Detection of Lower Hybrid Waves within a Plasma
by Microwave Scattering

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ABSTRACT

The spatial distribution and intensity of electrostatic waves injected into a hot plasma may be inferred from the scattering of millimeter microwaves. The report measurements on the 30° scattering of 8.6 mm microwaves by a 500 W, 2.45 GHz slow wave excited in a linear plasma by a phased array of two waveguides. From the magnitude of the scattered signal and auxiliary measurements with probes, we infer that a large fraction of the injected power penetrates to the center of the overdense test plasma.

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I. INTRODUCTION

Radio frequency heating near or above the lower hybrid frequency is a promising method of increasing the temperature of hot, toroidal plasmas.¹⁻³ Slow waves excited in the surface layers by waveguide arrays⁴⁻⁶ or loops⁷ propagate along resonance cones to the interior of the plasma torus, where they may damp and increase the kinetic energy of the plasma constituents. Wave penetration into a hot, dense toroidal plasma cannot be measured with a material probe, but has been inferred only indirectly, i.e., from the measured changes in plasma temperature.

In this paper we report the development of a microwave scattering technique to monitor the intensity and location of a lower hybrid beam injected into a linear test plasma, H-1, by a phased waveguide array. This technique, with appropriate modifications to allow for greater uncertainty in the position and wavenumber spectrum of the RF beam, could be adapted for use in hot, toroidal plasmas.

II. Test Plasma and Slow Wave Exciter

The experiment was carried out in a moderate density ($n \sim 2 \times 10^{12} \text{ cm}^{-3}$) linear argon plasma column 10 cm in diameter and 2 m long, produced by a 5 kW, 155 MHz discharge in an 8 kG solenoidal magnetic field⁸. Up to 500 W of power at 2.45 GHz was launched into the plasma column from a phased twin waveguide^{9,10} (Fig. 1).

This frequency is below the electron plasma frequency ω_{pe} but well above the lower hybrid frequency $\omega_{LH} = \omega_{pi} / (1 + \omega_{pe}^2 / \omega_{ce}^2)^{1/2}$, so that slow electrostatic waves may be excited in the plasma column and should penetrate to the interior of the plasma column if the wavelength parallel to the field is short and the magnetic field large.^{11,12} For our conditions $\omega \gg \omega_{LH}$, so that the waves can be described by the electrostatic dispersion relation $\omega / \omega_{pe} = k_{||} / k_{\perp}$ with the wavenumber parallel to the field, $k_{||} \ll k$. The term lower hybrid waves has been loosely applied to these slow waves in the literature, although they represent a branch of the well-known Gould-Trivelpiece modes.¹³ In any case, the mode we have studied is the same as that generated in any lower hybrid heating experiment.

In our low temperature test plasma, it is possible to map the position and wavenumber of the slow waves by means of a well-shielded triaxial probe. As shown in Fig. 2 the twin waveguide excited a broad band of wavelengths perpendicular to the magnetic field which were dispersed by the plasma column into a broad resonance cone structure. In fact, the short wavelength components of the spectrum bunch on the leading edge of the cone, while the long wavelength components are on the trailing edge (Fig. 2), as shown in previous work.⁹ From the probe alone, however, we are unable to ascertain either the power in the waves or the distribution of intensity in various wavelengths. These constraints can be removed by measurements of microwave scattering.¹⁴

III. The Scattering Apparatus

Two microwave horns were positioned at a fixed position displaced 20 cm axially from the waveguide array (Fig. 1). Space charge waves excited by the array perturbed the electron density

in a region midway between the plasma center and the far side of the plasma column. Signals scattered from the 2 W incident beam by $\sim 30^\circ$ were collected by the receiving horn and sent to a superheterodyne receiver. The frequency of the incident beam was 35 GHz, the mean scattering angle was 30° , and the half beam width of the two antennae was 8° . The frequency of the scattered beam was either 32.5 or 37.5 GHz, depending on the direction of the phase velocity with respect to the antennae.

A block diagram of the transmitting-receiving apparatus is shown in Fig. 3. The scattered signal was sent through a microwave bandpass cavity (30 MHz bandwidth) to remove the direct feedthrough from the transmitter, which would otherwise overload the detecting crystals. At the receiver the signal was mixed with the output from a Gunn diode, and the beat frequency was amplified in a 30 MHz IF amplifier before detection. Since the system requires that the difference in frequency between the klystron, the Gunn diode, and the magnetron be 30 MHz, we devised a convenient tuning indicator, consisting of two cascaded crystal mixers, shown within the dashed lines of Fig. 3. A calibration of the entire system was achieved by placing a copper reflecting plate at the position of the electrostatic waves and measuring the magnitude of the reflected signal.

IV. Scattering Theory

The scattering theory relevant to our experiment was first derived by Brillouin¹⁵ to account for the scattering of light by ultrasonic waves and was later generalized to a plasma medium by many authors.^{15,17} According to this theory the power scattered out of a monochromatic microwave beam by a density fluctuation Δn in a volume element $d\bar{V}$ into a unit solid angle $d\Omega$ is

$$\frac{dW}{dt} = \frac{cr_0^2}{8\pi^2} \left\{ \int \Delta n(\vec{V}) \exp [-i(\vec{k} + \vec{k}_s - \vec{k}_i) \cdot \vec{r}] dV \right\}^2 E^2 d\Omega, \quad (1)$$

and

$$\omega_s = \omega_i + \omega,$$

where r_0 is the classical electron radius, Δn is the amplitude of the density perturbation induced by the heating wave, E^2 is the amplitude of the electric field of the probing wave, and $\vec{k}$, $\vec{k}_s$, and $\vec{k}_i$ are the wavenumbers of the density perturbation, the scattered and incident microwave signal. To contribute significantly to the integral, $\vec{k}_i \approx \vec{k}_s + \vec{k}$, which is equivalent to the Bragg condition for scattering

$$|k| \approx 2|k_i| \sin \theta/2 \approx |k_i|/2, \quad (2)$$

where $\theta = 30^\circ$ is the scattering angle.

A numerical calculation of the scattering integral, given in Fig. 4, shows that for our conditions the spatial resolution is ~ 2 cm, and that the scattering apparatus is sensitive to wavelengths between 1.3 and 3 cm. These parameters are determined both by the spread of the microwave beam and the finite extent of the scattering volume, both of which are taken into account in the numerical calculation.

V. Experimental Results

We have detected a scattered signal at a power level about 200 times the noise level of the receiver (20,000°K). The amplitude of this signal can be affected in three different ways: Firstly, by changing the electron density. The density varies the

angle between the resonance cone and the magnetic field, $0 \approx \omega/\omega_{pe}$,² and therefore the radial position of the cone. Maximum signal amplitude was measured (Fig. 5) when the center of the resonance cone (the short wavelength component) was ~3 cm from the column center, i.e., just a few millimeters outside the expected position. The radial resolution was ~1.5 cm, i.e., slightly less than that suggested by the probe data (Fig. 2).

Secondly, we can vary the magnetic field, normally set at 8 kG, and show that there is essentially no wave penetration to the plasma core if $B < 4$ kG. This result was confirmed with the probe and is in agreement with the accessibility criterion for slow waves.^{11,12}

Thirdly, the scattering is sensitive to wavelength (Fig. 4). The wavelength spectrum of the waves can readily be varied by changing the phase between individual waveguides of the antenna array. The exact shape of the wavenumber spectrum can be calculated from the boundary value theory of Brambilla.⁵ In general, in-phase excitation of the waveguide elements favors the generation of long waves ($\lambda_{||} \sim 12$ cm) that do not penetrate the surface layers. Out-of-phase excitation favors the generation of short waves ($\lambda_{||} \sim 5-6$ cm, $\lambda_{\perp} \sim 1.3 - 1.7$ cm) that penetrate the surface layers to the position of the microwave horns. Only this component gives rise to microwave scattering. On this basis the scattering signal should peak at 180° phasing. One other factor, however, alters this behavior slightly. Phased arrays also exhibit left-right directional characteristics; as the phase is varied either above (or below) 180°, more (or less) if the microwave power is beamed along the plasma column in the direction of the scattering apparatus. The relative scattering signal expected from the

short wave component and predicted by the Brambilla theory⁵ is shown in the solid line of Fig. 6. Note that the peak in the scattered signal should occur at $\sim 200 - 210^\circ$.

The experimental results are also displayed in Fig. 6. There is a large variation of the scattered signal with phase, as expected. However, the shift of the peak from 180° is $\sim 45^\circ$, somewhat larger than that expected. The difference may result from a combination of factors, including error in the phase measurements and asymmetries introduced into the plasma column by the intrusion of the double waveguide. (The waveguide exciter was located between the rf plasma source and the scattering apparatus.) An experimental check of the scattering symmetry from microwave horns placed in the same cross section as the waveguide showed symmetry about 180° , as expected.

A measurement with the rf probe in the (normal) position of the scattering apparatus displayed a much larger shift in the peak amplitude to 270° . This results from two factors: (1) the upstream-downstream asymmetry of the phased array and (2) the enhanced sensitivity of the probe to long wavelengths (the detected signal is proportional to the rf potential squared and thereby to E^2/k^2).

The data shown in Fig. 6 were taken with microwave absorbers encircling the vacuum vessel near the scattering ports to avoid multiple scattering of the beam from the stainless steel vacuum walls. If the absorbers were removed, the peak signal level increased by $\sim 30\%$ and the minimum signal level increased by an order of magnitude. In addition to the normal upshifted signal near 37.5 GHz, a second signal downshifted in frequency by 2.5 GHz was detected at the receiver. We believe that these effects were caused by degradation of the wavenumber resolution by multiple scattering of the incident microwave beam

from the stainless steel vacuum chamber.

From the intensity of the scattered signal one can calculate the power flow in the space charge waves. The electric field of the space charge waves is related to the power flow through the

$$P = \frac{\bar{E}}{8\pi} \cdot \frac{\partial \omega \bar{\epsilon}}{\partial \omega} \cdot \bar{E} v_g A ,$$

where v_g is the group velocity $\approx \omega/k_{\parallel}$, A is the cross-sectional area of the cone, and $\bar{\epsilon}$ is the dielectric tensor. The perturbation may be obtained from the Poisson equation

$$i \bar{k} \cdot \bar{E} = 4\pi \Delta n e ,$$

where Δn is the variation of the electron density induced by the power wave. (Since $\omega \gg \omega_{LH}$, $\Delta n_{\perp}$ is only a small fraction of $\Delta n_{\parallel}$ and may be neglected.) One may then relate the power launched into the resonance cones to the electric field, the density perturbation, and the scattering power through equations (3), (4), and (5).

For our conditions, if all of the wave energy were scattered into short wavelengths, the ratio of the scattered power to the density wave to that from a perfect reflector located at the scattering volume should be 5.1×10^{-8} . Experimentally we have measured a scattering ratio of 1.0×10^{-8} , or about 20% of the theoretical level. [We estimate that $\Delta n \sim 3 \times 10^8 \text{ cm}^{-3}$ at $E_{\perp} \sim 140 \text{ V/cm}$]. From the theory of Brambilla,⁵ or from that about 1/3 of the RF power should be in the spectrum that penetrates to the plasma center and is within the range of our scattering apparatus ($2 \leq n_{\parallel} \leq 3$). Therefore we can expect for about 2/3 of the expected energy in this interval. This represents the first measurement inside the plasma column of the

radiated from a waveguide array.

VI. Discussion

The success of the scattering experiment in our test plasma is encouraging for experiments planned on toroidal plasmas. In particular, a 2 mm scattering experiment has been designed to detect lower hybrid waves injected into the Versator II tokamak at M.I.T.¹⁸ It is of some interest to estimate the signal to noise ratio expected in this experiment. The lower hybrid power density on Versator should be ~ 20 times greater than in H-1, the solid angle should be similar (the distance between horns is greater but the microwave beam should be better focussed), the microwave power density should be 10 times higher, and the noise level may be ~ 30 times greater. The expected signal to noise ratio should then be greater than 1000/1 on Versator, so that detection should pose no problem, provided most of the power penetrates the surface and the resonance cone is not strongly defocussed. Since it may not be possible to use rf probes to measure $\bar{k}$, it will be necessary to change both the scattering angle and the position of the scattering volume, in order to estimate both the wavelength spectrum and the intensity of the wave packet.

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FIGURE CAPTIONS

Fig. 1. Top and end views of the plasma column, the resonance cones excited by the twin-waveguide, and the 8.6 mm transmitting and receiving horns.

Fig. 2. Amplitude (lower curve) and phase (upper curves) of the 2.45 GHz resonance cone measured in the horizontal direction with a triaxial probe. The dashed line of the interferometer signal was taken after shifting the reference signal 180° with respect to the wave signal.

Fig. 3. Block diagram of the microwave transmitter and superheterodyne receiver. The double mixer inside the dashed lines is used to tune the difference frequency of the two klystrons and the high power magnetron to 30 MHz. The local oscillator could be changed to be sensitive either to 37.5 GHz or to 32.5 GHz. The direct single-pass signal was scattered at 37.5 GHz.

Fig. 4. Scattering integral (in Eq. 4) as a function of radial position and wavelength. In the calculation the horizontal width of the wave packet was assumed to be one wavelength; the vertical width, 4 cm; the distance between the vertex of the microwave horn and the scattering volume was 18 cm; the half width of the antenna beam was 8° .

Fig. 5. Variation of the 37.5 GHz scattered signal as a function of the radial position of the short wavelength component of the resonance cone (measured with the rf probe).

Fig. 6. Influence of waveguide phasing on the scattered microwave signal, experiment (full circles) and theory. The upstream-downstream asymmetry between 90° and 270° can easily be noted on the probe signal (dashed line).

Fig. 1. 776196

Fig. 2. 773981

Fig. 3. 776195

Fig. 4. 776198

Fig. 5. 773972

Fig. 6. 773973