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MEASURING AND MODELING SOLIDS MOVEMENT IN A LARGE, COLD FLUIDIZED BED TEST FACILITY

Third Quarterly Report for April 1-June 30, 1980

By
Thomas J. Fitzgerald
Robert V. Mrazek
Stephen D. Crane

June 1980

Work Performed Under Contract No. AC01-79ET13152

Oregon State University
Department of Chemical Engineering
Corvallis, Oregon

U. S. DEPARTMENT OF ENERGY

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TEST OF MAGNETIC FLOW MEASURING DEVICE

The magnetic particle mass flow measuring device described in the first quarterly report was tested using the set-up shown in Figure 1. Particles were magnetized by a pair of record/erase coils arranged coaxially with opposite polarities, so that the magnetic lines of force would be most dense at a plane midway between the two coils. Each coil was 2 inches in diameter and 1/2 inch thick. The spacing between the coils was 2 inches center to center. Each coil consisted of 40 turns of number 18 wire, giving an impedance at 2,000 hertz of approximately 5 ohms. The pickup coil was 3 inches in diameter and was situated midway between the two record/erase coils. It consisted of 2,000 turns of number 36 wire. A cardboard tube 1-1/2 inches in diameter was filled with 16-mesh iron particles and was pulled through the coil with a sinusoidally varying velocity by a mechanical drive system.

The electronic instrumentation is shown in block diagram form in Figure 2. Figure 3 shows the time sequence of record, read, and erase pulses. The record signal was a simple pulse of one polarity. The duration of the pulse was approximately 0.5 milliseconds. The erase signal was a 2 kHz square wave with an exponentially decaying envelope of about 20 milliseconds length. Figure 4 shows a typical output trace. The sinusoidally varying velocity is clearly represented in the output, indicating good linearity. Results for various velocities are tabulated below and shown in graphical form in Figure 5.

<u>Distance Travelled by iron mass (inches)</u>	<u>Frequency of Oscillation (Hz)</u>	<u>Record Amplifier Power (watts)</u>	<u>Output Signal Amplitude (volts)</u>
14	0.5	50	0.8
14	0.5	100	1.2
14	0.5	150	1.6
14	0.5	200	2.3
14	0.73	50	1.4
14	0.73	100	1.75
14	0.73	150	2.3
14	0.73	200	3.0
14	0.88	50	1.6
14	0.88	100	2.1
14	0.88	150	2.6
14	0.88	200	3.6

TABLE 1 Output signal from magnetic-solids mass flow detector for 3 frequencies and 4 amplifier power levels.

It can be seen that the output signals are proportional to the oscillating velocities (as they should be). The output level increases with amplifier power which indicates that saturation has not been achieved at the power levels used in this test. Saturation of material in the vicinity of the pick-up coil is necessary if one is to obtain a valid measure of the mass flow rate. Therefore in future tests the power or the pulse length will have to be increased or the coil size will have to be decreased.

Fig. 1. Experimental set up for the iron filled tube oscillating inside the sensing coil by means of a motor and a set of pulleys.

Fig. 2. Block diagram of electronics of the magnetic solids mass flow detector.

- Notes: 1. The sequence, Record → Delay → Window → Erase → Record-----, is controlled by the trailing edge of the preceeding function, indicated by ( ) on the block diagram.
2. Individual pulse widths are adjustable independently; the cycle time is the total time of all functions.
3. Amplitudes of RECORD and ERASE signals are adjustable independently; as well as together by the attenuator of the POWER AMPLIFIER.

Fig. 3. Timing diagram of control circuit.

Fig. 4 Typical record of output signal for a sinusoidally varying velocity.

Maximum excursion was 14 inches, frequency 0.88Hz, sampling frequency
25 per second.

Fig. 5. Response signal as a function of amplifier power does not show saturation behavior at power levels tested.

A SHORT STUDY OF POINT SOURCE TRACER GAS
DISPERSION IN A SCALED 3-DIMENSIONAL FLUIDIZED BED

An experimental study of the dispersion of a tracer gas from a point source in a three-dimensional cold, scaled, fluidized bed has been completed. A "meandering" plume model was applied to the bulk gas flow in an effort to separate the effects of dispersion and the movement of the gas plume. These effects will have an important influence on solids movement in the fluidized bed in the vicinity of the coal feed port(s). A number of statistical tests were applied to the experimental data to determine the significance of the measured variation in tracer concentration.

Generally, the analyses indicate that a large portion of the apparent gas dispersion coefficient should be attributed to the meander of the gas plume. A dispersion coefficient which is obtained by comparing the time-average tracer concentration at some location in the bed to the tracer inlet rate will overestimate the "real" turbulent/molecular dispersion contribution by as much as 60-70%. A complete description of the experimental measurements and statistical methods is given in the Appendix; as are more detailed conclusions.

A PROGRESS REPORT

A tube array 1' x 3' x 3' (0.30 x 0.91 x 0.91 m) using 1/2" diameter dummy heat exchange tubes has been constructed for Oregon State University's 3' x 3' cold fluidized bed test facility. The tube spacing can be varied from a tight 1.5" x 1.5" (3.8 x 3.8 cm) square pitch configuration to an open triangular pitch configuration. The tube diameter, spacing and pitch were modeled after an existing 6' x 6' (1.82 x 1.82 m) hot fluidized bed test facility at the Babcock and Wilcox Co. in Alliance, Ohio.

A SMALL SCALE EFFORT TO PRODUCE A WELL MIXED CIRCULATION PATTERN

The main thrust of the research which is being conducted by Oregon State University is to develop a predictive model which can be used in optimizing tube arrangements and coal feed point locations in a fluidized bed combustor.

Many experiments in the past have looked at the effect of equally spaced heat exchange tubes on bed circulation. However, there has not been an investigation of how to create desired circulation patterns by using an unequally spaced tube configuration. That is, one may be able to create a rapidly circulating and hence well mixed bed by placing a closely spaced tube array in one portion of a bed and a loosely spaced array in the other portion. A small scale effort has been initiated to examine this area.

Experiments will be conducted in a two-dimensional cold fluidized bed (19" x 5" x 72") using scaled dummy heat exchange tubes and scaled bed material. The tube pitch and spacing will be varied in an attempt to create a rolling well-mixed circulation pattern. Movies of each system will be used to make the comparisons.

A tube array which can be varied from a closely spaced square pitch configuration to a loosely spaced triangular configuration has been constructed. The diameter of the dummy heat exchange tubes is 1/2 inch (1.27 cm). The bed media of iron powder is currently being sieved in order to match the size distribution of the spent bed material from Babcock and Wilcox's 6' x 6' (1.83 x 1.83 m) hot fluidized bed located in Allcance, Ohio.

The experiments should commence around the second week of July.

APPENDIX

**A SHORT STUDY OF POINT SOURCE TRACER GAS DISPERSION IN A SCALED
3-DIMENSIONAL FLUIDIZED BED**

by

Clayton Gosmeyer

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NOMENCLATURE

<u>Symbol</u>	<u>Description</u>	<u>Dimensions</u>
A-G, A'-E'	Sensor locations, see Figure 1	--
A_{xy}	Probability density window area, Equation (7)	L^2
a, b	Positions relative to vertical (z) axis through injector	L
$c(x, y, z)$	Instantaneous tracer concentration	mole/ L^3
$\bar{c}, \bar{c}^2$	Time average tracer concentration, Equation (5,6)	mole/ L^3 , mole ² / L^9
$\bar{c}^*, \bar{c}^{2*}$	Dimensionless tracer concentration, Equation (A21) etc.	dimensionless
$\bar{d}_p$	Mean bed particle diameter $[\sum \frac{w_i}{d_{pmi}}]^{-1}$	L
D_x, D_y, D_z, D^1	Dispersion coefficients due to eddy/molecular effects	L^2/time
D_m^1, D_{mx}, D_{my}^1	Dispersion coefficient due to meander effect	L^2/time
P_i	Probability density estimate	L^{-2}
z_1, z_2	Distances along bulk flow direction relative to the injector (values in Figure 4)	L
$\dot{M}$	Tracer injection rate	mole/time
U_o	Superficial velocity	L/time
U_{mf}	Minimum fluidization velocity	L/time
U_f, U, U_z	Interstitial velocity U_o/ϵ_f	L/time

¹Note: $D_i = U_z \sigma_i^2 / 2z$

NOMENCLATURE (continued)

Greek Symbols

ϵ_p	voidage of packed bed material
ϵ_f	average fluidized voidage
ϵ	fluidized voidage exclusive of meander
ρ_s	solids density
ρ_b	bulk (piled) density
σ^2	variance

Dimensions

dimensionless
dimensionless
dimensionless
M/L^3
M/L^3
L^2

Introduction

The dispersion of a tracer gas into bulk gas flow is of interest in many problems involving fluidized bed reactions, including solids and gas mixing in the vicinity of a coal feed port and overall solids and gas movement.

Gifford (1959, 1960) proposed a "meandering" plume superimposed on an eddy diffusion concentration profile to describe overall exhaust stack plume dispersion. More recently Jovanovic (1979) developed a similar model to describe tracer gas dispersion in a fluidized bed. The latter author presented experimental and analytical results for the case of a line source of tracer gas in a 2-dimensional bed.

The meandering plume model for gas flow through a fluidized bed consists of the convolution of two effects, Figure 1. One effect is that of turbulent and molecular diffusion of a gas volume, element dV , as it moves through the bed. The other effect is that of lateral displacement of element dV by forces produced by bubble motion, bed internals, etc. This effect has been termed plume meander (Gifford (1960)) and is assumed not to result in additional mixing of the gas from element dV with the surrounding gas, but rather it represents a lineal displacement of all of the gas in the vicinity of dV relative to some fixed point.

To carry this one step further in an effort to shed more light on how gas and solids move in a fluidized bed an analytic and experimental study was initiated to examine gas dispersion from a point source in a 3-dimensional fluidized bed.

For this experimental work, the point of reference is taken as the location of a tracer gas injector, located immediately above and at the center of the air distribution plate. A 3-dimensional Cartesian coordinate system

Figure 1. Meandering plume dispersion model.

assumed to originate at the injector tip. The "z" direction is considered positive in the direction of the bulk gas movement. The "x" direction is parallel to a horizontal tube array which is immersed in the bed and the "y" direction is perpendicular to the axes of the tubes in the array.

The meander effect, based on this coordinate system, is assumed to be described by a joint Gaussian probability density function with independence between x and y location:

$$m(x,y,z) = \frac{U}{4\pi z D_{mx}^{1/2} D_{my}^{1/2}} \exp \left\{ \frac{-U}{4z} \left[\frac{x^2}{D_{mx}} + \frac{y^2}{D_{my}} \right] \right\} \quad (1)$$

where

$$D_{mx} = \frac{\sigma_{mx}^2 U}{2z}$$

$$D_{my} = \frac{\sigma_{my}^2 U}{2z}$$

$m(x,y,z)$ represents the fraction of time that volume elements dV , which originate at the tracer gas injector, will be found centered at (x,y,z) (after a long observation time). Note that Equation (1) does not depend on z , except through the dispersion coefficients D_{mx} and D_{my} .

The turbulent and molecular dispersion is assumed to be described by Fick's second law as:

$$\nabla^2 D_i c = U \frac{\partial c}{\partial z} \quad (2)$$

which for the present case may be reduced to:

$$D_x \frac{\partial^2 c}{\partial x^2} + D_y \frac{\partial^2 c}{\partial y^2} + D_z \frac{\partial^2 c}{\partial z^2} = U \frac{\partial c}{\partial z} \quad (3)$$

Following the method of Jovanovic (1979), the time averaged concentration measured at a point (x,y,z) is given by the convolution of the meander with the solution of the differential equation. Appendix A gives the solutions of the differential equation for both isotropic and anisotropic media.

The isotropic solution is:

$$c(x,y,z) = \frac{\dot{M}}{4\pi D(x^2+y^2+z^2)^{1/2}} \exp \left\{ \frac{-U[(x^2+y^2+z^2)^{1/2}-z]}{2n} \right\} \quad (4)$$

As mentioned the time average concentration measured at point (x,y,z) is given by

$$\bar{c}(x,y,z) = \iint_{-\infty}^{\infty} m(v,w,z) c(x-v,y-w,z) dv dw \quad (5)$$

The time average mean of the square of the instantaneous concentrations is given by

$$\bar{c}^2(x,y,z) = \iint_{-\infty}^{\infty} m(v,w,z) c^2(x-v,y-w,z) dv dw \quad (6)$$

Neither of these convolutions can be performed analytically using the diffusion equation solution given above.

A way to alleviate the convolution integration problem of Equations (4) and (5) is to assume an approximate form for the diffusion concentration profile which can be integrated.

A reasonable approximation (over the range of physical parameters en-

countered in the experimental apparatus) is given by

$$c(x,y,z) \cong \frac{\dot{M}}{4\pi D_z} \exp \left\{ \frac{-U}{4zD} (x^2 + y^2) \right\} \quad (7)$$

Equation (7) is for an isotropic medium. A graphical comparison of this equation with the exact solution (Equation (4)) is presented in Figure 2.

Using Equation (7):

$$D = \left[\frac{U\bar{a}^2}{z \ln \bar{c}^{2*}} - \frac{U\bar{a}^2}{z \ln \bar{c}^*} \right] \quad (8)$$

and

$$D_m = \frac{D}{2} \left[\frac{2 \ln \bar{c}^{2*} - \ln \bar{c}^{2*}}{\ln \bar{c}^{2*} - \ln \bar{c}^*} \right] \quad (9)$$

See Appendix A for details. $\bar{c}^*$ and $\bar{c}^{2*}$ are the normalized mean and mean square concentrations measured at point (a,a,z) with reference to (0,0,z).

The overall observed dispersion coefficient is given by

$$D_T = D + D_m \quad (10)$$

If it is assumed that the fluidized bed is an anisotropic medium with respect to turbulent/molecular dispersion then the solution to Equation (3) is:

$$c(x,y,z) = \frac{\dot{M}}{4\pi \{D_x D_y D_z \left[\frac{x^2}{D_x} + \frac{y^2}{D_y} + \frac{z^2}{D_z} \right] \}^{1/2}} \times \\ \times \exp \left\{ \frac{U}{2D_z^{1/2}} \left[\frac{z}{D_z^{1/2}} - \left(\frac{x^2}{D_x} + \frac{y^2}{D_y} + \frac{z^2}{D_z} \right)^{1/2} \right] \right\} \quad (11)$$

Figure 2. Comparison of exact and approximate solutions of the diffusion equation., continued.

where D_x , D_y and D_z are the turbulent/molecular dispersion parameters (analogous to D of the isotropic case) for the x , y and z directions respectively. The details are presented in Appendix A.

As before it was not found possible to use Equation (11) directly to perform the convolutions of Equations (5) and (6), consequently an approximation is used. The approximate form is given by Equation (12).

$$c(x,y,z) \cong \frac{\dot{M}}{4\pi z D_x^{1/2} D_y^{1/2}} \exp \left\{ \frac{-U}{4z} \left(\frac{x^2}{D_x} + \frac{y^2}{D_y} \right) \right\} \quad (12)$$

Figure 3 compares the approximate and exact solutions using extreme values for the physical parameters and with various axial turbulent/molecular dispersion coefficient values.

As indicated in Appendix A, if concentration measurements are made at $(a,0,z)$, $(0,b,z)$ and $(0,0,z)$ then the dispersion parameters are given by:

$$D_x = \frac{Ua^2}{2z} \left[\frac{1}{\ln \bar{c}_a^{2*}} - \frac{1}{\ln \bar{c}_a^*} \right] \quad (13)$$

$$D_y = \frac{Ub^2}{2z} \left[\frac{1}{\ln \bar{c}_b^{2*}} - \frac{1}{\ln \bar{c}_b^*} \right] \quad (14)$$

$$D_{mx} = \frac{D_x}{2} \left[\frac{2 \ln \bar{c}_a^* - \ln \bar{c}_a^{2*}}{\ln \bar{c}_a^{2*} - \ln \bar{c}_a^*} \right] \quad (15)$$

$$D_{my} = \frac{D_y}{2} \left[\frac{2 \ln \bar{c}_b^* - \ln \bar{c}_b^{2*}}{\ln \bar{c}_b^{2*} - \ln \bar{c}_b^*} \right] \quad (16)$$

The overall coefficients are then:

$$D_{Tx} = D_x + D_{mx} \quad (17)$$

Figure 3. Comparisons of exact and approximate solutions of the anisotropic diffusion equations.

Figure 3. Continued

and

$$D_{Ty} = D_y + D_{my} \quad (18)$$

Experimental Conditions

The sensor locations used for the experimental work is described in Figure 4. A perforated plate is used as the air distributor. The bed material consists of approximately spherical copper particles with a mean particle size of 215 μm (see Table 1). The probability density function for the bed material size distribution is shown in Figure 5. The bed weight is approximately 732 lbm.

The tracer gas is injected through a tube with an inside diameter of 0.03 inches. The injection rates are listed in Table 2.

The tracer gas concentrations are measured with photo-ionization detectors and data is collected for a period of five minutes. The output signals are digitized at approximately 100 times per second and logged by a NOVA minicomputer. Processing is done on a CDC CYBER computer.

Isotropic Analysis

Thirty-two data runs were made with sensor locations being of the type $(\pm a, \pm a, z)$ and $(0, 0, z)$. Sixteen runs were made at level $z_1 = 6\text{-}1/2$ inches above the tracer injector tip and 16 at $z_2 = 9\text{-}1/2$ inches above the tracer injector tip.

The sensor locations are of the type $|x| = |y|$ and therefore it must be assumed that the bed behaves as an isotropic dispersion medium in order to use the meandering plume model to extract the component dispersion coefficients. Equations (8), (9) and (10) were used to calculate D , D_m and D_T .

Notes:

1. Z₁ level is 6 1/2" above injector.
2. Z₂ level is 9 1/2" above injector.
3. Injector tip is 2 1/4" above plate.
4. First row of tubes is 5" above plate.
5. Sensors NE and SE are in the same tube, etc.
6. Tubes run north to south.

Sensor locations viewed from top

tube array viewed from end

view of plate from top

7. 18 rows of tubes vertically
8. 24 rows of tubes horizontally
9. sensors in 6th and 10th rows from the bottom and on the 5, 9, 13, 17th from the front
10. the central sensor is located on a vertical axis through the injector

Figure 4. Fluidized bed array and sensor layout.

Figure 5. Measured probability density of material size distribution.

Table 1. Bed material characteristics.

Tyler Mesh	d_p , μm	Weight Fraction
>10	-	$\emptyset$
10-12	1545	.0029
12-14	1300	.0987
14-20	1016	.1870
20-28	718	.0672
28-35	508	.1950
35-48	359	.1767
48-65	254	.0643
65-80	194	.0587
80-100	163	.0749
100-200	112	.0605
<200	37	.0145

$$\rho_s = 8160 \text{ kg/m}^3$$

$$\rho_b = 5660 \text{ kg/m}^3$$

$$\overline{d_p} = 315 \text{ } \mu\text{m}$$

$$\epsilon_p = 0.306$$

$$U_{mf} \times 0.24 \text{ m/sec}$$

U_o , ft/sec	ϵ_f	U_f , ft/sec
1.5	.654	2.29
3.0	.778	3.86
4.5	.837	5.38
6.0	.871	6.89

Table 2. Run data.

Run No.	Sensor Positions	Air Velocity, ft/sec	Bed Temp., °F	Tracer, g-mole/sec	Tape File Name
26	NEL,SEL,CL	1.5	90	2.64E-6	DAT 26
18	"	3.0	88	3.20	DAT 18
19	"	4.5	95	8.63	DAT 19
20	"	6.0	100	11.04	DAT 20
24	NWL,SWL,CL	1.5	84	2.64	DAT 24
23	"	3.0	99	2.64	DAT 23
22	"	4.5	102	7.94	DAT 22
21	"	6.0	102	11.04	DAT 21
27	NEU,SEU,C'U	1.5	90	2.64	DAT 27
30	"	3.0	103	4.31	DAT 30
33	"	4.5	106	8.28	DAT 33
36	"	6.0	107	22.16	DAT 36
28	NWU,SWU,C'U	1.5	90	2.64	DAT 28
29	"	3.0	102	6.15	DAT 29
34	"	4.5	106	9.53	DAT 34
35	"	6.0	107	22.16	DAT 35
55	EL,SL,CL	3.0	85	6.9	DAT 55
59	"	4.5	97	33.0	DAT 59
63	"	6.0	104	54.0	DAT 63
56	WL,NL,CL	3.0	89	6.9	DAT 56
60	"	4.5	97	33.0	DAT 60
64	"	6.0	104	54.0	DAT 64
74	CU,EU,SU	3.0	92	2.30	DAT 74
72	"	4.5	100	3.84	DAT 72
68	"	6.0	107	4.9	DAT 68
75	CU,WU,NU	3.0	92	2.30	DAT 75
71	"	4.5	100	3.84	DAT 71
69	"	6.0	107	4.9	DAT 69
76	CU,NU,WU	3.0	91	2.30	DAT 76

Table 3. Normalized mean and mean square concentration at each sensor location.

Control #	port	position	velocity	$\bar{C}^*$	$\bar{C}^{2*}$
1	A	NEL	1.5	0.0000	0.0000
2	B	SEL	1.5	0.0000	0.0000
3	A	NEL	3.0	0.0499	0.00468
4	B	SEL	3.0	0.0549	0.00480
5	A	NEL	4.5	0.0619	0.0171
6	B	SEL	4.5	0.0977	0.0341
7	A	NEL	6.0	0.0623	0.0256
8	B	SEL	6.0	0.0728	0.0444
9	D	NWL	1.5	0.0000	0.0000
10	E	SWL	1.5	0.0000	0.0000
11	D	NWL	3.0	0.0813	0.0102
12	E	SWL	3.0	0.0768	0.0135
13	D	NWL	4.5	0.103	0.0221
14	E	SWL	4.5	0.125	0.0300
15	D	NWL	6.0	0.143	0.0643
16	E	SWL	6.0	0.156	0.0638
17	A'	NEU	1.5	1.37E-5	3.85E-7
18	B'	SEU	1.5	0.0000	0.0000
19	A'	NEU	3.0	0.136	0.0480
20	B'	SEU	3.0	0.194	0.0647
21	A'	NEU	4.5	0.184	0.0966
22	B'	SEU	4.5	0.273	0.134
23	A'	NEU	6.0	0.137	0.0857
24	B'	SEU	6.0	0.177	0.100
25	D'	NWU	1.5	0.0000	0.0000
26	E'	SWU	1.5	0.0000	0.0000
27	D'	NWU	3.0	0.152	0.0389
28	E'	SWU	3.0	0.250	0.0942
29	D'	NWU	4.5	0.210	0.0953
30	E'	SWU	4.5	0.313	0.135
31	D'	NWU	6.0	0.241	0.113
32	E'	SWU	6.0	0.324	0.166
33	A	EL y	3.0	0.303	0.128
34	A'	SL x	3.0	0.415	0.175
35	D	WL y	3.0	0.289	0.104
36	D'	NL x	3.0	0.432	0.194

Table 3. continued

control	port	position	velocity	$\overline{C^*}$	$\overline{C^{2*}}$
37	A	EL y	4.5	0.288	0.146
38	A'	SL x	4.5	0.363	0.202
39	D	WL y	4.5	0.318	0.187
40	D'	NL x	4.5	0.517	0.386
41	A	EL y	6.0	0.216	0.0964
42	A'	SL x	6.0	0.285	0.174
43	D	WL y	6.0	0.346	0.209
44	D'	NL x	6.0	0.725	0.684 *
45	B	EU y	6.0	0.412	0.274
46	B'	SU x	6.0	0.394	0.216
47	E	WU y	6.0	0.589	0.162 *
48	E'	NU x	6.0	0.681	0.448 *
49	E	WU y	4.5	0.513	0.131 *
50	E'	NU x	4.5	0.554	0.459
51	B	EU y	4.5	0.579	0.682
52	B'	SU x	4.5	0.498	0.375
53	B	EU y	3.0	0.404	0.295
54	B'	SU x	3.0	0.315	0.155
55	E	WU y	3.0	0.474	0.077 *
56	E'	NU x	3.0	0.632	0.832
57	E'	NU x	3.0	0.532	0.285
58	E	WU y	3.0	0.244	0.044 *

Note: Starred values omitted from results, believed to be erroneous.

The results are presented in Table 4. The location codes conform to Figure 4 presented earlier.

Note that only one value for D and D_T was obtained for the 1.5 ft/sec runs. At this velocity there was insufficient meandering effect to move the tracer stream from the center of the bed to any of the outlying sensors. Consequently, it is not possible to calculate D or D_T for those runs and D_m was taken as zero for these runs. The 95% confidence interval about a data point with no degrees of freedom is not defined and the one value is probably not meaningful.

Anisotropic Analysis

For this study more sensor locations were added to the tube array so that measurements could be made at points of the type $(\pm a, 0, z)$, $(0, \pm a, z)$ and $(0, 0, z)$. Measurements at these points make it possible to use the meandering plume model to calculate the dispersion coefficients for an anisotropic dispersion medium.

Table 5 contains the results of these experiments. Equations (13), (14), (15) and (16) were used to calculate D , D_m and D_T .

Statistical Comparisons of the Results

There are several questions which need to be answered before many conclusions can be drawn. There appears to be several patterns in the data and therefore "Student's t " tests have been applied to various groups of results to see which of these apparent patterns are statistically significant.

The null hypothesis, H_0 , for each of these tests is that both groups of results under test are from the same normally distributed population with

Table 4. Isotropic assumption results.

Control Number	Superficial velocity, ft/sec	Location code	D_s , ft ² /sec	D_m , ft ² /sec	D_T , ft ² /sec
1	1.5	NEL	-	~0	-
2	1.5	SEL	-	~0	-
9	1.5	NWL	-	~0	-
10	1.5	SWL	-	~0	-
17	1.5	NEU	.00390	.00417	.00807
18	1.5	SEU	-	~0	-
25	1.5	NWU	-	~0	-
26	1.5	SWU	-	~0	-
3	3.0	NEL	.0655	.00873	.0742
4	3.0	SEL	.0700	.00668	.0767
11	3.0	NWL	.0803	.00839	.0887
12	3.0	SWL	.0700	.0167	.0867
19	3.0	NEU	.0524	.0240	.0763
20	3.0	SEU	.0745	.0184	.0929
27	3.0	NWU	.0679	.0130	.0809
28	3.0	SWU	.0908	.0191	.110
5	4.5	NEL	.0705	.0410	.111
6	4.5	SEL	.0831	.0503	.133
13	4.5	NWL	.110	.0263	.136
14	4.5	SWL	.121	.0278	.149
21	4.5	NEU	.0692	.0563	.125
22	4.5	SEU	.116	.0477	.164
29	4.5	NWU	.0914	.0446	.136
30	4.5	SWU	.153	.0293	.183
7	6.0	NEL	.0695	.0737	.143
8	6.0	SEL	.0481	.103	.152
15	6.0	NWL	.119	.0853	.204
16	6.0	SWL	.139	.0749	.214
23	6.0	NEU	.0522	.0845	.137
24	6.0	SEU	.0779	.0791	.157
31	6.0	NWU	.133	.0583	.191
32	6.0	SWU	.180	.0616	.241

Table 5. Anisotropic assumption results.

Control Number	Superficial velocity, ft/sec	Location Code	D_x , ft ² /sec	D_{mx} , ft ² /sec	D_{Tx} , ft ² /sec
34	3.0	SL	.125	.00116	.127
36	3.0	NL	.129	.00313	.133
54	3.0	SU	.0502	.0158	.0659
56	3.0	NU	-	-	-
57	3.0	NU	.120	.00067	.121
38	4.5	SL	.112	.0409	.153
40	4.5	NL	.144	.0908	.235
52	4.5	SU	.0881	.0642	.152
50	4.5	NU	.0868	.0929	.180
42	6.0	SL	.0893	.0689	.158
44	6.0	NL	-	-	-
46	6.0	SU	.114	.0315	.146
			D_y , ft ² /sec	D_{my} , ft ² /sec	D_{Ty} , ft ² /sec
33	3.0	EL	.0781	.0151	.0932
35	3.0	WL	.0810	.00869	.0896
53	3.0	EU	.0433	.0407	.0840
55	3.0	WU	-	-	-
58	3.0	WU	-	-	-
37	4.5	EL	.0880	.0366	.125
39	4.5	WL	.0857	.0496	.136
51	4.5	EU	-	-	-
49	4.5	WU	-	-	-
41	6.0	EL	.0894	.0402	.130
43	6.0	WL	.121	.0666	.187
45	6.0	EU	.0966	.0567	.153
47	6.0	WU	-	-	-

Table 6. "Student's t" test on various result groups.

Test #	N	$\bar{x}$	S^2	v	t_{calc}	$\pm t_{.975}$	Conclusion
A	2	.127	4.00E-6	2	1.20	± 4.30	accept H_0
	2	.0851	1.22E-3				
B	2	.128	2.56E-4	2	2.56	± 4.30	accept H_0
	2	.0869	1.32E-6				
C	4	.0939	1.34E-3	6	-0.475	± 2.78	accept H_0
	4	.111	2.45E-3				
D	2	.0651	1.65E-4	2	-3.42	± 4.30	accept H_0
	2	.157	5.52E-4				
E	8	.1023	1.97E-3	11	0.014	± 2.20	accept H_0
	5	.1020	1.71E-4				
F	8	.0776	1.76E-4	11	2.89	± 2.20	reject H_0
	5	.0528	2.15E-4				

Notes:

Test A = 3.0 ft/sec, anisotropic, lower level results for D_x vs. upper level results for D_x .

Test B = 4.5 ft/sec, anisotropic, lower level results for D_x vs. lower level results for D_y .

Test C = 6.0 ft/sec, isotropic, lower level results for D vs. upper level results for D.

Test D = 6.0 ft/sec, isotropic, NE and SE sensors vs. NW and SW sensors, for D.

Test E = 6.0 ft/sec, anisotropic vs. isotropic results, D vs. D_x and D_y .

Test F = 6.0 ft/sec, anisotropic vs. isotropic results, D_m vs. D_{mx} and D_{my} .

coefficients are the same at the .05 level of significance.

- (4) The isotropic and anisotropic results for the meander coefficients are the same at the .01 level of significance but not necessarily at the .05 level. That is, if only a .01 probability of a type I error in accepting H_0 is allowed then the results are from the same population. If a 5% error is allowed then they are not the same according to the "student's t" distribution. This raises the question of whether there is some anisotropy in the meander even though there doesn't appear to be much in the turbulent/molecular dispersion. Several additional hypothesis tests are presented in Table 7 in an attempt to answer this question. Comparisons are made between isotropic and anisotropic meander coefficients on a directional basis as well as on a height-in-the-bed basis. The results indicate that there may be some anisotropy in the meander but strong evidence is lacking. It appears that the meander favors a path parallel to the immersed tube array (i.e., north-south) and favors this path most strongly near the center of the array relative to the east-west axis (i.e., along the line $(x,0,z)$).

Because the results for a particular velocity appear to be homogeneous with regard to the anisotropic and isotropic experiments, the results for each velocity are lumped together and are presented in Table 8. Again "Student's t" tests have been applied to examine the relationships between superficial velocity and dispersion coefficients. These results are presented in Table 9. From these results, it appears that superficial velocity makes a difference except that the turbulent/molecular coefficient is nearly constant over the 4.5 to 6.0 ft/sec velocity range.

Table 7. "Student's t" test applied to various groups of meander coefficient results.

Test #	N	$\bar{x}$	S^2	v	t_{calc}	$\pm t_{.975}$	Conclusion
A	8	.0404	1.13E-4	11	-1.39	± 2.23	accept H_0
	4	.0545	4.62E-4				
B	8	.0404	1.13E-4	10	-3.15	± 2.23	reject H_0
	4	.0722	4.55E-4				
C	8	.0404	1.13E-4	8	-0.307	± 2.31	accept H_0
	2	.0431	4.23E-5				
D	4	.0722	4.55E-4	4	1.54	± 2.78	accept H_0
	2	.0431	4.23E-5				
E	2	.0502	3.50E-4	3	-0.251	± 3.18	accept H_0
	3	.0545	1.19E-4				
F	4	.00519	3.84E-5	5	-0.776	± 2.57	accept H_0
	3	.0215	1.21E-3				

Notes:

Test A = 4.5 ft/sec, all isotropic vs. anisotropic, lower meander coefficients.

Test B = 4.5 ft/sec, all isotropic vs. anisotropic, north-south meander coefficients (parallel array).

Test C = 4.5 ft/sec, all isotropic vs. anisotropic, east-west meander coefficients (cross-array).

Test D = 4.5 ft/sec, anisotropic north-south vs. anisotropic east-west meander coefficients.

Test E = 6.0 ft/sec, same coefficients as Test D.

Test F = 3.0 ft/sec, same coefficients as Test D.

Table 8. Means, variances and 95% confidence intervals vs. superficial velocity.

Velocity		D	D _m	D _T
1.5 ft/sec	$\bar{x} =$	0.00390	0.000521	0.00442
	$\sigma_s^2 =$	-	1.66E-6	-
	95% CI =	-	0.00253	-
	N =	1	8	1
3.0 ft/sec	$\bar{x} =$	0.0799	0.0134	0.0933
	$\sigma_s^2 =$	6.49E-4	9.78E-5	3.85E-4
	95% CI =	0.0499	0.0194	0.0384
	N =	15	15	15
4.5 ft/sec	$\bar{x} =$	0.101	0.0499	0.151
	$\sigma_s^2 =$	6.07E-4	4.01E-4	9.34E-4
	95% CI =	0.0483	0.0392	0.0599
	N =	14	14	14
6.0 ft/sec	$\bar{x} =$	0.102	0.0680	0.170
	$\sigma_s^2 =$	1.28E-3	3.36E-4	1.06E-3
	95% CI =	0.0700	0.0359	0.0638
	N =	13	13	13

Table 9. "Student's t" tests on final lumped results.

Test #	N	$\bar{x}$	S^2	v	t_{calc}	$\pm t_{.975}$	Conclusion
A	1	.00390	-		cannot test		reject H_0
	15	.0799	6.49E-4				
B	15	.0799	6.49E-4	27	-2.18	± 2.05	reject H_0
	14	.101	6.07E-4				
C	14	.101	6.07E-4	25	-0.082	± 2.06	accept H_0
	13	.102	1.28E-3				
D	8	.000521	1.66E-6	21	-3.50	± 2.08	reject H_0
	15	.0134	9.78E-5				
E	15	.0134	9.78E-5	27	-6.07	± 2.05	reject H_0
	14	.0499	4.01E-4				
F	14	.0499	4.01E-4	25	-2.35	± 2.06	reject H_0
	13	.0680	3.36E-4				
G	1	.00442	-		cannot test		reject H_0
	15	.0933	3.85E-4				
H	15	.0933	3.85E-4	27	-5.65	± 2.05	reject H_0
	14	.151	9.34E-4				
I	14	.151	9.34E-4	25	-1.51	± 2.06	accept H_0
	13	.170	1.06E-3				

Note:

Tests A through C = turbulent/molecular dispersion coefficients, 1.5 vs. 3.0, 3.0 vs. 4.5 and 4.5 vs. 6.0 ft/sec respectively.

Tests D through F = meander coefficients, same velocity pairs.

Tests G through I = overall coefficients, same velocity pairs.

A chi-square test has been applied to the groups of data which show the most apparent variation. This test is used to compare the class-frequency distribution of the experimental results with a normal class-frequency distribution in order to see if the variations may be ascribed to random error.

The "chi-square" test is given by:

$$\chi_{obs}^{2,\nu} = \sum \frac{k (f_{obs} - f_{exp})^2}{f_{exp}} \quad (21)$$

and

$$\nu = k - 3 \quad (22)$$

and

$$\chi_{.05}^{2,\nu} < \chi_{obs}^{2,\nu} < \chi_{.95}^{2,\nu} \quad (23)$$

This test was applied to the 15 experimental values for the meander coefficient at the 3.0 ft/sec superficial velocity. The "chi-square" value observed is 12.6 whereas the table value at the .05 level of significance is 21.0; consequently it may be claimed that the variations in these results is due to a random experimental error rather than some particular bias. However, the number of samples is not high enough to draw a concrete conclusion, $N \geq 30$ being taken as a requirement for good accuracy.

The results presented in Table 8 are listed with a 95% confidence interval based on the "chi-square" tests and that the variations in results within the lumped group may be taken as random experimental error. Consequently, the 95% C.I. is given as $\pm 1.96 \sigma_s$ where:

$$\sigma_s^2 = \frac{\sum (x - \bar{x})^2}{N} \quad (24)$$

Plots of the data are presented below.

Plots of the Results

Figure 6 shows the results for the turbulent/molecular dispersion coefficient, Figure 7 the meander coefficient and Figure 8 the overall coefficient. The data points on each plot represent the average value for each testing location and type. Each point is the average of from one to four measurements. The confidence interval is based on the standard deviation of all the data, and not on the standard deviation of the plotted means which would most likely show tighter intervals.

Figure 6. Molecular-turbulent dispersion coefficient vs. velocity

Figure 7. Meander dispersion coefficient vs. velocity.

Figure 8: Overall dispersion coefficient vs. velocity

Conclusions

A considerable fraction of the apparent gas dispersion coefficient of a fluidized bed is due to a non-dispersive meander of the bed gas as it travels up through the bed. The experimental results indicate that a dispersion coefficient which is obtained by measuring the time average concentration at some location in the bed and then comparing the concentration to the tracer inlet rate will overestimate the "real" turbulent/molecular dispersion coefficient by as much as 60-70%.

The meander coefficient was observed to vary from $0.0005 \text{ ft}^2/\text{sec}$ at a superficial velocity of 1.5 ft/sec (about $1.9 u_o/u_{mf}$) to about $0.068 \text{ ft}^2/\text{sec}$ at an air velocity of 6.0 ft/sec . The corresponding range in the turbulent/molecular dispersion coefficient was $0.004 \text{ ft}^2/\text{sec}$ to $0.10 \text{ ft}^2/\text{sec}$. The meander coefficient shows a continued increase with air velocity whereas the turbulent/molecular coefficient was very close to the same value for the two highest velocities.

There is some weak evidence of anisotropy in the meander dispersion coefficient. The turbulent/molecular coefficient seems to be independent of location.

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APPENDIX A

The two cases covered here are (1) isotropic, infinite moving medium and (2) anisotropic, infinite moving medium. Both cases are treated for a continuous point source located at the origin of a 3-dimensional Cartesian system. Only the steady-state solutions are presented.

Case 1. Isotropic medium, $D_x = D_y = D_z$:

Carslaw and Jaeger (1959) present the heat transfer analog solution for this case as their Equation 10.7(2). For velocity parallel to the x-axis and $t \rightarrow \infty$:

$$T(x,y,z) = \frac{q}{4\pi kR} \exp \left\{ \frac{-U_x(R-x)}{2\alpha} \right\} \quad 10.7(2)$$

where:

k = thermal conductivity

$$R^2 = x^2 + y^2 + z^2$$

α = thermal diffusivity, $k/\rho c_p$

U_x = velocity parallel to the x-axis

q = source heat rate (e.g., Btu/sec)

For the mass transfer analog under the conditions that velocity is parallel to the z-axis and $t \rightarrow \infty$, Equation 10.7(2) becomes:

$$c(x,y,z) = \frac{B'}{R} \exp \left\{ \frac{-U_z(R-z)}{2D_x} \right\} \quad (A1)$$

where:

$c(x,y,z)$ = concentration at (x,y,z)

D_x = diffusion (dispersion) coefficient

B' = constant to be determined

Define:

$\dot{M}$ = moles/time, tracer source rate

then a mass balance across the plane at $z=0$ gives:

$$\dot{M} = \int_A U_z c(x,y,0) dA \quad (A2)$$

where:

$$c(x,y,0) = \frac{B'}{r} \exp \left\{ \frac{-U_z r}{2D_x} \right\}$$

$$r^2 = x^2 + y^2$$

$$A = \pi r^2$$

$$dA = 2\pi r dr$$

then:

$$\dot{M} = \int_0^\infty 2\pi r U \frac{B'}{r} \exp \left\{ \frac{-U_z r}{2D_x} \right\} dr \quad (A3)$$

which gives:

$$B' = \frac{\dot{M}}{4\pi D_x}$$

so finally:

$$c(x,y,z) = \frac{\dot{M}}{4\pi D_x (x^2 + y^2 + z^2)^{1/2}} \exp \left\{ \frac{-U_z ([x^2 + y^2 + z^2]^{1/2} - z)}{2D_x} \right\} \quad (A4)$$

also:

$$c(0,0,z) = \frac{M}{4\pi D_x z} \quad (A5)$$

Case 2. Anisotropic medium, $D_x \neq D_y \neq D_z$:

The time dependent and/or steady-state solutions to this problem may exist in the literature, they were not found. The solution for the steady-state problem is derived here. The time-dependent problem is expressed as an integral solution which does not appear to be subject to analytic evaluation.

Following the method of Kelvin and starting with another equation from Carslaw and Jaeger (1959) for heat transfer in an anisotropic, static infinite medium with an instantaneous source of heat:

$$T(x,y,z) = \frac{Q(\rho c_p)^{3/2}}{8[\pi^3 t^3 k_x k_y k_z]^{1/2}} \exp \left\{ \frac{-\rho c_p}{4t} \left[\frac{(x-x')^2}{k_x} + \frac{(y-y')^2}{k_y} + \frac{(z-z')^2}{k_z} \right] \right\} \quad 10.2(8)$$

where:

(x',y',z') = point source location, relative to $(0,0,0)$

(x,y,z) = point of interest, relative to $(0,0,0)$

$Q\rho c_p$ = quantity of heat in impulse source (e.g., Btu's)

For the present problem with velocity of the medium parallel to the z-axis and a continuous heat source, the temperature at (x,y,z) at time "t" due to $\dot{q}dt'$ units of heat emitted at time "t'" is, by 10.2(8):

$$T(x,y,z) = \frac{\dot{q} dt' (\rho c_p)^{1/2}}{8(\pi^3 (t-t')^3 k_x k_y k_z)^{1/2}} \exp \left\{ \frac{-\rho c_p}{4(t-t')} \left[\frac{x^2}{k_x} + \frac{y^2}{k_y} + \frac{(z-U_z(t-t'))^2}{k_z} \right] \right\} \quad (A6)$$

where:

$\dot{q} dt' = Q \rho c_p$ = quantity of heat emitted in time dt'

$\dot{q}$ = rate of heating by the source (e.g., Btu/sec).

Then the temperature at time "t" due to heat emitted at (o,o,o) from time = 0 to time = t is:

$$T(x,y,z) = \lambda \int_0^t (t-t')^{-3/2} \exp \{ \beta(t') \} dt' \quad (A7)$$

where:

$$\lambda = \frac{\dot{q}}{8} \left[\frac{\rho c_p}{\pi^3 k_x k_y k_z} \right]^{1/2}$$

$$\beta(t') = \frac{-\rho c_p}{4(t-t')} \left[\frac{x^2}{k_x} + \frac{y^2}{k_y} + \frac{(z-U_z(t-t'))^2}{k_z} \right]$$

Substituting diffusivities for $k_i/\rho c_p$ and expanding $\beta(t)$:

$$T(x,y,z) = \lambda \exp \left\{ \frac{z U_z}{2 D_z} \right\} \int_0^t \frac{\exp \left\{ \frac{-1}{4(t-t')} \left[\frac{x^2}{D_x} + \frac{y^2}{D_y} + \frac{z^2}{D_z} \right] \right\}}{(t-t')^{3/2}} \times \\ \times \exp \left\{ \frac{-U_z^2(t-t')}{4 D_z} \right\} dt' \quad (A8)$$

Define:

$$w^2 = \frac{1}{4} \left[\frac{x^2}{D_x} + \frac{y^2}{D_y} + \frac{z^2}{D_z} \right]$$

$$\gamma = \lambda \exp \left\{ \frac{z U_z}{2 D_z} \right\}$$

$$\eta^2 = U_z^2 / 4 D_z$$

$$\xi^2 = w^2 / (t - t')$$

then:

$$\xi \Big|_{t'=0} = w/t^{1/2} \quad \text{and} \quad \xi \Big|_{t'=t} = w/0 = \infty$$

Substituting in (A8):

$$T(x, y, z) = \frac{2\gamma}{w} \int_{\frac{w}{t^{1/2}}}^{\infty} \exp \{ -\xi^2 - \eta^2(t-t') \} d\xi \quad (\text{A9})$$

Define:

$$\phi = \frac{4 D_z \eta^4 w^2}{U_z^2}$$

then:

$$\eta^2(t-t') = \frac{\phi}{\xi^2}$$

and:

$$T(x, y, z, t) = \frac{2\gamma}{w} \int_{\frac{w}{t^{1/2}}}^{\infty} \exp \{ -\xi^2 - \frac{\phi}{\xi^2} \} d\xi \quad (\text{A10})$$

Equation (A10) is the time dependent solution but the integral may not be analytic (although there is some resemblance to the form of an erfc).

For $t \rightarrow \infty$:

$$\frac{w}{t^{1/2}} \rightarrow 0$$

From Dwight (1961):

$$\int_0^{\infty} \exp \left\{ -a^2 x^2 - \frac{b^2}{x^2} \right\} dx = \frac{\sqrt{\pi}}{2a} \exp\{-2ab\}; [a, b > 0] \quad (860.25)$$

so:

$$T_{\infty}(x, y, z) = \frac{\gamma \sqrt{\pi}}{w} \exp\{-2\phi^{1/2}\} \quad (A11)$$

Substituting back:

$$T(x, y, z) = \frac{\dot{q}}{4\pi\rho c_p \left\{ \frac{k_x}{\rho c_p} \frac{k_y}{\rho c_p} \frac{k_z}{\rho c_p} \left[\frac{x^2}{D_x} + \frac{y^2}{D_y} + \frac{z^2}{D_z} \right] \right\}} \times \exp\left\{ -\frac{U_z}{2D_z^{1/2}} \left[\frac{z}{D_z^{1/2}} - \left(\frac{x^2}{D_x} + \frac{y^2}{D_y} + \frac{z^2}{D_z} \right)^{1/2} \right] \right\} \quad (A12)$$

The mass transfer analog is gotten by the same method as Equation (A4) and is:

$$c(x, y, z) = \frac{\dot{M}}{4\pi \{ D_x D_y D_z \left[\frac{x^2}{D_x} + \frac{y^2}{D_y} + \frac{z^2}{D_z} \right] \}^{1/2}} \times \exp\left\{ -\frac{U_z}{2D_z^{1/2}} \left[\frac{z}{D_z^{1/2}} - \left(\frac{x^2}{D_x} + \frac{y^2}{D_y} + \frac{z^2}{D_z} \right)^{1/2} \right] \right\} \quad (A13)$$

The centerline concentration is:

$$c(0,0,z) = \frac{\dot{M}}{4\pi(D_x D_y)^{1/2} z} \quad (A14)$$

This part of the appendix uses approximations to the preceding analytic solutions in order to apply the meandering plume model to the present experiments.

The meander probability density is taken as:

$$m(x,y,z) = \frac{1}{2\pi\sigma_m^2} \exp \left\{ \frac{-1}{2\sigma_m^2} (x^2 + y^2) \right\}$$

which, defining:

$$D_m = \frac{U\sigma_m^2}{2z}$$

can be rewritten as:

$$m(x,y,z) = \frac{U}{4\pi D_m z} \exp \left\{ \frac{-U}{4z D_m} (x^2 + y^2) \right\} \quad (A15)$$

A reasonable approximation to the tracer diffusion equation is:

$$c(x,y,z) = \frac{\dot{M}}{4\pi D z} \exp \left\{ \frac{-U}{4z D} (x^2 + y^2) \right\} \quad (A16)$$

The convolutions of the two give the time averages at measuring points (x,y,z) :

$$\bar{c}(x,y,z) = \iint_{-\infty}^{\infty} m(v,w,z) c(x-v, y-w, z) dv dw \quad (A17)$$

$$\bar{c}^2(x,y,z) = \iint_{-\infty}^{\infty} m(v,w,z) c^2(x-v, y-w, z) dv dw \quad (A18)$$

Using the relation:

$$\int_{-\infty}^{\infty} \exp[-p^2 \xi^2 + q\xi] d\xi = \frac{\sqrt{\pi}}{p} \exp\left[\frac{q^2}{4p^2}\right]$$

the convolutions become:

$$\bar{c}(x,y,z) = \frac{\dot{M}}{4\pi z(D + D_m)} \exp \left\{ \left(\frac{-U}{4zD} \right) (x^2 + y^2) \left(1 - \frac{D_m}{D + D_m} \right) \right\} \quad (A19)$$

and

$$\bar{c}^2(x,y,z) = \frac{\dot{M}^2}{16\pi^2 z^2 D(2D_m + D)} \exp \left\{ \left(\frac{-U}{2zD} \right) (x^2 + y^2) \left(1 - \frac{2D_m}{2D_m + D} \right) \right\} \quad (A20)$$

Defining:

$$\bar{c}^*(x,y,z) = \bar{c}(x,y,z) / \bar{c}(0,0,z)$$

$$\bar{c}^{2*}(x,y,z) = \bar{c}^2(x,y,z) / \bar{c}^2(0,0,z)$$

then:

$$\bar{c}^*(x,y,z) = \exp \left\{ \left(\frac{-U}{4zD} \right) (x^2 + y^2) \left(1 - \frac{D_m}{D + D_m} \right) \right\} \quad (A21)$$

$$\bar{c}^{2*}(x,y,z) = \exp \left\{ \left(\frac{-U}{2zD} \right) (x^2 + y^2) \left(1 - \frac{2D_m}{2D + D_m} \right) \right\} \quad (A22)$$

and for measurements at $(\pm a, \pm a, z)$ and $(0,0,z)$:

$$D = \left[\frac{Ua^2}{z \ln \bar{c}^{2*}} - \frac{Ua^2}{z \ln \bar{c}^*} \right] \quad (A23)$$

$$D_m = \frac{D}{2} \left[\frac{2 \ln \bar{c}^* - \ln \bar{c}^{2*}}{\ln \bar{c}^{2*} - \ln \bar{c}^*} \right] \quad (A24)$$

An anisotropic meander probability density is taken as:

$$m(x,y,z) = \frac{U}{4\pi z D_{mx}^{1/2} D_{my}^{1/2}} \exp \left\{ \frac{-U}{4z} \left(\frac{x^2}{D_{mx}} + \frac{y^2}{D_{my}} \right) \right\} \quad (A25)$$

and the diffusion profile as:

$$c(x,y,z) = \frac{\dot{M}}{4\pi z D_x^{1/2} D_y^{1/2}} \exp \left\{ \frac{-U}{4z} \left(\frac{x^2}{D_x} + \frac{y^2}{D_y} \right) \right\} \quad (A26)$$

performing the convolutions:

$$\bar{c}(x,y,z) = \frac{\dot{M}}{4\pi z [D_x + D_{mx}]^{1/2} [D_y + D_{my}]^{1/2}} \exp \left\{ \left[\frac{-U}{4z} \right] \left[\frac{x^2}{D_x} + \frac{y^2}{D_y} - \frac{x^2 D_{mx}}{D_x^2 + D_x D_{mx}} - \frac{y^2 D_{my}}{D_y^2 + D_y D_{my}} \right] \right\} \quad (A27)$$

and:

$$\bar{c}^2(x,y,z) = \frac{\dot{M}^2}{16\pi^2 z^2 [D_x^2 + 2D_x D_{mx}]^{1/2} [D_y^2 + 2D_y D_{my}]^{1/2}} \exp \left\{ \left[\frac{-U}{2z} \right] x \right. \\ \left. \times \left[\frac{x^2}{D_x} + \frac{y^2}{D_y} - \frac{2x^2 D_{mx}}{[D_x^2 + 2D_x D_{mx}]} - \frac{2y^2 D_{my}}{[D_y^2 + 2D_y D_{my}]} \right] \right\} \quad (A28)$$

Referring to the normalized forms of Equations (A27) and (A28) and for measurements made at $(\pm a, \pm a, z)$, $(\pm a, \pm 2a, z)$ and $(0, 0, z)$ then define:

$$c_1 = a^2(j+k) = \ln \bar{c}^* (a, a, z)$$

$$c_2 = a^2(c+d) = \ln \bar{c}^{2*} (a, a, z)$$

$$c_3 = a^2(j+4k) = \ln \bar{c}^* (a, 2a, z)$$

$$c_4 = a^2(c+4d) = \ln \bar{c}^{2*} (a, 2a, z)$$

then:

$$j = \frac{4c_1 - c_3}{3a^2} = -\frac{U}{4zD_x} + \frac{UD_{mx}}{4z[D_x^2 + D_x D_{mx}]}$$

$$k = \frac{c_3 - c_1}{3a^2} = -\frac{U}{4zD_y} + \frac{UD_{my}}{4z[D_y^2 + D_y D_{my}]}$$

$$c = \frac{4c_2 - c_4}{3a^2} = -\frac{U}{2zD_x} + \frac{2UD_{mx}}{2z[D_x^2 + 2D_x D_{mx}]}$$

$$d = \frac{c_4 - c_2}{3a^2} = -\frac{U}{2zD_y} + \frac{2UD_{my}}{2z[D_y^2 + 2D_y D_{my}]}$$

and

$$D_x = \frac{U}{2z} \left[\frac{j-c}{jc} \right] \quad (A29)$$

$$D_y = \frac{U}{2z} \left[\frac{k-d}{kd} \right] \quad (A30)$$

$$D_{mx} = \frac{D_x(c^2 - 2jc)}{2(c^2 + jc)} \quad (A31)$$

$$D_{my} = \frac{D_y(d^2 - 2kd)}{2(d^2 + kd)} \quad (A32)$$

Again using the normalized versions of Equations (A27) and (A28) and for measurements at (a,o,z), (o,b,z) and (o,o,z) then:

$$D_x = \frac{Ua^2}{2z} \left[\frac{1}{\ln \bar{c}_a^{2*}} - \frac{1}{\ln \bar{c}_a^*} \right] \quad (A33)$$

$$D_y = \frac{Ub^2}{2z} \left[\frac{1}{\ln \bar{c}_b^{2*}} - \frac{1}{\ln \bar{c}_b^*} \right] \quad (A34)$$

$$D_{mx} = \frac{D_x}{2} \left[\frac{2 \ln \bar{c}_a^* - \ln \bar{c}_a^{2*}}{\ln \bar{c}_a^{2*} - \ln \bar{c}_a^*} \right] \quad (A35)$$

and

$$D_{my} = \frac{D_y}{2} \left[\frac{2 \ln \bar{c}_b^* - \ln \bar{c}_b^{2*}}{\ln \bar{c}_b^{2*} - \ln \bar{c}_b^*} \right] \quad (A36)$$

APPENDIX B

Table B1 summarizes the results of three separate screen analyses performed on samples of the bed material. Samples 1 and 2 each weighed over 5 lb_m and sample 3 was just over 12 lb_m. The samples were composite grab samples. 1 and 2 were taken while emptying the bed and No. 3 was taken from the upper part of three barrels which contained all of the bed material.

The average probability density function estimate is given in the extreme right-hand column.

Two determinations of the copper solids density on 3 lb_m samples gave:

$$\bar{\rho}_s = \frac{8260 + 8050}{2} = 8160 \text{ kg/m}^3$$

Three determinations of bulk density, two on 3 lb_m samples and one on a 732 lb_m sample gave:

$$\bar{\rho}_b = \frac{(6)(6350 + 5420)}{(2)(738)} + \frac{(5660)(732)}{(738)} = 5660 \text{ kg/m}^3$$

The average packed bed voidage is then:

$$\epsilon_p = 1 - \frac{5660}{8160} = 0.306$$

Table B1. Bed Material Size Distribution

Tyler	d_p ,	Weight	P_i ,	Weight	P_i ,	Weight	P_i ,	Average
Mesh	μm	Fraction	μm^{-1}	Fraction	μm^{-1}	Fraction	μm^{-1}	$P_i, \mu\text{m}^{-1}$
>10	--	0	0	0	0	0	0	0
10-12	1545	.0049	1.81E-5	.0038	1.40E-5	0	0	1.61E-5
12-14	1300	.065	2.95E-4	.0790	3.57E-4	.152	6.91E-4	4.83E-4
14-20	1016	.164	4.70E-4	.208	5.96E-4	.189	5.42E-4	5.36E-4
20-28	718	.0580	2.36E-4	.0737	3.00E-4	.070	2.85E-4	2.73E-4
28-35	508	.181	1.03E-3	.201	1.15E-3	.203	1.16E-3	1.11E-3
35-48	359	.171	1.39E-3	.183	1.49E-3	.176	1.43E-3	1.44E-3
48-65	254	.0694	7.98E-4	.0585	6.72E-4	.065	7.47E-4	7.39E-4
65-80	194	.0678	2.05E-3	.0633	1.92E-3	.045	1.36E-3	1.78E-3
80-100	163	.0930	3.32E-3	.0777	2.78E-3	.054	1.93E-3	2.68E-3
100-200	112	.0981	1.31E-3	.0454	6.05E-4	.038	5.07E-4	8.07E-4
<200	37	.0273	3.69E-4	.0073	9.91E-5	.009	1.22E-4	1.97E-4

APPENDIX C

U_{mf} Calculation:

$$\begin{aligned} T_{air} &= 90\frac{1}{2}F \\ P_{air} &= 16.7 \text{ psia} \\ \rho_{air} &= 0.0851 \text{ lb}_m/\text{ft}^3 \\ M_{air} &= 1.23E-5 \text{ lb}_m/\text{ft-sec} \\ \rho_s &= 509 \text{ lb}_m/\text{ft}^3 \\ \bar{d}_p &= 315 \text{ } \mu\text{m} = 1.03E-3 \text{ ft} \end{aligned}$$

The Wen and Yu (1966) correlation is:

$$U_{mf} = \frac{M_g}{\bar{d}_p \rho_g} \left\{ \left[(33.7)^2 + 0.0408 \frac{\bar{d}_p^3 \rho_g (\rho_s - \rho_g) g}{\mu_g^2} \right]^{1/2} - 33.7 \right\}$$

and

$$U_{mf} = 0.79 \text{ ft/sec}$$

Jovanovic (1979) states that a good correlation for voidage versus fluidization velocity for slow bubbles ($U_{br}/U_f \approx 0.5$) is:

$$\frac{\epsilon - \epsilon_{mf}}{1 - c} = \frac{1}{2.5} \frac{U_0 - U_{mf}}{U_{mf}}$$

The work of Catipovic, et al (1978) indicates that this bed is just in the slow bubble regime. For lack of better data ϵ_{mf} is taken equal to $1.2 \epsilon_{packed}$.

$$\epsilon_{mf} = (1.2)(0.306) = .367$$

The fluidized voidages are then:

U_0 ft/sec	ϵ
1.5	.534
3.0	.701
4.5	.780
6.0	.826

An additional factor is the presence of the tube array. The actual voidage based on the superficial area is equal to ϵ for the 5" between the plate and the lowest tubes and $\epsilon_t + \epsilon - \epsilon_t \epsilon$ within the array region:

$$\epsilon_t = \frac{V_t}{V_0} = \frac{.871}{2.423} = 0.359$$

The array is 12.75" high so the approximate fluidized voidage is:

$$\epsilon_f = \frac{[0.359 + \epsilon - 0.359\epsilon] 12.75 + 5\epsilon}{17.75}$$

U_0 ft/sec	ϵ_f	U_f ft/sec
1.5	.654	2.29
3.0	.778	3.86
4.5	.837	5.38
6.0	.871	6.89

where:

$$U_f = \frac{U_0}{\epsilon_f}$$