the maximum power rating of a given reactor is often limited by stability characteristics, we devised TPF-1, a boiling water reactor transfer function digital code which gives the power amplitude and phase response to an oscillating reactivity input. In considering approximations to transfer functions, the ability of the residue method to represent a typical transform over a large range of frequencies is clearly shown. Since the stability methods used depend upon the linearization of the kinetic, hydraulic and thermodynamic equations, the question of their validity in representing the stability of a truly non-linear system is discussed.

A knowledge of the transient behavior of a reactor to accidents or power demand changes is necessary. We are investigating better numerical methods to be used in coding for digital computers the transient reactor equations. At present, Lancos' τ method seems to have some possibilities in greatly reducing the machine time required and its application to simple kinetics equations is presented.

Future work will immediately be more concerned with applying and improving the transfer function code and continuing investigation of methods for designing digital transient codes. Further work remains in basic bubble theory and more complete synthesis of neutronic, hydraulic and thermodynamic codes.

II. STATIC REACTOR BEHAVIOR

The requirement to use heavy and costly equipment needed to withstand the high pressures in a pressurized water reactor has led to the development of a reactor operating at water saturation pressures. The simplicity in supplying the steam to the turbine directly from the reactor without the use of intermediate heat exchangers and primary-secondary loops also has contributed to the appeal of boiling water reactors. The use of natural circulation instead of pumps to furnish the coolant throughput in the reactor provides a simplification uniquely required of a small, reliable power reactor. However, the incorporation of steam in the core adds a complication due to the effective density change of the water and subsequent alteration of neutron leakage and absorption rates. The limiting condition in boiling heat transfer rate, encountered when a steam blanket insulates the heat transfer surface at the inception of film boiling is affected by the amount of steam voids in the system. The attainment and maintenance of high power density under stable reactor operation are also dependent on the amount of steam voids in the system. Thus, to fully utilize the advantages inherent in boiling water reactors, the development of accurate analytical techniques to predict steam void distributions is apparent.

We have approached this problem by first reviewing the available literature on theory and experiments concerning steam void distributions. The best of the current methods were then incorporated into a boiling water reactor thermal and hydraulic code STREAC. Finally, taking a longer range viewpoint, an attempt was made to improve the basic physical theory of fluid flow and heat transfer in a boiling channel by studying the role of coalescence in bubble theory.

A. Status of Thermal and Hydraulic Information for Boiling Water Reactor Design

This is a discussion of the presently used techniques for determining the thermal and hydraulic characteristics of boiling water reactors. Special emphasis is given to the steam void distribution.

The steam volume fraction, α, is a function of system pressure, average velocity of each phase of the coolant mixture and the fraction of steam present. This may be shown by the continuity of mass equation and the definition of steam quality (fraction of steam).

$$x(z) = \frac{\rho_g V_g(z)\alpha(z)A}{\rho_g V_g(z)\alpha(z)A + \rho_f V_f(z)(1-\alpha(z))A} = \frac{\text{pounds of steam}}{\text{pounds of coolant}} \qquad (1)$$

Thus, in a channel of cross sectional area, A, operating at a saturation pressure and temperature corresponding to a fluid specific density ρ we arrive at the following equation for the steam volume fraction:

$$\alpha(z) = \frac{1}{1 + \frac{V_g(z)}{V_f(z)} \frac{\rho_g}{\rho_f} \frac{1-x(z)}{x(z)}} \quad . \tag{2}$$

The quality $x(z)$ may be determined if the power distribution is known using the following integral equation:

$$x(z) = \frac{1}{\rho_f V_f(o) A h_{fg}} \int_o^z q(z')dz' - \frac{h_f - hin}{h_{fg}} \quad . \tag{3}$$

It is assumed here that the inception of boiling occurs at the saturation temperature of the water. Experiments have been performed to study the effect of subcooled boiling on the steam volume fraction. The results of these tests are shown in Figure 1. It is apparent that there is considerable difference in void fraction plots with the addition of subcooling to the liquid entering the channels. The greatest change in the local void fraction is, of course, due to the shortening of the boiling length as inlet temperature is lowered; however, such examination does not produce as satisfactory a comparison of events as does the normalized void fraction distribution. The figure shows the normalized void fraction distributions for inlet subcooling temperatures from zero to 20°F. The effect of the inlet temperature on the steam void fraction may also be seen in Figure 2. Here may be seen the un-normalized plots of the data and the effect of subcooling on boiling length not possible on the normalized plots. The inlet temperature on each run differs by approximately 5°F. For saturated inlet temperature, the length of boiling corresponds to the length of channel and the void curve has the largest ordinate of those shown. Successive runs at lower inlet temperatures or higher subcooling, have shortened boiling lengths.

The effect of velocity upon steam void fraction is shown by typical curves in Figure 3. Inlet temperature was the saturation temperature on all runs. It is important to note that very little change in steam void fraction is experienced in going from 1.6 to 0.8 fps, while the exit steam weight fraction changes by a factor of about two. The apparent conclusion from this is that there is a considerable change of the steam and liquid velocity relationship; otherwise, a variation of about a factor of two in α would be expected.

It has been pointed out that, if the movement of the steam-liquid mixtures through the channels is considered as a steady-state phenomenon in which thermal equilibrium is maintained at every point, one may then consider the flow of either phase as a function of the weight fraction of the component, its density, its cross sectional area, and its mean velocity. With these assumptions, Equations (1), (2), and (3) have been derived, showing the resulting parameters of power density (power removal), continuity, and the distribution of steam voids to be functions of V_g/V_f. For a known flow, this will allow determination of density, power removal and the distribution of steam voids in a heated channel.

The parameters which have been found to be most strongly influencing the velocity slip ratio, V_g/V_f, are pressure, steam quality and liquid velocity. It has been found as a result of these tests that the velocity slip ratio decreases very rapidly with increase in pressure. It is suspected that the ratio V_g/V_f is following a direct proportionality with the ratio of specific volumes as pressure is varied. A similar behavior is obtained for any two-phase or two-component mixture where densities of the components are changed. Figure 4 shows the results of investigators in determining the dependence of slip ratio on pressure.

The trend of the data of slip ratio versus steam quality indicates a direct proportionality exists with an initial value for the ratio at the inception of steam formation. Typical results showing this relationship may be seen in Figure 5. A sample of the effect of inlet velocity is also shown.

Liquid velocity has been used as a parameter in describing the characteristics of the slip ratio in the form of the inlet velocity to the channel during quality flow conditions. Figures 6 and 7 show the effect of the inlet velocity on the slip ratio at pressures of 150 psig, 250 psig and 600 psig at various constant qualities. The effect of channel configuration has been found to be of negligible effect on the slip ratio during these experiments.

It should be pointed out at this time that the use of a constant slip ratio as a physical representation of the velocities of the gas and liquid phases originates with its ease of use in Equation (2). The previous material shows the variance of slip ratio with quality. However, the approximation of constant slip ratio in the calculation of average channel conditions is realistic provided the ratio is obtained at system pressure and inlet velocity conditions. Figure 8 shows the variation of slip ratio, V_f, V_g and V_g-V_f in a typical heated channel experiment with constant power distribution.

Recent work in this field has indicated a trend toward representing the slip as a function of pressure and local steam volume fraction. A theoretical prediction of steam slip based on a momentum model proposed by Levy yields the following relation for slip at low qualities:

$$\frac{V_g}{V_f} = \sqrt{\frac{\rho_f}{\rho_g}} \quad \sqrt{2a} \quad . \tag{4}$$

Resume

At the present, good agreement between calculated average void fraction and experimental quantities can be found using the formulation of Equations (1), (2), and (3) with constant slip ratio. Local void fraction should be calculated using a variable slip ratio and appropriate local subcooled void contribution. However, the exact form of the functionality of the slip ratio and subcooled boiling are unresolved at this time. Attempts to use the slip ratio in the following form

$$\frac{V_g}{V_f} = a + bx \quad , \tag{5}$$

where a and b are constants and x is quality along the channel, have given better results for predicting the local void fraction.

It is believed the present difficulty in obtaining good resolution in the experimental results is due to the unpredictability of the type of two-phase flow pattern involved; i. e., bubble flow, slug flow, annular flow, etc., Figure 9 shows an attempt to define the flow regimes.

Two-Phase Frictional Pressure Drop

The two-phase frictional pressure drop in a boiling channel is usually expressed as

$$\Delta P_{TP} = \frac{f_f G^2}{2g_c \rho_f D} \int_o^{L_B} R dL = \bar{R} \frac{f_f G^2 L_B}{2g_c \rho_f D} \tag{6}$$

where

$$R = \frac{(dP/dL)_{TP}}{(dP/dL)_{\text{liquid only}}} \quad . \tag{7}$$

The difficulty in evaluating the above equation is obtaining the correct two-phase friction factor multiplier R. There appears to be a relation between two-phase frictional pressure drop and the type of two-phase flow pattern involved; i.e., bubble flow, slug flow, annular flow, etc. The ability to predict this frictional pressure drop is complicated by the inability to predict these flow regimes.

A widely accepted work in this field is that of Martinelli and Nelson. They modified Martinelli's previous model to apply to steam-water mixtures and developed curves of the local pressure

$$\frac{(dP/dL)_{TP}}{(dP/dL)_{\text{liquid only}}} \tag{8}$$

gradient as a function of quality and pressure for the most common flow mechanism, that in which both phases are flowing turbulently. They also presented a curve of the mean pressure gradient in a uniformally heated boiling channel. The experimental data used came from low pressure, adiabatic, two-component flow in horizontal pipes and the pressure dependent parameters of the curves they present are largely interpolated from this low pressure data and the conditions at the critical point.

Lottes and Flinn proposed a simple annular flow model in which the mean two-phase friction multiplier, R, for a uniformly heated boiling channel is expressed as a function of the exit steam volume fraction. The basic assumptions of the model were:

a) the increased pressure drop is due to the increased velocity of the liquid phase only,

b) the friction factor is constant along the length of the channel,

c) the slip ratio V_g/V_f is constant along the length of the channel.

Tests used to verify these methods indicate that the Lottes and Flinn relations represent the data qualitatively though not quantitatively for 150 to 600 psig. The martinelli-Nelson correlation has shown better agreement but not at exit qualities less than about 2% for the same test pressures.

Resume

An attempt to apply a suitable multiplying factor to the Lottes and Flinn relations has been found to give satisfactory results. Using a 1.25 multiplying correction factor, this formulation becomes:

$$\bar{R} = \frac{1.25}{3}\left[1 + \frac{1}{1-\alpha_e(1-\psi)} + \left(\frac{1}{1-\alpha_e(1-\psi)}\right)^2\right] \quad (9)$$

where

$$\psi = \frac{V_g}{V_f}\,\frac{\rho_g}{\rho_f} \quad . \quad (10)$$

If the slip ratio is assumed equal to one, the above equation reduces to

$$\bar{R} = 1 + X_e\,\frac{v_{fg}}{v_f} \quad . \quad (11)$$

For adiabatic flow, the mean friction factor multiplier equals the local friction factor multiplier and the equation can be replaced by

$$R = \left(\frac{1-x}{1-\alpha}\right)^2 \quad . \quad (12)$$

Future Development Work

Multiregion physics calculations should be improved by the addition of multichannel engineering calculations. Such an engineering code should contain a variable slip ratio formulation as STREAC does, and subcooled boiling void contribution. At the present time, the variation of slip ratio along a heated channel is not accurately predictable but reasonable approaches are indicated in the previous sections. A good functional representation of subcooled boiling void distribution is not known and the effect should be included. Since the introduction of these voids occurs in the high reactivity region of the core, their significance is increased.

In natural circulation reactors, the two-phase pressure losses and gains require further refinement or possible reformulation. This is suggested because the present correlations and formula assume uniform power distributions and neglect the possible dependence on type of flow conditions; i. e., fog flow, annular flow, etc.

The effect of riser configuration is known qualitatively but quantitative evaluation is inadequate. This results from a poor understanding of the effect of changing flow area on steam volume fraction during the transition

of channel to riser sections. An attempt has been made at atmospheric pressure to describe this change at ANL using the following semi-theoretical correlation

$$\alpha_2 = \left[\frac{\left(\frac{1}{\alpha_1} - 1 \right)}{\left(\frac{A_1}{A_2} \right)^{0.2}} + 1 \right]^{-1} \qquad (13)$$

This change in the steam volume fraction is due to the slip ratio change as the liquid velocity changes. In conjunction with this is the pressure drop associated with expansion of area which is not predictable for two-phase flow.

Future development of analytical tools to calculate the above mentioned problems lies in a thorough experimental program and initiative to learn the basics of the two-phase flow phenomena.

Nomenclature

h	Enthalpy
hin	Channel inlet enthalpy
h_f	Enthalpy of saturated liquid
h_{fg}	Heat of vaporization
q'	Heat input per unit time per unit length
w	Flow rate
x	Quality = $\frac{w_g}{w_g + w_f}$
z	Distance along channel

Greek Letters

α	Steam void fraction
ρ	Density

(For Figure 9)

$$\lambda = \left[(\rho_g / 0.075)(\rho_f / 62.3) \right]^{1/2}$$

$$\psi = (73/J) \left[\mu_f (62.3/\rho_f)^2 \right]^{1/3}$$

μ	Viscosity

where J is the surface tension of the liquid in dynes per centimeter, G is the mass velocity of the gas phase, and L/G is the ratio of mass velocities of the liquid and gas phase.

Subscripts

f	Liquid phase
g	Gas phase
TP	Two-phase

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B. STREAC - A Boiling Water Reactor Thermal and Hydraulic Code

STREAC is an IBM-704 code designed to calculate the steady-state thermal and hydraulic behavior of one pass natural circulation boiling reactors. Given the power distribution along the axis of each box in a multichannel reactor under the assumption of axial and radial power separability, the code yields steam void distributions and inlet velocities for each box. Further information is also obtained on pressure differentials and velocity behavior for the hottest channel when the power is increased. A discussion of the average reactor calculation will help to illuminate the methods used.

For the average reactor calculation with a fixed feed-water enthalpy, the position of no net boiling or the "non-boiling length" is readily determined through the power required to raise the feed-water enthalpy to that of the saturated liquid. One then assumes an inlet velocity and that the quality of the steam above the non-boiling length is directly proportional to the excess of integrated power above that at the boiling length. Having thus determined the quality as a function of position, the slip ratios; i. e., the ratios of steam-to-liquid velocities, which for the given pressure are in tabular form as a function of inlet velocity and quality are also obtained as a function of position. The steam void distribution is then obtained from the definition of quality; i. e., the ratio of steam mass flow rate to that of total mass flow rate. Local two-phase friction factors are determined as in Equation (12) and are integrated to obtain an average factor. The acceleration pressure drop factor is assumed to be a function of exit void fraction and quality only. Since all the pressure losses including those due to two-phase friction, acceleration, and exit, entrance and downcomer losses are proportional to the square of the velocity and must be balanced by the bouyant effects of the steam, a new velocity is obtained. This entire process is repeated a sufficient number of times to make the difference between two successive velocities lower than some preset value.

For the multichannel case, one initially assumes that the inlet velocity for each box is that obtained from the average reactor calculation; here, however, each non-boiling length depends on the assumed velocity which determines the inlet enthalpy. Iterations are performed on the inlet velocity for each box in a manner very similar to that described above. These new velocities will yield a different total mass flow rate which in turn determines a new inlet water enthalpy. Using this new inlet enthalpy, the inlet velocities are again iterated to convergence. This process is repeated until the difference between successive total mass flow rates is less than some pre-assigned value.

For each box, the output includes steam void distribution, inlet velocity, average void fraction, exit void fraction, box flow rate, steam production rate, non-boiling length and the pressure loss term.

The power extrapolation routine determines the flow rate for the "hottest" box; i. e., that one producing the most power, for various powers within this box. Here the inlet enthalpy is assumed to be unchanged and the velocity iteration is carried out for only the one box. This information is useful as a stability criterion since beyond some critical power value the flow rate will decrease instead of increase and a given flow rate can correspond to two power values.

The report making this code available for external usage is nearing completion.

C. Coalescence in Bubble Theory

A glance at the literature devoted to the fluid flow and heat transfer in a boiling channel shows that a great deal of work has been done in this field. The major part of this work is based on rather crude theoretical models. Moreover, it would appear that further progress in this field is most likely to result from the analysis of more sophisticated models and from more careful experiments on simple systems. The theoretical work of Forester and Zuber[13] on the rate of growth of single bubbles in a superheated liquid stands out by virtue of its analytical integrity as well as for its power to predict experimental results. Forester and Zuber did not include in their analysis the phenomenon of bubble coalescence. It would seem useful to study bubble coalescence in turbulent two-phase flow.

A model of two-phase flow based upon turbulent flow of the liquid and treating the gas flow as bubbles by means of a Fokker-Planck type of equation seems physically realistic. The turbulent velocity components would provide the random forces acting on the bubbles. Thus, the bubbles would interact by coalescence and would grow in size by coalescence, by motion in a pressure gradient, and by evaporation of water at the bubble surface. Such a two-phase theory should predict the onset of plug flow from bubble flow. It would contain as a source term a bubble distribution which appears to be physically realistic for nucleate boiling. Such a theory might, in fact, be useful in studying the heat transfer at a surface in nucleate boiling. The theory should give some physical insight into such quantities as slip ratio, pressure drop in a fuel channel, etc. The following is an indication of how such a theory might be constructed.

It is simplest to consider first bubble growth by coalescence alone. The similarity of bubble coalescence to coagulation is at once apparent. However, one soon finds that coagulation theory cannot simply be taken

intact as a description of bubble coalescence. One can, however, make use of certain results arrived at in coagulation analysis[14].

When two particles describe Brownian motion independently of each other with diffusion constants D_i and D_k then the relative displacements of the two particles also follow the laws of Brownian motion with the diffusion constant $D_{ik} = D_i + D_k$. Furthermore, if R_{ik} represents the distance to which two particles must approach each other in order to coalesce into a single particle, the rate of coalescence of particles of type i and type k is given by

$$J_{i+k}dt = 4\pi D_{ik}\nu_i\nu_k\left(1 + \frac{R_{ik}}{(\pi D_{ik})^{1/2}}\right) dt \quad , \tag{14}$$

where ν_i, ν_k is the number density of particles of type i, type k respectively.

To describe bubble coalescence suppose $\nu(v,t)$ is the number density of bubbles with volume between v and v + dv, and suppose that when a bubble of volume v_1 and one of volume v_2 coalesce, the new bubble has volume $v_1 + v_2$ (this corresponds to a perfect gas law, isothermal process and neglect of surface tension forces). Then the rate of change of $\nu(v,t)$ with time due to coalescence is given by:

$$\frac{d\nu(v,t)}{dt} = \int_D^v c_1(v,v')\nu(v')\nu(v-v') - 2\int_o^\infty c_1(v,v')\nu(v')dv' \quad . \tag{15}$$

The first term on the right-hand side gives the increase in $\nu(v)$ by coalescence of bubble pairs of smaller volume; the second term gives the loss of bubbles in the range v, v + dv due to coalescence of a bubble in the range with another bubble.

The integral equation can be solved in closed form if $c_1(v,v')$ is a constant. Solutions with c_1 a constant show the qualitative features of coalescence. With c_1 constant and the initial distribution $\nu(v,o) = an_o e^{-av}$ the solution is:

$$\nu(v,t) = an_o(1 + n_o c_1 t)^{-2} \exp\left(\frac{-av}{1 + n_o c_1 t}\right) \tag{16}$$

where

$$n_o = \int_o^\infty \nu(v', o)\, dv' \quad , \tag{17}$$

the total number of bubbles at time $t=0$, and $\underline{a}$ is a constant characterizing the initial distribution.

The initial distribution

$$\nu_o = \nu(v, o) = n_o \delta(v - v_o) \tag{18}$$

where $\delta(v-v_o)$ is the Dirac δ function corresponds to an initial distribution of equal size bubbles. The solution for this case, again with a constant is:

$$\nu(v, t) = n_o (1 + n_o c_1 t)^{-2} \sum_{k=1}^{\infty} \frac{1}{n_o^{k-1}} \delta(v - k v_o) \left\{ \frac{n_o c_1 t}{1 + n_o c_1 t} \right\} k-1 \tag{19}$$

where n_o is defined as before and v_o is the volume of each bubble at time $t=0$.

The equations of coalescence given above can be modified to describe the case in which the distribution of bubbles of size v is not a continuous function of v. In particular, we may assume that only bubbles formed by an integral number of coalescences of bubbles of one size are present. It is hoped that this will lead to an analytical method for prediction of the onset of plug flow in a channel. If we assume that the dimensions of the channel are such that bubbles formed by $(n-1)$ collisions are the largest which may collide with any bubble, then Equation (15) is replaced by the equations

$$\frac{d\nu_i}{dt} = \begin{cases} \frac{1}{2} \sum_{k+j=i} c_{kj} \nu_k \nu_j - \nu_i \sum_{k=i}^{n-1} c_{ik} \nu_k \quad (i=1, \ldots, n-1) \, , \\ \\ \frac{1}{2} \sum_{k+j=i} c_{kj} \nu_k \nu_j \quad (i = n, \ldots, 2n-2) \, , \end{cases}$$

where $\nu_1(t)$ is the distribution of the smallest size bubble as a function of

time and $\nu_j(t)$ $(j=2, \ldots, 2n-2)$ is the distribution of bubbles of the volume of a bubble formed by the coalescence of j bubbles of the smallest size.

A good approximation to (20) is obtained if the c_{ij} are assumed constant and equal to the same number c. The equations can be simplified by making the change of coordinates:

$$(a) \quad x_j = \frac{\nu_j}{\nu_1(o)} \quad (j=1, \ldots, 2n-2)$$

$$t' = c\nu_1(o)\, t \quad .$$

Substituting (a) in (20), we obtain:

$$\begin{aligned} \dot{x}_i &= \frac{1}{2} \sum_{k+j=i} x_k x_j - x_i \Sigma x_j \quad (i=1, \ldots, n-1) \\ \dot{x}_i &= \frac{1}{2} \sum_{k+j=i} x_k x_j \quad (i=n-1, \ldots, 2n-2) \quad , \end{aligned} \tag{21}$$

where the dot signifies differentiation with respect to t'. Approximate solutions for $x_1, \ldots, x_{n-1}$ can be obtained by setting $\dot{x}_i = 0$ $(i=n-1, \ldots, 2n-2)$. Then upon adding Equations (21), one gets

$$\sum_{j=1}^{n-1} \dot{x}_j = -\frac{1}{2} (\Sigma x_j)^2 \quad , \tag{22}$$

which can be readily solved if initial conditions are known for the x_j. The result of (22) can then be substituted in the first of Equations (21) to obtain x_1. The other x_j's are then obtained in sequential order.

Exact solutions for (21) have been obtained for the case $n=4$ with the initial conditions

$$x_1(o) = 1, \quad x_j(o) = 0 \quad (j \neq 1) \quad .$$

These are plotted in Figure 10 together with approximate solutions obtained by the method above.

III. BOILING WATER REACTOR STABILITY

The maximum power rating of a given boiling water reactor is often determined by stability considerations since, especially with high conductivity plate fuel elements, fuel centerline or burnout temperature limits are not usually reached. Stable operation of a boiling reactor exists when the power oscillations are random in nature and only a few percent in magnitude during steady-state operation. There are two types of unstable operation which have been found in the Borax and Spert series of experiments. The first is characterized by a pronounced but non-divergent periodic power oscillation amounting to 10% or more. The second exhibits itself in a divergent oscillation leading to a scram or reactor damage. The condition of reactor operation is controlled by the interaction of various reactor design parameters. Among the significant factors are operating pressure, hydraulic characteristics, fuel time constants and the reactivity associated with steam voids. This last factor has been shown not to be a good criterion of instability. For example, Borax IV was found to be unstable with 0.4% reactivity in voids at atmospheric pressure yet stable at 300 psig with 5.5% reactivity in voids.

Theoretical models of the complete reactor system usually in a linear approximation of the transfer and feedback functions have been investigated with analogue computers with some degree of correspondence. Our transfer function code TPF-1 will serve the same function on digital computers. This code will help us to isolate the design parameters of significance and at the same time lead to a better description of stability behavior.

Since some of the equations governing the physical processes are non-linear, a study of applicability of non-linear stability was made.

A. Transfer Function

1. TPF-1 - A Transfer Function Code

The most successful method of predicting stability for boiling water reactors is to express conditions for stability in terms of the transfer functions of power to reactivity input. Analyses of experimental power-reactivity feedback transfer functions for a reactor operating at different power levels and pressures can allow the determination with reasonable success of the parameters of semi-empirical transfer functions. These have been useful in extending with some confidence the stable operating range of the EBWR[15]. At the same time, simple models with theoretically determined parameters have given good qualitative agreement with EBWR[16] and Borax cores at least at higher frequencies where the recirculation time of the reactor does not enter. These results give the

approximate location of a power resonance on the frequency scale, but magnitudes and phase shifts are not as well represented. As more detailed models requiring more time constants are used, improvement is noticed[17].

TPF-1, an IBM-704 transfer function code, has been programmed and is now being tested. Input to this code consists of various reactor design parameters (power level, coolant temperature, equilibrium pressure, etc.) and the angular frequencies of sinusoidally varying excess reactivities of fixed (but unspecified) amplitude. The output provides the magnitudes in decibels and the phase angles in radians of the open loop and closed loop transfer functions of a model of a boiling water reactor.

The equations represented in the program are Laplace transforms of linearized differential equations describing a reactor model which combines features of various models previously suggested in reactor literature. The output represents the steady-state response of the reactor power to a periodically varying excess reactivity. This allows the designer to apply the techniques of stability analysis developed for the design of servomechanisms[18] to the reactor model with various combinations of reactor parameters. It also provides a means of determining the effect of varying different parameters on the response of the system.

The code is limited to processes which occur within the pressure vessel only. The reactivity feedbacks considered are those caused by voids and fuel temperature Doppler effects. Void changes are induced by direct heat transfer, pressure flashing and condensation, and temperature and velocity effects.

Figure 11 is a block diagram for a model of a boiling water reactor. One notices that the figure can easily be separated into two distinct parts. There is a rectangular region bounded clearly by the line labelled $\Delta\alpha/\alpha_o$ and a region peripheral to that rectangle. The boxes lying in the rectangular region represent the phenomena occuring inside the reactor pressure vessel which affect the response of a natural circulation reactor while the boxes outside represent those effects which are due either to a forced circulation loop or to phenomena occuring external to the pressure vessel. The initial version of TPF-1 is restricted to the natural circulation portion of the figure.

The general form of the block diagram is based on one proposed by Iriarte[17], but it differs from his in three respects. First, it includes a loop which represents the fuel temperature Doppler effect on reactivity. Second, the transfer function for heat transfer from fuel to water is in a

generalized form which allows the use of an arbitrary number of time constands. Third, the effect of a layer of water (reflector) above the reactor core on the variations in subcooling has been included.

One can gain an understanding of the meaning of the code output by tracing the effects of a sinusoudal reactivity input through the block diagram. Assume that the reactor has been operating at an equilibrium power level P_o when a sinusoidally varying excess reactivity of small amplitude δk_{in} is supplied to the reactor. (The mechanism which would provide such an excess in reactivity does not matter for the stability analysis. One could think of its being supplied by a control rod motion, for example.) This excess reactivity will produce a change ΔP in the power which can be related to the reactivity through the standard kinetics equations. The relation expressed by the kinetics equations is represented in the diagram by the box labelled "kinetics".

The excess power generated will have two effects of interest. It will cause a temperature change in the reactor fuel, and the temperature change will in turn alter the amount of heat transferred per unit time into the water channels. A change in fuel temperature will have a direct effect on the reactivity which is indicated by the "Doppler" box. This gives the first contribution to the feedback reactivity.

In order to find the change in the rate of heat transfer, one must know the equation of heat transfer for the reactor. Except in the simplest of examples, some approximation must be made for the heat to power transfer function. A discussion of the methods of approximation will be found in the following article. Whatever the transfer function decided upon, it is designated by the "heat transfer" box.

The heat excess ΔQ will have a direct effect upon pressure through the change in enthalpy which it induces. Hence, the box labelled $Q_o \Delta p/\Delta Q$ with input from ΔQ. The output from this box is one of the contributions to an overall pressure change induced by the reactivity variation.

Heat entering the fluid channels will have two additional effects. That portion of heat introduced into the subcooled portion of the channel will raise the enthalpy of the feedwater and its effect is considered in the loop terminating at ΔQ_{SC}. The remaining portion of the heat produces voids directly.

A change in void content has a direct effect upon reactivity by its changing of the density of the moderator. This is the second contribution to feedback reactivity considered in the TPF-1 code. Note that in addition to changes in void content produced by changes in heat transfer, the diagram also has a contribution to void change due to pressure variations.

One of the most important effects of a void change is the resulting change in the inlet flow velocity of water into the core. Iriarte[17] has presented a good analysis of this effect but does not consider exchange of momentum between core and reflector. In our model, his equations have been modified to include this.

When water flashes into steam in the core, water is displaced from the core into the reflector. The movement of water increases momentum in the reflector while a corresponding decrease of momentum occurs in the core. Upon condensation of steam in the core, a reversal of the above effect takes place. This transfer of momentum between core and reflector causes variations in feedwater flow which are a major source of instability in boiling water reactors. The equation for the rate of change of momentum during a void change is based on a model proposed by J. A. Fleck, Jr.[19]. It appears in our model in the mathematical description of that portion of the block diagram labelled "core inlet velocity" and shows that the transient pressure during a void change is directly proportional to the height of water above the core. Future use of the digital code should indicate the effect of variations of the water height on reactor stability.

Variations in pressure and core inlet temperature account for the two other main loops in the block diagram. These loops are interrelated with each other and with the subcooling loop. However, only the pressure and heat flow portions of the diagram produce a direct effect on the void content.

When the changes in reactivity due to void changes and fuel temperature Doppler effect are added, the resulting change in reactivity is the total feedback reactivity $\Delta k_{F.B.}$. This is added to the original sinusoidal reactivity variation Δk_{in} to get Δk, the resultant input reactivity after transients have died away. The output of TPF-1 contains the magnitude and phase angle (relative to the sinusoidal reactivity input) of the ratio of $\Delta P/P_o$ to Δk_{in} in the steady state.

2. Numerical Approximations to Transfer Functions

Several methods of approximating transfer functions have been proposed in the literature. They are not all equally valid over a given frequency range, and in writing a transfer function code, one should choose that mode of representation which gives accurate results for a minimum of calculation over a suitably large choice of frequencies.

In particular, this problem was investigated with reference to the heat transport to power transfer function for a boiling reactor. The simplest approximation is that which assumes that a single time constant can be used to approximate the Laplace transform solution to the equation of heat conduction for the core and cladding. This procedure is equivalent to expanding the exact transfer function in a Taylor series in the transform variable and

neglecting all terms of degree two or higher. For this reason it is referred to as the "Taylor series" method in the accompanying numerical examples.

A second approach is to express the transfer function as a series in terms of its residues at its poles. In general, the heat transport to power transfer function will have a countable number of isolated singularities which can be ordered in terms of increasing distances from the origin. If the singularities are ordered as above and designated by s_1, s_2, ..., then the transfer function can be represented by the series

$$\Sigma\, (s-s_j)^{-1}\, \rho_j$$

where ρ_j $(j=1, 2, \ldots)$ are the residues of the transfer function at the points s_1, s_2, ... The series can be truncated after a suitable number of terms to obtain an approximation to the transfer function. This procedure has been suggested by Iriarte[17] and the relevant mathematics can be found in Churchill[21].

A third way is to take the Laplace transform of the equation of heat conduction and to solve the resulting differential equation by a finite difference scheme. Because this procedure places mesh points or nodes in the region of heat conduction, the method is often called a "nodal" method.

In order to compare the relative merits of these different approximations, the analytic Laplace transform solution to the equation of heat conduction was found for an infinite slab of uranium oxide one-half inch in thickness with a fifteen mil stainless steel cladding. The heat generation in the oxide was assumed to be uniform, and the rate of heat transfer across the face of the clad was taken proportional to the temperature difference between the outer clad face and the surrounding water. Each of the above approximations was then applied to the same problem. The results are presented in Figures 12 and 13 which give the phase angle and logarithm of the magnitude of the complex transfer function as functions of the angular frequency, ω, as the transform variable, s, ranges over the values $i\omega$.

Values obtained from the analytic solution are labelled "exact solution" in the figures. Two sets of calculations were performed with finite difference approximations. One has mesh points at the inner and outer boundaries of the clad. The other retained those mesh points and had an additional point in the interior of the oxide region. Series of residues were truncated at three and four terms each.

Both the residue series and finite difference methods can be applied for arbitrary values of ω. Because the radius of convergence of a Taylor series about $s=0$ of a function analytic at the origin in equal to the distance

from the origin of the first singularity of the function, such an approximation to the transfer function to any number of terms is limited in application to values of ω which are less in absolute value than that of the pole of smallest magnitude.

Since the first pole of the transfer function in the example is approximately $s = -.04$, the Taylor series about $s=0$ will not represent the transfer function for values of s with absolute values much larger than .04. Comparison of the results for the finite difference and residue series methods indicates that, for comparable amounts of labor spent in computation, the residue series method yields more accurate results. For small values of ω (in this case $\omega < .04$), the Taylor series approximation is by far the simplest and yields satisfactorily accurate results.

B. Non-Linear Stability

Since the stability criteria for a linear system, although well understood, do not necessarily imply that the original non-linear kinetic system is also stable, the question of non-linear stability was studied. In a linear system the ratio of response and input amplitudes is independent of the input amplitudes whereas for the non-linear system this ratio is amplitude dependent and spectrally more involved.

Several authors have considered the stability of the space-independent non-linear kinetics equations. Stuart and Harker[22] maintain that the equations for most reactors likely to be encountered can be transformed so as to put them into a form satisfying the hypotheses of their Theorem I. This theorem is essentially that which is the basis of Liapounov's First Method.

This method is a procedure for constructing a stable series solution to the kinetics equations, the existence of which is asserted by the theorem. However, a simple example of a stable reactor for which Theorem I does not apply is provided by a homogeneous reactor with constant power extraction.

Ergen, Lipkin and Nohel[23] have applied the Second Method of Liapounov to the cases of homogeneous reactors with conditions of constant cooling and Newton's law of cooling. The Second Method depends upon construction of certain positive definite functions of reactor variables and establishes stability or asymptotic stability of the solutions. It is in general too difficult to apply to be of practical use in the design of stable reactors.

A commonly used approach to the kinetics equations is the method of perturbation. It depends upon the assumption that the amplitude of the excess reactivity is small enough that product terms of excess reactivity and non-constant neutron density can be neglected. Solutions to the equations

are determined by power series in the magnitude of excess reactivity. Akcasu[24] has done this without including feedback effects of reactivity which leads him to the solution of linear equations with non-constant coefficients. The method is limited by the requirement of small reactivity variations.

Sandemier[25] has made an approach to the problem of stability under large variations in the magnitude of the excess reactivity. He applies a bias to the input reactivity of sufficient magnitude to insure that the average variation in neutron density remains constant. Under this condition, the shape of the variation of the neutron density remains constant and is well approximated by the fundamental component of its Fourier series representation. Thus, the shape of the variation in reactivity may be surmised, but the actual time variation with a reactivity variation of arbitrary magnitude (without bias) cannot be obtained. Sandemier's calculations for an analytical model of EBR-I indicate that a larger amplification between the reactivity insertion and the neutron flux exists if the amplitude of sinusoidal reactivity insertion is increased, a fact which has been observed experimentally by DeShong[15] for EBWR.

IV. TRANSIENT BEHAVIOR

In addition to knowledge of the stability of a reactor to small perturbations of the power level, the designer would often like to know the actual time behavior of the transient portion of the power. For although the reactor may be theoretically stable, the transient terms in the power following power demand changes or accidents may have a peak magnitude that is sufficiently high as to cause physical damage to the system.

Many people have found solutions to the reactor kinetics equations both analytically and numerically and for a variety of expressions of the reactivity as a function of time. For current design work, the existing analytical solutions are inadequate due either to over-simplification of the equations or to excessive awkwardness of the solutions for numerical evaluations. Present machine codes for solving the kinetics equation require an excessive amount of time to arrive at a satisfactory solution.

A. τ - Method for Kinetics Problems

In an attempt to provide a more rapid and accurate means for numerically solving the kinetics equations, tests are being made of the Lanczos τ-method[26]. This method makes use of the property that of all polynomials of degree n and leading coefficient 1 on the interval 0 to 1, the shifted Chebyshev polynomials have the minimum-maximum deviation from zero. For example, if one wishes to solve an equation of the form

$$Dy - a = 0 \quad , \tag{23}$$

where D is a linear differential operator and a is constant, he may try a polynomial expression for y and not try to satisfy the Equation (23) exactly but rather to have y satisfy the equation

$$Dy - a = \sum_{i=1}^{m} \tau_i T_{n+i-1}(x) \tag{24}$$

where the $T_k(x)$ are the shifted Chebyshev polynomials and the τ_i are constants which will be determined by the boundary values for the problem. In practice, more than two τ terms are almost never needed for a single equation. The solution y of (24) then leaves a remainder when substituted in the left side of (23) which oscillates about zero on the interval from zero to 1.

The adoption of the τ-method to sets of differential equations is fairly straight-forward. It is being investigated now for the kinetics equations to determine the ease of application and the accuracy of solutions.

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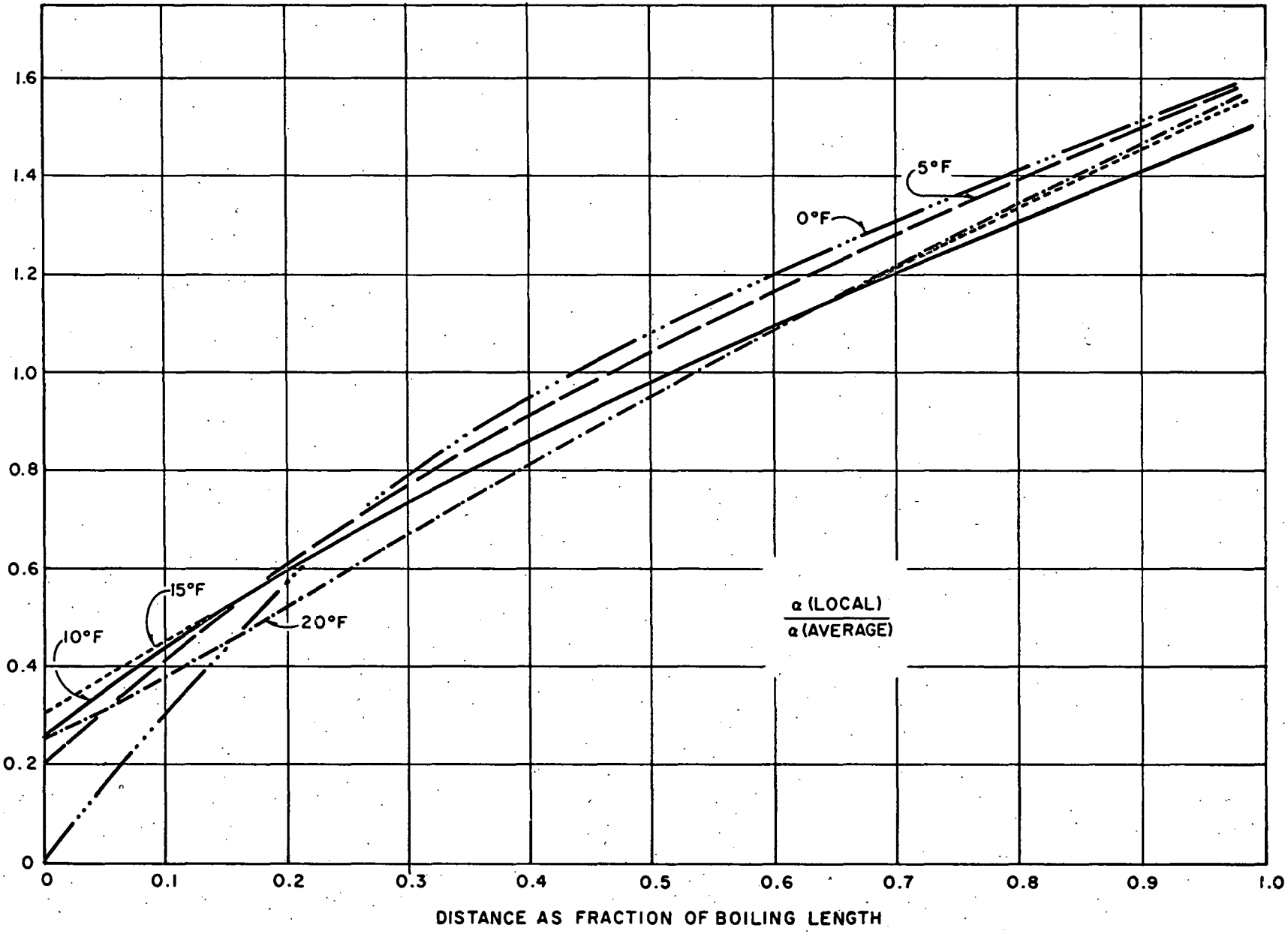
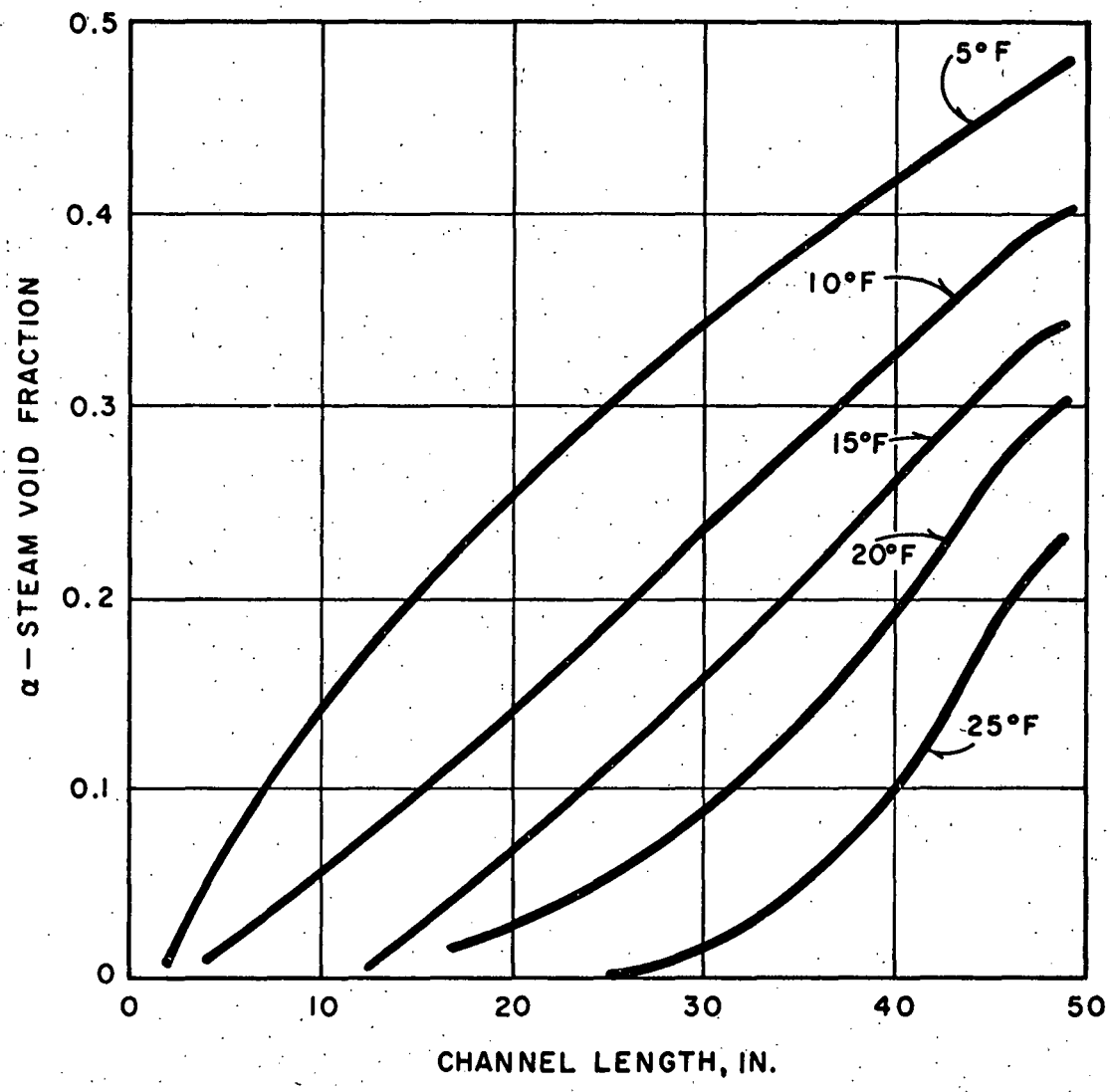


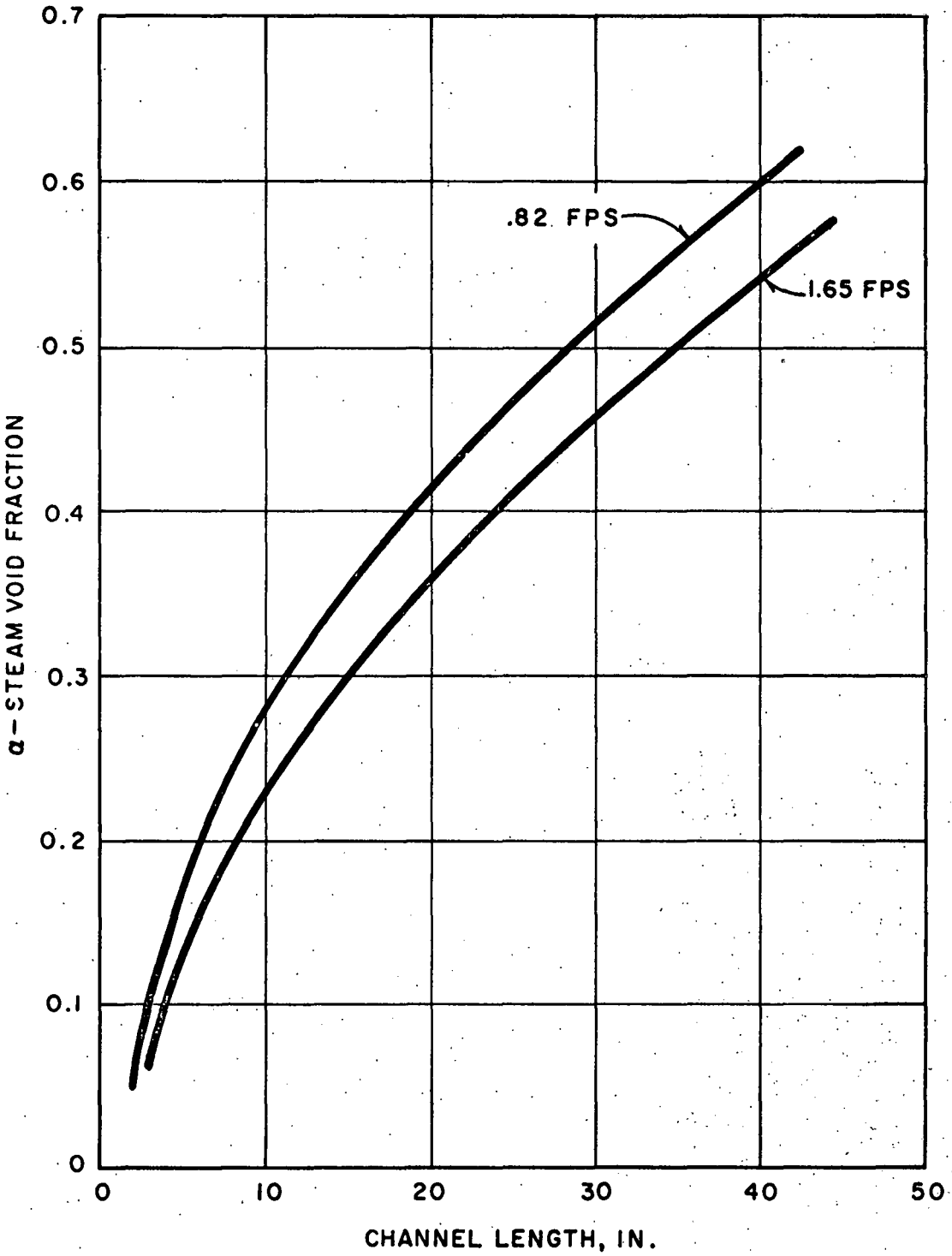
NORMALIZED VOID DISTRIBUTION PLOTS FOR VARIOUS INLET SUBCOOLINGS

Fig. 1



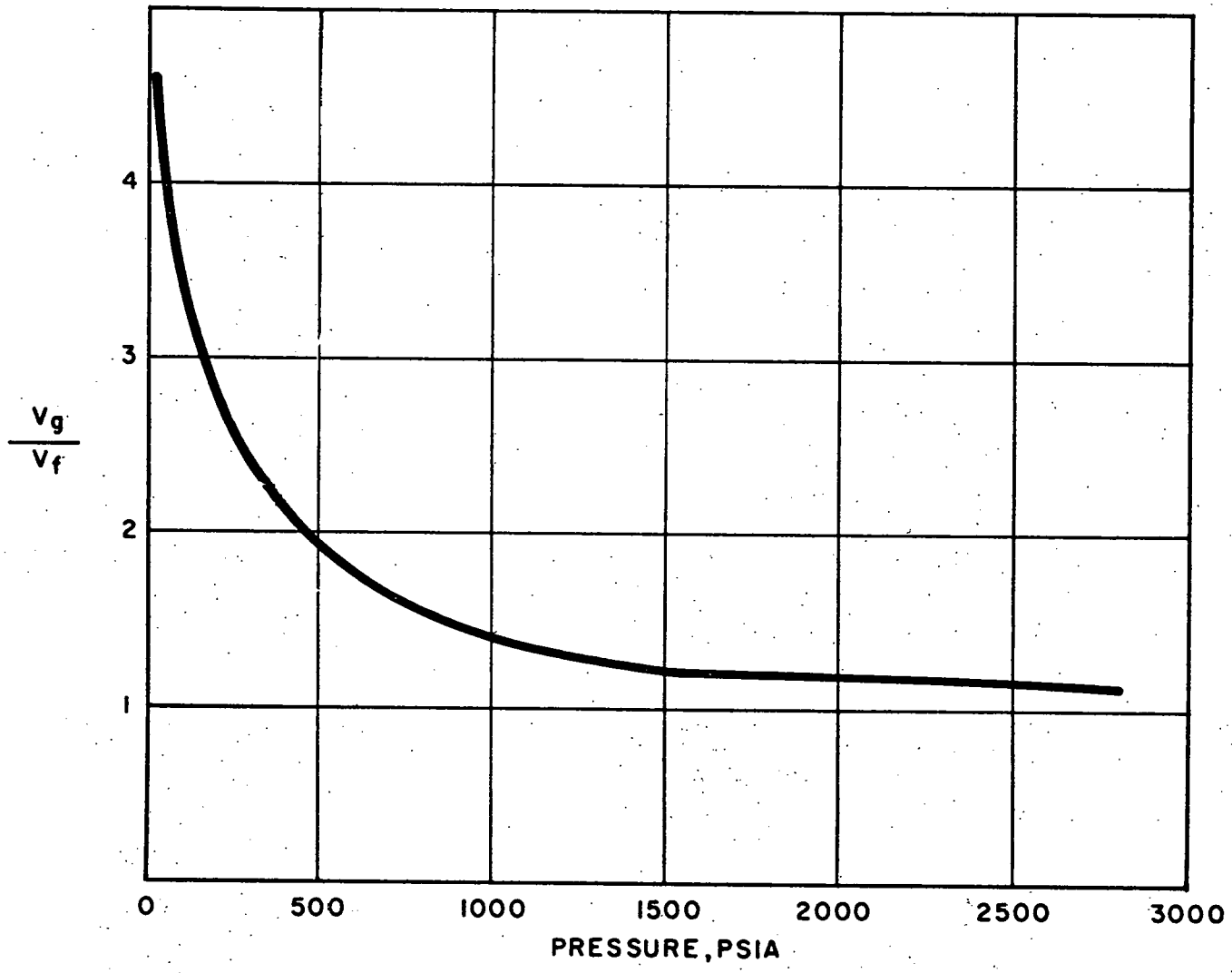
STEAM VOID FRACTION α VS DISTANCE FROM START OF HEATED LENGTH FOR VARIOUS INLET SUBCOOLING AMOUNTS

Fig. 2

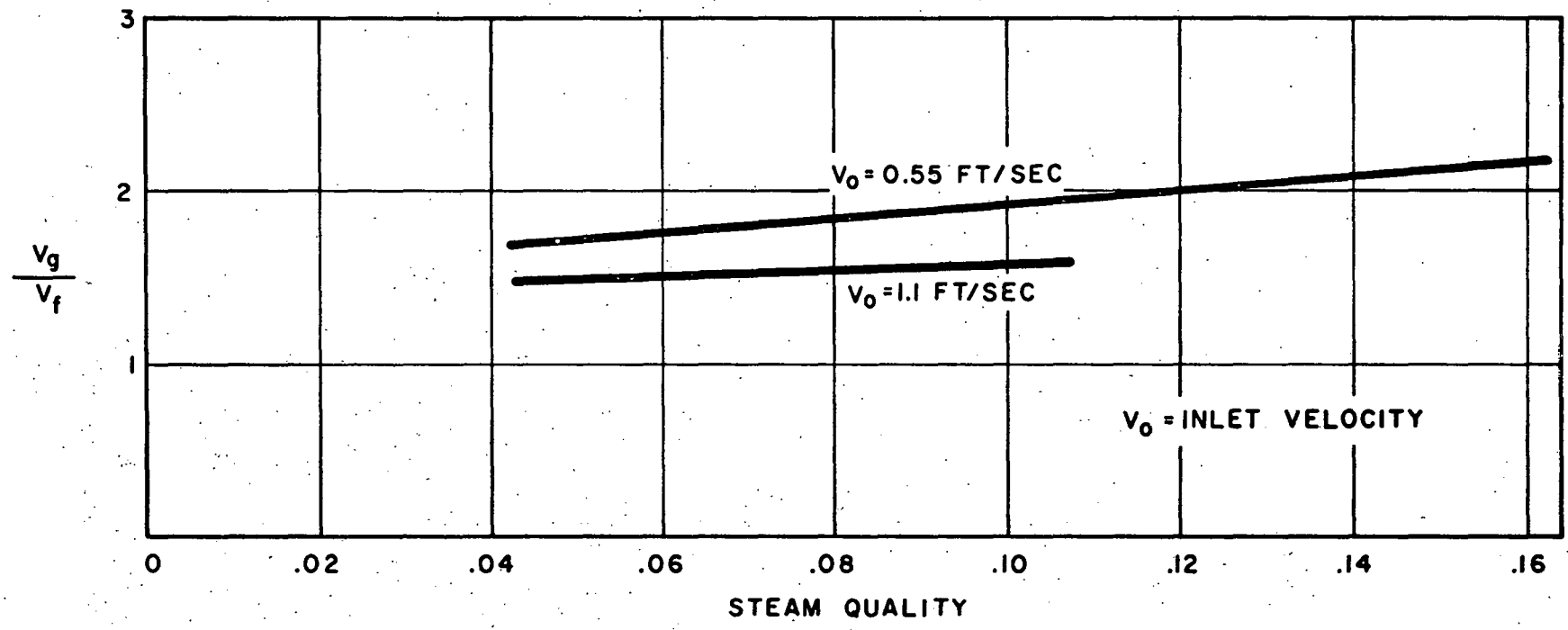


STEAM VOID FRACTION VS LENGTH FOR VARIOUS INLET VELOCITIES

Fig. 3

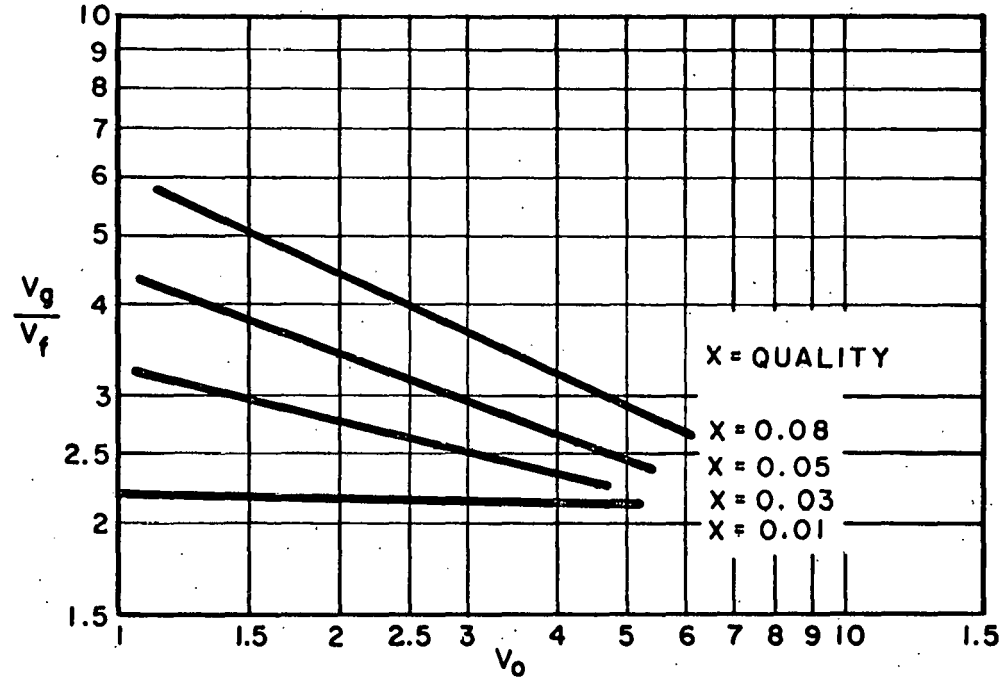


VELOCITY RATIO AS A FUNCTION OF PRESSURE $V_o \approx 1$ TO 3 FT/SEC
$X \approx .05$
Fig. 4

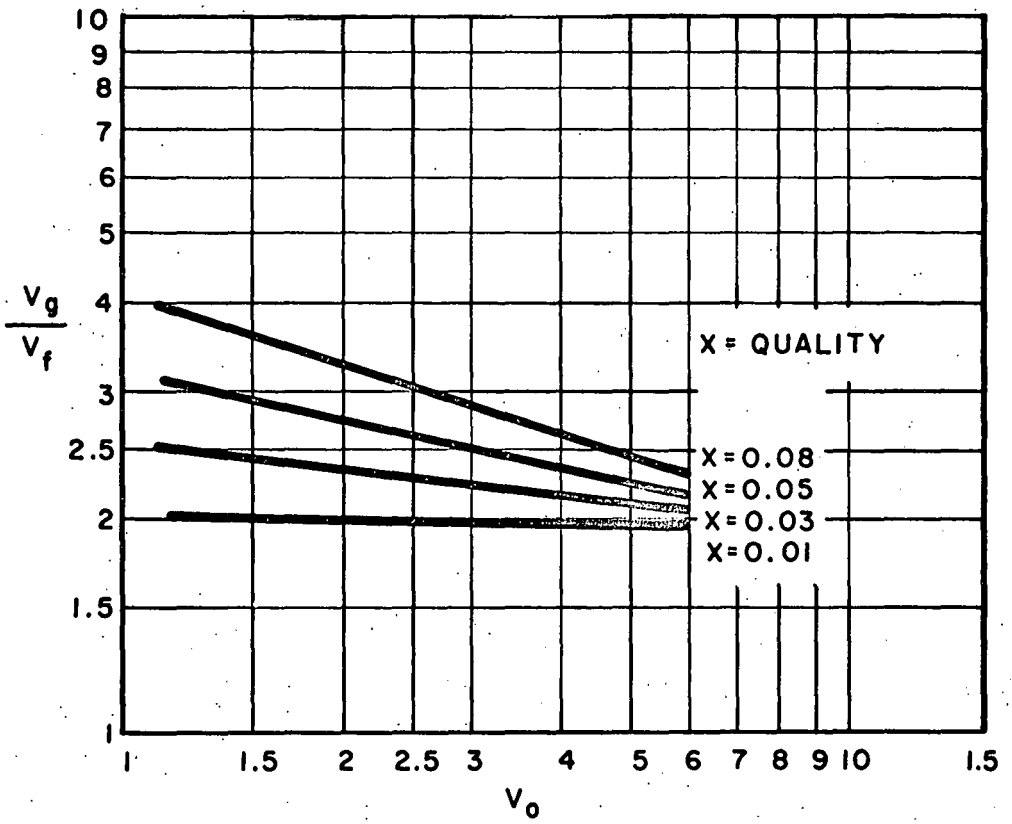


THE EFFECT OF INLET VELOCITY ON VELOCITY RATIO AT 1200 PSIA

Fig. 5



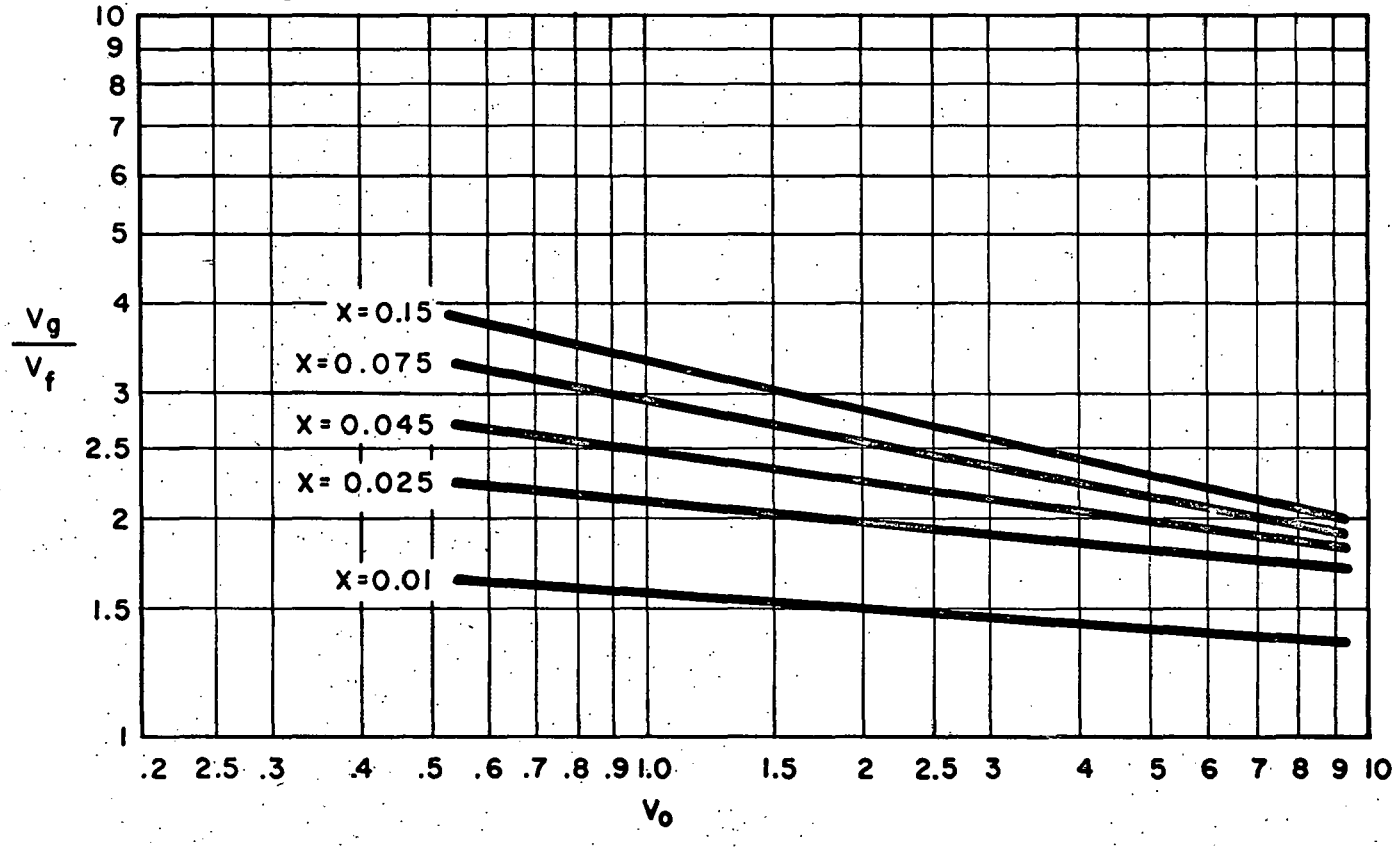
VELOCITY RATIOS VS INLET VELOCITY AT 150 PSIG



VELOCITY RATIOS VS INLET VELOCITY AT 250 PSIG

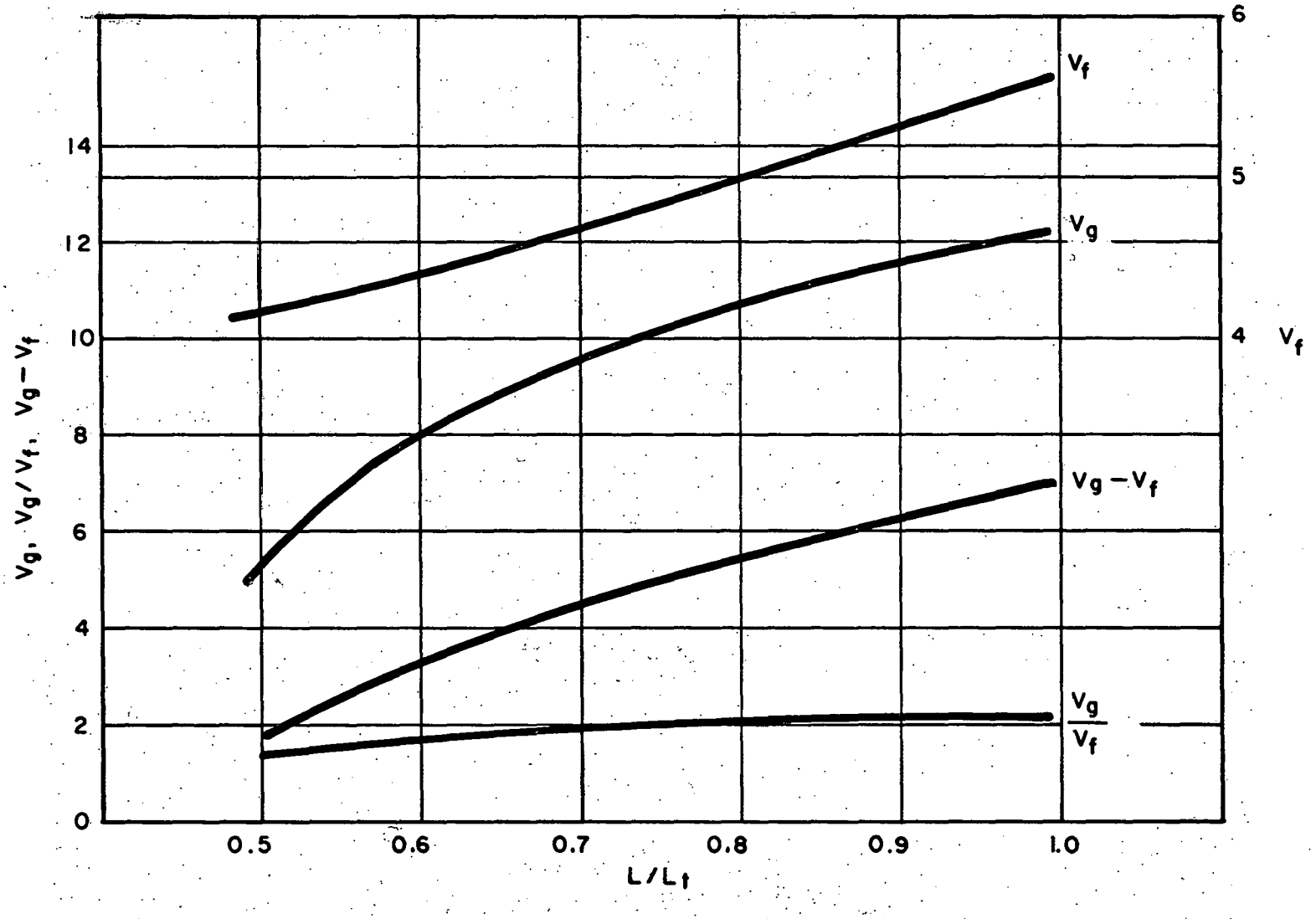
THE EFFECT OF INLET VELOCITY ON VELOCITY RATIO AT 150 & 250 PSIG

Fig. 6



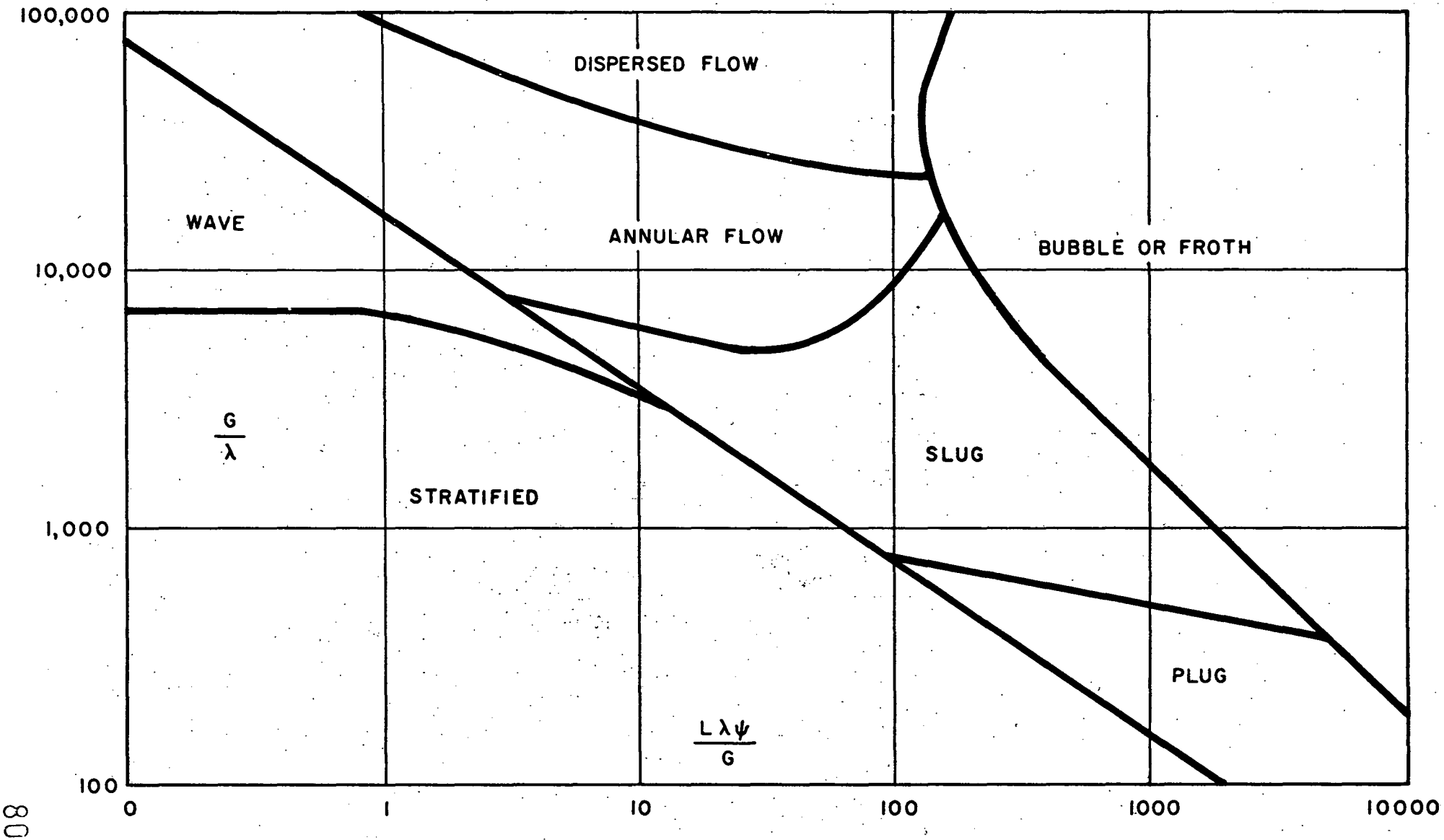
THE EFFECT OF INLET VELOCITY ON VELOCITY RATIO AT 600 PSIG

Fig. 7



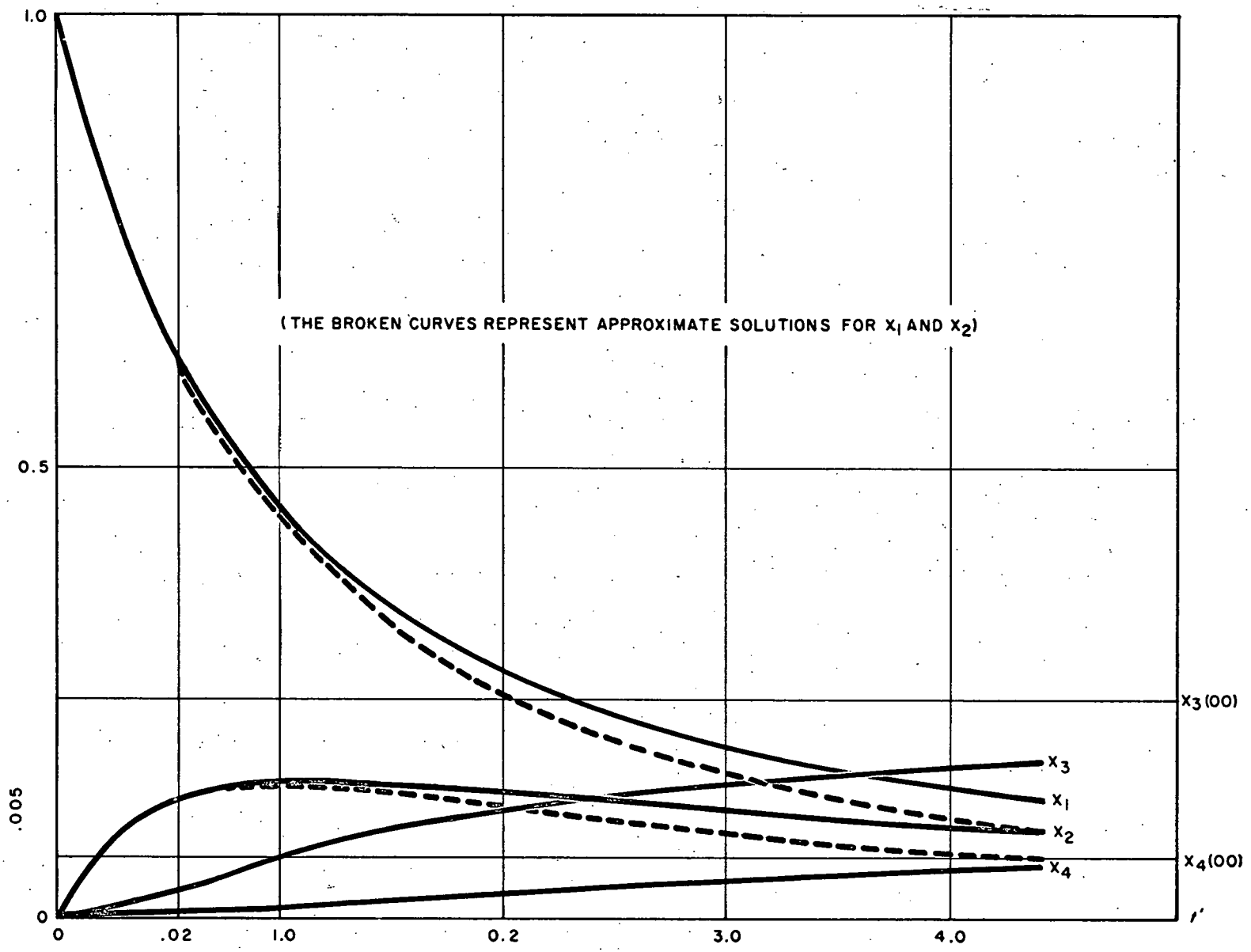
VARIATION OF LOCAL PHASE VELOCITIES, VELOCITY RATIO, AND VELOCITY DIFFERENCE FOR A TYPICAL BOILING RUN

Fig. 8



BAKER'S CORRELATION FOR FLOW-PATTERN REGIONS

Fig. 9



NORMALIZED BUBBLE DISTRIBUTIONS AS FUNCTIONS OF DILATED TIME

Fig. 10

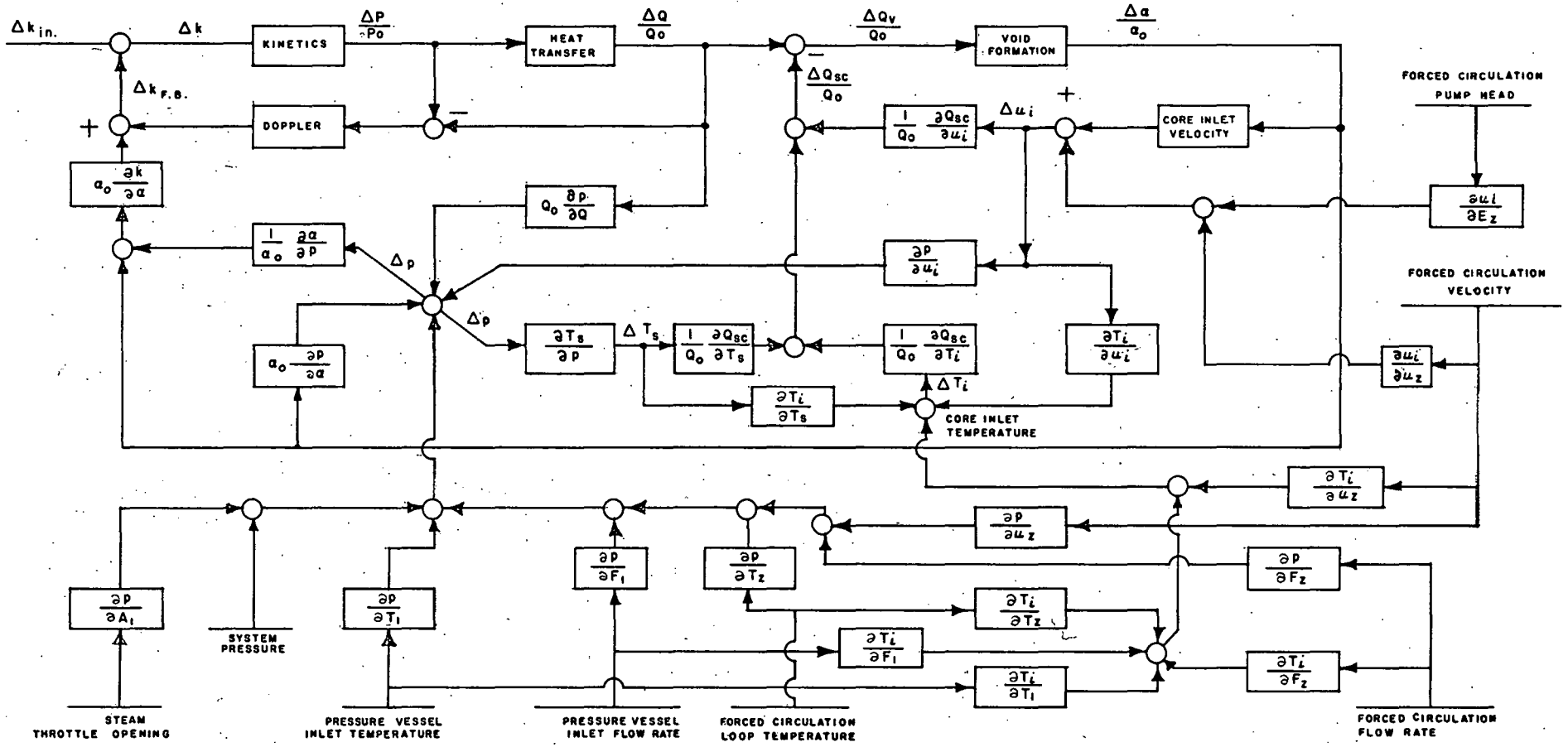


Fig. 11

BLOCK DIAGRAM FOR BOILING WATER REACTOR

Fig. 12 HEAT TRANSPORT TO POWER TRANSFER FUNCTION

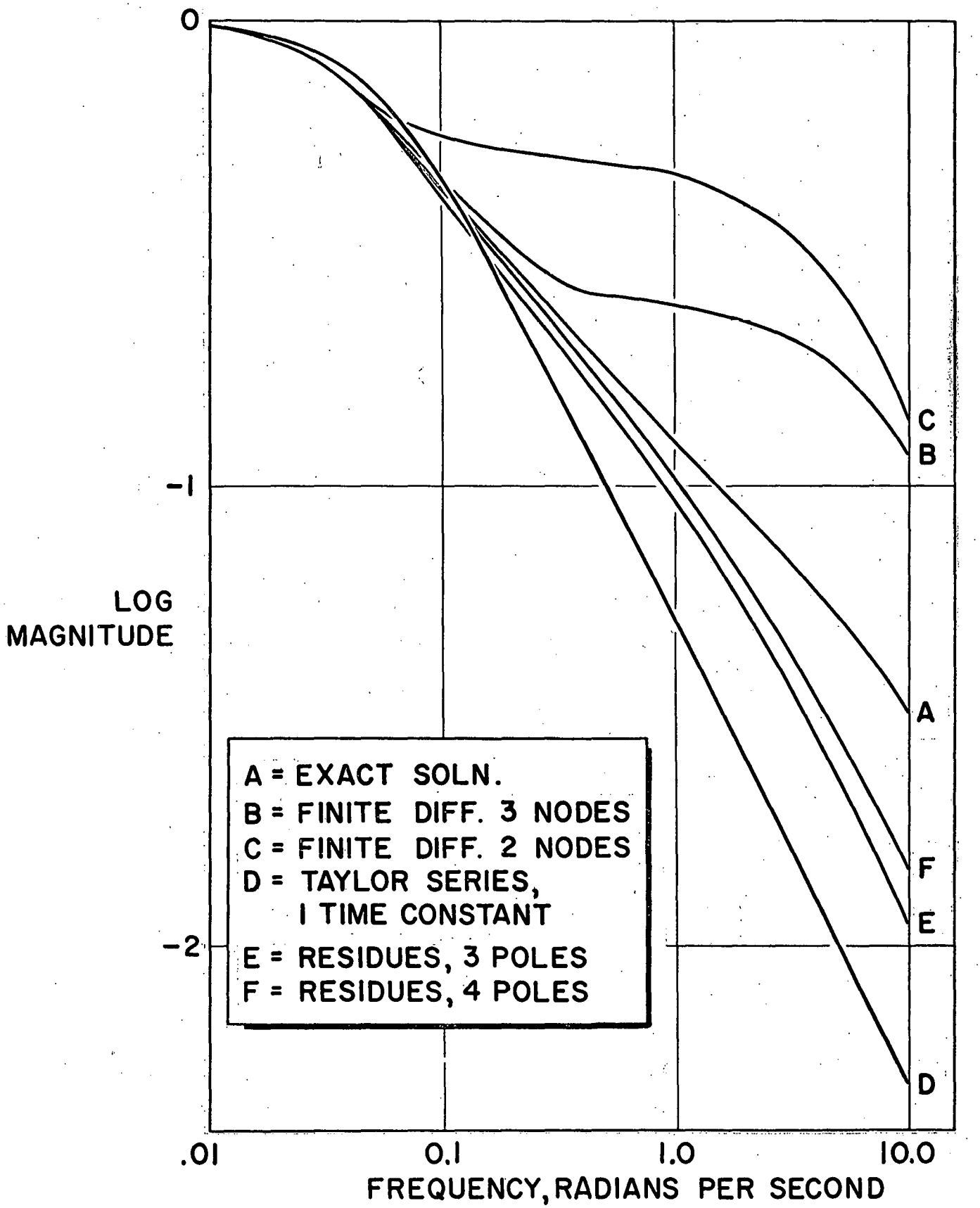


Fig.13 *HEAT TRANSPORT TO POWER TRANSFER FUNCTION*