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by

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GEOHERMAL RESERVOIR MODELS--CRACK PLANE MODEL

by

Robert D. McFarland

ABSTRACT

A brief description is given of a particular fluid-flow/heat-transfer model of the region near a vertically oriented crack produced by hydraulic fracture to obtain geothermal energy from dry hot rock. The model discussed in this report is in the plane of the crack. Some results of numerical calculations are presented.

INTRODUCTION

The concept of extracting heat from "dry" hot rock in the earth has been discussed in various references.¹⁻³ This report describes one numerical analysis model developed to obtain information about the outlet temperature and thermal power to be expected from artificial geothermal reservoirs, about working fluid pressure drops and about fluid flow patterns in the reservoir or crack. Models developed to date have all been essentially two dimensional in that heat transfer and fluid flow in the third dimension is assumed to not affect the "in-plane" calculations except in terms of sources or sinks of mass, heat, momentum or whatever.

The particular concept which has been dwelt on is that of a thin crack in dense rock, such as granite. The plane of this crack is perpendicular to the surface of the earth and is therefore said to be vertically oriented. The shape of the crack when viewed in the crack plane is assumed to be regular--circular, for instance (a "penny-shaped" crack). The "crack plane" is assumed to be truly planar and not three dimensional. Heat diffuses from the rock adjacent to the crack, essentially in a direction perpendicular to the crack plane to heat the working fluid circulating through the crack. Other diffusion processes (fluid mass, chemical species) take place in a similar fashion.

The problem, even simplified and idealized as it is here, is obviously a three-dimensional one,

but much useful information can be obtained from two-dimensional models at a drastically reduced level of expenditure in computer time. In fact, knowledge of the actual geometry of a geothermal reservoir system is needed before a more detailed model can be utilized.

At this time, two models have been developed using the same basic solution techniques. The first, described herein, is one in which attention is focused on gradients in the crack plane. A second model of processes in a plane perpendicular to the crack plane will be described in a later report.

BASIC COMPUTER PROGRAM EQUATIONS

The starting point for setting up the reservoir fluid flow models is a computer program called TEACH, developed at Imperial College.^{4,5} This is a generalized program designed to solve the full elliptic conservation equations in two dimensions for arbitrary boundary conditions. Although the details of this program are not within the province of this report, some description of the technique would be useful here.

Equations solved by the Imperial College method are of the form (axisymmetric coordinates)

$$\frac{1}{r} \left[\frac{\partial}{\partial z} (\rho u r \phi) + \frac{\partial}{\partial r} (\rho v r \phi) - \frac{\partial}{\partial z} \left(r \Gamma \frac{\partial \phi}{\partial z} \right) - \frac{\partial}{\partial r} \left(r \Gamma \frac{\partial \phi}{\partial r} \right) \right] = S \phi; \quad (1)$$

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where ϕ is any dependent scalar or vector parameter such as temperature (or enthalpy), pressure, a particular component of convective velocity, or chemical species concentration. Here u and v are the z and r components of convection velocity, ρ is the density and Γ is a general diffusion coefficient for whatever quantity ϕ represents.

S^ϕ is a general source term which is intended to be specified in the particular model along with the boundary conditions. In the solution of the finite difference equations the source term is assumed to be a linear function of the dependent variable, ϕ . That is,

$$S^\phi = S_1^\phi + S_2^\phi \cdot \phi \quad (2)$$

In this way time-dependent behavior can be included as well as laminar flow darcy drag factors and body force terms. A particular set of equations which may be solved are (here, in Cartesian coordinates):

(a) x-component of velocity (u)--i.e., the x-momentum equation,

$$\frac{\partial}{\partial x}(\rho u u) + \frac{\partial}{\partial y}(\rho v u) - \frac{\partial}{\partial x} \left(\mu \frac{\partial u}{\partial x} \right) - \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) = S^u, \quad (3a)$$

where μ is the absolute viscosity.

(b) y-component of velocity (v)

$$\frac{\partial}{\partial x}(\rho u v) + \frac{\partial}{\partial y}(\rho v v) - \frac{\partial}{\partial x} \left(\mu \frac{\partial v}{\partial x} \right) - \frac{\partial}{\partial y} \left(\mu \frac{\partial v}{\partial y} \right) = S^v. \quad (3b)$$

(c) Temperature (T), for constant specific heat

$$\frac{\partial}{\partial x}(\rho u T) + \frac{\partial}{\partial y}(\rho v T) - \frac{\partial}{\partial x} \left(\lambda \frac{\partial T}{\partial x} \right) - \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) = S^T; \quad (3c)$$

where λ is the standard thermal conductivity, λ , divided by the specific heat, C_p .

(d) Pressure (p). By assuming the Darcy relation for hydraulic diffusion;

$$u = -\frac{k}{\mu} \frac{\partial p}{\partial x}; \quad v = -\frac{k}{\mu} \frac{\partial p}{\partial y},$$

and inserting these in the equation of continuity:

$$-\frac{\partial}{\partial x} \left(\Gamma_x \frac{\partial p}{\partial x} \right) - \frac{\partial}{\partial y} \left(\Gamma_y \frac{\partial p}{\partial y} \right) = S^p, \quad (3d)$$

which is of the same form as the other equations except that the convective terms do not appear. In general, the coefficients, $\Gamma = \rho k' / \mu$, are obtained from the two momentum equations, (3a) and (3b).

The equations are set up in a two-dimensional finite-difference form, and are solved simultaneously--simultaneously in the physical sense, not in the mathematical sense. The method of solution is a semi-implicit sweeping of the grid. The sweeping process is carried out until residual source terms approach specified values. These residual source terms are accumulative source values over and above those specified, and represent a calculational deviation.

The iterative process may be controlled by altering the order in which the various equations are swept, the number of sweeps for each equation in a given iteration, and the under-relaxation factor for each dependent variable for two successive iterations.

MODEL IN PLANE OF ORIGINAL CRACK--DESCRIPTION

The first model to be made is one in the plane of the original hydrofracture, which is assumed to be vertically oriented (Fig. 1). In this model the fluid flow and thermal gradients of interest are those in the plane of the fracture, called the "crack plane." Contributions to the system behavior from gradients perpendicular to this plane--such as fluid drag at the crack surfaces and heat transfer from the adjacent rock--are treated as source terms which are determined in separate calculations. Fluid flow is confined to the fracture reservoir volume, i.e., the adjacent rock is considered to be impermeable. Fluid flow from the system could be included, but this would have to be a simple loss or gain in fluid without any truly three-dimensional interactions.

The basic finite-difference grid is rectangular, so the actual geometry of the fracture reservoir is controlled by varying the crack thickness perpendicular to the crack plane. This thickness is used in determining the fluid and heat flow area of the finite-difference equations. By varying this thickness

Fig. 1. Circular crack.

to zero at the crack edges, the effective rim of the model crack is determined within the limitations of the grid mesh size.

SOURCE TERMS--X-MOMENTUM

Source terms for the momentum equation in the x-direction (parallel to the earth's surface) consist of a pressure gradient term and a drag term. In Eq. (3a):

$$S^u = -\frac{\partial p}{\partial x} - C_D u, \text{ or}$$

$$S_1^u = -\frac{\partial p}{\partial x}; \quad S_2^u = C_D.$$

For laminar flow between parallel flat plates, a reasonable approximation to the flow in the crack, the drag coefficient is given by $C_D = \frac{12\mu}{w^2}$, where μ is the absolute viscosity of the fluid and w is the crack thickness. If any permeable medium--such as

sand, for instance--fills the crack opening, an additional resistance (usually dominant) is presented so that the general drag coefficient becomes, for laminar flow:

$$C_D = \left[\frac{1}{k} + \frac{12}{w^2} \right] \mu, *$$

where k is the permeability of the solid medium in the crack.

It can be shown that the terms on the left-hand side of Eqs. (3a) and (3b) are usually negligible for flow in these cracks of large diameter and small thickness. Then, Eq. (3a) reduces to

$$\frac{\partial p}{\partial x} + C_D u = 0;$$

from which Γ_x in Eq. (3d) is given by

$$\Gamma_x = \rho / C_D.$$

Y-MOMENTUM

In the y-direction (vertical), a body force term must be included in the source term expression for Eq. (3b),

$$S^v = -\frac{\partial p}{\partial y} - C_D v - b;$$

where $b = g\rho$, g is the acceleration due to gravity. We are only interested in the relative value of the body force, which will give us buoyancy forces. Then,

$$b = g(\rho - \rho_0) = -g\rho\beta(T - T_0); \text{ for small density changes}$$

$\beta = -\frac{1}{\rho} \frac{\partial \rho}{\partial T}$, is the average volumetric expansion of the fluid over the range T_0 to T , and

*The actual expression for laminar, incompressible flow between parallel flat plates with distributed flow resistance is

$$C_D = \mu/k [1 - \tanh(a)/a], \text{ where}$$

$$a = w/2 \sqrt{k}.$$

The above superposition equation, however, gives very close results.

T_0 is the reference temperature. For these calculations, T_0 is given the value of the average temperature at the local depth, to improve the stability of the calculations. The resulting values of pressure, therefore, show essentially no net effect of buoyancy forces. The real pressure change must be obtained by adding a component due to the average temperature as a function of depth.

ENERGY (TEMPERATURE)

For the temperature equation, the source term represents the flux of heat being conducted into the working fluid from the adjacent rock, and a transient term where the time derivative is taken to be approximated by $\frac{\partial T}{\partial t} = \frac{T - T^0}{\Delta t}$, where T^0 is the local temperature at the previous time step and Δt is the time increment. The source term is

$$S^T = \frac{1}{C_p} \left[2q_0/w - C_p \rho \left(\frac{T - T^0}{\Delta t} \right) \right]$$

where q_0 is the heat flux through the crack surface into the fluid assumed to be of the form $q_0 = h(T_R - T)$. Here, h is an overall heat transfer coefficient and T_R is the original local rock temperature, i.e., the temperature at a large horizontal distance from the crack.

There are several ways of determining h . A typical thermal diffusivity of granite is $10^{-6} \text{ m}^2/\text{s}$. This means that the thermal penetration depth is something like 100 m in 30 yrs. For large cracks, then, one-dimensional conduction perpendicular to the crack should be accurate. An analytic closed-form solution given in many references for conduction into (or out of) a semi-infinite solid from a constant temperature surface results in

$$h = \sqrt{\frac{\lambda_R C_R \rho_R}{t}};$$

where λ_R , C_R , ρ_R are the thermal conductivity, heat capacity, and density of the rock and t is time from start when the temperature is T_R everywhere in the solid. This can be applied to the present crack problem if the convective coefficient between rock surface and fluid is large, which is a good assumption, and if the surface temperature can be assumed to be constant. In Appendix A it is shown that,

although the surface temperature is not constant, the errors introduced by this assumption are within reason.

The best method of determining h would be a separate numerical integration of the one-dimensional heat conduction equation in the direction perpendicular to the crack plane. This would require a solution at each in-plane node, at each time. The increase in required computation time and core storage does not seem to be warranted by the increase in accuracy at this point in model development.

The method used in the calculations presented herein is another approximation based on the integral of heat extracted from the rock locally, and an approximate rock temperature profile. When the surface temperature is constant this method yields the value of h obtained above, as shown in Appendix A.

PRESSURE

In the pressure equation, (3d), the source term represents mass influx. The only mass sources used presently are at the inlet and outlet. The cold fluid from the earth's surface is considered to enter the crack as a "point" source of mass, and the hot fluid exiting the crack for the surface is a "point" sink. Actually, of course, these "points" are one grid element area each. The source terms are, then

$$S^P = \dot{m}/V_1 \text{ at the inlet element,}$$

$$S^P = -\dot{m}/V_0 \text{ at the outlet element,}$$

$S^P = 0$ elsewhere. V_1 and V_0 are the crack volume at the inlet and outlet elements and $\dot{m}$ is the total fluid mass flow rate. A hydraulic diffusion or conduction equation, similar to the above heat conduction equation, could be used here to determine local mass flow losses perpendicular to the crack. This has not been done in any of the present calculations.

SOME OMISSIONS

As pointed out above, the fluid inlet and outlet are large areas in the model, since a typical finite-difference grid element is a 50-m square. Because of this the fluid pressure losses at the inlet and outlet, which may be the biggest losses in an unpropped crack, are not adequately represented in the

present model. Another area of possible inaccuracy in the pressure drop calculation is that of fluid density changes due to diffusion of salts, etc., into the working fluid. This could conceivably offset any buoyancy effects due to the fluid temperature rise.

Except at very early times the transient terms for the momentum and mass equations are completely negligible and are not included. The transient term for energy is included in the analysis even though it, too, is very small.

CRACK THICKNESS VARIATION

In some calculations the crack thickness is varied with time according to the local thermal history. The basis for this is that the rock adjacent to the crack should contract as it is cooled, thereby opening the original crack. This is not an attempt to simulate possible thermal stress cracking. In fact, there is no accounting of increased surface area due to thermal contraction (or any other phenomenon) in this "crack-plane" model.

CALCULATION RESULTS--PROPERTIES

In the following calculation results for the "crack-plane" model, all rock and water properties are assumed to be constant except for the viscosity of water which is taken to be

$$\mu = 1/[1000 + 32(T-20)],$$

where T is in °C and μ is in Pa-s.*

Other properties used in the calculations are:

water density = 1000 kg/m³;

water conductivity = 0.63 W/m-K;

water heat capacity = 4200 J/kg-K;

volumetric thermal expansion coefficient of water:

$$\beta = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right) = 2 \times 10^{-4} \text{ K}^{-1};$$

rock density = 2700 kg/m³;

rock specific heat capacity = 1000 J/kg-K;

rock conductivity = 2.0 W/m-K.

When used, the linear thermal expansion coefficient for rock is assumed to be $8 \times 10^{-6} \text{ K}^{-1}$. The rock

properties used are only intended to be representative of the general class of dense granitic rock to be expected in the region of the geothermal reservoir.

SYMMETRICAL CRACK--THERMAL POWER

In Figs. 2-5, the history of outlet temperature and thermal power pickup is shown for a circular crack of 1 km diam (Fig. 1), operating at two different water flow rate levels. In these calculations no thermal contraction of the rock walls was allowed.

The crack is assumed to be symmetrical about a vertical centerline, that is, the water inlet and outlet both lie on the vertical centerline and the crack is circular as shown in Fig. 1. The crack thickness, perpendicular to the crack plane, is taken to have an elliptical shape with a maximum thickness of 3 mm at the center of the disk. The water inlet is centered 175 m above the bottom of the crack and the water outlet is 175 m below the top of the crack.

The initial rock temperature (the same as the rock temperature at a large horizontal distance from the crack, at any time) is taken to be 250°C at the center of the crack. Superimposed on this are various geothermal gradients so that the initial temperature at the water outlet is lower than the average. This gradient is assumed constant and negative, the temperature decreasing linearly in the upward direction. The water inlet temperature for these calculations was arbitrarily chosen at 65°C.

In Figs. 2 and 3, the water outlet temperature and thermal power are shown for a steady water flow rate of 144 kg/s (about 2300 gpm at the inlet) for geothermal gradients of 0 and 0.10°C/m and for different impedance conditions. Also shown for reference are results of uniform upward flow calculations using Eq. (A-5) with the same crack surface area, flow, etc.

With no new crack surface or fluid penetration perpendicular to the crack plane, it is seen that the thermal power is reduced to half its initial value of over 100 MW in roughly 1 yr. The effect of the geothermal gradient is to reduce the outlet temperature initially due to the lower rock temperature near the top of the crack, but this effect disappears as the rock is chilled. The addition of a proppant, such as sand, in the crack is seen to make a significant reduction in the power due to short-circuiting. A proppant with a permeability of 1 darcy (about

*1.0 Pa-s = 10 poise = $1.0 \frac{\text{kg}}{\text{m-s}}$.

Fig. 2. Circular crack with radius = 500 m. Elliptical cross section with 3 mm thickness at center. No thermal contraction. Water flow = 144 kg/s.

Fig. 3. Circular crack with radius = 500 m. Elliptical cross section with 3 mm thickness at center. No thermal contraction. Water flow = 144 kg/s.

$1 \mu m^2$) has a laminar flow impedance 6 orders of magnitude higher than a 3-mm open crack, so that the viscous drag forces completely dominate the flow in the crack.

At a water flow rate of 20 kg/s (320 gpm at the inlet) the maximum power level is down to around 15 MW, but the outlet temperature and power level stay up for a much longer time, as shown in Figs. 4 and 5. The power level, in fact, is not reduced to the 50% level until more than 30 yrs of operation for this case.

Fig. 4. Circular crack at low flow, radius = 500 m. No thermal contraction.

Fig. 5. Circular crack at low flow, radius = 500 m. No thermal contraction.

A calculation in which thermal contraction of the crack walls was allowed is shown in Fig. 6. This is the same crack, initially, as the previous cases with no proppant, a $0.10^\circ C/m$ geothermal gradient and a water flow rate of 144 kg/s. Also shown for comparison are repeat plots of the same case with no thermal contraction, with and without a proppant. At early times the performance is the same as the no proppant/no thermal contraction case, as it should be. After about 2 weeks, the performance of the thermal contraction case starts falling toward that of the case with proppant as the crack opens up. This indicates that there may be an optimum in the ratio of drag forces to buoyancy forces since the propped crack case represents a condition of essentially no

Fig. 6. 500-m-radius crack. Water flow = 144 kg/s. Geothermal gradient = $-0.10^{\circ}\text{C}/\text{m}$.

natural convection while the thermal contraction case, at later times, represents extreme natural convection as will be shown below. In fact, convergence could not be achieved at later times than those shown with thermal contraction because of the extreme amount of natural convection. This problem has been encountered repeatedly in these computations. If the ratio of buoyancy forces to viscous drag forces gets too large, a numerical instability occurs. Whether this is indicative of a physical instability is an interesting question which is not answered yet.

Other calculations made with linear thermal contraction of the rock adjacent to the crack surfaces show that the power always drops off more rapidly with time than the corresponding case with no thermal contraction. This occurs even when buoyancy is eliminated, indicating that some form of preferential channeling may take place.

FLOW PATTERNS

Flow patterns are shown in the form of streamlines in Figs. 7-17. In these figures, which show the right half of the crack plane, each pair of streamlines (including crack boundaries) bound 10% of the total water flow. On each of the figures is a term, "relative power," which is merely the thermal power divided by the analytic value for uniform flow calculated by Eq. (A-5).

Figure 7 shows the 500-m-radius crack with a flow of 144 kg/s, with no buoyancy. This is very similar to a case with an extremely high impedance

Fig. 7. Flow contours.

No buoyancy
Time = 4.17 Ms

Fig. 8. Crack thickness contours.

No buoyancy
With thermal contraction
Time = 30.0 Ms (347 days)
 $T_{out} = 101^{\circ}\text{C}$
Power = 22.0 MW
Relative power = 0.35
Flow = 144 kg/s

Fig. 9. Flow contours.

No buoyancy
Time = 30 Ms

Fig. 10. Crack thickness contours.

No proppant
With thermal contraction
Buoyancy
Time = 0.161 Ms (1.86 days)
 $T_{out} = 228^{\circ}\text{C}$
Power = 98.5 MW
Relative power = 0.88
Flow = 144 kg/s

Fig. 11. Flow contours.

No proppant
 With thermal contraction
 Buoyancy
 Time = 1.20 Ms (13.8 days)
 $T_{out} = 225^{\circ}\text{C}$
 Power = 96.8 MW
 Relative power = 0.87
 Flow = 144 kg/s

Fig. 12. Flow contours.

No proppant
 With thermal contraction
 Buoyancy
 Time = 3.34 Ms (38.7 days)
 $T_{out} = 204^{\circ}\text{C}$
 Power = 84.3 MW
 Relative power = 0.77
 Flow = 144 kg/s

Fig. 13. Flow contours.

No proppant
With thermal contraction
Buoyancy
Time = 7.79 Ms (90.2 days)
 $T_{out} = 176^{\circ}\text{C}$
Power = 66.8 MW
Relative power = 0.68
Flow = 144 kg/s

Fig. 14. Flow contours.

No proppant
With thermal contraction
Buoyancy
Time = 9.51 Ms (110 days)

Fig. 15. Crack thickness contours.

No proppant
 With thermal contraction
 Buoyancy
 Time = 0.161 Ms (1.86 days)
 $T_{out} = 222^{\circ}\text{C}$
 Power = 94.9 MW
 Relative power = 0.85

Fig. 16. Flow contours.

No proppant
 With thermal contraction
 Buoyancy
 Time = 9.51 Ms (110 days)
 $T_{out} = 132^{\circ}\text{C}$
 Power = 40.3 MW
 Relative power = 0.43
 Flow = 144 kg/s

Fig. 17. Flow contours.

as would be the case for a sand-filled crack. There is seen a tendency to short-circuit from inlet to outlet. This tendency becomes somewhat more pronounced with time, enhanced by the thermal contraction of the rock in the cooler regions as indicated by the crack thickness contours. At the time shown, the maximum crack thickness has increased from 3 mm to 9 mm and moved down 125 m to the water inlet as shown in Fig. 8. This figure shows lines of constant crack opening thickness in equal increments of thickness from zero at the outer boundary (A) to the maximum (J). Again, the crack thickness is elliptical at the start, with a maximum thickness of 3 mm at the crack center. Figures 9 and 10 show the flow streamlines and crack thickness contours at the end of about 1 yr for the same case. Here the maximum crack thickness is increased to 20 mm and the thickening of the crack near the vertical centerline is evident.

Figures 11 through 14 show flow patterns for a case with buoyancy and thermal contraction and without proppant in the crack. The development of a large recirculating zone can be seen due to the increasing dominance of the buoyancy forces as the crack opens up. The crack thickness contours at a later time are shown in Fig. 15. At this time step, the flow/thermal solution did not converge. There are, of course, flow streamlines within the recirculation zone which are not shown on the figures. These are closed, nested lines representing negative flow values.

In Figures 16 through 18 are shown results for a somewhat different water supply situation. Here, the water inlet is in the upper region of the crack very near the outlet. As shown, with no proppant, the dominant buoyancy forces create a downward and outward flow so that the performance of this crack is not excellent, but reasonably good. If a high impedance is imposed, by way of a proppant, the flow will short-circuit, which in this case is very bad. For a high impedance case (or no buoyancy) the relative power varies from 0.43 at about 2 days down to 0.11 at 1 yr.

PRESSURE CHANGE

In general, the pressure losses in an open (unpropped) crack should be small so that the question of self-pumping depends on the pressure drop

No proppant
With thermal contraction
Buoyancy
Time = 14.05 Ms (162.6 days)

Fig. 18. Crack thickness contours.

in the supply/return pipes, surface plant and at the inlet and outlet regions. The pressure rise due to water density change for the cases discussed herein is about 300 kPa (3 bars) at early times, decreasing as the outlet temperature decreases. For a propped crack there is little question that forced pumping will be necessary. The net pressure loss at 144 kg/s flow in a 500-m-radius crack filled with sand at a permeability of 1 darcy is on the order of 10^5 bars. The pumping power would be more than ten times the available power! Even at 100 darcy permeability and 10 kg/s flow the pressure drop would be about 100 bars

Fig. 19. Unsymmetric crack with crack thickness contours.

which would require a pump output power of about 100 kW.

UNSYMMETRIC CRACK

If the inlet and outlet are not on the same vertical line the calculation must encompass the entire crack region. Some results have been obtained using an oblong crack with the inlet and outlet 1000 m apart horizontally. This crack is shown in Fig. 19. A crack like this might be formed by growing two circular cracks together. Again, the initial crack thickness profile is elliptical with the maximum thickness of 3 mm on a line between the two semi-circle centers. Figure 19 also shows the effect of the finite-difference mesh size on the model geometry.

Figure 20 shows the thermal power history of this crack for a period of 1 yr, with and without a proppant, i.e., high flow impedance, in the crack. The power level is somewhat higher than the circular crack case at the end of the year because of the larger crack area. These calculations are for no thermal contraction. The curves for different inlet/outlet heights are initially quite different in power level reflecting the variation in rock temperature with height, but soon begin to converge. The difference between the high impedance and low impedance cracks is seen to be relatively small for this case, probably because of the horizontal displacement between the inlet and outlet which renders buoyancy of little effect. This probably would not be the case when thermal contraction of the rock is allowed. Again, however, calculational instabilities arise for the low impedance cases. In this instance, the convergence failures not only occur when the buoyancy-to-impedance force ratio gets large, but when the

Fig. 20. Unsymmetric crack, power history.

Fig. 21. Unsymmetric crack; flow contours for high impedance crack.

inlet and/or outlet is placed too low in the crack, for relatively low impedance cases.

Figures 21 through 23 show flow patterns for a high impedance crack, and a low impedance crack at two different times. These calculations place the inlet and outlet centers 275 m above the crack center.

PARAMETRIC RESULTS--SYMMETRIC CRACK

The foregoing results are for a relatively small crack, one which might be used for demonstration. For concept demonstration it is desirable to have a relatively rapid power decay so that the relation between analytical and actual performance--especially in such areas as the effect of thermal stress cracking--can be assessed in a reasonable time. The following parametric curves (Figs. 24-26) of power and temperature vs time demonstrate the ability of larger

Fig. 22. Unsymmetric crack; flow contours for low impedance crack, early time.

cracks to maintain a useful power level for a reasonable length of time under the pessimistic conditions of no secondary cracking and a relatively low ratio of buoyancy forces to viscous forces.

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Fig. 23. Unsymmetric crack, flow contours for low impedance crack, late time.

Fig. 24. Power histories for 500-m-radius crack, no thermal-stress cracking.

Fig. 25. Power histories for 1000-m-radius crack, no thermal-stress cracking.

Fig. 26. Power histories for 1500-m-radius crack, no thermal-stress cracking.

APPENDIX A

HEAT TRANSFER APPROXIMATION

Consider uniform one-dimensional flow upward between parallel plates of width W and height H , as in Fig. A-1. The total mass flow rate is $\dot{m}$, the fluid heat capacity is C and the fluid inlet temperature at the bottom is T_{in} . Heat is transferred to the fluid from both plates by transient conduction perpendicular to the x - y plane from a semi-infinite solid of initial temperature, T_R , and with thermal conductivity, heat capacity and density, λ_R , C_R , and ρ_R . It is assumed that the stay time of the fluid in the system is short compared with the time period of conduction in the solid, so that no fluid transients need be considered. This should be a very good assumption for geothermal cracks except at very early time. The other assumption, not so good, is that the heat flux into the fluid at the surfaces can be expressed by

$$q = h(T_R - T) ;$$

where h is an overall heat transfer coefficient which is a function of time only.

A heat balance across an increment dy gives

$$dQ = h(T_R - T) \cdot W dy \cdot 2 = C \dot{m} dT \quad \text{or}$$

$$\frac{dT}{dy} = \frac{2Wh}{C\dot{m}}(T_R - T) .$$

Assuming that T_R varies linearly with y ; $T_R = T_{RO} + My$ where M is generally negative, the solution is

$$T_{out} - T_{in} = \left(T_{RO} - T_{in} - \frac{\Delta T_R}{B} \right) (1 - e^{-B}) + \Delta T_R ; \quad (A-1)$$

$$\text{where } T_R = T_{R,H} - T_{RO} = MH ,$$

$$B = 2hWH/C\dot{m} .$$

If it is assumed that the fluid temperature is equal to the surface temperature and is roughly

Fig. A-1. Uniform upward crack flow.

constant with time, the solution given in many references (such as Carslaw and Jaeger, Second Edition, p. 60) for the conduction perpendicular to the surface is

$$T(Z) = T_0 + (T_R - T_0) \left[1 - \operatorname{erf} \left(\frac{Z}{2\sqrt{\alpha_R t}} \right) \right] \quad (A-2)$$

where $T_0 = T(Z=0)$, Z is the distance from the surface, α_R is the rock thermal diffusivity and t is the time from start when $T = T_R$ everywhere. From this, the heat flux into the fluid is

$$q_0 = \lambda_R \frac{\partial T}{\partial Z} \bigg|_{Z=0} = \sqrt{\frac{\lambda_R C_R \rho_R}{\pi t}} (T_R - T_0), \text{ or} \quad (A-3)$$

$$h = \sqrt{\frac{\lambda_R C_R \rho_R}{\pi t}} \quad (A-4)$$

The advantage of Eq. (A-4) is that it gives an estimate of the heat transfer rate into the fluid which is independent of the details of the fluid flow and temperature gradients in the x-y plane. Equation (A-1) with (A-4) is used to compare numerical results to determine the relative effectiveness of cracks for different flow characteristics and different geometries.

COMPARISON

In an internal Lawrence Livermore Laboratory memo of March 1973, A. L. Austin gives an exact solution to the combined problem shown in Fig. A-1 with conduction in the Z direction from semi-infinite solids of constant initial temperatures (no geothermal gradient). This solution can be written as

$$T_{out} - T_{in} = (T_R - T_{in}) \operatorname{erf} \left(\frac{1}{\sqrt{t^*}} \right); \text{ where} \quad (A-5)$$

$$t^* = \left(\frac{Cm}{WH} \right)^2 \frac{t}{\lambda_R C_R \rho_R},$$

while (A-1) combined with (A-4) gives, for $M = 0$:

$$T_{out} - T_{in} = (T_R - T_{in}) \left[1 - \exp(-2\sqrt{t^*}) \right]. \quad (A-6)$$

These equations are compared in Fig. A-2. The largest relative error in the approximation of Eq. (A-6) is 20% at a t^* of about 0.9. This is not a small error, but, considering the vast number of idealizations and approximations involved in these calculations, it does not seem so bad.

INTEGRAL APPROXIMATION

The error in (A-6) is due, primarily, to the assumption of constant fluid/surface temperature, so an approximation involving the history of the rock temperature should be better. Such an approximation can be obtained by noting that (for one-dimensional heat flow) the time integral of heat flux at the surface must be equal to the spacial integral of rock temperature change at a given time, i.e.,

$$Q''_0 = C_R \rho_R \int_0^\infty (T_R - T) dZ = \int_0^t q_0 dt'; \quad (A-7)$$

Fig. A-2. Uniform upward crack flow, constant rock temperature.

$$\text{where } q_o = \lambda \left(\frac{\partial T}{\partial Z} \right)_{Z=0} = \frac{dQ_o''}{dt}$$

What is needed, then, is the spatial temperature profile. An approximate temperature profile is taken from the form of Eq. (A-2), i.e.,

$$\frac{T - T_o}{T_R - T_o} = \text{erf}(\phi_o Z); \quad (\text{A-8})$$

where ϕ_o is an unknown function of time only. Putting this and the definition of q_o into (A-7) results in

$$\phi_o = \frac{C_R \rho_R}{\sqrt{\pi} Q_o''} (T_R - T_o) = \frac{\sqrt{\pi}}{2\lambda_R} \frac{1}{(T_R - T_o)} \frac{dQ_o''}{dt},$$

from which we obtain the relations

$$\left. \begin{aligned} q_o &= \frac{2\lambda_R C_R \rho_R}{\pi} (T_R - T_o)^2 / Q_o'' \\ \text{and} \\ (Q_o'')^2 &= \frac{4\lambda_R C_R \rho_R}{\pi} \int_0^t (T_R - T_o)^2 dt' \end{aligned} \right\} \quad (\text{A-9})$$

When $(T_R - T_o)$ is constant (i.e., T_o constant) with time, this reduces to Eq. (A-3), as it should.

Equation (A-9) can be evaluated numerically with the history of T_o known. This method is used to determine local heat sources in the crack-plane model. In addition, the same results can be (and are) used to determine local crack widening due to chilling the rock since

Fig. A-3. Uniform upward crack flow, constant rock temperature.

$$w - w(t=0) = 2\alpha \int_0^\infty (T_R - T) dz = \frac{2\alpha Q_o''}{C_R \rho_R}, \quad (\text{A-10})$$

where α is, in this case, the coefficient of linear thermal expansion of rock.

To compare this approximation with an exact solution, it is applied to the problem of Fig. A-1 and compared to the results of Eq. (A-5). In dimensionless form Eq. (A-9) are

$$\left. \begin{aligned} q_o^* &= \frac{8}{\pi} (1 - T^*)^2 / Q_o^* \\ (Q_o^*)^2 &= \int_0^{t^*} (1 - T^*)^2 dt^* \end{aligned} \right\} \quad (\text{A-11})$$

and the heat balance equation,

$$\frac{dT^*}{d\xi} = q_o^*$$

$$\text{where } q_o^* = \frac{2q_o WH}{C_m (T_R - T_{in})} = \frac{dQ_o^*}{dt}$$

$$T^* = (T_o - T_{in}) / (T_R - T_{in}),$$

$$\xi = y/H,$$

and t^* as defined above.

These equations were solved numerically for various values of t^* , and the resulting values of $T_{out}^* = T^*(\xi=1)$ are shown as data points in Fig. A-3.

Also shown in Fig. A-3 is the curve of exact results from Eq. (A-5). The agreement can be seen to be excellent for at least this problem.