SPECTRAL SHIFT CONTROL REACTOR
BASIC PHYSICS PROGRAM

Theoretical Analysis
Part I
Analysis of Experiments

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FOREWORD

Report BAW-1230 has been prepared as a part of the Spectral Shift Control Reactor Basic Physics Program. There are two, separately bound parts to the report. Part II describes the BPG computer code used extensively in the analysis of the experiments. Part I, presented here, contains the analysis of nuclear reactor experiments on heterogeneous lattices moderated by mixtures of light and heavy water. A description is given of the theoretical model, and calculated data are compared with those measured in the experimental program. From these results, the method is determined to be valid, the correlation of a wide variety of experiments is obtained, and fundamental information for large power reactor designs is provided. Also, several discrepancies are noted for further work.

Other details of the experimental data are described in the References.

CONTENTS

	Page
1. INTRODUCTION.	1
1.1. General Description of the Model	1
1.2. General Description of Experiments.	3
1.3. Summary of Conclusions	5
2. DETAILED DESCRIPTION OF THE THEORETICAL MODEL	7
2.1. Slowing Down and Leakage	7
2.2. Resonance Absorption in Fertile Materials	16
2.3. Resonance Absorption in U-235.	24
2.4. Effective Thermal Cross Sections	28
2.5. Computer Codes.	34
2.6. Cross Section Data	37
3. ANALYSIS OF UNIFORM LATTICE EXPERIMENTS	61
3.1. Criticality of Individual Core	61
3.2. Variation of Parameters with D ₂ O Concentrations	65
3.3. Comparison of Boron Poison and D ₂ O Concentration.	70
3.4. Epithermal Neutron Spectrum.	72
3.5. Variation of Parameters with Temperature.	74
3.6. Age and Leakage.	75
3.7. Conclusions for Uniform Lattices	91
4. ANALYSIS OF NON-UNIFORM LATTICE EXPERIMENTS	119
4.1. Two-Zone Core	119
4.2. Water Gaps and Control Blade	121
APPENDIX	A-1
REFERENCES	

List of Tables

Table

2.1. Calculated and Experimental Values of Resonance Integral.	18
2.2. Effective U-235 Resonance Integrals in Lattices.	27
2.3. Epithermal η for U-235 in a 1/E Spectrum	27

Tables (Cont'd)

Table	Page
2.4. Moderator-to-Fuel Thermal Flux Ratio	33
2.5. Resonance Parameters for U-238	42
2.6. Resonance Parameters for Th-232	44
3.1. Criticality of Uniform Lattices	62
3.2. Parameters of Uniform Lattices	63
3.3. Epithermal Neutron Spectra	73
3.4. Age in Th-Al-H ₂ O Lattices	79
3.5. Variation of Age with Energy of Detection	83
3.6. Influence of B ² on $\frac{1}{\phi} \frac{\partial \phi}{\partial B^2}$	83
3.7. Interaction Between Resonance Absorption, Slowing Down, and Leakage.	86
3.8. Buckling-Reactivity Changes	88
4.1. Parameters of Two-Zone Core VII	120
4.2. Reactivity Worth of Water Channels and Control Blade in Core III	123

List of Figures

Figure	
2.1. Comparison of Calculated Thermal Flux Shapes for Core III.	47
2.2. Critical Concentration of U-235 in Core of HRT with D ₂ O Reflector	48
2.3. Single Pin Resonance Integral as a Function of S/M for Uranium Oxide	49
2.4. Single Pin Resonance Integral as a Function of S/M for Thorium Oxide	50
2.5. Resonance Integral of U-238 Vs Effective Scattering per Atom	51
2.6. Resonance Integral of Thorium Vs Effective Scattering per Atom	52
2.7. Dancoff Factor for U-238 Lattice	53
2.8. Dancoff Factor for Th-232 Lattice.	54
2.9. Neutron Flux Profile Through 21.0 ev and 6.7 ev Resonances in U-238.	55
2.10. Effective U-235 Resonance Absorption Integral	56
2.11. Effective U-235 Resonance Fission Integral	57
2.12. Effect of Thermal Spectrum on an Average Cross Section	58
2.13. Thermal Spectrum Parameter for SSCR Lattices	59
2.14. Effect of Thermal Spectrum on Average Cross Sections in SSCR Lattices	60
3.1. Calculation Flowsheet	95
3.2. Critical Buckling of 4%-Enriched UO ₂	96

Figures (Cont'd)

Figure	Page
3.3. Critical Buckling of 93%-Enriched UO_2 - ThO_2 Lattices	97
3.4. Critical Buckling of Boron Poisoned Cores	98
3.5. Cadmium Ratio of U-238	99
3.6. Cadmium Ratio of Th-232	100
3.7. Cadmium Ratio of U-235 Fissions (4%-Enriched UO_2) . .	101
3.8. Cadmium Ratio of U-235 Fissions (U-235 in ThO_2)	102
3.9. Thermal Flux Ratios in U-238 Lattices	103
3.10. Thermal Flux Ratio in Th-232 Lattices	104
3.11. Radial Reflector Savings for Infinite Reflector	105
3.12. Radial Reflector Savings for a 14.22-cm Reflector with Boron Poison	106
3.13. Conversion Ratio	107
3.14. Critical Core Radius Vs D_2O Concentration	108
3.15. Critical Core Radius Vs Boron Concentration	109
3.16. % D_2O Vs Boron Concentration in Critical Reactors	110
3.17. Comparison of D_2O and Boron Poisoning for Equivalent Buckling	111
3.18. Reactivity Holddown of D_2O	112
3.19. Reactivity Holddown of Boron	113
3.20. Neutron Spectrum in Core III	114
3.21. Temperature Variation of Buckling in H_2O Lattice	115
3.22. Temperature Variation of Buckling in 81.5%- D_2O Lattice	116
3.23. Age in H_2O - D_2O Mixtures	117
3.24. Age in ThO_2 -Al- H_2O - D_2O Lattices	118
4.1. Radial Power Distribution in Core VII	125
4.2. Thermal Flux Distribution in Core VII	126
4.3. Flux Distribution Across Central Moderator Channel	127
4.4. Flux Distribution Across Off-Center Moderator Channel	128
4.5. Flux Distribution Across Off-Center Double Moderator Channel Containing Poison Control Blade	129

1. INTRODUCTION

1.1 General Description of the Model

The papers presented at the first two Geneva conferences and the extensive descriptions in textbooks on nuclear reactor theory show clearly that the nuclear characteristics of reactors moderated with H_2O and of those moderated with D_2O are well understood. For both systems, analytical methods and experimental data are available in sufficient quantity and quality to enable the reactor designer to predict the performance of large power reactors with good accuracy. However, the design methods differ for the two systems, and the reactor systems considered by the designers operate primarily on fission from thermal neutrons. Since the Spectral Shift Control Reactor (SSCR) uses mixtures of H_2O and D_2O for moderation, this system cannot be treated accurately by either H_2O or D_2O methods. Also, the SSCR depends on large amounts of resonance absorption for its operating principle, and the accuracy of resonance absorption data is appreciably less than for other aspects of the reactor design problem.

Recent investigations^{6,7,8} show that the Greuling-Goertzel⁹ approximation treats the slowing down of neutrons in mixtures of light and heavy water more accurately than either age theory^{10,12} or the Goertzel-Selengut¹² equations. Also, Nordheim¹⁴ has demonstrated the accurate calculation of resonance absorption in lumps and lattices from resonance parameters. Thus the incorporation of these developments into a multigroup scheme for computer calculations appears to be a suitable method to formulate the neutron problem in SSCR lattices. In addition, it is possible that experimental information interpreted in terms of this formulation will be useful in improving the accuracy of design calculations for reactors moderated with either pure H_2O or pure D_2O .

The general model has been developed in this program with the above possibilities in mind. The specific purpose of the model is to correlate the various experiments that have been performed using pure

and various mixtures of H_2O and D_2O as moderator in tightly packed lattices which have large amounts of resonance absorption. It is expected that this correlation will enable the reactor designer to proceed with accuracy in the design of large nuclear power plants using the Spectral Shift Control principle. The control of these reactors is accomplished primarily by resonance absorption in fertile material, which can be adjusted by varying the ratio of D_2O to H_2O in the moderator. Early in the life of a large power SSCR the H/D ratio in the moderator is low, so that the slowing down power ($\xi \Sigma_s$) is low. This causes many of the neutrons to be absorbed in the resonances of the fertile material. As the core life proceeds, some of the fissile material is burned and the fission product poisons build up, reducing the reactivity of the reactor. To compensate for this, the H/D ratio is increased, increasing the slowing down power and the resonance escape probability. This method of control has several important advantages over a control rod system:

1. Control absorptions are in fertile material rather than in control rods or other poisons, giving increased conversion ratio and core life.
2. More uniform power distributions are possible because the non-uniformities of localized heavy absorbers are eliminated, thus permitting higher power outputs.
3. More reactivity (or excess fissile material) can be held in the core, leading to longer core life.

The spectral shift principle is then the object of investigation, and the model is chosen to give reliable tests of the principle and to be a reliable basis for power reactor design. The wide range of experimental information to be encompassed by the model, and the availability of considerable quantities of accurate cross section data lend themselves to a very general formulation. It now seems possible to relate accurately the basic cross sections, a variety of integral experimental data, and final engineering design calculations through a single model. Specifically, the model chosen is one that can be used generally in reactor design work and one that also correlates accurately the following:

1. Cross section data (thermal, epithermal, fast).
2. Integral data (resonance integrals, age).

3. Criticality
4. Power and flux shapes (spatial).
5. Neutron balance (flux spectrum, cadmium ratios, etc.).

The model is based on 59-group representation of the basic cross sections. The BPG Code¹ has been written to perform an equivalent bare reactor calculation for many groups using the Greuling-Goertzel equations^{6,9} in the B_1 approximation. From the neutron balance of this calculation, the code computes equivalent four-group constants which are used in turn as input to the four-group code¹⁵ for one-dimensional systems or to PDQ¹⁶ for two-dimensional geometries. The details of this model are discussed in Chapter 2.

1.2. General Description of Experiments

To demonstrate the spectral shift principle and to obtain a good quantity of basic experimental information concerning spectral shift reactors, a varied series of physics experiments has been performed. These experiments are basic to the technology of heterogeneous reactors moderated and cooled by a variable mixture of light and heavy water. Because of the variety of the experiments, the results provide a good test of the theoretical model. Experiments were performed with two different types of fuel rods; a 4%-enriched UO_2 swaged in stainless steel tubes and a mixture of ThO_2 and fully-enriched U-235 oxide pellets in the atom ratio $N_{\text{Th}}/N_{25} = 15$ inside aluminum tubes. The volume fraction of moderator in all experiments was approximately 1/2. Detailed descriptions of the lattices are given in the Appendix. The types of experiments are described generally below, with more detailed information in the references.

1.2.1. Clean Lattices with 4%-Enriched UO_2 Fuel Rods^{2,3}

A series of critical experiments were performed in which cylindrical core radius was varied as the ratio of D_2O to H_2O in the moderator changed. Critical assemblies were obtained at 0, 49.66, 69.70, and 76.50% D_2O in the moderator. To extend the range of measurements with the limited number of fuel rods, exponential experiments were performed at 80.65 and 89.14% D_2O in the moderator. For each assembly the buckling was measured, and for most assemblies the following other data were obtained.

1. Cadmium ratio of U-235 fissions.
2. Cadmium ratio of U-238 captures.
3. Average ratio of thermal flux in the moderator to that in the fuel.
4. Radial reflector savings.

For several critical assemblies the reactivity worths of control blades and of differential water height were also measured.

1.2.2. Lattices with Boron Poison²

As a measure of reactivity worth to compare with D₂O addition, boric acid was dissolved in the moderator of a number of assemblies. In H₂O moderator, the variation of critical core radius with boric acid concentration was determined. For the full size core, the variation of critical boric acid concentration with D₂O concentration was measured from 0 to 76.50% D₂O in the moderator.

1.2.3. Clean Lattices with 93%-Enriched UO₂-ThO₂ Fuel Rods^{2,3,17}

A critical experiment was performed using fuel rods containing ThO₂ and U-235 in the atom ratio $N_{Th}/N_{235} = 15$, assembled in a lattice similar to the UO₂ fuel described above. The moderator contained 60.40% D₂O. These same fuel rods had been used previously in pure H₂O moderated lattices, one of which was nearly the same lattice pitch as measured here. To extend the range of D₂O in the moderator, three exponential experiments were performed at 55.38, 71.94, and 81.96% D₂O. In all assemblies, the buckling and cadmium ratio of U-235 fissions were measured. In several assemblies, the Th-232 cadmium ratio was measured, and in the critical assemblies, measurements were made of the radial reflector savings and of the ratio of thermal flux in the moderator to that in the fuel.

1.2.4. Age Measurements⁵

A lattice of ThO₂ fuel rods with the same configuration as the other assemblies was formed without U-235 so that age measurements could be made. Age determinations were made for 0, 49.77, 69.19, 79.19, and 90.0% D₂O in the moderator.

1.2.5. Hot Exponential Experiments⁴

The 4%-enriched UO₂ fuel rods were assembled in a pressurized exponential experiment. The variation of material buckling

with temperature was measured between 66 and 406 F for light water moderator and for 81.5% D₂O in the moderator.

1.2.6. Two-Zone Reactor²

As a test of the spatial calculations in the theoretical model, a two-region critical experiment was assembled using the 93%-enriched UO₂-ThO₂ fuel rods in a central (radial) zone and the 4%-enriched UO₂ fuel rods in an outer zone. Criticality was obtained for 81.16% D₂O in the moderator. Flux and fission density traverses were made, and the cadmium ratio of U-235 fissions and the ratio of thermal flux in the moderator to that in the fuel were measured in the asymptotic part of each zone. The cadmium ratio of thorium captures was measured in the inner zone.

1.2.7. Perturbation Experiments²

In a core with 76.50% D₂O in the moderator several measurements were made to determine the effects of gaps and control blades. Reactivity variations and local flux plots were measured for two sizes of moderator filled gaps, and for the introduction of a control blade into one of them.

1.3. Summary of Conclusions

The data obtained in the series of SSCR experiments and the theoretical analysis of the results demonstrate the spectral shift control principle clearly. The critical assemblies show that considerably larger amounts of fuel can be loaded into a core moderated with a mixture of D₂O and H₂O than in one moderated with H₂O alone. The measured cadmium ratios and the calculated neutron balances show that the excess reactivity associated with this extra mass is held up in the resonance absorption of the fertile material. This leads to a higher conversion ratio system than the same sized lattice controlled by thermal neutron poison.

Very good agreement is obtained between experimental and theoretical values for nearly all important parameters over the entire range of D₂O concentration in the moderator. This agreement confirms the use of the BPG-four-group treatment, and demonstrates the calculation of reactor criticality and other measured integral parameters from fundamental cross sections without the use of intermediate fitting functions.

The reactor leakage part of the calculational model is shown to be accurate by the good agreement obtained between calculated and experimental values of age, criticality, change of criticality with moderator height, and the comparison between boric acid poisoning and reactivity holddown with D_2O . Comparison of measured and calculated cadmium ratios indicates that the model gives good values of resonance absorption for U-238 and of resonance fission for U-235. The accurate criticality results also confirm this. More cross section and lattice cadmium ratio data are needed for thorium before definitive conclusions can be reached.

In two areas theory and experiment do not agree. Further analytical and experimental work is needed to obtain better results in the analysis of thermal flux ratios. There is definite disagreement between calculated and measured temperature defect, and effort should be exerted to resolve the discrepancy.

The good agreement between theory and experiment justifies the use of the model in this series of experiments, but the experiments were all performed with the same moderator volume fraction and fuel. Further confirmation would be useful at other moderator volume fractions and fuel enrichments.

2. DETAILED DESCRIPTION OF THE THEORETICAL MODEL

2.1. Slowing Down and Leakage

2.1.1. The Basic Model and Equations

An exact solution of the slowing down problem is possible for hydrogen moderator, and the Fermi age treatment is quite accurate for heavy elements. Somewhat more complex treatment is required for slowing down in deuterium moderator. In the Greuling-Goertzel approximation^{6,9} deuterium is treated in a hydrogen-like manner, computing the slowing down density, like hydrogen,

$$q_H = \int_0^u \Sigma_H(u') e^{u'-u} \phi(u') du'$$

to be

$$q_D = \frac{\xi_D}{\gamma_D} \int_0^u \Sigma_D(u') e^{u'-u/\gamma_D} \phi(u') du'.$$

Here, ξ is the average lethargy gain per collision ($\xi_D = 0.725$ for isotropic scattering) and γ is the Greuling-Goertzel constant which also can be computed easily from microscopic collision parameters ($\gamma_D = 0.583$ for isotropic scattering). For a finite reactor, the spatial variation of the flux can be separated from the energy variation in the equivalent bare reactor approximation. For small systems, the use of the B_1 expansion¹⁸ in the Fourier transformed equations leads to more accurate treatment of the leakage. The Greuling-Goertzel equations in the B_1 approximation are used as the basic model in the analysis of SSCR cores. These are contained in the BPG Code¹ which has been written for the Burroughs Datatron-205 computer. The code computes the criticality and neutron balance in a bare reactor using up to 80 energy groups (39 were used in the present work).

The neutron balance is reformulated by the computer to give four-group constants, which are used in turn in one- or two-dimensional multi-region calculations. MUFT-IV¹⁹ is a similar formulation, but the BPG permits the use of the Greuling-Goertzel approximation for many elements, contains the consistent form of the expansion approximation, and handles the data differently. BPG solves the following equations:

$$B J(u) + [\Sigma^A(u) + \Sigma^{in}(u)] \phi(u) = S(u) - \sum_k \frac{dq_k}{du} - \frac{dq_F}{du} + I(u) \quad (1)$$

$$[h(u)\Sigma^T(u) - \mu(u)\Sigma^S(u)] J(u) - \frac{B}{3} \phi(u) = \sum_m \frac{dp_m}{du} \quad (2)$$

$$\gamma_k(u) \frac{dq_k}{du} + q_k(u) = \xi_k(u) \Sigma_k^S(u) \phi(u) \left[1 - \frac{d\gamma_k}{du} \right] \quad (3)$$

$$q_F(u) = \sum_\ell \xi_\ell(u) \Sigma_\ell^S(u) \phi(u) \quad (4)$$

$$\zeta_m(u) \frac{dp_m}{du} + p_m(u) = \eta_m(u) \Sigma_m^S(u) J(u) \left[1 - \frac{d\zeta_m}{du} \right] \quad (5)$$

where

- u = lethargy
- $\phi(u)$ = flux
- $J(u)$ = current
- $q(u)$ = slowing down density from elastic scattering
- $p(u)$ = anisotropic component of slowing down density
- $I(u)$ = source from inelastic scattering
- B^2 = buckling
- $\Sigma^A(u)$ = absorption cross section
- $\Sigma^{in}(u)$ = inelastic scattering cross section
- $\Sigma^S(u)$ = elastic scattering cross section
- $\Sigma^T(u)$ = total cross section
- $S(u)$ = neutron source (usually from fission)
- $\mu(u)$ = average cosine of the scattering angle (laboratory system)
- $\xi(u)$ = average lethargy gain per collision
- $\gamma(u), \zeta(u), \eta(u)$ = slowing down parameters

$$h(u) = \frac{\alpha^2}{3} \left[\frac{\tan^{-1} \alpha}{\alpha - \tan^{-1} \alpha} \right], \text{ where } \alpha = \frac{B}{\Sigma^T(u)}$$

In the code it is possible to choose whether the Greuling-Goertzel approximation (subscript k) or the Fermi age approximation (subscript l) is to be used for slowing down from each nuclear species. The model used in the analysis chose all nuclei of mass 27 or less to be treated by Greuling-Goertzel and all others by Fermi age. Corrections to the slowing down density caused by anisotropy of scattering in the laboratory system are significant for the lighter elements. Equation (5) accounts for this and gives the consistent form of the B_1 approximation. If $p(u)$ is zeroed, the inconsistent form is obtained. In the analysis, the consistent form was used for slowing down by H and D, and the inconsistent form was used for all other nuclei.

2.1.2. Inelastic Scattering

Inelastic scattering is treated by a matrix transfer of neutrons between groups. Two matrices were investigated and negligible differences in age or reactivity were found by comparing them. One matrix assumes that neutrons scattered inelastically at lethargy u are distributed uniformly in lethargy up to a chosen threshold point (energy = 0.137 Mev). In the other matrix, which was used in this analysis, a better theoretical picture of the emission spectrum of the inelastic scattering process is taken into account ^{20,21,22} Three energy bands are distinguished.

1. Above 1.92 Mev the excitation levels are close together and the level scheme is considered as a continuum. The emission spectrum is a continuous function and the evaporation model is used.

2. Below 1.30 Mev, the excitation levels are spaced fairly far apart. A neutron suffering an inelastic collision loses a discrete amount of energy corresponding to the energy of one of these excitation levels, so that the emission spectrum is a discontinuous function. Cross sections for exciting single levels are given in Reference 21, and the emission spectra were computed by Amster's method. ²⁰

3. Between 1.30 and 1.92 Mev the excitation levels are close together, but the cross section for the levels below 1.30 Mev is

still important. The emission spectrum used in this band is a combination of an evaporation spectrum and a discontinuous one, each weighted by the cross section at 1.89 Mev (Reference 21).

This method is similar to that of Amster.²⁰ Only the matrix for U-238 has been computed. It is believed that this matrix also describes the inelastic process in thorium, because the level scheme of thorium is very similar to U-238. Since U-238 and Th-232 are the major contributors to inelastic scattering in SSCR lattices, this same matrix is used for all inelastic scattering.

2.1.3. Anisotropic Scattering

Anisotropic scattering in the center of mass system is an important effect at energies in the Mev range. A scheme has been set up to compute the variation of the scattering parameters in the BPG Code caused by the anisotropy. A two-parameter fit has been made to the measured angular distribution^{23,24} for each nuclear species, and from this the variation of $\mu(u)$, $\xi(u)$, $\gamma(u)$, $\zeta(u)$, $\eta(u)$ can be computed. Because of the uncertainties in most of the available data, this variation has been computed only for $\mu(u)$ for each element. In the present analysis, ξ , γ , ζ , and η are all assumed constant (for each element), but when better data are available the variation of these parameters with lethargy can be computed. The error in the analysis caused by these assumptions should be small, since the major variation, $\mu(u)$, is accounted for. Agreement of calculated ages with measured values, described later, substantiates this.

2.1.4. Fast Diffusion Constant

The diffusion equation as used in multigroup formulations has the form

$$D(u) B^2 \phi(u) + \Sigma_a(u) \phi(u) = \text{Source}.$$

For weak absorption the group diffusion constant

$$D = \frac{1}{3 \Sigma_s (1 - \mu)} . \quad (\text{Diffusion})$$

The equivalent coefficient in the P_1 expansion of the Boltzmann equation is

$$D = \frac{1}{3 (\Sigma_T - \mu \Sigma_s)} , \quad (P_1)$$

which applies also for weak absorption and an infinite medium. For thermal neutrons the first order correction for non-negligible absorption gives

$$D = \frac{1}{3(\Sigma_T - \mu\Sigma_s - 4/5 \Sigma_a)} \quad (\text{Thermal})$$

again valid for an infinite medium. For finite media the B_1 expansion leads to an equivalent diffusion coefficient (see the BPG equations (1) and (2) above),

$$D = \frac{1}{3(h\Sigma_T - \mu\Sigma_s)} \quad (B_1)$$

where h is greater than unity.

Because inelastic scattering is treated in a group model in a fashion similar to absorption, the effective absorption in the high energy groups is large. Thus it seems reasonable to apply a first order absorption correction to the equivalent diffusion coefficient similar to the one used for thermal neutrons. An investigation of this possibility has been made by performing the inverse Fourier transform on the B_1 expansion of a point source to obtain an effective attenuation length, identified as L in the relation

$$L^2 = \frac{D}{\Sigma_a}.$$

Various approximations of the flux function were made so that higher order terms in the transform variable $\frac{k}{\Sigma_T}$ could be included in the computed inverse transform. The first order expansion leads to the uncorrected diffusion theory result, and the next order expansion leads to the same form as the corrected result for thermal neutrons. Further investigation will be needed to determine the proper choice of the fast diffusion constant. In the course of analysis of the age experiments, a numerical comparison was made of the models to the experimental data. In BPG Equation (2) the value of

$$\Sigma_T = \Sigma_s + \beta(\Sigma_a + \Sigma_{in})$$

was computed for values of $\beta = 1$ (uncorrected) and $\beta = 0.2$ (corrected). The ages computed with $\beta = 0.2$ agreed with experiment slightly better than those with $\beta = 1$. However, because of the unresolved theoretical question, the basic analysis was made by using the uncorrected ($\beta = 1$) B_1 equations.

2.1.5. Group Boundaries

In the model used in the analysis, the energy variation of neutron flux was divided into 38 epithermal groups and one thermal group. Group cross sections were obtained by averaging BNL-325 cross sections²⁵ over an assumed $1/E$ flux in the group. The energy bounds of the groups are listed in the cross section library.²⁶ BPG Code calculations were performed for each region, and where spatial variations were computed with either one- or two-dimensional codes, a four-group equivalent scheme was derived from the neutron balance of the 39-group analysis. The four-group energy bounds are:

Group 1	10 ⁷ ev to 9210 ev
Group 2	9210 ev to 251.7
Group 3	251.7 ev to 0.4 ev
Group 4	Thermal

The equations solved by the four-group representation¹⁵ are:

$$D_1 \nabla^2 \phi_1 - W_1 \phi_1 + \frac{1}{k_{\text{eff}}} [f_1 \phi_1 + f_2 \phi_2 + f_3 \phi_3 + f_4 \phi_4] = 0$$

$$D_2 \nabla^2 \phi_2 - W_2 \phi_2 + \beta_1 \phi_1 = 0$$

$$D_3 \nabla^2 \phi_3 - W_3 \phi_3 + \beta_2 \phi_2 = 0$$

$$D_4 \nabla^2 \phi_4 - W_4 \phi_4 + \beta_3 \phi_3 = 0$$

In these equations the coefficients

D_i = diffusion

W_i = removal

β_i = transfer

f_i = fission source

are computed as averages from the BPG Code¹ to give the same neutron balance as that code does. To test group representation accuracy, a variety of calculational procedures were compared.

1. Two different choices of 39-group boundaries for a bare reactor calculation agreed on criticality within 1/2%. This indicates that the choice of boundaries is not a sensitive parameter. Final choice of the group boundaries is believed to reduce the uncertainty in computed criticality to considerably less than 1/2%.

2. By using the same group bounds, adjacent groups were joined to reduce the number of groups in bare reactor calculations of k_{eff} , giving the ratios

$$\frac{k(20 \text{ groups})}{k(39 \text{ groups})} = 1.011$$

$$\frac{k(11 \text{ groups})}{k(39 \text{ groups})} = 1.023 .$$

These results indicate that the larger number of groups is needed to reduce computing inaccuracies.

3. By using the same group boundaries as above, one-dimensional multigroup calculations (including effects of spatial variation) were performed for 20 groups and for 11 groups, giving

$$\frac{k(11 \text{ groups})}{k(20 \text{ groups})} = 1.008.$$

This result is practically the same as the bare reactor comparison.

4. A comparison was made between a four-group calculation with coefficients generated by BPG (in the P_1 approximation, 39 groups) and a 39-group, one-dimensional calculation using the same constants as BPG. The PIMG Code²⁷ was used for this purpose, and the ratio of k_{eff} was

$$\frac{k(39 \text{ groups})}{k(4 \text{ groups})} = 1.002.$$

The equivalent four-group fluxes from PIMG were compared to the BPG four-group fluxes and were found to be in good agreement.

(See Fig. 2.1.) Core III (76.5% D_2O) was used as the physical model.

5. Numerous calculations were made with the BPG-Four-Group scheme to determine the criticality of reflected reactors. It was noted in these that there was negligible difference in Four-Group k_{eff} values between cases in which the BPG calculations used zero buckling and those in which it used critical buckling. From this it is concluded that the choice of B^2 in the BPG calculation of group constants does not affect the spatial criticality calculation.

The main conclusions drawn from these calculations are that the BPG four-group representation is nearly as good a model as the equivalent 39-group multigroup representation, and that the representation is improved using 39 groups rather than 20 or less in the BPG Code.

2.1.6. Fission Source

The source of fast neutrons from fission which was used in the BPG Code was the usual prompt neutron spectrum.²⁸ The effect of the delayed neutrons on the reactor age was found to be negligible. Two multigroup calculations of criticality were performed, one using the prompt fission source, and the other using a source including delayed neutrons (both sources normalized to unity). The ratio of computed k_{eff} of the two cases was 1.001. For kinetics calculations, the effects of both delayed fission neutrons and of D (γ, n) neutrons were used², and changes (small but not negligible) in the inhour equation were noticed.

The fission source is correct to use in multiplying media, but in reflector regions where the primary source of fast neutrons comes from in-leakage, the choice of source is not so definite. In light-water systems, the major part of the reactor leakage has nearly a fission spectrum energy dependence, so it is reasonable to obtain effective four-group coefficients for reflector regions by averaging the cross section over an asymptotic spectrum with a fission source. This recipe was used in the analysis of the experiments. For comparison, a BPG four-group calculation of Core III was performed once using four-group reflector coefficients averaged over the spectrum resulting from a source which was the leakage from the core. The ratio

$$\frac{k_{\text{eff}}(\text{leakage})}{k_{\text{eff}}(\text{fission})} = 1.006 .$$

This is an appreciable effect, and represents some of the uncertainty inherent in the choice of a computational recipe. Since the computed k_{eff} and fluxes using the fission source in the reflector gave better agreement with the equivalent PIMG calculation (Fig. 2.1), the fission source was used. Further analysis may improve this part of the recipe.

2.1.7. Comparison of Leakage Model with Other Experiments

To test the validity of the BPG leakage model, several crucial experiments were selected for analysis. The experiments were homogeneous with spherical geometry, very few epithermal absorptions, and large leakage.

The HRT provides an excellent check for calculations for a pure D₂O moderator^{29,30}. In this reactor, U-235 is in solution in D₂O in the core region, and a pure D₂O reflector surrounds it. Since core k_{∞} is close to 2, the non-leakage probability is close to 0.5. Thus, calculations of this system are very sensitive to the leakage model. Experimental data are reported for four different temperatures, and the BPG four-group calculations for these cases agreed closely with experiment (Fig. 2.2). Thus, the model represents slowing down and leakage in a D₂O system fairly well.

Three small, bare H₂O-moderated critical assemblies³¹ served as good checks for the H₂O leakage model. Pertinent data are:

<u>Core</u>	<u>Radius, cm</u>	<u>UO₂F₂ · H₂O density, gm/cm³</u>	<u>U-235 density, gm/cm³</u>	<u>H/U-235 atom ratio</u>
A	16.0	1.160	0.129	198
B	27.9	1.050	0.1239	1088
C	34.9	1.045	0.0191	1363

The choice of extrapolation distance at the outer boundary is important for these cores. In the analysis model, λ_{tr} for each BPG group was weighted with the leakage in that group to determine the $\bar{\lambda}_{tr}$ for the assembly. Then, a constant extrapolation length $0.71 \bar{\lambda}_{tr}$ was used for all groups. A comparison calculation demonstrated that the reactor buckling computed in this manner gives nearly the same total core leakage as if each group had a different buckling based on the λ_{tr} in that group. Two other methods of obtaining the extrapolation distance and effective buckling have been investigated.³¹ The end-point method uses the boundary condition that the asymptotic solution for the thermal flux distribution goes to zero at the extrapolated end point. The current method chooses the linear extrapolation distance which causes the inward current to vanish. Comparison of the extrapolation distance, d , and the buckling, B^2 , by the three methods is shown by the following tabulation.

		<u>Core A</u>	<u>Core B</u>	<u>Core C</u>
End-Point Method	$d, \text{ cm}$	2.5	2.5	2.5
	$B^2, \text{ cm}^{-2}$	0.0288	0.0107	0.0070
Current Method	$d, \text{ cm}$	3.7	3.7	3.7
	$B^2, \text{ cm}^{-2}$	0.0244	0.0097	0.0065
$\bar{\lambda}_{\text{tr}}$ Method	$d, \text{ cm}$	2.73	2.78	2.84
	$B^2, \text{ cm}^{-2}$	0.0281	0.0105	0.0069

Criticality was computed by the BPG Code for each method. The results using the end-point and $\bar{\lambda}_{\text{tr}}$ methods agreed well, and both consistently gave results close to $k_{\text{eff}} = 1$. Another set of calculations compared the P_1 and B_1 approximations and showed the improved accuracy of the B_1 approximation in small core analysis.

<u>Approximation</u>	<u>Extrapolation method</u>	<u>Computed k_{eff}</u>		
		<u>Core A</u>	<u>Core B</u>	<u>Core C</u>
Consistent B_1	End Point	0.998	1.004	1.009
	Current	1.078	1.029	1.023
	$\bar{\lambda}_{\text{tr}}$	1.010	1.010	1.011
Consistent P_1	End Point	0.947	0.991	1.003
	Current	1.034	1.017	1.017
	$\bar{\lambda}_{\text{tr}}$	0.960	0.997	1.005

From these analyses it is concluded that the BPG Code in the B_1 approximation computes high leakage H_2O systems accurately if the leakage-averaged extrapolation recipe is used. Therefore, the model gives good results on high leakage systems both for H_2O and D_2O moderators.

2.2. Resonance Absorption in Fertile Materials

2.2.1. Resonance Absorption Model

In the multigroup formulation of the BPG Code, the groups are of finite width and generally are large compared with the

width of a single resonance. In U-238 and Th-232, a large part of the resonance absorption takes place in a few very strongly absorbing lines. This absorption is found in the code by computing the resonance escape probability for a single resonance and by obtaining the product of slowing down density and the absorption probability. If q_i is the slowing down density into group i , and if p is the resonance escape probability for a single peak,

$$\text{absorptions in the peak} = q_i (1-p)$$

$$= q_i \frac{NR}{(\xi \Sigma_s)_i},$$

where NR is the macroscopic resonance integral over the peak and $(\xi \Sigma_s)_i$ is the slowing down power in the group.

The model obtains resonance integrals of single peaks from the resonance parameters, computing a Doppler broadened Breit-Wigner shape. The effects of lump size and lattice mutual shielding are accounted by escape probabilities computed according to the methods of Nordheim¹⁴ and Dancoff and Ginsburg.³² The measured resonance parameters are related through these fundamental models to measured lattice cadmium ratios and criticalities with good accuracy. As an intermediate check, measured single pin resonance integrals agree well with computed values.

2.2.2. Single Lump Calculations

Resonance parameters for U-238 and Th-232 are shown in Tables 2.5 and 2.6. For the tabulated lines, the Doppler-broadened Breit-Wigner formula has been integrated to obtain the effective resonance integral for each line. For the unresolved region, average resonance parameters were obtained from the resolved data. The computer code,³³ which performs this calculation, also computes the lump escape probability and from this the effective lump resonance integral.³⁴ The choice of NR or of $NRIA$ approximations for each resonance line is important,³⁵ and the model follows the analysis of Nordheim.¹⁴ High energy (above 10^4 ev) capture and fission cross sections were integrated numerically over a $1/E$ spectrum to obtain that part of the resonance integral, and the cross section below the lowest energy resonance was assumed to be $1/v$. The calculated

values are compared with experiment in Table 2.1 (the $1/v$ part is excluded from this tabulation). The high energy region is taken from 10^7 to 10^4 ev, the unresolved region from 10^4 to 10^3 ev, and the resolved region is from 10^3 to 5.4 ev for U-238. For Th-232, the high energy region was from 10^7 to 10^4 ev, the unresolved region was from 10^4 to 10^3 ev, and the resolved region was from 10^3 to 20.0 ev.

The U-238 oxide experimental data are those of Hellstrand.³⁶ The experimental data for Th-232 single lumps were taken from metal data of Hellestrand³⁷ and interpreted for oxide. Since the calculated results for Th-232 did not agree with the measured single lump resonance integral, the final values used in the analysis were obtained by renormalization to fit the measured integral value. (See Table 2.1.) The discrepancy between the values from resonance parameters and those from measured integral experiments needs further fundamental investigation. Other experimental information for thorium oxide and thorium metal has been obtained by Pettus.³⁸ These results, those of Hellstrand for uranium oxide, and some preliminary results of Weitman and Hellstrand³⁹ are compared with calculated values for various lump sizes in Figures 2.3 and 2.4. Lump sizes are defined in terms of the surface-to-mass ratio, S/Max , where Max is the total oxide mass.

Table 2.1 Calculated and Experimental Values
of Resonance Integral

	U-238		Th-232		
	<u>Infinitely dilute</u>	<u>SSCR rod</u>	<u>Infinitely dilute</u>	<u>SSCR rod</u>	<u>Normalized SSCR rod</u>
High Energy	2.73b	2.73b	2.24b	2.24b	1.91b
Unresolved	3.95	1.89	4.45	3.45	2.94
Resolved	262.66	14.71	90.69	16.99	14.45
Total Calculated	269.34	19.33	97.38	22.68	19.30
Experimental	281	20.4	99	19.3	
Reference	28	36	38	37	

These comparisons of theoretical and experimental effective resonance integrals are good enough to indicate an adequate theory to relate measured resonance parameters to measured single lump effective resonance integrals for U-238. For Th-232, there is the discrepancy noted above. The relation to effective lattice resonance integrals is more complex.

2.2.3. Lattice Representation

The effective resonance integral of a single resonance peak in a single lump is a function of the escape probability from the lump. Dresner⁴⁰ and Levine⁴¹ have shown that since this escape probability is related to the surface-to-volume ratio, S/V , of the lump, a single lump resonance integral can be related to a homogeneous medium effective resonance integral through the variable

$$\sqrt{\sigma_p + \frac{S}{4NV}},$$

where, σ_p is the potential scattering in the lump per absorber atom and N is the density of absorber atoms in the lump. If a large number of lumps are assembled into a regular lattice, the mutual shielding between lumps is equivalent to a reduced escape probability from the lump. Bell⁴² has related this to an equivalent Dancoff³² factor, and very good correlations of single lump resonance integrals with lattice values have been shown by Levine⁴¹ and Drawbaugh⁴³, by using the effective variable

$$\sqrt{\sigma_p + \frac{\gamma S}{4NV}},$$

where γ is the Dancoff factor. The results indicate a nearly linear dependence of the resonance integral on this variable. The model used in the analysis uses this relation to determine the effective lattice resonance integral for the resolved and unresolved resonance lines. The high and low energy ($1/v$) absorption cross sections are assumed to be unaffected by the presence or absence of other pins, or by lumping effects. Figures 2.5 and 2.6 compare the theoretical calculation with experiment using this model.

2.2.4. Dancoff Calculations

Fukai⁴⁴ reviewed calculations of mutual shielding in close packed lattices and showed that his method and that of Dancoff

and Ginsburg³² are in closer agreement with the more accurate Monte Carlo results than is the method of Bell⁴². His work, however, has been performed for a two-region lattice cell (moderator and fuel). For clad fuel pins, Levine⁴¹ and Drawbaugh⁴³ have shown good agreement with the experiment using the Bell method, and in a special assembly of clad fuel pins, Pettus⁴⁵ has measured the Dancoff factor with changing moderator and finds good correlation with the Bell method. Further Monte Carlo work may clarify this situation.

A three-region calculation using the Dancoff-Ginsburg method has been performed with computer numerical integration⁴⁶ to account for the clad on the fuel pins. These results are in good agreement with calculated values using the Bell approximation for most of the SSCR lattices containing U-238. For the Th-232 lattices, the agreement is not as good. Figures 2.7 and 2.8 show the comparisons. The values are only for the lattices in these experiments where the metal-to-water ratio $M/W = 1$. Further work is needed to see if this same agreement occurs for other lattices. The Dancoff and Ginsburg formulation was standard in the model used for the analysis.

2.2.5. BPG Representations

The Greuling-Goertzel equations in the B_1 approximation are solved by the BPG Code as a number of difference equations. The neutron balance equation for each group (j) is given¹ as

$$\begin{aligned} B_j J_j + (\Sigma_j^A + A_j \Sigma_j^{\text{in}}) \phi_j + \sum_k q_k(u_j) + q_A(u_j) \\ = S_j + p_j \left[\sum_k q_k(u_{j-1}) + q_A(u_{j-1}) \right] + I_j. \end{aligned}$$

The absorption cross sections of U-238 and Th-232 are treated in different ways at different energies. In the high energy region above 10^4 ev, the cross sections are used directly and are added to the value of Σ_j^A . Between the low energy cutoff (0.4 ev) and the lowest resonance (5.4 ev for U-238, 20.0 ev for Th-232), the cross sections are $1/v$ and are treated in the same manner, adding them to the value of Σ_j^A . In the intermediate region, the effective resonance integrals, R_{ij} , of all peaks in the group are added, and the value of resonance escape for the group is computed as

$$P_j = \exp \left[\frac{-\alpha N \sum_i R_{ij}}{(\xi \Sigma_s)_j} \right],$$

where, N is the reactor atomic density of U-238 or Th-232. The U-238 and Th-232 values of $\sum_i R_{ij}$ for each group have been precomputed for several lump sizes and are stored in the cross section library of the code.²⁶ Appropriate Dancoff corrections for lattices are applied uniformly to all resonances by the factor α . This factor is defined by comparing the single pin resonance integral

$$R_{SP} = R_{SP}^H + R_{SP}^R = A \sqrt{\sigma_p + \frac{S}{4NV}}$$

broken into the high energy and resonance components, and is computed with the lattice value

$$R_L = R_L^H + R_L^R = A \sqrt{\sigma_p + \frac{\gamma S}{4NV}}$$

broken into similar components. Here, N is the lump atomic density of U-238 or Th-232. The high energy components R_L^H and R_{SP}^H must be the same, so that

$$R_L^R \equiv \alpha R_{SP}^R$$

defines α . This definition of α is the same as the quantity $\frac{P+\gamma}{P+1}$ described in the BPG part of this report.¹

2.2.6. Exact Calculation of Resonance Absorption

To estimate the accuracy of epithermal absorption calculations by the methods described above, several simple cases were compared with exact calculations. The slowing down and absorption of neutrons in deuterium moderator were computed exactly four cases of constant cross section. Numerical comparison of the exact results with the Greuling-Goertzel and Wigner approximations indicates that the Greuling-Goertzel calculation is more nearly correct for a low absorption cross section spread over a large lethargy band, and that the Wigner approximation is better for a strong absorber having a narrow lethargy spread. Both approximations give values of resonance escape probability which agree well with that computed from the exact formula. Starting with the exact expression for collision density

$$[\Sigma_s(u) + \Sigma_a(u)] \phi(u) = \int_{u + \ell \ln \alpha}^u \Sigma_s(u') \phi(u') \frac{e^{u' - u}}{1 - \alpha} du',$$

the approximate expressions for resonance escape probability are derived by Glasstone and Edlund ¹¹

$$p = \exp \left[- \int_{u_1}^{u_2} \frac{\Sigma_a}{\xi \Sigma_s + \gamma \Sigma_a} du \right] \quad (\text{Greuling-Goertzel})$$

and

$$p = \exp \left[- \int_{u_1}^{u_2} \frac{\Sigma_a}{\xi \Sigma_s + \Sigma_a} du \right] \quad (\text{Wigner})$$

With a moderator of mass number greater than unity, an analytic solution of the exact integral equation is possible if all cross sections are energy independent. Four cases for numerical calculation were selected to have constant cross sections over specified lethargy intervals.

Case	1	2	3	4
Σ_s (cm ⁻¹)	0.3	0.3	0.3	0.3
Σ_a (cm ⁻¹)	0.03	0.03	0.3	3.0
lethargy width of "peak"	$-\ell \ln \alpha$	$-2\ell \ln \alpha$	$\frac{-\ell \ln \alpha}{10}$	$\frac{-\ell \ln \alpha}{100}$

In all cases, Σ_s was constant between lethargy $0 \leq u \leq 19$, and Σ_a was set = 0 above and below the "resonance peak" between $u = 10$ and $u = 10$ + "peak" width. Cases 1, 3, and 4 were chosen so that the resonance integral $R = \Sigma_a \Delta u$ is the same. Case 2 was chosen to have twice the resonance integral as case 1. For a pure deuterium moderator, $\alpha = 1/9$. Numerical results for resonance escape probabilities are:

Case	1	2	3	4
Exact Solution	0.7560	0.5709	0.8575	0.9727
Wigner	0.7592	0.5763	0.8594	0.9728
Greuling-Goertzel	0.7554	0.5706	0.8453	0.9670

Since the narrow, heavily absorbing resonance lines of U-238 and Th-232 correspond approximately to case 4 where the Wigner approximation gives better results, the choice was made to treat these resonances in this fashion, as described in the previous section. Slowly varying absorption, on the other hand, is treated more accurately by the Greuling-Goertzel equations directly, and this recipe has been used for all other epithermal absorption.

These exact calculations were performed by using an approximate resonance peak model. If the exact shape of the peak is used, the calculations are more difficult and numerical approximations are useful. A calculation of this type was performed for the two lowest resonance peaks of U-238 by using the Doppler-broadened Breit-Wigner line shape. The calculation was for a homogenized moderator (80% D₂O, 20% H₂O) and UO₂ mixture with equal volumes. Figure 2.9 shows the flux shape in the neighborhood of the two lowest U-238 resonance peaks. Visible Placzek oscillations indicate that the flux is affected by a resonance peak over a range of nearly half a lethargy unit. Other investigations indicated that flux is affected by the resonance scattering in U-238 (neglected in this calculation). The results are also sensitive to the numerical mesh size used in the integration. A general investigation of these effects would be useful. The results here indicate the validity of the calculations, but they were not as accurate nor as simple as is required for the SSCR analysis model.

In the model, effective group cross sections for narrow resonances which are not so heavily absorbing are determined by

$$\bar{\sigma}_{aj} = \frac{\sum_i R_{ij}}{\Delta u_j} .$$

This treatment was used for U-235 resonances, where the sum is the over-all U-235 resonances in the group. Some small errors are present, notably in groups where there is fairly heavy absorption. Improvements to the model are possible, but these are complex and are so small numerically that they have been neglected in the present treatment. However, estimated correction factors can be obtained easily if the Wigner approximation is assumed. Then, the absorptions in a single, narrow slightly absorbing peak should be

$$\text{Abs} = \frac{\text{NR}}{\xi \Sigma_s} q \quad .$$

The BPG difference equations compute

$$\text{Abs} = \Sigma_a \phi = \frac{\text{NR}}{\Delta u} \phi \quad .$$

Relating ϕ to q through the BPG equations and redefining Σ_a to give the correct (Wigner) absorptions, the effective group cross section is

$$\sigma_a = \frac{R}{\Delta u} \frac{1}{1 - \frac{\text{NR}}{\xi \Sigma_s} \left(-\frac{\gamma}{\Delta u} + \frac{1}{2} \right)} \quad .$$

For several resonances in the same group, and if the uncorrected group cross section is defined by

$$\Sigma_0 = \sum_i \frac{\text{NR}_i}{\Delta u} \quad ,$$

an approximate corrected value is

$$\Sigma_a = \Sigma_0 \left(1 + \frac{\gamma \Sigma_0}{\xi \Sigma_s} \right) .$$

2.3. Resonance Absorption in U-235

2.3.1. Effective Resonance Integral

Because of the tight lattice and the small slowing down power in the SSCR cores, a large fraction of the absorptions in U-235 are at epithermal energies. Thus, the accurate calculation of U-235 resonance absorption and fission is important in the analysis of these cores. The effects of lumping and of lattice shielding on U-235 resonances were investigated in the same manner as were the fertile materials. Resonance parameters for the resolved resonances were used to compute the effective resonance integral of each line, assuming a Breit-Wigner shape and performing the integration by computer calculation.³³ The effect of mutual self-shielding as computed with the Bell approximation was used to obtain the effective homogeneous medium value of

$$\sqrt{\sigma_p + \frac{\gamma S}{4NV}}$$

used in the machine computation.

The available resolved U-235 resonance data ²⁵ only cover the energy region up to 20 ev, and numerical integration of the cross sections in the unresolved resonance region above this indicates that there is about as much resonance integral in the unresolved region as in the resolved. The amounts of infinitely dilute U-235 absorption and fission resonance integrals in various energy regions are:

<u>Energy range,</u> <u>ev</u>	<u>Absorption</u> <u>integral, b</u>	<u>Fission</u> <u>integral, b</u>
10 ⁷ - 20	196.2	127.7
20 - 1.1	164.0	82.3
1.1 - 0.4	<u>81.5</u>	<u>73.3</u>
Total	441.7	283.2

As an approximation, it was assumed that an amount of integral in the higher energy unresolved region equal to that in the resolved region was subjected to the same shielding reductions as occur in the resolved region (the resolved region calculated effects were doubled). The calculated curves of the U-235 effective resonance integrals as functions of the shielding parameter are shown in Figures 2.10 and 2.11, and the values of SSCR lattice resonance integrals as functions of % D₂O in the moderator are given in Table 2.2. The table shows the effect of self-shielding on U-235 resonance absorption in SSCR lattices causes a 10 to 20% reduction in the effective resonance integral. The main shielding is caused by lumping and lattice close-packing, and there is little variation with % D₂O in the moderator. There is enough U-235 in these lattices, even neglecting lumping effects, there is some homogenized shielding. Integrals calculated for homogenized cases are appreciably larger than for the corresponding lattices where lumping effects are accounted. As a lower limit, the effective resonance integral of U-235 in an infinite medium of pure UO₂ or ThO₂ fuel rod material was not much lower than the lattice value with moderator. Thus, there is a fair amount of scattering inside the fuel pins.

Because the change of the resonance integrals with changing moderator composition is not large, and because it is difficult to vary the cross sections (other than Th-232 and U-238) in the BPG Code, the model for analysis used the constant values $R_{\text{absolute}} = 400$ barns, and $R_{\text{fission}} = 275$ barns. These do not lead to very large errors in criticality. Where greater accuracy is needed in the analysis of fission cadmium ratios, the appropriate corrections were made from Table 2.2.

2.3.2. η for U-235

The average value of η in the epithermal region varies with U-235 lumping. If the definition

$$\bar{\eta}^{\text{res}} \equiv \nu_{25} \times \frac{\text{U-235 epithermal fissions}}{\text{U-235 epithermal absorptions}}$$

is adopted, then in a $1/E$ spectrum the value becomes

$$\bar{\eta}^{\text{res}} = \nu_{25} \frac{R_F}{R_A}.$$

Some values are given in Table 2.3 assuming $\nu = 2.43$. The spectra in the SSCR lattices, however, deviate considerably from the $1/E$ variation. Thus, the effective value of $\bar{\eta}^{\text{res}}$ based on the actual absorptions and fissions varies from reactor to reactor and is a function of the % D_2O in the moderator of the SSCR lattice. The BPG Code accounts for this variation by using many groups in the resonance absorption region. The shape of the spectrum is shown in Figure 3.20. For example, BPG calculations for Core V (69.7% D_2O) used U-235 cross sections which give

$$\nu \frac{R_F}{R_A} = 2.43 \frac{275}{400} = 1.67$$

averaged over a $1/E$ spectrum. The code actually computed the U-235 absorptions and fissions with the proper flux spectrum to give

$$\bar{\eta}^{\text{res}} = \nu \frac{\text{Fissions}}{\text{Absorptions}} = 1.61.$$

This reduction in η was similar for other lattices.

Table 2.2 Effective U-235 Resonance
Integrals in Lattices

Configuration	Fuel pins	% D ₂ O in moderator	$\sqrt{\sigma_p} + \frac{\gamma S}{4NV}$	Absorption integral, barns	Fission integral, barns
Lattice	UO ₂	0	33.8 *	384 **	259 **
Lattice	UO ₂	40	32.5	381	257
Lattice	UO ₂	70	31.0	376	255
Lattice	UO ₂	90	29.7	372	253
Single Pin	UO ₂	--	38.4	397	265
Homogenized Lattice	UO ₂	70	38.9	398	265
Homogenized Lattice	UO ₂	0	50.6	420	273
Infinite Fuel	UO ₂	--	18.8	329	234
Lattice	ThO ₂	0	32.2	380	259
Lattice	ThO ₂	40	30.4	374	255
Lattice	ThO ₂	70	28.1	366	251
Lattice	ThO ₂	90	26.1	359	248
Single Pin	ThO ₂	--	39.5	400	266
Homogenized Lattice	ThO ₂	70	32.0	379	257
Homogenized Lattice	ThO ₂	0	45.5	412	270
Infinite Fuel	ThO ₂	--	16.8	318	229

* Units of $\sqrt{\sigma_p} + \frac{\gamma S}{4NV}$ are (barns/absorber atom)^{1/2}.

** Lower energy bound on the integrals is 0.4 ev. Infinitely dilute integrals are R_A = 442 barns and R_F = 283 barns.

Table 2.3 Epithermal η for U-235
in a 1/E Spectrum

Configuration	$\sqrt{\sigma_p} + \frac{\gamma S}{4NV}$	$\bar{\eta}_{res}$
Infinite Dilution	∞	1.56
Single UO ₂ Pin, 4% enriched	38.4	1.62
Single ThO ₂ Pin, N ₀₂ /N ₂₅ = 15	39.5	1.62
Infinite UO ₂ , 4% enriched	18.8	1.73
Infinite ThO ₂ , N ₀₂ /N ₂₅ = 15	16.8	1.75

2.3.3. Energy Variation of ν

The number of neutrons per fission is generally assumed to be energy independent in thermal reactor calculations, and a constant value, 2.43, is used in the BPG Code. At high energies, there is an increase²⁸ in ν for U-235, and this changes the k_{eff} of SSCR lattices by as much as 1%. The fission cross section of U-235 has been increased in the high energy groups to account for this, so that $2.43 \times \sigma_F'(E)$ in the BPG Code is equal to the correct value of $\nu(E) \sigma_F(E)$ at the same energy. This correction accounts for the difference between the fission integral of 275 barns used in the code for computing criticality, and the integrals given in Table 2.2 used to compute cadmium ratios.

2.3.4. Cadmium Cutoff

The BPG Code assumes that neutrons with energies greater than a last group cutoff are in the slowing down spectrum, and that neutrons with energies less than this cutoff are all in a single thermal group. Although there is some question about the choice of this cutoff energy, the particular value does not affect the criticality calculation if the proper thermal cross section averages are obtained. For convenience in calculating measured cadmium ratios, the effective value of cadmium cutoff energy⁴⁷ is usually chosen as the breakpoint between the slowing down and thermal distributions. The calculated U-235 fission cadmium ratio is fairly sensitive to this choice. Changing the cutoff energy from 0.4 to 0.625 ev lowered the U-235 effective fission resonance integral from 275 to 236 barns. This variation represents the uncertainty in computing fission cadmium ratios and indicates the need for more work to determine effective cadmium cutoff energies in complex geometry. This analysis uses 0.4 ev as the cutoff energy.

2.4. Effective Thermal Cross Sections

2.4.1. Spatial and Energy Flux Variation

In loosely-packed graphite-moderated lattices, the assumptions of diffusion theory and Maxwell-Boltzmann energy distribution of thermal neutrons give accurate results.¹³ Usually calculations for these systems employ diffusion theory in a one-dimensional cell and assume a single velocity of neutrons. In close-packed lattices

moderated with H₂O or D₂O, none of these assumptions are valid. Higher order approximations of transport theory are needed to account for rapid changes of flux in dimensions less than a mean free path. The heterogeneities in these lattices must be considered in two or three dimensions for reasonable accuracy since these lattice spacings are approximately a mean free path. Also, the spectrum of thermal neutrons is altered by heavy absorption, by the variation of scattering and absorption cross sections with energy, and by spatial variations of cross sections. Therefore, the calculation of effective thermal cross sections for SSCR lattices is complex. However, the accuracy of criticality calculations is insensitive to spatial variations of thermal flux because the moderator and clad materials in the SSCR lattices absorb only a small fraction of the thermal neutrons. Most of the thermal neutrons, even in the borated moderator cases, are absorbed in the fertile and fissile materials. Several methods have been investigated to compute the ratio of moderator flux-to-fuel flux in an accurate fashion, but no single method has been very successful. A fairly simple approximation by Amouyal⁴⁸ fits most of the data as well as more complex methods and has the advantage of easy application. Probably the only way to obtain very accurate results is by using Monte Carlo techniques which can consider spatial and energy variations of fluxes and cross sections.

2.4.2. Thermal Spectrum

A Maxwell-Boltzmann distribution,

$$\phi(E) = A (KT)^{-3/2} E e^{-\frac{E}{KT}},$$

is correct for thermal neutrons in a medium which is not heavily absorbing. For a heavy mass moderator such as graphite, the effect of increasing absorption can be accounted by increasing the effective temperature, T, while retaining the distribution form. For a hydrogen moderator, the temperature motion of the moderating nuclei must be accounted, and this has been done by Wigner and Wilkins.⁴⁹ For the Maxwell-Boltzmann and the Wigner-Wilkins distributions, numerical averages of most of the important thermal neutron cross sections have been obtained. Tabulations of these values are

available ^{25, 50}, and graphs of the distribution functions have been reported. ⁵¹ For intermediate and heavy mass moderators, Wilkins ⁵² has extended the work and a number of distribution functions have been plotted. ⁵³

For a deuterium moderator, it is not clear which of these distributions describe the thermal neutron spectrum accurately. However, for SSCR lattices (up to and above 80% D₂O in the moderator) scattering by hydrogen dominates both the slowing down distribution and the thermal neutron distribution. Also, the Wigner-Wilkins distribution (H scattering) and the Wilkins distribution (heavy moderator) are nearly equivalent for the proper choice of parameters.

A comparison was made by using the heavy element distribution parameter, Δ , computed according to the relation

$$\Delta = \frac{2\Sigma_a}{\sum_i \frac{\Sigma_{si}}{m_i}}$$

for mixtures of moderating materials. Here, Σ_a is the effective absorption cross section in the reactor (relative to the flux in the moderator for lattices), Σ_{si} is the reactor scattering cross section of the i th element, and m_i is its mass relative to the neutron mass. If hydrogen is assumed to have mass one and $\sigma_s = 20.5$ barns, an effective value of

$$\Delta = \frac{\Sigma_a}{N_H} \times \frac{2}{20.5} = 0.1 \frac{\Sigma_a}{N_H}$$

is computed for a pure hydrogen moderator. Curves of the Wigner-Wilkins distribution ⁵¹ for $\Sigma_a/N_H = 3.73$ barns/atom and of the heavy element distribution ⁵³ for $\Delta = 0.4$ were compared and agreed very well. This agreement indicates an approximate relation between heavy moderator spectra and hydrogen spectra. Therefore, in lattices moderated mainly by hydrogen (such as the SSCR lattices), a reasonable first approximation of the thermal spectrum is either the heavy moderator spectrum with parameter Δ computed assuming hydrogen as mass one, or the Wigner-Wilkins spectrum with parameter Σ_a/N_H computed as ten times Δ . For $1/v$ cross sections, effective values are plotted as functions of the distribution parameter Δ in Figure 2.12.

By using the recipe with mass one for hydrogen, mass two for deuterium, and $\Sigma_a = 0.25 \text{ cm}^{-1}$, the value of Δ was computed for various D_2O concentrations in lattices with the moderator volume fraction = 0.5, (Fig. 2.13). Using this curve, the effective cross section is replotted as a function of % D_2O in Figure 2.14. These curves should be valid for the SSCR lattices because the 4%-enriched UO_2 lattices had $\Sigma_a = 0.30$, and the 93%-enriched $\text{UO}_2\text{-ThO}_2$ lattices had $\Sigma_a = 0.33$.

2.4.3. Comparison of Methods

Because the SSCR lattices are close packed (only 50% moderator by volume) and the dimensions inside a unit cell are of the order of a neutron mean free path, there are major difficulties in calculating thermal flux ratios. Since diffusion theory is not valid and P_3 may not have the accuracy needed, more complex, higher order expansions are needed. Also, the usual one-dimensional treatment of a cylindrical cell is not valid, and spatial variations in the second and third dimension may be as important as those in the first. This latter effect causes the difference between hexagonal and square lattices (at the same moderator volume fraction and composition) to be as important as a substantial change in moderator volume fraction or moderator composition. Also, separation of spatial and temperature effects is probably not valid.

To investigate these effects, thermal flux ratios for a number of lattices were calculated by using several different models, including a cylindrical geometry P_3 computer code⁵⁴. The effect of thermal spectrum is shown in Table 2.4 for two different calculations using this code. One calculation used cross sections averaged over a Maxwell-Boltzmann distribution and the other was averaged over the Wigner-Wilkins distribution for the lattice. A second model used the method of Amouyal⁴⁸ which treats the fuel lump by a P_3 approximation, uses a simpler recipe for the moderator portion of the cell. This method permits fairly accurate calculations to be made manually. It appears to agree better with experimental data for tight lattices than does the supposedly more accurate P_3 calculation. This may come, not from a better representation by the Amouyal method, but from the difficulty in making a one-dimensional problem out of what must inherently be a two-dimensional (or three-dimensional) one. The results are shown in Table 2.4.

The lattices in the table are similar to SSCR lattices since they contain tightly-packed cylindrical fuel pins moderated with water (H_2O only). Experimental data ⁵⁵ cover a fairly large range of the thermal flux ratio. All lattices were composed of aluminum clad fuel pins in hexagonal arrays.

2.4.4. Need for Further Work

The recipe used in the present analysis is to find effective cross sections averaged over the Wigner-Wilkins spectrum using the heavy moderator-to-hydrogen equivalence, and to obtain the spatial average of fluxes by the method of Amouyal. The agreement between theory and experiment, shown in Table 2.4, is good enough for criticality calculations on most SSCR lattices, but it is not sufficient for a more general application. The disagreement between P_3 and Amouyal methods indicates the need for more accurate calculations. Because the problem requires the consideration of several spatial dimensions and the effects of the energy spectrum, the most direct method is by Monte Carlo techniques. Since these techniques are expensive for today's computers, it seems reasonable to use a few Monte Carlo calculations and experiments as bases for comparison, and to develop recipes along the lines of this report (Amouyal spatial treatment, heavy moderator spectral model). The spatial problem should be investigated in two and three dimensions, and the spectral problem should be checked carefully to obtain more accurate effective cross sections. The spatial hardening of flux spectrum inside a fuel lump should also be investigated to determine the possibility of obtaining a simple representation.

Table 2.4. Moderator-to-Fuel Thermal Flux Ratios

Lattice number	Fuel	Fuel density, gm/cm ³	Enrichment (wt % U-235)	Pin OD in.	Moderator volume fraction	P ₃ Calculation *		Amouyal method *		Experiment **
						MB	WW	MB	WW	
1	U Metal	18.9	1.3	0.600	0.551	1.42	1.30	1.47	1.36	1.39 ± 0.04
2	U Metal	18.9	1.3	.600	.620	1.44	1.33	1.53	1.43	1.45 ± .04
3	U Metal	18.9	1.3	.600	.710	1.49	1.40	1.62	1.54	1.49 ± .04
4	U Metal	18.9	1.15	.600	.620	1.40	1.30	1.48	1.40	1.45 ± .04
5	U Metal	18.9	1.15	.600	.710	1.44	1.36	1.57	1.50	1.45 ± .04
6	U Metal	18.9	1.3	.387	.600	1.26	1.18	1.28	1.22	1.21 ± .04
7	U Metal	18.9	1.3	.387	.687	1.24	1.20	1.32	1.27	1.31 ± .04
8	UO ₂	7.53	1.3	.601	.465	1.13	1.10	1.14	1.12	1.09 ± .03
9	UO ₂	7.53	1.3	.601	.533	1.13	1.11	1.16	1.14	1.14 ± .03
10	UO ₂	7.53	1.3	.601	.588	1.14	1.12	1.17	1.15	1.16 ± .03
11	UO ₂	7.52	1.3	.388	.505	1.11	1.07	1.09	1.08	1.10 ± .01
12	UO ₂	7.52	1.3	.388	.560	1.11	1.07	1.09	1.08	1.10 ± .01
13	UO ₂	10.53	1.3	.383	.503	1.11	1.09	1.12	1.10	1.10 ± .01
14	UO ₂	10.53	1.3	.383	.560	1.11	1.09	1.13	1.11	1.13 ± .01
15	UO ₂	10.53	1.3	.383	0.631	1.11	1.10	1.14	1.13	1.13 ± 0.01

* MB columns were calculated using cross sections averaged over a Maxwell-Boltzmann distribution, and WW columns were calculated using cross sections averaged over the appropriate Wigner-Wilkins distribution for each lattice.

** See Reference 56

2.5. Computer Codes

Many problems have been solved by high speed digital computers. Since the particular programs used on the computers are integral parts of the method of analysis, a general understanding of their nature is important to an understanding of the theoretical methods. A short description is given for each code.

2.5.1. BPG

The BPG Code solves the multi-group equations for a bare reactor by using the exact slowing-down model for hydrogen, the Greuling-Goertzel approximation for deuterium and other light weight nuclei, and the Fermi age approximation for heavy nuclei. The choice of B_1 or P_1 expansions of the scattering kernel is available. The code computes the criticality and neutron balance in a bare reactor using up to 80 energy groups. Its primary purpose is to generate few-group constants to be used in one-, two-, or three-dimensional multiregion calculations. A set of 40-group cross sections has been prepared for use with the code.

Computer: Burroughs Datatron 205

References: deCoulin, G. A. G., Gates, L. D.,
Worley, W. R., SSCR Basic Physics
Program Theoretical Analysis Part II;
BPG Computer Program Report, BAW-
1230 Part II, 1961.

Roach, K. E. and Wittkopf, W. A.,
40-Group Neutron Cross Section
Library, BAW-158, June 1961.

2.5.2. Four-Group

The four-group program utilizes four-group diffusion theory to calculate the flux distribution and the effective multiplication factor of multiregion, one-dimensional reactors. The equations solved are

$$D_1 \nabla^2 \phi_1 - W_1 \phi_1 + \frac{1}{k_{\text{eff}}} [f_1 \phi_1 + f_2 \phi_2 + f_3 \phi_3 + f_4 \phi_4] = 0$$

$$D_2 \nabla^2 \phi_2 - W_2 \phi_2 + \beta_1 \phi_1 = 0$$

$$D_3 \nabla^2 \phi_3 - W_3 \phi_3 + \beta_2 \phi_2 = 0$$

$$D_4 \nabla^2 \phi_4 - W_4 \phi_4 + \beta_3 \phi_3 = 0.$$

These equations are solved spatially as a set of difference equations for up to 100 space points over 14 or fewer regions in slab, cylindrical, or spherical geometry. Boundary conditions at each interface give continuity of flux and current in each group. At the outer boundary, an extrapolation distance can be chosen; at the inner boundary, either symmetry or an extrapolation distance can be chosen.

Computer: Burroughs Datatron 205

Reference: Smith, E. J., Four-Group Program Report, Report Phy-26, The Babcock & Wilcox Co., September 1957.

2.5.3. PDQ

PDQ is a two-dimensional criticality and flux calculation for one to four neutron groups. The diffusion equations are solved by difference techniques for up to 20,000 mesh points in x-y or r-z geometry. Symmetry, zero flux, or zero current boundary conditions are used, and a logarithmic derivative condition may be applied at internal boundaries for control rod description.

Computers: IBM-704, Philco-2000

References: Bilodeau, et al., PDQ — An IBM-704 Code to Solve the Two-Dimensional Few-Group Neutron Diffusion Equations, WAPD-TM-70 (August 1957).

Cadwell, W. R., PDQ-3 (IBM-704), WAPD-TM-179 (March 1960).

Cadwell, W. R., PDQ-4 (Philco-2000), WAPD-TM-230 (June 1961).

2.5.4. P₃

The monoenergetic Boltzmann equation in cylindrical or slab geometry is solved in the P₃ approximation by a matrix solution in the P₃ Code. From 2 to 25 regions may be considered in the unit cell calculation. The code determines the neutron flux as a function of position in the cell and obtains a neutron balance based on the input spatial variation of source strength and cross sections. Boundary conditions are continuity of moment fluxes across interfaces and symmetry at the inner and outer boundaries.

Computer: Burroughs Datatron 205

Reference: Worley, W. R., P₃ Package Program Report, Report Phy-41, The Babcock & Wilcox Co., November 1959.

2.5.5. Dancoff

The Dancoff effect program calculates the factor C by which the effective surface of a given cylindrical fuel pin must be reduced to account for the shadowing effect of a single neighboring fuel pin. The basic equations of Dancoff and Ginsburg are used, and numerical integrations are performed to evaluate the expressions for a three-region problem (fuel, clad, and moderator). The total shielding of a pin in a lattice is the sum of contributions from all its neighbors.

Computer: Burroughs Datatron 205

References: Pitts, T., Dancoff Effect Program Report, DPR-054, The Babcock & Wilcox Co., May 20, 1958.

Dancoff, S.M. and Ginsburg, M., Surface Resonance Absorption in a Close Packed Lattice, CP-2157 (1944).

2.5.6. Resonance Integral

Given the appropriate resonance parameters for any number of resonance peaks, this program computes the infinitely dilute, the homogeneous, and the lumped resonance integrals for a given absorber. For the homogeneous and lumped cases, a provision is made to select the NR or the NRIA approximation for any peak. Values are computed by integrating the Doppler-broadened Breit-Wigner single level formula. Lumped values may be calculated by the method of Roe³⁴ or Levine⁴¹.

Computer: Burroughs Datatron 205

References: Roach, K.E., Calculation of Infinite Dilute, Homogeneous, and Lumped Resonance Integrals From the Resonance Parameters for the Individual Lines, BAW-TM-72, The Babcock & Wilcox Co., April 1961.

2.5.7. P1MG

The P_1 approximation is solved in multigroup form in this code to obtain the spatially dependent fluxes for up to 80 multigroups, up to 15 regions, and up to 200 mesh points. One-dimensional problems can be handled in slab, cylindrical, or spherical geometry. Region-averaged few-group cross sections are obtained, along with fluxes, currents, and slowing-down densities. Boundary conditions at the

interface are a continuence of flux and current. At the outer boundary, an extrapolation distance is used, and at the inner boundary, either symmetry or an extrapolation distance is used.

Computer: IBM-704

Reference: Bohl, H., Jr., et al., P1MG — A One-Dimensional Multigroup P₁ Code for the IBM-704, WAPD-TM-135, July 1959.

2.6. Cross Section Data

The theoretical model described in this report is used to relate the experimental information to basic cross sections. Generally, these cross sections have been measured fairly accurately, but there are some areas where appreciable uncertainties still exist. Fortunately, these uncertainties are not very important (except for some of the resonance absorption data) in SSCR lattice analysis.

2.6.1. Basis of Cross Sections

Wherever possible in the analysis, the information of BNL-325²⁵ was used. All thermal cross sections (absorption, scattering, fission, and transport) were taken directly from this collection, and then were averaged over the appropriate thermal neutron spectrum. Also, epithermal scattering cross sections were obtained from BNL-325. Epithermal absorption cross sections (except for U-238, Th-232, U-235, and stainless steel 304) were considered to be inversely proportional to neutron velocity. High energy cross sections were read directly from the curves. Important exceptions are noted below.

2.6.2. Resonance Parameters

U-238 resonance parameters were those given in BNL-325²⁵. Data for Th-232 have been measured recently at Columbia⁵⁶. In the resolved region, average values were used where parameters were missing. In the unresolved region, the neutron width Γ_n is given by

$$\Gamma_n = \bar{\Gamma}_n^0 \sqrt{E},$$

where $\bar{\Gamma}_n^0$ is the average reduced neutron width. $\bar{\Gamma}_n^0$ and the average capture width Γ_γ were averaged over all the measurements available. The average energy spacing between lines in the unresolved region was taken to be the average energy spacing between lines in the resolved region. The parameters used for U-238 are listed in Table 2.5, and those for Th-232 in Table 2.6. At energies above 10^4 ev, measured values of average cross section are believed to be more reliable than extrapolations from resonance parameters. Epithermal cross sections for U-238 were computed directly from the resonance parameters (Section 2.2). Values for Th-232 were computed from the listed resonance parameters, and were renormalized to agree with measured integral single lump data. Effective U-235 cross sections were determined similarly (Section 2.3) from BNL-325 resonance parameters. The cross sections at other energies were obtained directly from the curves.

2.6.3. Stainless Steel

An effective cross section for stainless steel is based on the average properties of seven 300-series stainless steels. The absorption cross section at 2200 meters per second was determined for each steel composition by using the neutron cross sections of BNL-325 for individual nuclei. The seven steels all have very nearly the same average thermal cross section (0.270 cm^{-1} at 2200 meters per second). For type-304 stainless steel (the clad material for the SSCR lattice), an effective microscopic cross section at 2200 meters per second of 3.125 barns is obtained if an atomic concentration $N_{ss} = 0.0864 \times 10^{24}$ atoms/cm³ is chosen.

An effective epithermal absorption cross section was determined from resonance integral values²⁸ for the individual constituents listed below.

<u>Constituent</u>	<u>Resonance Integral, barns</u>
Cr	2.6
Mn	12.0
Fe	2.3
Ni	3.2
Co	38.3

The average resonance integral of stainless steel found from these was 2.69 barns. All of these resonance integral values have a lower cutoff of 0.49 ev. Assuming a $1/v$ behavior below this, the value of 2.85 barns is obtained for stainless steel with the lower energy cutoff of 0.4 ev. The calculated $1/v$ portion of the resonance integral between 0.4 ev and 10 Mev is 1.54 barns. Therefore, the contribution of resonance absorptions is 1.31 barns. Most of the observed resonances in these materials occur in the energy range of 3000 to 9000 ev, so the cross section library was adjusted to give

$$\int_{3000 \text{ ev}}^{9000 \text{ ev}} \sigma_a \frac{dE}{E} = 1.31 \text{ barns,}$$

and the remaining resonance integral was spread out over all the groups as a $1/v$ -cross section.

2.6.4. Group Cross Sections

The complete 39-group set of cross sections and group bounds used in the analysis has been tabulated.²⁶ Average values of microscopic cross sections in each group were obtained assuming the flux within the group to be proportional to $1/E$. BNL-325 cross sections were used except for U-238, U-235, and Th-232.

2.6.5. Resonance Integrals

Several measurements of effective resonance integrals have been made for lumps of U-238 and Th-232 and their oxides. For this analysis, U-238 values were taken from Hellstrand³⁶ for single lumps.

$$R_{28} = 2.81 + 24.7 \sqrt{S/M} \quad (\text{metal})$$

$$R_{28} = 4.15 + 26.6 \sqrt{S/M_{\text{ox}}} \quad (\text{UO}_2)$$

S/M is the surface-to-mass ratio for metal, and S/M_{ox} is the surface-to-mass ratio for UO_2 . Calculations from resonance parameters give the relation

$$R_{28} = 5.84 + 21.98 \sqrt{S/M_{\text{ox}}} \quad (\text{UO}_2)$$

These relations are compared in Figures 2.3 and 2.5. For Th-232, measurements have been made by Pettus³⁸,

$$R_{02} = 0.8 + 2.11 \sqrt{S/M} \quad (\text{metal})$$

$$R_{02} = 3.1 + 26.9 \sqrt{S/M_{\text{ox}}} \quad (\text{oxide})$$

and by Hellstrand and Weitman^{37, 39}

$$R_{02} = 1.70 + 15.9 \sqrt{S/M} \quad (\text{metal})$$

$$R_{02} = 1.5 + 18.1 \sqrt{S/M_{\text{ox}}} \quad (\text{oxide})$$

Calculations from the resonance parameters give

$$R_{02} = 6.29 + 18.65 \sqrt{S/M_{\text{ox}}} \quad (\text{oxide})$$

These relations are compared in Figures 2.4 and 2.6.

2.6.6. Inelastic Scattering

The important inelastic scattering considered in this analysis are from U-238, Th-232, and stainless steel. Inelastic scattering cross sections as functions of energy for U-238 and stainless steel (assumed to be Fe) were taken directly from BNL-325. Th-232 data were obtained from Howerton's collection.⁵⁷ A single inelastic scattering matrix was used for all materials. (See Section 2.1.)

2.6.7. High Energy Elastic Scattering

At neutron energies in the Mev range, elastic scattering by many elements is highly anisotropic. Basic cross section data describing this anisotropy are collected in ANL-5800²⁸ and BNL-400²³, but this information is vague for many isotopes. For most elements tabulated for the BPG Code, the values of ω_1 from ANL-5800 were used to determine the average cosine of the scattering angle at high energies according to

$$\bar{\mu} (E) = \overline{\cos \theta} = \frac{1}{3} \omega_1 (E).$$

For deuterium and oxygen, differential scattering data from BNL-400 were integrated numerically to obtain the energy variation of the average cosine of the scattering angle

$$\bar{\mu} (E) = \frac{\int_{-1}^{+1} \sigma (E, \theta) \cos \theta \, d(\cos \theta)}{\int_{-1}^{+1} \sigma (E, \theta) \, d(\cos \theta)}.$$

The BPG Code permits the use of energy variation for a number of the parameters besides $\bar{\mu}(E)$. Preliminary investigations indicated that the analysis presented here is not very sensitive to these parameters, and that the cross section data available did not permit accurate calculation of the energy variations. Therefore, constant values were chosen for the slowing down parameters ξ , η , ζ , and γ in the BPG Code. The effects of energy variation of these parameters should be investigated more carefully when more experimental differential cross section data are available.

Table 2.5 Resonance Parameters for U-238

<u>E,</u> <u>ev</u>	<u>Γ_n,</u> <u>ev</u>	<u>Γ_γ,</u> <u>ev</u>	<u>Resonance</u> <u>integral, barns</u>
6.68	0.00148	0.025	127.89
10.2	.0000014	.02466	0.05
21.0	.009	.025	61.3
36.8	.033	.026	43.9
66.3	.023	.02	9.94
81.1	.0021	.02466	1.20
90	.00008	.02466	0.04
102.8	.072	.018	5.57
117	.0176	.017	2.58
145.9	.00081	.02466	0.15
165.7	.0034	.02466	0.44
190	.137	.022	2.14
209.1	.05	.026	1.60
238	.032	.02	0.89
264	.000023	.02466	0.01
274.5	.027	.022	0.66
292	.019	.019	0.46
312.5	.001	.02466	0.04
349	.045	.02	0.46
378	.0015	.02466	0.04
399	.01	.04	0.21
412	.017	.018	0.21
436	.014	.02	0.18
456	.0007	.02466	0.01
465	.007	.018	0.10
480	.0045	.035	0.07
491	.001	.2466	0.02
520	.0285	.02466	0.20
537	.054	.024	0.24
558	0.001	0.02466	0.01

Table 2.5 (Cont'd)

<u>E,</u> <u>ev</u>	Γ_n , <u>ev</u>	Γ_γ , <u>ev</u>	<u>Resonance</u> <u>integral, barns</u>
582	0.042	0.023	0.18
597	.066	.023	.20
606	.0006	.02466	.01
622	.039	.024	.16
630	.009	.02466	.07
663	.125	.026	.20
681	.0013	.02466	.01
696	.053	.025	.14
711	.017	.033	.09
724	.0147	.02466	.07
733	.0042	.02466	.03
767	.009	.02466	.05
783	.003	.02466	.02
793	.011	.2466	.05
826	.06	.032	.12
856	.13	.02466	.12
860	.06	.02466	.10
869	.0022	.02466	.01
898	.0013	.02466	.01
911	.09	.035	.13
930	.037	.02466	.07
942	.195	.025	.10
962	.19	.024	.09
985	.001	.02466	.004
997	0.4	0.03	0.11

Table 2.5 (Cont'd)

<u>E,</u> <u>ev</u>	<u>Γ_n,</u> <u>ev</u>	<u>Γ_γ,</u> <u>ev</u>	<u>Resonance</u> <u>integral, barns</u>
<u>Begin Unresolved Region D = 18.34 ev</u>			
1005.17	0.07232	0.02466	0.074/resonance
2000	.10196	.02466	.020
3000	.12488	.02466	.009
4000	.14421	.02466	.005
5000	.16122	.02466	.003
6000	.17661	.02466	.002
7000	.19077	.02466	.002
8000	.20392	.02466	.001
9000	.2163	.02466	.001
9990.83	0.22789	0.02466	0.001

Table 2.6 Resonance Parameters for Th-232

<u>E,</u> <u>ev</u>	<u>Γ_n,</u> <u>ev</u>	<u>Γ_γ,</u> <u>ev</u>	<u>Resonance</u> <u>integral, barns</u>
21.9	0.00239	0.034	18.99
23.6	.00413	.035	27.07
59.6	.00463	.021	14.36
69.7	.0392	.043	17.25
113.2	.0111	.042	2.79
121.0	.0185	.041	3.55
128.5	.0001	.040	0.03
129.4	.0030	.040	0.68
146.2	.0001	.040	0.02
154.6	.00022	.040	0.04
170.8	.0582	.048	3.68
193.0	0.0140	0.050	1.20

Table 2.6 (Cont'd)

<u>E,</u> <u>ev</u>	Γ_n , <u>ev</u>	Γ_{γ} , <u>ev</u>	<u>Resonance</u> <u>integral, barns</u>
196.8	0.00013	0.040	0.01
199.8	.00905	.040	0.75
203.0	.000043	.040	0.004
221.8	.023	.046	1.27
252.3	.024	.042	0.98
264.0	.015	.040	0.64
286.8	.021	.050	0.73
306.5	.020	.040	0.58
330.4	.055	.053	1.01
343.3	.030	.040	0.59
366.7	.035	.040	0.57
371.0	.024	.040	0.45
402.8	.0090	.040	0.19
413.4	.00010	.040	0.002
456.0	.00011	.04	0.002
465.0	.045	.04	0.40
491.0	.050	.04	0.38
513.0	.0035	.04	0.05
531.5	.015	.04	0.16
543.0	.001	.04	0.01
573.0	.025	.03	0.19
581.0	.0016	.04	0.02
601.5	.011	.04	0.10
621.5	.005	.04	0.05
660.5	.032	.04	0.17
669.0	.017	.04	0.11
679.5	.125	.04	0.27
692.0	.039	.04	0.17
705.0	.018	.04	0.10
717.5	.025	.04	0.12
746.0	0.160	.04	0.23

Table 2.6 (Cont'd)

<u>E,</u> <u>ev</u>	Γ_n , <u>ev</u>	Γ_γ , <u>ev</u>	<u>Resonance</u> <u>integral, barns</u>
783.0	0.010	0.04	0.05
810.0	.125	.04	.19
848.0	.023	.04	.06
872.0	.010	.04	.04
896.0	.030	.04	.09
950.0	.040	.04	.09
970.0	.0075	.04	.03
990.0	.024	.04	.06
998.0	0.070	0.04	0.10

Begin Unresolved Region D = 19.14 ev

1007.57	0.039	0.04	0.080/resonance
2000	.055	.04	.024
3000	.068	.04	.011
4000	.078	.04	.007
5000	.087	.04	.004
6000	.096	.04	.003
7000	.103	.04	.002
8000	.111	.04	.002
9000	.117	.04	.002
9990.43	0.124	0.04	0.001

Figure 2.1. Comparison of Calculated Thermal Flux Shapes for Core III

Figure 2.2. Critical Concentration of U-235 in Core of HRT
with D₂O Reflector

Figure 2.3. Single Pin Resonance Integral as a Function of S/M for Uranium Oxide

Figure 2.4. Single Pin Resonance Integral as a Function of S/M for Thorium Oxide

Figure 2.5. Resonance Integral of U-238 Vs Effective Scattering per Atom

Figure 2.6. Resonance Integral of Thorium Vs Effective Scattering per Atom

Figure 2.7. Dancoff Factor for U-238 Lattice

Figure 2.8. Dancoff Factor for Th-232 Lattice

Figure 2.9. Neutron Flux Profile Through 21.0 ev and
6.7 ev Resonances in U-238

Figure 2.10. Effective U-235 Resonance Absorption Integral

Figure 2.11. Effective U-235 Resonance Fission Integral

Figure 2.12. Effect of Thermal Spectrum on an Average Cross Section

Figure 2.13. Thermal Spectrum Parameter for SSCR Lattices

Figure 2.14. Effect of Thermal Spectrum on Average Cross Sections in SSCR Lattices

3. ANALYSIS OF UNIFORM LATTICE EXPERIMENTS

3.1. Criticality of Individual Cores

3.1.1. General Discussion

Numerical description of the cores measured in this program are given in the Appendix. More detailed descriptions are given in BAW-1231.² By using the previously described methods (Section 2), the criticality of nine clean lattices was computed (Table 3.1). The values in Table 3.1 were computed with the BPG Code using the measured critical buckling. The four-group and PDQ numbers were found by using four-group coefficients generated by the BPG Code. The results indicate that criticality is computed very well with this model. Several small deviations are discussed below.

Core IX – The difference between the BPG and the four-group calculations is caused by an error in calculating reflector savings. Measured and calculated reflector savings differ by an amount equivalent to a 10% mistake in measuring the buckling, and this is outside the limits of experimental error. (See Table 3.2.) Thus, the low value of the BPG calculation must come from accumulated errors in the core constants. These compensate the reflector savings error to give a good value of criticality computed by the BPG four-group combination.

Core II – The BPG and the four-group calculations show core has excess reactivity. Since the effect of reflector savings is small, the error comes from the calculated core properties. A likely source of error is the calculated thermal flux ratio (ϕ_M/ϕ_F) which is lower than the measured value. The measured value would bring the calculated k_{eff} almost exactly to 1.000.

TUPE 15A and Core VIII – The four-group calculation gives accurate criticality results, but the BPG predicts low values. As in Core IX, these effects come from two calculational errors; one in reflector savings and a compensating one in core constants (probably in the effective resonance integral for thorium).

Table 3.1. Criticality of Uniform Lattices

Core designation	% D ₂ O in moderator	Fertile material	k _{eff}		
			BPG	Four-Group	PDQ
I	0	U-238	0.998	1.008	-
II	0 B	U-238	1.017	1.017	-
III	76.50	U-238	0.996	1.000	0.999
III-21	73.77	U-238	0.993	0.998	-
III-22	73.77	U-238	0.994	0.996	-
IV	69.70	U-238	0.997	1.002	-
V	69.70*	U-238	0.997	0.999	-
VI	49.63*	U-238	1.004	1.006	-
VIII	60.40	Th-232	0.975	0.993	-
IX	49.66	U-238	0.982	1.003	-
TUPE 15A	0	Th-232	0.986	1.000	-

- - - - -
* Moderator contains boric acid

3.1.2. Calculation Flowsheet

The methods used in the calculations of criticality and of the reactor parameters are described in Section 2. For reference, an outline is given in Figure 3.1.

3.1.3. Reactor Parameters

Calculated and experimental values of the reactor parameters are compared in Table 3.2. The agreement between theory and experiment here indicates that the model is good for over-all system criticality and for the individual factors making up the criticality. Since agreement is still not perfect, minor modifications of the model are required.

Two values of computed buckling are listed in Table 3.2. The BPG value is for the just critical single-region reactor as computed with the BPG Code. The four-group value was found by fitting the radial group four (thermal) flux from the four-group criticality calculation (Table 3.1) to the function J_0 (Br).

Table 3.2. Parameters of Uniform Lattices

Core designation	% D ₂ O in moderator	B ²			Radial reflector savings		ϕ_M/ϕ_F		U-235 fission Cadmium ratio		Fertile material Cadmium ratio	
		Measured	BPG	Four-Group	Measured	Calculated	Measured	Calculated	Measured	Calculated	Measured	Calculated
		$\times 10^{-4} \text{ cm}^{-2}$			cm	cm						
I	0	88.0	87.5	85.3	7.4	7.9	1.27	1.21	4.76	4.84	1.239	1.248
II	0 B	17.2	22.6	17.2	6.2	6.2	1.27	1.21	-	-	-	-
III	76.50	10.77	10.25	10.26	25.7	28.8	1.17	1.17	2.14	2.07	1.061	1.067
III-21	73.77	14.33	13.28	13.56	23.8	26.8	-	-	-	-	-	-
III-22	73.77	14.09	13.28	13.86	20.3	21.4	-	-	-	-	-	-
IV	69.70	18.60	18.19	17.86	21.4	22.9	1.13	1.17	2.37	2.32	1.096	1.084
V	69.70*	14.23	13.80	14.06	11.6	12.2	1.15	1.17	2.29	2.27	-	-
VI	49.63*	15.78	16.68	15.56	6.7	7.4	1.21	1.18	2.80	2.77	-	-
VIII	60.40**	39.0	35.3	36.4	15.5	17.3	1.18	1.16	2.53	2.56	-	-
IX	49.66	44.0	40.2	39.8	11.5	13.7	1.20	1.18	3.02	3.07	1.121	1.131
TUPE 15A	0**	85.3	83.0	83.0	8.5	8.9	1.22	1.18	4.76	4.40	1.37	1.56

* Moderator contains boric acid

** Thorium lattice

This value of buckling was found to be the same as that required for the BPG calculation to give the same k_{eff} as the four-group calculation. Reflector savings values were found from the four-group bucklings and the measured core radii. Measured axial bucklings were used throughout the analysis.

The results for each core are listed in Tables 3.1 and 3.2.

Core I – All parameters and the over-all criticality are computed in good agreement with the experiment. There is some discrepancy between measured and calculated thermal flux ratios, but this has only a small effect on the reactivity.

Core II – Here, the same discrepancy in the thermal flux ratio is more important because of the boric acid in the moderator. This discrepancy can account for the high value of computed k_{eff} , but since the cadmium ratios were not measured in this core, the conclusion is not certain. Refinement of epithermal flux ratio calculations in the cell may alter the calculated criticality slightly since there is some boron absorption in the low epithermal energy range. In this manner, the criticality calculation may be improved by improving the determination of boron absorptions. The calculated value of reflector savings agrees well with the experiment.

Core III – Criticality is calculated well, and all parameters agree fairly well with the experiment. Computed reflector savings and the computed U-238 cadmium ratio are large. If corrected, both would tend to lower the computed k_{eff} . Loadings 21 and 22 should have the same critical buckling since only the reflector was changed. This indicates the uncertainty in the measured values of B^2 and the derived experimental reflector savings. Criticality is computed well. These three cores have an important fraction of the moderator composed of D_2O . About half the U-235 fissions and practically all the U-238 absorptions are epithermal. The fairly good agreement between theory and experiment of all parameters and criticality for these cores indicates that the model is good for high D_2O concentrations.

Core IV – Criticality and all parameters are calculated in good agreement with the experiment. There is a small discrepancy in the U-238 cadmium ratio. If the calculated resonance absorption were reduced slightly to agree better with the experiment, the calculated k_{eff} would also improve.

Core V – Criticality and all measured parameters are in good agreement with calculated values.

Core VI – Criticality and all measured parameters are in good agreement with calculated values.

Core VIII – The computed U-235 cadmium ratio and the thermal flux ratio agree well with experiment. The low value of k_{eff} computed by the BPG calculation could easily be caused by too much thorium resonance absorption (no measured cadmium ratio is available for comparison). Since the core is small, a small error in leakage can be important, and the small error in calculated reflector savings gives 1.5% of the 1.8% difference in k_{eff} between the two calculations.

Core IX – All calculated reactor parameters agree well with experiment except buckling and reflector savings. This is a small core, and a small error in leakage or reflector savings can give an appreciable difference in calculated k_{eff} or buckling. Since all other parameters are in good agreement and because the buckling difference is outside the experimental error, it is reasonable that the low k_{eff} calculated by the BPG Code be caused by too large a calculated leakage. This would cause the calculated reflector savings to be large.

Core TUPE 15A – Theory and experiment agree well except for the thorium-cadmium ratio and criticality. The measured value of thorium-cadmium ratio is uncertain, so the calculated value may be better. However, as in Core VIII, the calculated resonance absorption in thorium may be too high. If this were reduced, the k_{eff} value would be calculated closer to unity.

3.2. Variation of Parameters with D₂O Concentration

3.2.1. Buckling

Buckling measurements were made for the critical cores and for several exponential assemblies³ at a high D₂O fraction. Values for the clean lattices are shown in Figure 3.2 for the UO₂ fuel rods and in Figure 3.3 for the ThO₂ fuel rods. The calculated curve in both cases was the BPG value (Table 3.2). Agreement between calculated and experimental values appears to be very good for the UO₂ lattices. The calculated values for the ThO₂ lattices may all be low because the value of the thorium resonance integral used could easily have been too large

to account for the difference. More measurements of thorium cadmium ratio are needed to determine this. Comparison of calculated and measured bucklings for lattices containing boric acid poison in the moderator is shown in Figure 3.4. Here, the agreement is not as good as for the unpoisoned lattices and is probably caused by errors in the thermal neutron moderator-to-fuel flux ratio, or by neglecting spatial flux depressions in the low energy epithermal groups.

Buckling is a very sensitive parameter for over-all reactor comparison. Thus the fair agreement of theory and experiment over the entire range of D_2O and boric acid concentrations indicates that the theory is a good representation of the physical situation.

3.2.2. Cadmium Ratios of Fertile Materials

All U-238 cadmium ratios measured were in critical lattices (Table 3.2). Agreement is very good between calculated and experimental cadmium ratios, but there are still appreciable discrepancies in the values of C-1 (the ratio of thermal to epithermal activations) which is the important figure for the analysis of these cores. Figure 3.5 shows the nearly linear variation of the U-238 cadmium ratio with % D_2O in the moderator.

The calculated values were obtained using the BPG Code with cross sections formulated as described in Chapter 2. The effective resonance integral of a single fuel rod was computed to be 19.3 barns, and lattice mutual shielding effects reduced this to 16.9 barns for 0% D_2O and 15.1 barns for 80% D_2O .

For Th-232, there is much less experimental cadmium ratio data available, so conclusions are somewhat sketchier. The available data are in good agreement with calculations as shown in Figure 3.6. The calculated values were obtained in the same fashion as for U-238. The effective resonance integral of a single fuel rod was computed to be 19.3 barns, and the lattice mutual shielding effects reduced this to 15.2 barns for 0% D_2O and 12.3 barns for 80% D_2O .

There is considerable uncertainty in calculating the lattice mutual shielding effects, and this is important in SSCR lattices. The difference between calculations using the formulation of Dancoff and Ginsburg and those using the approximation of Bell are shown in Figures 2.7 and 2.8. The difference amounts to as much as 1.5 barns

in effective resonance integral for the thorium lattices, but there is very little difference for the U-238 lattices. Since the calculated and measured cadmium ratios are in good agreement, it can be concluded that the U-238 resonance integral formulation is correct using either of the mutual shielding calculations, and that the thorium resonance integral formulation is nearly correct using the Dancoff and Ginsburg calculation. The Bell approximation would give incorrect results for the thorium case (larger effective resonance integral), since if any changes were made they should reduce the thorium absorption in order to raise the calculated k_{eff} . One possibility that could aid in this direction is the use of a spatial flux depression for thorium in the low energy ($1/v$) range. There are also some effects of the non- $1/v$ character of the thorium thermal and low energy cross section which were neglected. A thorough re-evaluation of the thorium cross section is needed before these effects can be taken into account.

3.2.3. Cadmium Ratio of U-235 Fissions

The agreement between measured and calculated values is very good, as shown in Figures 3.7 and 3.8. This agreement is close for U-238 and Th-232 lattices, for critical and exponential experiments, for clean and for boron poisoned lattices, indicating that the theory is valid over a wide range of conditions. This agreement was obtained with between 5 and 10% resonance self shielding for U-235, and using a constant effective cadmium cutoff energy of 0.40 ev. A similar result could have been obtained using a constant, unshielded resonance integral and an effective cadmium cutoff energy which increases with the fraction D_2O in the moderator (the range 0.42 to 0.57 ev is sufficient to give good agreement with the measured cadmium ratios). Since each recipe has valid basis, a combination of the two probably represents the true physical picture.

The agreement of calculated and experimental fission cadmium ratios indicates that the U-235 fission cross section is represented properly. The good calculation of criticality for most cases indicates that the epithermal U-235 capture cross section cannot be very greatly in error, although this does not define the effective value.

3.2.4. Thermal Flux Ratio

Calculations using the method of Amouyal are described in Chapter 2. Wigner-Wilkins thermal spectra (1 deuterium atom = 0.1 hydrogen atom) were used to obtain effective cross sections. The results for the U-238 lattices are shown in Figure 3.9 and for Th-232 lattices in Figure 3.10. Calculated and experimental values are in rather poor agreement, and there is sufficient discrepancy to give appreciable criticality error for the boron poisoned cores. Both theory and experiment indicate, however, that the flux ratio is independent of boron poison up to 3.4 grams of boron per liter of moderator.

Improvements in the theoretical method may come from a more careful analysis of the thermal spectrum and of the exact geometry of the unit cell, since in these tight lattices the thermal spectrum and the geometry are not separable with accuracy. As an example, the spectrum must be appreciably harder in the interior of a strongly absorbing fuel rod than on its surface. In addition, the flux must be highly anisotropic in the neighborhood of the heavily absorbing fuel rods so that higher order expansions than the P_3 used in the present treatment may be needed. The external boundary of the unit cell in these lattices is square, and the fuel rod is cylindrical, so a complete treatment will probably require two or three dimensions. Because of the many complexities of the problem, it seems reasonable to attempt further improvements through Monte Carlo techniques utilizing the capacity of a large computer.

3.2.5. Reflector Savings

The BPG four-group method was used to determine the radial reflector savings (Section 3.1). Group coefficients for the reflector were obtained by averaging cross sections over the equilibrium spectrum obtained from a fission source in the reflector. For light water reactors, the leakage spectrum of neutrons into the reflector is similar to the fission spectrum, but this is not true for D_2O systems. Investigations utilizing the reactor leakage spectrum (calculated with the BPG Code) as the source for the reflector spectrum gave results not too different from those calculated using the spectrum from a fission source. Since the values calculated using the fission source were in fair agreement with experimental reflector savings, further work was not performed at this time.

The results are shown in Figure 3.11 for clean lattices and infinite reflectors, and in Figure 3.12 for the series of large cores in which D₂O and boric acid were used to balance criticality. The results give closer agreement for the borated cores, but this may be caused by a balancing of errors in the high energy constants with an error in the thermal constants, since a Maxwell-Boltzmann thermal neutron spectrum was assumed in all reflectors.

It is concluded that the computed reflector savings values are sufficiently accurate in all cases for good estimates of the criticality of large reactors, but the results leave some uncertainties in the interpretation of the criticality of some of the smaller cores. Greater accuracy may come from the use of better epithermal spectra in the reflector for obtaining epithermal four-group constants, or from the use of a greater number of groups.

3.2.6. Conversion Ratio

The measurement of cadmium ratios of fertile and fissile materials determines the conversion ratio of the lattice, providing thermal cross section data and average values of capture to fission ratio in the fissile material can be found. The relationship comes from the definition:

$$\text{Conversion Ratio} = \frac{\text{Fertile Absorptions}}{\text{Fissile Absorptions}}$$

Separating the absorptions into epicadmium (epi) and thermal (th) portions,

$$\text{Conversion Ratio} = \left(\frac{\text{Fertile}}{\text{Fissile}} \right)_{\text{th}} \frac{\left(\frac{\text{epi}}{\text{th}} + 1 \right) \text{ Fertile}}{\left(\frac{\text{epi}}{\text{th}} + 1 \right) \text{ Fissile}},$$

and the several ratios can be found separately. The thermal ratio of fissile to fertile absorptions is the ratio of the microscopic effective thermal cross sections. The ratio th/epi is the cadmium ratio minus one, which is measured directly for the fertile material. For U-235, only the fission has been measured so a calculated average capture to fission ratio is used (Section 2.3). Using the symbols C₂₈ for the U-238 cadmium ratio, and C₂₅ for the U-235 fission cadmium ratio, the final

equation for a low enriched uranium system is

$$\text{Conversion Ratio} = \frac{N_{28} \sigma_{28}}{N_{25} \sigma_{25}} \frac{1 + \frac{1}{C_{28-1}}}{1 + \frac{\eta_{th}}{\eta_{epi}} \frac{1}{C_{25-1}}}.$$

A similar equation applies for the Th-U-235 lattice. The following constant numerical values were used in the analysis:

$$\frac{N_{28}}{N_{25}} = 23.57$$

$$\frac{N_{02}}{N_{25}} = 15.00$$

$$\frac{\eta_{th}}{\eta_{epi}} = \frac{2.07}{1.61} = 1.285$$

$$\frac{\sigma_{25}}{\sigma_{28}} = 246.2$$

$$\frac{\sigma_{25}}{\sigma_{02}} = 88.23$$

The cross section ratios vary with thermal spectrum, but neglect of this variation produces a small effect when compared to other uncertainties in the analysis. The results are shown in Figure 3.13. Agreement between measured and calculated values is not very good, indicating the need for further work in determining conversion ratio. The main discrepancies between calculation and experiment are in the fertile material cadmium ratio. The lower values of conversion ratio for the thorium lattice indicate that the lattice spacing, fuel rod size, and ratio of fissile to fertile atoms is more nearly optimum for the uranium rods than for the thorium ones. Further studies are needed to determine the optimum conditions for the two fertile materials.

3.3. Comparison of Boron Poison and D₂O Concentration

To obtain a cross check on the reactivity worth of D₂O addition, a series of comparable experiments was performed with boric acid added to the moderator. In these two sets of experiments, the core radius (or critical buckling) was used as the intermediate measure of the variation. A third series of critical experiments balanced D₂O and boric acid

directly at constant core size. These three series of experiments are not a closed circle in that they do not check each other directly. Instead, the results form three different curves on a surface in a three-dimensional plot. The results do give a good test of the calculational model, and comparisons between theory and experiment are shown in Figures 3.14, 3.15, and 3.16. The agreement shown in Figures 3.14 is good, but the variation of radius with boron concentration for the H₂O core shown in Figure 3.15 indicates increasing disagreement between calculated and experimental values as the boron is increased. This error demonstrates the increasing importance of the thermal flux ratio, ϕ_M/ϕ_F . The calculated value of this ratio is probably in error for light water systems, as described previously. The third curve, balancing boron against D₂O at constant core radius, is calculated better, except for the H₂O case. This better agreement for the intermediate cores comes at least partly from the compensation of an error in reflector savings (computing too large a critical radius) by an error in thermal flux ratio (computing too small a critical radius).

It is also interesting to compare cases using a different extrapolation. A more useful procedure theoretically relates D₂O in the moderator to boron poison through the buckling. A curve of boric acid poisoning versus buckling has been shown earlier (Fig. 3.4) and also the variation of buckling with D₂O concentration (Fig. 3.2). The cross plot of boron poisoning at 0% D₂O versus % D₂O with no boron (where each gives equal buckling) is presented in Figure 3.17. Experimental data plotted on this curve are equivalent buckling points for the major critical cores, obtained from curves drawn through the experimental points in Figures 3.2 and 3.4. The calculated curve was obtained from the calculated curves of these figures. The agreement is fairly poor. In the cross plotting procedure, the disagreements between calculated and experimental values of the original curves cause deviations in the same direction. The final disagreement is caused primarily by the original disagreements for Core II and IX.

Reactivity holddown by D₂O and boron is important for reactor design. In order to obtain calculated values of reactivity holddown, we assume a one-group model for the light water lattice (no D₂O or boron) such that k_{eff} is given by

$$k_{\text{eff}} = \frac{k_{\infty}}{1 + M^2 B^2} = \frac{1 + M^2 B_{\text{critical}}^2}{1 + M^2 B^2}.$$

In this formulation we also assume a constant value of

$$M^2 = - \frac{1}{P_{\text{NL}}^2} \frac{P_{\text{NL}}}{B^2} = 43.4 \text{ cm}^2$$

(see Section 3.6). Equivalent values of k_{eff} for unpoisoned H_2O moderator are found from the bucklings of D_2O and poisoned H_2O lattices. These are plotted in Figures 3.18 and 3.19. Only the computed curves are plotted, since direct experimental data are not available.

3.4. Epithermal Neutron Spectrum

The basic calculation of the BPG Code determines the energy variation of the neutron flux during the slowing-down process. The calculation assumes that $\phi(u)$ is constant over each lethargy group, varying in discrete steps from group to group. Thus the calculated spectrum does not vary smoothly, and the effects of heavy resonance absorption depress the calculated flux over a wider energy range than is true. Figure 3.20 shows the calculated spectrum in Core III. Experimental values of flux spectrum were determined by foil activations, calibrated against an energy spectrum proportional to $1/E$, or $\phi(u) = \text{constant}$. The foils used and the energy of their major resonance activation are listed in Table 3.3. This table compares experimental and calculated points for five cores with varying amounts of D_2O in the moderator.

Table 3.3. Epithermal Neutron Spectra

Detector isotope		Rh-103	Au-197	W-186	Co-59	Mn-55	Se-80
Resonance energy, ev		1.26	4.91	18.8	132	337	2000
Core I	M	1.02	1.00	1.24	1.58	1.86	2.43
0% D ₂ O	C	0.92	1.00	1.14	1.35	1.38	1.48
Core IX	M	1.06	1.00	1.29	1.97	2.21	3.05
49.7% D ₂ O	C	0.89	1.00	1.24	1.62	1.68	1.87
Core VIII	M	1.04	1.00	1.22	1.86	2.13	2.92
60.4% D ₂ O	C	0.81	1.00	1.11	1.48	1.59	1.83
Core IV	M	1.05	1.00	1.44	2.43	-	4.34
69.7% D ₂ O	C	0.88	1.00	1.34	1.91	2.01	2.27
Core III	M	0.92	1.00	1.33	2.14	-	3.60
76.5% D ₂ O	C	0.88	1.00	1.40	2.10	2.22	2.55

M = Measured relative flux, $\phi(u) = E\phi(E)$
C = Calculated relative flux, $\phi(u) = E\phi(E)$

Both experimental and calculated flux spectra have been normalized to unity at the gold resonance energy (4.91 ev). The calculations indicate less resonance absorption for all cores than do the experimental data. Stated differently, the slowing down density is reduced at lower energies below that at higher energies, and the experimental data show a greater reduction than do the calculations for each core. Both experiment and calculation show that the spectrum hardens as the D₂O concentration of the moderator is increased. Again, the calculations indicate less variation with changing D₂O than do the experimental data. These differences are as yet unresolved, but it is encouraging to note that theory and experiment agree on the direction of variation, and that differences between theory and experiment are also in the same direction.

There is an apparent discrepancy in the measurements for Cores III and IV. Core III has a higher D₂O concentration than does Core IV and yet the measured values indicate a softer spectrum for Core III than for Core IV. This is in contradiction to the calculated spectra which show monotonic increased hardening with increased D₂O concentration.

The Core VIII results do not follow this trend, but since this is a thorium core and has a different U-235 concentration, a slightly different spectrum is expected. Some of these differences reflect the experimental difficulties encountered in the development of a new technique.

3.5. Variation of Parameters with Temperature

In normal practice, the nuclear characteristics of H₂O and D₂O moderated reactors are studied at room temperature, and extrapolations in the calculated results must be made to determine the characteristics at the elevated temperatures of power reactor operation. Measurements of the buckling as a function of temperature have been made in The Babcock & Wilcox Hot Exponential Facility using the U-238 oxide fuel rods in a first series of exponential experiments with H₂O moderator, and in a second series of hot exponentials with 81.5% D₂O in the moderator.

Calculations of the material buckling versus temperature have been made using the BPG Code, obtaining B_M^2 as that buckling which makes $k_{eff} = 1$. Temperature dependent resonance integrals were constructed from resonance parameters using the Resonance Integral Program.³³ Other variations with temperature were found using the methods of Chapter 2, taking into account the variation of moderator density with temperature. The results for the H₂O moderator are shown in Figure 3.21 and for the 81.5% D₂O moderator in Figure 3.22. Fair agreement between the calculations and the experimental data is obtained for the 81.5% D₂O case. This contrasts strongly with the H₂O case where agreement is much poorer.

There is appreciable difference between the calculated and experimental results for H₂O moderator. While the general trends are in the same direction, the calculations indicate about twice as much buckling change between 66 and 406 °F as is measured, and the difference amounts to about 5% in k_{eff} . This discrepancy appears to be out of the range of experimental uncertainty, although the experimental results are very strongly dependent on values of radial buckling.

On the other hand, the calculated value should be better than this difference indicates, because room temperature variation of buckling with D₂O concentration is predicted much better than this. In principle, variation of buckling with D₂O concentration should be similar to the variation with H₂O density. In addition, the variation of U-235 and U-238

cadmium ratios with D_2O concentration is predicted well; calculated variations of age with % D_2O agree well with measured values, and so do calculated and measured values of $\partial k_{eff}/\partial B^2$. Finally, the Doppler effect is small enough that even if a zero value were used, it could only change k_{eff} by about 1% or B^2 by about a third of the discrepancy. While these comments throw little light on a solution of the discrepancy, they do indicate that there is definite disagreement. The amount of this discrepancy is large enough to lead to appreciable inaccuracy in determining the temperature defect in large power reactors. For this reason, further work on high temperature effects is indicated.

A possible similar discrepancy has been noted in the critical Cores VIII and IX where small differences between measured and calculated bucklings were observed. These cores have intermediate amounts of D_2O in the moderator. In this region, the reduction of water density with temperature gives effects similar to addition of D_2O to the moderator. Experimental checks in the region of lower D_2O concentration (between 0 and 50% for $M/W \geq 1$) and for looser lattices ($M/W < 1$) at higher D_2O concentration, may throw further light on the situation.

3.6. Age and Leakage

3.6.1. Introduction

Using the BPG Code as the basic theoretical model for neutron slowing down, a number of calculations of the neutron age, $1/6 r^2$, have been performed. The relation between age and the non-leakage probability is found to be more complex for SSCR systems than for more thermal reactors, but the BPG model appears to relate them well and to correlate these integral measurements with basic cross section data.

Age is defined in the BPG Code ¹ as the output quantity, τ_{epi} , calculated from non-leakage probability

$$(1 + \tau_{epi} B^2)^{-1} = 1 - B \sum_j J_j,$$

where B^2 is buckling, J is proportional to the first moment of the Fourier transform of the flux, and j is microgroup number. This definition of age is rather restrictive and is useful only in calculation of fast and

epithermal neutron leakage. The value of τ_{epi} has no direct physical interpretation and is not equivalent to $1/6 \bar{r}^2$ for the SSCR lattices.

The B-1 approximation permits evaluation of rigorous mathematical expressions for the second moment of the flux.^{18, 58} To show this, we expand the Fourier transform of the flux in Legendre polynomials as

$$\int_{-\infty}^{+\infty} dz e^{iBz} \psi(\mu, u, B) = \sum_{i=0}^{\infty} \frac{2i+1}{4\pi} \phi_i(u, B) P_i(\mu).$$

Multiplying by $P_\ell(\mu)$ and integrating over μ , we obtain the individual components of the Fourier transformed flux.

$$\phi_\ell(\mu, B) = \int_{-\infty}^{+\infty} dz e^{iBz} \int_{-1}^{+1} d\mu P_\ell(\mu) \psi(\mu, u, z).$$

Even derivatives of the $\ell = 0$ equation (flux) are

$$\frac{\partial^{2m} \phi_0(u, B)}{\partial B^{2m}} = (-1)^m \int_{-\infty}^{+\infty} z^{2m} e^{iBz} \int_{-1}^{+1} d\mu \psi(\mu, u, z),$$

which become in the limit of zero buckling

$$\frac{\partial^{2m} \phi_0(u, 0)}{\partial B^{2m}} = (-1)^m \int_{-\infty}^{+\infty} z^{2m} \int_{-1}^{+1} d\mu \psi(\mu, u, z).$$

Now we note that the definition of the second moment of the flux is

$$\frac{1}{6} \bar{r}^2 \equiv \frac{1}{2} \frac{\int_{-\infty}^{+\infty} dz z^2 \int_{-1}^{+1} d\mu \psi(\mu, u, z)}{\int_{-\infty}^{+\infty} dz \int_{-1}^{+1} d\mu \psi(\mu, u, z)}$$

This is just the ratio of the $m = 1$ and $m = 0$ terms above. Therefore,

$$\frac{1}{6} \bar{r}^2 = -\frac{1}{2} \lim_{B^2 \rightarrow 0} \left[\frac{1}{\phi_0(u, B)} \frac{\partial^2 \phi_0(u, B)}{\partial B^2} \right].$$

Marshak⁵⁸ expands the $\ell = 0$ term in the Fourier transformed flux in a power series as

$$\phi_0(u, B) = \sum_{j=0}^{\infty} \frac{(-1)^j}{(2j)!} \phi_0^{(2j)}(u) B^{2j}.$$

We rewrite

$$\phi_0(u, B) = \phi_0^{(0)} - \frac{1}{2} \phi_0^{(2)} B^2 + \dots$$

and compare the derivatives as B^2 approaches zero:

$$\lim_{B^2 \rightarrow 0} \left[\frac{\partial^2 \phi_0(u, B)}{\partial B^2} \right] = -\frac{1}{2} \phi_0^{(2)}$$

and

$$\lim_{B^2 \rightarrow 0} \left[\frac{\partial^2 \phi_0(u, B)}{\partial B^2} \right] = -\phi_0^{(2)}$$

From these and the expression above for the second moment, we see that

$$\frac{\overline{r^2}}{6} = -\lim_{B^2 \rightarrow 0} \left[\frac{1}{\phi_0(u, B)} \frac{\partial \phi_0(u, B)}{\partial B^2} \right].$$

Since the B-1 approximation gives the correct values of the expansion coefficients $\phi_0^{(0)}$ and $\phi_0^{(2)}$, these equations should give correct values for the age.¹⁸

The age calculations reported here are performed by using the BPG Code in the B-1 approximation with Greuling-Goertzel slowing down for all elements with atomic weights $A \leq 27$ to evaluate

$$\tau = \frac{1}{\phi_0(u, B_1)} \times \frac{\phi_0(u, B_1) - \phi_0(u, B_2)}{B_2^2 - B_1^2},$$

and for the slowing down moment,

$$\tau_q = \frac{1}{q_0(u, B_1)} \times \frac{q_0(u, B_1) - q_0(u, B_2)}{B_2^2 - B_1^2}.$$

The linearity of $\phi(u, B)$ for small B has been investigated, and only a very small numerical error is introduced by using this difference technique if B_1^2 and B_2^2 are of the order of 10^{-22} and 10^{-6} cm^{-2} , respectively. Unless otherwise indicated, the group average of the flux $\phi_0(u, B)$ is taken in a microgroup with bounds 1.84 and 1.4 ev, while $q_0(u, B)$ is evaluated at 1.4 ev.

3.6.2. Comparison with Age Experiments

A number of experimentally investigated cases have been calculated with the method just described. Results for pure moderator are given in Figure 3.23, together with experimental values⁵⁹ and a curve based on Monte Carlo calculations.⁶⁰ Agreement with Monte Carlo calculations is excellent except for a slight discrepancy at the H_2O end which can be explained partly by small differences in cross section data. Disagreement between theory and Wade's experiments at the H_2O end has been discussed extensively in the literature. Later experiments by Doerner⁶¹ and by Pettus⁶² give values of $27.68 \pm 0.1 \text{ cm}^2$ and $27.0 \pm 0.9 \text{ cm}^2$, respectively, for the age to indium resonance in H_2O . The BPG calculated flux age is 26.8 cm^2 in reasonably good agreement with the experiments, while the Monte Carlo calculations give $26.0 \pm 0.3 \text{ cm}^2$. (Theoretical ages in Figure 3.23 are slowing down ages.)

Further preliminary checks were made by comparing calculated to experimental ages in lattices of thorium metal, aluminum and light water. The experimental values of Pettus⁶² are listed in Table 3.4 along with the calculated values. Although agreement is fair, there is enough discrepancy in some cases to indicate weak points in theory, experiments, or both. An improvement to the model gave the results listed as corrected age, and these are discussed later in this section.

Table 3.4. Age in Th-Al-H₂O Lattices

<u>Volume fraction</u>			<u>Age to 1.46 ev, cm²</u>		
<u>H₂O</u>	<u>Th</u>	<u>Al</u>	<u>Experiment</u> [*]	<u>Calculated</u>	<u>Corrected</u>
0.827	0.173	--	29.9 ± 3	29.7	30.9
0.710	0.290	--	35.6 ± 1.2	32.6	34.8
0.582	0.239	0.179	41.5 ± 1.4	40.4	43.1
0.564	0.436	--	42.4 ± 2	37.9	41.5
0.503	0.198	0.299	52.1 ± 2.1	47.6	50.7
0.433	0.327	0.240	52.4 ± 1.7	51.3	55.4

- - - - -
^{*} Including an estimated 5% correction for the neutron source tube.

Since the uncorrected model describes both homogeneous and lattice age data reasonably well, it was used to compute the values for lattices measured in this project. Ages were measured ⁵ in lattices containing ThO₂ cylindrical rods clad with aluminum and moderated with mixtures of H₂O and D₂O. Experimental and calculated ages for these lattices are compared in Figure 3.24. For the uncorrected calculations, agreement is satisfactory at the high D₂O end, but is not good at D₂O concentrations of 50% and below. Improvements to the model appear to improve this situation, as described below.

3.6.3. Improvements to the Model

The calculational model in use, as imbedded in the BPG Code, has several obvious shortcomings, none of which, however, appear to influence seriously the calculation of age in infinite media.

Probably the most important effect in terms of per cent influence on the age for lattices is the question of a transport theory improvement on the coefficient of the current in the second B-1 equation, as mentioned in Section 2.1. This improvement would be similar to transport improvements on ordinary diffusion theory. Two separate effects should be considered. Due to the fact that inelastic scattering in high energy groups is treated in the Code in a fashion similar to

absorption, an absorption correction is made in the calculation of Σ_T in the second B-1 equation, which then reads

$$h(u) [\Sigma^S (1 - \mu) + 0.2 (\Sigma^A + \Sigma^{in})] J(u) - \frac{B}{3} \phi(u) = \sum_m \frac{dp_m}{du}.$$

Furthermore, a scattering anisotropy correction (over and above the usual $1 - \mu$ factor) might be of value. Of these two effects, the absorption correction is probably most important. An illustration of the influence of the absorption correction is given in Figure 3.24, where results of age calculations on the ThO_2 - D_2O - H_2O -Al lattices are plotted. Results for calculations on Th-Al- H_2O lattices are given in Table 3.4. As can be seen from the figure and the table, the correction is fairly substantial, and the agreement between experimental and calculated values is appreciably improved. This better agreement indicates that a correction of this nature should be investigated further. However, the full theoretical justification for including the corrections is not definite, and the modifications have not been made to the present BPG method.

In its present state, the BPG Code is based on a bare reactor approximation, neglecting reflector moderation. A qualitative indication of the influence of this can be had from kinetic experiments on the water boiler,⁶³ where reflector moderated neutrons appear effectively as an extra delayed group. However, this effect is probably small and should have little influence in the cases under consideration since the ages in lattice and reflector differ little.

As mentioned in Section 2.1, variation of the scattering parameters ξ , γ , ζ , and η with lethargy is neglected. No quantitative estimate of the error introduced by this assumption is available. Again however, the agreement with Monte Carlo calculations and experiments for pure moderator over the range of D_2O concentrations affords an indication that no gross discrepancy is introduced.

3.6.4. Sources of Discrepancy

Correlation between BPG calculations and lattice age experiments is not entirely satisfactory. While agreement with Th-Al- H_2O age experiments is acceptable, there is enough of a discrepancy between calculated and experimental ThO_2 - D_2O - H_2O ages to indicate deficiencies in theory, experiment, or both. Introduction of the absorption

correction in the B-1 equations makes calculations agree very well with Th-Al-H₂O experiments, whereas the correlation with the ThO₂-D₂O-H₂O experiments is improved without giving entirely satisfactory results. Of the effects that could give rise to the remaining discrepancies, the most important experimentally are the finite lattice, lattice perturbation by foilholders, etc., and lattice anisotropy. On the theoretical side remain calculation of infinite instead of finite medium ages, homogenization of lattice (no anisotropy), and uncertainties in elastic and inelastic scattering cross sections (notably for oxygen and thorium).

Since the fission neutron spectrum stored in the BPG Code is a prompt spectrum, the age calculations do not account for the delayed neutrons. Although one would intuitively assume the effect of this on age to be small in most cases, it is not obvious for high leakage cores. If the total flux in a reactor at an energy E' is written as $\phi_p + \delta\phi_d$, it can be shown that

$$\tau(E') = \tau_p(E') + \frac{\delta\phi_d(E')}{\phi_p(E')} [\tau_d(E') - \tau_p(E')],$$

where higher order terms in $\delta\phi_d/\phi_p$ have been neglected. Here subscripts p and d refer to prompt and delayed neutrons, respectively, while the prefix δ on ϕ_d is used to indicate that the delayed flux is small compared to the prompt flux. As a first approximation,

$$\frac{\delta\phi_d(E')}{\phi_p(E')} = \beta \frac{1 + \tau_p(E') B^2}{1 + \tau_d(E') B^2},$$

where β is the delayed neutron fraction in fuel. For the lattices considered here, we generally have $\tau_d(1.4) \geq 0.4 \tau_p(1.4)$, $B^2 < 100 \cdot 10^{-4} \text{ cm}^{-2}$ and $\tau_p(1.4) < 100 \text{ cm}^2$. Thus the fractional contribution to τ at 1.4 ev from delayed neutrons will be less than β .

Some studies have been made on the importance of utilizing the full Greuling-Goertzel approximation for the slowing down process. Using the Fermi age approximation for slowing down by D changes the age by 5 to 15% in several cases. However, the age approximation seems to be adequate for all heavier nuclei in mixtures containing H and D. Neglecting the first order anisotropic component of the slowing down

density has more important consequences, leading to a 25% increase in the age in H₂O and a similar increase in the age for a Core I medium.

In comparing the calculations with experiments, the quantity

$$\frac{\partial \tau(E')}{\partial E'}$$

is important in evaluating E', the effective energy of detection.⁵ The BPG Code gives

$$\frac{\partial \tau_q(E')}{\partial E'} = -0.25 \frac{\text{cm}^2}{\text{ev}} \text{ at } E' = 1.4 \text{ ev}$$

or

$$\frac{\partial \tau_q(u')}{\partial u'} = 0.40 \text{ cm}^2 \text{ per unit lethargy at } u' = 15.8$$

for H₂O in good agreement with $0.42 \pm 0.1 \text{ cm}^2$ per unit lethargy as calculated in Reference 60. Calculated values of this quantity for the ThO₂-D₂O-H₂O lattices are shown in Table 3.5. A rapid change of τ_q with energy is evident for the high D₂O lattices. Slowing down age τ_q rather than flux age τ is used in this evaluation since the flux age is constructed from a group averaged flux, which is not very accurate in computing differential energy effects.

Calculations of the flux age are generally slightly less accurate than calculations of the slowing down age due to group averaging of fluxes. The difference between the two ages is the well-known last flight correction. Coveyou⁶⁰ has derived an expression for $\tau - \tau_q$ under fairly general conditions. The last flight corrections as obtained from the BPG calculations are in most cases approximately 0.3 to 1 cm² and agree well with Coveyou's equation. This indicates that the group averaging procedure used for the flux does not introduce significant errors in the age calculations.

The composite influence on $1/\phi \partial \phi / \partial B^2$ of neutron leakage and of the higher order expansion coefficients of the flux has been investigated by the calculation of $1/\phi \partial \phi / \partial B^2$ at high bucklings for several lattices. Some results are given in Table 3.6. The values of τ listed are those computed at $B^2 = 0$. The results of this table indicate that errors in the calculations caused by small deviations from zero buckling

are indeed small. The maximum B^2 used in computing τ was 10^{-6} cm^{-2} .

Table 3.5. Variation of Age with Energy of Detection

% D ₂ O in Lattice		0	49.77	69.19	79.19	90.0
$-\frac{\partial \tau}{\partial E} q$	1.46 ev	0.66	1.36	2.17	2.87	4.45

Lattice is ThO₂ fuel rods, 50% volume moderator

Units of age variation are cm²/ev

Table 3.6. Influence of B^2 on $\frac{1}{\phi} \frac{\partial \phi}{\partial B^2}$

Lattice	ThO ₂ -D ₂ O-H ₂ O 90% D ₂ O	ThO ₂ -D ₂ O-H ₂ O 0% D ₂ O	SSCR Core I 0% D ₂ O
B_1^2	$22.4 \times 10^{-4} \text{ cm}^{-2}$	$26.8 \times 10^{-4} \text{ cm}^{-2}$	$88.16 \times 10^{-4} \text{ cm}^{-2}$
$\frac{1}{\phi} \frac{\partial \phi}{\partial B_1^2}$	113 cm ²	40 cm ²	33 cm ²
τ	118 cm ²	43 cm ²	39 cm ²

The large number of cross sections involved in these computations prevents a systematic study of the influence of cross section uncertainties on the calculated age, but a semi-quantitative study has been made of some of the more important cross section effects.

A 60% increase in the inelastic scattering cross section for Th leads to a 2 to 3 cm³ decrease in calculated τ for the ThO₂-Al-H₂O-D₂O lattices. A change in inelastic matrix from one based on an approximation to the true inelastic matrix for U-238 to one giving equal probability in lethargy down to a suitable cutoff lethargy for the appearance of the scattered neutrons only changes the age in a typical lattice by 0.1%. With the present knowledge of inelastic scattering cross sections it is improbable that an error in this constant for Th or U could

cause more than a 1 to 2 cm² discrepancy in age for any of the experimental lattices.

Errors in the high energy scattering cross section for oxygen may have more important consequences. No investigations have been made of the influence of changes in the scattering characteristics of oxygen pending a revision of the present anisotropy constants. In pure water, the assumption of isotropic scattering by oxygen decreases the age by ~ 1 cm².⁶⁰ In the ThO₂-D₂O-H₂O and UO₂-D₂O-H₂O lattices the oxygen plays a prominent role in the slowing down process, so that a change in oxygen data may lead to noticeable changes in the calculated ages. This effect should be investigated further.

Neglecting entirely the resonance absorption in a 90% D₂O ThO₂-D₂O-H₂O lattice increases the age from 118 to 121 cm². Thus it does not appear that reasonable errors in resonance absorption cross sections should affect age calculations appreciably.

A discussion of perturbation of age experiments by various factors is found in Reference 5. The parallel and perpendicular measurements show no indication of anisotropy and no influence of foil location (in fuel or in moderator) if care to eliminate foilholder perturbations is taken. Table 3.6 indicates that preferential activation in the low energy wing of the 1.46 ev resonance in the detector foils has a greater effect at high D₂O concentrations than at the H₂O end. Calculations of the theoretical correction to age measurements for non-1.46 ev resonance activation in the foils⁵ show that this effect is very small. The finiteness of the experimental medium would act in a manner similar to absorption, that is, the age should be slightly decreased below the infinite medium age. This is in a direction to widen the gap between theory and experiment. From analogy with the influence of resonance absorption one would qualitatively expect it to be small, probably < 1 cm².

3.6.5. Leakage

The problem of space-energy separability in various approximations to the Boltzmann equation has been widely discussed in the literature (see e.g., Refs. 12, 55, 64). In general, fairly successful few-group schemes and four-factor type formulae have been devised on the basis of space-energy separability. Examples of simple

schemes are usually described by a criticality equation of the form

$$k_{\text{eff}} = k_{\infty} P_{\text{NL}}$$

and by the differential relation

$$\frac{1}{k_{\text{eff}}} \frac{\partial k_{\text{eff}}}{\partial B^2} = \frac{1}{P_{\text{NL}}} \frac{\partial P_{\text{NL}}}{\partial B^2}.$$

Leakage is usually described conceptually in terms of the Gaussian or Yukawa kernels; which for small L^2 are:

$$P_{\text{NL}} = e^{-M_G^2 B^2} \quad (\text{Gaussian})$$

$$P_{\text{NL}} = \frac{1}{1 + M_Y^2 B^2} \quad (\text{Yukawa}).$$

By differentiation, the migration area M^2 is given by:

$$M_G^2 = - \frac{1}{P_{\text{NL}}} \frac{\partial P_{\text{NL}}}{\partial B^2} \quad (\text{Gaussian})$$

$$M_Y^2 = - \frac{1}{P_{\text{NL}}^2} \frac{\partial P_{\text{NL}}}{\partial B^2} \quad (\text{Yukawa}).$$

These simple models, however, are not very applicable to the SSCR systems considered here. The main reason for this is the relatively strong influence of high leakage and heavy resonance absorption on the slowing down process. These effects are illustrated for some typical cases in Table 3.7.

Table 3.7. Interaction between Resonance Absorption, Slowing Down, and Leakage

Core	$B^2, 10^{-4} \text{ cm}^{-2}$	p	P_{NL}	$\tau (0.4 \text{ ev}), \text{ cm}^2$	$M_Y^2, \text{ cm}^2$	$\frac{1}{p} \frac{\partial p}{\partial P_{NL}}$
ThO ₂ -H ₂ O 90% D ₂ O	0	0.5517	1	121.4	108.7	0.1556
SSCR Core I	0	0.6431	1	39.9	37.2	0.0502
SSCR Core I	87.9	0.5948	0.737	--	43.4	0.0975
SSCR Core III	0	0.2863	1	83.1	70.3	0.1938
SSCR Core III	10.8	0.2856	0.9287	--	72.9	0.2155
SSCR Core IX	0	0.4458	1	58.1	52.6	0.1141
SSCR Core IX	44.0	0.4372	0.8025	--	59.5	0.1585
TRX 6*	0	0.6609	1	34.0	32.4	0.0760
TRX 6*	53.6	0.6522	0.8468	--	35.3	0.1011
TRX 7*	0	0.7436	1	30.8	30.5	0.0486
TRX 7*	58.2	0.7371	0.8444	--	32.9	0.0668
TRX 13*	0	0.7155	1	43.4	43.1	0.0504
TRX 13*	32.6	0.7112	0.8731	--	46.1	0.0535

* TRX cores are described in Table 2.4

Within the limitations inherent in the BPG model, the data in Table 3.7 support a number of important conclusions.

First, the concept of a simple multiplicative non-leakage probability is not applicable to cores with either a high leakage or a high resonance absorption. The table shows that τ and M_Y^2 (bearing in mind that M_Y^2 contains a small contribution from the diffusion length) disagree at zero buckling for SSCR lattices, and the value of M_Y^2 changes with both P_{NL} and p . Similar results were noted for M_G^2 . The relatively strong interaction between leakage and absorption in SSCR lattices is shown by the high values of $1/p \partial p / \partial P_{NL}$. These comments do not apply as aptly to the TRX cores, so we see that the concept of a simple

multiplicative non-leakage probability is a fair approximation for these lattices.

Second, the effect of epithermal fissions does not appear to be as important in these comparisons as implied by a statement in Reference 55. The table (particularly the first case with no fissions) indicates that the impossibility of using a simple model comes primarily from the interaction of resonance absorption and leakage with the slowing down of the neutrons.

Third, a comparison of SSCR Core I and TRX Core 6, which have similar amounts of resonance absorption but different leakages, shows that the slowing down process is more strongly influenced by the leakage than by the resonance absorption. This is also supported by the previously mentioned calculation in which a total neglect of resonance absorption in a highly absorbing lattice led to a change of only $\sim 3\%$ in τ . Thus, the calculation of a correct τ (or r^2) by a particular model does not necessarily mean that the model describes the slowing down process in a finite medium accurately. The measured values of τ present a fair test of the model, but since the relation between the τ and the leakage is very complex for SSCR lattices, an accurately measured τ may not insure a good evaluation of the leakage. Therefore, another more appropriate test for the leakage as evaluated by the BPG Code is described in the next paragraph.

3.6.6. Comparison with Leakage Experiments

As has been pointed out by Kouts,⁵⁵ an appropriate test for the theory is its ability to predict

$$\frac{1}{k_{\text{eff}}} \frac{\partial k_{\text{eff}}}{\partial B^2},$$

a quantity which is reasonably well defined and can be measured to fair accuracy. This test is especially severe in high leakage, high resonance absorption lattices. Table 3.8 compares calculated and measured values of this quantity for a number of lattices.

Table 3.8. Buckling-Reactivity Changes

Core	Calculated values, cm ²				Measured values, cm ²	
	τ (0.4 ev)	M_Y^2	M_G^2	$-\frac{1}{k_{eff}} \frac{\partial k_{eff}}{\partial B^2}$	$-\frac{1}{k_{eff}} \frac{\partial k_{eff}}{\partial B^2}$	M^2
SSCR I	39.9	43.4	32.0	32.5	31.9	--
SSCR III*	76.2	69.8	63.6	65.9	63.3	--
SSCR IV	74.7	68.7	61.1	63.4	60.2	--
SSCR IX	58.1	59.5	47.7	49.1	48.1	--
TRX 6	34.0	35.3	29.9	30.1	29.0	34.2
TRX 7	30.8	32.9	27.8	28.3	29.9	36.2
TRX 13	43.4	46.1	40.2	41.3	39.1	45.8

* Core loading 22, moderator contains 73.77% D₂O.

The agreement between experimental and calculated values of

$$\frac{1}{k_{eff}} \frac{\partial k_{eff}}{\partial B^2}$$

is somewhat better than might be expected in view of the uncertainties in determining the experimental values listed. These include experimental uncertainties in the period and height measurements, the uncertainties in effective delayed neutron fraction (β) needed to relate period to reactivity, and the uncertainty in determining the axial reflector savings in water height experiments. In addition, the values for the SSCR lattices were obtained from experiments which were originally intended merely as auxiliary aids to the main criticalities. However, the agreement is good enough for all lattices to indicate the usefulness of performing more experiments with greater accuracy at a later time. With greater experimental accuracy these measurements should be good tests of the theoretical leakage model, and would give information similar (although not identical) to that obtained from age measurements.

Table 3.8 also gives calculated values of various quantities usually associated with reactor leakage. These illustrate some of the characteristics of the leakage and absorption with the slowing down process.

The agreement between the calculated M_Y^2 and the measured M^2 (derived from a one-group analysis of the experiments) again indicates a fairly weak coupling between slowing down, leakage, and resonance absorption in the TRX cases. For the SSCR lattices, stronger dependence of the leakage on the resonance absorption is apparent. Furthermore, the resonance escape probability is affected more by leakage in these lattices than the TRX cases as is indicated by the values of

$$\frac{1}{p} \frac{\partial p}{\partial P_{NL}}$$

in Table 3.7.

An analysis of all these cores using the equation

$$k_{eff} = H p P_{NL}$$

reveals that the buckling dependence of k_{eff} is almost entirely attributable to the terms p and P_{NL} for both the TRX and SSCR lattices. This indicates that the fitting function H is almost independent of B^2 .

Therefore, in a conventional four factor representation, η , f , and ϵ would be materially less leakage dependent than p .

The Gaussian kernel results

$$M_G^2 = \frac{-1}{P_{NL}} \frac{\partial P_{NL}}{\partial B^2}$$

agree better for all cores with

$$\frac{-1}{k_{eff}} \frac{\partial k_{eff}}{\partial B^2}$$

than the Yukawa kernel results

$$M_Y^2 = - \frac{1}{P_{NL}^2} \frac{\partial P_{NL}}{\partial B^2}.$$

On the other hand, the Gaussian kernel values are in appreciably poorer agreement with the age than the Yukawa kernel values are. It should be useful to investigate the detailed physical reasons for this difference between Gaussian and Yukawa kernels.

3.6.7. Conclusions

Generally, the B_1 -Greuling-Goertzel and auxiliary equations, as formulated in the BPG Code with 39 energy groups, seem satisfactory for calculation of neutron slowing down and leakage in cores of the SSCR type. However, the results can be improved with additional work.

Some discrepancies between measured and calculated ages in ThO_2 -Al- H_2O - D_2O lattices are found at the low D_2O end. These are, however, small enough that foreseeable improvements in theory, experiment or both can close the gap. Main theoretical improvements would include cross section data (notably for O and Th) and possibly a transport type absorption correction to the coefficient of the current in the second B_1 equation. Generally, it seems improbable that these changes could have an effect of more than 3 to 4 cm^2 in the age for infinite media of the type considered here. The problem of age in pure moderators is believed to be well in hand. In order that lattice age experiments provide a really crucial test for the theory in its present stage of development, it is desirable that the experimental accuracy be increased.

A strong coupling between leakage slowing down, and resonance absorption is shown to exist in the SSCR lattices. This invalidates the usual few-group schemes with buckling independent lattice constants based on space energy separability of the flux. Due to these effects, the age concept is not as useful in reactivity calculations for these lattices since its use in a single infinite medium slowing down kernel to predict leakage is not justified. A further test of the theory is provided by experiments on the reactivity influence of changes in buckling (leakage and resonance absorption changes), and generally good agreement is found for a wide range of lattices. It is pointed out that, if a fair theoretical description of resonance absorption is used, these experiments test essentially the same parts of the theory as the age experiments. With improved accuracy, experimental value of $\partial\rho/\partial h$

may be as good a test of the slowing down model as measurement of age is.

3.7. Conclusions for Uniform Lattices

3.7.1. General Conclusions

A large amount of useful information about lattices moderated by mixtures of H_2O and D_2O has been obtained, and the theoretical model correlates nearly all of it with good accuracy. The experiments and the theory both demonstrate that large amounts of excess reactivity (or U-235 fuel loading) can be controlled by variable D_2O concentration. An increase in conversion ratio with increasing D_2O concentration is caused by increased epithermal resonance absorption in fertile material (or decreasing cadmium ratio). This demonstrates the concept of holding reactivity in fertile resonances rather than in thermal poison. These effects originally predicted by theory and now confirmed by experiment verify the principle of the SSCR.

3.7.2. Criticality

The BPG four-group model computes k_{eff} of SSCR critical assemblies very well. Only one lattice was calculated more than 1% off of $k_{eff} = 1$, and that was the boron poisoned case with a serious question about the correctness of the calculated thermal ϕ_M/ϕ_F . At intermediate amounts of D_2O in the moderator (about 50%) there may be compensating errors in reflector savings and in asymptotic core parameters which combine to give an accurate k_{eff} value. The bucklings of all room temperature exponential assemblies were calculated with about as good agreement as for the critical cases. Thus, the agreement of calculated and experimental eigenvalues covers the entire range of D_2O concentrations.

3.7.3. Resonance Absorption

Single pin resonance integrals for U-238 calculated from resonance parameters agree well with measurements of Hellstrand. For SSCR lattices, the calculated U-238 cadmium ratios are also in good agreement with measured values, even for cases with more than 90% epicadmium absorption. For thorium, the results are not as good, either in matching experimental single pin resonance integrals or in

determining lattice cadmium ratios. Here, more experimental and theoretical work is needed.

Since the effects of Dancoff mutual shielding between fuel pins are very important in these lattices, measurements here may be the best available information on accurate determination of this effect. Thus far, the Dancoff formulation has given better agreement between cadmium ratios, single pin values, and criticality than has the Bell approximation. These methods should be compared carefully in future experiments with varying lattice pitch.

The calculated cadmium ratios of U-235 fission are in very good agreement with experimental values. Self-shielding in the U-235 resonances (10 to 20% reduction) was taken into account in order to obtain the agreement. Further investigations are needed to determine the correct cadmium cutoff energy. This work does not determine the epithermal capture-to-fission ratio, but the usual cross section data apparently give this accurately enough for good criticality calculations for reactors with more than 50% epithermal fissions.

3.7.4. Conversion Ratio

Calculated and measured values are in fair agreement, but both are still too uncertain for very accurate reactivity life determinations in large spectral shift power reactors. The results obtained thus far do indicate that the combination of pitch, enrichment, and fuel rod size used in the lattices in this set of experiments are not optimum from a conversion ratio standpoint. Varying lattice pitch and obtaining an appreciably greater number of experimental data should improve the accuracy of results and permit better evaluation of the extent of agreement between theory and experiment.

3.7.5. Leakage

Since most cores studied in this series of experiments were small, they had appreciable leakage and provided good tests of the leakage model. The model calculates criticality very well, and good agreement is obtained between calculated and measured values of

$$\frac{1}{k_{\text{eff}}} - \frac{\partial k_{\text{eff}}}{\partial B^2}.$$

In addition, the model calculates ages in fair agreement with measured data in lattices of ThO₂ fuel rods (in D₂O and H₂O mixtures), in lattices of Th-Al-H₂O, and in pure moderator mixtures of H₂O and D₂O. These all represent an impressive array of confirmation for the model.

However, there are several areas that may change some of this. The four-group model does not now determine reflector savings very accurately. This may be improved by improving the spectrum in the reflector over which the four-group constants are averaged. A possible transport correction to the fast diffusion constant can improve agreement with the age experiments, but it may change criticality calculations slightly. Finally, improved cross section data should be used, including the energy variation of the Greuling-Goertzel parameters and improved inelastic cross section data for all materials.

3.7.6. Thermal Utilization

Except for boron poisoned cores, the thermal utilization is calculated with sufficient accuracy for the clean lattices. However, better results are needed to interpret the boron poisoned case and to evaluate power reactors after long periods of fuel burnup, particularly if appreciable quantities of plutonium are expected. Improvements are needed in methods to calculate the thermal flux ratio between moderator and fuel rod, and to determine the proper energy variation of the thermal spectrum. The methods used in this report to determine ϕ_M/ϕ_F do not agree well with the experimental data, although they do give fair agreement with measurements in some other lattices. Further theoretical work and more experimental data are needed.

3.7.7. Temperature Variation

Calculation of the temperature variation of buckling is in definite disagreement with data from the hot exponential experiments. The source of this disagreement is as yet unknown, but could possibly come from a number of small errors in the theoretical model which compensate each other at room temperature and at various D₂O concentrations, but which all lead the same direction as the temperature is raised. More theoretical analysis is needed, and more experimental data at elevated temperatures would be most helpful in solving this problem.

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Figure 3.1. Calculation Flowsheet

Figure 3.2. Critical Buckling of 4%-Enriched UO_2 Lattices

Figure 3.3. Critical Buckling of 93%-Enriched UO_2 - ThO_2 Lattices

Figure 3.4. Critical Buckling of Boron Poisoned Cores

Figure 3.5. Cadmium Ratio of U-238

Figure 3.6. Cadmium Ratio of Th-232

Figure 3.7. Cadmium Ratio of U-235 Fissions
(4%-Enriched UO_2)

Figure 3.8. Cadmium Ratio of U-235 Fissions
(U-235 in ThO₂)

Figure 3.9. Thermal Flux Ratios in U-238 Lattices

Figure 3.10. Thermal Flux Ratio in Th-232 Lattices

Figure 3.11. Radial Reflector Savings for Infinite Reflector

Figure 3.12. Radial Reflector Savings for a 14.22-cm Reflector with Boron Poison

Figure 3.13. Conversion Ratio

Figure 3.14. Critical Core Radius Vs D_2O Concentration

Figure 3.15. Critical Core Radius Vs Boron Concentration

Figure 3.16. % D₂O Vs Boron Concentration in Critical Reactors

Figure 3.17. Comparison of D₂O and Boron Poisoning for Equivalent Buckling

Figure 3.18. Reactivity Holddown of D₂O

Figure 3.19. Reactivity Holddown of Boron

Figure 3.20. Neutron Spectrum in Core III

Figure 3.21. Temperature Variation of Buckling in H₂O Lattice

Figure 3.22. Temperature Variation of Buckling in 81.5%-D₂O Lattice

Figure 3.23. Age in H₂O-D₂O Mixtures

Figure 3.24. Age in $\text{ThO}_2\text{-Al-H}_2\text{O-D}_2\text{O}$ Lattices

4. ANALYSIS OF NON-UNIFORM LATTICE EXPERIMENTS

4.1. Two-Zone Core

For large power reactors, the uniformity of power density is an important consideration. In order to test the capability of computing accurate power density distributions with the theoretical model, a two-zone core was assembled. Core VII had an inner radial zone of thorium fuel rods and an outer radial zone of uranium fuel rods, each region having the same lattice configuration as the other SSCR single region cores. The assembly was critical with 81.16 % D₂O in the moderator. Because the inner zone was in the form of a square rather than a circular cylinder, and the outer boundary was a circle, a two-dimensional calculation was required to obtain the correct geometrical representation. Criticality was computed using BPG generated four-group constants in the PDQ Code. This calculation gave $k_{\text{eff}} = 0.98$ for the critical assembly, a disagreement somewhat larger than noted for the single-region lattices. This could be caused by inaccuracies in determining the transition between regions, but it is more likely that small errors in determining the asymptotic parameters could account for it, since this core has higher D₂O percentage in the moderator than does any other critical core. The cause may be a large value of thorium resonance integral in the central region, since the single region thorium cores were both calculated with low k_{eff} .

Comparison of parameters in the asymptotic flux regions of the two zones is shown in Table 4.1. Agreement of all measured parameters with calculated values is very good, giving fair confidence to computed values of other reactor characteristics. The experimental uncertainty of the Th-232 cadmium ratio is sufficiently large (± 0.03) that agreement is still good with the calculated higher value (1.14) that would be required to change the calculated k_{eff} to 1.00. As noted before, the thorium resonance integral may have been chosen a little large, so it could be reduced slightly and still be consistent with both lattice and single pin measurements. Only a 5% change in resonance integral would be required. Thus

this analysis indicates that measurement of criticality in SSCR lattices may be a sensitive indicator of fertile material resonance integral.

Table 4.1. Parameters Of Two-Zone Core VII

<u>Zone</u>	<u>Inner</u>		<u>Outer</u>	
Fertile material	Th-232		U-238	
Equivalent radius	28.84 cm *		68.04 cm	
% D ₂ O in moderator	81.16		81.16	
<u>Asymptotic parameters</u>	<u>Calculated</u>	<u>Measured</u>	<u>Calculated</u>	<u>Measured</u>
ϕ_M/ϕ_F	1.15	1.16	1.16	1.15
U-235 Fission cadmium ratio	1.86	1.83	1.91	1.92
U-238 cadmium ratio	--	--	1.054	--
Th-232 cadmium ratio	1.128	1.12	--	--

* Actual configuration was square (52 rods $\times$ 52 rods).

The calculated power density distribution is compared with a radial experimental determination in Figure 4.1. The thermal flux shape is shown in Figure 4.2. Agreement is fair, indicating that the theoretical method represents the gross aspects of the transition from zone to zone reasonably well. Because of the large fraction of epithermal fissions in this core, thermal flux plots do not give adequate representations of the power distribution, and in particular, may not determine the correct location of "hot spots" nor the correct maximum power density. This particular core does not have the optimum gross power distribution, so that the comparison of calculations with experiment is indicative of the validity of calculations of this type rather than a demonstration of good power density distribution. In viewing these two curves it should be noted that the calculated power density curve includes the effects of U-238 fissions in the outer region, while the measured data do not. The

calculated thermal flux curve was continuous and did not predict the observed thermal flux peaking at the interface. A small moderator gap at the interface was apparently not accounted for properly in the calculation.

4.2. Water Gaps And Control Blade

A series of perturbations were made to Core III (76.50% D₂O) by removing fuel rods to put water gaps of various sizes in the center of the core. Then, a poison control blade was inserted. Because of the complex geometry involved, the criticality of these configurations was calculated using the two-dimensional PDQ Code in four groups, with group coefficients generated by BPG. Thermal cross sections were averaged over a Maxwell-Boltzmann distribution. The calculated criticalities are compared with experiment in Table 4.2. The exact configurations are described in BAW-1231.²

There is fair agreement between calculated and measured reactivity worths. Better agreement might be obtained by the use of thermal cross sections averaged over a Wigner-Wilkins thermal neutron spectrum. This would increase the calculated control blade worth without changing the calculated clean channel worths appreciably. The convergence of the PDQ calculations may not have been sufficiently good to expect better agreement between calculated and experimental Δk . The uniform core was calculated with the two different meshes (PDQ geometry), and a difference of 0.0015 is shown in the table. This is about the same as the disagreement in Δk .

Flux variations across the single channel are shown in Figure 4.3, those across the double channel in Figure 4.4, and those across the double channel with the control blade inserted in Figure 4.5. Measurements were made with bare and cadmium-covered gold foils. Calculated thermal flux distributions are in fairly good agreement with the measured data for all three cases. This agreement, along with the Δk agreement, supports the conclusion⁶⁵ that few-group, two dimensional diffusion theory, with fast group constants obtained from multigroup Fourier transform calculations, gives adequate prediction of thermal flux distributions near water gaps. For epithermal flux, the calculated peaking in the channel and at the edges of the channel is appreciably less than the measured peaking in all three cases. This could result from anisotropy in the flux

that diffusion theory does not account for, or it could be the difference between the flux at the gold resonance (measured epithermal distribution) and the average over the wide energy range (0.4 ev to 251.7 ev) chosen for group 3 in the four-group scheme. There is also the possibility that activation of the gold resonance (4.91 ev) is partly suppressed by the lowest U-238 resonance (6.68 ev) in the lattice but not in the gap, thus indicating a higher epithermal flux peaking than would occur at other energies where U-238 interference is less. Further calculational effort and perhaps some more experimental data are needed to reach a valid conclusion concerning the peaking of the epithermal flux in and near water gaps. In these high D₂O lattices, about half the power is in epithermal fissions, so the peaking of the epithermal flux near a gap is much more important than it would be in most other reactor configurations.

Table 4.2. Reactivity Worth Of Water Channels And Control Blade In Core III

Core configuration	PDQ geometry	Calculated k_{eff}^*	Δk Change for perturbation	
			Calculated	Experimental
Uniform Core III	Quarter Symmetry	0.9987	0.012	0.013
Central Channel, One Fuel Rod Wide	Quarter Symmetry	1.0107		
Uniform Core III	Full Core	1.0002	0.0136 -0.0123	0.016 -0.016
Off-Center Channel, Two Fuel Rods Wide	Full Core	1.0138		
Off-Center Channel, Boron Blade Inserted	Full Core	1.0015		

* Experimentally, excess reactivity was controlled by water height variation. Calculated values were all for the same water height.

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Figure 4.1. Radial Power Distribution in Core VII

Figure 4.2. Thermal Flux Distribution in Core VII

Figure 4.3. Flux Distribution Across Central Moderator Channel

Figure 4.4. Flux Distribution Across Off-Center Moderator Channel

Figure 4.5. Flux Distribution Across Off-Center Double Moderator Channel Containing Poison Control Blade

APPENDIX

Description of Cores

Detailed descriptions of the cores are given in BAW-1231. Some of the information is repeated here for reference.

	<u>Fuel Rods</u>	
Fertile Material	U-238	Th-232
Rod OD, in.	0.4755 ± 0.0015	0.308 ± 0.001
Wall Material	Stainless Steel	Al
Wall Thickness, in.	0.0160 ± 0.0005	0.014 ± 0.001
Fuel OD, in.	0.444 ± 0.002	0.260 ± 0.002
Active Fuel Length, in.	66.7 ± 0.3	60.0 ± 0.1
Fuel Density, gm/cm ³	9.46 ± 0.10	8.33 ± 0.14
Uranium, wt % U-235	4.020 ± 0.005	93.17
Atom Ratio, Th-232/U-235	0	15.00 ± 0.05

Lattice Properties

Fertile Material	U-238	Th-232
Fuel Rod Pitch, in.	0.595	0.387
Lattice Array	Square	Square
Volume Fractions		
Moderator	0.4982	0.5025
Fuel	0.4373	0.3545
Clad	0.0645	0.0863
Void	0	0.0566
<u>Homogenized Atomic Densities</u>		
(atoms/cm ³ × 10 ⁻²⁴)		
U-235	0.0003752	0.0004206
U-238	0.008845	0.0000349
Th-232	0	0.006310
Stainless Steel	0.005569	0
Al	0	0.005206
O*	0.01844	0.03153
	+ 0.4982 N ₀ ⁰	+ 0.5025 N ₀ ⁰
H*	0.4982 N ₀ ^H	0.5025 N ₀ ^H
D*	0.4982 N ₀ ^D	0.5025 N ₀ ^D

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 *See following table for N₀⁰, N₀^H, N₀^D

Pure Moderator Atomic Densities

atoms/cm³ × 10⁻²⁴

<u>Mole % D₂O in moderator</u>	<u>N₀^O</u>	<u>N₀^H</u>	<u>N₀^D</u>
0	0.03337	0.06674	0
10	.03336	.06004	.00667
20	.03335	.05335	.01334
30	.03333	.04666	.02000
40	.03332	.03998	.02666
50	.03330	.03330	.03330
60	.03329	.02663	.03995
70	.03328	.01997	.04659
80	.03327	.01331	.05322
90	.03325	.00665	.05985
100	0.03324	0	0.06648

These numbers are based on pure H₂O and D₂O densities at 68 F of 0.9983 and 1.1054 gm/cm³.

Critical Core Properties

<u>Core designation</u>	<u>Fertile material</u>	<u>Mole % D₂O</u>	<u>Grams boron per liter of moderator</u>	<u>Core radius, cm</u>	<u>Core height, cm</u>	<u>Radial reflector thickness, cm</u>
I	U-238	0	0	18.75	154.02	∞
II	U-238	0	3.39	59.71	146.65	∞
III	U-238	76.50	0	61.98	152.92	14.22 + 5 in. Paraffin
III-21	U-238	73.77	0	50.76	~139	25.44
III-22	U-238	73.77	0	61.98	110.78	14.22
IV	U-238	69.70	0	40.46	151.02	35.74
V	U-238	69.70	0.422	61.98	144.99	14.22
VI	U-238	49.63	1.79	61.98	147.56	14.22
VII-A	Th-232	81.16	0	----	141.28	----
VII-B	U-238	81.16	0	68.04	141.28	8.16
VIII	Th-232	60.40	0	25.94	118.40	50.26
IX	U-238	49.66	0	26.31	150.55	49.89
TUPE 15A	Th-232	0	0	18.37	127.21	∞

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