Project 0272
Technical Report No. 2

MASTER

Westinghouse Atomic Power Divisions

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CONTROL PROCEDURES FOR XENON-INDUCED X-Y
INSTABILITIES IN LARGE PRESSURIZED
WATER REACTORS

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February 1969

Prepared for the New York Operations Office
U. S. Atomic Energy Commission
Contract AT(30-1)-3680

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ACKNOWLEDGMENT

The authors wish to express their appreciation to Drs. D. L. Miller, J. E. Olhoeft, G. G. Poncelet, and to Messrs. A. M. Christie, M. J. O'Boyle and P. M. Wood for their valuable discussions and suggestions throughout the study. They also wish to thank Miss Mary Anna Rowsick and Mr. A. D. Gray, II for their assistance in the preparation and running of the computer code.

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ABSTRACT

Using digital simulation techniques, various methods were investigated for the control of xenon-induced spatial oscillations in large, pressurized water reactor (PWR) cores. The oscillations are studied in a three-zone core, eleven feet in diameter, in a plane at right angles to the coolant flow. It is shown that successful control can be achieved by a single insertion of one appropriate rod selected from a number of suitably located control rods. The effect of varying the duration and position of rod insertion is discussed, and various alternate control procedures are proposed and evaluated.

SECTION 1

INTRODUCTION

In the larger PWR cores currently being designed, there is an increased probability of xenon-induced spatial power oscillations. Thus, operating procedures must be formulated to ensure continuous safe operation, even in the presence of incipient divergent oscillations. This report describes a possible strategy for the detection and control of these oscillations.

Xenon-induced oscillations can arise as a result of a perturbation to the normal equilibrium operation. The oscillation may be caused by a condition where the x-y flux distribution remains in an unbalanced state for a relatively short period of time compared to the xenon-iodine time constants. If the reactor system is inherently unstable with respect to xenon oscillations, this unbalance, if not corrected, can eventually result in unacceptable x-y power tilts.

The stability characteristics of a reactor system are determined by various core parameters such as physical size, power density, power distribution, flux level, and the reactivity coefficients. The results of Poncelet and Christie^{1} in their threshold analysis of xenon oscillations have established the functional relationships of core stability as these parameters vary. The stability of a particular core will be modified by detailed considerations such as fuel and poison geometry, control rod insertion and fuel burnup. A typical core has been chosen for detailed study and the results obtained indicate a particular response to control which will not necessarily duplicate the response of other cores. However, the overall pattern of behavior in the core studied is expected to apply to many large cores. This report indicates the steps necessary to devise a satisfactory control procedure for similar potentially unstable pressurized water reactors.

The calculational methods used in this study are presented in Section 2. A discussion of the two-dimensional, two-group diffusion theory code which was used in the analysis is included along with a description of the models which represent the pointwise Doppler and the pointwise enthalpy effects in the calculations. A discussion of possible errors and limitations in the computational model is included.

In Section 3 the reactor core selected for this study is defined. Some typical design data and a summary of the thermal-hydraulic data are tabulated. This section also describes the geometric representation used in the digital simulations, and outlines the methods employed for preparing the nuclear constants used in the calculations.

The core chosen for this study is normally stable to xenon oscillations, and a convergent oscillation which results from a brief insertion of a control rod is described. Since a divergent oscillation represents the most meaningful condition for a control procedure study, an artificially unstable system was also considered. This is accomplished through elimination of the moderator density feedback in the calculations. An operationally convenient procedure for eliminating divergent oscillations, if they should occur, is presented.

In large PWR cores the means available for controlling xenon oscillations in the x-y plane are limited to the insertion of one, or possibly two, full-length control rods. A procedure involving the use of these rods should have the capability of limiting power density variations in any part of the core to within a specified operating band. The particular strategy discussed here is one where control action is reduced to a minimum, i.e., stability is achieved with as little control as possible.

SECTION 2

ANALYTICAL MODEL

2.1 CODE DESCRIPTION

In recent years a considerable effort has been directed toward the development of effective optimal control systems for the control of power oscillations in large reactor systems. Particularly, the problem of controlling spatial oscillations induced by a redistribution of xenon has received considerable attention. One approach which has been taken by Stacey^{2} for example, is to develop a dynamic programming computing strategy based on the optimality principle. This method does not generally lead to a simple control procedure, particularly for oscillations in the x-y plane. In the analysis reported below a detailed digital simulation is used to study the behavior of the reactor core throughout the xenon transients. A digital mathematical model facilitates a reasonable (and more realistic) description of core geometry, loading patterns, and control elements. In this study, a two-dimensional, two-group diffusion theory program which includes pointwise Doppler and enthalpy effects is used.

The code used for this analysis solves numerically the two-group diffusion equations for each mesh rectangle in a detailed picture of the core which has 7 mesh intervals across each fuel assembly. For time step calculations, such as in the study of xenon-induced spatial oscillations, the fluxes are held constant and the iodine and xenon equations are solved analytically. The equilibrium condition is calculated by setting the derivative terms to zero so that equilibrium is obtained in the first time step. The pointwise Doppler and pointwise enthalpy feedback mechanisms are presented in the following sections.

2.2 FEEDBACK MECHANISMS

The major reactivity feedback effects in PWR cores are due to variations in fuel temperature, the Doppler effect, and variations in the moderator density. These feedbacks are reflected in variations in cross sections; primarily the fast removal and fast absorption cross sections due to the Doppler effect and the fast removal and thermal absorption and fission due to changes in the moderator density. A detailed general treatment of these feedback effects can be found in Reference 1.

A local correction for the Doppler effect is made by changing the fast absorption cross section according to the following model:

$$\Sigma_a (P) = a_o + \frac{a_1 P}{1 + a_2 P}$$

where P is the power density relative to a mean full power value of 1.0. Clearly,

$$\Sigma_a (1) = a_o + \frac{a_1}{1 + a_2}$$

which when substituted, leads to

$$\Sigma_a (P) = \Sigma_a (1) + \frac{a_1 P}{1 + a_2 P} - \frac{a_1}{1 + a_2}.$$

Thus, the code needs an equilibrium full-power absorption cross section and the two coefficients, a_1 and a_2 which are obtained by fitting the above expression to a plot of $\Sigma_a (P)$ versus power. The $\Sigma_a (P)$'s are computed using the spectrum code LEOPARD,^{3} in which the fuel temperatures are varied while power and moderator temperature are held constant to eliminate moderator density effects in the absorption rates.

The correction to the fast removal cross section is achieved through the moderation cross section which is defined as

$$\Sigma_m = \frac{\Sigma_a}{\ln(1 + \Sigma_a/\Sigma_r)},$$

or in terms of the resonance escape probability, p , which is defined as

$$p = \frac{\Sigma_r}{\Sigma_a + \Sigma_r} = e^{-\Sigma_a/\Sigma_m}.$$

Consequently, the removal cross section can be expressed in terms of p and Σ_a , i.e.,

$$\Sigma_r = \frac{p}{1 - p} \Sigma_a.$$

The moderator density feedback effect is realized through a local correction to the group constants in each mesh rectangle by first calculating a local enthalpy rise and then a mean coolant density in each channel. The model assumes that pressure remains constant, and that there is no cross-flow, no slip, and no subcooled boiling.

For the x-y geometry, i.e., for a plane at right angles to the coolant flow, the model defines a mean coolant density as

$$\bar{\rho} = \frac{\int \rho \phi dz}{\int \phi dz}.$$

But ϕdz can be expressed as

$$\phi dz = \frac{D_e G dh}{4 \kappa \Sigma_f},$$

where D_e is the hydraulic diameter, G the mass flow rate, and $\kappa \Sigma_f$ the power generated over channel length dz . This leads to a relationship expressing the mean density in terms of the channel enthalpy rise, i.e.,

$$\bar{\rho} = \frac{\int \rho dh}{\int dh},$$

where the limits on the integral are the inlet and outlet enthalpies. There is a proportionality existing between the channel enthalpy rise and the local power density.

The analytic treatment of these integrals uses a quadratic fit to generate subcooled density as a function of enthalpy, i.e.,

$$\rho(h) = a + bh + ch^2; \quad h < h_f$$

$$\rho(h) = \frac{1}{v_f(1 - \chi) + v_g\chi}; \quad h \geq h_f$$

where χ is the mass quality, v_f is the specific volume of saturated liquid, and v_g is the specific volume of saturated vapor.

After some manipulation, and remembering that $dh = h_{fg} d\chi$, it can be shown that a linear approximation quite adequately expresses the mean coolant density in terms of the average enthalpy rise in a channel, i.e.,

$$\bar{\rho} = \rho(\bar{h}).$$

Having computed the mean water density in a channel, the code calculates the changes in the water and boron number densities followed by a modification to the cross sections. In the case of the absorption cross section, the corrected value is calculated from

$$\Sigma_a(h) = \Sigma_{a_0} + \delta N^W \sigma_a^W + \delta N^B \sigma_a^B,$$

where Σ_{a_0} is the macroscopic absorption cross section corresponding to a relative power of 1.0; δN^W , δN^B the changes in number densities of water and boron, and σ_a^W , σ_a^B the microscopic absorption cross sections of water and boron, respectively.

The identical treatment is used to correct the transport and moderation cross sections with the exception that σ_a^B is equal to zero. The moderation cross section was defined above.

The code also compensates for the hardening of the fast group spectrum resulting from any reduction in the density of the water. The spectrum factor, α , is defined as

$$\alpha = (\Delta\Sigma/\Sigma)/(\Delta\rho/\rho)$$

and is applied to D_1 , Σ_{a_1} , and $(v\Sigma_f)_1$.

SECTION 3

DESCRIPTION OF THE REACTOR SYSTEM

3.1 CORE DESCRIPTION

Throughout this study a typical, large pressurized water reactor (PWR) has been used as a basis for the calculations. The core, containing 193 fuel assemblies, is arranged to form a unit that is roughly cylindrical in shape. The active fuel length of the core is 12 feet and the equivalent diameter is 11 feet, which results in a length to diameter ratio of 1.085. The fuel assembly design is of the Rod Cluster Control (RCC) type. The core employs a nonuniform checkerboard-like fuel loading and has a combination of soluble boron and burnable poison for control of excess reactivity required for burnup. The fuel loading scheme of the first core uses a checkerboard pattern for the low and intermediate enrichments where the center assembly contains a low enrichment. The high enrichment assemblies are loaded toward the outside of the core. A typical set of U-235 enrichments are 2.15, 2.65, and 3.10 w/o. Figure 3-1 illustrates the checkerboard fuel loading pattern employed in large PWR'S.

The burnable poison rods, 1160 in all, are distributed throughout the core in clusters located in vacant RCC guide tubes. They contain about 12.5 w/o B_2O_3 . Figure 3-2 shows the pattern of burnable poison distribution assumed.

The presence of burnable poison rods in large PWR's eliminates a positive moderator temperature coefficient at operating temperature conditions during the initial phase of the first fuel cycle. A negative moderator temperature coefficient is desirable in various postulated accident conditions where positive reactivity insertion would otherwise occur. It has also been established through extensive threshold analysis that a negative coefficient gives greater spatial stability with respect to xenon-induced x-y power oscillations.

Figure 3-1. Checkerboard Fuel Loading Pattern in Large PWR Cores

Figure 3-2. Burnable Poison Distribution in a Typical PWR Core
(Number of Poison Pins per Assembly)

The particular burnable poison distribution present in these calculations has an initial reactivity worth of 7.2% $\Delta\rho$, which is equivalent to about 700 ppm soluble boron. The resulting initial boron concentration is 1100 ppm. With this combination of soluble boron and burnable poison the moderator temperature coefficient is calculated to be about $-0.5 \times 10^{-4} (\text{°F})^{-1}$.

Some typical core design data pertaining to a large PWR are listed in Table 3-1. In Table 3-2 a summary of the thermal-hydraulic data is listed. In order to preserve consistency these data were employed in all calculations.

TABLE 3-1

TYPICAL CORE DESIGN DATA FOR A LARGE PWR

Number of fuel assemblies in core	193
Number of burnable poison rods	1160
Effective core diameter (ft)	11
Fuel assembly pitch (in.)	8.466
Length of active fuel (in.)	144
Fuel rods per assembly (in.)	204
Pitch (in.)	0.563
Fuel rod outer diameter (in.)	0.422
Cladding thickness (in.)	0.0243
Pellet diameter (in.)	0.3669

TABLE 3-2
THERMAL-HYDRAULIC DATA FOR A LARGE PWR

System pressure (psia)	2250
Coolant inlet temperature (°F)	543
Core average temperature (°F)	572.9
Average temperature rise in core (°F)	67.8
Total flow rate (lb/hr)	134 x 10 ⁶
Average heat flux (Btu/hr-ft ²)	217,000
Average fuel temperature (°F)	1516
Average core power density (W/cc)	104.5

The cores investigated in these calculations are all surrounded by a 1.5-inch baffle and an effectively infinite water reflector.

3.2 GEOMETRIC REPRESENTATION

Since computer time increases with increasing number of spatial mesh points, it becomes clear that a model with relatively few mesh intervals per fuel assembly must be used since this study requires a large number of calculations. The model chosen for representing an RCC element is consistent with the one currently used in half-core and full-core x-y diffusion calculations. The representation referred to is commonly known as the slot model and is described below.

In the slot model the structure and moderator associated with the water holes and gaps between the assemblies are homogenized into four water slots which contain no fuel. The remaining area contains the fuel equivalent to the 204 fuel pins. When an assembly contains burnable poison, this too is homogenized into the slot. The model permits a 15 x 15 fuel assembly to be described by as few as 7 mesh intervals along each coordinate, whereas 15 intervals are needed in the conventional discrete representation of one mesh interval per lattice pitch. A comparison of the slot representation and the conventional

model of one interval per pitch has shown agreement on the location and magnitude of the peak power to within 0.6 percent. The discrepancy in predicting reactivity was found to be negligible. Thus, the slot representation of the RCC elements can be used in two-dimensional diffusion calculations with no significant loss of accuracy in predicting reactivity and power distribution for the cores. For illustrative purposes the models are shown in Figures 3-3 and 3-4.

It is noted that the mesh intensity used in this analysis exceeds by a factor of 3 the intensity found adequate in Reference 1. This mesh is consistent with already established models that have been developed to give good agreement with static experiments. Further, the spatial mesh requirement for control studies may be more stringent than for the study of free-running oscillations.

3.3 CROSS SECTION DATA

In preparing the macroscopic cross section tables for the calculation, the LEOPARD^[3] code is used. The macroscopic, few-group constants for a fuel region are obtained by doing multigroup spectrum calculations on an equivalent unit fuel cell; each cell consists of a fuel pellet region, a moderator region plus a fictitious extra region which is made up from water holes, interassembly gaps, etc. The microscopic cross sections required to do a boron criticality search, a water density correction, and a xenon depletion are taken from the fuel spectrum LEOPARDS for the average enrichment. The macroscopic group constants for the non-lattice regions in the core are computed using LEOPARD with their own "soft spectrum" as opposed to the harder fuel spectrum.

In the case where the slots contain burnable poison, the procedure for arriving at a set of macroscopic group constants has been verified by comparison with critical experiments. A unit burnable poison cell is defined similar to a unit fuel cell, i.e., the burnable poison is inserted in the pellet region and analogous cladding and moderator regions are defined. The resulting constants from LEOPARD are used in fine-mesh unit assembly spatial calculations in which the burnable poison is represented explicitly, and one

Figure 3-3. Geometric Representation of the Slot Model

Figure 3-4. Geometric Representation of the Coarse Discrete Model

in which the burnable poison is removed. The group constants for the coarser slot model are determined by varying the removal and the thermal absorption cross sections in such a way as to reproduce the results obtained from the fine-mesh calculation.

SECTION 4

CONTROL PROCEDURES FOR XENON-INDUCED X-Y INSTABILITIES IN LARGE PWR CORES

4.1 SECTION SUMMARY

The initial phase of this study was concerned with determining the stability characteristics of large PWR cores at various conditions. This analysis revealed that at nominal conditions the 11-foot-diameter cores are stable provided pointwise Doppler and pointwise moderator density feedbacks are accounted for. An unstable system was then artificially created by eliminating the feedback mechanism of the moderator density in the calculations. Section 4.3 describes the detailed two-dimensional analysis with and without moderator density feedback. It also describes the two modes of oscillation studied; namely, the oscillation about a diagonal axis of symmetry ($\pm 45^\circ$) and the oscillation about a principal axis of symmetry (0° or 90°).

Section 4.4 reports on the control of x-y xenon-induced spatial oscillations. A set of operational restrictions are defined for the reactor systems, i.e., the allowable power asymmetry conditions are defined. A group of eight control rods is identified as xenon control rods which can be operated individually. The effectiveness of these rods is then demonstrated by applying control to an unstable system. A control procedure for both geometric modes is outlined and definite bounds on the time duration of corrective action are established. A procedure for detecting x-y oscillations is described; however, the specification of a actual detection system is outside the scope of this study. Also included is a brief description of procedures which do not provide the required control for the oscillation about the diagonal axis of symmetry.

Lastly, Section 4.5 includes cautionary notes on possible errors in the methods. In particular, the effects of the finite length timestep and the convergence criterion are discussed with reference to earlier work on uncontrolled oscillations.

4.2 DEFINITION OF STABILITY INDEX

With the assumption that the oscillation can be approximated by one mode, the flux or power perturbation $\delta\phi$ can be written as a function of time t as,

$$\delta\phi = Ae^{bt} \sin \frac{2\pi t}{\tau}$$

where A is a constant and τ is the period of the oscillation, and b is called the stability index^{1}. Therefore:

$$b = \frac{1}{t_2 - t_1} \ln \frac{\delta\phi_2}{\delta\phi_1}$$

where $t_2 - t_1$ is the time differential between successive flux peaks, and $\delta\phi_1$ and $\delta\phi_2$ are consecutive maxima in the perturbed flux at times t_1 and t_2 . Properly used, this relationship should be applied to consecutive peaks in the same region of the core. However, this is not feasible in detailed two-dimensional digital analysis for cost reasons. Therefore, instead of using consecutive maxima, this work has defined $\delta\phi_1$ as:

$$\delta\phi_1 = \frac{\phi_{\max} - \phi_{\min}}{0.5(\phi_{\max} + \phi_{\min})} ,$$

where $\phi_{\max}$ and $\phi_{\min}$ are the maximum and the minimum values of the flux at time t_1 at symmetrically opposed points on the axis of perturbation. $\delta\phi_2$ is similarly defined at time t_2 , which is one-half cycle later. This definition

reduces the calculation required by one-half cycle, although the result could be quite inaccurate in the presence of higher modes.

4.3 X-Y ANALYSIS WITHOUT CONTROL

The stability characteristics of cores similar to that described in Section 3 have been determined from detailed two-dimensional digital calculations in which the geometry and material compositions are explicitly represented. The feedback mechanisms due to pointwise Doppler and moderator density variations were represented as described in Section 2.3. The cores had a combination of soluble boron and burnable poison rods to give a calculated moderator coefficient of approximately $-0.5 \times 10^{-4} \Delta\rho/^{\circ}\text{F}$. The results of this analysis indicate that the large, 11-foot-diameter PWR cores are stable with respect to xenon-induced spatial oscillations in a plane at right angles to the coolant flow at average core conditions. Figure 4-1 shows the behavior of the stability index as a function of power density.

The reactor systems were perturbed by inserting a local poison in the form of a control rod in the peripheral region of the core. The poison remained in the core for one hour and was then withdrawn. This sufficiently distorts the flux distribution across the core to initiate a xenon transient. Subsequent oscillations were examined using a time-step length of two hours. The reactivity worth of the perturbation was approximately $0.1\% \Delta\rho$ which is equivalent to the reactivity insertion resulting from a 20°F change in coolant temperature in the reactor.

Figure 4-2 is an example of this type of oscillation for a power density of 84.5 watts/cc. The stability index, as defined earlier, for this core is -0.034 hr^{-1} (stable). It can be concluded from this phase of the study that the 11-foot-diameter cores need no corrective action for this oscillation. This is evident from Figure 4-2 in which it can be seen that the change in the power distribution associated with this perturbation and the resulting convergent oscillation is only a few percent.

Having calculated that the reactor is stable under nominal operating conditions, attention is directed toward finding an artificial means for making

Figure 4-1. Stability Index versus Power Density,
Including Moderator Density Feedback

Figure 4-2. X-Y Power Peaking versus Time after Perturbation for 85 W/cc Core
(With Moderator Density Feedback)

these cores unstable so that a meaningful study of the effectiveness of a control system can be accomplished. Clearly, a relatively large negative reactivity feedback effect must be eliminated so that instability might be achieved. Such an effect is realized by reducing the moderator density feedback to zero in the calculations. In addition, a high-power-density core was selected for the further analysis since this also enhances instability. Thus, all subsequent calculations were carried out on a 104.5 W/cc power density core with no moderator density feedback.

The overall geometric shape of the cores implies that two modes of oscillation must be analyzed. These modes are an oscillation about a principal axis of symmetry (0° or 90°) and an oscillation about a diagonal axis of symmetry ($\pm 45^\circ$). The two geometries are illustrated in Figures 4-3 and 4-4, respectively.

Using the same procedure for perturbing the cores as described previously, two calculations were made in which an oscillation was set up in each of the two geometric modes shown in Figures 4-3 and 4-4. These figures also indicate the locations of the control rods used as perturbations. The two oscillations are shown in Figures 4-5 and 4-6 where the x-y power peaking in symmetrically opposed assemblies are plotted as a function of time following the removal of the perturbation. Both models produced a diverging oscillation; however, the half-core with diagonal symmetry was clearly the more divergent. The stability indices were calculated to be $+0.004 \text{ hr}^{-1}$ and $+0.014 \text{ hr}^{-1}$ for the half-core with symmetry about a principal axis and the half-core with diagonal symmetry, respectively.

Since the diagonal mode of oscillation proved to yield the more unstable condition, the emphasis in this model was placed in the analysis of remedial control procedure. It was judged that if a successful control procedure could be found for this mode of oscillation, a procedure could also be found for the less severe oscillation mode. This, indeed, proved to be the situation.

Figure 4-3. Half-core Geometry with Symmetry about Principal Axis (0° or 90°)

Figure 4-4. Half-core Geometry with Diagonal Symmetry ($\pm 45^\circ$)

Figure 4-5. X-Y Power Peaking versus Time After Perturbation
 (104.5 W/cc Core, Principal Axis Symmetry, No Moderator Density Feedback)

Figure 4-6. X-Y Power Peaking versus Time after Perturbation
(104.5 W/cc Core, Diagonal Symmetry, No Moderator Density Feedback)

4.4 CONTROL OF X-Y XENON-INDUCED SPATIAL OSCILLATIONS

4.4.1 The Control and Detection System

In large PWR cores the means available for the control of xenon-induced spatial oscillations in a plane at right angles to the coolant flow are limited to the insertion of one, or possibly two, full-length control rods. An important part of this study was concerned with identifying which rods were most suitable for the control of xenon during an oscillation. Operational considerations are the governing factors in the selection of the appropriate rods. Intuitively, these rods should be located on the axes where the most probable oscillations might occur so that no additional asymmetry is introduced when corrective action is taken. The two geometric modes were defined in Section 4.3.

The capability of any effective control procedure should be to limit power density variations in any part of the core to within a specified percentage of its value before an oscillation begins. Consequently, a criterion for initiating remedial control action will be governed by power asymmetry conditions detected by the nuclear instrumentation. This study has chosen a quadrant power tilt of ten percent as a criterion for initiating remedial action. Quadrant tilt is defined as the ratio of the power in one quadrant of the core to the average power. When the tilt reaches ten percent, the appropriate xenon control rod is selected by the operator and inserted in the region where the local power density is increasing. Figure 4-7 shows a typical situation where the quadrant tilt is ten percent. The situation corresponds to the oscillation shown in Figure 4-6 at 16 hours, just before a xenon control rod is inserted into the core on a diagonal. The corresponding assembly-wise power distribution is shown in Figure 4-8. The decisions made on which control rod to use, and when, and how long to use it, are based on the assumption that there exists a reliable detection system which can show the behavior of the gross power distribution in the core. Let us assume, therefore, that there are four out-of-core detectors located symmetrically at the core "corners". i.e., on the diagonals about which the most unstable oscillation occurs. At these locations, the instrumentation will detect power

Figure 4-7. Comparison of Outer Assembly Power Tilt with Actual Quadrant Tilt

Figure 4-8. Power Distribution Just Before Corrective Rod is Inserted

distribution changes near the core periphery. Therefore, gross changes in the x-y power distribution can be detected.

To show that these peripheral assemblies, eight in all, adequately represent the behavior of the core during a xenon transient, the average power densities in the outer two assemblies on the diagonal and adjacent to the instrumentation were compared to the detailed core power distribution during a calculated xenon oscillation. An excellent agreement was found between the power in the outer assemblies and the core power distribution. A typical example is shown in Figure 4-7.

A similar comparison was carried out for the power distributions calculated in obtaining the plots in Figures 4-2, 4-5, and 4-6, as well as in Figures 4-11 and 4-12, which are discussed later and include periods of rod insertion. For these power distributions the x-y hot-channel factor, F_{xy} , can be predicted accurately by the quantity $F_o (\phi_{max}/\phi_{avg})$ whether or not the xenon control rod is inserted. The individual parameters are defined as follows: F_{xy} is the power produced in the hot assembly relative to that produced by the average assembly, (ϕ_{max}/ϕ_{avg}) is the value of the power in the outer assemblies in the hot quadrant relative to the average of the power in the outer assemblies in the four quadrants, and F_o is the unperturbed value of F_{xy} . Plotting the quantity $F_o (\phi_{max}/\phi_{avg})$ versus F_{xy} (Figure 4-9), a near-perfect correlation is obtained between what the outer assemblies indicate and how the core is behaving. The perfect correlation is indicated by the 45° line.

4.4.2 Control with a Single Insertion Period

The initial attempts to control the diagonal mode of oscillation indicated that the magnitude of the effect of the control rod on the core radial hot-channel factor was sensitive to the particular location of the rod. In fact, the two outermost control rods on the diagonal both produced unacceptable radial hot-channel factors within a short period of time following their insertion into the core. This effect was found to be less severe for the other oscillation mode. Figure 4-10 shows the control rods which have proved the

Figure 4-9. Simulated Detector Response at Core Corner

Figure 4-10. X-Y Xenon Control Rod Arrangement

most effective for xenon control in the cores studied. Each control rod is worth approximately 0.15 percent reactivity. Consider these rods to be designated x-y xenon control rods; then the following procedure is defined:

- a. When the power in one quadrant of the core exceeds the average of the four quadrants by a set amount, fully insert the x-y xenon control rod closest to the peak.
- b. When the power tilt again approaches the set limit, withdraw the rod. (The rod may be withdrawn at any earlier time but a repetition of the procedure may be required.)

The effectiveness of the procedure is demonstrated by applying control to the divergent oscillations shown in Figures 4-5 and 4-6. For the diagonal mode of oscillation, the xenon control rod located on the diagonal in the quadrant where the power was increasing, was inserted. The xenon was then allowed to redistribute for four hours before the corrective rod was withdrawn. The subsequent behavior of the system is illustrated in Figure 4-11 which shows the power peaking in a horizontal plant as a function of time throughout the operation. The oscillation following the withdrawal of the rod is seen to be damped and the stability index computed from these results is changed from $+0.014 \text{ hr}^{-1}$ to -0.004 hr^{-1} .

The fact that the oscillation converges after the rod has been removed from the core indicates that the new power shape, or xenon distribution, is such that the stability characteristics of the core are significantly altered. This is presumably a temporary state brought about by the redistribution of xenon and iodine and is a phenomenon which is very important to the reactor operator. The significance of this effect, if confirmed, is that repeated and continual action to control an unstable system is not required, but corrective action need only be taken once following a significant perturbation.

In a similar fashion, this procedure is verified for the mode of oscillation set up about the other axis of symmetry (0° or 90°). Previous work has shown this mode to have a greater degree of stability and, in fact, the quadrant power tilt never reaches the ten-percent limit during the first oscillation period. The implication is, of course, that corrective action

Figure 4-11. X-Y Power Peaking versus Time after a Perturbation
 (Xenon Rod, 4 Hour, Diagonal Mode)

is not required until some future oscillation cycle due to the slow growth rate of this mode. Since extensive computer time is required to reach this situation, it was decided to apply control during the first cycle during which the power tilt reaches its peak value of about seven percent. The desired control was achieved by following the same procedure and using the appropriate rod, as shown in Figure 4-10. The power peaking in this case is plotted in Figure 4-12 and the stability index for the oscillation shown after the rod withdrawal is -0.009 hr^{-1} .

4.4.3 Control with Repeated Insertion Period

Clearly, the cases described above need no further corrective action for the time duration investigated. It is seen that the four-hour time interval allowed for xenon redistribution has had a sufficient effect on the core's stability characteristics to change the system from an unstable to a stable condition. Note, however, that the four-hour time interval used in these calculations represents a maximum time rather than an optimum time. By a maximum time is meant that any rod insertion extending beyond four hours results in a violation of the operational restrictions imposed. The x-y hot-channel factor rather than the quadrant tilt determines when the rod must be withdrawn. In Figure 4-13 the x-y peaking factor is plotted as a function of rod insertion time. It is observed that the F_{xy} after four hours reaches a value of 1.40, which is seven percent above the steady-state value of 1.31.

As implied by Figure 4-13, the four-hour insertion time represents a maximum allowable length of time a control rod may be left in the core for corrective action during this xenon transient. This immediately implies the possibility of pulling the corrective rod too soon, with the result that the subsequent oscillation still being divergent. Intuitively, if there is a single control action that tends to damp out a diverging oscillation, then there should exist a set of repeated actions that will accomplish a greater effect.

The consequences of a shorter period of insertion were evaluated on the diagonal mode of oscillation when the initial corrective action was terminated after two hours. The resulting oscillation has a stability index of

Figure 4-12. X-Y Power Peaking versus Time after Perturbation
(Xenon Rod, 4 Hour, Principal Axis Mode)

Figure 4-13. X-Y Power Peaking Factor versus Duration of Rod Insertion

approximately zero; i.e., a sustained oscillation is created where the x-y power peaking factor oscillates between 1.20 and 1.40. To return this system to a steady-state condition further action is required and two possibilities have been examined. First, the same control rod was inserted again two hours after the initial action was terminated and left inserted for two additional hours. This proved unsuccessful as a more divergent oscillation resulted following the withdrawal of the rod. In the second case, the control action was commenced half an oscillation period later; i.e., the control was applied to the peak when it had shifted over to the symmetric quadrant of the core. This proved highly successful as the subsequent behavior of the core indicates that the system has been returned to steady state. The power peaking as a function of time for this case is plotted in Figure 4-14.

An alternate control procedure can therefore be defined, namely:

- a. When the power in one quadrant of the core exceeds the average of the four quadrants by a set amount, fully insert the x-y xenon control rod closest to the peak for a period of two hours.
- b. When the power is growing in the opposite quadrant, insert the symmetrically placed rod for a period of two hours.
- c. Repeat as necessary.

4.4.4 Other Procedures

For the amplitude of oscillation described above it has been demonstrated that successful control can be achieved with insertion times ranging from two to four hours, and within this range success is not critically dependent upon time duration of rod insertion. It has also been established that the four-hour insertion represents a maximum time in this analysis; i.e., any insertion times longer than four hours produced unacceptable power peaks during the insertion. Figure 4-13 illustrates the effect on x-y power peaking as a function of insertion time. In any particular case, the upper limit will be set by the operational restrictions applicable to the reactor system.

Figure 4-14. X-Y Power Peaking versus Time after Perturbation
(Xenon Rod, Two 2-Hour Insertions, Diagonal Mode)

It was mentioned that part of this study was concerned with identifying the most suitable xenon control rods. The results of this effort, of course, are illustrated in Figure 4-10 which shows possible xenon control rod positions. This analysis also considered the use of other rods. It was found that any outer rod located on a diagonal depressed the quadrant power as seen by the detectors to such an extent that the power tilt became intolerable after a relatively short period of time. In addition, a divergent oscillation resumed following the withdrawal of these rods. Control was also attempted by inserting a rod in the quadrant where the power density was decreasing, the object being to burn out the xenon peak more rapidly. This procedure failed primarily because unacceptable power asymmetry conditions resulted when the rod was inserted.

Other procedures were investigated involving the use of rods not on the axis of symmetry, and the simultaneous use of two rods. These procedures either produced an initially unacceptable power distribution or failed to alter the divergent character of the oscillation.

4.5 LIMITATIONS IN THE METHOD OF ANALYSIS

Throughout this analysis the approach of digital simulation was consistently employed when studying xenon-induced spatial oscillations. Although digital simulations facilitate detailed descriptions of core geometries, loading patterns, and control rods, there are definite disadvantages to this method. The most important sources of error in this type of an analysis are the finite length time-step and the pointwise convergence criterion^{1}. In the case of finite time mesh an error is introduced because the spatial flux distribution is assumed to be constant in time for each time-step, thus leading to an incorrect computation of xenon and iodine number densities. This can result in an underestimation of the core stability index and an overestimation of the period; however, the errors decrease as the index approaches zero.

Similar inaccuracies are associated with the pointwise convergence criterion employed in diffusion theory codes. In the present analysis, the time-step

length was chosen to be two hours and the pointwise flux convergence criterion of the order of 10^{-4} , the latter having been established as adequate for most static digital diffusion theory calculations although somewhat coarse for oscillations. The finite length time-step is necessary by the very nature of the model. The choice of the convergence criterion was directly dictated by economic considerations as two-dimensional calculations with pointwise feedback effects included are very time consuming. Typically, one time-step takes 40 minutes on the CDC 6600 computer, depending on the degree of the asymmetry of the flux distribution.

In spite of the indicated limitations to the model employed throughout this work, a good quantitative feeling for the problem of controlling xenon-induced spatial oscillations is realized. The significance of these limitations is somewhat reduced because the unstable system studied was artificially created by eliminating any moderator density effects in the calculations and the purpose of the study is to identify problems of a general nature.

SECTION 5

SUMMARY AND CONCLUSIONS

This report has examined in detail various possible control strategies for the control of xenon-induced spatial oscillations in large PWR cores. The report is in two parts, one describes the calculational methods and the physical makeup of the cores and the other discusses the analysis which defines and evaluates the various control procedures. The entire study is carried out in the x-y plane, i.e., in the plane perpendicular to the direction of the coolant flow.

Digital simulation techniques were the basic calculational tool employed in the analysis. Although this approach has certain drawbacks, it is preferred over other methods since it readily accommodates a realistic treatment of reactivity feedback mechanisms and a detailed spatial description of the relatively complex geometry of large PWR's. The particular spatial representation used for describing the individual fuel assemblies is consistent with the models currently being used in digital diffusion theory calculations for the design of large PWR cores.

Cores of the present size and power density have been shown to be stable provided the pointwise Doppler and moderator density feedbacks are included. Here, an unstable system was created by eliminating the moderator density effect in the calculation. A divergent oscillation was obtained. The effectiveness of various control strategies was then studied using this system as a reference.

Power tilts caused by a gross redistribution of xenon can be detected adequately by a set of detectors located on the outside of the core at the corners, i.e., on the diagonals. These detectors read the power in the outer assemblies on the diagonal, and it is demonstrated that a near-perfect correlation exists between these assemblies and the quadrant power. Likewise, the

x-y power peaking factor is predicted using the known detector readings and the steady-state peaking factor. A near-perfect correlation is indicated at all times during the control procedure.

The means available for the control of xenon oscillations in PWR cores are limited to the use of one or two full-length control rods. This analysis identifies a group of eight rods which are best suited for this purpose. These x-y xenon rods are all located on the major axes of symmetry, i.e., on the principal axes and diagonal axes. Their selection is based on operational restrictions as well as on their capability of stabilizing the cores with respect to xenon oscillation.

A control procedure has been defined which requires a minimum of control action. The procedure involves the single insertion of one of a specified set of control rods for a time period which is not critical and may vary from two to four hours. It was shown that the reference divergent oscillation became convergent after insertion of an appropriate control rod for four hours. The same result was achieved with an initial two-hour insertion followed by a second two-hour insertion in the symmetric quadrant of the core half an oscillation period later.

SECTION 6

REFERENCES

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