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THE ELECTRIC-DIPOLE GAMMA-RAY STRENGTH
FUNCTION FOR HEAVY EVEN-EVEN NUCLEI

by

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ABSTRACT

Gamma-ray spectra following neutron capture have been observed in a large number of neutron resonances in twelve even-even compound nuclei between Nd¹⁴⁴ and Hg²⁰². Absolute values of the transition probabilities of the highest energy E1 gamma-ray transitions have been obtained for most resonances observed and averaged over all resonances considered in a given nuclide. The resonance-average values so obtained yield transition probabilities which are characteristic of the particular transitions considered with relatively good precision. This is in contrast to the values obtained from similar studies of thermal-neutron-capture gamma-ray spectra which are beset with inherent and rather severe uncertainties. The dependence of the experimental values on the energy of the radiated gamma ray and the mass of the radiating nucleus is consistent with both the semi-classical prediction $A^{2/3} E_\gamma^{-3}$ and the recent prediction of the giant-dipole-resonance approximation, $A^{8/3} E_\gamma^{-5}$. Further, if either of these mass-energy factors is removed, the resulting reduced widths are observed to be closely proportional to the spacing of neutron resonances, as predicted by both theories. It thus becomes significant to define an E1 gamma-ray strength function as the resonance-average value of the partial width divided by the average spacing between neutron resonances and the appropriate energy factor and to study its dependence on the atomic mass of the radiating nucleus. The E1 gamma-ray strength functions have been constructed according to both proposed energy dependences and are found to follow the predicted mass dependence, although there are suggestions of peaks near the closed neutron shells. The absolute value of the strength function constructed according to the giant-dipole-resonance approximation is in reasonable agreement with the theoretical prediction, but there is violent disagreement in the other case. The

implications of this and other aspects of the results are discussed. Since the techniques used to analyze the data are for the most part new, they are dealt with in considerable detail. The pertinent theory and the techniques used to obtain the data are also discussed.

INTRODUCTION

The study of neutron-capture gamma-ray spectra may be expected to yield three essentially different kinds of physical information. First, the primary gamma rays may be used as a tool to study properties of the initial state or states excited by the neutron capture.^{1,2,3} Second, the cascade gamma rays may be studied as a tool to determine energies and, in favorable cases, spins, parities, and lifetimes of low-lying nuclear states.^{4,5,6} Third, the gamma rays may be studied for themselves to determine properties of the radiative-transition probabilities.^{5,7} A large number of experiments have been performed in the past to yield information of all three types. Before 1957, nearly all such experiments were performed with thermal neutrons,⁸ because of the very large thermal neutron fluxes which are, and have been, available at nuclear reactors. However, significant advances in the last five years in both the gamma-ray detectors⁹ and the analyzers¹⁰ used to accumulate the data have made it possible to study the spectra of gamma rays following capture in neutron resonances.

Thus, concerning information of the first type above, it has become possible to assign spins and parities to individual neutron resonances^{1,2,11} by studying their capture-gamma-ray spectra, and to determine the isotope in which these resonances occur.^{4,12} Also, it has been demonstrated that the individual gamma-ray transition probabilities exhibit interference effects³ between resonances, of the same type as have previously been observed for neutron scattering and fission.

For the investigation of low-lying states fed by transitions from the capture state, the study of thermal-capture gamma-ray spectra remains the most powerful technique, because

the available flux is several orders of magnitude higher at thermal energy than at higher energies.^{4,5} However, at least one new state (the 1.27-MeV state in Hg^{200}) has been discovered by resonance-capture experiments¹³ after being missed in intensive thermal-capture investigations, presumably because of the unfavorable spin (0^-) of the state which dominates the thermal capture in Hg^{199} .

For the third type of investigation mentioned above (the study of the properties of the transition probabilities from highly excited nuclear states), it might seem that the large thermal fluxes available would prevent the results from resonance-capture experiments from being competitive. This, however, is decidedly not the case. For reasons which will be discussed in the following section, transition probabilities determined from thermal-neutron-capture gamma-ray experiments, although generally of relatively high precision in themselves,¹⁴ are almost completely unreliable for determining any but the broadest dependences of the transition probabilities on the various nuclear parameters. After examining the reasons for this unfortunate fact, it will be clear that the uncertainties inherent in the thermal-capture results can be largely removed by studying the gamma-ray spectra produced by capture in the peaks of resonances and obtaining mean transition probabilities averaged over as many resonances as possible. The study of resonance-capture gamma-ray spectra thus provides the only known experimental means of directly obtaining reliable values for radiative-transition probabilities from highly excited nuclear states.

In the present work, gamma-ray spectra following resonance neutron capture have been observed in thirteen even-odd target nuclides ranging from Nd^{143} to Hg^{201} . Spectra were observed in all neutron resonances that could be resolved with a resolution of 100 nsec/m. A total of 24 resonance-averaged transition probabilities have been obtained. The choice of target nuclides was dictated by the

necessity of making unambiguous assignments of the multipolarity of the observed gamma rays. This requires that the low-energy level scheme of the compound nucleus be well investigated. All of the target nuclides investigated capture s-wave neutrons to form negative-parity resonances which may then decay by E1 transitions to well-known low-lying even-parity states characteristic of even-even nuclei. Further, the energies of the final states are relatively well spaced and usually quite well determined. Thus the differences in energy of the observed gamma rays are known a priori with high precision, a fact which, as will be seen later, greatly improves the precision of the analysis. There exist in nature only 26 stable nuclides of the type required for the realization of these advantages, that is, that have even Z, odd N, and odd parity. Of these, 10 were rejected because of the scarcity of neutron resonances. The investigation of some of these (notably Cr^{53} , Fe^{57} , Zn^{67} , and Se^{77}) may become practical with somewhat greater neutron-energy resolution, although investigations at higher neutron energies will have to resolve the possibility of p-wave interactions. Two nuclides (Gd^{157} and Dy^{163}) were not included because of difficulties in resolving the neutron resonances from resonances in other isotopes in the normal element. These nuclides could be investigated if separated isotopes were available in 100-gram quantities. Two other nuclides (Os^{187} and Os^{189}) were not included because their resonance-capture gamma-ray spectra are being investigated by another member of the Argonne fast-chopper group in an independent investigation. The present investigation thus is meant to be a fairly exhaustive study of the characteristics of high-energy E1 gamma-ray transitions.

Section I contains a brief summary of some basic slow-neutron spectroscopy necessary for the interpretation of the data. Also included in this section is a review of the theory of electromagnetic transitions from highly excited nuclear states and, as mentioned above,

a discussion of the uncertainties inherent in the results of thermal-neutron-capture experiments. The details of the experimental arrangement are given in Sec. II. Section III then gives a general discussion of the techniques used to analyze the data. Since no measurements of the absolute intensities have yet been published, this discussion is quite detailed. The various sources of systematic and statistical uncertainty are carefully examined. The special techniques needed in particular cases are discussed in Sec. IV, where the data and the results of the analysis are presented. Section V contains a summary of the final results and a comparison of these results with the theoretical predictions given in Sec. I. In closing, Sec. VI presents a summary of the experiment and a discussion of the conclusions that may be drawn from the results.

I. THEORY

We are considering radiative transitions from nuclear states just above the neutron binding energy to levels near the ground state in the radiating nucleus. The main concern in this section is the derivation of an expression for the probability of such transitions. It is desirable, however, to begin with a phenomenological discussion of the initial states, the neutron resonances. Although none of the detailed properties of the neutron resonances are used in the derivation of the radiative-transition probabilities, this discussion serves to introduce and define a number of quantities which will be used throughout the remainder of the paper.

(a) Slow-Neutron Resonances

When neutrons are incident on a nucleus $^{15,16} X^A$ they may either undergo potential scattering, with cross section σ_p , or form a compound nucleus X^{A+1} . The compound nucleus, if formed, is a highly-excited system having total excitation energy equal to the neutron binding energy B_n plus the kinetic energy E_n of the neutron (in the center-of-mass system). For nonfissionable nuclei, and for low neutron energies, the compound nucleus may lose this energy by only two processes: it may emit a neutron, with probability per unit time $\Gamma_n/\hbar$, leaving the nucleus X^A in its ground state; or it may emit a photon with probability per unit time $\Gamma_{\gamma_i}/\hbar$, leaving the nucleus X^{A+1} in a bound state with excitation energy $B_n + E_n - E_{\gamma_i}$, where E_{γ_i} is the energy of the emitted photon. The total probability per unit time for decay by emission of electromagnetic radiation is obtained by summing this quantity over all bound states in the compound nucleus i.e.,

$$(1) \quad \frac{\Gamma_Y}{\hbar} = \sum_{i=0}^N \frac{\Gamma_{Y_i}}{\hbar} = \left(\begin{array}{l} \text{Total probability per unit time} \\ \text{for emission of electromagnetic} \\ \text{radiation} \end{array} \right) .$$

Finally, the total probability per unit time for the decay of the compound nucleus is given by

$$(2) \quad \frac{\Gamma}{\hbar} = \frac{\Gamma_n}{\hbar} + \frac{\Gamma_Y}{\hbar} .$$

The foregoing discussion should be regarded as defining the various partial widths that appear in the cross-section formulae below.

The total cross section for the interaction of slow neutrons with nonfissionable nuclei is given, for isolated resonances, by the single-level Breit-Wigner formula

$$(3) \quad \sigma_T = 4\pi R^2 + \frac{\sigma_0}{1+x^2} \left[\frac{\Gamma_n}{\Gamma} + \frac{2R}{\lambda_0} x \right] + \frac{\sigma_0}{1+x^2} \frac{\Gamma_Y}{\Gamma} \sqrt{\frac{E_0}{E}} ,$$

where $\sigma_0 = 4\pi\lambda_0^2 g \Gamma_n / \Gamma$,

$$x = 2(E - E_0) / \Gamma ,$$

Γ = full width of resonance at half maximum,

Γ_Y = partial width for photon emission (resonance capture),

Γ_n = partial width for neutron emission (resonance scattering),

R = effective radius of the target nuclei for neutrons,

λ_0 = wavelength of the neutron at exact resonance,

E_0 = neutron energy at exact resonance,

$$g = (2J+1)/2(2I+1),$$

J = spin of the resonance, and

I = spin of the target nucleus.

The first term in Eq. (3) is the potential-scattering cross section σ_p . The second term, the resonance scattering term σ_n , includes a term linear in x which arises from coherent interference between the potential and resonance scattering. The third term is the resonance-capture cross section σ_γ .

Implicit in the form of Eq. (3) for the total cross section is the assumption that only neutrons with $\ell = 0$ (s-wave) may interact with appreciable cross section. This assumption is almost certainly valid for the nuclei and neutron energies considered in this report. If it is assumed that the s-wave and p-wave scattering strength functions are equal,* then the ratio of the average p-wave total cross section to the average s-wave total cross section is given for moderately slow neutrons by

$$(4) \quad \frac{\sigma_T(\ell=1)}{\sigma_T(\ell=0)} \approx 3 \left(\frac{R}{\lambda} \right)^2 \approx 10^{-5} E_n \text{ (in electron volts),}$$

the last expression being computed for $R = 8.2 \text{ F}$, a value typical of the nuclei considered here. Thus for neutron energies of a few hundred electron volts or less, the existence of a p-wave resonance of observable size is seen to be quite improbable. Further, most of the

* Actually, the rather sparse experimental evidence that exists indicates that the p-wave strength function is smaller than the s-wave strength function by a factor of at least 3 in the range of nuclides considered in this work.

resonances considered in this work have been investigated previously by total-cross-section measurements¹⁸ and the measured values of Γ_n are too large to be consistent with their being formed by p-wave interactions. We therefore may have considerable confidence that $l = 0$ for all of the observed neutron resonances treated. As a consequence, the spins of the observed resonances are restricted to

$$(5) \quad J = I \pm \frac{1}{2}$$

and the parity of the resonances is in all cases the same as the parity of the ground state of the target nucleus.

Before proceeding to a discussion of the radiative-transition probabilities, we digress for a moment to consider one of the inherent uncertainties in the transition probabilities determined from thermal-capture experiments.

If more than one resonance of the same spin contributes significantly to the total cross section, as is often the case in off-resonance regions, an additional term appears in the expression for the total cross section. This term arises from interference between the contributing resonances. For the case of neutron scattering, this interference term has the form

$$(6) \quad \sigma_{n.int.} = \sum_{i \neq j} \theta_{n_{ij}}^2 \sigma_{0i} \sigma_{0j} \frac{\Gamma_{ni}}{\Gamma_i} \frac{\Gamma_{nj}}{\Gamma_j}^{-\frac{1}{2}} \frac{1+x_i x_j}{(1+x_i^2)(1+x_j^2)},$$

where the sum is over all pairs of resonances with the same spin.

The new parameter $\theta_{n_{ij}}$ may be regarded as a phase factor between the probability amplitudes $(\Gamma_{ni}/h)^{1/2}$ and $(\Gamma_{nj}/h)^{1/2}$. Note that these amplitudes occur again in $(\sigma_{0i})^{1/2}$ and $(\sigma_{0j})^{1/2}$. Thus, as a result of the fact that the entrance and exit channels are the same for elastic scattering, there are no random fluctuations in sign of the scattering interference terms. A similar interference term appears formally in the radiative-capture cross section given by

$$(7) \quad \sigma_{\gamma \text{ int.}} = \sum_{i \neq j} \theta_{n_{ij}} \left[\sigma_{0i} \sigma_{0j} \frac{\sqrt{E_{0i} E_{0j}}}{E} \right]^{\frac{1}{2}} \frac{1+x_i x_j}{(1+x_i^2)(1+x_j^2)} \sum_{k=1}^N \theta_{\gamma k, ij} \left(\frac{\Gamma_{\gamma ki}}{\Gamma_i} \frac{\Gamma_{\gamma kj}}{\Gamma_j} \right)^{\frac{1}{2}},$$

where $\Gamma_{\gamma ki}/\hbar$ is the radiative-transition probability from the i th neutron resonance to the k th excited state in the compound nucleus. However, in this case the entrance and exit channels are distinct, so that the $\theta_{n_{ij}}$ may take on the values ± 1 , presumably with equal probability,¹⁹ for each pair (i, j) and the $\theta_{\gamma k, ij}$ take on the values ± 1 randomly for each pair (i, j) and each k . Since the sum over k involves a very large number of terms,* the contribution to the total cross section vanishes. On the other hand, in an experiment in which a particular gamma-ray transition is distinguished, interference effects should be observed in the partial capture cross section for the particular transition. This, in fact, has been observed experimentally³ for the ground-state transition following capture in Pt¹⁹⁵. In this case it is observed that, at thermal energies, the net effect of interference between several resonances is to reduce the ground-state transition probability to about a third of what it would be if there were no interference. The thermal-capture result for this particular transition thus contains a systematic error of a factor three. The situation is even worse in the case of capture in Hf¹⁷⁷. Here, there apparently is almost complete destructive interference for the transition from the state formed by the capture of thermal neutrons to the first excited state. The measured transition probability is zero,¹⁴ with an upper limit only a tenth of the observed transition probability for the same transition in the lowest energy resonance, which contributes approximately

* Using reasonable level-density formulae, for example the ones used in reference 11, we find that on the order of 10^4 to 10^5 levels in the nuclei considered in this work are accessible from the capture state by E1 transitions alone.

50% of the thermal capture. Thus the thermal-capture results are seen to provide very unreliable values for the radiative-transition probabilities, even for the resonances that contribute predominantly to the thermal capture. A second, even more severe source of inherent uncertainty in the thermal-capture results will be discussed after the derivation of the expressions for the radiative-transition probabilities in the next section.

(b) Radiative-Transition Probabilities

The formal theory of multipole radiation has been treated in detail by several authors.^{17,20,21} Instead of repeating these details here, this discussion will merely summarize two essentially different approaches to the problem. The first, due to Sachs and Austern,²¹ is the most straightforward, and perhaps the most generally valid. For the present purposes it serves to show explicitly that the familiar dependence of the dipole-transition probabilities on the cube of the gamma-ray energy is a general result, independent of any assumptions about the radiating system other than that its dimensions be small compared with the wavelength of the emitted radiation. The second approach, due to Weisskopf,¹⁷ is the semiclassical one. One considers classical fields with quantum mechanical sources. The fields are expressed initially as expansions in terms of eigenfunctions of the total angular momentum and its z component with definite parity, so the approach is particularly useful for keeping track of the dependence on the multipole order. Both of these direct approaches give results in terms of matrix elements involving unknown nuclear wave functions. These matrix elements may be estimated, but only very crudely. Fortunately, a new, indirect approach given by Axel²² avoids mention of the matrix elements entirely. This approach will be discussed in some detail.

Electromagnetic radiation may be treated as arising from the interaction of two quantum-mechanical systems, the photons in the radiation field and the radiating charges. The interaction is assumed to be weak so the Hamiltonian may be expanded in powers of $\vec{A}$, the vector potential of the photon. Thus

$$(8) \quad H\{\vec{A}\} = H_0 + H_1\{\vec{A}\} + H_2\{\vec{A}\} + \dots,$$

where H_0 is the Hamiltonian in the absence of an electromagnetic field, H_1 depends linearly on $\vec{A}$, H_2 depends quadratically on $\vec{A}$, and so on. The curly brackets are used to indicate that the H_n are functionals of the vector field $\vec{A}(\vec{r})$. It can be shown that H_2 is the interaction Hamiltonian for processes involving two photons, H_3 determines three-photon processes, etc., so that for simple emission and absorption only $H_0 + H_1\{\vec{A}\}$ needs to be considered. The transition probability is now given by the usual result from first-order time-dependent perturbation theory, namely,

$$(9) \quad dP_{if} = \frac{2\pi}{\hbar} |\langle i | H_1\{\vec{A}_{u,\omega}\} | f \rangle|^2 d\rho_f,$$

where i denotes the radiating system in initial state i , and f denotes the radiating system in final state f plus a photon whose polarization is in the direction of the unit vector $\vec{u}$, is propagated in direction $\vec{\omega}/\omega$, and which has energy $\hbar\omega$. The density of final states is the density of final photon states and is given by

$$(10) \quad d\rho_f = \frac{V}{(2\pi)^3} \frac{(\hbar\omega)^2}{(\hbar c)^3} d\Omega.$$

This expression is normalized in a box of volume V , and $d\Omega$ is an element of solid angle which includes the propagation vector $\vec{u}$. The vector potential is a plane wave (also normalized in a box of volume V), namely,

$$(11) \quad \vec{A}_{u, \omega} = \left(\frac{2\pi}{V} \frac{\hbar^2 c^2}{\hbar \omega} \right)^{\frac{1}{2}} \vec{u} \exp(-i\hbar \vec{\omega} \cdot \vec{r} / \hbar c).$$

Since H_1 is linear in $\vec{A}$, the constant coefficients may be extracted from the matrix element. This gives

$$(12) \quad dP_{if} = \frac{1}{2\pi\hbar} \frac{\hbar\omega}{\hbar c} |\langle i | Q | f \rangle|^2 d\Omega,$$

where Q is the operator

$$(13) \quad Q = H_1 \{ \vec{u} \exp(-i\hbar \vec{\omega} \cdot \vec{r} / \hbar c) \}.$$

The exponent is now expanded in powers of $\vec{\omega} \cdot \vec{r} / c$,^{*} and use is made of the assumed gauge invariance of the Hamiltonian to arrive at

$$(14) \quad Q = \sum_{\ell=1}^{\infty} \left(\frac{i\omega}{c} \right)^{\ell} (D_{\ell} + M_{\ell}),$$

where

$$(15) \quad D_{\ell} = \frac{1}{\ell!} \sum_j e_j (\vec{u} \cdot \vec{r}_j) (\vec{\omega} \cdot \vec{r}_j / \omega)^{\ell-1}$$

and

$$M_{\ell} = \frac{-\ell}{(\ell+1)!} H_1 \{ [(\vec{u} \times \vec{\omega} / \omega) \times \vec{r}] (\vec{\omega} \cdot \vec{r} / \omega)^{\ell-1} \}.$$

It can now be shown that D_{ℓ} is an element of a symmetric tensor of rank ℓ and parity $(-1)^{\ell}$, and that M_{ℓ} is an element of a tensor of rank ℓ but with parity $(-1)^{\ell+1}$. Thus D_{ℓ} and M_{ℓ} transform under rotations

^{*} Note $\vec{\omega} \cdot \vec{r} / c \leq r / \lambda$. The coordinates $\{r\}$ of the charged particles in the system are assumed to be much smaller than λ , the wavelength of the emitted radiation.

like linear combinations of spherical harmonics of order ℓ and lower. Since states i and f have definite spins J_i and J_f , and definite parities $\pi_i \pi_f = \pm 1$, the matrix elements of D_ℓ and M_ℓ between these states will vanish unless

$$(16) \quad |J_i - J_f| \leq \ell \text{ and } \pi_i \pi_f = (-1)^\ell, \quad \text{Electric } 2^\ell\text{-pole}$$

$$(17) \quad |J_i - J_f| \leq \ell \text{ and } \pi_i \pi_f = (-1)^{\ell+1}. \quad \text{Magnetic } 2^\ell\text{-pole}$$

Because of the rapid convergence of the expansion of the exponential in Eq. (13), only the leading term consistent with the above selection rules is expected to contribute for a given change in spin and parity. Thus, substituting Eqs. (14) and (15) into (12) gives

$$(18) \quad dP_{if,\ell}(\text{elect.}) = \frac{1}{2\pi\hbar} \left(\frac{\hbar\omega}{\hbar c} \right)^{2\ell+1} |\langle i | D_\ell | f \rangle|^2 d\Omega$$

if

$$\Delta J = \ell \text{ and } \pi_i \pi_f = (-1)^\ell;$$

and

$$(19) \quad dP_{if,\ell}(\text{mag.}) = \frac{1}{2\pi\hbar} \left(\frac{\hbar\omega}{\hbar c} \right)^{2\ell+1} |\langle i | M_\ell | f \rangle|^2 d\Omega$$

if

$$\Delta J = \ell \text{ and } \pi_i \pi_f = (-1)^{\ell+1}.$$

Of course, there is no $\ell = 0$ radiation so dipole radiation is predominant if $\Delta J = 0$. Gamma-ray transitions between two $J = 0$ states are strictly forbidden.

Thus the dependence of the 2^ℓ -pole transition probabilities on the $2\ell + 1$ power of the gamma-ray energy is seen to be due to what are essentially phase-space factors. Additional energy dependences arising from the wave functions of the initial and final states are, of course, not excluded.

It is significant to note that the electric-multipole transition probabilities do not depend on the precise form of the interaction Hamiltonian, but only on its gauge invariance, whereas this is not the case for the magnetic-multipole transition probabilities.

In this approach the detailed dependence of the transition probabilities on the multipole order would arise from an explicit expression for the operators D_ℓ and M_ℓ in terms of the spherical harmonics. The Wigner-Eckhart theorem can then be used²⁰ to separate the geometric factors from quantities that depend on detailed properties of the nucleus. The latter quantities are contained in the reduced matrix element, which must be estimated by one means or another to permit quantitative comparison between the theory and experiment.

In the approach of Weisskopf, the dependence on the multipole order is made explicit from the beginning. The electric field is assumed to have the form

$$(20) \quad \vec{\mathcal{E}}(\vec{r}, t) = \vec{\mathcal{E}}(\vec{r}) e^{-i\omega t} + \text{c. c.},$$

where c. c. stands for the complex conjugate, and a similar expression is assumed for the magnetic field $\vec{H}(\vec{r}, t)$. The time-independent part expanded in multipoles is then

$$(21) \quad \vec{\mathcal{E}}(\vec{r}) = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} \left[a_E(\ell, m) \vec{\mathcal{E}}_E(\ell, m; r) + a_M(\ell, m) \vec{\mathcal{E}}_M(\ell, m; r) \right],$$

where $\mathcal{E}_E(\ell, m; r)$ has parity $(-1)^\ell$ and $\mathcal{E}_M(\ell, m; r)$ has parity $(-1)^{\ell+1}$. Requiring that the multipole fields $\mathcal{E}_E(\ell, m; r)$, etc., satisfy the free-space Maxwell equations, and that they also be eigenfunctions of the total angular momentum and of its z component, with eigenvalues $\ell(\ell+1)$ and m, respectively, is sufficient to determine explicit

expressions. The amplitudes $a_E(\ell, m)$ and $a_M(\ell, m)$ of the multipole field are determined by assuming a source of varying charge density

$$(22) \quad \rho(r, t) = \rho(r) e^{-i\omega t} + \text{c. c.},$$

a source of varying current density

$$(23) \quad \vec{j}(r, t) = \vec{j}(r) e^{-i\omega t} + \text{c. c.},$$

and a source of varying magnetization density

$$(24) \quad \vec{M}(r, t) = \vec{M}(r) e^{-i\omega t} + \text{c. c.}$$

to take into account the effect of the intrinsic spins of the radiating particles. The amplitudes are then found by demanding that $\vec{E}(r, t)$ and $\vec{H}(r, t)$ satisfy Maxwell's equations with these sources. The asymptotic solutions of the radial part of the equation are used. The energy emitted per second is then found for pure multipole radiation by computing the Poynting vector for the appropriate field components. It is found convenient to use

$$(25) \quad \langle |S_{\ell m}| \rangle_{\text{time av}} = \frac{c}{2\pi} |H_{\ell m}(r)|^2 = \frac{c}{2\pi} |a_E(\ell, m)|^2 |H_E(\ell, m; r)|^2$$

for the electric 2^ℓ -pole radiation with a similar expression for the magnetic 2^ℓ -pole radiation. The total energy emitted per second is then given by the integral of these quantities over a sphere. The transition to quantum mechanics is made in two steps. First, since the radiation is emitted in quanta of energy $\hbar\omega$ rather than continuously, one obtains the number of quanta emitted per second as the total energy emitted per second divided by the energy per quantum. Finally, the quantum mechanical expressions for the source terms must be used, namely,

$$(26) \quad \rho(r) = \sum_{n=1}^Z e_n \phi_f^* \phi_i$$

$$(27) \quad \vec{j}(r) = \sum_{n=1}^Z \frac{e_n}{2M_n} [\phi_f^* \vec{p}_n \phi_i + (\vec{p}_n \phi_f)^* \phi_i], \quad \vec{p} = \frac{\hbar}{i} \vec{\nabla}$$

$$(28) \quad \vec{M}(r) = \sum_{n=1}^A \frac{e_n}{2M_n c} \mu_n \{ \phi_f^* \vec{\sigma}_n \phi_i \},$$

where in Eq. (28) μ_n is the magnetic moment of the radiating particle in Bohr magnetons, $\vec{\sigma}_n$ is the Pauli spin operator, and the curly brackets signify a sum over the two spin orientations without an integration over $\vec{r}$. The final results obtained by performing all of the above indicated operations may be written

$$(29) \quad T_{if}^{(\ell, m)}{}_E = \frac{8\pi(\ell+1)}{\ell[(2\ell+1)!!]^2} \frac{k^{2\ell+1}}{\hbar} |Q_{\ell m} + Q'_{\ell m}|^2,$$

$$(30) \quad T_{jf}^{(\ell, m)}{}_M = \frac{8\pi(\ell+1)}{\ell[(2\ell+1)!!]^2} \frac{k^{2\ell+1}}{\hbar} |M_{\ell m} + M'_{\ell m}|^2,$$

where the matrix elements are given by

$$(31) \quad Q_{\ell m} = e \sum_{n=1}^Z \int r_n^\ell Y_{\ell m}^*(\theta_n, \phi_n) \phi_f^* \phi_i d\tau,$$

$$(32) \quad Q'_{\ell m} = \frac{-ik}{\ell+1} \frac{e\hbar}{2Mc} \sum_{n=1}^A \mu_n \int r_n^\ell Y_{\ell m}^*(\theta_n, \phi_n) \text{div}(\phi_f^* \vec{r}_n \times \vec{\sigma}_n \phi_i) d\tau,$$

$$(33) \quad M_{\ell m} = \frac{1}{\ell+1} \frac{e\hbar}{Mc} \sum_{n=1}^Z \int r_n^\ell Y_{\ell m}^*(\theta_n, \phi_n) \text{div}(\phi_f^* \vec{L}_n \phi_i) d\tau, \quad \vec{L}_n = -i\vec{r}_n \times \vec{p}_n,$$

$$(34) \quad M'_{\ell m} = \frac{e\hbar}{2Mc} \sum_{n=1}^A \mu_n \int r_n^\ell Y_{\ell m}^*(\theta_n, \phi_n) \text{div}(\phi_f^* \vec{\sigma}_n \phi_i) d\tau.$$

To obtain the transition probabilities $\Gamma_{\gamma i}/\hbar$, one must sum these expressions over the polarizations and ordinarily average over initial spins.

To proceed further, various estimates are made of the various matrix elements. For example, from Eqs. (31) and (32), on the assumption that the divergence operator may approximately be replaced by $1/r$ and that $|\vec{M}| \approx e\hbar/Mc$, it follows that

$$(35) \quad \frac{Q'_{\ell m}}{Q_{\ell m}} \approx \frac{\hbar\omega}{Mc^2(\ell+1)}$$

so that $Q'_{\ell m}$ may be neglected in what follows. (It is, of course, not surprising that spin effects give negligible contributions to electric-multipole radiation.)

The so-called single-particle estimate can now be obtained by evaluating $Q_{\ell m}$ for a simple independent-particle model. Since such a model describes each state by the collection of quantum numbers of the individual nucleons, and since the operators in Eqs. (26)–(28) depend on the coordinates of a single nucleon at a time, all of the matrix elements (31)–(34) vanish for orthogonality reasons unless states i and f differ only in the excitation of a single nucleon. Thus, only a single term in the sums need be considered. If it is assumed that $\ell=0$ in the final state and that no spin flip is involved, and if properly normalized wave functions are used, the result is

$$(36) \quad Q_{\ell m} = \frac{e}{\sqrt{4\pi}} \int_0^\infty r^\ell u_a(r) u_b(r) r^2 dr,$$

where u_a and u_b are the normalized radial wave functions. It is now assumed that u_a and u_b are constants, equal to $(3/R^3)^{1/2}$ for $r < R$, and zero outside this range. For this case, Eq. (36) becomes

$$(37) \quad Q_{\ell m} \approx \frac{3e}{\sqrt{4\pi}} \frac{R^\ell}{\ell+3}.$$

For transitions between neighboring levels, this result is expected to be valid within about an order of magnitude. However, drastic errors may occur if the levels differ greatly in excitation. To approximately account for the gross dissimilarity in u_a and u_b in this case, a qualitative argument is then given to show that the average value of $|Q_{\ell m}|^2$ is expected to be inversely proportional to the density of levels at the energy of the state of higher excitation, i.e.,

$$(38) \quad \langle |Q_{\ell m}|^2 \rangle_{av} = (q_{\ell m})^2 \rho^{-1}(E_{upper}).$$

This assertion is well supported, both theoretically²³ and experimentally²⁴ for levels of fairly high excitation. Since the same dependence occurs naturally in Axel's approach,²² to be discussed next, the qualitative argument of Weisskopf will not be repeated here. Now, if Eq. (38) holds at all excitations, then for low-lying levels for which (37) is approximately valid the quantity $(q_{\ell m})^2$ is given by

$$(39) \quad (q_{\ell m})^2 = \rho(E_{low}) \frac{e^2}{4\pi} \left(\frac{3}{\ell+3} \right)^2 R^{2\ell},$$

where $\rho(E_{low})$ is the density of low-lying levels between which transitions may occur by electric 2^ℓ -pole radiation (combining levels). Then for all excitations Eq. (38) gives

$$(40) \quad \langle |Q_{\ell m}|^2 \rangle_{av} = \frac{e^2}{4\pi} \left(\frac{3}{\ell+3} \right)^2 R^{2\ell} \frac{\rho(E_{low})}{\rho(E_{upper})}.$$

Finally it is assumed that the sum over polarizations can be approximated by multiplying this result by $(2\ell + 1)$. Then substituting Eq. (40) into (31) gives

$$(41) \quad \langle \Gamma_Y(E, \ell) \rangle_{av} = \frac{18(\ell+1)(2\ell+1)}{\ell(\ell+3)^2 [(2\ell+1)!!]^2} e^2 R^{2\ell} \left(\frac{\hbar\omega}{\hbar c} \right)^{2\ell+1} \frac{\rho(E_{low})}{\rho(E_{upper})}.$$

An estimate similar to the type used to derive Eq. (35) shows that the magnetic 2^ℓ -pole transition probability is of the same form as (41) but is reduced by a factor of about $10(\hbar/McR)^2$. Inserting numbers into (41) and the corresponding expression for $\langle \Gamma_Y(M, \ell) \rangle$ gives

$$(42a) \quad \Gamma_{\gamma i}(E1) = 2.0 \times 10^{-1} A^{2/3} E_{\gamma i}^3 D(E)/D_0,$$

$$(42b) \quad \Gamma_{\gamma i}(E2) = 2.4 \times 10^{-7} A^{4/3} E_{\gamma i}^5 D(E)/D_0,$$

$$(42c) \quad \Gamma_{\gamma i}(M1) = 6.3 \times 10^{-2} E_{\gamma i}^3 D(E)/D_0,$$

$$(42d) \quad \Gamma_{\gamma i}(M2) = 7.5 \times 10^{-8} A^{2/3} E_{\gamma i}^5 D(E)/D_0,$$

etc., where $E_{\gamma i}$ is in MeV, $\Gamma_{\gamma i}$ is in eV, and the value used for the nuclear radius is $R = 1.2 A^{1/3}$ F. The level densities have been replaced by their reciprocals, the level spacings. Using this notation, $D(E)$ is the spacing (in the vicinity of the radiating state) of levels which have the same spin and parity as the initial state, and D_0 is the spacing (near the final state) of levels that combine with the final state by transitions of the particular multipolarity considered.

Transition probabilities observed experimentally are frequently expressed in Weisskopf units defined as the ratio of the

experimental value to the numerical estimate given in Eqs. (42). To make this a significant procedure, it would be desirable to have some a priori value for D_0 , the spacing of low-lying combining levels. Weisskopf's original value was 0.5 MeV, the assumption being that actual low-lying nuclear levels could be approximated by single-particle wave functions. The experimental data are best fitted, however, with a value of D_0 some 10 to 100 times this big. It has become the custom, therefore, to speak of D_0 as the spacing of single-particle levels, which is of the correct order of magnitude. It should be emphasized, however, that this is a purely arbitrary device which has as yet received no theoretical support. The dependence on the level density in Weisskopf's original argument arises from considerations of the degeneracy of single-particle levels, and is not in any way connected with their spacing. In any case, it is clear that the difficulties that arise in the above theory are of a more fundamental nature and not of a type that may be corrected by multiplying by some normalizing constant.

The principal deficiency in the Weisskopf theory is its failure to take account of correlations in the nucleon coordinates in evaluating the matrix element $Q_{\ell m}$. In the single-particle model the nucleons are assumed to be completely uncorrelated. It is known that this is not the case in actual nuclei even at fairly high excitations. In the case of complete correlation between the neutron and proton coordinates (for example, if the nucleus consists entirely of clusters—all of which have the same mass and charge) the center of charge is fixed at the center of mass and there can be no electric-dipole radiation at all. The actual nucleus is no doubt intermediate between these two extreme cases. This point was in fact emphasized by Weisskopf, who even used some estimates of the influence of excitation on the correlations to give a qualitative description of the photonuclear giant dipole resonance.

(It is currently believed that the giant dipole resonance arises from extreme anticorrelation between the coordinates of the neutrons and protons.)

These considerations have been put on a quantitative basis in a new approach due to Axel. First a relation will be derived between the transition probabilities and the average cross section for the inverse capture process, i.e., nuclear photodisintegration. This cross section is, of course, intimately related to $|Q_{\ell m}|^2$ by the reciprocity law. However, instead of attempting to evaluate the effect of correlations between nucleon coordinates and the influence of excitation energy on this effect, use is made of an empirical law for the photon absorption cross section. This is the classical Lorentz shape, which is known²⁵ to accurately fit the experimentally observed dependence of the photon absorption cross section on the excitation energy. An accurate account of the effects of nucleon correlations would, of course, have to reproduce this result. Thus the final expression for the transition probabilities implicitly includes the effects of nucleon correlations without explicitly considering the matrix elements.

Axel begins by writing down a dispersion-type formula for the cross section for absorption of monoenergetic photons into an isolated quasi-stationary state in the compound nucleus. His expression is

$$(43) \quad \sigma_a = 4\pi\lambda^2 g \frac{\Gamma_{\gamma i}}{\Gamma} \frac{1}{1 + [(2/\Gamma)(E_\gamma - E_0)]^2},$$

where λ is the photon wavelength $\hbar c/E_\gamma$, E_0 is the energy of the single excited state that is assumed to dominate the absorption of photons of the particular energy E_γ , and g is the appropriate statistical factor, which in this case is

$$(44) \quad g = (2J + 1) / [(2I + 1)(2\ell)],$$

where J is the spin of the excited state and I is the spin of the initial ground state. The average absorption cross section for a beam of photons with an energy spread ΔE which includes N excited states is obtained by integrating a sum of N terms of the form (43) over ΔE . The result is

$$(45) \quad \langle \sigma_a \rangle = \frac{2\pi^2 \hbar^2 c^2}{E_\gamma^2} g \frac{\langle \Gamma_{\gamma i} \rangle}{D},$$

where

$$(46) \quad D = \Delta E / N$$

and

$$(47) \quad \langle \Gamma_{\gamma i} \rangle = \frac{1}{N} \sum_{i=1}^N \Gamma_{\gamma i}.$$

This expression is still valid when account is taken of the Doppler broadening of the excited states as a result of the thermal motion of the target nuclei. The expression for the ground-state transition probability is thus

$$(48) \quad \frac{\langle \Gamma_{\gamma 0} \rangle}{D} = \frac{1}{2\pi^2 \hbar^2 c^2 g} E_\gamma^2 \langle \sigma_a \rangle.$$

Actually, this expression is of general validity and is not dependent on a particular form, such as Eq. (43), for the absorption cross section for monoenergetic photons. It may be

derived from the theorem of detailed balance.* The average photon absorption cross section may, of course, be expressed in terms of the matrix element $Q_{\ell m}$ defined in Eq. (31). The expression is

$$(49) \quad \langle \sigma_a \rangle_{av} = \frac{8\pi^3 (\ell+1)(2\ell+1)}{\ell [(2\ell+1)!!]^2} \left(\frac{E_\gamma}{\hbar c} \right)^{2\ell-1} g \frac{\langle |Q_{\ell,1}|^2 \rangle_{av}}{D_\ell(E_\gamma)},$$

where $D_\ell(E_\gamma)$ is the spacing of levels that may be excited by absorption of electric 2^ℓ -pole radiation of energy E_γ . Now, based on the previous discussion of the effect of correlations of the value of $|Q_{\ell m}|^2$, we may, following Weisskopf,¹⁷ make the qualitative prediction that for electric-dipole absorption there is a general increase in $\langle \sigma_a \rangle_{av}$ because of the factor E_γ . However, for low excitations there are expected to be strong correlations between the coordinates of the neutrons and protons so that $|Q_{1,1}|^2$ is very small. As the excitation increases, an occasional neutron may be expected to become on the average more separated from the protons, and/or an occasional proton may become on the average more separated from the neutrons. Either separation would allow an increase in the asymmetry of the charge distribution. The quantity $|Q_{1,1}|^2$ therefore becomes larger and $\langle \sigma_a \rangle_{av}$ increases faster than linearly with E_γ . This effect continues until, at the giant dipole resonance, the charge distribution becomes as asymmetric as possible in the final state and $|Q_{1,1}|^2$, and thus $\langle \sigma_a \rangle_{av}$, reaches its maximum value. This represents the extreme anticorrelation between the

* See, for example, chapter XII and appendix B of reference 17. This reference considers the absorption of polarized photons into a spinless nucleus. The statistical factor in Eq. (44) arises from consideration of these factors. The quantity 2ℓ appears in g rather than the usual $(2\ell+1)$ because the magnetic substate $m=0$, which corresponds to longitudinal polarization, is not allowed for the photons.

coordinates of neutrons and protons in the sense in which the term "correlation" is used in this discussion. Above the giant dipole resonance $\langle \sigma_a \rangle_{av}$ decreases for a number of reasons. First, the degree of anticorrelation between nucleon coordinates decreases, the maximum having been reached. Also, near the giant dipole resonance the long-wavelength limit ceases to be a good approximation. (For $E_\gamma = 19.7$ MeV, $\lambda = 10$ F, a value greater than most nuclear radii.) In this case this entire treatment breaks down and the significance of considering pure multipole fields becomes dubious. In any case, it is clear that for excitations on the order of or below the giant dipole resonance, $\langle \sigma_a \rangle_{av}$ follows $|Q_{1,1}|^2$ and that a formula closely approximating $\langle \sigma_a \rangle_{av}$ as a function of gamma-ray energy is an equally good description of the way in which the correlation between the coordinates of neutrons and protons affects the magnitude of $|Q_{1,1}|^2$.

Now it is known²⁵ that the photon-energy dependence of the giant dipole resonance is quite accurately given by a classical Lorentz line, at least for energies a few MeV above the neutron binding energy. The behavior of the electric-dipole absorption cross section near the (γ, n) threshold is unfortunately masked by strong magnetic-dipole and electric-quadrupole absorption. However, Axel now makes the reasonable assumption that the Lorentz shape gives the electric-dipole part of the absorption cross section everywhere. That is, he assumes that

$$(50) \quad \langle \sigma_a \rangle_{av} = \left(\frac{0.013 \text{ A}}{\Gamma_g} \right) \frac{E_\gamma^2 \Gamma_g^2}{(E_g^2 - E_\gamma^2)^2 + E_\gamma^2 \Gamma_g^2} \text{ (in barns)},$$

where Γ_g is the full width at half maximum and E_g is the peak energy of the giant dipole resonance. The factor $(0.013 \text{ A}/\Gamma_g)$ is the peak cross section required to satisfy the dipole sum rule²⁶

$$(51) \quad \int_0^{\infty} \sigma_a dE_{\gamma} = 0.021 A,$$

where A is the atomic mass number, σ_a is to be expressed in barns, and E_{γ} is in MeV. Combining Eq. (50) with (48) then gives

$$(52a) \quad \frac{\langle \Gamma_{\gamma 0} \rangle}{D} = \frac{0.013 A}{2\pi^2 \hbar^2 c^2 g} \frac{E_{\gamma}^4 \Gamma_g}{(E_g^2 - E_{\gamma}^2)^2 + E_{\gamma}^2 \Gamma_g^2},$$

which is the desired expression. For numerical purposes,

$$(52b) \quad \frac{\langle \Gamma_{\gamma 0} \rangle}{D} = 1.7 \times 10^{-6} \frac{A}{g} \frac{E_{\gamma}^4 \Gamma_g}{(E_g^2 - E_{\gamma}^2)^2 + E_{\gamma}^2 \Gamma_g^2},$$

where E_{γ} , E_g , and Γ_g are in MeV. Equations (49) and (50) may be combined to express the electric-dipole matrix element explicitly as

$$(53) \quad \frac{\langle |Q_{1,1}|^2 \rangle_{av}}{D(E)} = \frac{(0.013 A)}{g} \frac{3\hbar c}{16\pi^3} \frac{E_{\gamma} \Gamma_g}{(E_g^2 - E_{\gamma}^2)^2 + E_{\gamma}^2 \Gamma_g^2},$$

where $D(E)$ is also in MeV and $\hbar c = 19.7 \text{ MeV-barn}^{1/2}$.

Figure 1 shows a comparison of the variation with excitation energy of $(q_{\ell m})^2$, defined in Eqs. (38) and (40) for the single-particle model, with the same quantity defined by (53). Each is normalized so that it satisfies the sum rule (51). This normalization is achieved for the single-particle estimate by arbitrarily assuming $(q_{\ell m})^2$ cuts off at some energy, a procedure which is required in order for the sum rule to converge at all.

Fig. 1. Comparison of the square of the electric-dipole matrix element calculated from the single-particle estimate (dashed lines) and an assumed Lorentz shape for the giant dipole resonance (solid line).

Two values of D_0 were assumed: the value of 0.5 MeV originally proposed by Weisskopf, and 10 MeV, which is the empirical value currently used to force agreement between the single-particle estimate and experiment. The curves are calculated for a nucleus with $A = 190$ and $R = 1.2 A^{1/3}$. The giant-dipole-resonance parameters used were $E_g = 15$ MeV and $\Gamma_g = 5$ MeV. It was assumed that $g = 3/2$, corresponding to a spin-zero ground state. Clearly the single-particle matrix element calculated with $D_0 = 0.5$ MeV is much too large. Further, in order for this matrix element to satisfy the sum rule, it is necessary to require that the dipole matrix element should vanish in the region of the peak of the giant dipole resonance. The situation is much improved by using $D_0 = 10$ MeV. Notice that in the region between 8 and 10 MeV the single-particle estimate in this case is in reasonable agreement with the calculation for the giant dipole resonance. Since the value $D_0 = 10$ MeV was chosen to make the single-particle estimate agree with experimental values of transition probabilities for transitions of about this energy, the giant-dipole-resonance calculation is expected to agree well with experiments. Further, the giant-dipole-resonance calculation involves no adjustable parameters, such as D_0 , so that agreement with experiment may be considered a convincing argument that Eq. (53) is a good approximation for the dipole matrix element at excitations near the neutron binding energy.

(c) Porter-Thomas Distribution

The theoretical discussion will conclude with a brief discussion of the second source of inherent uncertainty in the values of transition probabilities obtained from thermal-neutron-capture experiments. This is important since this is also the major source of uncertainty in the resonance-average values reported here.

The problem under discussion is the distribution of transition probabilities for transitions to a particular final state as sampled from resonance to resonance in the compound nucleus. The discussion will closely follow that of Porter and Thomas.²⁷

It is to be expected that the wave function of the compound nucleus, representing (as it does) a state of some 8 MeV excitation, will be extremely complex. Then the matrix element involved in the calculation of any of the partial widths for the decay of the compound nucleus will be an integral over configuration space of a wildly fluctuating function of the variables of integration. We now assume that the configuration space can be divided into cells, small enough that there will be a large number of the cells within the volume of configuration space corresponding to the compound nucleus, but large enough that there will be several alternations of the sign of the integrand within the volume of each cell. In this case a cell is equally likely to make a positive or a negative contribution to the matrix element, regardless of the contribution from each other cell. If this is true, and it is the only requisite for what follows, then it follows from the "central limit" theorem of statistics that the distribution of the sum (i.e., the transition probability amplitudes) will be a normal distribution with zero mean.²⁸ The squares of the matrix elements, which are proportional to the partial radiation widths, will then be distributed as a chi-squared distribution with one degree of freedom²⁸ (the Porter-Thomas distribution) in the variable $x = \Gamma_{\gamma i} / \langle \Gamma_{\gamma i} \rangle$, where $\langle \Gamma_{\gamma i} \rangle$ is

the mean value that would be obtained by averaging over a very large number of resonances of the same type. The form of this distribution is

$$(54) \quad P(x)dx = (2\pi x)^{-\frac{1}{2}} \exp(-x/2)dx.$$

It is such that a single partial radiation width selected at random from the distribution has a probability 0.1 of being 62.5 times smaller than the mean, or less, and the same probability of being more than 2.7 times the mean. The 90% confidence limits thus include a range of variation of more than two orders of magnitude. Clearly the true mean value is determined only very poorly by the measurement of a transition probability in a single resonance. The average of a particular partial width over n resonances has the distribution of a sum of the squares of n variables, each of which has a Gaussian distribution with zero mean and a constant variance. The distribution of such a quantity is a chi-squared distribution with n degrees of freedom, ²⁸ i.e.,

$$(55) \quad P_n(x)dx = \Gamma(\frac{1}{2}n)^{-1} (nx/2)^{(n/2)-1} \times \exp(-nx/2) d(nx/2).$$

Unfortunately this is still a rather broad distribution. For n greater than 30 it approaches a Gaussian distribution with standard deviation approximately equal to $(2n-1)$. The two 90% confidence limits for the mean determined as an average taken over n resonances are plotted as a function of n in Fig. 2. For a typical nuclide investigated in this work, it has been possible to examine the

Fig. 2. The 90% confidence factors, defined by the equations appearing in the insert above, for the chi-squared distribution with n degrees of freedom for n between 1 and 30.

spectra in 8-10 resonances. Therefore the true mean value of the transition probabilities has been determined only within an uncertainty of about $\pm 60\%$. This, however, is a considerable improvement over the orders-of-magnitude uncertainty inherent in the thermal-capture results. Of course, if more than one resonance contributes to the thermal capture, the Porter-Thomas fluctuations will be reduced. The distribution in this case is complicated by the fact that the result is a weighted average, the weights being the fraction of the total thermal capture contributed by each resonance. However, in a recent survey of experimental values of the partial radiation widths of the transitions following thermal capture, Bartholomew⁵ has shown that the collection of all values reported before September 1961 is consistent with the chi-squared distribution for $n = 2$, but that the results for cases in which the thermal capture is dominated by a single resonance are consistent with the Porter-Thomas distribution. For intermediate and light nuclei having no resonances at low neutron energies, it would seem that the thermal values might be quite good. In these cases the thermal-capture contributions from several resonances are important. The interference terms [Eq. (7)], being randomly positive and negative for the different contributing resonances, will tend to cancel. This is all very fortunate since these are just the cases in which resonance-average values of the partial radiation widths are not, and will not be, available. A few thermal-capture results of this type will be included in Figs. 31-34 of Sec. VI, where the theory presented in part(b) of this section is compared with experiment.

II. THE EXPERIMENT

The experimental facility consists of three parts: a source of timed neutrons, a gamma-ray detector, and a gamma-ray pulse-height analyzer. The neutron source consisted of the Argonne fast chopper operating at the Argonne research reactor CP-5 with a neutron flight path of 25 m. The fast chopper has been described in detail elsewhere.²⁹ Rotor III, designed to give a high intensity, was used. The over-all time-of-flight resolution width was 0.1 $\mu\text{sec/m}$ when the chopper was operated at 15 000 rpm, and 0.25 $\mu\text{sec/m}$ at 5000 rpm. Bursts of neutrons formed by the fast chopper drift a distance of 25 m and are incident on a sample of the material under investigation. The samples were rectangular in form and were inclined at an angle of approximately 20° with respect to the beam direction. The resolution widths quoted above include the uncertainty that the depth of the sample introduces into the time of flight.

The gamma-ray detector consisted of a NaI(Tl) crystal, 8 in. in diameter and 6 in. long, mounted on a 12-in. EMI 9545 A photomultiplier tube. The detector was located in a well-shielded room outside the reactor room, the wall facing the reactor and the ceiling consisting of one foot of heavy concrete. In the early part of the experiment, the detector was additionally shielded from stray radiation from the reactor by 4 in. of lead and from neutrons scattered by the sample by a 3-in. thickness of B_4C in a can. Later, the can of B_4C was replaced by a slab of B_4C , 4 in. thick, suspended in paraffin. In the final stage of the experiment, the entire crystal-photomultiplier assembly was encased in a tight-fitting lead shield 2 in. thick, open at the front; and this in turn was surrounded by 2 in. of B_4C . The effectiveness of the different types of shielding will be discussed in the next section.

Two different systems were used during the experiment to record the pulse-height spectra. In the early part of the experiment, a four-spectrum analyzer, of the type described by Carpenter and Bollinger,¹¹ was used. A schematic diagram of this arrangement is shown in Fig. 3. Components connected by dashed lines are included only for the observation of coincidence spectra. Pulses from the large crystal, which was used to study the high-energy region of the gamma-ray spectra in all cases, are amplified and fed into the pulse-height-to-time converter. The converter is triggered when certain arbitrary conditions on the pulse heights from one or both crystals are satisfied. This permits the associated amplified pulse to be transformed into a pair of output pulses ("start" and "stop"), for which the difference in time is proportional to the pulse height of the input pulse. The "start" pulse is now required to pass through a gate which may impose a second set of conditions on the scintillation pulse heights. If these conditions are met, the "start" pulse is used to trigger a 1024-channel time analyzer, split into 4 groups of 256 channels each, arranged so that the addresses of these four groups advance in parallel. The "start" pulse is also used as the detector input of a 100-channel time analyzer, which is triggered by a photocell pulse just before the burst. Thus the spectrum of all gamma rays whose pulse heights satisfy whatever restrictions are imposed is recorded (as a function of the neutron time of flight) by the 100-channel analyzer. In addition, if a count is recorded in one of four selected ranges of channels on the 100-channel analyzer, a pulse is produced which opens a corresponding gate in the time-gate sorter through which the "stop" pulse may pass into the corresponding group of 256 channels in the 1024-channel analyzer. In this way four gamma-ray spectra, corresponding to four selected neutron energy intervals, are recorded simultaneously. Examples of the data taken with this system will be shown in Secs. III and IV.

Fig. 3. Experimental arrangement for observing neutron-resonance-capture gamma rays by use of the 4-spectrum analyzer. The dashed lines connect components that are employed only for observing the resonance-capture spectra of coincident gamma rays.

The principal deficiency of the four-spectrum analyzer is that the gamma-ray spectra may be recorded in only four neutron energy intervals simultaneously. As will be seen in Sec. III, it is usually necessary to record a background spectrum on both sides of the resonance or resonances being investigated. Thus the gamma-ray spectra may usually be examined in only two resonances at a time. Besides requiring that a large number of runs be made on each sample, this limitation also restricts the precision with which the data may be analyzed. This will be discussed in detail in Sec. III.

Approximately midway through the course of the experiment, the system described above was replaced by a new data-recording system which permits the simultaneous recording of the gamma-ray spectra in each of 256 channels. This system, the Argonne three-parameter analyzer,¹⁰ so reduced the running time and increased the precision of the data analysis that nearly all of the earlier work was repeated with the new system.

A schematic diagram of the experimental arrangement employing the three-parameter analyzer is shown in Fig. 4. Except for the manner of recording the data, the system is identical with the system previously discussed. The detector is shown encased in the lead shield, although much of the data were taken with the same shielding as shown in Fig. 3. With this system the scintillation pulses are fed directly into the analyzer. There they are amplified and converted into eight-bit binary numbers which correspond to channel numbers 0-255. In this case, a clock in the analyzer is started by the photocell pulse just before the rotor slits open and is stopped by the trigger pulse which, as before, is formed when certain arbitrary conditions on the pulse heights are met. After the arrival of the photocell pulse, the starting of the clock may be delayed (in steps of 64 channels) up to 1024 channels. When a trigger pulse is received, the digital numbers corresponding

Fig. 4. Experimental arrangement for observing neutron-resonance-capture gamma rays by use of the 3-parameter analyzer. Only two parameters are recorded for "singles" runs of the type described in this report.

to the scintillation pulse height and the reading of the clock (a crystal-controlled scaler) are recorded on magnetic tape. The tape recorder is so arranged that it advances only when an event is recorded.

Consequently the events are packed with a uniform density, which is about 120 events per inch. Thus a full tape, which is 3600 ft long, contains some 5 million events. These tapes are then removed to a separate search station for analysis. The tapes are passed through the tape reader in the search station at 150 in. per second so that a single pass requires about 5 min. During each pass, four spectra of one of the parameters may be stored in ferrite-core memories. Each spectrum is associated with an arbitrary set of conditions on the other parameter. All four sections of the memory may be read out in the form of an xy plot, typewritten table, punched paper tape, or any combination of these in about 20 additional minutes. Thus, once the data tape is obtained, a few hours suffice to obtain the pulse-height spectra associated with all desired neutron energy intervals included in the 256 neutron channels. Examples of the data obtained with this system will be shown in Secs. III and IV.

As its name implies, the 3-parameter analyzer is more versatile than it needed to be for the present experiment. An additional parameter (for example, the pulse height of a coincident gamma ray) may also be stored on the magnetic tape. Also, a third analogue quantity (for example, the pulse height of a third coincident gamma ray) can be stored instead of the digital reading from the clock scaler.

III. DATA ANALYSIS

This section is a detailed consideration of the general problems that are encountered in the analysis of the data to obtain absolute values of the partial radiation widths. For illustrative purposes, the actual data obtained for capture in the resonances in neodymium will be treated. Specific problems that arise in particular cases for other nuclei will be dealt with as they come up in Sec. IV.

There are usually four steps in the analysis. First, the experimental spectra must be corrected for background. A consideration of the sources of the background, the methods used to correct for it, and evidence for the validity of these methods will be discussed in part (a). Second, for each neutron resonance the intensities of the gamma-ray transitions that contribute to the spectral region being considered must be determined relative to the intensity of an arbitrarily chosen standard line. This is done by a least-squares fitting of the data to a linear superposition of appropriate response functions. The technique used to obtain the response functions and the details of the least-squares fitting are described in part (b). Third, the relative intensities are converted into quantities proportional to the number of photons emitted per neutron capture by dividing by a quantity which is proportional to the total number of neutron captures in the energy interval considered. One such a quantity is the total number of detected gamma rays. The justification for this statement and the techniques used when more than one resonance contributes to the neutron energy interval considered are contained in part (c). Finally, the conversion of these normalized relative intensities into absolute intensities requires that the intensity of at least one gamma-ray transition in

one neutron resonance in the nuclide in question should be determined relative to the intensity of a gamma-ray transition of known absolute intensity. The details of this part of the experiment are included in part (d) of this section. Once the absolute intensities are obtained, the partial widths can be found as the products of the absolute intensities and the total radiation width Γ_γ . Part (e) is a discussion of the uncertainty in the final values of $\langle \Gamma_{\gamma i} \rangle$ from all sources.

(a) Background Correction

Since the three-parameter analyzer provides gamma-ray spectra for a wide continuous range of neutron energies, a spectrum of all detected gamma rays versus neutron energy will usually include a number of off-resonance regions where the intensity of gamma rays from resonance capture is negligibly small. The gamma-ray spectra in these neutron energy regions thus are composed entirely of background radiation and may be used to correct the resonance-capture spectra. However, to perform the corrections with confidence, it is necessary to understand the nature of the background and its sources, and to be sure that the character of the background does not change in the vicinity of the neutron resonances.

Neodymium is a particularly happy choice for background investigations, since the resonance-capture gamma rays are, for the most part, of lower energy than the prominent high-energy lines in the background spectrum. The regions of interest in the level schemes of the neodymium isotopes are shown in Fig. 5. Both Nd^{143} and Nd^{145} have ground states with negative parity and spin 7/2. Thus the states in Nd^{144} and Nd^{146} formed by the capture of s-wave neutrons are also of negative parity and have spins of either 3 or 4. E1 transitions will be observed from both the 3^- and

4^- resonances to final states of spin 3^+ and 4^+ . In addition, there will be E1 transitions from the 3^- resonances to final 2^+ states, and from the 4^- resonances to final 5^+ states (although no low-lying states of this character are known for the nuclei considered in this work).

According to Eqs. (16) and (17), transitions to the 0^+ ground state would have to be either E3 or M4 from the 3^- resonances and M4 or E5 from the 4^- resonances. Since there are very many final states available for more favorable

transitions, it is expected that

transitions of such high multipolarity will be so weak that they will be completely unobservable in this type of experiment.* The odd-N isotopes of neodymium, formed by capture in the even-N isotopes, all have binding energies of less than 6 MeV. Thus the highest energy that will be observed for gamma rays resulting from resonance capture is about 7.1 MeV for the 3^- resonances and about 6.5 MeV for the 4^- resonances.

Spectra of all detected gamma rays (plots of N_γ vs neutron energy) are shown in Fig. 6. The 4-spectrum analyzer was used for the low-energy neutron resonances while the 3-parameter analyzer was used for the higher neutron energies. The runs were made simultaneously in

Fig. 5. Low-lying levels in Nd^{144} and Nd^{146} fed by primary transitions from slow-neutron resonances.

* In fact, no transitions of multipolarity higher than quadrupole [reference 5] have ever been observed from highly excited states, even in thermal-capture experiments for which the flux is several orders of magnitude greater.

Fig. 6. Total intensity of detected gamma rays versus neutron energy for neodymium. Gamma-ray spectra were observed in the intervals indicated by pedestals.

this case. The upper spectrum was obtained with the 100-channel analyzer as explained in Sec. II. The lower spectrum was obtained when the 3-parameter analyzer tape was read with no restrictions on the gamma-ray pulse height. The mass numbers of the isotopes in which neutrons are captured to form the various resonances are indicated. Isotopic identifications are taken from reference 18. A new resonance at about 7.5 eV, indicated by a question mark, had not been previously detected. The numbered brackets correspond to intervals in which gamma-ray spectra were observed. The dashed line indicates the total number of gamma rays in each channel from background radiation of all types.

The characteristics of the background are most easily investigated by use of the data from the three-parameter analyzer. Gamma rays in interval B1, corresponding to neutron energies from about 31 to 33 eV, arise almost entirely from various sources of background. The spectrum observed in this interval is shown in Fig. 7. Also shown are the spectra observed in intervals B2 and B3 which are predominantly (more than 95%) background. The most striking features of the background spectra are the numerous strong lines.* Because of the importance of an accurate background correction, a considerable effort has been made to understand and, if possible, eliminate these lines. An interpretation of their source,

* The break in the spectra between 3 and 4 MeV is caused by the action of the dual discriminator arrangement shown in Fig. 4. Discriminator 1 is set to accept all pulse heights above some arbitrary level, about 3.8 MeV in this case. Discriminator 2, which is set to accept pulse heights above some lower level, about 2 MeV here, will only produce trigger pulses some fraction of the time, depending on the scale factor (a factor of 8 for the spectra shown) set on the fast scaler. This is done so that the low-energy data may be collected, since it is useful for normalization purposes but not to the extent that it use up excessive space on the magnetic tape.

Fig. 7. Background spectra observed in off-resonance regions for the run made on neodymium. The origin of the prominent transitions, indicated by vertical lines, is discussed in the text.

which is consistent with all pertinent experimental evidence* is as follows. The 7.64-MeV line arises from capture in iron contained in the heavy concrete that forms the walls and ceiling of the detector room. The neutrons involved are predominantly fast neutrons which leak through the rotor at all angular positions, are scattered from the beam by the sample, by air, and to a small extent by the aluminum windows in the flight tube,** and are moderated to thermal energies by multiple scattering. The line at about 7 MeV arises from fast neutrons scattered into the NaI crystal, predominantly by the sample, and captured in the 2.85-keV neutron resonance in sodium. Since the capture is internal, the crystal represents a 4π detector for the resulting radiation and the full energy (corresponding to the neutron binding energy of 6.96 MeV) is observed. Also observed in the high-energy region is a line at 6.49 MeV, which may be explained as arising from escape of the 0.47-MeV quanta produced in 74% of the cascades²⁹ following capture in sodium, and a 6.00-MeV line which is also observed in the thermal-capture spectrum of iron. Prominent lines are observed in the low-energy region at 2.75 MeV, from the 15-hr activity induced by the $\text{Na}^{23}(n,\gamma)\text{Na}^{24}$ reaction, and at 2.22 MeV, from capture in hydrogen in paraffin blocks located at various positions around the sample to reduce the number of neutrons scattered to the walls and ceiling. The rest of the background

* A wide variety of experiments were performed. For example, spectra were observed with the reactor down, with the reactor up with various filters in the beam, with a carbon scatterer in the sample position, with the sample removed, with directional shielding on the crystal, with a polonium-beryllium neutron source, etc., in addition to the work described later in this section.

** The lead collimator shown in the flight tube in Figs. 3 and 4 was installed to reduce the neutron flux in the detector room at positions other than the sample. It is located 10 m from the entrance to the detector room to reduce the effect of neutrons scattered by the collimator.

spectrum is structureless and arises from a variety of sources, such as cosmic rays, scattered radiation from the reactor, activity induced in the sample, escape of various photons in the cascades following neutron capture in the crystal, etc.

The shielding used for the runs on neodymium was of the type shown in Fig. 3, with the 3-in. can of B_4C for neutron shielding. It was later found that the background component due to capture in sodium could be significantly reduced by replacing this can by 4 in. of B_4C suspended in paraffin (the purpose of the paraffin being to increase the moderation of the scattered fast neutrons) and most subsequent runs were then made with neutron shielding of this type. The shielding shown in Fig. 4 was even more effective in that it also removed most of the background component due to iron, and much of the lower energy structureless background. This arrangement, however, was used only for the runs on hafnium.

Having investigated the nature of the background in off-resonance regions, it remains to check that it is not altered in the vicinity of the resonances. This check is especially important since the scattering of neutrons by the sample is an important source of background. Since the scattering cross section increases greatly in the neutron resonances, it must be established that it is in fact fast neutrons (not resonance-energy neutrons) that are responsible for this background.

One important argument that this is the case may be presented on the basis of the data already shown in Fig. 6. If resonance-energy neutrons were scattered into the crystal, the time-of-flight spectra would be expected to show peaks corresponding to capture in the resonances in iodine. However, prominent iodine resonances at 20.5, 31.4, and 37.8 eV and other neutron energies definitely do not appear in Fig. 6. Further evidence for the

Fig. 8. The neutron-energy dependence of the intensity of three components in the background spectra. Component a arises mostly from events induced by cosmic rays. Component b comes from the capture of fast neutrons by iron after they are moderated to thermal energies. Component c results from the capture of fast neutrons by sodium in the NaI crystal after they are moderated to 2.85 keV.

correctness of the above interpretation of the sources of background is afforded by the spectra of Fig. 8, in which the gamma-ray intensities are plotted as a function of neutron energy for each of the intervals A, B, and C indicated in Fig. 7. Interval A contains predominantly gamma rays from events induced by cosmic rays, although there are contributions from capture in iron (from the tail of the 7.64-MeV transition as well as from relatively weak transitions of 9.30, 8.87, and 8.34 MeV), and possibly weak contributions from other neutron-capture gamma rays of undetermined origin. In any case, the contribution to the background from gamma rays in this interval is clearly not altered in the vicinity of the neutron resonances. The same is obviously true of the background

component included in interval B, which arises predominantly from the capture of moderated neutrons in iron. The rise in intensity with increasing neutron energy (decreasing time of flight) is due to the tail of the distribution of moderation times for the intense burst of fast neutrons released when the rotor slits are open.

There is some structure in the neutron-energy spectrum of gamma rays in interval C. However, in addition to the 6.96-MeV component from capture of scattered neutrons in the crystal, interval C also contains contributions from the 7.1-MeV transition associated with capture in the 3^- resonances in neodymium. In part (b) of this section it will be shown that the resonances at 103, 138, and 186 eV have strong transitions of this energy, so that the structure seen in Fig. 8 may be entirely explained as arising from this source. In any case, the fact that the background component associated with the scattering of neutrons by the sample shows no trace of an increase at 43 eV, 56 eV, and other neutron energies at which there are strong resonances is convincing evidence that the B_4C neutron shield is completely adequate for resonance-energy neutrons, and that the background spectrum is not altered in the vicinity of neutron resonances.

However, as shown in Fig. 8, the three background components vary in different ways with neutron energy. It is therefore not sufficient to employ a single background spectrum measured at one neutron energy to correct a spectrum measured at some quite different energy. However, for fairly small changes in neutron energy, the variation in intensity of all three components is approximately linear. Thus, a sufficiently accurate background spectrum can be obtained for a given resonance by finding the intensity of the background radiation in each gamma-ray pulse-height channel by making a linear interpolation between spectra measured on opposite sides of the resonance. This method was

used in all cases. Background spectra were taken as close as possible to the resonance under investigation. Note, however, that for low neutron energies, say below about 40 eV, the variation with neutron energy is approximately the same for all three components. Thus, if necessary, single background spectra may be used in this region.

One final point concerning the background correction remains, namely, the corrections that must be applied if one is forced to use background spectra measured in intervals where there are some contributions from resonance capture. A discussion of these additional corrections is deferred to part (c) of this section.

(b) Relative Intensities

Having obtained a set of spectra measured in the resonances and a set of background spectra measured in off-resonance regions, the spectra of resonance-capture gamma rays are given by

$$(56) \quad n_{ir} = N_{ir} - (C_{1r} B_{1ir} + C_{2r} B_{2ir}) ,$$

where $\{n_{ir}\}$ is the corrected spectrum of gamma rays for the r th resonance, $\{N_{ir}\}$ is the corresponding raw spectrum, $\{B_{1ir}\}$ and $\{B_{2ir}\}$ are background spectra measured on either side of the r th resonance, and C_{1r} and C_{2r} are constants that include the linear interpolation constants and the normalizations for the number of time-of-flight channels in which the various spectra were measured. The true spectra $\{n'_{ir}\}$, which would be the same as the $\{n_{ir}\}$ except for counting statistics, may be expressed as

$$(57) \quad n'_{ir} = \sum_{k=1}^n a_{kr} R(j_k, i) ,$$

where $\{a_{kr}\}$ is the relative intensity of the k th transition (whose energy corresponds to channel j_k , where j_k is not necessarily an integer) and $R(j_k, i)$ is the contribution the i th channel would receive from the k th transition if the latter had unit intensity. The sum is over all transitions that produce gamma rays in the spectral region being considered.

Now if the response function $R(j_k, i)$ is known for each k , the relative intensities may easily be obtained by a straightforward least-squares fit. Thus one finds the set $\{a_{kr}\}$ for which the expression

$$(58) \quad \sum_i \frac{[n_{ir} - \sum_{k=1}^n a_{kr} R(j_k, i)]^2}{(\Delta n_{ir})^2}$$

is a minimum. The Δn_{ir} in expression (58) are the standard deviations given by

$$(59) \quad \Delta n_{ir} = [N_{ir} + C_{1r}^2 B_{1ir} + C_{2r}^2 B_{2ir}]^{\frac{1}{2}}.$$

The problem of finding the relative intensities thus becomes principally one of finding the response functions. For gamma rays of about 8 MeV, the response function for a large NaI crystal may, with very good precision, be written in the form

$$(60) \quad R(j, i) = R(j-i)$$

for $(j - i)$ corresponding to small energy variations. That is, for small energy changes, the only significant change in the response function is a simple translation of the entire spectrum. For the 8×6 -in. crystal used in this work, it is an experimental fact that this is also a good approximation for moderately large energy

shifts. In the present discussion it is merely assumed that the response function for all gamma-ray energies transitions of all energies considered here may be obtained by such a simple translation of the response function measured for gamma rays of any energy in the vicinity of 8 MeV. The errors introduced in the values of the relative intensities by the failure of this assumption will be discussed in part (e) of this section.

The response of the system to 7.64-MeV gamma rays may easily be obtained by observing the spectrum of gamma rays resulting from the capture of thermal neutrons in iron. This spectrum is shown by the open circles in Fig. 9. There are several other well isolated and

Fig. 9. The response (solid circles) of the experimental system to monoenergetic (7.64-MeV) gamma rays. This curve was obtained by correcting the observed spectrum (open circles) resulting from the capture of thermal neutrons in iron. The positions of minor lines in the spectrum are indicated by arrows and labeled with the intensities of the transitions relative to the intensity of the 7.64-MeV transition which is taken as 100.

relatively weak lines in the spectrum which, however, have well-known energies and intensities. The positions of these lines, and their intensities relative to the intensity of the 7.64-MeV transition which is taken as 100, are indicated in the figure. When the intensities and positions are known, the contributions from these weaker lines may easily be subtracted to yield the response function for 7.64-MeV gamma rays, shown by solid circles in Fig. 9.

The response function for all lines may now be obtained by translation once the line positions j_k are known. Unfortunately, even if the energy calibration of the system were precisely known, the neutron binding energies are not known precisely enough to allow an independent determination of the line positions. The line positions must be determined from the data.

For well-resolved transitions, this may easily be accomplished in principle by finding the values of $\{j_k\}$ for which chi-squared for the problem is a minimum. Expression (58), evaluated with the best-fit values $\{a_{kr}\}$ is χ^2 . One proceeds by selecting a neutron resonance in which a particular transition is strong and evaluating χ^2 for a series of positions of the dominant line, leaving the line positions of all the other transitions fixed. The value of j_k that corresponds to the minimum in the resulting curve of χ^2 versus line position is chosen. The procedure is then repeated for other spectra where other transitions are dominant until the line positions of all transitions being considered are fixed.

While straightforward in principle, this procedure is complicated in practice by the fact that the spectra in all resonances in a given nuclide must be fitted with the same set of line positions.* Further, the differences in line positions must be

* The maximum variation in neutron energy considered here is only 3×10^{-5} of the total excitation.

consistent with whatever is known of the differences in the energies of the final states. This usually requires a considerable amount of adjustment to obtain a compromise set of line positions for which χ^2 is acceptable for all the spectra. The situation is made much more difficult if two or more lines are not resolved, since in this case one can never be certain that a single transition is dominating a given spectral region. However, because it is known a priori that the line positions are precisely the same for all spectra (since they are all recorded simultaneously) and because different lines are alternately strong and weak in different spectra (because of the Porter-Thomas fluctuations), quite precise values of the line positions may be obtained by minimizing χ^2 for all of the spectra simultaneously. That is, expression (58) is summed over all resonances in the same nuclide and the resulting expression

$$(61) \quad \sum_r \sum_i \left\{ \frac{[n_{ir} - \sum_{k=1}^n a_{kr} R(j_k, i)]^2}{(\Delta n_{ir})^2} \right\}$$

is minimized simultaneously with respect to all of the $\{a_{kr}\}$ as well as the $\{j_k\}$. Knowledge of differences in the line positions, obtained from existing level schemes of low-lying levels, is fed into the problem as a set of linear constraints on the line positions. Such knowledge considerably improves the precision of the determinations of line position. The problem is coded for the IBM-704 computer.

Thus by use of this technique, not only are all spectra fitted with the same set of line positions (which are consistent with known differences in energies of the final states) but the line positions are "best" values in the sense that χ^2 for any given spectrum cannot be reduced without increasing χ^2 by a greater amount in one or more other resonances.

The spectra obtained for the resonances in neodymium, and the fits obtained by the above procedure, are shown in Figs. 10 and 11. Only the transitions shown in Fig. 5 were considered in the fit and the differences in spacing were constrained according to the energies of the final states given there. Vertical lines are shown in Figs. 10 and 11 at the determined line positions.

The derived relative intensities for the various transitions are given in Table I. The errors given are standard errors.

TABLE I. The relative intensities in spectra observed in the neutron resonances in neodymium. The designations of the neutron energy intervals are the same as shown in Figs. 6 and 7. The relative intensities were obtained by use of a response function having 100 counts per channel at its peak.

Nd ¹⁴³				
Interval	E _n (eV)	a _k		
		7.118 MeV	6.501 MeV	6.259 MeV
3	56	0.05 ± 0.20	0.17 ± 0.35	0.40 ± 0.35
7	132	0.05 ± 0.13	0.49 ± 0.22	4.36 ± 0.22
8	138	0.43 ± 0.15	0.00 ± 0.25	1.27 ± 0.25
9	186 + 193	4.66 ± 0.10	2.45 ± 0.17	0.69 ± 0.17
Nd ¹⁴⁵				
Interval	E _n (eV)	a _k		
		7.112 MeV	6.512 MeV	
1'	4.4	3.31 ± 0.16	15.66 ± 0.29	
2'	43	0.71 ± 0.15	31.24 ± 0.28	
2	43	0.14 ± 0.15	9.06 ± 0.11	
4	87	0.23 ± 0.09	1.50 ± 0.09	
5	97	0.14 ± 0.04	0.08 ± 0.08	
6	103	0.77 ± 0.05	4.27 ± 0.11

Fig. 10. Gamma-ray spectra observed in the neutron resonances in Nd^{143} . The spectra are labeled with the energies of the neutron resonances and with the interval designations taken from Fig. 6.

Fig. 11. Gamma-ray spectra observed in neutron resonances in neodymium-145. The spectra are labeled with the energy of the neutron resonance and the number of the neutron-energy interval (taken from Fig. 6) in which they were observed. The spectra in intervals 1' and 2' were taken with the equipment shown in Fig. 3. The spectra in intervals 2, 3, 4, 5, and 6 were taken with the equipment shown in Fig. 4.

These intensities are relative in the sense that the ratio of any two values taken in the same spectrum is equal to the ratio of the absolute intensities for the same transitions. The response function is arbitrarily normalized to have a peak height of 100.

(c) Normalized Relative Intensities

The relative intensities a_k , as defined above, are related to the absolute intensities I_k by the relation

$$(62) \quad a_k = N_c I_k \epsilon_k p_k,$$

where N_c is the total number of neutron captures in the neutron energy interval in which the particular spectrum was measured, ϵ_k is the total efficiency for detecting gamma rays of the energy considered, and p_k is the quotient obtained when the ratio of the peak height of the response function to its area is divided by the arbitrary peak height of the response function used in the analysis. The absolute intensities are expressed in photons per capture. The product $\epsilon_k p_k$ is a constant from resonance to resonance. Further, the assumption that the response function $R(j_k, i)$ can be expressed in the form $R(j_k - i)$ implies that $\epsilon_k p_k$ is assumed to be constant for all transitions.* Thus, dividing the a_k by a quantity that is proportional to the number of neutron captures in the appropriate neutron-energy interval normalizes the intensities in the sense that they are all proportional to the absolute intensities of the transitions involved with the same proportionality constant.

* Recent Monte-Carlo calculations [reference 30] have shown that ϵ_k varies less than 2% for the case of a collimated beam of gamma rays incident on an 8 X 6-in. NaI crystal for an energy variation of 6-8 MeV.

The total number of pulse heights detected in channel i may be written

$$(63) \quad N_{\gamma i} = N_c \sum_{k=1}^n I_k \epsilon_k R(j_k, i),$$

where the sum is over all lines in the spectrum. This expression is identical to (61) when (62) is used for the relative intensities, except that now the response function is required to have unit area (to prevent superfluous factors from appearing). Thus, if Eq. (63) is summed over all channels in the spectrum, the response function for all lines will sum to unity, except for those low-energy lines affected by the discriminator setting. The resulting expression

$$n_{\gamma} = \sum_{i=1}^{\max} N_{\gamma i} \approx N_c \sum_k I_k \epsilon_k$$

provides a relation between the total number of captures and the easily obtained quantity n_{γ} . The quantities I_k are subject to the Porter-Thomas fluctuations discussed in Sec. I(c). However, the sum is expected to be very nearly constant from resonance to resonance in the same nuclide since a great many terms are involved. Except for the effect of the low-level discriminator setting (which should be constant from resonance to resonance) and the slowly varying function ϵ_k , the sum is proportional to the average multiplicity $\bar{\nu}$ of gamma rays, namely,

$$\bar{\nu} = \sum_{\text{all } k} I_k.$$

This quantity is expected to be even more constant than the total radiation width,^{*} since it involves not only a sum over primary transitions, but also secondary, tertiary, etc., transitions that arise in the countless cascades that follow captures in the neutron resonances. The effects of variation of the total detection efficiency ϵ_k and of the low-level discriminator will be negligible, provided that the spectra in the different resonances have the same gross structure. For example, it is necessary that the discriminator exclude a constant fraction of the pulse heights in all spectra. Thus, for resonances in the same isotope we have

$$(64) \quad N_c = k n_\gamma,$$

where the proportionality constant depends on the fraction of gamma-ray pulse heights excluded by the low-level discriminator, the gamma-ray multiplicity, the geometric properties of the system, and the gross shape of the spectra. This relationship has been experimentally verified by Moxon³¹ who showed, by a direct measurement of both N_c and n_γ that, for eleven resonances in Pt^{195} , k is a constant within $\pm 5\%$ when the low bias is set to accept all pulse heights above 2 MeV.

The normalized relative intensities are defined by

$$(65) \quad A_k = a_k / n_\gamma.$$

The normalization areas n_γ and the normalized relative intensities are given in Table II for the resonance spectra in neodymium.

It should be emphasized that the commonly employed technique of using the total intensity in some restricted low-energy

* Carpenter and Bollinger [reference 11] have shown, for example, that although the total radiation widths of two resonances in mercury-201 differ by the anomalously large factor of 2, their multiplicities vary by less than 25%.

TABLE II. Normalization areas and normalized relative intensities for spectra in the neutron resonances in neodymium. Also shown are the resonance-average values of the normalized relative intensities and of the corresponding partial radiation widths.

Nd ¹⁴³				
E _n (eV)	n _γ	A ₁	A ₂	A ₃
56	58 160	0.09 ± 0.34	0.29 ± 0.60	0.69 ± 0.60
132	40 180	0.12 ± 0.32	1.22 ± 0.55	10.85 ± 0.55
138	27 530	1.56 ± 0.54	0.00 ± 0.91	4.61 ± 0.91
186	—	5.08 ± 0.34	0.67 ± 0.58	1.22 ± 0.58
193	—	1.39 ± 0.27	3.22 ± 0.44	-0.18 ± 0.44
	$\overline{A}_k$ (Rel.)	2.0 ^{+5.6} _{-1.0}	1.1 ^{+3.8} _{-0.5}	3.9 ^{+8.1} _{-1.6}
	$\overline{\Gamma}_k$ (10 ⁻³ eV)	1.7 ^{+4.8} _{-0.8}	1.0 ^{+3.3} _{-0.5}	3.4 ^{+7.1} _{-1.4}
Nd ¹⁴⁵				
E _n (eV)	n _γ	A ₁	A ₂	
4.4	127 250	0.72 ± 0.04	3.42 ± 0.08	
43.1	139 060	0.10 ± 0.11	6.52 ± 0.08	
87	11 820	0.68 ± 0.34	4.23 ± 0.42	
97	6 150	0.81 ± 0.32	0.49 ± 0.65	
103	59 210	1.30 ± 0.08	7.21 ± 0.19	
	$\overline{A}_k$ (Rel.)	0.8 ^{+2.1} _{-0.4}	4.6 ^{+9.7} _{-2.1}	
	$\overline{\Gamma}_k$ (10 ⁻³ eV)	0.5 ^{+1.2} _{-0.5}	2.6 ^{+5.5} _{-1.2}	

region of the gamma-ray spectra to obtain normalization factors is without justification and can in fact compound any errors due to fluctuations in the gamma-ray multiplicity. In the single case that has been thoroughly investigated,¹¹ it has been shown that fluctuations in the gamma-ray multiplicity in the resonances in Hg^{201} are due to fluctuations in the intensities of the highest energy transitions, which are unusually strong. Resonances with weak high-energy gamma rays also have higher multiplicities, since a smaller fraction of the primary transitions proceed directly to low-lying levels. Thus, if this should be a general situation, as seems reasonable, values of a_k/n_γ would be systematically too low for spectra in which the high-energy transitions are weak, and systematically too high for those whose high-energy transitions are strong. If now the high-energy regions of the spectra are intentionally excluded, as is the common practice, these systematic errors will clearly be increased. For most nuclides, however, the effect will be slight, since the high-energy region contains only a small fraction of the total number of counts.

The proper normalization areas are, then, the total number of counts resulting from capture in the resonance being considered. It often happens that this quantity cannot be directly obtained because of contributions from other resonances to the energy interval involved. If resonances in other isotopes are responsible, the only effect will be to alter the normalization. In such a case, a simple process of unpeeling, i.e., subtracting the $N_\gamma(E_n)$ of the other isotopes from the experimental curve, will yield the proper normalization areas. If, however, the contributing resonances and the resonance under investigation are in the same isotope, the values of a_k as well as n_γ will be affected. In this case, the analysis may still be performed, provided that the contribution from each resonance may be determined by unpeeling, although now it is necessary to consider possible interference effects.

An example of the analysis of two unresolved resonances in the same isotope is afforded by the pair of resonances in Nd^{143} at about 190 eV. The N_γ spectrum, corrected for background, is shown in Fig. 12. The nominal values for the resonance energies are 186 and 193 eV, a spacing of 7 eV. The resolution width at this energy is approximately 9 eV, a value much larger than the total width of either resonance. Thus, for unpeeling purposes, the experimental spectrum is effectively the convolution of the resolution function, which is known to be symmetric, with two delta functions located at the resonance energies. Thus, in this case, the unpeeling is made possible by requiring that both resonances be symmetric and have the same shape. Simple considerations show that these conditions actually over-determine the two resonance shapes. In the unpeeling of n points in the $N_\gamma(E_n)$ spectrum, there will be $2n$ unknowns, the number of counts contributed by each resonance to each channel. The requirement that the individual contributions add up to the experimental total imposes n restrictions; the requirement that both shapes are the same gives n restrictions; and symmetry gives $n/2$ more. However, instead of solving analytically to obtain the shapes, a trial and error unpeeling by hand is sufficient since a solution, if found, must be unique. The shapes obtained are shown by dashed lines in Fig. 12. It is now a simple matter to obtain the contribution each resonance makes to any desired energy interval.

Fig. 12. The intensity of all detected gamma rays (corrected for background) versus neutron-time-channel number for neodymium in the vicinity of 190 eV. The dashed lines show the contribution from individual resonances at 186 and 193 eV.

For these two resonances, spectra were measured in the four intervals shown in Fig. 12. These spectra, and the fits obtained by the methods described in part (b) are shown in Fig. 13. The derived relative intensities and the contributions to each interval from each resonance are shown in Table III. The derived relative intensities in each interval are now assumed to be given by

$$(66) \quad a_{ij} = A_{i,186} n_{j,186} + A_{i,193} n_{j,193},$$

where a_{ij} is the relative intensity for the i th transition measured in the j th interval, $A_{i,186}$ is the normalized relative intensity of the i th

Fig. 13. Gamma-ray spectra observed in four neutron-energy intervals in the vicinity of 190 eV. The spectra are labeled with the numbers of the intervals (taken from Fig. 12) in which they were observed.

TABLE III. The relative intensities and the normalization areas for four intervals taken across two unresolved neutron resonances in neodymium-143. The interval designations are taken from Fig. 12. The normalization areas are the areas under the dashed curves in Fig. 12. The relative intensities were obtained by use of a response function having 100 counts per channel in its peak.

Int.	$n_{\gamma 186}$	$n_{\gamma 193}$	a_1	a_2	a_3
9a	10 420	1 047	0.48 ± 0.05	0.07 ± 0.09	0.09 ± 0.09
9b	12 072	6 556	0.75 ± 0.05	0.33 ± 0.08	0.16 ± 0.08
9c	3 452	15 650	0.41 ± 0.05	0.48 ± 0.08	0.01 ± 0.08
9d	355	10 980	0.13 ± 0.05	0.41 ± 0.08	-0.02 ± 0.08

transition for the 186-eV resonance, $n_{j,186}$ is the number of detected gamma rays contributed to the j th interval by the 186-eV resonance, and the other quantities are similarly defined. We now consider all pairs of intervals and solve the two equations for $A_{i,186}$ and $A_{i,193}$. If interference effects are negligible, the results for the A_i should be the same, within statistical errors, for all six pairs. For this particular pair of resonances, this is in fact the case. The values of the normalized relative intensities given in Table II for these resonances are the least-squares solutions to the four equations [Eq. (66) for $j = 1, 2, 3, 4$] for the four neutron-energy intervals. They are consistent with the values obtained for any pair of spectra. We conclude that, in the neutron-energy intervals considered, the interference term [Eq. (7)] is small in comparison with the main capture cross section included in Eq. (3)—although interference must be present because the strong transitions to the 2^+ final states in both resonances indicate that both resonances are 3^- .

As mentioned in part (b) of this section, it is often necessary to employ background spectra which include contributions from resonance capture. The corrections necessary in this case are precisely analogous to the corrections required when a resonance-capture spectrum contains contributions from other resonances—except, of course, that the spurious contributions to the spectra corrected for background are now negative rather than positive. The methods used are the same as those just treated and will not be discussed further.

(d) Absolute Intensities

The final absolute intensities are obtained by normalizing data obtained in a separate part of the experiment. It is, of course, only necessary to determine the absolute intensity of a single transition

in a single resonance in a given nuclide since the normalized relative intensities are all proportional to the absolute intensities with the same normalization constant. Therefore, having observed the gamma-ray spectra for all resolved resonances in the main part of the experiment, one may choose for the final normalization a resonance whose properties are most suitable for the precise determination of the normalization constant.

The normalization experiment consisted of a series of short runs, of some 10 to 20 hr each, on the samples of the various materials being investigated, the neutron flux being carefully monitored throughout this time. Considerable care was taken to insure that the experimental arrangement remained identical for all the samples investigated and that the factors which could influence the energy dependence of the neutron flux (e. g., the rotor speed, the position of the axis of rotation of the rotor, and the collimator setting) remained constant throughout the normalization runs, a total of some 100 to 200 hr. The neutron flux was measured, as a function of neutron energy, before and after the series of runs on the various samples and at one or more intermediate times by use of a "boron plate" counter known to have a neutron detection efficiency that varies as $(E)^{-1/2}$ for the range of neutron energies of interest here.

Equation (62) may be written

$$(67) \quad I_k = k a_k / N_c.$$

The number of captures N_c in the neutron energy interval used to obtain the relative intensity a_k may be expressed in terms of the parameters of the resonance involved, the neutron flux, and a constant which is independent of the material being investigated. To see this, imagine that the material under investigation is in the form of a slab with depth t and thickness n (in atoms per barn).

Let there be a beam of neutrons incident on the front face of the slab with total flux ϕ_0 (neutrons/sec). If we neglect, for the moment, the possibility that neutrons may be captured after they are once scattered, then the number of captures in a sheet of thickness dx located at a depth x from the front face is

$$(68) \quad dN_c(E) = \phi_0(E) \exp(-n\sigma x/t) \sigma_Y n \, d(x/t).$$

The total number of neutrons of energy E captured in the slab is then

$$(69) \quad N_c(E) = \phi_0(E) \frac{\sigma_Y(E)}{\sigma(E)} \{1 - \exp[-n\sigma(E)]\}.$$

Since multiple scattering has been neglected, this is an underestimate of the actual number of captures. A rough estimate of the increase in $N_c(E)$ due to multiple scattering may be made as follows. Equation (69) may be interpreted as the product of the total number of neutrons that interact at least once in the sample, $\phi_0(1 - T)$, and the fraction of the initial interactions that are radiative captures, σ_Y/σ . The number of neutrons scattered at least once is then $\phi_0(1 - T)\sigma_n/\sigma$. The calculation of the fraction of these scattered neutrons that interact again in the sample is complicated by such things as the loss of some neutrons through the sample edges, the fact that the sample is inclined to the beam, and the fact that the neutrons lose energy by an amount which depends on the center-of-mass angle through which they are scattered. We assume that a fraction α of the initially-scattered neutrons interact again with the sample. Then a fraction σ_Y/σ of these will be captured and a fraction σ_n/σ will be again scattered. The moderation of the neutrons is ignored and the same fraction α of the doubly-scattered neutrons is assumed to interact at least once more with the sample. Continuing in this way leads to

$$(70) \quad N_c(E) = \phi_0(E) \frac{\sigma_Y(E)}{\sigma(E)} \left(1 - e^{-n\sigma(E)}\right) \left[1 + a \frac{\sigma_n}{\sigma} + \left(a \frac{\sigma_n}{\sigma}\right)^2 + \dots\right].$$

The fraction of scattered neutrons interacting again in the sample may approach unity in the peaks of the resonances. However, for normalization purposes we will consider only resonances such that $\sigma_n/\sigma \approx \Gamma_n/\Gamma \ll 1$ in the peaks. Although σ_n/σ may approach unity in the wings of the resonances, a becomes quite small in these regions for the sample thicknesses used. Thus $a \sigma_n/\sigma$ is small everywhere and it is a good approximation to neglect quadratic and higher power terms in Eq. (70). For $a \sigma_n/\sigma$ we may use the value Γ_n/Γ attained at the peaks of the resonances since this is the region in which most captures occur, and obtain

$$(71) \quad N_c(E) = \phi_0(E) \left(1 + \frac{\Gamma_n}{\Gamma}\right) \frac{\sigma_Y(E)}{\sigma(E)} \left(1 - e^{-n\sigma(E)}\right).$$

The largest value of Γ_n/Γ for any resonances used in the normalization is 0.138 for the 33.5-eV resonance in Hg^{199} and for the 56-eV resonance in Nd^{143} .

The total number of captures detected in a neutron-energy interval $E_1 \leq E_n \leq E_2$ is now

$$(72) \quad N_c = \int_{E_1}^{E_2} dE \int_0^\infty N_c(E') R(E', E) dE',$$

where $R(E', E)$ is the neutron-energy resolution function. If the size of the energy interval is allowed to increase indefinitely, the order of integration may be interchanged and the integral over the resolution function carried out to yield unity. Thus, the total number of captures detected in a very large energy interval is independent of the neutron-energy resolution. This is clearly also true if the interval

$E_1 \leq E_n \leq E_2$ is made only large enough that all data points excluded from the interval are completely resolved. However, in general such an interval is still larger than one would like to use in order to obtain a favorable signal-to-background ratio, and in some cases it is so large that it overlaps other nearby resonances. The effect of resolution on the number of captures detected in the energy interval used to obtain the normalization spectrum may be obtained from the data. We define the factor f as the ratio obtained when the total number of detected gamma rays (after subtraction of background) in the interval used to obtain the spectrum is divided by the quantity S_γ defined as the total number of detected gamma rays (with background subtracted) in the interval $E_1 \leq E_n \leq E_2$ chosen large enough to include all points affected by the resolution. Then

$$(73) \quad N_c = f \left(1 + \frac{\Gamma_n}{\Gamma} \right) \int_{E_1}^{E_2} \phi_0(E) \frac{\sigma_\gamma(E)}{\sigma(E)} \left(1 - e^{-n\sigma(E)} \right) dE.$$

This expression includes resolution exactly and multiple-scattering effects to a good approximation provided that $\Gamma_n/\Gamma \ll 1$.

The neutron flux $\phi_0(E)$ is a slowly varying function (varying approximately as $E^{-1/2}$) and may be removed from the integral in Eq. (73). Experimentally, it is proportional to the product of a neutron-energy-dependent factor Φ (measured by the "boron plate" counter) and the number of monitor counts M during the normalization run (measured by a fission counter located near the collimator in the reactor). Thus, in terms of quantities that are determined by the experiment or that may be calculated from known parameters, Eq. (67) may be written

$$(74) \quad I_k = C \frac{a_k}{M\Phi f J_\gamma},$$

where

$$(75) \quad J_{\gamma} = \left(1 + \frac{\Gamma_n}{\Gamma}\right) \int_{E_1}^{E_2} \frac{\sigma_{\gamma}(E)}{\sigma(E)} \left[1 - e^{-n\sigma(E)}\right] dE.$$

This integral is evaluated numerically by a program which includes the effects of Doppler broadening of the various cross sections involved.

It is clear from the definition that the factor f is equal to the quotient obtained when the same normalization area n_{γ} as was used to obtain the normalized relative intensities is divided by the resolution-independent sum S_{γ} . Thus Eq. (74) can be rearranged to give

$$(76) \quad a_k/n_{\gamma k} = A_k = \left(\frac{M\Phi}{C} \frac{J_{\gamma}}{S_{\gamma}} \right) I_k.$$

For a given run the quantity in parentheses is a constant for resonances in the same nucleus. Thus

$$(77) \quad J_{\gamma 1}/J_{\gamma 2} = S_{\gamma 1}\Phi_2/S_{\gamma 2}\Phi_1.$$

This provides a means of obtaining a value of J_{γ} for a resonance with a particularly suitable gamma-ray spectrum, even though the value of Γ_n/Γ for the resonance may be so large that Eq. (75) is a poor approximation. The only requisite for obtaining $S_{\gamma 1}/S_{\gamma 2}$ is that a resonance with suitable Γ_n/Γ be observed in the same run.

Some nuclides, however, contain no available resonance whose gamma-ray spectrum includes a strong enough transition to permit a good determination of a_k . In this case the following procedure may be used. The constants M and C are clearly the same for all resonances observed in the same run,

regardless of the nuclide in which they occur. Thus Eq. (76) may be used to give

$$(78) \quad \left(\frac{I_k}{A_k} \right)_1 = \left(\frac{I_k}{A_k} \right)_2 \left(\frac{\Phi_J}{S_\gamma} \right)_2 \left(\frac{S_\gamma}{\Phi_J} \right)_1 ,$$

where the numerical subscripts refer to resonances 1 and 2 which may be in different nuclides, but which are observed in the same run. Now suppose resonance 2 is chosen to have a gamma-ray spectrum containing a strong transition so that Eq. (74) may be used to obtain a relatively precise value of the corresponding absolute intensity. Suppose also that the ratio Φ_2/Φ_1 of neutron fluxes may be determined for the run made in the main part of the experiment. Then all quantities appearing on the right-hand side of Eq. (78) may be calculated or measured for this main run. Equation (78) then gives the absolute intensity per unit normalized relative intensity in resonance 1 for the run made in the main part of the experiment; and, since the normalized relative intensities are all proportional to the corresponding absolute intensities (with the same proportionality constant) this is sufficient to determine the absolute intensities of all transitions considered.

The constant C serves to normalize the flux and the amplitude of the response function to absolute units. Otherwise it depends only on the experimental arrangement, which is carefully kept the same for all samples investigated during a series of normalization runs. The numerical value of C is determined by including in the series of normalization runs a measurement of the spectrum in a resonance that contains a line of known intensity. The standard line chosen is the ground-state transition from the 11.9-eV resonance in Pt^{195} . The capture rate in this resonance is very high, so that data with good statistical accuracy may be obtained in a short time. The resonance is well resolved and has

the quite suitable value $\Gamma_n/\Gamma = 0.06$. Further, the ground-state transition has an energy of 7.92 MeV, which is in the neighborhood of the energies of all the transitions investigated. It dominates the high-energy region of the spectrum, being 354 keV above the transition to the first excited state and being 12 times as strong as this transition. In a series of normalization runs of the type just described, the absolute intensity of this ground-state transition was determined by comparison with the ground-state transition from the 33.5-eV resonance in Hg^{199} , which had a previously measured³² absolute intensity of 0.035 photons/capture.

The value obtained for the ground-state transition in Pt^{195} is 0.048 photons/capture. All values of partial radiation widths determined in this work depend directly on this number. Future refinements in this quantity will have the effect of multiplying all partial widths reported here by the same correction factor. It is estimated that the value 0.048 photons/capture may be uncertain by as much as $\pm 30\%$, most of this uncertainty arising from uncertainties in the intensity of the ground-state transition from the 33.5-eV resonance in Hg^{199} . This source of uncertainty, which has a systematic effect on all the derived partial widths, is not included explicitly in the uncertainties quoted for the various partial widths in this report.

The constant C is now determined by using $I = 0.048$ photons/capture in Eq. (74) with the other quantities determined as discussed above. Table IV shows the results of the normalization runs on all nuclides investigated. Two series of runs were made. The results for run 1 were obtained with the equipment shown in Fig. 4, while the results for run 2 were obtained with the equipment shown in Fig. 3. It should be mentioned in this regard that the 4-spectrum analyzer is completely adequate for the normalization runs, since only one or two resonances need to be investigated, and

TABLE IV. Normalization data for all nuclides investigated. The first column lists the target isotope. The second column lists the energy of the neutron resonance selected for the normalization. The next four columns list parameters appearing in Eqs. (74) and (75). The seventh column gives the excitation number of the final state involved in the transition having relative intensity a_k . The eighth column lists the absolute intensity, in photons per capture, of the transition to the k th excited state. The ninth column gives the total radiation width, in units of 10^{-3} eV, for the resonance with energy E_n ; and the tenth column lists the partial radiation width, also in units of 10^{-3} eV.

Isotope	E_n (eV)	Γ_n/Γ	M $\ddagger$ f	J_γ	a_k	k	I_k	Γ_γ	Γ_k
<u>Series 1</u>									
Nd ¹⁴³	56	0.138	a	0.715	a	—	0.0058	150	0.87
Nd ¹⁴⁵	4.4	0.029	—	0.250	—	—	—	—	—
Nd ¹⁴⁵	43.1	—	0.41	0.548 ^b	1.52 ± 0.07	2	0.0819	48	3.93
Sm ¹⁴⁷	3.4	0.015	5.80	0.265	0.31 ± 0.03	1	0.0024	61	0.15
Sm ¹⁴⁹	5.0	0.029	a	0.324	a	—	0.0016	67	0.11
Gd ¹⁵⁵	2.0	0.003	2.18	0.219	0.29 ± 0.04	0+1	0.0071	110	0.78
Yb ¹⁷¹	13.5	0.111	a	0.518	a	—	0.0016	80	0.13
Yb ¹⁷³	4.6	0.004	8.10	0.119	3.08 ± 0.11	1+2	0.0381	80	3.05
Pt ¹⁹⁵	11.9	0.061	1.35	1.940	10.51 ± 0.13	0	0.048000	155	7.44
<u>Series 2</u>									
Hf ¹⁷⁷	2.4	0.132	9.85	0.632	4.17 ± 0.11	?	0.0264	60	1.58
W ¹⁸³	7.6	0.029	2.00	0.464	1.58 ± 0.04	0	0.0671	67	4.50
Hg ¹⁹⁹	33.5	0.138	2.51	1.814	4.01 ± 0.10	0	0.0350	289	10.1
Hg ²⁰¹	70.8	0.020	1.35	0.249	1.69 ± 0.06	1	0.1981	456	90.3
Pt ¹⁹⁵	11.9	0.061	3.81	2.115	9.82 ± 0.15	0	0.048000	155	7.44

^a Equation (78) was used to calculate the absolute intensity per unit normalized relative intensity.

^b Calculated by Eq. (77) by use of the J_γ determined for the 4.4-eV resonance.

since the relative intensities of the high-energy transitions in the observed spectra have already been determined.

For neodymium, the only normalization spectrum obtained was that in the 43-eV resonance. For this resonance $\Gamma_n/\Gamma = 0.56$, so that Eq. (75) cannot be used for the capture integral. The spectrum of N_γ versus neutron energy for the 4.4-eV resonance was therefore measured at the same time and Eq. (77) was used to calculate $J_{\gamma 1}$ for the 43-eV resonance. Equation (75) was used to calculate $J_{\gamma 2}$ for the 4.4-eV resonance, which has $\Gamma_n/\Gamma = 0.029$. The value of J_γ for the 43-eV resonance which is obtained from Eq. (75) by use of the resonance parameters listed in reference 18 is 2.10 eV. The discrepancy between this and the value of 0.548 eV listed in Table IV is most probably caused by quite large errors in the reported parameters. For example, the total radiation width is given as $200 \pm 110 \times 10^{-3}$ eV for the 43-eV resonance and as $48 \pm 5 \times 10^{-3}$ eV for the 4.4-eV resonance. Based on existing knowledge of fluctuations in the total radiation widths,¹¹ actual fluctuations of this size are not expected for neodymium. A value of Γ_γ on the order of 50×10^{-3} eV for the 43-eV resonance would therefore be more reasonable and would remove the discrepancy between the values of J_γ calculated by the two different methods.

Equation (78) was used to calculate the absolute intensity corresponding to unit normalized relative intensity in the 56-eV resonance, and hence in all the resonances in Nd^{143} .

(e) Uncertainties

The standard errors in the relative intensities a_k depend on the standard errors (Δn_{ir}) in the individual data points in the spectral region considered, as given by Eq. (59), and on the goodness of fit of the spectra to the "true" spectra defined in Eq. (57). The standard errors are calculated by the usual equation

$$(79) \quad (\Delta a_{kr})^2 = \frac{\chi_r^2}{\nu - 1} (H_r)_{jj}^{-1},$$

where $(H_r)_{jj}^{-1}$ are the diagonal elements of the inverse of the matrix defined by

$$(80) \quad (H_r)_{jm} = \sum_i (\Delta n_{ir})^{-2} R(E_i, E_{kj}) R(E_i, E_{km}),$$

and $\nu = N - M$ is the number of degrees of freedom in the appropriate χ^2 distribution, N being the number of data points in the spectral region considered and M the number of lines used to fit the spectrum. The quantity χ_r^2 is obtained from Eq. (58) by substituting the best-fit values of a_{kr} obtained in the minimization of Eq. (61). Whenever $\chi_r^2/(\nu - 1) < 1$, it is replaced by unity in Eq. (79) for purposes of calculating errors.

There may also be systematic errors in the derived relative intensities because of the failure of Eq. (60). However, the existence of such errors should be exhibited by the occurrence of systematically large values of χ_r^2 for spectra containing strong lines with energies greatly different from 7.64 MeV, the energy of the iron transition used as the reference spectrum. No such systematic effect was observed for any of the spectra considered in this work. The 6.25-MeV transition in Nd^{143} represents the largest deviation from 7.64 MeV of any case considered. Note in Fig. 10 the quite acceptable fits obtained for the various spectra in Nd^{143} . Particularly note the fit obtained for the spectrum in the resonance at 132 eV, which has good statistical accuracy and which is dominated by the 6.25-MeV transition. Thus there is a complete lack of evidence of any significant change in the line shape. Therefore, this source of error may be ignored, especially since its effects are already largely accounted for in the values of χ_r^2 obtained.

In calculating the normalized relative intensities by Eq. (65), errors may be introduced as the result of fluctuations in the gamma-ray multiplicity, as reflected by the failure of Eq. (64). This effect has been investigated only for resonances in Pt^{195} , where it was found³¹ that Eq. (65) is valid within about $\pm 5\%$. For lack of any exact information, a statistical uncertainty of $\pm 5\%$ is included in the average values of the partial widths from this source.

In the calculation of the absolute intensities by Eq. (74), the quantities M , Φ , and f are determined with uncertainties of less than $\pm 1\%$, so that errors from these sources may be ignored. The uncertainty in I_k arises predominantly from uncertainties in the neutron resonance parameters used to calculate J_γ from Eq. (75). The parameters used were taken from reference 18. The uncertainty that errors in these parameters necessarily introduce is considerably diminished by the fact that Eqs. (77) and (78) permit the resonance whose parameters are best determined to be selected for the normalization in a given nuclide, provided that this resonance is otherwise suitable. However, in many cases it still is necessary to use parameters with large quoted uncertainties.

The expression for the uncertainty in J_γ due to uncertainties in the resonance parameters may be obtained in a straightforward way but is complicated by the fact that the parameters are interdependent through the expressions $\sigma_0 = 4\pi \chi_0^2 g \Gamma_n / \Gamma$, and $\Gamma = \Gamma_n + \Gamma_\gamma$. Further, the parameters listed in reference 18 are Γ_n and Γ_γ , whereas the directly measured quantities in the area analysis used to obtain the parameters are σ_0 and Γ . Thus it is difficult if not impossible to assess the errors in σ_0 and Γ from the quoted errors in Γ_n and Γ_γ . However, a simple expression, adequate for estimating errors, may be obtained by the following means. First, since Γ_n / Γ is small for the resonances considered, errors in the factor $(1 + \Gamma_n / \Gamma)$ are ignored. Next, σ_γ / σ is approximated

by Γ_Y/Γ and removed from the integral. The remaining integral is just the area A_E obtained in the area analysis of the total-cross-section data. The error in the resulting expression

$$(81) \quad I_k = C \frac{a_k}{M \Phi f \left(1 + \frac{\Gamma_n}{\Gamma}\right) \frac{\Gamma_Y}{\Gamma} A_E}$$

is given by

$$(82) \quad \left(\frac{\Delta I_k}{I_k}\right)^2 = \left(\frac{\Delta a_k}{a_k}\right)^2 + \left(\frac{\Delta \Gamma_Y/\Gamma}{\Gamma_Y/\Gamma}\right)^2 + \left(\frac{\Delta A_E}{A_E}\right)^2 + \left(\frac{\Delta C}{C}\right)^2.$$

For strong isolated resonances of the type used for normalization, these areas may easily be obtained with errors from all sources of less than $\pm 10\%$. The areas calculated with parameters derived from the same areas should therefore have comparable precision. Therefore, unless other information is available, the value $(\Delta A_E/A_E) = 0.1$ is used as being probably an overestimate of the error from this source. The error in the remaining factor Γ_Y/Γ is small for resonances with small Γ_n/Γ , being equal to $\Delta(\Gamma_n/\Gamma)$, and may be ignored in comparison with $(\Delta A_E/A_E)$. Thus the total standard error in the absolute intensity, as calculated by Eq. (74), is

$$(83) \quad (\Delta I_k/I_k)^2 = (\Delta a_k/a_k)^2 + (\Delta A_E/A_E)^2 + (\Delta C/C)^2.$$

The errors in the partial radiation widths $\Gamma_k = I_k \Gamma_Y$ are somewhat greater. Note that Γ_Y cancels when this is substituted in Eq. (81) so that

$$(84) \quad (\Delta \Gamma_k/\Gamma_k)^2 = (\Delta a_k/a_k)^2 + (\Delta \Gamma/\Gamma)^2 + (\Delta A_E/A_E)^2 + (\Delta C/C)^2.$$

The error in the constant C is taken to result only from errors in the relative intensities measured for platinum during the normalization runs, and does not include the uncertainty in the absolute intensity of 0.048 photons/capture to which all results are normalized.

The total experimental error in the derived resonance-average values of the partial radiation widths, from all sources considered above, is typically on the order of $\pm 20\%$. Unfortunately, this error is small compared with the uncertainty in the true mean values of the partial radiation widths as determined by the data. This added uncertainty results from the Porter-Thomas distribution of widths as discussed in Sec. I(c). To give a numerical example, suppose the resonance-average value of some partial radiation width, determined by averaging over eleven resonances, is $(5 \pm 1) \times 10^{-3}$ eV. From Fig. 2, the 90% confidence factors for $n = 11$ are 2 and $1/1.6$. Thus if only this source of uncertainty is included, the experimental result should be expressed as $5^{+5}_{-1.9} \times 10^{-3}$ eV. If, for illustrative purposes only, this is expressed as $(6.55 \pm 3.45) \times 10^{-3}$ eV, and if the distribution of experimental values is assumed to be normal, then the 20% uncertainty in the experimental average increases the total uncertainty in the determination of the true mean value to $(6.55 \pm 3.69) \times 10^{-3}$ eV or $5^{+5.2}_{-2.2} \times 10^{-3}$ eV. This, of course, is not a valid procedure, but it illustrates the fact that the usual experimental errors may usually be ignored in comparison with the Porter-Thomas uncertainties.

There is one more source of uncertainty which is by no means negligible, namely, the uncertainty due to lack of knowledge of the spins of the various resonances. Some spins are known from previous work¹⁸ and many more are determined with considerable confidence by the work reported here. For example, in a nucleus for which the resonance spins may be 3^- or 4^- , the observation of a strong transition to a final 2^+ state is good evidence that the

resonance spin is 3^- . Similarly, a strong transition to a final 4^+ state from a resonance which may have spin 2^- or 3^- indicates that the resonance spin is 3^- , and so on. However, the absence of a transition to a given final state tells nothing of the resonance spin, since the transition may be one of the large number of weak transitions expected on the basis of the Porter-Thomas distribution. Therefore, unless there is evidence to the contrary, it is assumed that the spins of the resonances investigated are distributed at random between the two possible values in such a way that the density of states is proportional to $2J + 1$,^{*} where $J = I \pm \frac{1}{2}$. Now in this case, if N resonances are selected at random, the expectation values of m_+ and m_- , the numbers of levels with spin $I + \frac{1}{2}$ and $I - \frac{1}{2}$, respectively, are

$$\langle m_+ \rangle = N \frac{I + \frac{1}{2}}{2I + 1}, \quad (85)$$

$$\langle m_- \rangle = N \frac{I - \frac{1}{2}}{2I + 1}.$$

The standard deviation in the distribution of both quantities is

$$\Delta m_{\pm} = \frac{[NI(I + \frac{1}{2})]^{1/2}}{2I + 1}. \quad (86)$$

Thus if, for example, transitions to a given final state are allowed from resonances with spin $I + \frac{1}{2}$ but not from resonances with spin $I - \frac{1}{2}$, then the average value of the partial radiation width is calculated by

$$\overline{\Gamma}_k = \frac{\sum_r \Gamma_{kr}}{\langle m_+ \rangle}, \quad (87)$$

* This dependence of the level density on spin is well established both theoretically and experimentally. See, for example, references 23 and 24.

and the uncertainty in this quantity (from all sources except Porter-Thomas fluctuations) is

$$(88) \quad \frac{\Delta \bar{\Gamma}_k}{\bar{\Gamma}_k} = \frac{\sum_r (A_{kr})^2}{\sum_r A_{kr}^2} + \frac{I(I+1)}{N(2I+1)}.$$

A slightly different procedure is required if transitions to a given final state are allowed from resonances of both spins. The transitions occur with different average probabilities from resonances with different spin because the transition probabilities depend on the spacing of levels that have the same spin and parity as the radiating state. This factor, being the reciprocal of the level density, introduces an inverse dependence on $(2J+1)$. The product of $(2J+1)$ and the average value of the partial width for the k th transition will be denoted by G_k . The experimental average obtained by averaging over all resonances is then given by

$$(89) \quad \bar{\Gamma}_{ke} = \frac{1}{m_+ + m_-} \left[\sum_{r=1}^{m_+} \frac{G_k}{2I+2} x_{kr} + \sum_{r=1}^{m_-} \frac{G_k}{2I} x_{kr} \right],$$

where x_{kr} is the ratio obtained when the partial width of the k th transition in the r th resonance is divided by the mean value of this partial width and is distributed with a Porter-Thomas distribution. Since the mean value of x_{kr} is unity, the expectation value of $\sum_{r=1}^m x_{kr}$ is equal to the expectation value of m . The use of Eqs. (85) thus reduces (89) to

$$(90) \quad \langle \bar{\Gamma}_{ke} \rangle = G_k / (2I+1).$$

An analytic form for the distribution of the experimental averages has not been obtained. Instead, a Monte-Carlo calculation was performed to actually reproduce a distribution of quantities calculated

by Eq. (89). The numbers m_+ and m_- were chosen from the appropriate binomial distribution and values of x_{kr} were chosen from the Porter-Thomas distribution. Equation (89) was then evaluated for a total of 10 000 combinations of numbers chosen randomly in this way and the resulting distribution function was plotted. The 90% confidence values in the experimental averages were taken as the 10% and 90% points of this curve. The 90% confidence factors in the experimental averages determined by Eq. (87) were also found by a similar Monte Carlo calculation.

Neodymium represents a case in which the data indicate that the binomial distribution of spin states is not a good approximation for the limited number of resonances observed. Referring to Table II, at least three out of the five resonances in Nd^{143} have spin 3^- (since strong transitions to the 2^+ first excited state are seen in their spectra) and, for the same reason, at least four of the five resonances in Nd^{145} have spin 3^- . Consider first the case of Nd^{145} . The resonance at 43 eV could have a spin of either 3^- or 4^- and the relative probabilities of the two possibilities are not changed by the fact that the spins of the other resonances are known. Thus its spin is 4^- with probability 9/16, and in this case the average value of the normalized relative intensity of the transition to the first excited state is $0.88^{+2.4}_{-0.4}$, where the uncertainties are taken from Fig. 2 with $n = 4$. The spin is 3^- with probability 7/16 and in this case the average for A_1 is $0.72^{+1.5}_{-0.3}$, where the uncertainties are now determined from Fig. 2 with $n = 5$. The experimental value is now taken as the weighted average of these two individual averages with uncertainties determined by the weighted averages of the minima and maxima given for the individual averages. The result is $A_1 = 0.8^{+2.1}_{-0.4}$.

The data for Nd^{143} are treated in the same way, except that now there are two resonances with unknown spin. The probability

that the spin of both resonances is 4 is $81/256$. The probability that one resonance has spin 3 and the other spin 4 is $63/256$ (with the same probability for the same combination of spins but with the assignment reversed) and the probability that both resonances have spin 3 is $49/256$. The weighted average obtained for A_1 is $2.0^{+5.6}_{-1.0}$, the uncertainties being determined as before. The averages for the other transitions are obtained in a similar way, except that now additional spin factors, such as occur in Eq. (89), must be included when the average is taken over resonances assumed to have different spins. The result, in this case, is of the form given in Eq. (90). For all spin $-\frac{7}{2}$ nuclides investigated, the result as obtained in the form of Eq. (90) is referred to resonances with spin 3^- by multiplication by $(2I + 1)/(2J + 1) = 8/7$. The final results obtained for the resonance averages of the normalized relative intensities for transitions from resonances of spin 3^- are listed in Table II. The corresponding average partial radiation widths, obtained by use of the normalization data shown in Table IV, are also given in Table II.

IV. RESULTS

In this section the experimental data obtained with the equipment described in Sec. II will be given. Unless otherwise stated, the techniques described in detail in Sec. III were used to obtain the resonance-average values of the partial radiation widths $\langle \Gamma_k \rangle$.

(a) Neodymium

The sample used for the measurements consisted of 506 g of Nd_2O_3 powder mixed with aluminum dust and packed in a sample holder with dimensions $8 \times 8 \times 1/4$ in. thick. The sample thickness in the direction normal to the face of the sample holder was 0.00453 Nd atoms/barn. The runs made were described in Sec. II. The total running time was 80 hr. The final values obtained for $\langle \Gamma_k \rangle$ are given in Table II for three transitions in Nd^{143} and two transitions in Nd^{145} .

(b) Samarium

The sample consisted of 385 g of Sm_2O_3 powder mixed with aluminum dust and packed in the standard $8 \times 8 \times 1/4$ in. sample holder. The sample thickness in the direction normal to the face of the sample holder was 0.00323 Gd atoms/barn.

Gamma-ray spectra were observed in the neutron resonances between 2.5 and 17 eV, the neutron time-channel widths being 3 μsec and the rotor speed being 5500 rpm. The total running time was 23 hr. This run was included as part of the first series of normalization runs. The intensity of all detected gamma rays as a function of neutron energy is plotted in the upper graph in Fig. 14. Previously undetected resonances were observed at the

Fig. 14. Plot of the intensity of all detected gamma rays versus neutron energy for samarium. The resonances are labeled with the target nuclide in which they occur (reference 18).

positions of the arrows labeled with question marks. Gamma-ray spectra were observed in neutron resonances between 18 and 49 eV with neutron time-channel widths of 0.67 μ sec and a rotor speed of 16 000 rpm. The total running time was 58 hr. The intensity of all detected gamma rays is plotted as a function of neutron energy in the lower graph of Fig. 14. This run was made immediately following the run on the lower energy neutron resonances without altering the placement of the sample or any of the electronic settings. The relation between the absolute intensities and the normalized relative intensities obtained for the run at low neutron energies is therefore assumed to hold also for the run at high neutron energies.

A gamma-ray spectrum was obtained at each of the neutron energy intervals indicated by pedestals in Fig. 14. The energies of the low-lying levels³³ which are fed by direct transitions from the neutron resonances are shown in Fig. 15. From this diagram it is clear that data of very high quality would be required in order to obtain reasonably precise values of the relative intensities of transitions to states in the vicinity of the second excited state in Sm^{150} . Unfortunately, the relative intensities of all the transitions indicated in Fig. 15 are found to be quite small in the spectra obtained in all neutron energy intervals examined except those labeled 1, 2, and 3 in Fig. 14. Therefore, $\langle \Gamma_k \rangle$ was determined only for transitions to the first excited state in Sm^{150} . In the case of Sm^{148} , the spin of the

Fig. 15. The energies of low-lying samarium levels which are fed by direct transitions from slow-neutron resonances (references 33 and 29).

second excited state is unknown. Further, since Sm^{148} is a spherical nucleus, it is expected that the familiar $(0^+, 2^+, 4^+)$ triplet should be found in the vicinity of the second excited state.³⁴ Therefore, $\langle \Gamma_k \rangle$ for Sm^{148} also was determined only for transitions to the first excited state. Values of the relative intensity and the normalized relative intensity for this transition are given in Table V.

The intensities of transitions to the first excited state in Sm^{150} from neutron resonances in Sm^{149} at 0.0976, 0.873, 4.98, and 6.48 eV are not included in Table V or in the calculation of the resonance averages since these resonances are known³⁵ to have spin 4^- . The intensities determined for these resonances (zero within statistical errors) are consistent with this spin assignment. Also not included in Table V are the intensities for transitions from the resonances at 39.9 eV in Sm^{147} and at 40.2 eV in Sm^{149} . These resonances were not sufficiently resolved from each other to allow normalization areas to be determined. They were, however, well enough resolved from the resonance at 41.5 eV in Sm^{149} that the method discussed on page 59 could be applied to obtain the normalized relative intensity in this case. The resonance shape and peak position were determined for the 41.5-eV resonance by observing the intensity of the transition to the first excited state as a function of neutron time of flight.

The resonance-average values of the normalized relative intensities were determined by finding the sum of all values of A_1 obtained for each nuclide and dividing this sum by the expected number of resonances with spin 3, given by $\langle m_- \rangle$ in Eq. (85). The 90% confidence factors were determined by a Monte Carlo calculation of the distribution of this quantity.

TABLE V. Relative intensities a_1 and normalized relative intensities A_1 for transitions to the first excited states in samarium-148 and 150 from neutron resonances in samarium-147 and 149. Also given are the resonance average values $\langle A_1 \rangle$ of the normalized relative intensities and the corresponding partial radiation widths $\langle \Gamma_1 \rangle$.

Sm ¹⁴⁷			
Interval	E _n (eV)	a ₁	A ₁
1	3.4	0.28 ± 0.03	1.00 ± 0.11
	18.3	0.00 ± 0.09	0.01 ± 0.16
	27.1	0.05 ± 0.04	0.32 ± 0.27
2	29.8	0.63 ± 0.05	2.55 ± 0.21
	32.1	0.07 ± 0.05	0.25 ± 0.18
		$\langle A_1 \rangle$	= 1.9 ^{+11.6} _{-1.1} (relative units)
		$\langle \Gamma_1 \rangle$	= 0.28 ^{+1.72} _{-0.10} · 10 ⁻³ eV
Sm ¹⁴⁹			
Interval	E _n (eV)	a ₁	A ₁
	12.2	0.05 ± 0.06	0.77 ± 1.06
	14.8	0.11 ± 0.09	1.02 ± 0.87
	23.2	-0.02 ± 0.08	-0.34 ± 1.38
	25.2	0.21 ± 0.08	0.88 ± 0.34
	26.1	0.03 ± 0.07	0.42 ± 0.98
	30.9	0.01 ± 0.06	0.14 ± 0.85
	34.1	0.04 ± 0.08	0.32 ± 0.63
3	41.5	^a	3.46 ± 0.38
	45.0	0.00 ± 0.10	0.00 ± 0.30
		$\langle A_1 \rangle$	= 1.7 ^{+3.5} _{-0.9} (relative units)
		$\langle \Gamma_1 \rangle$	= 0.18 ^{+0.37} _{-0.09} · 10 ⁻³ eV

^a See text for a discussion of the method used to obtain A_1 .

(c) Gadolinium

The total intensity of detected gamma rays is plotted as a function of neutron energy in Fig. 16 for the measurements performed on gadolinium. The sample consisted of 450 g of Gd_2O_3 powder mixed with aluminum dust and packed in a standard $8 \times 8 \times 1/4$ in. sample holder. The sample thickness in the direction normal to the face of the sample holder was 0.00374 Gd atoms/barn. Data for neutron energies between 1.4 and 13 eV were obtained with the rotor turning at 5660 rpm and with neutron time-channel widths of 4 μsec . The total running time was 31 hr. The neutron energy region from 12 to 50 eV was investigated with the rotor turning at 16 000 rpm and with neutron time-channel widths of 1 μsec . The total time required for this run was 51 hr.

Gamma-ray spectra were observed in all neutron energy intervals indicated by pedestals in Fig. 16. These spectra were analyzed to obtain the intensities of only the transitions to the ground state and the first excited state in Gd^{156} , shown in Fig. 17. No attempt was made to analyze the spectra observed in the resonances in Gd^{157} for two reasons. First, only four neutron resonances were observed in Gd^{157} so that resonance-average values of the partial radiation widths would be quite uncertain because of Porter-Thomas fluctuations. Second, normalization of the relative intensities is complicated for three of the resonances because of resolution difficulties. For example, the resonance in Gd^{157} at 44.3 eV is not separated from a resonance in Gd^{155} , that also has a resonance energy of 44.3 eV. The resonance at 17.1 eV in Gd^{157} was found to be unresolved from a new resonance at about 16.6 eV whose gamma-ray spectrum indicates that it is probably also in Gd^{157} . The intensities obtained for the spectrum in the 17.1-eV resonance could not be normalized because of the uncertain isotopic assignment of the

Fig. 16. The intensity of all detected gamma rays versus neutron energy for gadolinium.

new resonance. Similarly, the spectrum observed in the 2.90-eV resonance in Gd^{157} could not be normalized because of the suggestion of previously undetected and unidentified resonances in the neutron energy range from 2.5 to 3 eV. (For the same reason, no attempt was made to analyze the spectra observed in the 2.64-eV resonance in Gd^{155} .) The resonance at 25.5 eV is well isolated. However, its isotopic assignment has not been previously reported. Its gamma-ray spectrum contains a prominent transition at about 7.92 MeV, which may be interpreted as the ground-state transition in Gd^{158} . However, it could also be interpreted as the transition to the 0^+ state²⁹ at 540 keV in Gd^{156} . Thus no normalized relative intensities could be unambiguously determined for spectra in resonances observed in Gd^{157} .

The intensities of the transitions to the 0^+ state at 540 keV in Gd^{156} , shown in Fig. 17, were not determined because of the possibility of confusing this transition with the ground-state transition from an unreported resonance in Gd^{157} . The relative intensities and normalized relative intensities obtained for the transitions to the ground state and first excited state in Gd^{156} are given in Table VI. Also given are the resonance-average values $\langle A_k \rangle$ and $\langle \Gamma_k \rangle$. Neither the table nor the calculation of the averages include values obtained for the spectra in resonances that were so weak that the error in the normalized relative

Fig. 17. The energies of low-lying levels in gadolinium-156 which are fed by direct transitions from slow-neutron resonances (reference 29).

TABLE VI. Relative intensities and normalized relative intensities for transitions to the ground state and first excited state in gadolinium-156. Also shown are the resonance averages A_k and Γ_k .

Gd^{155}					
Interval	E_n (eV)	a_0	a_1	A_0	A_1
1	2.01	0.55 ± 0.12	0.17 ± 0.14	1.71 ± 0.25	0.50 ± 0.28
2	6.49	0.21 ± 0.04	0.33 ± 0.04	0.27 ± 0.05	0.43 ± 0.05
3	7.95	0.01 ± 0.04	0.08 ± 0.04	0.03 ± 0.11	0.21 ± 0.11
4	11.67	a	a	7.00 ± 0.80	3.00 ± 0.90
5	12.10	a	a	0.30 ± 0.25	0.04 ± 0.25
6	14.7	-0.05 ± 0.09	0.21 ± 0.10	-0.13 ± 0.23	0.55 ± 0.26
7	20.2	0.01 ± 0.02	0.01 ± 0.02	0.11 ± 0.18	0.06 ± 0.20
8	21.3	-0.04 ± 0.04	0.45 ± 0.04	-0.22 ± 0.22	2.75 ± 0.23
10	29.9	a	a	0.02 ± 0.40	-0.13 ± 0.38
11	30.5	a	a	-0.03 ± 0.42	0.80 ± 0.39
14	37.3	-0.02 ± 0.10	0.32 ± 0.11	-0.12 ± 0.60	1.92 ± 0.66
15	44.3	a	a	4.28 ± 0.62	10.68 ± 0.67
$\langle A_k \rangle$ (relative units)		$2.9^{+13.2}_{-1.4}$		$2.3^{+2.2}_{-0.8}$	
$\langle \Gamma_k \rangle (10^{-3} \text{ eV})$		$1.0^{+4.7}_{-0.5}$		$0.8^{+0.8}_{-0.4}$	

^a See text for discussion of the method used to determine the normalized relative intensities.

intensities exceeded one unit. If all normalized relative intensities obtained were included in the calculation of the averages, the averages would be changed very little, but the uncertainties from counting statistics would be increased so much that they would no longer be negligible in comparison to the uncertainties from the Porter-Thomas fluctuations. The methods used to obtain the averages and the 90% confidence factors have been discussed previously.

(d) Ytterbium

Previous investigations¹⁸ of the neutron resonances in ytterbium have yielded the energy and isotopic identification of only a single resonance, the 0.60-eV resonance in Yb¹⁶⁸. Therefore, two additional types of runs were required in this case in order to determine the energies of the neutron resonances, establish the isotope in which each investigated resonance occurs, and determine the resonance parameters of at least one resonance in each nuclide investigated.

The isotopic identification of the various resonances was performed by means of the "sum-coincidence" method.¹² In this method the sums of the pulse heights of coincident gamma rays are recorded. The three-parameter analyzer was used to record the data with the detector arrangement as shown in Fig. 3. The maximum energy observed in the sum spectra will be, of course, the maximum energy that can be emitted in a two-step cascade from the neutron resonances. For resonances in all of the ytterbium isotopes this is the neutron binding energy in the compound nuclei formed by neutron capture. Thus the maximum energy observed in the sum spectra will be 8.02 MeV for resonances in Yb¹⁷¹, 7.49 MeV for resonances in Yb¹⁷³ (see Fig. 19), and 6.7 MeV or less for resonances in the even-mass nuclei.¹⁴ These differences in

energy may be quite easily detected so that a run of only 24 hr was sufficient to unambiguously determine the isotopic identification of all resonances investigated. The results of this investigation are given in Tables VII and VIII.

In order to determine the parameters of resonances in Yb^{171} and Yb^{173} a transmission measurement was performed.³⁶ A flight path of 60 m was used so that the resolution width was 30 nsec/m. The 4.6-eV resonance was well enough resolved that all of the resonance parameters could be determined. The spin was assigned on the basis of the strength of the radiative transition from the resonance to the 4^+ second excited state. The value of $(0.080 \pm 0.010) \times 10^{-3}$ eV obtained for the total radiation width of the 4.6-eV resonance was assumed for the other resonances as well. The results of the area analysis of the transmission data are shown in Table IX.

The sample used for all of the runs on ytterbium consisted of 400 g of Yb_2O_3 powder mixed with aluminum dust and packed in a standard $8 \times 8 \times 1/4$ in. sample holder. The sample thickness in the direction normal to the face of the sample holder was 0.00298 Yb atoms/barn. The gamma-ray spectra in the neutron resonances between 4.1 and 22 eV were investigated with the rotor turning at 5075 rpm and with neutron time-channel widths of 2 μsec . The total running time was 59 hr. The neutron resonances between 22 and 200 eV were investigated with the rotor turning at 15 000 rpm and with neutron time-channel widths of 1 μsec . The total time required for this run was 60 hr. The intensity of all detected gamma rays is shown as a function of neutron energy in Fig. 18 for both runs.

Gamma-ray spectra were observed in all neutron energy intervals indicated by pedestals in Fig. 18. The energies²⁹

TABLE VII. Relative intensities and normalized relative intensities of the highest energy transitions in ytterbium-172 and 174, from slow-neutron resonances in ytterbium-171 and 173. Also shown are the resonance averages $\langle \Gamma_k \rangle$ and $\langle A_k \rangle$.

Yb ¹⁷¹					
Interval	E _n (eV)	a ₀	a ₁	A ₀	A ₁
2	13.5	0.55 ± 0.14	0.01 ± 0.16	0.76 ± 0.19	0.01 ± 0.22
4	29.6	-0.04 ± 0.11	0.45 ± 0.12	-0.26 ± 0.71	2.93 ± 0.75
6	36.5	-0.12 ± 0.12	0.69 ± 0.13	-1.47 ± 1.42	8.25 ± 1.49
8	44.0	0.08 ± 0.09	0.02 ± 0.10	0.59 ± 0.72	0.16 ± 0.76
10	57.1	a	a	0.55 ± 0.56	6.67 ± 0.58
10	58.5	a	a	0.24 ± 0.45	0.63 ± 0.47
11	86.9	a	a	-5.1 ± 4.7	2.6 ± 5.0
11	89.1	a	a	16.2 ± 4.6	-1.9 ± 4.8
13	113	a	a	3.29 ± 0.48	1.65 ± 0.51
14	117	a	a	3.93 ± 0.69	1.18 ± 0.72
15	134	0.81 ± 0.08	1.24 ± 0.09	10.31 ± 1.07	15.81 ± 1.13
				⟨A _k ⟩ (relative units)	4.0 ^{+4.6} _{-1.6} 4.4 ^{+5.0} _{-1.8}
				⟨Γ _k ⟩ (10 ⁻³ eV)	0.51 ^{+0.58} _{-0.20} 0.57 ^{+0.65} _{-0.23}

Yb ¹⁷³					
Interval	E _n (eV)	a ₁	a ₂	A ₁	A ₂
1	4.6	5.07 ± 0.11	4.45 ± 0.13	14.68 ± 0.31	12.88 ± 0.38
3	18.4	2.52 ± 0.17	0.34 ± 0.21	2.90 ± 0.20	0.39 ± 0.24
5	33.1	1.61 ± 0.07	0.08 ± 0.08	4.43 ± 0.19	0.21 ± 0.21
7	37.7	0.58 ± 0.05	0.24 ± 0.05	2.03 ± 0.16	0.83 ± 0.19
9	48.2	0.25 ± 0.05	-0.05 ± 0.06	1.64 ± 0.32	-0.31 ± 0.38
12	102	0.48 ± 0.10	0.19 ± 0.19	4.69 ± 1.03	1.96 ± 1.20
				⟨A _k ⟩ (relative units)	5.1 ^{+8.8} _{-2.2} 4.7 ^{+45.0} _{-2.3}
				⟨Γ _k ⟩ (10 ⁻³ eV)	0.57 ^{+0.94} _{-0.25} 0.51 ^{+4.98} _{-0.25}

^a See text for discussion of methods used to obtain A_k .

TABLE VIII. Energies of resonances assigned to ytterbium-171 (column 171), 173 (column 173) or to an even-mass isotope of ytterbium (column e-e). The spectra in these resonances were not examined because the resonances are insufficiently resolved to allow normalization areas to be determined.

E_n (ev)		
171	173	e-e
8.1	63	8.3
65	157	42
72		75
83		79
182		145
205		

TABLE IX. Resonance parameters determined for neutron resonances in ytterbium by transmission measurement. The spins are assigned on the basis of the strength of transitions observed in the gamma-ray spectra.

Isotope	E_0 (eV)	σ_0 (barns)	Γ (10^{-3} eV)	$2g\Gamma_n$ (10^{-3} eV)	J
173	4.6	965 ± 80	80.3 ± 0.7	0.27 ± 0.02	3^-
171	13.5	9070 ± 800	85.4 ± 2.0	7.9 ± 0.8	1^-
173	18.4	$22\,400 \pm 2000$	118 ± 15	36 ± 3	(3^-)

of low-lying levels that are fed by direct transitions from the slow-neutron resonances are shown in Fig. 19. The gamma-ray spectra observed in the various neutron resonances are shown in Figs. 20 and 21. The spectra shown in Fig. 20, obtained in the neutron resonances in Yb^{171} , are similar to the spectra observed in the resonances in W^{183} . The spectra shown in Fig. 21 for the resonances in Yb^{173} are similar to the spectra seen in the resonances in Gd^{155} and Hf^{177} . These spectra, particularly those shown in Fig. 20, afford a good illustration of the necessity of a priori knowledge that the energy calibrations are identical for all spectra. That is, were it

not for the fact that all the spectra shown (with the exception of the spectrum obtained in interval 2) were measured simultaneously, a shift in the apparent peak position caused by fluctuations in the relative intensities of the two highest energy transitions would be virtually impossible to distinguish from a shift caused by a drift of the gain of the scintillation amplifier or of the photomultiplier voltages.

The relative intensities and normalized relative intensities obtained for the two highest energy transitions in the neutron-resonance-capture spectra in ytterbium are given in Table VII. The resonance averages $\langle A_k \rangle$ and $\langle \Gamma_k \rangle$, also given in Table VII, were obtained by methods already discussed. The spectra taken in intervals 10, 11, 13,

Fig. 19. Low-lying levels in ytterbium fed by primary transitions from slow-neutron resonances in ytterbium-171 and 173. The energy of the second excited state in ytterbium-174 is an estimate based on the systematics of levels in neighboring nuclides.

Fig. 20. Resonance-capture gamma-ray spectra for ytterbium-171.

Fig. 21. Resonance-capture gamma-ray spectra for ytterbium-173.

and 14 were normalized after unpeeling the $N_{\gamma}(E_n)$ spectrum in the manner described in Sec. III(c). This unpeeling was made possible in intervals 10 and 11 by the fact that the gamma-ray spectrum of only one of the two resonances involved in each case contains strong high-energy transitions. Thus a measurement of the intensity of high-energy gamma rays as a function of neutron energy yields the peak position and shape of one resonance in each case. The other resonance shape was then assumed to be symmetric but not necessarily of the same width. The unpeeling of the resonances at 113 and 117 eV (intervals 13 and 14) was performed by a least-squares fit of the $N_{\gamma}(E_n)$ spectrum to the neutron energy resolution function, which was taken as given by the well-separated peak at 88 eV (interval 12).

(e) Hafnium

The hafnium sample used for the main runs consisted of 1150 g of hafnium filings contained in a standard $8 \times 8 \times 1/4$ in. sample holder. The sample thickness in the direction normal to the face of the sample holder was 0.00940 Hf atoms/barn. The $N_{\gamma}(E_n)$ spectra obtained are shown in Fig. 22. The low-neutron-energy run required 16 hr. The rotor speed was 4910 rpm and the width of the neutron time channels was 6 μ sec. For the high-neutron-energy run, the rotor speed was 16 000 rpm, the width of the neutron time channels was 0.667 μ sec, and the total time required was 74 hr. The energies of low-lying levels in Hf^{178} that may be reached by primary transitions from the neutron resonances in Hf^{177} are shown in Fig. 23. The relative intensities and normalized relative intensities obtained for transitions in spectra measured in the intervals indicated by pedestals in Fig. 22 are given in Table X. The analysis of these data required no considerations that have not been previously discussed.

Fig. 22. Intensity of all detected gamma rays versus neutron energy for hafnium.

TABLE X. Relative intensities and normalized relative intensities for transitions to the first and second excited states in hafnium-178 from slow-neutron resonances in hafnium-177.

Hf ¹⁷⁷					
Int	E _n (eV)	a ₁	a ₂	A ₁	A ₂
1	1.10	0.56 ± 0.01	0.24 ± 0.02	0.25 ± 0.01	0.11 ± 0.01
2	2.38	-0.01 ± 0.01	0.15 ± 0.01	<0.005	0.04 ± 0.003
3	5.9	0.40 ± 0.01	0.35 ± 0.02	0.66 ± 0.02	0.57 ± 0.03
—	6.6	0.02 ± 0.01	0.02 ± 0.02	0.02 ± 0.01	0.02 ± 0.02
—	8.8	-0.04 ± 0.02	0.07 ± 0.03	-0.06 ± 0.03	0.11 ± 0.05
4	11	0.07 ± 0.02	0.02 ± 0.02	0.08 ± 0.02	0.02 ± 0.02
7	22.2	-0.02 ± 0.03	0.29 ± 0.03	-0.01 ± 0.01	0.13 ± 0.02
9	25.9	0.01 ± 0.02	0.05 ± 0.02	0.02 ± 0.06	0.15 ± 0.06
$\langle A_k \rangle$ (relative units)				0.27 ^{+1.61} _{-0.15}	0.14 ^{+0.17} _{-0.06}
$\langle \Gamma_k \rangle$ (10 ⁻³ eV)				0.38 ^{+2.27} _{-0.21}	0.20 ^{+0.24} _{-0.08}

Notice the asymmetric shapes of the neutron resonances at 1.10 and 2.38 eV in Fig. 22. It is true that the capture cross section is asymmetric because of the $(E_0/E)^{\frac{1}{2}}$ factor [Eq. (3)]. However, this would be exactly canceled for a perfectly resolved resonance by the change in neutron flux per neutron time channel. Two interpretations of the observed asymmetries are possible. First, both resonances being considered have a small value of Γ_n/Γ but are at a low enough energy that σ_0 is large. Hence the factors that multiply the flux in Eq. (69) may closely approach unity over an extended neutron-energy range in the vicinity of the resonance peak. In this case, the number of captures is simply equal to the number of incident neutrons and the $N_\gamma(E_n)$ spectrum in the peak of the resonance follows the change in the neutron flux. A second possible explanation is that multiple scattering is contributing appreciably to the observed capture rate.³⁷ The data are in fact consistent with the first explanation.

However, because of the possibility of the second, a different, thinner sample was prepared for use in the normalization runs. This sample consisted of 200 g of HfO_2 mixed with powdered sulfur and packed in a standard $8 \times 8 \times \frac{1}{4}$ in. sample holder. The sample thickness of this sample in the direction normal to the face of the sample holder was 0.00139 Hf atoms/barn.

Fig. 23. Energies of low-lying levels in hafnium-178 fed by primary transitions from slow neutron resonances in hafnium-177.

(f) Tungsten

The main runs on tungsten were performed in connection with another experiment.³⁸ The four-spectrum analyzer, shown in Fig. 3, was used to record the data. Therefore, since only two or three spectra could be measured in a given run, and since the highest energy transitions are unresolved (the spectra are comparable to those shown in Fig. 20), a considerable amount of rather tedious analysis was required.* The energies of directly-fed low-lying levels are shown in Fig. 24. The $N_{\gamma}(E_n)$ spectra are shown in Fig. 25. The relative intensities and normalized relative intensities are given in Table XI. The resonance averages were obtained by methods discussed previously.

(g) Platinum

The data shown here for platinum are also taken from reference 38. The $N_{\gamma}(E_n)$ spectra are shown in Fig. 26, the level scheme of directly-fed low-lying levels is shown in Fig. 27, and the relative intensities and normalized relative intensities are given in Table XII. Again, the methods used to obtain the resonance

Fig. 24. Low-lying levels in tungsten-184 fed by primary transitions from slow neutron resonances in tungsten 183.

* This analysis was performed by L. M. Bollinger and J. P. Marion. It is described in detail in reference 38. The author would like to express his thanks to Dr. Bollinger and Miss Marion for permission to include the results of this analysis in this report prior to publication elsewhere.

Fig. 25. Intensity of all gamma rays as a function of neutron energy for tungsten.

TABLE XI. Normalized relative intensities for transitions from neutron resonances in tungsten-183 to the ground state and first excited state in tungsten-184. The resonances investigated are all known (reference 18) to have spin 1^- . These results were obtained in connection with another investigation. (See footnote, page 100.)

Interval	E_n (eV)	A_0	A_1
1	7.5	10.00 ± 0.24	0.00 ± 0.19
2	27	0.53 ± 0.08	4.84 ± 0.10
3	41	0.38 ± 0.27	2.35 ± 0.34
4	46	3.85 ± 0.58	0.17 ± 0.58
5	65	3.90 ± 1.11	4.73 ± 1.39
$\langle A_k \rangle$ (relative units)		$3.7^{+7.9}_{-1.7}$	$2.4^{+5.1}_{-1.1}$
$\langle \Gamma_k \rangle (10^{-3} \text{ eV})$		$1.7^{+3.5}_{-0.8}$	$1.1^{+2.3}_{-0.5}$

Fig. 26. Intensity of all gamma rays as a function of neutron energy for platinum.

averages are the same as those discussed previously. The resonance-capture gamma-ray spectra obtained in the intervals indicated by pedestals in Fig. 26 are shown in Fig. 28. The spectrum obtained in the resonance at 11.9 eV, shown at the top of the figure, contains the transition used as the standard to which all absolute intensities reported here are normalized. Therefore, the resonance averages for platinum are subject only to uncertainties caused by Porter-Thomas fluctuations and by error in the value 0.0480 photons per capture used for the absolute intensity of the standard line.

(h) Mercury

The sample used for the runs on mercury consisted of 328 g of HgS packed in a $8 \times 4\frac{1}{2} \times \frac{1}{4}$ in. sample holder. The sample thickness in the direction normal to the face of the sample holder was 0.00420 Hg atoms/barn. The intensity of all detected gamma rays as a function of neutron energy is shown in Fig. 29 for the single run made on mercury. The energy range from 31 to 800 eV was covered with 1- μ sec neutron time channels and with a rotor speed of 16 000 rpm. The energies of low-lying levels fed by direct transitions from the slow-neutron resonances in mercury-199 and 201 are shown in Fig. 30. The data were analyzed to obtain transition probabilities for all transitions shown in the figure. The relative intensities and normalized relative intensities obtained for the four highest

Fig. 27. Low-lying levels in platinum-196 fed by primary transitions from slow-neutron resonances in platinum-195.

TABLE XII. Normalized relative intensities for the three highest energy transitions in platinum-196 from spin 1^- neutron resonances in platinum-195. The results are taken from reference 38.

Interval	E_n (eV)	A_0	A_1	A_2
1	11.9	10.00 ± 0.15	0.86 ± 0.15	0.25 ± 0.17
2	19.6	1.84 ± 0.05	0.91 ± 0.07	0.96 ± 0.07
3	67.4	8.60 ± 0.25	7.49 ± 0.32	0.44 ± 0.32
4	112	2.33 ± 0.12	2.85 ± 0.20	1.55 ± 0.20
5	120	6.04 ± 0.17	0.49 ± 0.17	1.92 ± 0.20
6	140	1.18 ± 0.10	0.96 ± 0.17	10.71 ± 0.32
7	151	1.11 ± 0.12	0.27 ± 0.15	0.32 ± 0.17
8	155	0.71 ± 0.07	0.07 ± 0.10	0.47 ± 0.12
9	189	3.05 ± 0.20	0.44 ± 0.27	4.03 ± 0.34
<hr/>				
$\langle A_k \rangle$ (relative units)		$3.9^{+4.5}_{-1.5}$	$1.6^{+1.8}_{-0.6}$	$2.3^{+2.7}_{-0.9}$
$\langle \Gamma_k \rangle (10^{-3} \text{ eV})$		$1.9^{+2.2}_{-0.7}$	$0.8^{+0.9}_{-0.3}$	$1.1^{+1.3}_{-0.4}$

energy transitions in Hg^{200} are given in Table XIII. The calculation of the resonance averages is simplified in this case since the spins of all resonances investigated are known¹¹ to be 1^- . The uncertainties quoted are the 90% confidence limits for $n = 3$, as shown in Fig. 2.

The relative intensities and normalized relative intensities obtained for transitions in Hg^{202} are shown in Table XIV. Since it is known¹¹ that the total radiation width is not a constant from resonance to resonance in Hg^{201} , it is necessary to compute the partial radiation widths before taking the resonance average in this case. These are given also in Table XV. The resonance averages $\langle \Gamma_k \rangle$ are computed as usual by means of Eqs. (87) and (90). (Of course the expression for $\langle m_- \rangle$ given by Eq. (85) is to be used in (87) in this case since ground-state transitions are possible from resonances with spin $I - \frac{1}{2} = 1$, but not from resonances with spin $I + \frac{1}{2} = 2$.) The 90% confidence limits are determined as before by means of a Monte Carlo calculation of the distribution function for the experimental values of the $\langle \Gamma_k \rangle$.

Fig. 28. Capture gamma-ray spectra obtained in spin 1^- neutron resonances in platinum-195.

Fig. 29. Intensity of all gamma rays as a function of neutron energy for mercury.

Fig. 30. Low-lying levels in mercury-200 and 202 fed by primary transitions from slow-neutron resonances in mercury-199 and 201.

TABLE XIII. The relative intensities and normalized relative intensities for transitions to the ground state and to the first, third, and fourth excited states in mercury-200 from neutron resonances in mercury-199 with spin 1^- .

Hg^{199}					
Interval	E_n (eV)	a_0	a_1	a_3	a_4
1	33.5	6.54 ± 0.20	0.38 ± 0.25	-0.50 ± 0.26	1.66 ± 0.28
4	175	0.06 ± 0.02	0.55 ± 0.03	0.58 ± 0.08	-0.02 ± 0.08
6	263	0.28 ± 0.02	0.25 ± 0.02	0.03 ± 0.06	0.30 ± 0.06
		A_0	A_1	A_3	A_4
1	33.5	1.68 ± 0.05	0.10 ± 0.06	-0.13 ± 0.07	0.43 ± 0.07
4	175	0.09 ± 0.04	0.90 ± 0.05	0.94 ± 0.12	-0.03 ± 0.13
6	263	2.06 ± 0.13	1.89 ± 0.05	0.25 ± 0.43	2.27 ± 0.43
$\langle A_k \rangle$ (relative units)		$1.3^{+5.3}_{-0.7}$	$1.0^{+4.0}_{-0.5}$	$0.4^{+1.5}_{-0.2}$	$0.9^{+3.7}_{-0.5}$
$\langle \Gamma_k \rangle (10^{-3} \text{ eV})$		$7.7^{+31.7}_{-4.0}$	$5.8^{+23.8}_{-3.0}$	$2.1^{+8.8}_{-1.1}$	$5.3^{+22.0}_{-2.8}$

TABLE XIV. Relative intensities, normalized relative intensities, and partial radiation widths for the three highest energy transitions in mercury-202 from slow-neutron resonances in mercury-201.

Hg^{201}				
Interval	E_n (eV)	a_0	a_1	a_2
2	43	-0.01 ± 0.03	0.21 ± 0.04	0.22 ± 0.04
3	71	-0.02 ± 0.04	2.63 ± 0.05	0.70 ± 0.06
5	210	1.48 ± 0.04	0.42 ± 0.05	0.16 ± 0.06
		A_0	A_1	A_2
2		-0.01 ± 0.07	0.54 ± 0.09	0.55 ± 0.11
3		-0.06 ± 0.13	8.88 ± 0.17	2.37 ± 0.19
5		3.31 ± 0.08	0.94 ± 0.11	0.35 ± 0.12
	Γ (10^{-3} eV)	Γ_0 (10^{-3} eV)	Γ_1 (10^{-3} eV)	Γ_2 (10^{-3} eV)
2	290	-0.06 ± 0.46	0.35 ± 0.06	0.36 ± 0.07
3	456	-0.59 ± 1.32	90.33 ± 1.68	24.12 ± 1.96
5	509	37.62 ± 0.92	10.69 ± 1.17	3.97 ± 1.37
$\langle \Gamma_k \rangle (10^{-3} \text{ eV})$		33^{+825}_{-20}	45^{+202}_{-24}	13^{+58}_{-7}

TABLE XV. Summary of experimental values of $\langle \Gamma_k \rangle$. Also given are the ratios obtained when the $\langle \Gamma_k \rangle$ are divided by the cubes of transition energies (the energy dependence predicted by the single-particle model) and the fifth power of the transition energy (as predicted by the giant-dipole-resonance result).

Target isotope	J_i^π	$D(J_i)$ (eV)	J_k^π	$E_{\gamma k}$ (MeV)	$\langle \Gamma_k \rangle$ (10^{-3} eV)	$\frac{\langle \Gamma_k \rangle \text{ (eV)}}{E_{\gamma k}^3 \text{ (MeV}^3\text{)}} \times 10^6$	$\frac{\langle \Gamma_k \rangle \text{ (eV)}}{E_{\gamma k}^5 \text{ (MeV}^5\text{)}} \times 10^8$
Nd ¹⁴³	3 ⁻	75	2 ⁺	7.12	1.7 ^{+4.8} _{-0.8}	4.6 ⁺¹³ _{-2.2}	9.3 ⁺²⁶ _{-4.4}
			4 ⁺	6.50	1.0 ^{+3.3} _{-0.5}	3.6 ⁺¹² _{-1.8} 7.8 ^{+6.6} _{-2.7}	8.6 ⁺²⁸ _{-4.3} 18 ⁺¹⁵ ₋₄
			3 ⁺	6.25	3.4 ^{+7.1} _{-1.4}	13.9 ⁺²⁹ _{-5.7}	35.5 ⁺⁷⁴ _{-14.6}
Nd ¹⁴⁵	3 ⁻	56	2 ⁺	7.11	1.0 ^{+2.0} _{-0.5}	2.8 ^{+5.6} _{-1.4} 12.1 ⁺¹⁴ _{-4.7}	5.5 ⁺¹¹ _{-2.8} 31 ⁺⁴⁰ ₋₁₃
			4 ⁺	6.51	5.4 ⁺¹² _{-2.5}	19.6 ⁺⁴² _{-9.1}	46.1 ⁺⁹⁸ _{-21.3}
Sm ¹⁴⁷	3 ⁻	18	2 ⁺	7.56	0.3 ^{+1.7} _{-0.2}	0.65 ^{+4.0} _{-0.4} 0.65 ^{+4.0} _{-0.4}	1.1 ^{+7.0} _{-0.7} 1.1 ^{+7.0} _{-0.7}
Sm ¹⁴⁹	3 ⁻	5.9	2 ⁺	7.78	0.2 ^{+0.4} _{-0.1}	0.38 ^{+0.8} _{-0.2} 0.38 ^{+0.8} _{-0.2}	0.6 ^{+1.3} _{-0.3} 0.6 ^{+1.3} _{-0.3}
Gd ¹⁵⁵	1 ⁻	5.5	0 ⁺	8.52	1.0 ^{+4.7} _{-0.5}	1.7 ^{+7.6} _{-0.8} 1.4 ^{+1.2} _{-0.5}	2.2 ⁺¹⁰ _{-1.1} 2.0 ^{+1.4} _{-0.7}
			2 ⁺	8.43	0.8 ^{+0.8} _{-0.4}	1.3 ^{+1.3} _{-0.7}	1.9 ^{+1.9} _{-0.9}
Yb ¹⁷¹	1 ⁻	9.0	0 ⁺	8.02	0.5 ^{+0.6} _{-0.2}	1.0 ^{+1.1} _{-0.4} 1.1 ^{+0.8} _{-0.4}	1.5 ^{+1.7} _{-0.6} 1.7 ^{+1.4} _{-0.6}
			2 ⁺	7.94	0.6 ^{+0.7} _{-0.3}	1.1 ^{+1.3} _{-0.5}	1.8 ^{+2.1} _{-0.7}
Yb ¹⁷³	3 ⁻	15	2 ⁺	7.40	0.6 ^{+1.0} _{-0.3}	1.4 ^{+2.3} _{-0.6} 1.4 ^{+1.6} _{-0.5}	2.6 ^{+4.2} _{-1.1} 2.6 ^{+3.4} _{-1.0}
			4 ⁺	7.14	0.5 ^{+5.0} _{-0.3}	1.4 ⁺¹⁴ _{-0.7}	2.7 ⁺²⁷ _{-1.3}

[illegible]

V. DISCUSSION

Table XV summarizes the main results of the experiment. The symbol of the target nucleus is given in column 1. The second column lists the spin of the neutron resonances which are the initial states for the transitions considered. As mentioned in Sec. III (e), the value obtained by averaging the normalized relative intensities over resonances which may have either spin $I + \frac{1}{2}$ or $I - \frac{1}{2}$ is a quantity whose expectation value is inversely proportional to $2I + 1$, provided that E1 transitions of the energy being considered are possible from either initial spin state. The value obtained in this manner is then multiplied by the ratio $(2I + 1)/(2J_i + 1)$ to yield a quantity which is proportional to the resonance-average value of the corresponding partial radiation width $\langle \Gamma_k \rangle$ for resonances with spin J_i . The spacings of levels of spin J_i at the neutron binding energy are listed in column 3.* The spins of the final states involved in the various transitions are listed in column 4, and the transition energies are listed in column 5. Column 6 lists the values of $\langle \Gamma_k \rangle$ obtained in Secs. II and III. The uncertainties quoted are the 90% confidence limits shown in Fig. 2. They depend strongly on the number of resonances in which usable gamma-ray spectra may be obtained. In cases in which the resonance spins are unknown, the 90% confidence factors are increased and are determined by a Monte Carlo calculation, as discussed in Sec. III (e). The increase in the uncertainties as a result of lack of knowledge of the resonance spins also depends strongly on the number of terms included in the average. According to the discussion given in Sec. I (b), values of $\langle \Gamma_k \rangle$ obtained for transitions to different final states in the same nuclide should depend only on the transition energy involved

*Values of the level spacing were taken from references 18 and 24 except that the value listed for Yb was determined in this work, and the value listed for Pt was determined from data kindly supplied by J. R. Bird in a private communication.

and not, for example, on the spin of the final state. This hypothesis may be tested by observing the constancy of the ratios obtained when the various values of $\langle \Gamma_k \rangle$ are divided by the appropriate energy factors. For the single-particle estimate, these energy factors are simply the cubes of the transition energies [Eq. (42a)]. The ratios $\langle \Gamma_k \rangle / E_{\gamma k}^3$ for the various transitions are listed in column 7. The uncertainties quoted are unfortunately too large to permit a very rigorous testing of the E_{γ}^3 law. However, except for the single case of Nd^{145} , the ratios agree within the uncertainties for all transitions in a given nuclide. Therefore, it may be assumed for purposes of further comparison that the ratio $\langle \Gamma_k \rangle / E_{\gamma k}^3$ may be averaged over transitions to different final states in the same nuclide to obtain a quantity that contains considerably less uncertainty from Porter-Thomas fluctuations. These averages are listed in column 8. The energy factors that should be used to remove the energy dependence in $\langle \Gamma_k \rangle$ are somewhat more involved for the giant-dipole-resonance result [Eq. (52)]. Unfortunately, the parameters E_g and Γ_g of the giant dipole resonance have not been determined for most nuclides studied in this work. Therefore, in order to test the general validity of Eq. (52), it will be assumed^{22, 26} that Γ_g is a constant equal to 5 MeV and that E_g is given by $80 A^{-1/3}$ MeV. On the basis of these values for the parameters of the giant dipole resonance, it is easy to show that for excitations of the order of 7 MeV, Eq. (52b) can be closely approximated by²²

$$(91) \quad \frac{\langle \Gamma_k \rangle}{D} = K E_{\gamma k}^5 A^{8/3},$$

where

$$K = 6.1 \times 10^{-14}.$$

Both $\langle \Gamma_k \rangle$ and D are to be expressed in the same units and $E_{\gamma k}$ is to be expressed in MeV. Values calculated by this equation differ by less than 5% from values calculated by Eq. (52b) for all gamma-ray energies considered in this report. [In determining the numerical factor in Eq. (91) it

is assumed that the statistical factor g , defined in Eq. (44), is $3/2$. This corresponds to transitions in nuclei with ground-state spins of zero.] Thus for the purpose of testing the general validity of the expression obtained for the resonance-average values of the partial radiation widths, the energy dependence may be removed by forming the ratios of the $\langle \Gamma_k \rangle$ and the fifth power of the transition energy. These ratios are shown in column 9 of Table XV. Again the ratios are constant for transitions to all final states considered in a given nucleus except for the case of Nd^{145} . (No significance is attached to the difference in the values obtained for the two transitions in Nd^{145} since 95% confidence intervals overlap.) Thus for purposes of further comparison, uncertainties caused by Porter-Thomas fluctuations may be greatly reduced by forming the averages of the quantities listed in column 9 over all transitions studied in a given nuclide. The results of this averaging are listed in column 10.

Both the single-particle estimate and the giant-dipole-resonance result predict that $\langle \Gamma_k \rangle$ should be directly proportional to the spacing of neutron resonances lying in the vicinity of the initial state and having the same spin and parity as the initial state. A test of this prediction is shown in Figs. 31 and 32 where the quantities listed in columns 8 and 10 in Table XV are plotted against the resonance spacing listed in column 3. The predicted dependence on the atomic mass ($A^{2/3}$ for the single-particle estimate and $A^{8/3}$ for the giant-dipole-resonance result) has been removed in each case. Also shown in the figures are values obtained for transitions in Cu^{64} , Cu^{66} , Ga^{72} , and Ag^{108} from states excited by the capture of thermal neutrons.¹⁴ Several resonances are believed to contribute significantly to the thermal capture in these cases. The error bars for these points indicate the 90% confidence limits based on the assumption that three resonances contribute equally to the thermal capture. The sets of points in both figures are acceptably fitted with straight lines with slope equal to unity. Although the fractional root-mean-square deviation from the mean is somewhat smaller for the points in Fig. 31 (0.52) than for the points in Fig. 32 (0.60), it

Fig. 31. The average values of the partial radiation widths with the single-particle-model mass-energy dependence removed versus the spacing of neutron resonances with the same spin and parity as the radiating state. The solid line passes through the mean value of the ratio of the ordinates to the abscissas of the plotted points which indicates the parameter D_0 in Eq. (42a) is 51 MeV.

Fig. 32. The average values of the partial radiation widths with the mass-energy dependence predicted by the giant-dipole-resonance result removed versus the neutron resonance spacing. The dashed line is the relation predicted by the giant-dipole-resonance result. The solid line passes through the mean value of the ratio of the ordinates to the abscissas of the plotted points.

is not possible to choose between the two proposed energy dependences on the basis of these data. However, the dashed line in Fig. 32 is the relation between the plotted quantities that is predicted by Eq. (91). As emphasized in Sec. I (b), this expression contains no adjustable parameters so that the indicated agreement with experiment may be considered good evidence for the validity of the giant-dipole-resonance approach to the problem of the radiative de-excitation of nuclear states in the vicinity of the neutron binding energy. The solid lines in Figs. 31 and 32 are lines with slopes of unity drawn through the mean values of the quantities obtained

when the ordinates of the plotted points are divided by the average spacing between neutron resonances. This mean value (0.0039 for the points shown in Fig. 31) indicates that the adjustable parameter D_0 in Eq. (42a), which is sometimes identified with the spacing of single-particle levels, is equal to 51 MeV. This is 3 or 4 times the value typically^{5, 39} used to force agreement between the single-particle estimate and the results of thermal-capture experiments. The derived value for D_0 depends directly, of course, on the constant factor in Eq. (42a), and unfortunately there is no universal agreement as to the number that should be used. For example, in place of 0.20 used in Eq. (42a), reference 5 uses 0.11. In reference 39, no attempt is made to approximate the sum over photon polarizations with the result that the factor $(2\ell + 1)$ is omitted from Eq. (41). It would seem, however, that arguments over factors of 2 or 3 are pointless unless one attempts to attach some physical significance to D_0 , and, as emphasized in Sec. I (b) of this work and in reference 22, such an attempt is highly questionable. The value of the ratio $\langle \Gamma_k \rangle / E^3 A^{2/3}$ is not subject to these ambiguities. This ratio, termed the "radiation strength" in reference 5 and denoted by k_{E1} was found there to be equal to 0.0032 by taking the average of the ratio over all partial radiation widths reported for thermal-neutron-capture gamma rays before September, 1961. This result agrees well with the value 0.0039 quoted above for the transitions examined in this work. On the other hand, the data compiled in reference 39 are for a selected group of fairly strong transitions (the Porter-Thomas distribution of partial radiation widths had not been confirmed at the time these data were collected) and indicate a value of $\langle k_{E1} \rangle$ of about 0.008.

The solid line in Fig. 32 indicates an experimental value $K = 3.2 \times 10^{-14}$ for the numerical constant in Eq. (91), about half the predicted value. In view of the incomplete nature of both the experimental and theoretical knowledge of the behavior of the dipole adsorption cross section in the vicinity of the neutron binding energy, a discussion of this discrepancy

would appear to be premature at this time. Furthermore, the data are not of high enough precision to guarantee that the discrepancy is in fact real. However, one may conclude on the basis of the data shown in Figs. 31 and 32 that the average values of the partial radiation widths are closely proportional to the spacings of neutron resonances lying in the vicinity of the radiating state and having the same spin and parity as the radiating state.

It therefore becomes significant to define an E1 gamma-ray strength function as the average value of the quotient obtained when the partial radiation widths for E1 transitions are divided by the average spacing between neutron resonances and by an appropriate energy factor. The strength function so defined should be independent of both the excitation energy of the initial state and the energy of the emitted gamma rays. Figures 33 and 34 show this E1 gamma-ray strength function defined with energy factors appropriate to the single-particle model (Fig. 33) and to the giant-dipole-resonance result (Fig. 34). The points shown by open circles are obtained from columns 8 and 10 of Table XV by dividing by the neutron-resonance spacings listed in column 3. The solid circles are points obtained from thermal-capture results discussed previously. The point indicated by the triangle was obtained by Huizenga *et al.*⁴⁰ for a group of fissionable nuclei with masses near 238. When this additional point is included, the data seem to favor the giant-dipole-resonance result. This serves to illustrate the extreme importance of data obtained by averaging over a very large number of resonances. (The result for mass 238 is an average over some 100 keV. Unfortunately, the method used is applicable only to fissionable nuclei.)

It is interesting to note that there is no large systematic increase in the strength function in the region of deformed nuclei. It is known that the width of the giant dipole resonance increases in this region.²⁶ The constant K in Eq. (91) is directly proportional to the width of the giant dipole resonance. Therefore, a strict application of Eq. (52b) would predict an increase in the strength function in the region of deformed nuclei. There is

an indication that the strength function for Gd^{156} (a highly deformed nucleus) is larger than for Sm^{150} (a nearly spherical nucleus). However, the strength functions for Yb^{172} , Yb^{174} , and Hf^{178} , all of which are nuclei with large eccentricities, do not appear to be systematically high. It does appear that nuclei very near the closed neutron shells have gamma-ray strength functions systematically higher than those of neighboring nuclei. This behavior of the strength function has not yet been investigated theoretically. If the effect is real, it could be a manifestation of the increase in the single-particle character of the initial-state and final-state wave functions near closed shells.

Fig. 33. The E1 gamma-ray strength function computed according to the single-particle model. The dashed line represents the predicted $A^{2/3}$ dependence.

Fig. 34. The E1 gamma-ray strength function computed according to the giant-dipole-resonance result. The dashed line represents the predicted $A^{8/3}$ dependence.

VI. SUMMARY

The resonance-average values of the partial radiation widths $\langle \Gamma_k \rangle$ of a total of twenty-six E1 transitions have been measured in a total of twelve nuclei. The values obtained are rather uncertain because of the Porter-Thomas fluctuations in the limited number of terms that could be included in the averages. Nevertheless, since the effects of interference are eliminated (by considering only spectra measured in the peaks of resonances) and the effects of Porter-Thomas fluctuations reduced (by averaging over at least a few resonances in each case) the results obtained in this work represent the most reliable values so far obtained for the nuclides considered for purposes of making quantitative comparisons with theoretical predictions.

The experimental values of $\langle \Gamma_k \rangle$ are interpreted in terms of two theoretical expressions, both of which predict that $\langle \Gamma_k \rangle$ should be directly proportional to the spacing D of neutron resonances with the same spin and parity as the radiating state. According to the single-particle estimate,¹⁷ the ratio $\langle \Gamma_k \rangle / D$ is proportional to $E_\gamma^3 A^{2/3}$, where E_γ is the transition energy and A is the atomic mass of the radiating nucleus. The average value obtained for the proportionality constant in this expression is 0.0039 if E_γ and D are expressed in MeV and $\langle \Gamma_k \rangle$ is in eV. This agrees quite well with the value 0.0032 obtained for the same quantity by Bartholomew,⁵ who averaged the results obtained for some 180 E1 transitions observed in thermal-neutron-capture gamma-ray spectra. The expression for the reduced electric-dipole matrix element obtained by the single-particle estimate requires the explicit assumption that the coordinates of the nucleons are completely uncorrelated. The necessity of making this highly unphysical assumption may be removed. The same reduced matrix element appears formally in the expression for the cross section for the inverse capture process — the photoexcitation of the nucleus. Thus the reduced electric-dipole matrix element may be expressed in terms of a classical Lorentz line, if it is assumed that the photoexcitation cross section in

the region of the neutron binding energy can be approximated by an extrapolation of this line. This seems reasonable since the Lorentz line is known to be a good approximation for the photoexcitation cross section for energies a few MeV above the neutron binding energy. In the absence of experimental values for the width and peak energy of the giant dipole resonance in the nuclei investigated, an expression adequate for testing the general validity of this hypothesis may be obtained²² by assuming the giant-dipole-resonance width is a constant equal to 5 MeV and that the peak energy is given by $80 A^{-1/3}$ MeV, the empirical law. With these assumptions, the value of the ratio $\langle \Gamma_k \rangle / D$ obtained by extrapolating the Lorentz line to the vicinity of 7 MeV excitation is closely proportional to $E^5 A^{8/3}$. The predicted proportionality constant is 6.1×10^{-14} if $\langle \Gamma_k \rangle$ and D are expressed in the same units and E is expressed in MeV. The experimental value obtained for this proportionality constant is 3.2×10^{-14} , which may be considered fair agreement. It should be emphasized that the predicted value depends on the dipole sum rule and contains no adjustable parameters.

The values obtained for the ratios $\langle \Gamma_k \rangle / E^3$ and $\langle \Gamma_k \rangle / E^5$ show no systematic dependence on the final states involved in the transitions. (Since the formal connection between the capture cross section and the photoexcitation cross section exists only for transitions involving the ground state, this perhaps indicates that the giant-dipole resonance result has a more general validity than is implicit in its derivation.) If it is assumed that these ratios are indeed independent of the final state involved (the values obtained are not precise enough to allow a definite statement to this effect), then the uncertainty caused by the Porter-Thomas fluctuations may be reduced by averaging the ratios over all transitions observed in a given nuclide. These averages are then divided by the appropriate power of the atomic mass and found to be closely proportional to D , as predicted by both theoretical expressions. Finally, the $E1$ gamma-ray strength function is defined as the quotient obtained when $\langle \Gamma_k \rangle$ is divided by D and the appropriate power of the transition energy. The result is a function of A .

The strength functions defined according to both theoretical expressions follow the predicted mass dependences in a general way. However, there seem to be systematic increases in the immediate vicinity of the closed neutron shells. If this effect is real it is perhaps a manifestation of the increased single-particle character (i.e., a reduction in the degree of correlation of the coordinates of one of the nucleons with the coordinates of the other nucleons) of the initial and final states involved in the transitions near the closed shells. The use of the complete expression for $\langle \Gamma_k \rangle / D$ based on the extrapolation of the Lorentz line predicts that $\langle \Gamma_k \rangle / D$ is almost directly proportional to the width of the giant dipole resonance. Since it is known²⁶ that this width increases in the region of deformed nuclei, it should be expected that the gamma-ray strength function also increases in this region. This effect would be rather difficult to observe with data as uncertain as the present results. However, within the uncertainties imposed by the Porter-Thomas distribution, no systematic trend of this type can be detected.

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