35
3-20-78
Specified Data

MASTER

WAPD-TM-1274
DOE RESEARCH AND
DEVELOPMENT REPORT

A MODEL TO ESTIMATE RADIATION DOSE COMMITMENTS TO THE WORLD POPULATION FROM THE ATMOSPHERIC RELEASE OF RADIONUCLIDES

(LWBR Development Program)

FEBRUARY 1978

CONTRACT EY-76-C-11-0014

**BETTIS ATOMIC POWER LABORATORY
WEST MIFFLIN, PENNSYLVANIA**

Operated for the U. S. Department of Energy by
WESTINGHOUSE ELECTRIC CORPORATION

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Special External Distribution

**A MODEL TO ESTIMATE RADIATION DOSE COMMITMENTS
TO THE WORLD POPULATION FROM THE ATMOSPHERIC
RELEASE OF RADIONUCLIDES
(LWBR Development Program)**

J. L. Rider and S. K. Beal

February 1978

Contract EY-76-C-11-0014

Printed in the United States of America
Available from the
National Technical Information Service
U. S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22151

NOTICE
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

NOTE

This document is an interim memorandum prepared primarily for internal reference and does not represent a final expression of the opinion of Westinghouse. When this memorandum is distributed externally, it is with the express understanding that Westinghouse makes no representation as to completeness, accuracy, or usability of information contained therein.

BETTIS ATOMIC POWER LABORATORY
WEST MIFFLIN, PENNSYLVANIA
Operated for the U. S. Department of Energy by
WESTINGHOUSE ELECTRIC CORPORATION

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States, nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

TABLE OF CONTENTS

	<u>Page</u>
I. INTRODUCTION	1
II. GENERAL DISCUSSION	1
III. MODEL DESCRIPTION	4
A. Dose Commitment Pathways	4
1. Immersion	4
2. Surface Deposits	5
3. Ingestion	5
a. Ingestion During Deposition	6
b. Ingestion Subsequent to Deposition	6
4. Inhalation	7
B. Air Concentrations	7
1. During Releases	7
2. Subsequent to Releases	10
C. Ground Concentrations	12
1. During Releases	12
2. Subsequent to Releases	17
D. Final Dose Commitment Equations	21
1. Immersion Dose Commitments	22
2. Surface Deposit Dose Commitments	25
3. Ingestion Dose Commitments	26
a. Ingestion of Food Contaminated Directly by Deposition	26
b. Ingestion of Food Produced in Contaminated Soil	28
4. Inhalation Dose Commitments	29
IV. MODEL APPLICABILITY AND USE	31
V. SUMMARY	32
VI. ACKNOWLEDGEMENTS	33
VII. REFERENCES	33

NOMENCLATURE

<u>Symbol</u>	<u>Definition</u>
A	Total activity concentration in the air—Ci/m ³
A _a	Surface area of the region affected by the releases—m ²
B _a	Breathing rate of the people in the region affected by the releases—m ³ /hr
C	Concentration of activity on the surface of the ground during the time interval that the activity is being released—Ci/m ²
C*	Concentration of activity on the surface of the ground after the releases of activity have stopped—Ci/m ²
D _a	Dose commitment due to the inhalation of radionuclides—rem
D _i	Dose commitment due to the ingestion of radionuclides—rem
D _m	Dose commitment due to immersion in air containing radionuclides—rem
D _s	Dose commitment due to a surface deposit of radioactive material—rem
F _f	Activity contained in a unit mass of the food f at the time of consumption—Ci/gm
G	Subscripted term used in the general equations which define the air and ground activity concentrations during the time interval that releases are occurring—defined by Equation (33)
H	Subscripted term used in the general equations which define the air and ground activity concentrations during the time interval that releases are occurring—defined by Equation (31)
K _a	Dose commitment conversion factor defining the dose accumulated during a given time interval due to the inhalation of a unit quantity of activity at the beginning of the time interval—rem/Ci
K _i	Dose commitment conversion factor defining the dose accumulated during a given time interval due to the ingestion of a unit quantity of activity at the beginning of the time interval—rem/Ci
K _m	Dose rate conversion factor defining the dose rate resulting from immersion in air containing a unit activity concentration—rem/hr per Ci/m ³
K _s	Dose rate conversion factor defining the dose rate resulting from standing on a surface deposit containing a unit surface activity concentration—rem/hr per Ci/m ²
K ₀	Coefficient of the exponential term of the expression used to define the relationship between the resuspended activity in the air and the ground activity concentration—m ⁻¹
K ₁	Constant term of the expression used to define the relationship between the resuspended activity in the air and the ground activity concentration—m ⁻¹

<u>Symbol</u>	<u>Definition</u>
L	Subscripted term used in the general equation which defines the ground activity concentrations during the time interval that releases are occurring—defined by Equation (51)
M	Subscripted term used in the general equation which defines the ground activity concentration during the time interval that releases are occurring—defined by Equation (52)
N	Subscripted term used in the general equation which defines the ground activity concentration during the time interval that releases are occurring—defined by Equation (53)
P	Population in the region affected by the releases of any given time
P_0	Population in the region affected by the releases at the time the releases start
Q	Constant rate of continuous release of radioactive material—Ci/hr
q_f	Mass rate at which the f-th food type is being consumed by the people in the region affected by the releases—gm/hr
R	Subscripted term used in the general equations which define the air and ground activity concentrations after releases have stopped—defined by Equation (41)
S	Subscripted term used in the general equations which define the air and ground activity concentrations after releases have stopped—defined by Equation (42)
T_{fa}	Beginning of the time interval during which radioactive nuclides are depositing upon the f-th food measured from the time at which releases start—hr
T_{fb}	End of the time interval during which radioactive nuclides are depositing upon the f-th food measured from the time at which releases start—hr
T_I	Beginning of the time interval over which the dose commitments are to be determined measured from the time at which the release of active material starts—hr
T_L	End of the time interval over which the dose commitments are to be determined measured from the time at which the release of active material starts—hr
T_S	Time that the release of active material stops measured from the time that the releases begin—hr
t	Time measured from the start of the release of active material—hr
t_d	Time measured from the moment that deposition of radioactive material occurs—hr
U	Subscripted term used in the general equation which defines the ground activity concentration after the release of active material has stopped—defined by Equation (54)
V_a	Volume of the atmosphere of the region affected by the releases— m^3
v	Deposition velocity defining all non-precipitation deposition mechanisms—m/hr
W_f	Amount of activity taken up per unit mass of the f-th food due to the deposition of a unit amount of activity per unit area— m^2/gm

<u>Symbol</u>	<u>Definition</u>
W_i^*	Amount of activity taken up per unit mass of the i-th food due to a unit activity concentration in the soil per unit thickness of the root bearing layer— m^2/gm
Y	Subscripted term in the general equation which defines the ground activity concentration after the release of active material has stopped—defined by Equation (55)
Z	Height of the atmosphere of the region affected by the releases measured from the surface of the ground— m
z	Vertical distance measured from the ground surface— m
α	Loss coefficient term representing all (sum of) the ground activity concentration removal mechanisms which are dependent upon the ground activity concentrations— hr^{-1}
β	Resuspension factor relating the amount of activity which is resuspended in the air to the activity concentration on the surface of the ground— m^{-1}
γ	Loss coefficient term representing all (sum of) the deposition mechanisms from the activity concentration in the air— m/hr
ϵ	Subscripted term used in the general equations which define the air and ground activity concentrations during the time period that releases are occurring—defined by Equation (32)
λ	Radioactive decay constant— hr^{-1}
λ_a	Loss rate constant of the exponential term of the expression used to define the relationship between the resuspended activity in the air and the ground surface activity concentration— hr^{-1}
λ_p	Population growth rate constant— hr^{-1}
λ_r	Loss rate constant due to resuspension— hr^{-1}
λ_s	Precipitation scavenging rate constant— hr^{-1}
λ_w	Loss rate constant defining the effect of weathering, migration, and erosion— hr^{-1}
$\bar{\lambda}$	Loss rate constant term representing all (sum of) the removal mechanisms from the activity concentrations in the air— hr^{-1}
τ	Time measured from the moment that releases stop— hr
ϕ	Rate at which activity is depositing on a unit surface area— $Ci/hr\ m^2$
χ	Activity concentration in the air during the time interval that activity releases are occurring— Ci/m^3

SymbolDefinition

χ^*	Activity concentration in the air after the activity releases have stopped—Ci/m ³
ψ	Rate at which people in the region affected by the releases are ingesting activity via contaminated food—Ci/hr

Subscripts

f	Refers to the specific food type being consumed
g	Refers to the organ which is subjected to the radiation doses
i	Refers to the released radionuclides
j	Refers to a specific daughter in a summation term over all daughters
ℓ	Refers to the daughters of a released radionuclide i
m	Refers to a specific term in a summation
n	Refers to a specific term in a summation

This report describes a model developed to estimate the radiation dose commitments received by the population of the world beyond the immediate vicinity of a facility which releases radionuclides into the atmosphere. This model considers the dose commitments arising from immersion in the atmosphere containing the released material, ingestion of contaminated food, inhalation of gaseous and suspended radioactivity and exposure to ground deposits. The dose commitments from each of these pathways is explicitly considered for each radioactive nuclide released into the atmosphere and for each daughter of each released nuclide. This is accomplished via the calculation of the specific air and ground concentrations in the world regions directly affected by each released nuclide and its daughter using the release rate of only the parent. This is considered to be a significant improvement over other models in which the effect of daughter radionuclides must be approximated by separate releases.

A MODEL TO ESTIMATE RADIATION DOSE COMMITMENTS TO THE WORLD POPULATION FROM THE ATMOSPHERIC RELEASE OF RADIONUCLIDES (LWBR Development Program)

J. L. Rider and S. K. Beal

I. INTRODUCTION

The Light Water Breeder Reactor (LWBR) program was undertaken to develop the technology necessary to use a light water breeder reactor based on the thorium-uranium fuel cycle as a means to provide an alternate energy source and to improve the utilization of the available nuclear fuel resources. To fully assess the benefits and the impacts which may result from the use of LWBRs, an environmental impact statement (Reference 1) was prepared for an LWBR and its supporting fuel cycle industry ranging from the mining and milling facilities through the reprocessing and refabrication facilities.

As part of the overall effort to prepare the LWBR environmental statement it was necessary to estimate the radiation dose commitments accrued by man from the release of radionuclides from the various facilities which comprise the hypothetical LWBR industry. Estimates of these dose commitments were made for the maximum individual, the population in the vicinity of the radionuclide releases, and the world population. These estimates were made for liquid and airborne releases of radioactive material from each facility of an LWBR industry. This report presents the model developed for use during the preparation of the LWBR environmental statement to estimate the radiation dose commitments to the population of the world beyond the immediate vicinity of the point of release of radioactive material to the atmosphere. An earlier report (Reference 2) has been issued which described the model used in the LWBR program to define the dose commitments to the maximum individual and to the local population in the vicinity of the atmospheric release of radioactive material. The primary purpose of this report is to provide the details of the derivation of the model used and thus permit others who are involved in similar analyses to judge the feasibility and validity of using this model in their specific analyses.

II. GENERAL DISCUSSION

The initial behavior of material released into the atmosphere is in general dictated by the meteorological conditions in the immediate vicinity of the release point. As the released material moves away from the release point it is dispersed vertically and laterally. As the distance from the release point increases and correspondingly the time since the release occurred increases, the released material is affected less by the local meteorological conditions and more by the general weather patterns which exist in the region of the world which contains the release point. As the time subsequent to release increases further, the released material comes under the influence of the world weather patterns. Under the world meteorological patterns, the released material continues to disperse as it is transported around the world until it has encircled the earth and returned to its release point in about 30 days. During this period the released material has

spread over approximately a 40 to 50° latitude band. As the released material continues to disperse it becomes essentially uniformly distributed throughout the world's atmosphere in about two years.* In addition to dispersion throughout the atmosphere, the released radioactive material is subjected to radioactive decay and, if released in particulate form, to the scavenging effects of rain, snow, and sleet, and to dry ground deposition mechanisms. Thus if material is released into the atmosphere in the United States, its immediate behavior is defined by the local weather conditions in the vicinity of the release point. However, as the released material moves beyond 80 to 100 kilometers from the release point it comes under the general influence of the North American weather patterns and is dispersed over the North American continent. As the released material is further dispersed and moves to points 2000 to 3000 kilometers away from the release point its behavior is determined by the northern hemisphere's meteorological conditions as it encircles the globe in the next 30 days or so. Subsequent to this period the released material is affected at an increasing rate by world meteorological conditions and is gradually dispersed throughout the world's atmosphere until, at the end of about two years, the material remaining in the atmosphere is uniformly dispersed in the world's atmosphere.

The preceding discussion is based primarily upon the behavior of a relatively long-lived gaseous nuclide. If the released material is in particulate form, it will be affected by both wet (rain, etc.) and dry (soil absorption, etc.) deposition processes in addition to the radioactive decay and dispersion mechanisms which affect material released in a gaseous form. The effect of dry deposition processes upon particulates released into the atmosphere is strongly dependent upon the stability characteristics of the wind, the height of the release point, and the dry deposition velocity during the first few hours subsequent to the release. During this period while the released particulates are subject to the local meteorological conditions, the concentration of particulates in the atmosphere at 80 km (~50 miles) from the release point can range from being depleted by as much as 95 percent for ground level releases of particulates with high deposition velocities (1 cm/sec) in highly unstable meteorological conditions to essentially no depletion for 100 m high stack releases into extremely stable weather conditions of particulates with low deposition velocities (0.1 cm/sec).** However, as the released particulates leave the region dominated by the local meteorological conditions and are subjected to the influence of the weather patterns associated with a larger segment of the world, any effects associated with stable wind patterns and release heights become small and the particulates in the atmosphere are depleted by dry deposition mechanisms. The effect of wet deposition processes upon particulates released into the atmosphere is generally not as effective in depleting the particulate concentration in the atmosphere in the vicinity of the release point as the dry deposition mechanisms (Reference 4). However, as the released material moves away from the release point and is subjected to precipitation deposition mechanisms over a broader area and a longer period of time, the depletion of the particulates in the atmosphere is enhanced. After a period of approximately six days, the particulate concentration in the atmosphere may be reduced by 30 percent due solely to wet deposition processes associated with an average annual rainfall of 30 to 40 inches.† Thus, the combination of wet and dry deposition processes will result in the relatively rapid depletion (i.e., within a few days) of particulate material released into the atmosphere from points within 100 m of the surface. For example, particulates released into the atmosphere on the west coast of the United States would deposit within the North American continent and particulates released into the atmosphere in the eastern portion of the United States would not reach the European continent. Thus, during the period that particulates are being released into the atmosphere, radioactive material will be in the air and on the ground in a region encompassing approximately six days travel time. Subsequent to particulate releases, this radioactive material will remain on the ground in this region and a reduced amount will be dispersed in the air in this region due to the resuspension of the material which had been deposited earlier.

As a result of this behavior of material released into the atmosphere, the people who live in the regions affected by the releases (i.e., extending from those who live within a few thousand kilometers of the release point to the entire world population depending upon the actual material released) will be exposed to the

*This general behavior of material released to the atmosphere is based upon that postulated for gaseous material in Reference 3.

**These depletion values are based upon material presented in Reference 4 and upon calculations performed by the authors using the local radiation dose model defined in Reference 2.

†This wet deposition depletion value was obtained from an unpublished report by L. Machta, G. J. Ferber, and J. F. Heffter of the Air Resources Laboratories of the National Oceanic and Atmospheric Administration in June 1973, entitled "Local and World Wide Pollutant Concentrations and Population Exposures from a Continuous Point Source."

effect of the released radioactivity. This exposure will arise from being immersed in the air containing radioactive material (including the material which was resuspended subsequent to earlier deposition), breathing this contaminated air, eating food such as fruits and leafy vegetables on which active nuclides were deposited, drinking the milk or eating the meat of animals pastured on grass subjected to the deposition of radioactive material and exposure to surfaces on which radioactive material has been deposited. The model described in this report was developed to estimate the radiation exposure which would be accrued by people who live beyond the immediate vicinity (i.e., 80 to 100 km) of radioactive releases into the lower regions of the atmosphere (i.e., within approximately 100 m of the surface).

The model uses the concept of a dose commitment to define the effect of radiation upon man. A dose commitment is defined to be the total dose received during a specific period of time as the direct result of exposure to radioactive material. This exposure may occur via any of many pathways such as immersion in air containing radioactive material, and ingestion of food upon which radioactive material had deposited. Further, the effect of an exposure may last well beyond the duration of the exposure. For example, essentially the entire dose accrued by eating food which contains radioactive material is caused by the decay of the ingested material and its daughters at times subsequent to the actual act of ingestion. Thus, a 70-year dose commitment is the total dose that would be acquired by an individual or by a group of people in a 70-year period as the direct result of exposure to radioactive material. The dose commitments from the various pathways are additive. Thus, the total dose commitment is the sum of the dose commitments from all of the pathways by which man may be exposed to radiation.

The model examines the dose commitments which will be accrued from each radionuclide released and from all daughters of the released nuclide which are formed subsequent to the release. This is accomplished in the model through the solution of differential equations which define the air and ground concentrations of the released nuclide and their daughters as a function of time. The daughters of each released nuclide are subjected to the same processes (i.e., decay, dispersion, dry deposition, etc.) as the released nuclide. However, the source term for each daughter is the decay of its precursor in the decay chain while the source term for the released nuclide is the rate at which the nuclide is being released.

This model examines the effect of the released radionuclides and their daughters upon people who are removed from the vicinity of the releases (i.e., beyond 80 kilometers). Thus, local meteorological and standard diffusion deviation data cannot be used to explicitly define the dispersion of the released material and its daughters in the overall region affected by the releases. In this model each radionuclide and its daughters are assumed to be dispersed uniformly throughout the volume of the atmosphere associated with the region which is affected by the release. This dispersion is assumed to occur instantaneously upon the release of the given radionuclide and the formation of a daughter nuclide. The affected region is considered to be dependent upon the nuclide released. If the released nuclide is a particulate, the size of the affected region is also dependent upon the height at which the releases occur, the meteorological stability conditions and the weather patterns within approximately six days travel time from the release point. Thus, if a long-lived gaseous nuclide such as krypton-85 with a 10.7-year half-life is released, the affected region would be the entire world; if a shorter-lived gaseous nuclide such as xenon-133 with a 5.3-day half-life is released, the affected region would be the hemisphere in which the release occurred; and if a relatively short-lived gaseous nuclide such as xenon-135 with a 9.2-hour half-life is released, the affected region would be within a few thousand kilometers. Conversely, the maximum size of a region affected by the release of a particulate would not extend beyond the first traverse of a major body of water such as an ocean. In general, the region affected by the release of particulates would be restricted to a few thousand kilometers (essentially the smallest region affected by a gaseous nuclide). In cases where particulates are released from ground level sources in unstable meteorological conditions, the region affected by the release would not extend beyond a few hundred kilometers from the release point. Thus, the use of this model to adequately estimate the dose commitments received by people located beyond the immediate vicinity of a release requires that each individual released nuclide and its daughters be explicitly examined to define the region affected by the release.

Using this uniform dispersion in the region affected by the release, the model depicts the time dependent behavior of the air and ground concentrations of the released nuclides and their daughters. These concentrations are defined throughout the period over which the dose commitments are to be obtained covering both the period while releases are occurring and the period subsequent to their creation. The model considers the effect of radioactive decay, dry deposition mechanisms and precipitation in defining the time dependent behavior of these concentrations. It further permits the effect of resuspension and weathering to be examined.

Other features of the model are the capability to obtain dose commitments for various organs such as the bone, lung, GI tract, etc., in addition to the total body dose and the ability to permit the population in a region to vary exponentially with time. The model also includes the ability to vary the time at which the initial exposure occurs relative to the start of the release and the time intervals over which the exposure to the radiation environment continues.

III. MODEL DESCRIPTION

This section of the report describes the model equations and their derivation. This information is presented in four major segments. The first of these segments defines the dose commitments which are received by the people located in the region affected by a release due to the presence of a radioactive nuclide in the environment of that region. The second and third of these segments define the air and ground concentrations of the released nuclides and their daughters in the regions affected by the releases both during the time that the releases are occurring and after the releases have stopped. The fourth major segment combines the first three to define the detailed equations used in the model for the pathway dose commitments as a function of time for the people who live in the world beyond the 80-kilometer region immediately affected by releases.

A. Dose Commitment Pathways

1. Immersion

The dose commitment D_{mgi} that people in a given region affected by a release can receive to the g -th organ over a time interval from T_I to T_L due to immersion in air containing the radionuclide i is given by the integral over that time interval of the product of the conversion factor K_{mgi} relating the dose rate due to immersion in a cloud containing the nuclide i to the concentration of the nuclide i in the cloud, the number of people $P(t)$ in the region affected by the release, and the concentration $A_i(t)$ of the nuclide i in the air.

$$\therefore D_{mgi} = \int_{T_I}^{T_L} K_{mgi} P(t) A_i(t) dt \quad (1)$$

The concentration of the nuclide i in the atmosphere of the region affected by the release consists of two components. The first component $\chi_i(t)$ is that material which is dispersed throughout the region immediately upon its release or its formation as a daughter. The second component is the concentration of the nuclide i in the atmosphere of the affected regions from material which was deposited previously and has been resuspended. This latter component is given by the product of the ground concentration $C_i(t)$ of the i -th nuclide in the region affected by the release and the factor $\beta_i(t_{di})$ relating the amount of the i -th nuclide which is resuspended into the air to the amount of the i -th nuclide which is on the ground. This factor is a function of the time elapsed subsequent to the deposition of the i -th nuclide.

$$\therefore A_i(t) = \chi_i(t) + \beta_i(t_{di}) C_i(t) \quad (2)$$

Substituting the expression given in Equation (2) into Equation (1), the dose commitment that the people located in the region affected by the release can receive over a time interval from T_I to T_L due to immersion in air containing the radionuclide i is

$$D_{mgi} = \int_{T_I}^{T_L} K_{mgi} P(t) [\chi_i(t) + \beta_i(t_{di}) C_i(t)] dt \quad (3)$$

2. Surface Deposits

The dose commitment $D_{s_{gi}}$ that the people in the region affected by the releases can receive to the g-th organ over a time interval from T_I to T_L from a surface deposit of a radionuclide i is given by the integral over that time interval of the product of the conversion factor $K_{s_{gi}}$ relating the dose rate due to standing on a surface which has a deposit of the nuclide i, the number of people in the region affected by the release, and the concentration of the nuclide i on the ground in the affected region.

$$\therefore D_{s_{gi}} = \int_{T_I}^{T_L} K_{s_{gi}} P(t) C_i(t) dt \quad (4)$$

3. Ingestion

There are two basic pathways associated with the ingestion of radionuclides which have been released to the atmosphere or have formed via radioactive decay processes subsequent to the release of this precursor. The first of these pathways is the eating of food which becomes contaminated during the period that deposition of the released material or its daughters is occurring. The dose commitments associated with this pathway are dependent upon the total rate at which material is depositing on the ground. The second ingestion pathway is the eating of food which has become contaminated from the material which has accumulated in the soil from earlier deposition. The dose commitments associated with this pathway are dependent upon the concentration of the radioactive material on the ground.

The dose commitment $D_{I_{gi}}$ that people in the region affected by the releases can receive to the g-th organ over a time interval from T_I to T_L from the ingestion of food containing quantities of the radioactive nuclide i is given by the integral over the time interval of the product of the conversion factor $K_{I_{gi}}$ relating the dose accrued over that time interval due to the ingestion of a unit amount of activity of the nuclide i, the number of people in the region affected by the releases, and the rate $\psi_i(t)$ at which people in the affected region are ingesting activity via food which contains the radionuclide i.

$$\therefore D_{I_{gi}} = \int_{T_I}^{T_L} K_{I_{gi}} P(t) \psi_i(t) dt \quad (5)$$

The rate at which people in the region affected by the releases are ingesting activity via the specific food f which is contaminated by the nuclide i is given by the product of the mass rate $q_f(t)$ by which the f-th food is being consumed by the people in the affected region and the activity $F_{fi}(t)$ contained in a unit mass of the food f at time of consumption. The total rate at which people in the region affected by the releases are ingesting activity of the radionuclide i is given by the sum over all foods consumed of the activity ingesting rates associated with the f-th food.

$$\therefore \psi_i(t) = \sum_f q_f(t) F_{fi}(t) \quad (6)$$

Substituting Equation (6) into Equation (5) gives

$$D_{I_{gi}} = \int_{T_I}^{T_L} K_{I_{gi}} P(t) \sum_f q_f(t) F_{fi}(t) dt \quad (7)$$

a. *Ingestion During Deposition*—During the period of time that deposition of the released nuclides and their daughters is occurring, the total activity of the nuclide i contained in a unit mass of the food f at the time of ingestion is given by the integral of the rate at which activity of the radionuclide i is being taken up by the specific food f . The rate at which activity is being taken up by the f -th food is given by the product of the amount W_{fi} of activity of the i -th nuclide taken up per unit mass of the f -th food for a unit activity deposition of the i -th nuclide per unit area and the rate $\phi_i(t)$ at which the nuclide i is depositing per unit area. The integral is carried out over the time that the nuclide i is depositing upon the food f .

$$\therefore F_{fi}(t) = \int_{T_{fa}(t)}^{T_{fb}(t)} W_{fi} \phi_i(t) dt \quad (8)$$

The limits in the preceding integral are denoted as functions of food type and time; since, in general, the time at which deposition on a given food crop can begin or end may vary. This is apparent when considering long-term continuous releases and a food crop which is planted and harvested in successive stages to extend the availability of the crop.

The rate at which the i -th nuclide is depositing per unit area in the region affected by the releases is given by the sum of the rate at which the i -th nuclide is depositing on the ground by dry deposition processes and the rate at which it is being scavenged from the atmosphere by rain, snow, and sleet.

$$\therefore \phi_i(t) = v_i \chi_i(t) + \int_0^Z \lambda_{si} \chi_i(t) dz \quad (9)$$

where

v_i = the dry deposition velocity, and
 λ_{si} = the precipitation scavenging coefficient.

Combining Equations (8) and (9) and substituting into Equation (7) yields the following expression for the dose commitment that the people in the region affected by the release receive over a time interval from T_I to T_L from ingesting food contaminated by the radionuclide i during the period that deposition is occurring.

$$D_{igi} = \int_{T_I}^{T_L} K_{igi} P(t) \left\{ \sum_f q_f(t) \int_{T_{fa}(t)}^{T_{fb}(t)} W_{fi} \left[v_i \chi_i(t) + \int_0^Z \lambda_{si} \chi_i(t) dz \right] dt \right\} dt \quad (10)$$

b. *Ingestion Subsequent to Deposition*—The total activity of the radionuclide i contained in a unit mass of the food f at the time of ingestion due to material which had deposited previously is given by the product of the amount W_{fi}^* of the i -th nuclide taken up per unit mass of the f -th food for a unit concentration of the i -th nuclide in the soil per unit thickness of the root bearing layer and the ground activity concentration of the i -th nuclide in the region affected by the releases.

$$\therefore F_{fi}(t) = W_{fi}^* C_i(t) \quad (11)$$

Substituting the above expression into Equation (7), the following equation for the dose commitment that the people in the region affected by the release receive over a time interval from T_I to T_L from ingesting food contaminated by the radionuclide i subsequent to its deposition on the ground is obtained.

$$D_{igi} = \int_{T_I}^{T_L} K_{igi} P(t) \sum_f q_f(t) W_{fi}^* C_i(t) dt \quad (12)$$

4. Inhalation

The dose commitment D_{agi} that people in the region affected by the releases can receive to the g -th organ over a time interval from T_I to T_L from the inhalation of the nuclide i is given by the integral over that time interval of the product of the conversion factor K_{agi} relating the dose accrued over that time interval to the inhalation of a unit amount of activity of the nuclide i , the rate B_a at which air is inhaled, the number of people in the affected region and the concentration of the nuclide i in the air in the affected region.

$$\therefore D_{agi} = \int_{T_I}^{T_L} K_{agi} B_a P(t) A_i(t) dt \quad (13)$$

Substituting the expressions given in Equation (2) for $A_i(t)$ into Equation (13), the dose commitment that people in the affected region can receive over a time interval from T_I to T_L due to the inhalation of the radionuclide i is given by

$$D_{agi} = \int_{T_I}^{T_L} K_{agi} B_a P(t) [\chi_i(t) + B_i(t_{di})C_i(t)] dt \quad (14)$$

B. Air Concentrations

The concentration of a nuclide in the air in the region affected by the releases is found by applying the principle of the conservation of mass to the nuclide in the atmosphere of the affected region. Thus, the total quantity of activity (i.e., product of the decay constant and the atom concentration) of a given nuclide in the air of the affected region at the time $t + \Delta t$ is equal to the amount of activity of the given nuclide in that portion of the atmosphere associated with the affected region at the time t plus the amount of the nuclide activity which appears in the affected region during the time interval Δt less the amount of activity of the given nuclide which leaves the affected region in the Δt time interval. During the period of time that releases are occurring, activity of a given radionuclide may be introduced into the region affected by the release either by the release of the given nuclide from one or more facilities or by its production via the decay of its precursor which is present in the air of the affected region at the time t . Subsequent to releases stopping, activity of a given nuclide may be introduced into the affected region only via the decay of its precursor. Activity of a nuclide may be removed from the atmospheric region affected by the releases during the Δt time interval by the radioactive decay of the nuclide, by the precipitation scavenging effects of rain, snow, and sleet, and by the processes of dry deposition on the ground surfaces of the affected region. As noted previously, the material released into the region will be assumed to be uniformly dispersed throughout the affected region immediately upon release.

1. During Releases

The amount of the activity of a released radionuclide and of each of its daughters in the atmosphere of the region affected by the release at the time $t + \Delta t$ subsequent to the start of the release during the period that releases are occurring is given by

$$\begin{aligned} V_a X_{i0} \Big|_{t+\Delta t} &= V_a X_{i0} \Big|_t + Q_i \Delta t - V_a \lambda_{i0} X_{i0} \Big|_{t+\Delta/2} \Delta t \\ &\quad - V_a \lambda_{si0} X_{i0} \Big|_{t+\Delta/2} \Delta t - v_{i0} A_a X_{i0} \Big|_{t+\Delta/2} \Delta t \end{aligned}$$

$$\begin{aligned}
V_a X_{i1} \Big|_{t+\Delta t} &= V_a X_{i1} \Big|_t + V_a \lambda_{i0} X_{i0} \Big|_{t+\Delta/2} \Delta t - V_a \lambda_{i1} X_{i1} \Big|_{t+\Delta/2} \Delta t \\
&\quad - V_a \lambda_{s11} X_{i1} \Big|_{t+\Delta/2} \Delta t - v_{i1} A_a X_{i1} \Big|_{t+\Delta/2} \Delta t \\
V_a X_{i2} \Big|_{t+\Delta t} &= V_a X_{i2} \Big|_t + V_a \lambda_{i2} X_{i1} \Big|_{t+\Delta/2} \Delta t - V_a \lambda_{i2} X_{i2} \Big|_{t+\Delta/2} \Delta t \\
&\quad - V_a \lambda_{s12} X_{i2} \Big|_{t+\Delta/2} \Delta t - v_{i2} A_a X_{i2} \Big|_{t+\Delta/2} \Delta t \\
&\quad \vdots \\
V_a X_{i\ell} \Big|_{t+\Delta t} &= V_a X_{i\ell} \Big|_t + V_a \lambda_{i\ell} X_{i\ell-1} \Big|_{t+\Delta/2} \Delta t - V_a \lambda_{i\ell} X_{i\ell} \Big|_{t+\Delta/2} \Delta t \\
&\quad - V_a \lambda_{s1\ell} X_{i\ell} \Big|_{t+\Delta/2} \Delta t - v_{i\ell} A_a X_{i\ell} \Big|_{t+\Delta/2} \Delta t ,
\end{aligned} \tag{15}$$

where the subscript ℓ refers to the daughter of the released nuclide. Thus $\ell = 0$ refers to the released nuclide itself, $\ell = 1$ represents the first daughter of the i -th released nuclide, $\ell = 2$ refers to the second daughter of the released nuclide i , etc. V_a is the volume of the atmosphere of the affected region; A_a is the surface area of the affected region; Q_i is the rate at which the nuclide i is being released; and $\lambda_{i\ell}$ is the decay constant of the ℓ th daughter of the nuclide i .

Dividing through by $V_a \Delta t$, letting Δt approach zero and rearranging yields the following differential equations which define the air concentrations of the released nuclide and each of its daughters in the region affected by the release as a function of time subsequent to the beginning of the release.

$$\begin{aligned}
\frac{d X_{i0}(t)}{dt} + \left(\lambda_{i0} + \lambda_{s10} + \frac{v_{i0} A_a}{V_a} \right) X_{i0}(t) &= \frac{Q_i}{V_a} \\
\frac{d X_{i1}(t)}{dt} + \left(\lambda_{i1} + \lambda_{s11} + \frac{v_{i1} A_a}{V_a} \right) X_{i1}(t) &= \lambda_{i1} X_{i0}(t) \\
\frac{d X_{i2}(t)}{dt} + \left(\lambda_{i2} + \lambda_{s12} + \frac{v_{i2} A_a}{V_a} \right) X_{i2}(t) &= \lambda_{i2} X_{i1}(t) \\
&\quad \vdots \\
\frac{d X_{i\ell}(t)}{dt} + \left(\lambda_{i\ell} + \lambda_{s1\ell} + \frac{v_{i\ell} A_a}{V_a} \right) X_{i\ell}(t) &= \lambda_{i\ell} X_{i\ell-1}(t) .
\end{aligned} \tag{16}$$

At the beginning of the release period the concentration of the released nuclides and of each of its daughters is zero. Thus, the boundary conditions for the preceding set of differential equations is given by

$$\begin{aligned}
 \chi_{i0}(0) &= 0 \\
 \chi_{i1}(0) &= 0 \\
 \chi_{i2}(0) &= 0 \\
 &\vdots \\
 \chi_{i\ell}(0) &= 0
 \end{aligned} \tag{17}$$

Letting

$$\bar{\lambda}_{i\ell} = \lambda_{i\ell} + \lambda_{si\ell} + v_{i\ell} A_a / V_a \tag{18}$$

and solving the set of differential equations defined by Equation (16) using the boundary conditions specified by Equation (17), yields the following set of equations which define the air concentrations of the released nuclide and of each of its daughters in the region affected by the release during the period of time that the release is occurring as a function of time subsequent to the start of the release.

$$\begin{aligned}
 \chi_{i0}(t) &= \frac{Q_i}{\bar{\lambda}_{i0} V_a} \left[1 - e^{-\bar{\lambda}_{i0} t} \right] \\
 \chi_{i1}(t) &= \frac{\lambda_{i1} Q_i}{V_a} \left[\frac{1}{\bar{\lambda}_{i0} \bar{\lambda}_{i1}} - \frac{e^{-\bar{\lambda}_{i0} t}}{\bar{\lambda}_{i0} (\bar{\lambda}_{i1} - \bar{\lambda}_{i0})} + \frac{e^{-\bar{\lambda}_{i1} t}}{(\bar{\lambda}_{i1} - \bar{\lambda}_{i0}) \bar{\lambda}_{i1}} \right] \\
 \chi_{i2}(t) &= \frac{\lambda_{i1} \lambda_{i2} Q_i}{V_a} \left[\frac{1}{\bar{\lambda}_{i0} \bar{\lambda}_{i1} \bar{\lambda}_{i2}} - \frac{e^{-\bar{\lambda}_{i0} t}}{\bar{\lambda}_{i0} (\bar{\lambda}_{i1} - \bar{\lambda}_{i0}) (\bar{\lambda}_{i2} - \bar{\lambda}_{i0})} \right. \\
 &\quad \left. + \frac{e^{-\bar{\lambda}_{i1} t}}{(\bar{\lambda}_{i1} - \bar{\lambda}_{i0}) \bar{\lambda}_{i1} (\bar{\lambda}_{i2} - \bar{\lambda}_{i1})} - \frac{e^{-\bar{\lambda}_{i2} t}}{(\bar{\lambda}_{i2} - \bar{\lambda}_{i0}) (\bar{\lambda}_{i2} - \bar{\lambda}_{i1}) \bar{\lambda}_{i2}} \right] \\
 &\vdots \\
 \chi_{i\ell}(t) &= \frac{\lambda_{i1} \lambda_{i2} \dots \lambda_{i\ell} Q_i}{V_a} \left[\frac{1}{\bar{\lambda}_{i0} \bar{\lambda}_{i1} \dots \bar{\lambda}_{i\ell}} \right. \\
 &\quad \left. - \sum_{j=0}^{j=\ell} \frac{(-1)^j e^{-\bar{\lambda}_{ij} t}}{(\bar{\lambda}_{ij} - \bar{\lambda}_{i0}) (\bar{\lambda}_{ij} - \bar{\lambda}_{i1}) \dots (\bar{\lambda}_{ij} - \bar{\lambda}_{i(j-1)}) \bar{\lambda}_{ij} (\bar{\lambda}_{ij+1} - \bar{\lambda}_{ij}) (\bar{\lambda}_{ij+2} - \bar{\lambda}_{ij}) \dots (\bar{\lambda}_{i\ell} - \bar{\lambda}_{ij})} \right] \tag{19}
 \end{aligned}$$

2. Subsequent to Releases

The concentration $\chi_{i\ell}^*$ of the activity of a released radionuclide and of each of its daughters in the atmosphere of the region affected by the release at the time $\tau + \Delta\tau$ during the period after the release has stopped (i.e., $t > T_S$) is given by

$$\begin{aligned}
 \chi_{i0}^*|_{\tau+\Delta\tau} &= \chi_{i0}^*|_{\tau} - \lambda_{i0}\chi_{i0}^*|_{\tau+\Delta\tau/2}\Delta\tau - \lambda_{si0}\chi_{i0}^*|_{\tau+\Delta\tau/2}\Delta\tau - \frac{v_{i0}A_a}{V_a}\chi_{i0}^*|_{\tau+\Delta\tau/2}\Delta\tau \\
 \chi_{i1}^*|_{\tau+\Delta\tau} &= \chi_{i1}^*|_{\tau} + \lambda_{i1}\chi_{i0}^*|_{\tau+\Delta\tau/2}\Delta\tau - \lambda_{i1}\chi_{i1}^*|_{\tau+\Delta\tau/2}\Delta\tau - \lambda_{si1}\chi_{i1}^*|_{\tau+\Delta\tau/2}\Delta\tau \\
 &\quad - \frac{v_{i1}A_a}{V_a}\chi_{i1}^*|_{\tau+\Delta\tau/2}\Delta\tau \\
 \chi_{i2}^*|_{\tau+\Delta\tau} &= \chi_{i2}^*|_{\tau} + \lambda_{i2}\chi_{i1}^*|_{\tau+\Delta\tau/2}\Delta\tau - \lambda_{i2}\chi_{i2}^*|_{\tau+\Delta\tau/2}\Delta\tau - \lambda_{si2}\chi_{i2}^*|_{\tau+\Delta\tau/2}\Delta\tau \\
 &\quad - \frac{v_{i2}A_a}{V_a}\chi_{i2}^*|_{\tau+\Delta\tau/2}\Delta\tau \\
 &\vdots \\
 \chi_{i\ell}^*|_{\tau+\Delta\tau} &= \chi_{i\ell}^*|_{\tau} + \lambda_{i\ell}\chi_{i\ell-1}^*|_{\tau+\Delta\tau/2}\Delta\tau - \lambda_{i\ell}\chi_{i\ell}^*|_{\tau+\Delta\tau/2}\Delta\tau - \lambda_{si\ell}\chi_{i\ell}^*|_{\tau+\Delta\tau/2}\Delta\tau \\
 &\quad - \frac{v_{i\ell}A_a}{V_a}\chi_{i\ell}^*|_{\tau+\Delta\tau/2}\Delta\tau
 \end{aligned} \tag{20}$$

where

$$\tau = t - T_S \tag{21}$$

Dividing the preceding set of equations by $\Delta\tau$, letting $\Delta\tau$ approach zero and rearranging yields the following set of differential equations which define the air concentration of the released nuclide and each of its daughters in the region affected by the release as a function of time after the release has stopped.

$$\begin{aligned}
 \frac{d\chi_{i0}^*(\tau)}{d\tau} + \left(\lambda_{i0} + \lambda_{si0} + \frac{v_{i0}A_a}{V_a} \right) \chi_{i0}^*(\tau) &= 0 \\
 \frac{d\chi_{i1}^*(\tau)}{d\tau} + \left(\lambda_{i1} + \lambda_{si1} + \frac{v_{i1}A_a}{V_a} \right) \chi_{i1}^*(\tau) &= \lambda_{i1}\chi_{i0}^*(\tau) \\
 \frac{d\chi_{i2}^*(\tau)}{d\tau} + \left(\lambda_{i2} + \lambda_{si2} + \frac{v_{i2}A_a}{V_a} \right) \chi_{i2}^*(\tau) &= \lambda_{i2}\chi_{i1}^*(\tau) \\
 &\vdots \\
 \frac{d\chi_{i\ell}^*(\tau)}{d\tau} + \left(\lambda_{i\ell} + \lambda_{si\ell} + \frac{v_{i\ell}A_a}{V_a} \right) \chi_{i\ell}^*(\tau) &= \lambda_{i\ell}\chi_{i\ell-1}^*(\tau)
 \end{aligned} \tag{22}$$

At the time that the releases stop (i.e., $\tau = 0$), the air concentrations of the released nuclide and of each of its daughters defined by the above set of equations will be the same as those defined during the time that releases are occurring at $t = T_S$. Thus, the boundary conditions for the preceding set of differential equations are given by

$$\begin{aligned} \chi_{i0}^*(0) &= \chi_{i0}(T_S) \\ \chi_{i1}^*(0) &= \chi_{i1}(T_S) \\ \chi_{i2}^*(0) &= \chi_{i2}(T_S) \\ &\vdots \\ \chi_{i\ell}^*(0) &= \chi_{i\ell}(T_S) \end{aligned} \quad (23)$$

The set of differential equations defined by Equation (22) is solved using the boundary conditions specified by Equation (23), the $\bar{\lambda}_{i\ell}$ term as defined by Equation (18), and the relationship between τ and $t - T_S$ as given in Equation (21). This results in the following set of equations which define the air concentrations of the released nuclide and each of its daughters in the region affected by the release during the period after releases have stopped as a function of time subsequent to the start of the releases.

$$\begin{aligned} \chi_{i0}^*(t) &= \chi_{i0}(T_S) e^{-\bar{\lambda}_{i0}(t-T_S)} \\ \chi_{i1}^*(t) &= \frac{\lambda_{i1} \chi_{i0}(T_S)}{(\bar{\lambda}_{i1} - \bar{\lambda}_{i0})} e^{-\bar{\lambda}_{i0}(t-T_S)} + \left(\chi_{i1}(T_S) - \frac{\lambda_{i1} \chi_{i0}(T_S)}{\bar{\lambda}_{i1} - \bar{\lambda}_{i0}} \right) e^{-\bar{\lambda}_{i1}(t-T_S)} \\ \chi_{i2}^*(t) &= \frac{\lambda_{i1} \lambda_{i2} \chi_{i0}(T_S)}{(\bar{\lambda}_{i1} - \bar{\lambda}_{i0})(\bar{\lambda}_{i2} - \bar{\lambda}_{i0})} e^{-\bar{\lambda}_{i0}(t-T_S)} + \frac{\lambda_{i2}}{(\bar{\lambda}_{i2} - \bar{\lambda}_{i1})} \left(\chi_{i1}(T_S) - \frac{\lambda_{i1} \chi_{i0}(T_S)}{(\bar{\lambda}_{i1} - \bar{\lambda}_{i0})} \right) e^{-\bar{\lambda}_{i1}(t-T_S)} \\ &\quad + \left(\chi_{i2}(T_S) - \frac{\lambda_{i2} \chi_{i1}(T_S)}{(\bar{\lambda}_{i2} - \bar{\lambda}_{i1})} + \frac{\lambda_{i1} \lambda_{i2} \chi_{i0}(T_S)}{(\bar{\lambda}_{i2} - \bar{\lambda}_{i1})(\bar{\lambda}_{i2} - \bar{\lambda}_{i0})} \right) e^{-\bar{\lambda}_{i2}(t-T_S)} \\ &\vdots \\ \chi_{i\ell}^*(t) &= \sum_{j=0}^{j=\ell} \frac{\lambda_{ij+1} \lambda_{ij+2} \dots \lambda_{i\ell} e^{-\bar{\lambda}_{ij}(t-T_S)}}{(\bar{\lambda}_{i\ell} - \bar{\lambda}_{ij})(\bar{\lambda}_{i\ell-1} - \bar{\lambda}_{ij}) \dots (\bar{\lambda}_{ij+1} - \bar{\lambda}_{ij})} \\ &\quad \left[\sum_{m=0}^{m=j} \frac{(-1)^{j+m} \lambda_{im+1} \lambda_{im+2} \dots \lambda_{ij} \chi_{im}(T_S)}{(\bar{\lambda}_{ij} - \bar{\lambda}_{ij-1})(\bar{\lambda}_{ij} - \bar{\lambda}_{ij-2}) \dots (\bar{\lambda}_{ij} - \bar{\lambda}_{im})} \right] \end{aligned} \quad (24)$$

C. Ground Concentrations

The concentration of a nuclide on the ground of the region affected by the releases is found by applying the principle of the conservation of mass to the ground surface of the affected region. Thus, the total quantity of activity of a given radionuclide on the ground of the affected region at the end of a given time interval is equal to the amount of activity of the given nuclide on the ground at the beginning of the time interval plus the amount of the given nuclide activity which appears on the ground surfaces of the affected region during the time interval less the amount of activity of the given nuclide which leaves the ground surfaces of the affected region during the time interval.

Activity may appear on the ground surfaces of the region affected by the release from the scavenging by precipitation (i.e., rain, snow, sleet, etc.) of the radioactive material in the atmosphere of the affected region and from the deposition on the ground of the active nuclides in the atmosphere of the affected region via the various dry deposition mechanisms. Activity of a given nuclide may also be produced on the ground surfaces of the region affected by the release from the decay of its precursor which had been deposited or produced on the ground earlier. Activity may be removed from the ground surfaces associated with the region affected by the release by radioactive decay, by the weathering of the material on the surface and by the resuspension of the material on the surface.

1. During Releases

The total amount of activity of the ℓ -th daughter of the released radionuclide i on the ground surface of the region affected by the release at the time $t + \Delta t$ during the period that the release is occurring is obtained by summing the amount of activity of the ℓ -th daughter which is present on the ground of the affected region at the beginning of the Δt time interval and the ℓ -th daughter of the released nuclide i which is produced by the decay of the $\ell - 1$ daughter during the Δt time interval. To this is added the activity of the ℓ -th daughter which is deposited on the ground surface of the region affected by the release of the radionuclide i due to precipitation scavenging and to dry deposition mechanisms during the Δt time interval. From this sum is subtracted the amount of activity of the ℓ -th daughter which decays during the Δt time interval, the amount of activity of the ℓ -th daughter which is removed from the ground surface during the time interval by erosion, migration and weathering and the amount of activity of the ℓ -th daughter on the ground which is resuspended into the air of the affected region during the Δt time interval.

Thus, the amount of activity of the released radionuclide i and of each of its daughters on the ground surfaces of the region affected by the release at the time $t + \Delta t$ subsequent to the start of the release during the period that releases are occurring is given by

$$\begin{aligned}
 A_a C_{i0} \Big|_{t+\Delta t} &= A_a C_{i0} \Big|_t + A_a \int_0^Z \lambda_{si0} X_{i0} \Big|_{t+\Delta/2} dz \Delta t + A_a v_{i0} X_{i0} \Big|_{t+\Delta/2} \Delta t - A_a \lambda_{i0} C_{i0} \Big|_{t+\Delta/2} \Delta t \\
 &\quad - A_a \lambda_{wi0} C_{i0} \Big|_{t+\Delta/2} \Delta t - A_a \lambda_{ri0} C_{i0} \Big|_{t+\Delta/2} \Delta t \\
 A_a C_{i\ell} \Big|_{t+\Delta t} &= A_a C_{i\ell} \Big|_t + A_a \lambda_{i\ell} C_{i\ell} \Big|_{t+\Delta/2} \Delta t + A_a v_{i\ell} X_{i\ell} \Big|_{t+\Delta/2} \Delta t + A_a \int_0^Z \lambda_{si\ell} X_{i\ell} \Big|_{t+\Delta/2} dz \Delta t \\
 &\quad - A_a \lambda_{i\ell} C_{i\ell} \Big|_{t+\Delta/2} \Delta t - A_a \lambda_{wi\ell} C_{i\ell} \Big|_{t+\Delta/2} \Delta t - A_a \lambda_{ri\ell} C_{i\ell} \Big|_{t+\Delta/2} \Delta t
 \end{aligned}$$

$$\begin{aligned}
A_a C_{i2} \Big|_{t+\Delta t} &= A_a C_{i2} \Big|_t + A_a \lambda_{i2} C_{i1} \Big|_{t+\Delta/2} \Delta t + A_a v_{i2} \chi_{i2} \Big|_{t+\Delta/2} \Delta t + A_a \int_0^Z \lambda_{si2} \chi_{i2} \Big|_{t+\Delta/2} dz \Delta t \\
&\quad - A_a \lambda_{i2} C_{i2} \Big|_{t+\Delta/2} \Delta t - A_a \lambda_{wi2} C_{i2} \Big|_{t+\Delta/2} \Delta t - A_a \lambda_{ri2} C_{i2} \Big|_{t+\Delta/2} \Delta t \\
&\quad \vdots \\
A_a C_{i\ell} \Big|_{t+\Delta/2} &= A_a C_{i\ell} \Big|_t + A_a \lambda_{i\ell} C_{i\ell-1} \Big|_{t+\Delta/2} \Delta t + A_a v_{i\ell} \chi_{i\ell} \Big|_{t+\Delta/2} \Delta t + A_a \int_0^Z \lambda_{si\ell} \chi_{i\ell} \Big|_{t+\Delta/2} dz \Delta t \\
&\quad - A_a \lambda_{i\ell} C_{i\ell} \Big|_{t+\Delta/2} \Delta t - A_a \lambda_{wi\ell} C_{i\ell} \Big|_{t+\Delta/2} \Delta t - A_a \lambda_{ri\ell} C_{i\ell} \Big|_{t+\Delta/2} \Delta t
\end{aligned} \tag{25}$$

where

λ_w = the loss coefficient defining the effect of weathering, migration, and erosion,

λ_r = the loss coefficient due to resuspension,

$\chi_{i\ell}(t)$ represents a uniformly distributed concentration of the ℓ -th daughter of the released radionuclide i in the atmosphere of the region affected by the release of the nuclide i and $\lambda_{si\ell}$ is a precipitation scavenging coefficient applicable to the entire region affected by the release of the nuclide i . Thus, neither of these terms is a function of distance above the ground. Therefore, the integral term in the preceding set of equations becomes

$$\int_0^Z \lambda_{si\ell} \chi_{i\ell}(t) dz = Z \lambda_{si\ell} \chi_{i\ell}(t) \tag{26}$$

Using the above relationship to define the values of the integrals in the set of equations specified by Equation (25) and letting

$$\alpha_{i\ell} = \lambda_{i\ell} + \lambda_{wi\ell} + \lambda_{ri\ell} , \tag{27}$$

and

$$\gamma_{i\ell} = v_{i\ell} + Z \lambda_{si\ell} , \tag{28}$$

then the following differential equations defining the ground concentration of the released nuclide i and of each of its daughters in the region affected by the release of the nuclide i as a function of time subsequent to the beginning of the release may be found by dividing Equation (25) through by $A_a \Delta t$, letting Δt approach zero and rearranging.

$$\frac{d C_{i0}(t)}{dt} + \alpha_{i0} C_{i0}(t) = \gamma_{i0} \chi_{i0}(t)$$

$$\frac{d C_{i1}(t)}{dt} + \alpha_{i1} C_{i1}(t) = \gamma_{i1} \chi_{i1}(t) + \lambda_{i1} C_{i0}(t)$$

$$\begin{aligned}
\frac{d C_{i2}(t)}{dt} + \alpha_{i2} C_{i2}(t) &= \gamma_{i2} \chi_{i2}(t) + \lambda_{i2} C_{i1}(t) \\
&\vdots \\
\frac{d C_{i\ell}(t)}{dt} + \alpha_{i\ell} C_{i\ell}(t) &= \gamma_{i\ell} \chi_{i\ell}(t) + \lambda_{i\ell} C_{i\ell-1}(t)
\end{aligned} \tag{29}$$

The values for $\chi_{i\ell}(t)$ in the above set of equations are given by Equation (19). However, these solutions for $\chi_{i\ell}(t)$ can be expressed in the simpler form given below

$$\begin{aligned}
\chi_{i0}(t) &= H_{i0} - \epsilon_{i0} G_{i00} e^{-\bar{\lambda}_{i0} t} \\
\chi_{i1}(t) &= H_{i1} - \epsilon_{i1} \left(G_{i10} e^{-\bar{\lambda}_{i0} t} + G_{i11} e^{-\bar{\lambda}_{i1} t} \right) \\
\chi_{i2}(t) &= H_{i2} - \epsilon_{i2} \left(G_{i20} e^{-\bar{\lambda}_{i0} t} + G_{i21} e^{-\bar{\lambda}_{i1} t} + G_{i22} e^{-\bar{\lambda}_{i2} t} \right) \\
&\vdots \\
\chi_{i\ell}(t) &= H_{i\ell} - \epsilon_{i\ell} \sum_{j=0}^{j=\ell} G_{i\ell j} e^{-\bar{\lambda}_{ij} t}
\end{aligned} \tag{30}$$

where

$$H_{i\ell} = \frac{\lambda_{i1} \lambda_{i2} \dots \lambda_{i\ell}}{\bar{\lambda}_{i0} \bar{\lambda}_{i1} \dots \bar{\lambda}_{i\ell}} \frac{Q_i}{V_a} \tag{31}$$

$$\epsilon_{i\ell} = \lambda_{i1} \lambda_{i2} \dots \lambda_{i\ell} \frac{Q_i}{V_a} \tag{32}$$

$$G_{i\ell j} = \frac{(-1)^j}{(\bar{\lambda}_{ij} - \bar{\lambda}_{i0})(\bar{\lambda}_{ij} - \bar{\lambda}_{i1}) \dots (\bar{\lambda}_{ij} - \bar{\lambda}_{ij-1}) \bar{\lambda}_{ij} (\bar{\lambda}_{ij+1} - \bar{\lambda}_{ij})(\bar{\lambda}_{ij+2} - \bar{\lambda}_{ij}) \dots (\bar{\lambda}_{i\ell} - \bar{\lambda}_{ij})} \tag{33}$$

Substituting the expressions for $\chi_{i\ell}(t)$ given in Equation (30) into the differential equations specified in Equation (29) yields the following:

$$\begin{aligned}
\frac{d C_{i0}(t)}{dt} + \alpha_{i0} C_{i0}(t) &= \gamma_{i0} H_{i0} - \gamma_{i0} \epsilon_{i0} G_{i00} e^{-\bar{\lambda}_{i0} t} \\
\frac{d C_{i1}(t)}{dt} + \alpha_{i1} C_{i1}(t) &= \lambda_{i1} C_{i0}(t) + \gamma_{i1} H_{i1} - \gamma_{i1} \epsilon_{i1} \left(G_{i10} e^{-\bar{\lambda}_{i0} t} + G_{i11} e^{-\bar{\lambda}_{i1} t} \right)
\end{aligned}$$

$$\frac{d C_{i2}(t)}{dt} + \alpha_{i2} C_{i2}(t) = \lambda_{i2} C_{i1}(t) + \gamma_{i2} H_{i2} - \gamma_{i2} \epsilon_{i2} \left(G_{i20} e^{-\bar{\lambda}_{i0} t} + G_{i21} e^{-\bar{\lambda}_{i1} t} + G_{i22} e^{-\bar{\lambda}_{i2} t} \right)$$

•
•
•

$$\frac{d C_{i\ell}(t)}{dt} + \alpha_{i\ell} C_{i\ell}(t) = \lambda_{i\ell} C_{i\ell-1}(t) + \gamma_{i\ell} H_{i\ell} - \gamma_{i\ell} \epsilon_{i\ell} \sum_{j=0}^{j=\ell} G_{i\ell j} e^{-\bar{\lambda}_{ij} t} \quad (34)$$

At the beginning of the release period the concentration of the released nuclide i and of each of its daughters on the ground surfaces of the region affected by the release of the nuclide i is zero. Thus, the boundary conditions for the preceding set of differential equations is given by

$$C_{i0}(0) = 0$$

$$C_{i1}(0) = 0$$

$$C_{i2}(0) = 0$$

•
•
•

$$C_{i\ell}(0) = 0 \quad (35)$$

Solving the set of differential equations given by Equation (34) using the boundary conditions specified in Equation (35) yields the following set of equations which define the ground concentration of the released nuclide i and of each of its daughters in the region affected by the release of the nuclide i during the period of time that the release is occurring as a function of time subsequent to the start of the release.

$$\begin{aligned} C_{i0}(t) &= \frac{\gamma_{i0} H_{i0}}{\alpha_{i0}} - \frac{\gamma_{i0} \epsilon_{i0} G_{i00}}{\alpha_{i0} - \bar{\lambda}_{i0}} e^{-\bar{\lambda}_{i0} t} - \left(\frac{\gamma_{i0} H_{i0}}{\alpha_{i0}} - \frac{\gamma_{i0} \epsilon_{i0} G_{i00}}{\alpha_{i0} - \bar{\lambda}_{i0}} \right) e^{-\alpha_{i0} t} \\ C_{i1}(t) &= \frac{\gamma_{i0} H_{i0} \lambda_{i1}}{\alpha_{i0} \alpha_{i1}} + \frac{\gamma_{i1} H_{i1}}{\alpha_{i1}} - \frac{\gamma_{i1} \epsilon_{i1} G_{i11}}{\alpha_{i1} - \bar{\lambda}_{i1}} e^{-\bar{\lambda}_{i1} t} - \left[\frac{\gamma_{i0} \epsilon_{i0} G_{i00} \lambda_{i1}}{(\alpha_{i0} - \bar{\lambda}_{i0})(\alpha_{i1} - \bar{\lambda}_{i0})} + \frac{\gamma_{i1} \epsilon_{i1} G_{i10}}{\alpha_{i1} - \bar{\lambda}_{i0}} \right] e^{-\bar{\lambda}_{i0} t} \\ &\quad - \frac{\lambda_{i1}}{(\alpha_{i1} - \alpha_{i0})} \left[\frac{\gamma_{i0} H_{i0}}{\alpha_{i0}} - \frac{\gamma_{i0} \epsilon_{i0} G_{i00}}{\alpha_{i0} - \bar{\lambda}_{i0}} \right] e^{-\alpha_{i0} t} + \left[\frac{\gamma_{i0} H_{i0} \lambda_{i1}}{\alpha_{i1}(\alpha_{i1} - \alpha_{i0})} - \frac{\gamma_{i1} H_{i1}}{\alpha_{i1}} \right. \\ &\quad \left. - \frac{\gamma_{i0} \epsilon_{i0} \lambda_{i1}}{\alpha_{i1} - \alpha_{i0}} \left(\frac{G_{i00}}{\alpha_{i1} - \bar{\lambda}_{i0}} \right) + \gamma_{i1} \epsilon_{i1} \left(\frac{G_{i10}}{\alpha_{i1} - \bar{\lambda}_{i0}} + \frac{G_{i11}}{\alpha_{i1} - \bar{\lambda}_{i1}} \right) \right] e^{-\alpha_{i1} t} \end{aligned}$$

$$\begin{aligned}
C_{i2}(t) = & \frac{\gamma_{i0} H_{i0} \lambda_{i1} \lambda_{i2}}{\alpha_{i0} \alpha_{i1} \alpha_{i2}} + \frac{\gamma_{i1} H_{i1} \lambda_{i2}}{\alpha_{i1} \alpha_{i2}} + \frac{\gamma_{i2} H_{i2}}{\alpha_{i2}} - \left[\frac{\gamma_{i0} \epsilon_{i0} G_{i00} \lambda_{i1} \lambda_{i2}}{(\alpha_{i0} - \bar{\lambda}_{i0})(\alpha_{i1} - \bar{\lambda}_{i0})(\alpha_{i2} - \bar{\lambda}_{i0})} \right. \\
& + \frac{\gamma_{i1} \epsilon_{i1} G_{i10} \lambda_{i2}}{(\alpha_{i1} - \bar{\lambda}_{i0})(\alpha_{i2} - \bar{\lambda}_{i0})} + \frac{\gamma_{i2} \epsilon_{i2} G_{i20}}{\alpha_{i2} - \bar{\lambda}_{i0}} \left. \right] e^{-\bar{\lambda}_{i0} t} - \left[\frac{\gamma_{i1} \epsilon_{i1} G_{i11} \lambda_{i2}}{(\alpha_{i1} - \bar{\lambda}_{i1})(\alpha_{i2} - \bar{\lambda}_{i1})} + \frac{\gamma_{i2} \epsilon_{i2} G_{i21}}{\alpha_{i2} - \bar{\lambda}_{i1}} \right] e^{-\bar{\lambda}_{i1} t} \\
& - \frac{\gamma_{i2} \epsilon_{i2} G_{i22}}{\alpha_{i2} - \bar{\lambda}_{i2}} e^{-\bar{\lambda}_{i2} t} - \frac{\lambda_{i1} \lambda_{i2}}{(\alpha_{i1} - \alpha_{i0})(\alpha_{i2} - \alpha_{i0})} \left[\frac{\gamma_{i0} H_{i0}}{\alpha_{i0}} - \gamma_{i0} \epsilon_{i0} \left(\frac{G_{i00}}{\alpha_{i0} - \bar{\lambda}_{i0}} \right) \right] e^{-\alpha_{i0} t} \\
& + \frac{\lambda_{i2}}{\alpha_{i2} - \alpha_{i1}} \left[\frac{\gamma_{i0} H_{i0} \lambda_{i1}}{\alpha_{i1}(\alpha_{i1} - \alpha_{i0})} - \frac{\gamma_{i1} H_{i1}}{\alpha_{i1}} - \frac{\gamma_{i0} \epsilon_{i0} \lambda_{i1}}{\alpha_{i1} - \alpha_{i0}} \left(\frac{G_{i00}}{\alpha_{i1} - \bar{\lambda}_{i0}} \right) + \gamma_{i1} \epsilon_{i1} \left(\frac{G_{i00}}{\alpha_{i1} - \bar{\lambda}_{i0}} \right. \right. \\
& \left. \left. + \frac{G_{i11}}{\alpha_{i1} - \bar{\lambda}_{i1}} \right) \right] e^{-\alpha_{i1} t} - \left[\frac{\gamma_{i0} H_{i0} \lambda_{i1} \lambda_{i2}}{\alpha_{i2}(\alpha_{i2} - \alpha_{i0})(\alpha_{i2} - \alpha_{i1})} - \frac{\gamma_{i1} H_{i1} \lambda_{i2}}{\alpha_{i2}(\alpha_{i2} - \alpha_{i1})} + \frac{\gamma_{i2} H_{i2}}{\alpha_{i2}} \right. \\
& - \frac{\gamma_{i0} \epsilon_{i0} \lambda_{i1} \lambda_{i2}}{(\alpha_{i2} - \alpha_{i0})(\alpha_{i2} - \alpha_{i1})} \left(\frac{G_{i00}}{\alpha_{i2} - \bar{\lambda}_{i0}} \right) + \frac{\gamma_{i1} \epsilon_{i1} \lambda_{i2}}{\alpha_{i2} - \alpha_{i1}} \left(\frac{G_{i10}}{\alpha_{i2} - \bar{\lambda}_{i0}} + \frac{G_{i11}}{\alpha_{i2} - \bar{\lambda}_{i1}} \right) \\
& \left. \left. - \gamma_{i2} \epsilon_{i2} \left(\frac{G_{i20}}{\alpha_{i2} - \bar{\lambda}_{i0}} + \frac{G_{i21}}{\alpha_{i2} - \bar{\lambda}_{i1}} + \frac{G_{i22}}{\alpha_{i2} - \bar{\lambda}_{i2}} \right) \right] e^{-\alpha_{i2} t}
\end{aligned}$$

$$\begin{aligned}
C_{i\ell}(t) = & \sum_{j=0}^{j=\ell} \frac{\gamma_{ij} H_{ij} \lambda_{ij+1} \lambda_{ij+2} \dots \lambda_{i\ell}}{\alpha_{ij} \alpha_{ij+1} \dots \alpha_{i\ell}} \\
& - \sum_{j=0}^{j=\ell} e^{-\bar{\lambda}_{ij} t} \left[\sum_{m=j}^{m=\ell} \frac{\gamma_{im} \epsilon_{im} G_{imj} \lambda_{im+1} \lambda_{im+2} \dots \lambda_{i\ell}}{(\alpha_{im} - \bar{\lambda}_{ij})(\alpha_{im+1} - \bar{\lambda}_{ij}) \dots (\alpha_{i\ell} - \bar{\lambda}_{ij})} \right] \\
& + \sum_{j=0}^{j=\ell} \frac{(-1)^{j+1} \lambda_{ij+1} \lambda_{ij+2} \dots \lambda_{i\ell}}{(\alpha_{ij+1} - \alpha_{ij})(\alpha_{ij+2} - \alpha_{ij}) \dots (\alpha_{i\ell} - \alpha_{ij})} e^{-\alpha_{ij} t} \\
& + \sum_{n=0}^{n=j} \frac{(-1)^n \gamma_{in} H_{in} \lambda_{in+1} \lambda_{in+2} \dots \lambda_{ij}}{\alpha_{ij}(\alpha_{ij} - \alpha_{in})(\alpha_{ij} - \alpha_{in+1}) \dots (\alpha_{ij} - \alpha_{ij-1})} \\
& + \sum_{n=0}^{n=j} \frac{(-1)^{n+1} \gamma_{in} \epsilon_{in} \lambda_{in+1} \lambda_{in+2} \dots \lambda_{ij}}{(\alpha_{ij} - \alpha_{in})(\alpha_{ij} - \alpha_{in+1}) \dots (\alpha_{ij} - \alpha_{ij-1})} \sum_{m=0}^{m=n} \frac{G_{inm}}{\alpha_{ij} - \bar{\lambda}_{im}}
\end{aligned} \tag{36}$$

where the values of $H_{i\ell}$, $\epsilon_{i\ell}$, and G_{imj} are defined by Equations (31), (32), and (33) respectively.

2. Subsequent to Releases

The procedure to define the total quantity of activity of the released nuclide and each of its daughters which are on the ground surfaces of the region affected by the release during the period after releases have stopped is the same as that used while the releases are occurring. The deposition and production mechanisms are the same during releases as they are after the releases have stopped and the same is true of the removal mechanisms of decay, weathering, and resuspension. The only difference lies in the expressions used to define the air concentrations in the region affected by the releases in the period after releases have stopped.

Thus, the amount of activity of the released radionuclide i and of each of its daughters on the ground surfaces of the region affected by the release at the time $\tau + \Delta\tau$ after the releases have stopped is given by

$$\begin{aligned}
 A_a C_{i0}^* \Big|_{\tau + \Delta\tau} &= A_a C_{i0}^* \Big|_{\tau} + A_a \int_0^Z \lambda_{si0} \chi_{i0}^* \Big|_{\tau + \Delta\tau/2} dz \Delta\tau + A_a v_{i0} \chi_{i0}^* \Big|_{\tau + \Delta\tau/2} \Delta\tau \\
 &\quad - A_a \lambda_{i0} C_{i0}^* \Big|_{\tau + \Delta\tau/2} \Delta\tau - A_a \lambda_{wi0} C_{i0}^* \Big|_{\tau + \Delta\tau/2} \Delta\tau - A_a \lambda_{ri0} C_{i0}^* \Big|_{\tau + \Delta\tau/2} \Delta\tau \\
 A_a C_{i1}^* \Big|_{\tau + \Delta\tau} &= A_a C_{i1}^* \Big|_{\tau} + A_a \int_0^Z \lambda_{si1} \chi_{i1}^* \Big|_{\tau + \Delta\tau/2} dz \Delta\tau \\
 &\quad + A_a v_{i1} \chi_{i1}^* \Big|_{\tau + \Delta\tau/2} \Delta\tau + A_a \lambda_{i1} C_{i0}^* \Big|_{\tau + \Delta\tau/2} \Delta\tau - A_a \lambda_{i1} C_{i1}^* \Big|_{\tau + \Delta\tau/2} \Delta\tau \\
 &\quad - A_a \lambda_{wi1} C_{i1}^* \Big|_{\tau + \Delta\tau/2} \Delta\tau - A_a \lambda_{ri1} C_{i1}^* \Big|_{\tau + \Delta\tau/2} \Delta\tau \\
 A_a C_{i2}^* \Big|_{\tau + \Delta\tau} &= A_a C_{i2}^* \Big|_{\tau} + A_a \int_0^Z \lambda_{si2} \chi_{i2}^* \Big|_{\tau + \Delta\tau/2} dz \Delta\tau \\
 &\quad + A_a v_{i2} \chi_{i2}^* \Big|_{\tau + \Delta\tau/2} \Delta\tau + A_a \lambda_{i2} C_{i1}^* \Big|_{\tau + \Delta\tau/2} \Delta\tau - A_a \lambda_{i2} C_{i2}^* \Big|_{\tau + \Delta\tau/2} \Delta\tau \\
 &\quad - A_a \lambda_{wi2} C_{i2}^* \Big|_{\tau + \Delta\tau/2} \Delta\tau - A_a \lambda_{ri2} C_{i2}^* \Big|_{\tau + \Delta\tau/2} \Delta\tau \\
 &\quad \vdots \\
 A_a C_{iL}^* \Big|_{\tau + \Delta\tau} &= A_a C_{iL}^* \Big|_{\tau} + A_a \int_0^Z \lambda_{siL} \chi_{iL}^* \Big|_{\tau + \Delta\tau/2} dz \Delta\tau \\
 &\quad + A_a v_{iL} \chi_{iL}^* \Big|_{\tau + \Delta\tau/2} \Delta\tau + A_a \lambda_{iL} C_{iL-1}^* \Big|_{\tau + \Delta\tau/2} \Delta\tau - A_a \lambda_{iL} C_{iL}^* \Big|_{\tau + \Delta\tau/2} \Delta\tau \\
 &\quad - A_a \lambda_{wiL} C_{iL}^* \Big|_{\tau + \Delta\tau/2} \Delta\tau - A_a \lambda_{riL} C_{iL}^* \Big|_{\tau + \Delta\tau/2} \Delta\tau
 \end{aligned} \tag{37}$$

Noting that $\chi_{i\ell}^*(t)$ represents a uniformly dispersed concentration of the ℓ -th daughter of the released radionuclide i and thus is not a function of the distance above the ground, the integral term in the preceding set of equations becomes

$$\int_0^Z \lambda_{si\ell} \chi_{i\ell}^*(t) dz = Z \lambda_{si\ell} \chi_{i\ell}^*(t) \quad (38)$$

Using the above relationship to define the values of the integrals and the expressions for $\alpha_{i\ell}$ and $\gamma_{i\ell}$ given by Equations (27) and (28), respectively, the following set of differential equations defining the ground concentrations of the released nuclide i and of each of its daughters in the region affected by the release of the nuclide i as a function of time after releases have stopped may be found by dividing the expressions in Equation (37) through by $A_a \Delta\tau$, letting $\Delta\tau$ approach zero and rearranging.

$$\begin{aligned} \frac{d C_{i0}^*(\tau)}{d\tau} + \alpha_{i0} C_{i0}^*(\tau) &= \gamma_{i0} \chi_{i0}^*(\tau) \\ \frac{d C_{i1}^*(\tau)}{d\tau} + \alpha_{i1} C_{i1}^*(\tau) &= \gamma_{i1} \chi_{i1}^*(\tau) + \lambda_{i1} C_{i0}^*(\tau) \\ \frac{d C_{i2}^*(\tau)}{d\tau} + \alpha_{i2} C_{i2}^*(\tau) &= \gamma_{i2} \chi_{i2}^*(\tau) + \lambda_{i2} C_{i1}^*(\tau) \\ &\vdots \\ \frac{d C_{i\ell}^*(\tau)}{d\tau} + \alpha_{i\ell} C_{i\ell}^*(\tau) &= \gamma_{i\ell} \chi_{i\ell}^*(\tau) + \lambda_{i\ell} C_{i\ell-1}^*(\tau) \end{aligned} \quad (39)$$

where τ is defined by Equation (21) as $t - T_S$.

The values for $\chi_{i\ell}^*(\tau)$ in the preceding set of equations are given by Equation (24). However, these solutions for $\chi_{i\ell}^*(\tau)$ can be expressed in the following simpler form.

$$\begin{aligned} \chi_{i0}^*(\tau) &= R_{i00} S_{i00} e^{-\bar{\lambda}_{i0}\tau} \\ \chi_{i1}^*(\tau) &= R_{i10} S_{i00} e^{-\bar{\lambda}_{i0}\tau} + R_{i11} (S_{i10} + S_{i11}) e^{-\bar{\lambda}_{i1}\tau} \\ \chi_{i2}^*(\tau) &= R_{i20} S_{i00} e^{-\bar{\lambda}_{i0}\tau} + R_{i21} (S_{i10} + S_{i11}) e^{-\bar{\lambda}_{i1}\tau} + R_{i22} (S_{i20} + S_{i31} + S_{i22}) e^{-\bar{\lambda}_{i2}\tau} \\ &\vdots \\ \chi_{i\ell}^*(\tau) &= \sum_{j=0}^{j=\ell} R_{i\ell j} e^{-\bar{\lambda}_{ij}\tau} \left[\sum_{m=0}^{m=j} S_{ijm} \right] \end{aligned} \quad (40)$$

where

$$R_{i\ell j} = \frac{\lambda_{ij+1} \lambda_{ij+2} \dots \lambda_{i\ell}}{(\bar{\lambda}_{i\ell} - \bar{\lambda}_{ij})(\bar{\lambda}_{i\ell-1} - \bar{\lambda}_{ij}) \dots (\bar{\lambda}_{ij+1} - \bar{\lambda}_{ij})} \quad (41)$$

and

$$S_{ijm} = \frac{(-1)^{j+m} \lambda_{im+1} \lambda_{im+2} \dots \lambda_{ij} \chi_{im}(T_S)}{(\bar{\lambda}_{ij} - \bar{\lambda}_{ij-1})(\bar{\lambda}_{ij} - \bar{\lambda}_{ij-2}) \dots (\bar{\lambda}_{ij} - \bar{\lambda}_{im})} \quad (42)$$

Substituting the expressions given in Equation (40) for $\chi_{i\ell}^*(\tau)$ into the differential equations specified in Equation (39) yields

$$\begin{aligned} \frac{d C_{i0}^*(\tau)}{d\tau} + \alpha_{i0} C_{i0}^*(\tau) &= \gamma_{i0} R_{i00} S_{i00} e^{-\bar{\lambda}_{i0}\tau} \\ \frac{d C_{i1}^*(\tau)}{d\tau} + \alpha_{i1} C_{i1}^*(\tau) &= \lambda_{i1} C_{i0}^*(\tau) + \gamma_{i1} R_{i10} S_{i00} e^{-\bar{\lambda}_{i0}\tau} + \gamma_{i1} R_{i11} (S_{i10} + S_{i11}) e^{-\bar{\lambda}_{i1}\tau} \\ \frac{d C_{i2}^*(\tau)}{d\tau} + \alpha_{i2} C_{i2}^*(\tau) &= \lambda_{i2} C_{i1}^*(\tau) + \gamma_{i2} R_{i20} S_{i00} e^{-\bar{\lambda}_{i0}\tau} + \gamma_{i2} R_{i21} (S_{i10} + S_{i11}) e^{-\bar{\lambda}_{i1}\tau} \\ &\quad + \gamma_{i2} R_{i22} (S_{i20} + S_{i21} + S_{i22}) e^{-\bar{\lambda}_{i2}\tau} \\ &\vdots \\ \frac{d C_{i\ell}^*(\tau)}{d\tau} + \alpha_{i\ell} C_{i\ell}^*(\tau) &= \lambda_{i\ell} C_{i\ell-1}^*(\tau) + \gamma_{i\ell} \sum_{j=0}^{\ell} R_{i\ell j} e^{-\bar{\lambda}_{ij}\tau} \left[\sum_{m=0}^{m=j} S_{ijm} \right] \end{aligned} \quad (43)$$

At the time that the release stops ($\tau = 0$), the ground concentrations of the released nuclide i and of each of its daughters defined by the preceding set of differential equations will be the same as those defined during the time that releases are occurring at $t = T_S$. Thus the boundary conditions for the preceding set of differential equations are

$$\begin{aligned} C_{i0}^*(0) &= C_{i0}(T_S) \\ C_{i1}^*(0) &= C_{i1}(T_S) \\ C_{i2}^*(0) &= C_{i2}(T_S) \\ &\vdots \\ C_{i\ell}^*(0) &= C_{i\ell}(T_S) \end{aligned} \quad (44)$$

The set of differential equations defined by Equation (43) is solved using the boundary conditions specified in Equation (44) and the relationship between τ and $t - T_S$ as given by Equation (21). This results in the following set of equations which define the ground concentrations of the released nuclide i and each of its daughters in the region affected by the release during the period after the release has stopped as a function of time subsequent to the start of the release.

$$C_{i0}^*(t) = \left[C_{i0}(T_S) - \frac{\gamma_{i0} R_{i00} S_{i00}}{\alpha_{i0} - \bar{\lambda}_{i0}} \right] e^{-\alpha_{i0}(t-T_S)} + \frac{\gamma_{i0} R_{i00} S_{i00}}{\alpha_{i0} - \bar{\lambda}_{i0}} e^{-\bar{\lambda}_{i0}(t-T_S)}$$

$$C_{i1}^*(t) = \frac{\lambda_{i1}}{\alpha_{i1} - \alpha_{i0}} \left[C_{i0}(T_S) - \frac{\gamma_{i0} R_{i00} S_{i00}}{\alpha_{i0} - \bar{\lambda}_{i0}} \right] e^{-\alpha_{i0}(t-T_S)} + \left[C_{i1}(T_S) - \frac{\lambda_{i1} C_{i0}(T_S)}{\alpha_{i1} - \alpha_{i0}} \right. \\ \left. + \frac{S_{i00}}{\alpha_{i1} - \bar{\lambda}_{i0}} \left(\frac{\lambda_{i1} \gamma_{i1} R_{i00}}{\alpha_{i1} - \alpha_{i0}} - \gamma_{i1} R_{i10} \right) - \frac{\gamma_{i1} R_{i11} (S_{i10} + S_{i11})}{\alpha_{i1} - \bar{\lambda}_{i1}} \right] e^{-\alpha_{i1}(t-T_S)} \\ \left. + \frac{S_{i00}}{\alpha_{i1} - \bar{\lambda}_{i0}} \left[\frac{\lambda_{i1} \gamma_{i1} R_{i00}}{\alpha_{i0} - \bar{\lambda}_{i0}} + \gamma_{i1} R_{i10} \right] e^{-\bar{\lambda}_{i0}(t-T_S)} + \frac{S_{i10} + S_{i11}}{\alpha_{i1} - \bar{\lambda}_{i1}} \gamma_{i1} R_{i11} e^{-\bar{\lambda}_{i1}(t-T_S)} \right]$$

$$C_{i2}^*(t) = \frac{\lambda_{i1} \lambda_{i2}}{(\alpha_{i1} - \alpha_{i0})(\alpha_{i2} - \alpha_{i0})} \left[C_{i0}(T_S) - \frac{\gamma_{i0} R_{i00} S_{i00}}{\alpha_{i0} - \bar{\lambda}_{i0}} \right] e^{-\alpha_{i0}(t-T_S)} + \frac{\lambda_{i2}}{\alpha_{i2} - \alpha_{i1}} \left[C_{i1}(T_S) \right. \\ \left. - \frac{\lambda_{i1}}{\alpha_{i1} - \alpha_{i0}} C_{i0}(T_S) + \frac{S_{i00}}{\alpha_{i1} - \bar{\lambda}_{i0}} \left(\frac{\lambda_{i1} \gamma_{i0} R_{i00}}{\alpha_{i1} - \alpha_{i0}} - \gamma_{i1} R_{i10} \right) - \frac{(S_{i10} + S_{i11})}{\alpha_{i1} - \bar{\lambda}_{i1}} \gamma_{i1} R_{i11} \right] e^{-\alpha_{i1}(t-T_S)} \\ \left. + \left[C_{i2}(T_S) - \frac{\lambda_{i2} C_{i1}(T_S)}{\alpha_{i2} - \alpha_{i1}} + \frac{\lambda_{i1} \lambda_{i2} C_{i0}(T_S)}{(\alpha_{i2} - \alpha_{i0})(\alpha_{i2} - \alpha_{i1})} - \frac{S_{i00}}{\alpha_{i2} - \bar{\lambda}_{i0}} \left(\frac{\lambda_{i1} \lambda_{i2} \gamma_{i0} R_{i00}}{(\alpha_{i2} - \alpha_{i1})(\alpha_{i2} - \alpha_{i0})} \right. \right. \right. \\ \left. \left. - \frac{\lambda_{i2} \gamma_{i1} R_{i10}}{\alpha_{i2} - \alpha_{i1}} + \gamma_{i2} R_{i20} \right) + \frac{(S_{i10} + S_{i11})}{\alpha_{i2} - \bar{\lambda}_{i1}} \left(\frac{\lambda_{i2} \gamma_{i1} R_{i10}}{\alpha_{i2} - \alpha_{i1}} - \gamma_{i2} R_{i21} \right) \right. \\ \left. \left. - \frac{(S_{i20} + S_{i21} + S_{i22})}{\alpha_{i2} - \bar{\lambda}_{i2}} \gamma_{i2} R_{i22} \right] e^{-\alpha_{i2}(t-T_S)} + S_{i00} \left[\frac{\lambda_{i1} \lambda_{i2} \gamma_{i0} R_{i00}}{(\alpha_{i0} - \bar{\lambda}_{i0})(\alpha_{i1} - \bar{\lambda}_{i0})(\alpha_{i2} - \bar{\lambda}_{i0})} \right. \right. \\ \left. \left. + \frac{\lambda_{i2} \gamma_{i1} R_{i10}}{(\alpha_{i1} - \bar{\lambda}_{i0})(\alpha_{i2} - \bar{\lambda}_{i0})} + \frac{\gamma_{i2} R_{i20}}{\alpha_{i2} - \bar{\lambda}_{i0}} \right] e^{-\bar{\lambda}_{i0}(t-T_S)} + (S_{i10} + S_{i11}) \left[\frac{\lambda_{i2} \gamma_{i1} R_{i11}}{(\alpha_{i1} - \bar{\lambda}_{i1})(\alpha_{i2} - \bar{\lambda}_{i1})} \right. \right. \\ \left. \left. + \frac{\gamma_{i2} R_{i21}}{\alpha_{i2} - \bar{\lambda}_{i1}} \right] e^{-\bar{\lambda}_{i1}(t-T_S)} + \frac{(S_{i20} + S_{i21} + S_{i22})}{\alpha_{i2} - \bar{\lambda}_{i2}} \gamma_{i2} R_{i22} e^{-\bar{\lambda}_{i2}(t-T_S)} \right]$$

⋮

$$C_{i\ell}^*(t) = \sum_{j=0}^{j=\ell} \frac{\lambda_{ij+1} \lambda_{ij+2} \dots \lambda_{i\ell}}{(\alpha_{ij+1} - \alpha_{ij})(\alpha_{ij+2} - \alpha_{ij}) \dots (\alpha_{i\ell} - \alpha_{ij})}$$

$$\left[\sum_{m=0}^{m=j} \frac{(-1)^{j+m} \lambda_{im+1} \lambda_{im+2} \dots \lambda_{ij} C_{im}(T_S)}{(\alpha_{ij} - \alpha_{ij-1})(\alpha_{ij} - \alpha_{ij-2}) \dots (\alpha_{ij} - \alpha_{im})} \right]$$

$$\begin{aligned}
& - \sum_{m=0}^{m=j} \frac{(-1)^{j+m}}{\alpha_{ij} - \lambda_{im}} \left\{ \sum_{k=0}^{k=m} S_{imk} \right\} \\
& \left\{ \sum_{n=m}^{n=j} \frac{(-1)^{n+m} \gamma_{in} R_{inm} \lambda_{in+1} \lambda_{in+2} \dots \lambda_{ij}}{(\alpha_{ij} - \alpha_{ij-1})(\alpha_{ij} - \alpha_{ij-2}) \dots (\alpha_{ij} - \alpha_{in})} \right\} e^{-\alpha_{ij}(t-T_S)} \\
& + \sum_{j=0}^{j=\ell} \left[\sum_{n=j}^{n=\ell} \frac{\gamma_{in} R_{inj} \lambda_{in+1} \lambda_{in+2} \dots \lambda_{i\ell}}{(\alpha_{in} - \lambda_{ij})(\alpha_{in+1} - \lambda_{ij}) \dots (\alpha_{i\ell} - \lambda_{ij})} \right] \\
& \left[\sum_{m=0}^{m=j} S_{ijm} \right] e^{-\lambda_{ij}(t-T_S)}, \tag{45}
\end{aligned}$$

where the values of R_{inj} and S_{ijm} are defined by Equations (41) and (42), respectively, and the value of $C_{im}(T_S)$ is obtained by evaluating Equation (36) at $t = T_S$.

D. Final Dose Commitment Equations

This segment of the report defines the model equations actually used to estimate the dose commitments received by man in the region of the world which is affected by the release of a radionuclide. These final model equations are developed by simplifying and combining the general equations presented previously which defined the dose commitments and the air and ground concentrations. In addition, the following assumptions were used in the development of these final equations.

1. The time dependence of the population growth in the region affected by the release is assumed to be represented by an exponential term which uses the number of people in the affected region at the time releases start as an initial base and a growth constant which is spatially independent.

$$\therefore P(t) = P_0 e^{+\lambda_p t} \tag{46}$$

2. The time and/or spatial dependence of the deposition velocity, the precipitation scavenging coefficient, the resuspension coefficient, and the weathering coefficient are neglected.
3. The factor which relates the amount of material which is resuspended to the amount which is on the ground is assumed to be dependent upon the time at which releases start rather than the time at which deposition occurs. This factor is assumed to be represented by the sum of a constant term and an exponentially decaying term in the manner defined in Reference 5 and given below.

$$\beta(t) = K_0 e^{-\lambda_a t} + K_1 \tag{47}$$

4. Average rates at which various foods are consumed are used rather than the time dependent rates which reflect the seasonal variations of food consumption.
5. The time dependence of the times during which a given radionuclide is depositing upon a food crop is neglected.
6. The integral over the time interval from T_I to T_L can be expressed as the sum of two integrals—one from T_I to T_S , the time that the releases stop, and the other from T_S to T_L . The choice of T_S as a break point in the overall interval is required since the expressions for the air and ground concentrations prior to T_S differ from those which define the air and ground concentrations subsequent to T_S .

Given the preceding assumptions, the total dose commitment from all released radionuclides and their daughters to all the people in the region affected by the release can be found by summing the dose commitments of each released nuclide and of each daughter of each released nuclide.

The final model equations for each of the dose commitment pathways are developed in the remainder of this section.

1. Immersion Dose Commitments

The dose commitment over the time interval from T_I to T_L to the g -th organ of all the people in the region affected by the release due to immersion in air containing the ℓ -th daughter of the released nuclide i is obtained by modifying Equation (3) to reflect the assumptions and approximations stated previously and is given by

$$D_{mgi\ell} = \int_{T_I}^{T_S} K_{mgi\ell} P_0 e^{\lambda_p t} \left[\chi_{i\ell}(t) + (K_0 e^{-\lambda_a t} + K_1) C_{i\ell}(t) \right] dt + \int_{T_S}^{T_L} K_{mgi\ell} P_0 e^{\lambda_p t} \left[\chi_{i\ell}^*(t) + (K_0 e^{-\lambda_a t} + K_1) C_{i\ell}^*(t) \right] dt \quad (48)$$

The values for $\chi_{i\ell}(t)$ and $\chi_{i\ell}^*(t)$ in the above equation are given in Equation (30) and Equation (40), respectively, and the values for $C_{i\ell}(t)$ and $C_{i\ell}^*(t)$ in the above equation are given in Equation (36) and Equation (45), respectively. However, the expressions for the ground concentrations during and after the releases can be simplified into the following form.

$$C_{i\ell}(t) = \sum_{j=0}^{j=\ell} \left(L_{ij\ell} - M_{ij\ell} e^{-\bar{\lambda}_{ij} t} + N_{ij\ell} e^{-\alpha_{ij} t} \right) \quad (49)$$

$$C_{i\ell}^*(t) = \sum_{j=0}^{j=\ell} \left(U_{ij\ell} e^{-\alpha_{ij}(t-T_S)} + Y_{ij\ell} e^{-\bar{\lambda}_{ij}(t-T_S)} \right) \quad (50)$$

where

$$L_{ij\ell} = \frac{\gamma_{ij} H_{ij} \lambda_{ij+1} \lambda_{ij+2} \dots \lambda_{i\ell}}{\alpha_{ij} \alpha_{ij+1} \dots \alpha_{i\ell}} \quad (51)$$

$$M_{ij\ell} = \sum_{m=j}^{m=\ell} \frac{\gamma_{im} \epsilon_{im} G_{imj} \lambda_{im+1} \lambda_{im+2} \dots \lambda_{i\ell}}{(\alpha_{im} - \bar{\lambda}_{ij})(\alpha_{im+1} - \bar{\lambda}_{ij}) \dots (\alpha_{i\ell} - \bar{\lambda}_{ij})} \quad (52)$$

$$N_{ij\ell} = \frac{(-1)^{j+1} \lambda_{ij+1} \lambda_{ij+2} \dots \lambda_{i\ell}}{(\alpha_{ij+1} - \alpha_{ij})(\alpha_{ij+2} - \alpha_{ij}) \dots (\alpha_{i\ell} - \alpha_{ij})} \sum_{n=0}^{n=j} \frac{(-1)^n \gamma_{in} \lambda_{in+1} \lambda_{in+2} \dots \lambda_{ij}}{(\alpha_{ij} - \alpha_{in})(\alpha_{ij} - \alpha_{in+1}) \dots (\alpha_{ij} - \alpha_{ij-1})} \left(\frac{H_{in}}{\alpha_{ij}} - \epsilon_{in} \sum_{m=0}^{m=n} \frac{G_{inm}}{\alpha_{ij} - \bar{\lambda}_{im}} \right) \quad (53)$$

$$U_{ij\ell} = \frac{\lambda_{ij+1} \lambda_{ij+2} \dots \lambda_{i\ell}}{(\alpha_{ij+1} - \alpha_{ij})(\alpha_{ij+2} - \alpha_{ij}) \dots (\alpha_{i\ell} - \alpha_{ij})} \sum_{m=0}^{m=j} (-1)^{j+m} \left[\frac{C_{im}(T_S) \lambda_{im+1} \lambda_{im+2} \dots \lambda_{ij}}{(\alpha_{ij} - \alpha_{ij-1})(\alpha_{ij} - \alpha_{ij-2}) \dots (\alpha_{ij} - \alpha_{im})} - \frac{1}{\alpha_{ij} - \bar{\lambda}_{im}} \left\{ \sum_{k=0}^{k=m} S_{imk} \right\} \right] \left\{ \sum_{n=m}^{n=j} \frac{(-1)^{n+m} \gamma_{in} R_{inm} \lambda_{in+1} \lambda_{in+2} \dots \lambda_{ij}}{(\alpha_{ij} - \alpha_{ij-1})(\alpha_{ij} - \alpha_{ij-2}) \dots (\alpha_{ij} - \alpha_{in})} \right\} \quad (54)$$

$$Y_{ij\ell} = \left[\sum_{n=j}^{n=\ell} \frac{\gamma_{in} R_{inj} \lambda_{in+1} \lambda_{in+2} \dots \lambda_{i\ell}}{(\alpha_{in} - \bar{\lambda}_{ij})(\alpha_{in+1} - \bar{\lambda}_{ij}) \dots (\alpha_{i\ell} - \bar{\lambda}_{ij})} \right] \left[\sum_{m=0}^{m=j} S_{ijm} \right] \quad (55)$$

where the values of H_{in} , ϵ_{in} , and G_{inm} are defined by Equations (31), (32), and (33), respectively, the values of R_{inj} and S_{ijm} are defined by Equations (41) and (42), respectively, and the value of $C_{im}(T_S)$ is obtained by evaluating Equation (36) at $t = T_S$.

Substituting the expressions given in Equations (30), (40), (49) and (50) for $\chi_{i\ell}(t)$, $\chi_{i\ell}^*(t)$, $C_{i\ell}(t)$, and $C_{i\ell}^*(t)$, respectively, into Equation (48) and rearranging terms yields

$$D_{mgi\ell} = K_{mgi\ell} P_0 \left[H_{i\ell} \int_{T_I}^{T_S} e^{\lambda_p t} dt - \epsilon_{i\ell} \sum_{j=0}^{j=\ell} G_{i\ell j} \int_{T_I}^{T_S} e^{-(\bar{\lambda}_{ij} - \lambda_p)t} dt \right. \\ \left. + K_0 \sum_{j=0}^{j=\ell} L_{ij\ell} \int_{T_I}^{T_S} e^{-(\lambda_a - \lambda_p)t} dt - K_0 \sum_{j=0}^{j=\ell} M_{ij\ell} \int_{T_I}^{T_S} e^{-(\bar{\lambda}_{ij} + \lambda_a - \lambda_p)t} dt \right. \\ \left. + K_0 \sum_{j=0}^{j=\ell} N_{ij\ell} \int_{T_I}^{T_S} e^{-(\alpha_{ij} + \lambda_a - \lambda_p)t} dt + K_1 \sum_{j=0}^{j=\ell} L_{ij\ell} \int_{T_I}^{T_S} e^{\lambda_p t} dt \right]$$

$$\begin{aligned}
& - K_1 \sum_{j=0}^{j=\ell} M_{ij\ell} \int_{T_I}^{T_S} e^{-(\bar{\lambda}_{ij}-\lambda_p)t} dt + K_1 \sum_{j=0}^{j=\ell} N_{ij\ell} \int_{T_I}^{T_S} e^{-(\alpha_{ij}-\lambda_p)t} dt \\
& + \sum_{j=0}^{j=\ell} R_{i\ell j} \left(\sum_{m=0}^{m=j} S_{ijm} \right) e^{\bar{\lambda}_{ij}T_S} \int_{T_S}^{T_L} e^{-(\bar{\lambda}_{ij}-\lambda_p)t} dt \\
& + K_1 \sum_{j=0}^{j=\ell} U_{ij\ell} e^{\alpha_{ij}T_S} \int_{T_S}^{T_L} e^{-(\alpha_{ij}-\lambda_p)t} dt + K_1 \sum_{j=0}^{j=\ell} Y_{ij\ell} e^{\bar{\lambda}_{ij}T_S} \int_{T_S}^{T_L} e^{-(\bar{\lambda}_{ij}-\lambda_p)t} dt \\
& + K_0 \sum_{j=0}^{j=\ell} U_{ij\ell} e^{\alpha_{ij}T_S} \int_{T_S}^{T_L} e^{-(\alpha_{ij}+\lambda_a-\lambda_p)t} dt \\
& + K_0 \sum_{j=0}^{j=\ell} Y_{ij\ell} e^{\bar{\lambda}_{ij}T_S} \int_{T_S}^{T_L} e^{-(\bar{\lambda}_{ij}+\lambda_a-\lambda_p)t} dt \Big] \quad (56)
\end{aligned}$$

Performing the indicated integrations in the preceding equation results in the following equation to define the dose commitment over the time interval from T_I to T_L to the g -th organ of all the people in the region affected by the release due to immersion in air containing the ℓ -th daughter of the released nuclide 1.

$$\begin{aligned}
D_{mgi\ell} &= K_{mgi\ell} P_0 \left[\left(H_{i\ell} + K_1 \sum_{j=0}^{j=\ell} L_{ij\ell} \right) \frac{(e^{\lambda_p T_S} - e^{\lambda_p T_I})}{\lambda_p} - K_0 \frac{(e^{-(\lambda_a-\lambda_p)T_I} - e^{-(\lambda_a-\lambda_p)T_S})}{\lambda_a - \lambda_p} \right] \sum_{j=0}^{j=\ell} L_{ij\ell} \\
& - \sum_{j=0}^{j=\ell} \frac{(\epsilon_{i\ell} G_{i\ell j} + K_1 M_{ij\ell}) (e^{-(\bar{\lambda}_{ij}-\lambda_p)T_I} - e^{-(\bar{\lambda}_{ij}-\lambda_p)T_S})}{\bar{\lambda}_{ij} - \lambda_p} \\
& - K_0 \sum_{j=0}^{j=\ell} \frac{M_{ij\ell} (e^{-(\bar{\lambda}_{ij}+\lambda_a-\lambda_p)T_I} - e^{-(\bar{\lambda}_{ij}+\lambda_a-\lambda_p)T_S})}{\bar{\lambda}_{ij} + \lambda_a - \lambda_p} \\
& + K_0 \sum_{j=0}^{j=\ell} \frac{N_{ij\ell} (e^{-(\alpha_{ij}+\lambda_a-\lambda_p)T_I} - e^{-(\alpha_{ij}+\lambda_a-\lambda_p)T_S})}{\alpha_{ij} + \lambda_a - \lambda_p} \\
& + K_1 \sum_{j=0}^{j=\ell} \frac{N_{ij\ell} (e^{-(\alpha_{ij}-\lambda_p)T_I} - e^{-(\alpha_{ij}-\lambda_p)T_S})}{\alpha_{ij} - \lambda_p}
\end{aligned}$$

$$\begin{aligned}
& + \sum_{j=0}^{j=\ell} \left(K_1 Y_{ij\ell} + R_{i\ell j} \sum_{m=0}^{m=j} S_{ijm} \right) e^{\bar{\lambda}_{ij} T_S} \frac{(e^{-(\bar{\lambda}_{ij} - \lambda_p) T_S} - e^{-(\bar{\lambda}_{ij} - \lambda_p) T_L})}{\bar{\lambda}_{ij} - \lambda_p} \\
& + K_0 \sum_{j=0}^{j=\ell} \frac{U_{ij\ell} e^{\alpha_{ij} T_S} (e^{-(\alpha_{ij} + \lambda_a - \lambda_p) T_S} - e^{-(\alpha_{ij} + \lambda_a - \lambda_p) T_L})}{\alpha_{ij} + \lambda_a - \lambda_p} \\
& + K_0 \sum_{j=0}^{j=\ell} \frac{Y_{ij\ell} e^{\bar{\lambda}_{ij} T_S} (e^{-(\bar{\lambda}_{ij} + \lambda_a - \lambda_p) T_S} - e^{-(\bar{\lambda}_{ij} + \lambda_a - \lambda_p) T_L})}{\bar{\lambda}_{ij} + \lambda_a - \lambda_p} \\
& + K_1 \sum_{j=0}^{j=\ell} \frac{U_{ij\ell} e^{\alpha_{ij} T_S} (e^{-(\alpha_{ij} - \lambda_p) T_S} - e^{-(\alpha_{ij} - \lambda_p) T_L})}{\alpha_{ij} - \lambda_p} \Bigg] \quad (57)
\end{aligned}$$

2. Surface Deposit Dose Commitments

The dose commitment over the time interval from T_I to T_L to the g -th organ of all the people in the region affected by the release due to a surface deposit of the ℓ -th daughter of the released radionuclide i is obtained by modifying Equation (4) to reflect the assumptions and approximations stated previously and is given by

$$D_{sgil} = \int_{T_I}^{T_S} K_{sgil} P_0 e^{\lambda_p t} C_{i\ell}(t) dt + \int_{T_S}^{T_L} K_{sgil} P_0 e^{\lambda_p t} C_{i\ell}^*(t) dt \quad (58)$$

Using the expressions for $C_{i\ell}(t)$ and $C_{i\ell}^*(t)$ given in Equations (49) and (50), respectively, and rearranging yields the following:

$$\begin{aligned}
D_{sgil} = K_{sgil} P_0 & \left[\left(\sum_{j=0}^{j=\ell} L_{ij\ell} \right) \int_{T_I}^{T_S} e^{\lambda_p t} dt - \sum_{j=0}^{j=\ell} M_{ij\ell} \int_{T_I}^{T_S} e^{-(\bar{\lambda}_{ij} - \lambda_p) t} dt \right. \\
& + \sum_{j=0}^{j=\ell} N_{ij\ell} \int_{T_I}^{T_S} e^{-(\alpha_{ij} - \lambda_p) t} dt + \sum_{j=0}^{j=\ell} U_{ij\ell} e^{\alpha_{ij} T_S} \int_{T_S}^{T_L} e^{-(\alpha_{ij} - \lambda_p) t} dt \\
& \left. + \sum_{j=0}^{j=\ell} Y_{ij\ell} e^{\bar{\lambda}_{ij} T_S} \int_{T_S}^{T_L} e^{-(\bar{\lambda}_{ij} - \lambda_p) t} dt \right] \quad (59)
\end{aligned}$$

Performing the integrations defined in the preceding equation results in the following equation to define the dose commitment over the time interval from T_I to T_L to the g -th organ of the people in the region affected by the release due to a surface deposit containing the ℓ -th daughter of the released radionuclide i .

$$D_{sgi\ell} = K_{sgi\ell} P_0 \left[\left(\sum_{j=0}^{j=\ell} L_{ij\ell} \right) \frac{(e^{\lambda_p T_S} - e^{\lambda_p T_I})}{\lambda_p} - \sum_{j=0}^{j=\ell} \frac{M_{ij\ell} (e^{-(\bar{\lambda}_{ij} - \lambda_p) T_I} - e^{-(\bar{\lambda}_{ij} - \lambda_p) T_S})}{\bar{\lambda}_{ij} - \lambda_p} \right. \\ + \sum_{j=0}^{j=\ell} \frac{N_{ij\ell} (e^{-(\alpha_{ij} - \lambda_p) T_I} - e^{-(\alpha_{ij} - \lambda_p) T_S})}{\alpha_{ij} - \lambda_p} + \sum_{j=0}^{j=\ell} \frac{U_{ij\ell} e^{\alpha_{ij} T_S} (e^{-(\alpha_{ij} - \lambda_p) T_S} - e^{-(\alpha_{ij} - \lambda_p) T_L})}{\alpha_{ij} - \lambda_p} \\ \left. + \sum_{j=0}^{j=\ell} \frac{Y_{ij\ell} e^{\bar{\lambda}_{ij} T_S} (e^{-(\bar{\lambda}_{ij} - \lambda_p) T_S} - e^{-(\bar{\lambda}_{ij} - \lambda_p) T_L})}{\bar{\lambda}_{ij} - \lambda_p} \right] \quad (60)$$

where the values of $L_{ij\ell}$, $M_{ij\ell}$, $N_{ij\ell}$, $U_{ij\ell}$ and $Y_{ij\ell}$ are defined by the expressions given in Equations (51), (52), (53), (54), and (55), respectively.

3. Ingestion Dose Commitments

a. Ingestion of Food Contaminated Directly by Deposition—The dose commitment over the time interval from T_I to T_L to the g -th organ of the people in the region affected by the release due to the ingestion of food which is contaminated directly by the deposition of a given radionuclide is given by Equation (10). Applying the approximations and assumptions presented previously to Equation (10), the dose commitment from the ingestion of food crops which are directly contaminated by the deposition of the ℓ -th daughter of the released radionuclide i is given by

$$D_{tgi\ell} = \int_{T_I}^{T_S} K_{tgi\ell} P_0 e^{\lambda_p t} \left\{ \sum_f q_f \int_{T_{fa1}}^{T_{fb1}} W_{fi\ell} \left[v_{i\ell} \chi_{i\ell}(t) \right. \right. \\ + \left. \left. \int_0^Z \lambda_{si\ell} \chi_{i\ell}(t) dz \right] dt + \int_{T_S}^{T_L} K_{tgi\ell} P_0 e^{\lambda_p t} \left\{ \sum_f q_f \right. \right. \\ \left. \left. \int_{T_{fa2}}^{T_{fb2}} W_{fi\ell} \left[v_{i\ell} \chi_{i\ell}^*(t) + \int_0^Z \lambda_{si\ell} \chi_{i\ell}^*(t) dz \right] dt \right\} dt \right\} \quad (61)$$

Since the precipitation scavenging coefficient $\lambda_{si\ell}$ and the air concentrations $\chi_{i\ell}(t)$ and $\chi_{i\ell}^*(t)$ are not dependent upon Z , the integrals over Z in the preceding equation reduce to the following

$$\int_0^Z \lambda_{si\ell} \chi_{i\ell}(t) dz = \lambda_{si\ell} \chi_{i\ell}(t) Z, \quad (62)$$

$$\int_0^Z \lambda_{si\ell} \chi_{i\ell}^*(t) dz = \lambda_{si\ell} \chi_{i\ell}^*(t) Z \quad (63)$$

Using the results given in Equations (62) and (63) and rearranging the terms of Equation (61) yields the following expression:

$$D_{fgi\ell} = K_{fgi\ell} P_0(v_{i\ell} + Z\lambda_{si\ell}) \left[\left\{ \sum_f q_f W_{fi\ell} \int_{T_{fa1}}^{T_{fb1}} \chi_{i\ell}(t) dt \right\} \int_{T_I}^{T_S} e^{\lambda_p t} dt \right. \\ \left. + \left\{ \sum_f q_f W_{fi\ell} \int_{T_{fa2}}^{T_{fb2}} \chi_{i\ell}^*(t) dt \right\} \int_{T_S}^{T_L} e^{\lambda_p t} dt \right] \quad (64)$$

Performing the indicated integration over the T_I to T_S and the T_S to T_L intervals and substituting the expressions for $\chi_{i\ell}(t)$ and $\chi_{i\ell}^*(t)$ given in Equations (30) and (40), respectively, results in the following equation.

$$D_{fgi\ell} = K_{fgi\ell} P_0(v_{i\ell} + Z\lambda_{si\ell}) \left[\frac{e^{\lambda_p T_S} - e^{\lambda_p T_I}}{\lambda_p} \sum_f q_f W_{fi\ell} \right. \\ \left. \int_{T_{fa1}}^{T_{fb1}} \left(H_{i\ell} - \epsilon_{i\ell} \sum_{j=0}^{\ell} G_{ij\ell} e^{-\bar{\lambda}_{ij} t} \right) dt + \frac{e^{\lambda_p T_L} - e^{\lambda_p T_S}}{\lambda_p} \sum_f q_f W_{fi\ell} \right. \\ \left. \int_{T_{fa2}}^{T_{fb2}} \sum_{j=0}^{\ell} R_{ij\ell} \left(\sum_{m=0}^m S_{ijm} \right) e^{-\bar{\lambda}_{ij}(t-T_S)} dt \right] \quad (65)$$

Performing the indicated integrations over the time intervals during which deposition of the ℓ -th daughter of the i -th released nuclide on the f -th food type occurs results in the following expression for the dose commitment received by the people in the region affected by the release due to the ingestion of food upon which deposition occurs.

$$D_{fgi\ell} = K_{fgi\ell} P_0(v_{i\ell} + Z\lambda_{si\ell}) \left[\frac{e^{\lambda_p T_S} - e^{\lambda_p T_I}}{\lambda_p} \sum_f q_f W_{fi\ell} \left\{ H_{i\ell} (T_{fb1} - T_{fa1}) \right. \right. \\ \left. \left. - \epsilon_{i\ell} \sum_{j=0}^{\ell} G_{ij\ell} \frac{e^{-\bar{\lambda}_{ij} T_{fa1}} - e^{-\bar{\lambda}_{ij} T_{fb1}}}{\bar{\lambda}_{ij}} \right\} + \frac{e^{\lambda_p T_L} - e^{\lambda_p T_S}}{\lambda_p} \sum_f q_f W_{fi\ell} \right. \\ \left. \sum_{j=0}^{\ell} R_{ij\ell} e^{\bar{\lambda}_{ij} T_S} \frac{e^{-\bar{\lambda}_{ij} T_{fa2}} - e^{-\bar{\lambda}_{ij} T_{fb2}}}{\bar{\lambda}_{ij}} \sum_{m=0}^m S_{ijm} \right] \quad (66)$$

where the values of $H_{i\ell}$, $\epsilon_{i\ell}$, $G_{i\ell j}$, $R_{ij\ell}$, and S_{ijm} are given by the expressions in Equations (31), (32), (33), (41), and (42), respectively.

b. Ingestion of Food Produced in Contaminated Soil—The dose commitment over the time interval from T_I to T_L to the g -th organ of the people in the region affected by the release due to the ingestion of food produced in soil which contains the ℓ -th daughter of the released radionuclide i is obtained by modifying Equation (12) to reflect the assumptions and approximations stated previously and is given by

$$D_{tgi\ell} = \int_{T_I}^{T_S} K_{tgi\ell} P_0 e^{\lambda_p t} \sum_f q_f W_{fi\ell}^* C_{i\ell}(t) dt + \int_{T_S}^{T_L} K_{tgi\ell} P_0 e^{\lambda_p t} \sum_f q_f W_{fi\ell}^* C_{i\ell}^*(t) dt \quad (67)$$

Substituting the expressions given in Equations (49) and (50) for $C_{i\ell}(t)$ and $C_{i\ell}^*(t)$, respectively, and rearranging yields the following equation.

$$D_{tgi\ell} = K_{tgi\ell} P_0 \sum_f q_f W_{fi\ell}^* \sum_{j=0}^{j=\ell} \left[\int_{T_I}^{T_S} (I_{ij\ell} e^{\lambda_p t} - M_{ij\ell} e^{-(\bar{\lambda}_{ij} - \lambda_p)t} + N_{ij\ell} e^{-(\alpha_{ij} - \lambda_p)t}) dt \right. \\ \left. + \int_{T_S}^{T_L} (e^{\alpha_{ij} T_S} U_{ij\ell} e^{-(\alpha_{ij} - \lambda_p)t} + e^{\bar{\lambda}_{ij} T_S} Y_{ij\ell} e^{-(\bar{\lambda}_{ij} - \lambda_p)t}) dt \right] \quad (68)$$

Performing the indicated integration in the preceding equations results in the following expression for the total dose commitment to the g -th organ of the people in the region affected by the release due to the ingestion of food which is grown in soil which contains the ℓ -th daughter of the released radionuclide i .

$$D_{tgi\ell} = K_{tgi\ell} P_0 \sum_f q_f W_{fi\ell}^* \sum_{j=0}^{j=\ell} \left[L_{ij\ell} \frac{e^{\lambda_p T_S} - e^{\lambda_p T_I}}{\lambda_p} - M_{ij\ell} \frac{e^{-(\bar{\lambda}_{ij} - \lambda_p) T_I} - e^{-(\bar{\lambda}_{ij} - \lambda_p) T_S}}{\bar{\lambda}_{ij} - \lambda_p} \right. \\ \left. + N_{ij\ell} \frac{e^{-(\alpha_{ij} - \lambda_p) T_I} - e^{-(\alpha_{ij} - \lambda_p) T_S}}{\alpha_{ij} - \lambda_p} + U_{ij\ell} e^{\alpha_{ij} T_S} \frac{e^{-(\alpha_{ij} - \lambda_p) T_S} - e^{-(\alpha_{ij} - \lambda_p) T_I}}{\alpha_{ij} - \lambda_p} \right. \\ \left. + Y_{ij\ell} e^{\bar{\lambda}_{ij} T_S} \frac{e^{-(\bar{\lambda}_{ij} - \lambda_p) T_S} - e^{-(\bar{\lambda}_{ij} - \lambda_p) T_I}}{\bar{\lambda}_{ij} - \lambda_p} \right] \quad (69)$$

where the values of $L_{ij\ell}$, $M_{ij\ell}$, $N_{ij\ell}$, $U_{ij\ell}$, and $Y_{ij\ell}$ are defined by the expressions given in Equations (51), (52), (53), (54), and (55), respectively.

4. Inhalation Dose Commitments

The dose commitment over the time interval from T_I to T_L to the g -th organ of the people in the region affected by the release due to the inhalation of air containing the ℓ -th daughter of the released nuclide i is obtained by modifying Equation (14) in accordance with the assumptions and approximations stated previously and is given by

$$D_{agi\ell} = \int_{T_I}^{T_S} K_{agi\ell} B_a P_0 e^{\lambda_p t} \left[\chi_{i\ell}(t) + (K_0 e^{-\lambda_a t} + K_1) C_{i\ell}(t) \right] dt \\ + \int_{T_S}^{T_L} K_{agi\ell} B_a P_0 e^{\lambda_p t} \left[\chi_{i\ell}^*(t) + (K_0 e^{-\lambda_a t} + K_1) C_{i\ell}^*(t) \right] dt \quad (70)$$

Substituting the expressions given in Equations (30), (40), (49), and (50) for $\chi_{i\ell}(t)$, $\chi_{i\ell}^*(t)$, $C_{i\ell}(t)$, and $C_{i\ell}^*(t)$, respectively, into the preceding equation and rearranging terms yields

$$D_{agi\ell} = K_{agi\ell} B_a P_0 \left[H_{i\ell} \int_{T_I}^{T_S} e^{\lambda_p t} dt - \epsilon_{i\ell} \sum_{j=0}^{j=\ell} G_{ij\ell} \int_{T_I}^{T_S} e^{-(\bar{\lambda}_{ij} - \lambda_p)t} dt \right. \\ + K_0 \sum_{j=0}^{j=\ell} L_{ij\ell} \int_{T_I}^{T_S} e^{-(\lambda_a - \lambda_p)t} dt - K_0 \sum_{j=0}^{j=\ell} M_{ij\ell} \int_{T_I}^{T_S} e^{-(\bar{\lambda}_{ij} + \lambda_a - \lambda_p)t} dt \\ + K_0 \sum_{j=0}^{j=\ell} N_{ij\ell} \int_{T_I}^{T_S} e^{-(\alpha_{ij} + \lambda_a - \lambda_p)t} dt + K_1 \sum_{j=0}^{j=\ell} L_{ij\ell} \int_{T_I}^{T_S} e^{\lambda_p t} dt \\ - K_1 \sum_{j=0}^{j=\ell} M_{ij\ell} \int_{T_I}^{T_S} e^{-(\bar{\lambda}_{ij} - \lambda_p)t} dt + K_1 \sum_{j=0}^{j=\ell} N_{ij\ell} \int_{T_I}^{T_S} e^{-(\alpha_{ij} - \lambda_p)t} dt \\ + \sum_{j=0}^{j=\ell} R_{i\ell j} \left(\sum_{m=0}^{m=j} S_{ijm} \right) e^{\bar{\lambda}_{ij} T_S} \int_{T_S}^{T_L} e^{-(\bar{\lambda}_{ij} - \lambda_p)t} dt \\ + K_0 \sum_{j=0}^{j=\ell} U_{ij\ell} e^{\alpha_{ij} T_S} \int_{T_S}^{T_L} e^{-(\alpha_{ij} + \lambda_a - \lambda_p)t} dt + K_0 \sum_{j=0}^{j=\ell} Y_{ij\ell} e^{\bar{\lambda}_{ij} T_S} \int_{T_S}^{T_L} e^{-(\bar{\lambda}_{ij} + \lambda_a - \lambda_p)t} dt \\ \left. + K_1 \sum_{j=0}^{j=\ell} U_{ij\ell} e^{\alpha_{ij} T_S} \int_{T_S}^{T_L} e^{-(\alpha_{ij} - \lambda_p)t} dt + K_1 \sum_{j=0}^{j=\ell} Y_{ij\ell} e^{\bar{\lambda}_{ij} T_S} \int_{T_S}^{T_L} e^{-(\bar{\lambda}_{ij} - \lambda_p)t} dt \right] \quad (71)$$

Performing the indicated integrations in the preceding equations results in the following equation to define the dose commitment over the time interval from T_I to T_L to the g -th organ of the people in the region affected by the release due to the inhalation of air containing the ℓ -th daughter of the released radionuclide i .

$$\begin{aligned}
D_{agi\ell} = & K_{agi\ell} B_a P_0 \left[\left(H_{i\ell} + K_1 \sum_{j=0}^{j=\ell} L_{ij\ell} \right) \frac{(e^{\lambda_p T_S} - e^{\lambda_p T_I})}{\lambda_p} \right. \\
& - K_0 \frac{(e^{-(\lambda_a - \lambda_p) T_I} - e^{-(\lambda_a - \lambda_p) T_S})}{\lambda_a - \lambda_p} \sum_{j=0}^{j=\ell} L_{ij\ell} \\
& - \sum_{j=0}^{j=\ell} \frac{(\epsilon_{i\ell} G_{ij\ell} + K_1 M_{ij\ell}) (e^{-(\bar{\lambda}_{ij} - \lambda_p) T_I} - e^{-(\bar{\lambda}_{ij} - \lambda_p) T_S})}{\bar{\lambda}_{ij} - \lambda_p} \\
& - K_0 \sum_{j=0}^{j=\ell} \frac{M_{ij\ell} (e^{-(\bar{\lambda}_{ij} + \lambda_a - \lambda_p) T_I} - e^{-(\bar{\lambda}_{ij} + \lambda_a - \lambda_p) T_S})}{\bar{\lambda}_{ij} + \lambda_a - \lambda_p} \\
& + K_0 \sum_{j=0}^{j=\ell} \frac{N_{ij\ell} (e^{-(\alpha_{ij} + \lambda_a - \lambda_p) T_I} - e^{-(\alpha_{ij} + \lambda_a - \lambda_p) T_S})}{\alpha_{ij} + \lambda_a - \lambda_p} \\
& + K_1 \sum_{j=0}^{j=\ell} \frac{N_{ij\ell} (e^{-(\alpha_{ij} - \lambda_p) T_I} - e^{-(\alpha_{ij} - \lambda_p) T_S})}{\alpha_{ij} - \lambda_p} \\
& + \sum_{j=0}^{j=\ell} \left(K_1 Y_{ij\ell} + R_{ij\ell} \sum_{m=0}^{m=j} S_{ijm} \right) e^{\bar{\lambda}_{ij} T_S} \frac{(e^{-(\bar{\lambda}_{ij} - \lambda_p) T_S} - e^{-(\bar{\lambda}_{ij} - \lambda_p) T_L})}{\bar{\lambda}_{ij} - \lambda_p} \\
& + K_0 \sum_{j=0}^{j=\ell} \frac{U_{ij\ell} e^{\alpha_{ij} T_S} (e^{-(\alpha_{ij} + \lambda_a - \lambda_p) T_S} - e^{-(\alpha_{ij} + \lambda_a - \lambda_p) T_L})}{\alpha_{ij} + \lambda_a - \lambda_p} \\
& + K_0 \sum_{j=0}^{j=\ell} \frac{Y_{ij\ell} e^{\bar{\lambda}_{ij} T_S} (e^{-(\bar{\lambda}_{ij} + \lambda_a - \lambda_p) T_S} - e^{-(\bar{\lambda}_{ij} + \lambda_a - \lambda_p) T_L})}{\bar{\lambda}_{ij} + \lambda_a - \lambda_p}
\end{aligned}$$

$$+ K_1 \sum_{j=0}^{j=L} \frac{U_{ijL} e^{\alpha_{ij} T_S} (e^{-(\alpha_{ij} - \lambda_p) T_S} - e^{-(\alpha_{ij} - \lambda_p) T_L})}{\alpha_{ij} - \lambda_p} \quad (72)$$

where the values of H_{iL} , ϵ_{iL} , and G_{iLj} are defined by Equations (31), (32), and (33), respectively, the values of L_{ijL} , M_{ijL} , N_{ijL} , U_{ijL} , and Y_{ijL} are given by Equations (51), (52), (53), (54), and (55), respectively, and the values for R_{ijL} and S_{ijm} are defined by Equations (41) and (42), respectively.

IV. MODEL APPLICABILITY AND USE

The proper use of the model requires that each nuclide released from each facility to be examined by the model must have individual consideration. This must be done to ensure that the actual region affected by each released nuclide and its associated daughter set is adequately represented in the model. In general, the released nuclides and their daughters will fall into broad categories in which all the nuclides in a given category may be treated in a similar manner. Thus, the regions affected by particulates fall into a few size classes within a few thousand kilometer range of the release point, while gaseous nuclide releases fall into size classes of the affected regions which vary from a few thousand kilometers from the release point to the entire world.

The actual choice of the regions affected by the releases will not only be dependent upon the nuclides released but also upon the type of problem being examined. If the model is being used to estimate the dose commitments to the world which may be accrued from known releases from a given facility during a specific time period, the meteorological conditions which existed during that time interval can be used to define the regions which would be affected by each of the released nuclides and their daughters. On the other hand, if the model is being used to estimate the world dose commitments which may be encountered due to the expected releases from a proposed facility during some future typical time period, then the general meteorological conditions which have prevailed in the past for this type of time period can be used to estimate the regions which would be affected by the released nuclides and their daughters.

When the overall region which will be affected by the release of a specific nuclide and its daughters has been defined, it may be desirable to subdivide this affected region into a series of smaller sequential subregions to reflect the varying activity dispersion as a function of travel time from the release point. The need for this subdivision process of the affected regions becomes evident when the release of a long-lived gaseous isotope such as krypton-85 is considered. The entire world is affected by the release of a nuclide of this type. However, there is a two-year period subsequent to its release before it has been uniformly dispersed throughout the world's atmosphere. During this two-year period, there are subregions in which the population has been subjected to much higher air activity concentrations than the air activity concentrations associated with the entire world. Thus, to adequately estimate the total world dose commitment from a release of this nature, the overall world region should be subdivided into a relatively small region defined by a few days travel time from the release point and another region to represent the dispersion that occurs between a few days and a month after the release. Further, two more regions, representing the portions of the world in which the gaseous nuclides are distributed during the period which extends from one month to two years after release, should possibly be considered in addition to the entire world for the period subsequent to two years. The effect of the release upon the local population within approximately 100 kilometers of the release point may be found using the model for local dose commitments defined in Reference 2.

In general, when regions are treated in this manner, the populations in all smaller sequential regions should be excluded from the populations of all subsequent sequential regions until the released material has encircled the earth and the smaller regions are thus encompassed by a larger region with a lower air concentration. Further, when defining the air concentrations of nuclides subject to deposition and removal mechanisms other than decay while being transported in the atmosphere, the material which has been removed in a given sequential subregion should not be considered available for dispersion in the subsequent sequential regions. Thus, the amount of radioactivity which is assumed to be uniformly dispersed in a given region per unit time should be reduced by the fraction of that radioactivity which has been removed by processes other than decay in all preceding regions.

In addition to the considerations noted above, special care must be given to ensure that the dispersion of specific released nuclides, such as carbon-14 and tritium which rapidly come into equilibrium with the carbon and the hydrogen in the atmosphere, the upper layers of the oceans, and the biosphere, are treated adequately. A simple way to treat carbon-14 and tritium is given in Reference 1. Furthermore, the removal of nuclides which are chemically active by these chemical reactions, of nuclides with high ground adsorption properties, etc. should be considered in defining the regions affected by the releases and in defining the air radioactivity concentrations in these affected regions.

Some features which were considered during the development of this model but which were not incorporated in the model are presented below. Consideration of each of the items in this list should be given for each application of this model to a specific situation.

1. The contribution to the dose commitments resulting from the deposition of resuspended radioactivity (i.e., material which had been deposited and then subsequently resuspended) on the food crops which are produced in the regions affected by the releases is not incorporated in the model. The neglect of this term could result in an underestimation of the ingestion dose commitments by as much as 14 percent (Reference 1).
2. The model does not include the ability to permit some portion of the ingestion dose commitment to be obtained from the ingestion of food which is produced in a region other than the specific one being examined. The AIRWAY program will permit some portion of the diet of the people in the affected region to be obtained from an area which produces noncontaminated food, but no provisions are made to incorporate an external source of contaminated food.
3. The pathway associated with the deposition of material released into the atmosphere onto lakes, ponds, rivers, streams, etc. is not considered in this model. Thus, the dose commitments which may be received by the people in the region affected by the releases due to the contamination of their drinking water, their aquatic food such as fish and crustaceans, the water used for the irrigation of their crops, etc., from the deposition of airborne material are not estimated by this model. In addition, the model does not consider the consumption of wildlife which may contain radioactivity in its edible tissues due to the ingestion and inhalation of the released radioactivity.
4. The resuspension factor, used to define the amount of deposited activity which is resuspended, is a function of the time subsequent to the deposition of the material which is resuspended. However, to reduce the complexity of the model, the assumption was made that the time dependence of the resuspension factor can be represented as measured from the time that releases start rather than from the time of actual deposition.
5. During the model development consideration was given to the inclusion of procedures to develop the dose commitment conversion factors for ingestion and inhalation as a function of the time interval over which the dose commitments are to be calculated. Currently the model and the AIRWAY computer program only have values of the 70-year dose commitment conversion factors available for use.

V. SUMMARY

The preceding sections presented the equations developed for use in the LWBR environmental statement to estimate the dose commitment over a given time interval to a given organ of the population in the entire region affected by the atmospheric releases of a radionuclide and may be used for any assessment of dose commitments in these regions. These equations define the dose commitments to the world resulting from a released radionuclide and each of its daughters and the sum of these dose commitments provides the total dose commitment accrued from the release of a given radionuclide. If more than one radionuclide is released from a facility, then the sum of the dose commitments from each released nuclide and from each daughter of each released nuclide is the total dose commitment to the world from that facility. Furthermore, if more than one facility is considered as part of an industry, then the sum of the dose commitments from the individual facilities represents the total world dose commitment associated with that industry.

The actual solutions to these equations are carried out by the AIRWAY computer program. The writing of this computer program entailed defining in detail the specific representations of the various parameters such as scavenging coefficients, resuspension factors, deposition velocities, dose commitment conversion factors, and food uptake factors, in addition to providing specific numerical values for these types of parameters. The program permits the examination of more than one released nuclide at a time and performs the necessary summing to obtain the total dose commitment to the world accrued from the specified releases.

The program also provides the fraction of the total dose commitment from each isotope examined (i.e., all released radionuclides and all daughters of each released radionuclide).

Thus, a model has been developed to estimate the radiation dose commitments to the population of the world beyond the immediate vicinity of the point of release of radionuclides into the atmosphere. This model is amenable to solution via a computer program and contains a number of features which are not known to be available elsewhere. The most significant of these features is the ability to calculate automatically the dose commitments arising from the daughters of the released radionuclides which form subsequent to the release via radioactive decay. This model is based primarily upon the law of the conservation of mass and upon the relationship that radiation dose rates and dose commitments are directly proportional to the concentrations of the radionuclides.

VI. ACKNOWLEDGEMENTS

During the preparation of the LWBR environmental statement, many individuals contributed to the development of this model to estimate the dose commitments which would be received by the world's population from the atmospheric release of radionuclides. However, particular acknowledgement should be given to J. P. Witherspoon of the Environmental Sciences Division of the Oak Ridge National Laboratory for his aid in defining food ingestion pathways. In addition, considerable effort was performed by M. M. Baker and W. F. Grof to check and verify the model equations. Finally, the authors wish to thank L. R. Foulke for his support and encouragement throughout the development of the model.

VII. REFERENCES

1. Light Water Breeder Reactor Program, Final Environmental Statement, ERDA-1541, June 1976.
2. J. L. Rider and S. K. Beal, "A Model to Estimate the Local Radiation Doses to Man from the Atmospheric Release of Radionuclides," WAPD-TM-1273, December 1976.
3. L. Machta, et. al., "Regional and Global Scale Dispersion of Krypton-85 for Population Dose Calculations," Proc. Symposium on the Physical Behavior of Radioactive Contaminants in the Atmosphere, pages 411 - 426, IAEA, Vienna, 1974.
4. Regulatory Guide 1.111, "Methods for Estimating Atmospheric Transport and Dispersion of Gaseous Effluents in Routine Releases from Light-Water-Cooled Reactors," U. S. Nuclear Regulatory Commission, Office of Standards Development, March 1976.
5. L. R. Anspaugh, et. al., "Evaluation of the Resuspension Pathway Toward Protective Guidelines for Soil Contamination with Radioactivity," Lawrence Livermore Laboratory, IAC-SM-184/13, May 19, 1974.