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PLASMA CURRENT PROFILE SHAPING WITH RF-CURRENT DRIVE

David A. Ebst and Kenneth Evans, Jr.

Argonne National Laboratory, Fusion Power Program
9700 South Cass Avenue, Building 205
Argonne, Illinois 60439 U.S.A.

1. INTRODUCTION

As our understanding of rf current drive in tokamaks improves we should seek to employ this technique in the optimum fashion to enhance the attractiveness of tokamak reactor economics. Work at ANL throughout the past year has identified several areas in which the tokamak concept could be improved relative to the STARFIRE design.^{1,2} Many of these improvements derive from the possibility of operating in the second stability regime.^{3,4} In particular, we note that plasmas with toroidal betas in excess of twenty per cent have been found theoretically stable to ideal MHD modes in high aspect ratio tokamaks with quite modest toroidal currents. In our opinion a tokamak reactor may be most attractive if it embodies the four following characteristics. First, high beta is essential to reducing cost, since considerable capital is invested in the large toroidal field magnets. Second, high aspect ratio ($A \geq 5$) will facilitate the use of efficient fast wave current drive^{5,6} and could simplify reactor maintenance operations. Third, low toroidal current (≤ 5 MA) will greatly reduce the cost of the equilibrium field coil (EFC) system and attendant start-up power supplies and will reduce the consequences of a plasma disruption. Fourth, steady state operation is likely to reduce costs and increase reliability relative to pulsed operating cycles.

The key to achieving these laudable reactor goals may well be rf current drive. At very high beta, tokamak equilibria require very broad, or even hollow, current density profiles. As suggested in earlier studies,^{2,5} it may be possible to tailor the current density in an rf-driven

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discharge by careful selection of the wave properties. Furthermore, a judicious choice of current density profile can also result in small total toroidal current; in addition to the advantages cited above, this also helps by reducing the rf current drive power, which thus improves the reactor's power balance.

In this paper we report the first calculation of rf current drive in a toroidal geometry. The result is self-consistent in that the tokamak magnetic field generated by the rf-driven current is used to compute the wave trajectory and spatial damping in the plasma. In the next section we derive the quasilinear theory in an axisymmetric torus. In Section 3 we describe a numerical solution to this problem and investigate rf-generated equilibria in a reactor-relevant geometry ($A=6.0$; major radius, $R_0=5.25$ m; elongation, $k=1.6$; triangular boundary). By suitably adjusting the rf/plasma parameters a wide range of equilibria can be created. Although we have not optimized our rf-generated equilibria (in particular, we are limited at present to a narrow spectrum) we find evidence that equilibria can be sustained which should lead to attractive tokamak reactors.

2. TOROIDAL RF-CURRENT DRIVE THEORY

Toroidal equilibrium (MHD pressure balance) is found by solving the Grad-Shafranov equation for the poloidal flux, ψ :

$$-4\pi^2 (u_0 R^2 \frac{dp}{d\psi} + F \frac{dF}{d\psi}) = R^2 \nabla^2 \psi = \left(\frac{\nabla^2 \psi}{k^2} \right) = -2\pi R u_0 j_z. \quad (1)$$

We desire a solution for ψ in cylindrical coordinates, R, Z , and from this the total tokamak magnetic field may be computed as $\underline{B} = F \nabla \zeta + \nabla \zeta \times \nabla \psi / 2\pi$, where $\nabla \zeta = \hat{\zeta}/R$, $\nabla \psi = \hat{\psi} 2\pi R B_p$, $F = R B_z$, and B_p and B_z are respectively the poloidal and toroidal field components. We define $\hat{\zeta}$ to be a unit vector parallel to the toroidal current and $\hat{\psi}$ to be outward normal to a flux surface.

In the earliest treatments⁷ the pressure, $p(\psi)$, and diamagnetism, $F(\psi)$, were chosen somewhat arbitrarily and independently of each other. [Alternatively, $p(\psi)$ and the safety factor, $q(\psi) = \frac{F(\psi)}{2\pi} \oint \frac{dF}{R^2 B}$, can be specified.] However, in a true steady-state rf-driven tokamak there is a specific relationship between $p(\psi)$ and $F(\psi)$. To see this we note that the current density parallel to $\underline{B}$ is

$$j_z = \frac{-2\pi F}{B} \frac{dp}{d\psi} - \frac{2\pi B}{u_0} \frac{dF}{d\psi},$$

where $B = |\underline{B}|$. Multiplying by B and taking the flux surface average, we find

$$\frac{1}{u_0} \frac{dF}{d\psi} + \left[\frac{1}{\langle B^2 \rangle} \frac{dp}{d\psi} \right] F + \frac{\langle j_{\parallel} B \rangle}{2\pi \langle B^2 \rangle} = 0 \quad (2)$$

where $\langle X \rangle \equiv \oint \frac{dr X}{B} / \oint \frac{dr}{B}$; x denotes the poloidal direction. This ordinary differential equation may be solved for $F(\psi)$ once $p(\psi)$ and the averaged quantities are known. In our calculations we select model functions for the plasma temperatures $\{T_e(\psi) \text{ and } T_i(\psi)\}$ and density $\{n_e(\psi) = n_i(\psi) = n(\psi)\}$, so the pressure is $p(\psi) = n(\psi) [T_e(\psi) + T_i(\psi)]/f_p$; $f_p = 0.9$, which allows for a small impurity content. Specifically, we choose $p(\psi) = p_0 \tilde{\psi}^a$, $T_e(\psi) = T_{e0} \tilde{\psi}^{aeT}$, $T_i(\psi) = T_{i0} \tilde{\psi}^{aeT}$, and $n(\psi) = n_0 \tilde{\psi}^n$, where $\tilde{\psi} = (\psi - \psi_b)/(\psi_a - \psi_b)$ and ψ_a and ψ_b are respectively the flux at the magnetic axis and the limiter boundary. More elaborate $p(\psi)$ profiles are conceivable,^{3,4} but this simple form will prove instructive for our purposes, especially in the absence of reliable transport predictions of reactor density and temperature profiles.

It remains for us to demonstrate that $\langle j_{\parallel} B \rangle / \langle B^2 \rangle$ in Eq. [2] can also be found if the temperature and density profiles are known. The derivation of this relationship is given by Ehst⁸ and will be only briefly outlined here. For weak damping of low frequency waves (angular frequency $\omega = 2\pi f \ll \Omega_e$, the electron cyclotron frequency) the drift kinetic equation for electrons can be reduced to

$$\nu_{\parallel} \tilde{B} \cdot \nabla \tilde{f} - C(\tilde{f}) = - \underline{v}^d \cdot \nabla f_M - \frac{\partial}{\partial v} \cdot \underline{f} \quad (3)$$

where $\tilde{f}$ is the perturbation from a Maxwellian, f_M , C is the linearized collision operator, $\underline{v}^d$ is the magnetic (neoclassical) drift⁹ and $\underline{f}$ is the quasilinear flow in phase space from rf-induced diffusion. We have assumed a pure steady state so there is no DC electric field which would generate a Spitzer current. High phase speed waves have an index of refraction parallel to $\underline{B}$ of $n_{\parallel} \ll c/v_e$, where $v_e = \sqrt{T_e/m_e}$ is the electron thermal speed, and for these waves the neoclassical and quasilinear perturbations are separated in velocity space. In Eq. [3] may be readily solved to yield

$$\langle j_{\parallel} B \rangle / \langle B^2 \rangle = H(\psi) + G(\psi) \quad .$$

$H(\psi)$ is the neoclassical contribution, which is only significant in the banana regime, when it becomes the bootstrap current.⁹ In the present

study we ignore bootstrap currents ($H \equiv 0$) in order to focus attention on the quasilinear current drive problem. For narrow power spectra and wave absorption with only a few passes through the plasma, the resulting expression is

$$\frac{\langle j_{\parallel B} \rangle}{\langle B \rangle} = G(\psi) = A \frac{T_e}{n_e} \left[\frac{\tilde{j}/\tilde{p}_d}{B} \right] x_0 \langle p_d \rangle, \quad [4]$$

where $\langle p_d \rangle$ is the average wave power density absorbed on a flux surface and $A = 19.2 \times 10^{18} / \ln \Lambda$, where $\ln \Lambda$ is the Coulomb logarithm. (All units are SI except temperatures, which are in KeV.) The quantity in brackets is evaluated at the poloidal location where the (narrow) wave packet crosses the flux surface.

The derivation of $G(\psi)$ is based on the self-adjointness property of the linearized collision operator;¹⁰ moreover, linear wave damping facilitates the use of a ray tracing calculation of $\langle p_d \rangle$. In the linear limit the normalized current to power density ratio is known to be¹¹ $\tilde{j}/\tilde{p}_d = 2 + 1.4 u^2 + 3.0/u$ for electron Landau damping with singly charged ions, where $u = c/n_i v_e$. A full numerical treatment requires ray tracing in arbitrarily shaped flux surfaces for low frequency, high phase velocity waves. This procedure has been carried out for the fast (magnetosonic) wave,^{5,6} which is a prime candidate for reactor plasmas due to its good penetration to high density.

3. NUMERICAL RF-GENERATED EQUILIBRIA

Our code solves the plasma dispersion relation for $\Omega_i \ll \omega \ll \Omega_e$, where Ω_i is a typical ion cyclotron frequency, so ion cyclotron damping is weak. We examine a single ray (only one mode number in the spectrum) launched at the outboard midplane. The initial poloidal mode number, n_p , is zero. After we specify the frequency, the initial n_i , n_p , and spatial location, the dispersion relation determines the radial wave vector and the evolution of the ray's path and wave vector, $\underline{k}$, through the plasma.¹² The local plasma quantities (B , n_e , n_i , T_e , T_i) are computed from a knowledge of the flux surface geometry, $\psi(R, Z)$.

The calculation proceeds iteratively, as follows. An initial guess for $F(\psi)$ is used in Eq. [1] to obtain an initial $\psi(R, Z)$, subject to the boundary condition $\psi = \text{constant} \equiv \psi_b$ at the boundary of a prescribed shape. We use a simple model, $F = F_b [1 - \delta \tilde{\psi}]$, where $F_b = F(\psi_b)$, to generate the initial guess. (The final solution is independent of the δ , γ combination chosen.) Ray tracing proceeds on the initial $\psi(R, Z)$ distribution, and the function $G(\psi)$ is computed as given in Eq. [4]. This is

substituted as the inhomogeneous term in Eq. [2], and the ordinary differential equation is solved for $F(\psi)$, with the boundary condition $F(\psi_b) = F_b$. This new $F(\psi)$ is inserted back into Eq. [1], and the calculation repeats until $F(\psi)$ converges.

This calculation is illustrated in Figs. 1 and 2 for a typical pressure profile $p(\psi) = p_0 \tilde{\psi}^\alpha$ with $p_0 = 1.08$ MPa and $\alpha = 1.4$. The field on axis is $B_0 = 3.66$ T, so $F_b = 19.215$ T-m, and an initial $F(\psi)$ guess with $\gamma = 1.7$ and $\delta = 0.147$ results in the initial $\psi(R, Z)$ contours in Fig. 1a. Having solved Eq. [1], the other relevant plasma parameters are easily found;⁷ for this initial guess we find

$$\text{average toroidal beta} = \bar{p}/B_0^2/2\mu_0 \equiv \beta_t = 0.0813 \quad ,$$

$$\text{total toroidal current} = \int \int d\mathbf{r} \, d\mathbf{r}_\chi \, j_t \equiv I_t = 3.55 \text{ MA} \quad ,$$

$$\text{magnetic axis radius} (\psi = \psi_a) \equiv R_{\text{mag}} = 5.47 \text{ m} \quad ,$$

where the bar indicates a volume average. The initial current density profile in the midplane and $q(\psi)$ are illustrated in Fig. 1b. The two terms in Eq. [1] which contribute to j_t are distinguished (dashed - $dp/d\psi$; dotted - $FdF/d\psi$) from the total current density (solid). The ray trajectory is also shown in Fig. 1b for $f=400$ MHz, $n_i = 2.4$, and incoming wave heating power of $P_{\text{edge}} = 50$ MW. For plasma peak values of $T_{e0} = 18$ keV, $T_{i0} = 12.5$ keV, and $n_0 = 2.0 \times 10^{20} \text{ m}^{-3}$ with profile parameters $e_T = 0.25$ and $e_n = 1 - e_T = 0.75$ the function $G(\tilde{\psi})$ is also displayed. The dotted curve is the exact G , whereas the solid curve is an averaged curve chosen to simulate the spreading effects expected when broader spectra are broadcast from a realistic antenna into a tokamak plasma. A practical advantage of averaging G is that this fills in the "hole" in the function near the magnetic axis ($\tilde{\psi} = 1$); otherwise, with only one n_i in the spectrum, it is difficult to get a broad $G(\tilde{\psi})$ which supplies current drive both at the axis and near the limiter ($\tilde{\psi} = 0$). Convergence is fairly fast. By the ninth iteration we have convergence to $\beta_t = 0.0786$, $I_t = 2.45$ MA, and $R_{\text{mag}} = 5.52$ m. Close examination of Fig. 2 shows the flux contours have shifted slightly, and the $q(\psi)$ and $j_t(R)$ profiles have assumed the form dictated by the fast wave current drive physics.

A study of current drive and equilibrium generation was performed by varying the different plasma and wave parameters which are inputs to this calculation. We will highlight the findings from this survey.

If the peak pressure, p_0 , is increased with $\tilde{\psi}^a$ held fixed one expects the peak and average betas to rise. This is shown clearly in Fig. 3 where β_c varies roughly linearly with p_0 for fixed wave parameters and plasma temperature. It is quite striking, however, that the ratio I_c/P_{edge} decreases as β_c increases. This is mainly due to the inverse relationship of local current drive efficiency with density in Eq. [4]. The power of our two-dimensional, self-consistent current drive calculation is that it converts the local $\tilde{j}/\tilde{p}_d$ calculated by Fisch and Karney¹¹ into global quantities of interest to reactor designers. The corollary conclusion, shown in Fig. 3, is that I_c/P_{edge} may be a misleading figure of merit for noninductive current drive.

The difficulty of associated trends in I_c/P_{edge} with equilibrium β_c values is that both quantities are dependent on profile effects. This is seen graphically in Fig. 4, where all parameters and peak densities and temperatures are constant but $T_e(\tilde{\psi})$, $T_i(\tilde{\psi})$, and $n(\tilde{\psi})$ are varied in width [with $\alpha = \alpha(e_n + e_T) = \text{constant}$]. As the temperature profile broadens and density narrows the local current density peaks (see Fig. 2) reside in a surface region which becomes increasingly favorable to current drive: higher surface temperature and lower surface density (Eq. [4]). The effect is two-fold. First, larger j_c results, which leads to an increase in I_c . More importantly, the increase in j_c near the limiter permits a larger pressure gradient to exist at the limiter. This, of course, raises pressure everywhere inside the plasma and increases β_c . This type of result supports the rationale of the lower hybrid current driven equilibrium proposed for STARFIRE.^{1,2} That is, a hollow current density minimizes the I_c/P_{edge} requirement and puts current density near the limiter where it is most useful for sustaining large plasma pressure.

The variation of I_c/P_{edge} with T_{e0} is nearly linear, as shown in Fig. 5. This is a reasonable expectation based on the functional dependence in Eq. [4]. Also, β_c increases with T_{e0} , even though p_0 is held constant. As before, the j_c peaks, localized away from the plasma center, increase and support higher local pressure gradients, broadening the pressure profile to yield higher β_c .

Figure 6 shows that, for a given target plasma, slower phase speeds produce higher β_c . Again this is associated with the shift of j_c to the plasma periphery. The variation with wave frequency, f , is given in Fig. 7. At the highest frequencies the ray trajectory does not bend closely towards the magnetic axis, and in consequence j_c is concentrated nearer the outer flux surfaces, which results in higher β_c .

The variations with wave power are seen in Fig. 8. To a great extent I_t varies linearly with P_{edge} , as expected from Eq. [4]. The increase of β_t with P_{edge} likewise is due to increases in j_t which support the larger local pressure gradients.

CONCLUSIONS

The toroidal equilibria which we have found have not been examined for stability. In general we expect a multiray treatment will be necessary to have the flexibility to adjust the j_t and $q(\psi)$ profiles to have MHD stability. Moreover, the rf hardware in a reactor will most likely require tunability in order to properly program the equilibria through a stable start-up sequence. [Figure 9 illustrates a sequence of one-dimensional equilibria calculated for STARFIRE² with the requisite n_i temporal variation. The lower hybrid current drive in this early work was necessarily inconsistent with the two-dimensional MHD equilibrium sequence displayed.]

We are encouraged that a wide range of equilibria have been found with a single ray calculation. This offers the hope that tokamak stability may be enhanced relative to inductive current drive by a clever selection of wave driver parameters in the future. We note that there are already reports^{13,14} of experimental success in modifying tokamak MHD stability with rf current drive.

Our ultimate hope is to properly adjust the equilibrium to achieve the second stability regime. The benefits are enormous. For the reactor studied here ($R_0 = 5.25$ m, $A = 6.0$, $\kappa = 1.6$) we find an equilibrium with $\beta_t = 0.10$ would produce net electric power $P_{net} = 350$ MW, provided $P_{edge} = 60$ MW. This would require toroidal coils with a maximum field $B_H = 10.2$ T. If even higher β_t is stable with $\kappa \geq 2$ the reductions in B_H could be substantial.

Clearly the motivations for studying rf current drive extend beyond the desire for steady state operation.

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REFERENCES

1. C. C. Baker et al., "STARFIRE - A Commercial Tokamak Fusion Power Plant Study," Argonne National Laboratory Report, ANL/FPP-80-1 (1980).
2. D. A. Ebst, C. D. Boley, K. Evans, Jr. et al., J. Fusion Energy 2 (1982) 83.
3. M. S. Chance, S. C. Jardin, and T. H. Stix, Phys. Rev. Lett. 51 (1983) 1963.
4. J. Manickam, R. C. Grimm, and M. Okabayashi, Phys. Rev. Lett. 51 (1983) 1959.
5. M. Abdou et al., "A Demonstration Tokamak Power Plant Study, Interim Report," Argonne National Laboratory Report, ANL/FPP/TM-154 (1982), Chapter 2.
6. Y.-K. M. Peng, P. H. Rutherford et al., "FED-A, An Advanced Performance FED Based on Low Safety Factor and Current Drive," Oak Ridge National Laboratory Report, ORNL/FEDC-83-1 (1983), Section 4.3.
7. J. D. Callen and R. A. Dory, Phys. Fluids 15 (1972) 1523.
8. D. A. Ebst, Nucl. Fusion (to be published).
9. F. L. Hinton and R. D. Hazeltine, Rev. Mod. Phys. 48 (1976) 239.
10. T. M. Antonsen, Jr. and K. R. Chu, Phys. Fluids 25 (1982) 1295.
11. N. J. Fisch and C. F. F. Karney, Phys. Fluids 24 (1981) 27.
12. D. W. Ignat, Phys. Fluids 24 (1981) 1110.
13. C. Gormezano et al., Sixth Top. Conf. RF Plasma Heating, Pine Mountain, Georgia (1985).
14. R. Bell et al., Sixth Top. Conf. RF Plasma Heating, Pine Mountain, Georgia (1985).

(a)

(b)

Figure 1. Initial equilibrium with $A=6.0$, $\kappa=1.6$, $\alpha=1.4$, $p_0=1.08$ MPa, $B_0=3.66$ T: (a) flux surface contours and (b) toroidal current density, safety factor, wave trajectory and $G(\psi) = \langle j_{\parallel} B \rangle / \langle B^2 \rangle$ for a fast wave with $f=400$ MHz, $n_{\parallel}=2.4$, and $P_{edge}=50$ MW incident on a deuterium plasma with $T_{e0}=18$ KeV, $T_{i0}=12.5$ KeV, and $n_0=2.0 \times 10^{20} \text{ m}^{-3}$.

(a)

(b)

Figure 2. Final converged equilibrium for the parameters of Figure 1.

Figure 3. Beta and current drive efficiency vs. peak pressure for fixed $T_{eo} = 18$ KeV, $T_{io} = 42.9$ KeV, $\alpha = 1.4$, $e_r = 0.25$, $f = 400$ MHz, $n_i = 2.4$, and $P_{edge} = 50$ MW.

Figure 4. Beta and I_f / P_{edge} vs. profile widths at fixed pressure ($\alpha = 1.4$, $p_0 = 1.08$ MPa), peak $T_{eo} = 18$ KeV, $T_{io} = 12.5$ KeV, $n_0 = 2.0 \times 10^{20} \text{ m}^{-3}$ and wave parameters $f = 400$ MHz, $n_i = 2.4$, and $P_{edge} = 50$ MW.

Figure 5. Beta and current variation with T_{20} at fixed $P_0=1.08$ MPa, $\alpha=1.4$, $n_{e0}=n_{e0}$, $2.0 \times 10^{20} \text{ m}^{-3}$, $e_T=0.25$, $f=400$ MHz, $n_i=2.4$, and $P_{edge}=50$ MW.

Figure 6. Beta and current vs. n_i for fixed $\alpha=1.4$, $T_{e0}=18$ KeV, $T_{i0}=12.5$ KeV, $n_0=2.0 \times 10^{20} \text{ m}^{-3}$, $e_T=0.25$, $f=400$ MHz, and $P_{edge}=50$ MW.

Figure 7. Beta and current vs. f for fixed $\alpha=1.4$, $T_{eo}=15.5$ KeV, $n_0=2.0 \times 10^{20}$, $e_T=0.25$, $n_i=2.4$, and $P_{edge}=60$ MW.

Figure 8. Beta and current vs. P_{edge} at various T_{eo} for fixed $\alpha=1.4$, $P_0=1.68$ GPa, $n_0=2.0 \times 10^{20}$ m^{-3} , $e_T=0.25$, $f=400$ MHz, and $n_i=2.4$.

(9a)

Figure 9. Startup current drive sequence for the STARFIRE reactor: (a) two dimensional model MHD equilibria with poloidal beta and diamagnetism (α , δ) increasing in time; (b) one dimensional lower hybrid MHD equilibrium sequence with decreasing central n_j required to match j_c evolution in (a).

$$\begin{aligned}\tau &= 4 \text{ ns} \\ n_1^C &= 4.7 \\ \Delta n_1 &= 0.8 \\ P_{ref} &= 3.0 \text{ MW} \\ I_{ref} &= 0\end{aligned}$$

$$\begin{aligned}\tau &= 9 \text{ ns} \\ n_1^C &= 3.1 \\ \Delta n_1 &= 1.2 \\ P_{ref} &= 17 \text{ MW} \\ I_{ref} &= 2.3 \text{ MA}\end{aligned}$$

$$\begin{aligned}\tau &= 15 \text{ ns} \\ n_1^C &= 2.1 \\ \Delta n_1 &= 0.8 \\ P_{ref} &= 55 \text{ MW} \\ I_{ref} &= 13.5 \text{ MA}\end{aligned}$$

$$\begin{aligned}\tau &= 300 \text{ ns} \\ n_1^C &= 1.63 \\ \Delta n_1 &= 0.46 \\ P_{ref} &= 90 \text{ MW} \\ I_{ref} &= 14.8 \text{ MA}\end{aligned}$$

$$\begin{aligned}\tau &= 380 \text{ ns} \\ n_1^C &= 1.63 \\ \Delta n_1 &= 0.46 \\ P_{ref} &= 90 \text{ MW} \\ I_{ref} &= 14.8 \text{ MA}\end{aligned}$$

(9b)