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STATUS REPORT ON LAMB WAVES

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I. INTRODUCTION

This status report presents the theoretical and experimental aspects of Lamb waves that have been investigated as part of the research program of the Nondestructive Testing Group of the Metallurgy Division. The list of the several types of elastic waves that may be generated in an elastic plate usually includes just shear, longitudinal, and surface waves. In recent years, however, another type of elastic wave motion, known as Lamb waves, has been receiving recognition. Lamb waves designate that type of stress waves in elastic plates theoretically predicted by Horace Lamb and announced in 1916 at a meeting of the Royal Society of London. Nothing further was done with the subject, at least as far as literature references indicate, until 1945, when Floyd A. Firestone and Daniel S. Ling, who were then at the University of Michigan, undertook an extensive theoretical and experimental study. They were the first intentionally to produce such waves in plates, verifying, after 26 years, Lamb's theory, and the first to bring out those properties of Lamb waves which make them unique when contrasted with other elastic stress waves.

II. THEORETICAL ASPECTS

In developing the theory, it is assumed that the medium is perfectly elastic, homogeneous, and isotropic. The motion of the particle takes place in two dimensions: u and v are the horizontal and vertical displacement of the particle. The origin of the coordinate system is taken in the medial plane of the plate: x is the coordinate along the plate in the direction along which the wave travels, and the y axis is normal to this. The plate is infinite and has a thickness d ; it is to be considered free.

The following quantities are defined:

P_{yy} = stress acting on a plane normal to the y axis in the y direction

P_{xy} = stress acting on a plane normal to the z axis in the y direction

λ = Lamé's constant

μ = rigidity modulus

u = displacement of the particle along the x axis

v = displacement of the particle along the y axis

ρ = density of medium.

The stress strain relations are

$$\begin{aligned} P_{yy} &= \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + 2\mu \frac{\partial u}{\partial y} \quad ; \\ P_{xy} &= \mu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \quad . \end{aligned} \quad (1)$$

There exist potential functions ϕ and ψ such that

$$\begin{aligned} u &= \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y} \quad ; \\ v &= \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x} \quad , \end{aligned} \quad (2)$$

where

$$\begin{aligned} \frac{\partial^2 \phi}{\partial t^2} &= \frac{\lambda + 2\mu}{\rho} \nabla^2 \phi \quad ; \\ \frac{\partial^2 \psi}{\partial t^2} &= \frac{\mu}{\rho} \nabla^2 \psi \quad . \end{aligned} \quad (3)$$

We see that ϕ is the potential function associated with longitudinal waves and ψ that associated with shear waves. We assume plane wave motion so that the particle motion is independent of z , and also that ϕ and ψ are periodic with respect to time and distance. The latter dependence will be taken care of by the factor

$$e^{i\sigma t} \quad \text{and} \quad e^{i\epsilon x}$$

where

$$\epsilon = 2\pi/\lambda' \quad .$$

The time factor $e^{i\sigma t}$ will be omitted in the discussion.

Define

$$h^2 = \frac{\rho \sigma^2}{\lambda + 2\mu} \quad ; \quad k^2 = \frac{\rho \sigma^2}{\mu} \quad , \quad (4)$$

so that

$$\begin{aligned} (\nabla^2 + h^2)\phi &= 0 \quad ; \\ (\nabla^2 + k^2)\psi &= 0 \end{aligned} \tag{5}$$

We also define

$$\alpha^2 = \epsilon^2 - h^2 \tag{6}$$

$$\beta^2 = \epsilon^2 - k^2 \quad . \tag{7}$$

Then (5) can be written:

$$\begin{aligned} \frac{\partial^2 \phi}{\partial y^2} &= \alpha^2 \phi \quad ; \\ \frac{\partial^2 \psi}{\partial y^2} &= \beta^2 \psi \quad . \end{aligned} \tag{8}$$

We see that solutions to (8) are

$$\phi = (A \cosh \alpha y + B \sinh \alpha y) e^{i\epsilon x}$$

and

$$\psi = (D \cosh \beta y + C \sinh \beta y) e^{i\epsilon x} \quad ,$$

but we should also note that

$$\begin{aligned} \phi &= (A \cosh \alpha y) e^{i\epsilon x} \quad ; \\ \psi &= (B \sinh \beta y) e^{i\epsilon x} \end{aligned} \tag{9}$$

and

$$\begin{aligned} \phi &= (C \sinh \alpha y) e^{i\epsilon x} \quad ; \\ \psi &= (D \cosh \beta y) e^{i\epsilon x} \end{aligned} \tag{10}$$

are also solutions.

Lamb designated solutions of type (9) as symmetric wave motion and of type (10) as antisymmetric wave motion. Using (2) and (8) and substituting into the stress-strain equations, we get the stress-strain equations in terms of quantities that have been defined:

$$\frac{P_{yy}}{\mu} = (\epsilon^2 + \beta^2)\phi - 2i\epsilon \frac{\partial \psi}{\partial y} \quad ;$$

$$\frac{P_{xy}}{\mu} = 2i\epsilon \frac{\partial \phi}{\partial y} + (\epsilon^2 + \beta^2)\psi \quad .$$

We shall treat only the symmetric case; the procedure is identical for the antisymmetric case. Using (9), that is, the symmetric-type solution and substituting into the last expression for the stress-strain equation, we get

$$\frac{P_{yy}}{\mu} = [A(\epsilon^2 + \beta^2) \cosh \alpha \frac{1}{2}d - B\beta 2i\epsilon \cosh \beta \frac{1}{2}d] e^{i\epsilon x} \quad ;$$

$$\frac{P_{xy}}{\mu} = \pm [2iA\alpha \sinh \alpha \frac{1}{2}d + B(\epsilon^2 + \beta^2) \sinh \beta \frac{1}{2}d] e^{i\epsilon x} \quad .$$

Now with the boundary condition that at the surface $y = \frac{1}{2}d$, the stress must be zero, that is, $P_{yy} = P_{xy} = 0$. We get, after eliminating the constants A and B,

$$\frac{2i\epsilon\beta \cosh \beta \frac{1}{2}d}{(\epsilon^2 + \beta^2) \cosh \alpha \frac{1}{2}d} = \pm \frac{(\epsilon^2 + \beta^2) \sinh \beta \frac{1}{2}d}{2i\epsilon\alpha \sinh \alpha \frac{1}{2}d}$$

or, finally, the frequency equation:

$$\frac{\tanh \beta \frac{1}{2}d}{\tanh \alpha \frac{1}{2}d} = \frac{4\epsilon^2 \alpha \beta}{(\epsilon^2 + \beta^2)^2} \quad .$$

Replacing the values for α , ϵ , and β in the above equation, remembering that

$$V_L = \sqrt{\frac{\lambda + 2\mu}{\rho}} \quad ; \quad V_s = \sqrt{\frac{\mu}{\rho}}$$

and that by definition the phase velocity is $V = \sigma/\epsilon$, we can write

$$\frac{\tanh \pi f d \sqrt{\frac{V_{\rho}^2 - V^2}{V_s^2 V^2}}}{\tanh \pi f d \sqrt{\frac{V_L^2 - V^2}{V_L^2 V^2}}} = \frac{4 \sqrt{\left(1 - \frac{V^2}{V_L^2}\right) \left(1 - \frac{V^2}{V_s^2}\right)}}{\left(2 - \frac{V^2}{V_s^2}\right)^2} \quad . \quad (11)$$

This is the frequency for the symmetric-type motion in terms of frequency, plate thickness, shear velocity, and longitudinal velocity.

The equation as it stands now is single-valued and will give just one mode of vibration having a phase velocity less than both longitudinal and shear velocity. The equation, however, can become multivalued in two ways. First, the wave velocity can exceed the shear velocity. Then $V_s < V < V_L$, so that

$$\frac{\tan \pi fd \sqrt{\frac{V^2 - V_s^2}{V_s^2 V^2}}}{\tan \pi fd \sqrt{\frac{V_L^2 - V^2}{V_L^2 V^2}}} = \frac{4 \sqrt{\left(1 - \frac{V^2}{V_L^2}\right) \left(\frac{V^2}{V_s^2} - 1\right)}}{\left(2 - \frac{V^2}{V_s^2}\right)^2} \quad (12)$$

Secondly, the wave velocity can exceed both the shear and longitudinal velocity. Then $V_s < V_L < V$, so that

$$\frac{\tan \pi fd \sqrt{\frac{V^2 - V_s^2}{V_s^2 V^2}}}{\tan \pi fd \sqrt{\frac{V^2 - V_L^2}{V_L^2 V^2}}} = \frac{-4 \sqrt{\left(\frac{V^2}{V_L^2} - 1\right) \left(\frac{V^2}{V_s^2} - 1\right)}}{\left(2 - \frac{V^2}{V_s^2}\right)^2} \quad (13)$$

Equations (11), (12), and (13) give an infinite number of modes of vibration. The higher the root of the equation, the higher the mode of the vibration. These equations are readily evaluated for any material for which the shear and longitudinal velocities are known. One chooses a value of V , the wave velocity, and then looks for the values of the frequency-thickness (fd) product that will satisfy the equation. It is this dependence of the phase velocity on the fd product which make Lamb waves so unique, for there is no such dependence for shear, longitudinal, or surface waves.

Figure 1 illustrated the plotting of this relationship, that is, of phase velocity versus frequency-thickness product, for aluminum and for the symmetric (Figure 1a) and antisymmetric (Figure 1b) types of motion. The essential features of these graphs remain the same for different metals. The phase velocity of the Lamb wave mode for a given metal depends only on the frequency-thickness product. For any frequency-thickness product, there are a finite number of modes that can be generated, but the number of possible modes increases as the fd product increases. For example, in aluminum with a frequency-thickness product of 2.0×10^5 in./sec, 3 asymmetric and 3 symmetric modes can be generated, whereas at a frequency-thickness product of 5.0×10^5 in./sec, 6 asymmetric and 6 symmetric modes can be generated.

Figure 2 illustrates the plotting of group velocity versus frequency-thickness product for the symmetric (Figure 2a) and antisymmetric (Figure 2b) modes in aluminum. These graphs are easily plotted, for the relation between phase and group velocity is well known.

Fig. 1a. Waves in Aluminum Plate - Phase Velocity Plotted Against Product of Frequency and Plate Thickness. Symmetrical Type. The dotted lines refer to the asymptotic values of the 2nd, 3rd, 4th, etc., modes. (From Sperry Technical Report 50-6001)

Fig. 1b. Waves in Aluminum Plate - Phase Velocity Plotted Against Product of Frequency and Plate Thickness. Asymmetrical Type. The dotted lines refer to the asymptotic values of the 2nd, 3rd, 4th, etc., modes. (From Sperry Technical Report 50-6001)

Fig. 2a. Waves in Aluminum Plate - Group Velocity Plotted Against Product Frequency and Plate Thickness. Symmetrical Type. (From Sperry Technical Report 50-6001)

Fig. 2b. Waves in Aluminum Plate - Group Velocity Plotted Against Product Frequency and Plate Thickness. Asymmetrical Type. (From Sperry Technical Report 50-6001)

Let us now go into further detail concerning the particle and plate vibrations themselves. Lamb's grouping of these infinite number of modes into symmetric and antisymmetric is a designation which depends on the direction of the particle displacement. This is illustrated in Figure 3. With the symmetric type, there is mirror symmetry with respect to the medial plane through the plate. When a particle in the upper half of the plate is moving up and to the right, the particle occupying the equivalent position in the lower half is moving down and to the right; the corresponding components parallel to the plane of the plate move in the same direction, the corresponding perpendicular components move in the opposite sense. The situation is reversed for the antisymmetric type. When a particle in the upper half of the plate is going up and to the right, the corresponding particle in the lower half is going up and to the left. The corresponding components parallel to the plane of the plate move in the opposite direction, the corresponding perpendicular components move in the same direction. The two components - parallel and perpendicular to the plate - combine to give a particle orbit which is elliptical. The particles in the lower and upper half of the plate trace orbits with the same major and minor axes, but the direction of traversing the ellipses is different. Whether the major axis of the ellipse is perpendicular to the surface or to the direction of propagation depends on the ratio of thickness to the wavelength of the vibration.

Fig. 3. Directions of Particle Displacements for the Symmetric and Antisymmetric Modes

Let us now consider the motion of the plate itself. First, as u and v are functions of x , y , and t (the time) and as the disturbance is moving in the x direction, they are functions of the form

$$u = U(y) \cos (\epsilon x - \sigma t) \quad ;$$

$$v = V(y) \sin (\epsilon x - \sigma t) \quad .$$

Figure 4 shows a section of the plate on which has been drawn the direction of the motion of the particles. This diagram is a picture at one instant of time as this is, in reality, a progressive wave motion, and we must realize that this drawing moves to the right with a phase velocity of the particular mode. The curved lines drawn in the compartments are "lines of motion," that is, a tangent to these curves gives the direction of the

motion of a particle located at the point of tangency. As the diagram and the lines of motion move to the right, a particle located at a given point in the plate will have different lines - or different directions of motion at different times. It is to be emphasized that the particles are not travelling around these curved lines or "lines of motion." The motion of each particle itself is a very small ellipse about its equilibrium position; that the "lines of motion" just tell in what direction the particle is moving.

Fig. 4. Vibration Pattern of a Symmetrical Wave

Although the "lines of motion" give no information concerning the magnitude of the displacements, they are related to u and v in the following way. If u is large and v is small at a given point, then the line of motion should be practically horizontal at that point. If v is large and u is small, the line of motion should be nearly vertical.

The plate is divided into compartments bounded by vertical and horizontal lines. In each compartment we have shown only one "line of motion," but really each point should have a line of motion passing through it. They have been left out for the sake of clarity. It should now be evident from the discussion of the "lines of motion" and their relation to u and v , the displacements, that the horizontal lines bounding the compartments are those lines along which $v = 0$, that is, there is no vertical displacement of the particles on these lines. The vertical lines bounding the compartments are lines on which $u = 0$, that is, there is no horizontal displacement of the particles on these lines. Thus, sections of the plate are divided into boxes, along whose boundaries the motion is entirely horizontal or entirely vertical.

Now, u and v can vanish in two ways: $U(y)$ or $V(y)$ can be zero, or the cosine and sine term can vanish. If the cosine equals zero, then U vanishes, and the lines along which $u = 0$ give the vertical boundaries of the compartments shown by solid lines in Figure 4. Equally, if $V(y)$ is zero, v vanishes, and the lines along which $v = 0$ give the horizontal boundaries of the compartments shown by solid lines in Figure 4. These compartments are bisected vertically and horizontally. The horizontal bisector shown by the dotted line on Figure 4 is that line along which $u = 0$ which occurs when $U(y) = 0$. The vertical bisector is that line along which $v = 0$ which occurs when the sine terms vanishes; it is not shown on Figure 4.

In summary, then, both the horizontal and vertical displacements are sinusoidal in form along any given horizontal line in the plate with an amplitude dependent on the depth.

There are also some conditions under which the plate surfaces will not vibrate in either the vertical or the horizontal direction. These conditions will not be derived but simply stated. Any symmetric or anti-symmetric mode that travels in a plate with a phase velocity equal to longitudinal velocity will not vibrate the plate surface in the vertical direction. Any symmetric or antisymmetric mode that travels in a plate with a phase velocity equal to $\sqrt{2}$ times the shear velocity will not vibrate the plate surface in the horizontal direction.

There has been some tendency to confuse Lamb waves and Rayleigh surface waves. They are not the same. By a surface wave, we mean a wave which travels along surfaces with its field confined to the surface; it is exponentially attenuated in a direction normal to the surface; the disturbance exists for only a few wave lengths below the surface. This is not characteristic of Lamb waves. They exist throughout the body of the plate and not just at the surface - they are not exponentially attenuated normal to the plate surfaces and, hence, are not surface waves. Nevertheless, it is true that there is some definite relationship between Lamb waves and Rayleigh waves. We have derived the period equation for the first modes:

$$\frac{\tanh \beta f}{\tanh \alpha f} = \frac{4\epsilon^2 \alpha \beta}{(\epsilon^2 + \beta^2)^2} \quad .$$

Now, for waves which are very short compared with the thickness f , the values of αf and βf are large, and the equation will tend to the form

$$(2\epsilon^2 - k^2)^2 - 4\epsilon^2 \alpha \beta = 0 \quad ,$$

which is recognized as the equation to determine the period of Rayleigh waves. In other words, in the limit $d/\lambda^1 \rightarrow \infty$, the phase velocity of the first Lamb wave modes will approach Rayleigh wave velocity. This is evident in Figure 1 - the plot of phase velocity versus frequency-thickness. Here, we can see that the first modes are already starting to approach the surface wave velocity.

III. EXPERIMENTAL ASPECTS

A. Equipment

The source of the energy is a High Power Pulsed Oscillator Model PG-650-C, manufactured by the Arenberg Ultrasonics Laboratories, Inc. It is a variable-frequency, pulse-modulated radiofrequency (rf) source

giving high power, and extreme phase and frequency stability. The oscillator covers a continuously variable range from 5 to 80 megacycles/sec in 7 bands by use of plug-in coils. Operation from 0.05 Mc to 150 Mc is possible with special coils. The frequency is stable to 2% or less. The output is free from generator noise and hash during the off-pulse time, and rf leakage is detectable only with great difficulty. The pulse has a rise and fall time of $0.3 \mu\text{sec}$ maximum above 20 Mc and within 3 cycles at lower frequencies. The pulse length is continuously variable from 1.5 to $15.0 \mu\text{sec}$ and can be observed with less than 5% droop. There is a Helipot delay potentiometer for covering 3 overlapping ranges to 200, 1100, and 11,000 μsec , and a separate delayed trigger output for synchronizing any standard cathode-ray oscilloscope with a delayed and expanded sweep, when operating on internal trigger. The power level is such that transmission at 1 Mc through 6 in. of air or 1 in. of Teflon is possible when using barium titanate transducers.

At the receiving side of the system there is an Arenberg Preamplifier Model PA-620 which resonates at all frequencies from 6 to 70 Mc with the fixed capacity of the crystals. It serves to narrow the pass band and to improve the signal-to-noise ratio, and provides a means for varying the band width. This unit operates with the Arenberg Wide Band Amplifier Model WA-600. The amplifier covers a 6-to-60-Mc range without any adjustments or change in if strips. It has a gain of 80-90 db and a video band width of 10 Mc. The output, either rf or video, from the amplifier is viewed on a Tektronix Oscilloscope Type 531A.

The transducers used are the Sperry Immersion Testing Search Units, Style 50A1338, Type SI. They are 5-Mc quartz crystals having a $\frac{3}{8}$ -in. diameter. Angular settings of the transducers up to 30° are made by means of a Curtiss-Wright Precision Manual Manipulator. These settings are read on quadrant-type scales that have magnifiers. The system is accurate to 0.5° .

B. Experimental Setup

Two transducers - one the transmitter and the other the receiver - were submerged in water to achieve suitable coupling. They were mounted in special water-tight housings which were fitted with hollow support shafts. Coaxial cable for the necessary electrical connections to the crystal was run through the hollow shaft. Each transducer mounting and shaft was supported by an arm rigidly attached to one carriage of a small lathe. To this was attached the Curtiss-Wright Precision Manual Manipulator so that the angles of the transducers relative to the plate could be very accurately set and measured. The transducers also could be moved in any desired direction, and the distance moved was determined by the reading dials on the lathe.

A rubber shield was mounted on the tank and positioned to prevent reflections from the front surface of the plate from reaching the receiver. The experimental setup is shown in Figures 5 and 6.

Fig. 5. Experimental Setup

Fig. 6. Experimental Setup

C. Preliminary Experiments

A set of aluminum plates, 4x12 in., and varying in thickness from 15.0 to 65.0 mils, were cut from stock 1100 aluminum sheets. A plate was mounted in the water tank and carefully aligned for parallelism with the transducer faces. Two of Lamb's conditions have been violated. The plates are not infinite nor are they free. However, it was found that the violation of these conditions did not affect the results.

Lamb waves were generated in plates in the following manner. A longitudinal plane sound wave with velocity V_L was sent through the water to strike the plate at an angle θ_i , as measured from the normal. The velocity with which the line of contact of any wave front ran along the surface of the plate was

$$\frac{V_L}{\sin \theta_i} = V_a \quad .$$

If this velocity, V_a , is made the same as the phase velocity of a Lamb wave mode, the mode will be generated in the plate by a resonance process. The wave will travel down the plate and radiate from the plate, along its whole length, with an exit angle θ_E equal to θ_i . A receiving transducer positioned at the angle θ_E will detect this energy. This received pulse is then amplified and displayed on a cathode ray oscilloscope.

The family of curves shown in Figures 1 and 2 illustrate the phase velocity versus frequency-thickness product and the group velocity versus frequency-thickness product for the symmetric and antisymmetric modes in aluminum. In these figures have been plotted the values of phase velocity and group velocity that have been experimentally measured at several discrete values of frequency-thickness products. The agreement of these velocities is seen to be quite good, with the exception of those regions where the curves of phase velocity become almost vertical. The reason for this is easily understood. In these regions, a single point is not being selected out; there is, rather, a small spread of phase velocities that satisfy the frequency-thickness relation. This makes the readings in these regions somewhat ambiguous.

It was previously stated that any mode that travels in a plate with a phase velocity equal to the longitudinal velocity will not vibrate the plate surface in the vertical direction. Such a mode will not attenuate rapidly, as water cannot support the shearing motion and very little energy is imparted to the water. Also, any mode which travels in a plate with a phase velocity equal to $\sqrt{2}$ times the shear velocity will not vibrate the plate in the horizontal direction. Such a mode will attenuate very rapidly, for it imparts its energy to the water very quickly. In a given material, then, the attenuation of a Lamb wave mode depends only on its phase velocity or on the relative amplitudes of the horizontal and vertical components of

displacements. In Figure 7, a plot of the attenuation measurements that were made for aluminum, A/A_0 is plotted against distance of travel of the mode in the plate, where A_0 is the amplitude of the mode at zero distance of travel in the plate, and A the amplitude of the mode at any distance of travel in the plate. In aluminum the longitudinal velocity is 2.45×10^5 in./sec and the $\sqrt{2}$ shear velocity is 1.724×10^5 in./sec. From the graph, it is seen that the mode which has a measured phase velocity of 2.34×10^5 in./sec is the least attenuated and that the mode which has a measured phase velocity of 1.796×10^5 in./sec is the most attenuated. The attenuation of some of the other modes is also shown.

Lamb waves are body waves, that is, a mode, once it is generated, should travel throughout the entire plate and be radiated from both surfaces. Thus it should be possible to detect Lamb waves when the receiver and sender are on the opposite sides of the plate. The measurements of phase velocity made by this method verify this fact. There are, however, many more signals picked up by the receiver when this method is used, and it makes the detection of Lamb waves experimentally difficult. Consequently, although this is a possible method, it is not to be recommended from a practical viewpoint.

D. Applications

The experiments described above substantiate the main concepts associated with the theory of Lamb waves. Subsequent investigations have been concerned with the applicability of Lamb waves to the field of Nondestructive Testing.

One study was made to determine the feasibility of Lamb waves to bond testing. An area of nonbond will appear as a thin plate, and a Lamb wave mode will be generated over this region when the proper frequency-thickness condition is satisfied. Over a region of good bond, the frequency-thickness relationship will no longer be satisfied, and the Lamb wave will no longer be generated. Thus, the appearance of a Lamb wave mode indicates a nonbond.

To test this concept, a plate $4.0 \times 3.0 \times 0.333$ in. was cut out of an aluminum (1100) block, $8.0 \times 3.0 \times 3.0$ in., and then glued back into position. This arrangement is representative of a nonbond area whereas that over the solid block is representative of a good bond area. As the block is scanned, the Lamb wave mode generated over the plate should disappear over the solid block. It was found that the mode generated did not entirely disappear over the region of good bond; it merely underwent an 80 to 90 per cent decrease in amplitude. Theoretically, the frequency-thickness curves extend to infinity, but the practical experimental limits were not known. These results show that, with sufficient amplification of the received pulse, Lamb waves can be generated and detected at comparatively large frequency-thickness products. The great decrease in amplitude is to be expected; the available energy is now shared over the many more modes that occur at large frequency-thickness products.

Fig. 7

Plot of the Attenuation of Lamb Wave Modes in Aluminum

Figure 8, Part A, shows the second symmetric mode being generated over the plate or nonbond region. Figure 8, Part B, shows this same mode

when the transducers are over the solid block or bonded region. Further tests are being made to determine how small an area of nonbond can be detected.

(A) Nonbonded Area

(B) Bonded Area

Fig. 8. Lamb Wave Mode Over a "Nonbond" and Over a "Bond"

The phase velocity of a Lamb wave mode in a given material is dependent on the elastic constants of that material. Thus, if the material undergoes a physical change, the phase velocity of the Lamb waves should also change. A series of tests were made in an attempt to correlate extreme stress in an aluminum plate and the phase velocity of the Lamb wave modes. The plates were cut in a form recommended for use in a Tatnall-Krause fatigue machine and were subjected to stress under various loads until they reached their breaking point. The phase velocity of the Lamb wave modes was checked during the stressing process and after the plate had broken. Deviations in these readings from the theoretical value were no greater than the deviations noted in any number of successive readings of the phase velocity. The magnitude of any structural change occurring during the stressing process was too small to be observed in its effect on the phase velocity; the measurements of phase velocity had three-place significance.

Another attempt to apply Lamb waves to nondestructive testing, and this met with no success, was that of testing welds. There are physical differences between the weld area and the parent metal; as a Lamb wave is propagated from the metal and through the weld, such physical differences might affect the velocity, attenuation, and scattering of the mode. Thin (0.065 in.) plates of aluminum (1100) were welded together. The welded region was made smooth and of the same thickness as the plates. The velocity, attenuation, and scattering of various Lamb wave modes were measured in the weld and in the parent metal; a correlation was to be made between the strength of the weld and the magnitude of the changes in these

quantities. Readings over the nonweld and welded areas were repeated a number of times. Differences in readings that occurred between the nonwelded and welded areas were no greater than differences in any successive number of readings over the nonwelded area. Nothing of any significance could be found in the data.

It is hoped that, in the future, the unique properties of Lamb waves will be utilized to advantage in the field of Nondestructive Testing.

BIBLIOGRAPHY

1. Lamb, Horace, On Waves in an Elastic Plate, Proc. Roy. Soc. (London) A93, 114.
2. Firestone, F. A., and D. S. Ling, Methods and Means of Generating and Utilizing Vibrational Waves in Plates, U. S. Patent No. 2,536,128 (Jan. 2, 1950).
3. Firestone, F. A., and D. S. Ling, Report of the Propagation of Waves in Plates - Lamb and Rayleigh Waves, Sperry Products, Inc., Technical Report 50-6001.
4. Worlton, D. C., Ultrasonics Testing with Lamb Waves, Sperry Products, Inc. Reprint 50-892.
5. Worlton, D. C., Lamb Waves at Ultrasonic Frequencies, HW-60662.
6. Lord Rayleigh (J. W. Strutt), On Wave Propagation along the Plane Surface of an Elastic Solid, Proceedings of London Mathematical Society, 17, 1-11 (1885).
7. Sanders, F. H., Transmission of Sound through Thin Plates, Can. J. Research 17, 179 (1939).
8. Schock, A., "Schallreflesior, Schalbrechung und Schallugung," Ergebnisse der Exakten Naturwissenschaften, Vol. 23, p. 127 (1950).
9. Beauyard, L., G. Collette, and C. Droeut, The Effect of Dispersion and Velocity of Propagation of Lamb Waves during the Ultrasonic Testing of Mild Steel Sheet, British Journal of Non-Destructive Testing, 1 (1) (July 1959).
10. Gregsby, T. N., and E. J. Tajchman, Properties of Lamb Waves Relevant to the Ultrasonic Inspection of Thin Plates, IRE Transactions on Ultrasonic Engineering, Vol. UE-8, pp. 26-33 (March 1961).
11. Kolsky, H., Stress Waves in Solids, Oxford University Press, London (1953).
12. Hereter, T. F., and R. H. Boet, Sonics, John Wiley & Sons, Inc., New York.