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*Pion Scattering
to 8- Stretched States in ^{60}Ni*

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MASTER

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PION SCATTERING
TO 8^- STRETCHED STATES IN ^{60}Ni

by

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Pion Scattering to 8^- Stretched States in ^{60}Ni

Thesis directed by Professor R. Jerome Peterson

Using the Energetic Pion Channel and Spectrometer (EPICS) at the Los Alamos Meson Physics Facility (LAMPF), differential cross sections for pion scattering were measured for ten previously known $J^\pi = 8^-$ stretched states in ^{60}Ni . A possible new pure isoscalar stretched state was also found. The data were taken near the $\Delta_{3,3}$ -resonance using 162 MeV incident pions and scattering angles of 65° , 80° , and 90° for π^+ and 65° and 80° for π^- .

For ^{60}Ni the isoscalar ($\Delta T = 0$) strength was found to be much smaller than the isovector ($\Delta T = 1$) strength, as has been found previously in other nuclei, and the isovector strength was found to be only 25% of that expected from a simple sum rule. The isoscalar and isovector amplitudes of each stretched transition were found by comparing the pion scattering cross sections with previous electron scattering data.

The analysis of the ^{60}Ni data found that the use of Woods-Saxon (WS) wave functions in the theoretical calculations gave much better agreement with data than the use of the usual harmonic oscillator (HO) wave functions. The WS theory gave better predictions of: 1) the angle at which the π^- and π^+ angular distributions are maximum, 2) the ratios of π^- to π^+ cross sections for pure isovector states (which were much larger than unity), and 3) the absolute size of the cross sections for all states (so that the normalization factor necessary to arrive at agreement of theory with data was closer to unity). The theoretical calculations used the distorted

wave impulse approximation, including new methods for unbound states. The sensitivities of the calculations to input parameters were investigated.

This analysis using WS wave functions was extended to five other nuclei (^{12}C , ^{14}C , ^{16}O , ^{28}Si , and ^{54}Fe) on which both pion scattering and electron scattering have been done. A significant improvement in arriving at a normalization factor close to unity was found when WS wave functions were consistently used for analyzing both pion and electron inelastic scattering data. This is the first time that a consistent analysis in several nuclei has been done using WS wave functions.

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CHAPTER I

INTRODUCTION

The purpose of this thesis is to understand better why the experimental cross sections found by pion and electron scattering to stretched states—one of the easier cases to study—have not been explained by past theoretical approaches.

To achieve this a new experiment was performed and a new theoretical approach was tried. The pion scattering experiment measured cross sections for the 8^- stretched states in ^{60}Ni and the resulting nuclear matrix elements were compared with those known from previous electron scattering experiments on ^{60}Ni . The new theoretical approach used Woods-Saxon wave functions (instead of the usual simple harmonic oscillator wave functions) in the extreme-single-particle model (as a standard) to analyze the stretched states in ^{60}Ni as well as in five other nuclei.

This chapter first outlines the problem to be studied, including a definition of a stretched state and why it is one of the easier cases to study. It then explains the standard experimental approaches used in the past and the theoretical adjustments used to try to improve the explanation of the data. Finally, the purpose of this experiment is given and the need to use Woods-Saxon wave functions is explained.

The Problem to be Studied

Quenching of magnetic excitations. There has been a systematic failure of theory to explain the experimental strength of nuclear magnetic excitations [Lin 84], i.e., those of unnatural parity and involving a spin-flip. A considerable number of experiments have been performed to look at these excitations in medium-energy inelastic scattering and in charge-exchange reactions. The magnetic excitations studied have usually been either M1 and Gamow-Teller (GT) transitions or stretched states. The isovector spin-flip strength seems to be systematically quenched (the experimental cross sections are less than theoretical predictions), but there is no general agreement on the quenching mechanism, and it may be that various mechanisms contribute differently for transitions of high and low multipolarity, or in low and high mass nuclei. There are fewer data on isoscalar spin-flip excitations, but results from pion scattering indicate an even larger quenching for these transitions, especially in higher mass nuclei [Gee 84]. Stretched states excited by transfer reactions are not quenched the same as when excited by inelastic scattering either; therefore, the particle is not just coupled to the ground state of the $(A - 1)$ nucleus to form the stretched state [Gee 84; Pet 87].

The loss of magnetic strength for low multipoles such as observed for 1^+ states in Gamow-Teller (p,n) reactions [Boh 81; Suz 81] and in electron scattering [Knu 80; Ric 82] could be partially accounted for by including mixing with the delta-particle nucleon-hole states in the nucleus, but many competing processes serve the same end, so including Δ -N effects

is not obviously justified. On the other hand, in high-spin magnetic multipole excitations (to stretched states) to be discussed next, mixing with the subnucleon degrees of freedom is not likely because delta-hole states are thought to affect low-multipolarity modes much more than those of high multipolarity [Ost 82].

Stretched states. It is expected that there will be fewer difficulties in understanding the magnetic transition strength of stretched excitations because they are of simpler construction. The particle-hole configuration of a stretched state is coupled to the maximum possible angular momentum and formed by the highest spin orbitals of adjacent oscillator shells and thus there is only one $1\hbar\omega$ transition possible for forming this state. Nuclear configuration mixing should then be minimal due to the uniqueness of the maximally stretched particle-hole configuration. Transitions to states of stretched configuration, $(j_p j_h^{-1})^J$, use $j_p = l_p + 1/2$, $j_h = l_h + 1/2$, and $J = j_p + j_h = L + 1$. For example, in ^{60}Ni the stretched state is due to the configuration, $(g_{9/2} f_{7/2}^{-1})^{8^-}$.

The stretched states for any particular nucleus can be divided into $T_<$ states (where the transition, $T_i \rightarrow T_f$, is for nuclear states of isospin $T_0 \rightarrow T_0$) and $T_>$ states (where the transition is for $T_0 \rightarrow T_0 + 1$). This uses the definition, $T_0 = (N - Z)/2$, for the ground state isospin of a nucleus with N and Z equal to the number of neutrons and protons in the nucleus.

It is also necessary to explain isoscalar and isovector transitions. In a pure isoscalar ($\Delta T = 0$) transition, the neutron and proton contribute completely symmetrically. In a pure isovector ($\Delta T = 1$) transition, the neutron and proton contribute completely antisymmetrically. It is also possible

to have combinations of these two types of transitions. These structure effects are embodied in Z coefficients (to be discussed in Chapter V): Z_0 and Z_1 for isoscalar and isovector and Z_n and Z_p for neutron and proton. The relation between the Z coefficients is $Z_0 = (Z_n + Z_p)/\sqrt{2}$ and $Z_1 = (Z_n - Z_p)/\sqrt{2}$. The $T_<$ states can be formed by either isoscalar or isovector transitions; however, $T_>$ states can only be formed by isovector transitions.

Other related problems. Several other problems related to magnetic excitations will be briefly mentioned here, but not addressed in this thesis. One is the unexplained splitting of stretched state energy levels; the other is the use of magnetic excitations in elucidating the strong interaction.

A complete and satisfactory explanation for the often-observed wide splitting of the stretched state energy levels (beyond that of splitting into a $T_<$ and a $T_>$ state) has not been found. Collective effects such as particle-vibration coupling and static deformations have been suggested as reasons for this splitting. Goncharova and Matveev [Gon 85] have made predictions for ^{60}Ni that there is an overlap in energy between the $T_<$ and $T_>$ states. However, Lindgren et al. [Lin 81b] have suggested that the centroids of the $T_<$ and $T_>$ states have a well defined separation, as expected from considerations of the nuclear symmetry energy. Peterson et al. [Pet 87] have shown that the strengths of $T_>$ 8^- states in ^{60}Ni have a Gaussian distribution.

Moffa and Walker [Mof 74] have reported that structure deduced from (e, e') to stretched states could be used in conjunction with hadron

scattering reactions to extract information on the high momentum components of the effective interaction of the projectile with the nucleus. The first systematic comparison of the (e, e') and (p, p') transition strengths [Lin 79] for the excitation of unnatural parity states of stretched configurations verified that this observation was correct for proton scattering in self-conjugate nuclei. This comparison can be used to determine the tensor component of the nucleon-nucleon force and the spin-orbit component of the pion-nucleon force in the nucleus. The tensor component is particularly interesting because in the low q limit its microscopic origin evolves naturally from one-pion exchange considerations and in the high q region there are significant contributions from one-rho exchange [Bro 81]. Since it is not known whether the most useful theory of strong interactions is described by nucleons interacting through pion and rho exchange or quarks interacting through gluon exchange, measurements on the tensor force in the nucleus are valuable in discriminating between such models [Lin 84].

Studies such as the present thesis are thus closely related to a wide range of concepts, united in an understanding of the nature of stretched excitations.

Previous Experimental Approaches

In the past, electron and hadron scattering have both been used to study magnetic excitations. Petrovich et al. [Pet 86] give an excellent review of the use of electron and hadron scattering as probes of nuclear structure. Complementary information from the two types of probes can be used to separate effects due to reaction dynamics from effects due to nuclear structure. Petrovich and Love [Pet 81] have summarized the cross

section formulae for (e, e') , (p, p') , (p, n) , and $(\pi^\pm, \pi^{\pm'})$ and illustrated how these processes are interrelated through particle, spin, and current transition densities. Excitations of stretched states involve only the spin transition density, leading to a particularly direct connection between the pion, electron, and proton scattering reactions.

The electron scattering reaction is the easiest to use because it is well understood and described by quantum electrodynamics. The electromagnetic interaction has a small coupling constant ($1/137$) and thus there are small distortion effects on the scattering. This “magnetic” scattering to unnatural parity states is due to the interaction between the electron current and the magnetic moments of the protons and neutrons and the magnetic moment caused by the orbital motion of the protons. When electron scattering is used to study stretched states, the scattering is done at back angles (where only the transverse magnetic form factor is important) to excite mainly the isovector transitions.

The pion and proton scattering reactions involve the strong interaction with a large coupling constant (~ 1) and thus large distortion effects. When pion scattering is used to study stretched states, incident pions near the $\Delta_{3,3}$ -resonance energy (180 MeV incident energy) are used in order to excite mainly isoscalar transitions (see Appendix A). Thus the pion data complement the information from electron scattering.

Previous Theoretical Approaches

In calculating the theoretical nuclear matrix elements, several models have been used. The simplest model is the extreme-single-particle-hole-model (ESPM) [Lin 78; Lin 84], which assumes that a single nucleon is

promoted from a ground state orbital to some excited state orbital. Other more sophisticated calculations have been done using the random phase approximation (RPA) [Lin 78; Lin 81b; Wis 85; Yen 80; Zar 77], a recoil corrected continuum shell model (RCCSM) for unbound excited states [Hal 81], and the shell model calculations of Cohen-Kurath, Millener-Kurath, and Lawson [Gee 84; Hic 84; Hol 85; Plu 84; Plu 85]. These more sophisticated models result in smaller theoretical cross sections and thus less quenching by comparison to data.

In the ESPM model description, only a single particle-hole configuration is involved in the excitation of a stretched state. Although multi-particle- multi-hole-configurations can produce physical ground states which are not pure closed shell and excited states which are not of the single particle-hole type, these additional components in the wave function cannot be connected by one body spin and orbital current operators. The only effect is that the transition strength is reduced and fragmented; the observed cross section remains proportional to the single-particle matrix element corresponding to the stretched configuration.

All the theoretical models predict larger cross sections than are found experimentally; thus other projectile-nucleus interactions have been suggested. Valence shell fragmentation effects include: particle-vibration coupling [Kre 80; Kre 86], nuclear deformation [Zam 84], extending the shell model configuration space [Amu 83], and decreasing the occupation number distributions in the nuclear ground states near the Fermi energy [Jam 85; Pan 84; Pap 86]. Core polarization has been considered, including admixtures of $1\hbar\omega$, $2\hbar\omega$, $3\hbar\omega$, etc. [Ham 80; van 87; Yok 87]. Other

possibilities include rho exchange [Bro 81; Gee 84], 2-particle 2-hole configurations [Gee 84], and a change in g' [Coh 85]. Whereas most of these effects improve the agreement between theory and experiment, the inclusion of meson exchange currents (MEC) in the analysis of the electron scattering data [Deh 85; Kre 86] results in yet further disagreement between experiment and theory [Lin 86].

Rather than enter into any sophisticated studies of the nuclear structure, results from the present work will be compared only to the extreme-single-particle-hole-model, as a simple basis of comparison.

The Purpose of the Present Experimental Approach

This section describes the unique features that must be taken into account when using the pion as a probe. It also explains why ^{60}Ni was chosen as the nucleus in which to study stretched states.

Use of the pion as a probe. The pion and its interaction with nucleons is unique and much can be learned about the nucleus by using the pion as a probe. Because of the many possibilities in using this probe, three meson factories have been built: LAMPF in Los Alamos, NM, SIN in Switzerland, and TRIUMF in Vancouver, BC. Details of the pion-nucleus (π -A) interaction are described by Ingram [Ing 82], by Lock and Measday [Loc 70], and by Cence [Cen 69]. A brief description of some features of the pion are given here.

In the quark picture, a pion is a meson made up of a quark and an anti-quark. Thus in π -A interactions, the particle has no internal structure that needs to be included except at high energies, and no exchange corrections are necessary because the pion is not a nucleon.

Pions exist in three charge states: π^- , π^0 , π^+ and thus are isovector particles ($T = 1$). The charged pions ($\pi^\pm$) have a mass of 139.57 MeV, a lifetime of 2.60×10^{-8} sec, and they decay by $\pi \rightarrow \mu \nu_\mu$ ($\sim 100\%$). The neutral pion (π^0) has a mass of 134.96 MeV, a lifetime of 0.83×10^{-16} sec, and it decays by $\pi^0 \rightarrow \gamma\gamma$ ($\sim 99\%$). In π -A interactions, recoil effects are small since the pion has only about $1/7^{th}$ the mass of the nucleon. Because the pion has three charge states, the π -A interaction can include single charge exchange (SCX) and double charge exchange (DCX) reactions in addition to elastic and inelastic scattering. Especially useful is the fact that pion scattering can be used as a symmetric probe for scattering reactions $[(\pi^-, \pi^{-'})$ and $(\pi^+, \pi^{+'})]$, as in the present study, and for SCX reactions $[(\pi^-, \pi^0)$ and $(\pi^+, \pi^0)]$.

Pions are pseudoscalar particles, i.e., they have zero spin and negative parity ($J^\pi = 0^-$). Thus they are bosons and can be emitted and absorbed. Pions are also the lightest known strongly interacting particles. These two facts result in a theory where the pions mediate the the long-range part of the strong force as first predicted by Yukawa [Yuk 35] in 1935. The angular momentum algebra and the parameterization of the basic pion-nucleon (π -N) interaction is simplified because pions have zero spin. The π -A interaction is strongly surface peaked, because pions are strongly interacting, but by varying the incident energy of the pion, different depths in the nucleus can be probed.

A very useful feature in π -A interactions is the $\Delta(1232)$ scattering resonance where the scattering reaction is particularly easy to understand. The delta resonance ($\Delta_{3,3}$) in a π -N interaction occurs at a total energy of 1232 MeV when an intermediate state is formed where the isospin is $3/2$

($2T = 3$) and the spin is $3/2$ ($2J = 3$). (See Appendix A for more details.) This $\Delta_{3,3}$ -resonance occurs when a pion with a kinetic energy of $180 \text{ MeV} \pm 60 \text{ MeV}$ is incident on a nucleon. Experimental cross sections for this energy range are shown in Figure 1.1.

The isospin relations that are derived in Appendix A for π -N interactions at the $\Delta_{3,3}$ -resonance show that the π^+p (π^-n) reaction has a cross section that is nine times larger than the π^-p (π^+n) reaction, and the isoscalar/isovector ratio of cross sections is 4. If this behavior were to continue in the case of π -A interactions, ratios of π^+ to π^- cross sections would be an excellent way to distinguish the neutron and proton pieces of nuclear ground state and transition densities. As derived in equation (A.1) of Appendix A, the ratio of neutron amplitude to proton amplitude (A_n/A_p) for pion scattering could be used to do this. A pure neutron (proton) transition should have $A_p = 0$ ($A_n = 0$) and thus the ratio should be $R = 1/9$ ($R = 9$). For pure isoscalar or isovector transitions, A_n and A_p should be equal and thus the ratio should be $R = 1$. In fact, the ratio R for known states has not always been found to be as expected, revealing new details of the isospin content of nuclear transitions [See 81].

Use of ^{60}Ni to study stretched states. In the past, stretched states in a number of different nuclei have been studied. It has been useful to compare the quenching of stretched states in these different nuclei to see the effects of altering how the orbitals are filled, how the energy levels are split, whether the stretched state is bound or unbound, whether the nucleus is spherical or non-spherical, what the total J of the state is, and what the ground state isospin is.

Figure 1.1 Total cross section for $\pi^+ p$ scattering, for $\pi^- p$ elastic scattering, and for charge exchange. The solid lines give resonant $J=3/2$, $T=3/2$ contributions. The pion laboratory kinetic energy is given by T_π [Mon 81]. See also Lock and Measday [Loc 70].

There are several reasons why ^{60}Ni was chosen in this experiment as the target for exciting stretched states by pion scattering. 1) Electron scattering to the stretched states in ^{60}Ni has already been done [Lin 81b], so the isoscalar and isovector amplitudes can be determined by combining the results of these two reactions. 2) The (α, t) stripping reaction to the stretched states in ^{60}Ni has been done [Pet 87], so coherent scattering and incoherent stripping results can be compared. 3) Both electron and pion scattering have been done to the 8^- stretched states in ^{54}Fe . In the two nuclei, the $1f_{7/2}$ orbital is filled differently, the stretched states are split differently, and the nucleon binding energies are different. The cross sections for the stretched states in the two nuclei can now be compared to see the results of these differences. 4) The ^{60}Ni nucleus is unique among the nuclei studied by pion scattering to stretched states in having neutrons outside the major shell from which nucleons are promoted to the stretched state, i.e., having $p_{3/2}$ neutrons beyond the high spin $f_{7/2}$ core.

The New Theoretical Approach

For simplicity, almost all the theoretical calculations in the past have used a harmonic oscillator (HO) potential ($V = Cr^2 + V_0$) to calculate the nuclear wave functions of the hole-state and the particle-state contributing to the stretched state (for both electron scattering reactions [Hic 84; Hyd 87; Lin 78; Lin 81a; Lin 81b; Plu 84; Plu 85; Yen 80] and pion scattering reactions [Hal 81; Hol 81; Hol 85; Olm 79]). This potential is not realistic for two reasons. 1) Stretched states lie at fairly high excitation energies, sometimes above the proton and/or neutron separation energies, and a HO potential cannot treat these unbound states correctly. 2) The

implicit isospin symmetry used in HO calculations is questionable, since neutron and proton wave functions may differ substantially due to the difference in binding energy for the two nucleons. A more realistic potential is the Woods-Saxon potential ($V = V_0[1 + e^{(r-R)/a}]^{-1}$), where these two problems are taken care of. The shape of the HO and WS potentials is compared in Figure 1.2 for a typical case.

The effects of using a Hartree-Fock (HF) potential [Lic 79; Wis 85] and a Woods-Saxon (WS) potential [Hic 84; Hyd 87; Kre 80; Lic 79; Olm 79] have been mentioned in the past, but not thoroughly tested. This thesis and a related paper [Cla 87] (see Appendix D) have thoroughly tested the use of WS wave functions in analyzing pion and electron scattering reactions, respectively. These tests include a comparison with previous results using HO wave functions. The neutron and proton transition densities to stretched states were calculated using radial wave functions computed for nucleons in a WS potential by the computer code DWUCK4 [Kun 87]. For cases where the nucleon is unbound, the excited nucleon is treated as a resonance by a method well-established for single-nucleon stripping reactions [Vin 70].

Besides using WS wave functions to analyze pion scattering to stretched states in ^{60}Ni , this thesis looks at the effect of using WS wave functions to analyze pion scattering for five other nuclei. These results are compared to results from using WS wave functions to analyze electron scattering to these same nuclei to establish a sense of the systematics. Both electron and pion scattering to stretched states have been done in a total of only these six nuclei: ^{12}C [Hic 84; Mor 79; Hal 81], ^{14}C [Plu 85; Hol 81],

Figure 1.2 Comparison of a Woods-Saxon potential (dot-dashed line) with a harmonic oscillator potential (solid line) for ground state neutrons in ^{60}Ni .

^{16}O [Hyd 87; Hol 80], ^{28}Si [Yen 80; Olm 79], ^{54}Fe [Lin 81a; Gee 84], and now ^{60}Ni [Lin 81b].

Thesis Outline

Chapter II of this dissertation describes the LAMPF facilities where the pion scattering experiment on ^{60}Ni was done. Chapter III describes the data acquisition and analysis and gives the cross sections for the stretched states in ^{60}Ni excited by inelastic pion scattering. Chapter IV gives a discussion of non-relativistic scattering theory and its application to electron and pion-nucleus scattering. In Chapter V the theoretical predictions (using both HO and WS wave functions) for pion scattering are compared to the data from this experiment. Pion and electron scattering reactions are then compared to separate the structure effects from the reaction dynamics. Finally, Chapter VI gives a summary of the thesis.

CHAPTER II

THE LAMPF FACILITIES

This chapter and the next describe the pion scattering experiment performed on the Energetic Pion Channel and Spectrometer (EPICS) at the Clinton P. Anderson Meson Physics Facility (LAMPF) in Los Alamos, New Mexico. The experiment was performed using 275 hours of beam time during the summer of 1986. This chapter gives a general summary of the experimental apparatus and Chapter III describes the details specific to LAMPF experiment #984.

The LAMPF facility is shown in Figure 2.1 and the LAMPF experimental areas are shown in Figure 2.2. The secondary pion beam at EPICS is produced from a primary proton beam in the half-mile long LAMPF linear accelerator (linac). The summary description of the linac and EPICS hardware and the computer software given here will be based on several previous descriptions [See 81; Kiz 84; Sei 85; Gil 85; Mit 86].

LAMPF Accelerator

The three sections of the LAMPF accelerator and the A-1 production target are described in detail by Livingston [Liv 77].

The first section contains three injector systems for H^+ , H^- , and polarized H^- beams. The H^+ and either the H^- or $\bar{H}^-$ beams can be accelerated at the same time. Each system contains an ion source of high-purity hydrogen gas, a high-voltage Cockcroft-Walton generator, and an

Figure 2.1 The Clinton P. Anderson Meson Physics Facility (LAMPF) [LAM 82].

Figure 2.2 The LAMPF experimental areas including EPICS [LAM S2].

accelerating column. In this section the beam is accelerated to 750 keV and is bunched by passing through sets of radio-frequency (rf) cavities. The bunching gives a macro-time structure to the beam of 120 pulses per second with each pulse being approximately $900 \mu\text{s}$ long. The resulting duty factor is $\frac{900 \mu\text{s}}{8333 \mu\text{s}} \approx 11\%$.

The second section is a 62 m long Alvarez-type drift tube linac. Its operating frequency of 201.25 MHz gives each macro-pulse a 5 ns micro-time structure. An rf alternating electric field is set up in four cylindrical vacuum tanks to accelerate the particles to a kinetic energy of about 100 MeV. The final section is a resonantly-side-coupled type of linac with an operating frequency of 805 MHz and final energy of 800 MeV. LAMPF is operated with a beam current as high as 1 mA.

Pions for EPICS and the Low Energy Pion (LEP) channel are produced by the passage of a primary proton beam through a 3 cm thick graphite production target, designated A-1. Graphite is used because of its thermal characteristics and its low Z . The thermal characteristics and low density of carbon make it a good target for heat dissipation, but even so it is also water cooled and rotated. The relatively low Z of carbon helps reduce multiple scattering of the incident proton beam and is also used to maximize the number of pions produced per unit energy loss in the target. The final benefit is because pion production cross sections increase as $Z^{1/3}$ [Coc 72], whereas energy loss is proportional to Z .

Energetic Pion Channel and Spectrometer (EPICS)

The entire EPICS system is shown in Figure 2.3 with a schematic of the channel in Figure 2.4 and a schematic of the spectrometer in Fig-

Figure 2.3 The energetic pion channel and spectrometer (EPICS) located at LAMPF [Kiz 84]. The area to the left of the scattering chamber is in concrete shielding.

Figure 2.4 The EPICS channel and its optical mode [Kiz 84]. The solid (dashed) line illustrates the optics for the vertical (horizontal) mode.

Figure 2.5 Block diagram of the EPICS spectrometer optics and detector system [Kiz 84].

ure 2.5. Details of the system have been previously described [Thi 70; LAM 80]. The channel has an energy range of 70-300 MeV and the spectrometer has a momentum range of 100-750 MeV/c.

The channel. The channel views the pion production target at an angle of 35° relative to the incident beam. This angle represents a compromise between high intensity (maximal at 0°) and space requirements for movement of the spectrometer. The channel has a path length of 15.24 m. Physically, there are several elements in the channel including four dipole magnets (BM01-BM04), three trim magnets (FM01-FM03), four collimating jaws (FJ01-FJ04), a proton separator, and a proton absorber.

Momentum selection in the channel is done by the four dipole magnets that bend the particle trajectories in a vertical plane. Nuclear magnetic resonance (NMR) probes are used to measure the fields in BM01-BM04. The channel provides for point-to-point focusing in the vertical plane and point-to-parallel focusing in the horizontal plane. Point-to-point focusing means that if a monoenergetic divergent beam emerges from a point on the production target, these particles will be focused to a point in the focal plane (at the scattering target). Point-to-parallel focusing means that particles which come from a given point on the production target emerge on parallel trajectories at the focal plane.

All four sets of adjustable uranium collimating jaws were left wide open during this experiment, because beam collimation reduces intensity, which was undesirable in this low count rate experiment. The first jaw, FJ01, determines the acceptance of the channel. It consists of only a vertical jaw, which was fixed at an opening of 6 inches. With the jaws fully open, the

channel beam at the scattering target has a vertical dispersion of 10 cm/%, a momentum spread of $\pm 1\%$, a divergence of 100 mrad, and a solid angle of 3.4 msr. Using 162 MeV pions with the jaws wide open, the flux is $\sim 10^8 \pi^+$ /sec and $\sim 10^7 \pi^-$ /sec.

The scattering chamber. The pion beam incident on the scattering target is about 20 cm high by 8 cm wide. The beam is momentum dispersed in the vertical direction in such a way that momentum is correlated with vertical position with $\Delta p/p = 2\%$ FWHM over the 20 cm vertical spot size giving 0.1%/cm momentum spread. The EPICS spectrometer was designed with position sensitive detectors in front of the bending magnets to be able to determine accurately where a particle comes from on the scattering target. This feature also means that particle momentum can be determined accurately, resulting in a momentum uncertainty of 0.02% FWHM. To get this same resolution with a monochromatic beam would involve a decrease in the intensity by a factor of 200. This feature (coupled with the large physical spot size of the beam) also allowed data to be taken simultaneously on several (strip) targets during this experiment.

Located at the focal plane of the channel, at the center of the scattering chamber, is a target ladder containing up to four full-size scattering targets (22.9 cm x 15.2 cm). Different targets are selected by moving the target ladder vertically. The target ladder can also be rotated relative to the channel beam so as to bisect the scattering angle, thus minimizing the path length traversed through the target.

The spectrometer. The spectrometer rotates about a pivot (at the focal plane of the channel) through an angular range of -10° to

120°. It has a solid angle of about 10 msr and a flight path of 12.5 m. The EPICS spectrometer operates in the VHV mode: vertical dispersion, horizontal scattering, and vertical analysis. The spectrometer contains three quadrupole magnets (QM01-QM03) for focusing scattered pions at the front focal plane, a front detector system of wire chambers (F1-F4), two dipole magnets (BM05-BM06) to momentum-analyze the pions onto the rear focal plane, and a rear detector system of wire chambers (R5, R6, R9, R10) and scintillators (S2-S9).

In the discussion of the spectrometer, z is in the direction of the beam, x is in the dispersion direction (positive down at the target), and y is in the non-dispersive direction such that (x, y, z) forms a right-handed coordinate system.

The passage of a particle through the spectrometer can be treated in analogy with the passage of light through an optical system. Given x , θ (dx/dz), y , ϕ (dy/dz), l (the path length difference from the central ray), and δ (the momentum difference from the central ray, where $\delta = 100 \times \frac{p - p_0}{p_0}$) for a particle at one point in the system, it should be possible to calculate these variables for that particle at another point in the system. Using a transport matrix formalism, it is possible to calculate the positions/angles of a particle at the scattering target given its positions/angles at the front focal plane. It is also possible to calculate the particle momentum and scattering energy loss given its positions/angles at the rear focal plane. The transport matrix that allows one to do this is based on a set of polynomials used by the computer and stored in a file named POL.DAT.

The EPICS spectrometer is not dispersion matched to the channel and therefore the momentum of the incident particle must be measured in order to determine the momentum loss in the scattering process. For this reason there are three quadrupole magnets and a set of wire chambers between the scattering target and the spectrometer bending magnets. The focusing of this triplet is point-to-point in the vertical plane, with a magnification of -1 , and parallel-to-point in the horizontal plane. The set of wire chambers placed near the focal point of the triplet is used to measure x , θ , y , and ϕ of each particle and then calculate its momentum.

The two spectrometer dipole magnets each bend scattered particles through an angle of $\sim 60^\circ$, resulting in a momentum dispersion of $\Delta p/p \approx 14\%$ or $4 \text{ cm}/\%$ at the rear focal plane. This momentum spread of the spectrometer allows a large region of excitation energy to be viewed at one spectrometer setting. The optics of the dipole magnets are point-to-point in the x - z plane and point-to-parallel in the y - z plane. The magnetic fields are measured by NMR probes.

Beam Monitoring and Particle Detection

The relative intensity of the incident pion beam is monitored in three ways: 1) A charge integrating toroidal coil (1ACM02) is located in the proton beam upstream of the A-1 target. For a fixed channel and production target, output from the coil is proportional to the flux of pions in the EPICS channel. 2) "Beam on Target" (BOT) is an ionization chamber in the A-1 target cell that measures the flux of secondary particles due to the proton beam striking the target. Its signal is proportional to the product of the H^+ current and the thickness of the A-1 target. 3) The ion

chamber (IC1) is located in the scattering chamber on the same axis as the incoming channel beam. This ion chamber is used to monitor the pion intensity incident on the scattering target. Both ion chambers (BOT and IC1) contain a mixture of 90% argon and 10% CO₂ gas and are operated at atmospheric pressure. The ratios 1ACM02/BOT and 1ACM02/IC1 did not vary by more than 5% during the experiment. Thus the steering of the proton beam onto the production target was constant, giving the EPICS channel a constant pion flux.

The scattered-particle detection system at EPICS is composed of three types of particle detectors: scintillators, a Čerenkov detector, and wire chambers. These are divided into two sets, the front detectors before the dipole magnets, and the rear detectors after the dipole magnets as seen in Figure 2.5.

Each long scintillator is connected to a photo tube at each end in order to do mean-timing and to calculate mean pulse heights. This is important in counters of this length in order to get output signals that are reasonably independent of the position of impact on the scintillator. The two ends of the scintillators are designated S_N or S_P for negative or positive delta (low or high momentum side of the spectrometer).

The Čerenkov counter is located at the rear of the spectrometer between S2 and S3. The signals from this counter can be used to identify electrons. This is most useful for very low count rate experiments in which a majority of events might be due to electrons. Such was not the case during this experiment and so the Čerenkov counter was not used.

The wire chambers. The position-sensitive detectors used at EPICS are delay-line readout drift chambers designed at LAMPF [Ate 81; Mor 78; Mor 82]. These multiwire proportional chambers (MWPC) measure 90 cm by 30 cm and are mounted with the long dimension along the dispersion direction of the spectrometer.

Originally, four pairs of chamber planes were used to measure unambiguously x , θ , y , and ϕ at the front focal plane (after the quadrupole magnets) and another four pairs at the rear focal plane (after the bending magnets). A pair of planes was necessary in measuring position to remove a left-right ambiguity as explained later. Pairs of planes are separated by 0.95 cm, and an aluminized mylar foil connects to the cathode frame providing a ground plane between wire planes.

Currently, only four chamber planes are used at each focal plane and the left-right ambiguity is removed another way. Two planes measure two x positions and the other two planes measure two y positions. The θ and ϕ angle measurements require two sets of two planes, separated along the particle trajectory axis. Average x , θ , y , and ϕ are calculated for each event from the four raw positions.

In each wire plane the drift cell size or anode (cathode) wire spacing is 8 mm (see Figure 2.6). The anode wires are coupled to a delay line rather than using an individual wire readout and the field-defining cathode wires are bussed together in two sets. The cathode is at ground and the anode is held at a positive high voltage relative to it (typically 2150 V). The passage of a charged particle through the chamber produces an avalanche of ions: anions (electrons) that are collected at the nearest anode wire and cations (ionized gas molecules) that are collected by the nearest cathode

Figure 2.6 A schematic view of wires in a drift chamber [Mit 86]. The * denotes passage of a particle.

wire. Outputs are obtained from both ends of the anode delay line and from both bussed cathode lines. The raw anode signals are small (only 1-3 mV in amplitude) compared to the raw scintillator pulse heights of approximately 100-500 mV.

Signals from both ends of the delay lines are used to obtain position information. Suppose that there are N wires in the chamber and that the detected particle produced a signal on the m^{th} wire as counted from the left of the chamber. If the characteristic time per wire spacing on the delay line is t_c (2.5 ns/cm) and the amount of time for the avalanche to drift from the point of impact to the anode wire is t_d (20 ns/mm), then the time measured at the left end of the delay line is $T_L = mt_c + t_d$ and the corresponding time on the right end of the line is $T_R = (N - m)t_c + t_d$. Both T_L and T_R are referenced to a time signal from a “prompt” scintillator. This scintillator must be near the chambers so that fluctuations in the start time due to particles of different momenta are minimal.

The difference of the two times, $T_L - T_R = 2mt_c - Nt_c$, identifies the wire that was struck. The sum of the two times, $T_L + T_R = 2t_d + Nt_c$, is independent of which wire was struck, but is proportional to the electron drift distance from the ionizing particle to the wire (plus an offset) after comparing to some “prompt” time to get drift time information.

To apply the drift time correction, it is necessary to know on which side of a wire the particle passed. This is the so called left-right ambiguity. Originally, this left-right ambiguity was eliminated by having two planes of anode wires, offset from each other by half a wire spacing. Currently, the left-right ambiguity can be eliminated using a single wire plane, rather than a pair. The new method uses the cathode wires bussed together in two

groups designated even and odd, i.e., every other cathode wire is bussed together. As an example (see Figure 2.6), if a particle passes on the right side of an anode wire, then the nearer (odd) cathode wire will get a larger induced pulse than the further (even) cathode wire. The difference of the induced pulse heights (odd-minus-even) is positive in this case. In general the difference is positive if the particle passes on the right side of the anode wire and negative on the left. The left-right ambiguity is thus removed. The final position resolution of these chambers is $125\ \mu\text{m}$ FWHM.

The front detectors consist of four drift chambers with F4 at the focus of the quadrupoles and a scintillator upstream of the front chambers. The S1 scintillator was not used in this experiment, however, because it is in front of the other spectrometer detectors and thus degrades resolution.

The rear detectors include a set of rear wire chambers (R5, R6, R9, and R10) with R5 at the rear focal plane, a Čerenkov detector to identify electrons, two scintillators (S2 and S3), a slab of Lucite between S2 and S3 used to range out protons, and a muon-rejector. An event is required to generate pulses in both S2 and S3 for it to be considered a valid event. The pulse heights in S2 and S3 and the time of flight between S2 and S3 are used for particle identification.

The muon-rejection system. Pions decay to muons and electrons with a half life of 2.6×10^{-8} sec (in their rest frame). Muons from decay in the spectrometer are removed by the muon-rejection system. The design of the muon-rejector uses the fact that, at a given momentum, muons have a $\sim 20\%$ greater range in matter than pions. This is because pions are absorbed by nuclear interactions while muons, which are not

strongly interacting particles, pass through. The muon-rejector consists of scintillators S4 through S9 separated by graphite blocks used as absorbers. Carbon was chosen as the absorbing material because of its relatively long radiation length (due to its low Z), large nuclear cross section (equal for π^+ and π^-), and low cost.

The muon-rejector is designed so that at a particular central spectrometer energy, pions with a shorter range in matter will not trigger a particular scintillator, whereas muons will. The signal from the scintillator is then used to veto events associated with muons instead of pions. The thickness of graphite before each scintillator is optimized to remove pions with a particular energy (100 MeV pions before S4, 140 MeV pions before S5, 180 MeV for S6, 220 MeV for S7, 260 MeV for S8, and 300 MeV for S9). A small gap before S4 allows the insertion of aluminum plates to induce energy loss and adjust the muon-rejector for intermediate energies. The graphite blocks are wedge shaped to compensate for the variation of particle momenta across the focal plane.

Computer Hardware and Software

The data acquisition system for EPICS consists of: an online VAX computer which runs under DEC's VAX VMS operating system, several CAMAC crates containing CAMAC electronic modules, a Microprogrammable Branch Driver (MBD) interfacing the VAX and the CAMAC crates, computer peripherals (tape drive, disk drive, and terminals), and the LAMPF standard data acquisition software package "Q" for managing the data acquisition and display of the experimental data [Kel 78; Ama 79].

A schematic of the EPICS electronics setup is shown in Figure 2.7. Raw signals from the chambers pass through amplifiers and fast discriminators in the experimental area and then the scintillator signals and chamber signals are passed via coaxial cables to the counting house. In the counting house the electronic hardware is quite standard and is rarely changed. The signals from both scintillators and chambers are processed using the fast electronics in the NIM bins. They are delayed as necessary, fanned out for distribution, pulse shaped, amplified, and discriminated. Meantiming is also done on the scintillator signals. The results provide pulse height, timing, scaler, and logical information.

These NIM standard pulses are then fed to CAMAC modules for digitizing. The modules contain Time to Digital Converters (TDCs), Analog to Digital Converters (ADCs), and scalers. Scintillator signals go through both ADCs and TDCs and chamber pulses go through TDCs. The $S2 \cdot S3$ (the output of a logical “and” unit with inputs from S2 and S3) is used as the start for the TDCs and the chamber signals are used as the stop. Unlike the NIM bins, which are just power supplies for the electronic units in them, CAMAC crates are set up so that the individual units can all talk to the crate controller over the crate’s data-way and different crates can communicate over the branch highway.

To read out and then clear the information in the CAMAC crates, a hardware trigger signal must be present; hardware trigger = $EVENT \cdot \overline{BUSY}$. This occurs if: a trigger signal is present indicating that an event has occurred, and a computer-not-busy signal has been generated by the LAMPF trigger module to indicate that it is ready to accept data. The EVENT trigger is an electronic determination that a real event has occurred

and is defined as the coincidence of signals from S2 and S3 and any two of the front wire chambers ($\text{EVENT} = \text{S2} \cdot \text{S3} \cdot \text{FRONT}$). One purpose of the hardware trigger is to minimize the number of bad events written to tape. In addition to the above, a run gate signal must be present indicating that a run is in progress (RUN) and a beam gate signal must be present indicating the presence of a macro-pulse on beam line A (BG).

The LAMPF Q system controls the process of reading in the approximately 50 data words associated with each event, writing them to magnetic tape, and analyzing the event online if computer time is available ("may process" mode of online data acquisition). In addition to reading the CAMAC modules when a trigger signal is present, the Q system reads various scalers at fixed time intervals and stores all quantities in a scaler file.

Data acquisition and replay were implemented using a specialized EPICS version of the Q system. This system is a flexible framework of programs written at LAMPF by the MP-1 computer group for the use of all experimenters there. The Q version used for this experiment was written to be compatible with the Digital Equipment Corporation (DEC) VAX VMS operating system.

Figure 2.8 shows the basic architecture of the Q system. The general structure is discussed here, and its specific use for this experiment is described in Chapter III. The analyzer is the heart of the Q system and contains all the software that actually processes data on an event-by-event basis and calculates all necessary quantities (see Appendix B for some information on building the analyzer). The general purpose management region

Figure 2.8 The structure of the Los Alamos Q system [Ama 86].

is a large common block through which all the Q subsystems access information. The histogram plotting package (HPL), which is intimately connected with the data testing package, allows one to select and histogram subsets of the raw data which pass certain user-defined criteria. The dynamic parameter array contains all the variables used in calculations, such as calibration offsets, kinematic parameters and processing flags. "Data word" is a name given to the quantities dealt with in the software. Examples are given in Appendix B, BLK001.TXT.

The following two sections describe in more detail the various steps in the data analysis, leading to a missing mass (excitation energy) histogram of good pion events which can be used to calculate scattering cross sections.

The EPICS Analyzer

The two types of events at EPICS are the scaler event (event 8) and the data event (event 6). The scaler event is processed by a standard EPICS subroutine PROC8. This processing consists mostly of Q system overhead; the function of the subroutine is simply to increment current scaler values stored in core memory by the values read. An electronic timer is set to trigger a scaler event every 30 seconds. Some scalers of importance are shown in Appendix B, SLRTTL.TXT.

Data event processing, by the subroutine PROC6 (also a standard EPICS subroutine), examines each event and calls subroutines to calculate all chamber and scintillator quantities from the outputs of the TDSs and ADCs, calculate trajectories and momenta, use the test file to test the various quantities, compute the missing mass associated with each event,

and arrange the data in bins for storage and display as one- and two-dimensional histograms. Because an extensive amount of CPU time can be used in processing individual events, on line data are acquired with event 6 in “may process” mode. The availability of CPU time determines whether or not an event is processed. In a typical run during the experiment, over 95% of all events were processed on line.

In the following discussion, the subscripts will refer to the usual convention for reactions, $A(a, b)B$ [for this experiment $^{60}\text{Ni}(\pi, \pi')^{60}\text{Ni}$]. The “ δ ” will be defined as the % difference between the momentum of a particular particle and the central momentum setting:

$$\delta = 100 \times \frac{p_{part} - p_{cent}}{p_{cent}} . \quad (2.1)$$

The analyzer finds $x_f, y_f, \theta_f, \phi_f, x_r, y_r, \theta_r, \phi_r$ from the chamber signals. From these measured quantities it calculates θ_{sc}, p_a , and p_b . The final quantity of interest is the “missing mass” (excitation energy) of the residual nucleus. This is calculated as follows.

To first order $x_{tgt} = -x_f$ and $y_{tgt} = -y_f$. However, due to aberrations in the quadrupole fields and misalignment of the system, x_{tgt} , y_{tgt} , θ_{tgt} , and ϕ_{tgt} are calculated in the analyzer as polynomials of up to fourth order in the front chamber positions and angles, x_f, y_f, θ_f , and ϕ_f . This uses the transport matrix for the spectrometer quadrupoles. The technique for determining the coefficients in the polynomial is outlined by Boudrie et al. [Bou 79]. These coefficients are stored in a file called POL.DAT which is read by the analyzer when it is initialized.

The scattering angle is given by: $\theta_{sc} = \theta_{sp} + \phi_{tgt}$, where θ_{sp} is the angle of the spectrometer with respect to the incident beam direction.

The momentum, p_a , of the incoming particle is calculated from its position at the target, x_{tgt} . The incoming momentum is $p_a = (\delta_{ch} + 1) \cdot p_{ch}$, where $\delta_{ch} = x_{tgt}/D_{ch}$, $D_{ch} = 10 \text{ cm}/\%$ is the dispersion of the channel, and p_{ch} is the central momentum of the channel beam (determined by the magnetic field of the channel bending magnets: $p_{ch} \approx B_{ch}/43.4 \frac{\text{Gauss}}{\text{MeV}/c}$).

The outgoing momentum, $p_b = (\delta_{sp} + 1) \cdot p_{sp}$, is calculated from quantities at the front and rear wire chambers. The central momentum of the spectrometer, p_{sp} , is calculated from the magnetic field of the spectrometer bending magnets ($p_{sp} \approx B_{sp}/25.63 \frac{\text{Gauss}}{\text{MeV}/c}$). To first order: $\delta_{sp} = (x_r + x_f)/D_{sp}$. This is made up of a magnification between front and rear chambers of -1 and the dispersion of the spectrometer, $D_{sp} = 4 \text{ cm}/\%$, giving: $x_r = -1 \times x_f + D_{sp} \times \delta_{sp}$. In practice, however, δ_{sp} is calculated as a polynomial in x_f , y_f , θ_f , ϕ_f , and x_r .

At the beginning of a run the subroutine KINSET asks for the reaction, target angle, and energy loss in the unrotated target. The reaction information is used to look up masses in a table. The target angle and unrotated energy loss determine the total energy loss in the target. The target energy loss and recoil energy are used to correct the energy of the incident and scattered pion.

The subroutine, CALKIN, is now called to calculate the missing mass of the event using θ_{sc} , p_a , and p_b plus the input kinematic information. First, the momentum transfer, p_B , is calculated from θ_{sc} , p_a , and p_b . The quantities E_a and E_b are calculated from $E = \sqrt{p^2 + m^2}$. Then missing mass, Q , is calculated from

$$(Q + m_B)^2 + p_B^2 = E_B^2, \quad (2.2)$$

where conservation of energy is used: $E_B = E_a + m_A - E_b$.

Data Testing and Display

The data acquisition system is set up so that once an event has been processed, i.e. the various relevant quantities calculated, any data word may be tested using the program ALLTST [Ama 79]. The set of tests defined in the test file are used to gate various histograms and to calculate quantities used for monitoring and normalization of the data. The output lists the tests and the number of events which have passed each test. A sample test file is given in Appendix B.

Tests on raw or calculated data words are called microtests, and tests on the logical combinations of previous tests are called macrotests. Examples of data words are given in Appendix B, BLK001.TXT, and examples of tests are given in Appendix B, EXP984REP.TST.

A microtest can be a BIT test which is true (passed) if a specified bit equals 1 in a specified data word, a GATE test which is true if the specified data word is between some user-defined limits, or a BOX test which is a combination of two GATE tests. The GATE and BOX tests can be either DIRECT or INDIRECT. The limits for DIRECT tests are set in the test file. Limits for INDIRECT tests are set on a displayed histogram; thus, they are often used for tests that will vary during the experiment.

Test #112 for the strip target is used as an example to explain a microtest: 112,GAT,233,-470,980 ;GATE 2 (60Ni GATE). This GATE test is on the XTGT data word #233. By looking at the histogram of XTGT, it was obvious that everything in bins -470 to 980 was coming from the ^{60}Ni target. The information after the “;” is comments.

Macrotests are logical ANDs and ORs on combinations of previous microtests and/or macrotests. One special macrotest is the IOR test, (IOR,1,-1,) which is always true since it is an OR of a true and false result. Since cumulative test results are compiled, this test is used to count the number of times the tests are executed.

The test file is divided into two blocks or loops that are called sequentially by the analyzer. Block one includes only tests on raw data words (signal in muon-rejection scintillators, odd-minus-even signal, and particle identification or PID). If the PID test is not true, the calculation of derived quantities and the execution of the remainder of the tests can be skipped, saving much CPU time. If an event passes the PID test, the analyzer proceeds to calculate the positions in the drift chambers. Then it calls the second block of tests on these derived quantities.

A good event must pass the PID test, the good-chamber tests, and background-rejection tests. Good events may be from pion, muon, or electron events.

The PID test is a box cut consisting of two indirect gates on the (geometrical) mean energy loss in S2 and S3 versus the time-of-flight between S2 and S3. This box cut will eliminate protons (but not muons or electrons) and some events with unreasonable pulse heights or flight times. Protons, for the same momentum as pions, have a smaller velocity and hence a longer time of flight.

The good-chamber tests are of two types, drift-time tests and drift-difference tests. The drift-time tests are gates (100 ns) on the drift times from all the chamber planes. The drift-difference tests are applied to pairs of chamber planes by comparing the positions calculated to a straight line

trajectory. These tests eliminate events which did not generate signals in one or more chamber planes, events for which two particles passed through the chambers simultaneously, and events where a pion has decayed between two chambers ($\approx 0.5 - 1\%$ of all events). The results of these tests are used to calculate the chamber and drift efficiencies.

There are six background rejection tests. Four of these tests are on x_{tgt} , y_{tgt} , θ_{tgt} , and ϕ_{tgt} . Histograms are made of these quantities and gate cuts are made at the fairly sharp edges on each histogram. Events outside these sharp edges are assumed to be due to background.

The other two background rejection tests are on θ_{chk} and ϕ_{chk} . These quantities are the difference between the angles measured at the rear chambers and those calculated for the rear focal plane from information measured at the front chambers using an empirically determined polynomial [Bou 79]. These tests eliminate at least 95% of the muons resulting from the decay of pions in the spectrometer. For 160 MeV pions, about 45% of the pions decay in the spectrometer. When a pion decays in flight, muons are emitted in a forward cone. Good pion events have θ_{ch} and ϕ_{ch} distributions no wider than 20 mrad, while the muon decay cone has an opening angle of approximately 460 mrad at $T_\pi = 160$ MeV. Thus, events falling outside the 20 mrad gate are assumed to be muon events. Once again, histograms are made of these quantities and gates are set at the fairly sharp edges.

The θ_{chk} and ϕ_{chk} will not eliminate muons due to pion decay before the front chambers or muons elastically scattered in the scattering chamber. An additional muon-rejection test, based on the muon-rejection hardware already discussed, is used to eliminate most of these muons.

Finally, x_{tgt} gates can be placed on the strip targets and their corresponding missing mass spectra can be separately histogrammed.

The display package is used to construct and display both one-dimensional histograms and two-dimensional dot plots. Histograms and dot plots are defined by the data words to be histogrammed or plotted, lower and upper limits of display, a binning factor, and the tests defined in the test file which the event must pass (see Appendix B, file `EXP984REP.HST`). Good pion events, which have passed the good-event test and the muon-rejection test, are histogrammed in `MM_PAS`. The final peak areas were extracted from these missing mass histograms.

Magnet Drift

The effect of magnetic field drift during long runs was looked at carefully in this experiment, because resolution was so important in finding the stretched states with very small cross section. Normally, as the channel and spectrometer magnetic fields drift, the peak positions shift slightly on the spectra. A new stabilization algorithm has recently been included in the `EPICS` analyzer to compensate for this drift. The user tells the analyzer where the centroid of a strong peak should be located and the energy width and number of events over which to average to find where the centroid actually is. After every event, the analyzer compares the present position of the centroid to where it should be. The difference in energy between the two is used as an energy offset for that event. During this experiment, the centroid of the most recent 100 events within ± 0.5 MeV of where the elastic peak (sometimes the 2^+ peak in ^{60}Ni) should be located was used to determine the offset.

To see how much the magnets drifted, the offsets from one run were periodically recorded (during replay) and then averaged over every 1000 events (about every 15 minutes). The results are shown in the upper diagram of Figure 2.9. The run had 24,000 events over a period of about 6 hours. During this time the spectra drifted by 40 keV. The result of this drift can be seen in the dashed-line peak of the lower diagram in Figure 2.9. If the stabilization offset is included, the result is a sharper peak with an improvement in resolution of about 7 keV as seen in the solid-line peak of the diagram in Figure 2.9.

This particular run was done between 3 pm and 9 pm in July. Thus a cooling of the magnets in the evening can explain the drift. Magnetic field readings were taken at the beginning and end of the run showing that the channel field increased by 0.2 Gauss and the spectrometer field increased by 1.1 Gauss. This is as expected, since the channel is better insulated by concrete shielding. The corresponding change in the magnetic fields is 4 keV and 37 keV, respectively. This calculated change is quite close to the offset found for the stabilization.

Figure 2.9 Magnet field drift (upper) and effect of stabilization algorithm (lower). In the lower plot 1 channel is about 10 keV. The dashed line shows an elastic peak without using the stabilization algorithm; the solid line is with.

CHAPTER III

DATA ACQUISITION AND ANALYSIS

This chapter, describing specific details of LAMPF experiment #984, is a continuation of Chapter II where general information is given about the experimental apparatus. This chapter describes the data acquisition and replay, the energy calibration, and the cross section calculations. It finishes by giving π^- and π^+ cross sections for the states of interest in ^{60}Ni .

Targets

During this experiment three targets (C, CH_2 , and ^{60}Ni) were run in four arrangements: a 10 cm $\times$ 20 cm sheet of C (a standard graphite target used at EPICS), a 10 cm $\times$ 20 cm sheet of CH_2 (also a standard plastic EPICS target), four 10 cm $\times$ 5 cm strips of ^{60}Ni , and a combination (strip) target made of three 10 cm $\times$ 5 cm strips of ^{60}Ni plus one 10 cm $\times$ 5 cm strip of CH_2 . In the strip target, the 1/4 frame CH_2 target was situated furthest in the $-x$ direction in the target frame. The densities and energy losses for each target are listed in Table 3.1.

For this experiment, 40.5 g of ^{60}Ni powder (originally from Oak Ridge National Laboratory) was borrowed from Argonne National Laboratory. The plan was to make the ^{60}Ni into eight 5 cm $\times$ 10 cm $\times$ 0.1 mm foils with an areal density of about 100 mg/cm² and a mass of approximately 5 g each. To make the thin foils it was necessary to repeatedly heat and roll the

Table 3.1 Density and energy loss of targets used in this experiment. Energy losses are for 162 MeV pions.

target	density (mg/cm ²)	$-dT/\rho dx$ (MeV-cm ² /g)	$-dT$ (keV)
⁶⁰ Ni	94.3	1.52	72
C	131.4	1.97	129
CH ₂ (full)	73.68	2.33	86
CH ₂ (strip)	84.0	2.33	98

foils. In the process three of the foils cracked. Of the five remaining foils, four were used to fill the 10 cm $\times$ 20 cm target frame. The final (average) areal density of the ⁶⁰Ni targets came out to be 94.3 mg/cm² $\pm$ 2%. The uncertainty is the variation in density over each foil and the variation in average density between foils.

The ⁶⁰Ni was enriched from natural Ni which is made up of 68% ⁵⁸Ni, 26% ⁶⁰Ni, 1% ⁶¹Ni, 4% ⁶²Ni, and 1% ⁶⁴Ni. Because ⁶⁰Ni is an enriched isotope, there is a question of how much of the other isotopes are left, particularly ⁵⁸Ni. To check the amount of impurity, data were taken on the Ni target at 30°, where the cross sections for the first 2⁺ and 3⁻ states are at a maximum [Olm 80]. For ⁵⁸Ni the 2⁺ state is at 1.454 MeV and the 3⁻ state is at 4.475 MeV [Pek 84]; whereas, for ⁶⁰Ni the corresponding energies are 1.333 MeV and 4.040 MeV [Aub 79]. The result of the 30° run was that any peak at 4.5 MeV could have (7 $\pm$ 5)% of the number of counts in the 4.0 MeV peak. The conclusion is that the nickel sample must be (93 $\pm$ 5)% ⁶⁰Ni. The production runs on ⁶⁰Ni at 65°, 80°, and 90° also show no peak at 1.5 MeV or 4.5 MeV. The possible impurity of the ⁶⁰Ni target was not included in the calculation of the cross sections.

Data Acquisition

This experiment was done near the $\Delta_{3,3}$ -resonance energy with the channel magnets set to give 162.0 ± 0.1 MeV incident pions. The spectrometer field was set so that pions causing an excitation energy of 12.8 MeV (approximately the average energy of the stretched states) would have a central momentum with $\delta = 0\%$. The elastic peak was then at $\delta \approx +6\%$.

Strip targets of Ni and CH₂ were initially used for both π^+ and π^- runs at a scattering angle of 80° . This was to insure that the energy calibration of the Ni spectra could be done correctly using the C spectra. Both targets would have exactly the same channel and spectrometer setting and any magnet drift would be the same. Once the calibration seemed satisfactory, a full frame Ni target was used to speed up data acquisition and a full frame C target was inserted about twice a day as a calibration check. During the data analysis, it was found that all carbon calibration runs at a given scattering angle were similar. Therefore the calibration runs were added together to give three summed spectra, one for each scattering angle.

Most nickel data runs lasted no more than 4 hours, so that if any problem appeared in a run, no large amount of data would have to be discarded. Data runs were taken at three angles for π^+ (65° —9 runs in 44 hours; 80° —17 runs in 59 hours— 90° : 12 runs in 38 hours) and two angles for π^- (65° —12 runs in 38 hours; 80° —19 runs in 79 hours).

At each angle and pion sign a normalization run was taken. The channel was set at the appropriate energy (104 to 124 MeV, depending on angle) to keep the $^1\text{H}(\pi, \pi')$ elastic peak in the same channel of the Mmass histogram for all runs. The H in the CH₂ target was used for

the normalization and a C target was run to subtract background. The normalization factor was found by comparing the yield in the elastic peak to previous experimental π -p cross sections.

An acceptance scan was taken in 9 runs by changing the spectrometer field in steps to move the elastic peak across the rear focal plane. To get high count rates, a π^+ beam scattering at an angle of 65° from a carbon target was used.

Replay

After data acquisition at LAMPF, the data tapes were replayed using the Q system set up on the University of Colorado Nuclear Physics VAX 11/780. The analyzer, text package, histogram parameters, and PRM files were optimized for this particular experiment before replay (see Appendix B).

Two slightly different analyzers were used in replaying the runs from this experiment because the electronics for the rear chambers went bad twice during the two-week experiment. For runs where all rear wire chambers were working correctly, the analyzer removed the left-right ambiguity by checking the odd-minus-even outputs of the anode wires. For runs where a rear wire chamber was not working, the analyzer removed the ambiguity by using the old method based on pairs of wire chambers being offset by half a wire spacing. For both analyzers the stabilization routine was included. Some details of building the analyzer are given in Appendix B.

The test file and histogram setup file used for this experiment are given in Appendix B. The purpose of the tests is explained in the previous chapter, but the basis for setting some of the test values is given below.

The left-right tests (#2-13) are used to remove the left-right ambiguity. The tests are based on the odd-minus-even signal histogram with an example shown in Figure 3.1. One edge of the gate is set based on the minimum in the histogram. For the two front chambers this minimum varied by up to 25 channels when the pion polarity and the scattering angle changed. These gates were thus set differently during replay for these different conditions.

The PID test (#15) is used to remove protons. A two dimensional plot of time-of-flight versus mean pulse height in S2 and S3 is shown in Figure 3.1. The PID box encloses the events that passed this test.

The muon rejection test (#1) checks if a signal occurred in the proper muon rejection scintillators, S4-S9. If a signal did not occur, the particle is a pion (test #100), otherwise it is a muon (test #101). For data runs and calibration runs, test #1 was BIT,2,1 to accept 162 MeV pions but reject muons that activate scintillator S5. For normalization runs, test #1 was BIT,2,0 to accept 102 to 124 MeV pions (depending on the thickness of aluminum) but reject muons that activate S4.

When the strip targets were used, gates were set on the XTGT histogram to separate events from the two targets (test #111-112). A 1 mm gap separated the two targets, but 30 channels (about 3 mm) was allowed for the gap. A sample XTGT histogram with one edge of the gate set for a strip target is shown in Figure 3.1.

Figure 3.1 Sample one- and two-dimensional histograms and sample cuts. Histograms shown here are a time-of-flight (y) versus pulse height (x) histogram for the S2 and S3 scintillators (PID: top), a strip target histogram with C on the left and ^{60}Ni on the right (XTGT: center left), an odd-minus-even histogram for left-right ambiguity (LRX2: center right), and a histogram of the angular deviation of a particle at the front chambers (THFRNT: bottom).

To improve resolution, gates were set to limit the angle of the pion trajectory at the front wire chambers (tests #121, 122, 129). The resolution can be decidedly improved by removing pions that deviate by a large angle (θ_f) in the x -direction from the central ray. Decreasing the size of the gate would improve resolution but decrease the statistics. To be able to find the small stretched states above the background, it was necessary to maximize the ratio between the peak height and the background variation. This is approximately equal to maximizing the ratio of $\frac{\text{peak area/resolution}}{\sqrt{\text{events}}}$. A nickel run was replayed using a number of different gate sizes. From the replayed histograms the area and width of the elastic peak were determined and the total number of counts in the histogram was found. Using these data, the above ratio was maximum for the gates as shown in Figure 3.1. These gates are not symmetric about $\theta_f = 0$, apparently because the polynomials used in ray tracing are not equally accurate for rays on either side of the central ray.

The stabilization routine in the analyzer used test #145 to determine which channels in the missing mass histogram to include in the routine. Fifty channels (0.5 MeV) on either side of the chosen peak were included. This routine used the elastic peak in all cases except for the ^{60}Ni π^- runs at 80° . In that case the 2^+ state was used because it had a larger cross section. For the strip targets the carbon elastic peak was used.

Some of the parameters in the PRM file had to be adjusted for each run, such as the peak on which to run the stabilization routine, the scattering angle, the pion energy loss in the target, and the magnetic fields in the channel and spectrometer (average for the run rather than just the field at the beginning). For runs where the old analyzer was used to remove

the left-right ambiguity, I135, I136, I139, and I140 had to be changed from 3 to 1. The nucleus for which to do kinematic calculations also had to be input. Carbon runs to be used in calibrating the nickel spectra were replayed using the appropriate nickel kinematic shifts. Some of the values contained in the PRM files are list in Appendix B under ITTL.TXT and RTTL.TXT.

The final results of the replay were the missing mass histograms (MM_PAS) used for finding peak areas to calculate cross sections. Events in these histograms were from data word #292 and had passed test #137, 138, or 139. For a 1/4 frame target (CH₂) the MMCPAS histogram based on test #137 was used. Similarly, a 3/4 target (N) used #138, and a full (F) target used #139. Events in the MM_PAS histograms have passed the good event test #93, the pion event (P) test #1, and the front angle cut (A) test #129. The histogrammed data word #242 has been optimized using the stabilization (S) routine.

For peak fitting, the missing mass histograms from Q were transferred to histograms in SPECTR, a University of Colorado peak fitting code written by Dale Prull.

Cross Section Calculations

Cross sections were calculated using:

$$\left. \frac{d\sigma}{d\Omega} \right|_{cm} = \frac{(area/atom)(\%reacting)}{(solid\ angle)(efficiencies)} \times (Jacobian) .$$

The $area/atom$ is:

$$\frac{area}{atom}(\mu b) = \frac{at.wt.(g/mole)}{tgt.dens./\cos\theta_{tgt}(g/cm^2) \cdot A_0 \cdot N} \times 10^{30}(\mu b/cm^2) ,$$

where A_0 is Avogadro's number, N is the number of atoms per molecule (1 for Ni and C; 2 for H in CH_2), and $\cos\theta_{tgt}$ is half the scattering angle. The target density is adjusted by $\cos\theta_{tgt}$ to account for the target not being perpendicular to the beam. The $\%reacting$ is:

$$\%reacting = yield(out) / \#pions(in) ,$$

where *yield* is the number of counts in the histogram peak and $\#pions = IC1 \times \#pions/IC1 \times (tgt. fraction)$. The value of *IC1* is proportional to the pion flux and comes from scaler #61, $\#pions/IC1$ is the proportionality constant and comes from normalizing to a known elastic cross section, and the *tgt. fraction* (for strip targets) is found from the accep'tance scan. The *solid angle* is approximately 10 msr, but varies with δ and is also found from the acceptance scan. The *efficiencies*, to be explained next, are made up of the survival fraction, the chamber and drift efficiencies, the computer live time, and the loops per event. The *Jacobian*, to convert from lab cross section to center-of-mass cross section, is calculated from the kinematics of the reaction.

Both the solid angle (acceptance) and survival fraction vary with excitation energy. A computer code, explained in the next section, was used to make these adjustments on the Q histograms before transferring them to SPECTR histograms.

The other *efficiencies*,

$$EFF = CHEff \cdot DReff \cdot CLT \cdot loops/event ,$$

are calculated and tabulated at the end of each run. The chamber efficiency is based on the number of good events that generated signals in all planes. It is defined as:

$$CH_{eff} = \prod_{i=1}^{12} \frac{(all\ fired) \cdot (PID)}{(all\ fired\ except\ i) \cdot (PID)} = \frac{test\#46}{test\#'s71-86} ,$$

where the numerator is the number of times in a run that test #46 was passed and the denominator is the number of times test #71 or 72 or ... or 86 was passed. The chamber efficiency for this experiment was 0.95 ± 0.01 . The drift efficiency is based on the number of good events for which the drift time correction was made correctly (e.g., the pion did not decay in flight), and is defined as:

$$DR_{eff} = \frac{good\ chambers}{chambers\ fired} = \frac{test\#90}{test\#46} .$$

The drift efficiency was 0.91 ± 0.01 for π^- and 0.86 ± 0.02 for π^+ , except for runs where a wire chamber was bad, in which case the efficiency was about 0.76. The computer livetime is the fraction of time the computer is available to take data from the MBD:

$$CLT = \frac{EVENTS \cdot \overline{BUSY}}{EVENTS} = \frac{scaler\#2}{scaler\#1} .$$

The computer livetime was 0.98 ± 0.01 for data runs, and smaller for calibration runs. The *loops/event* is the fraction of events analyzed:

$$loops/event = \frac{events\ analyzed}{EVENTS \cdot \overline{BUSY}} = \frac{test\#16}{scaler\#2} .$$

This fraction was less than 1 during data acquisition in "may process" mode but equal to 1 during replay in "must process" mode.

Combining the above equations gives the equation used in calculating cross sections from SPECTR histograms

$$\left. \frac{d\sigma}{d\Omega} \right|_{cm} = \frac{(at.wt.)(\cos\theta_{tgt})(JAC)}{(tgt.dens)(N)} \times \frac{1}{tgt. fraction} \cdot \frac{1}{IC1 \cdot EFF} \cdot norm \cdot yield, \quad (3.1)$$

where *norm* includes the conversion between cm^2 and μb (10^{30}), Avogadro's number (6.023×10^{23}), the spectrometer solid angle (≈ 0.01 sr), and the $\#pions/IC1$ ($\approx 2 \times 10^6$). The procedure for finding *norm* is explained in a following section.

Acceptance and Survival Fraction Corrections

An acceptance correction is necessary because the spectrometer solid angle depends on the x_{tgt} position and the position in the dispersion direction on the rear focal plane. To determine the correction, an acceptance scan was done, where the spectrometer field was changed in steps of $\delta \approx 2\%$ from -8% to $+8\%$. To compare the different runs, the number of counts in the elastic peak at each delta setting was adjusted by the efficiencies and the IC1 scaler. This adjusted number of counts (in arbitrary units) as a function of δ is shown in Figure 3.2 for the 1/4, 3/4, and full targets. The shape of the acceptance curve is somewhat different for the strip targets than for the full targets because the acceptance varies as a function of x_{tgt} . From this scan, the *tgt. fraction* was determined to be 0.204 for the 1/4 target and 0.780 for the 3/4 target. (A fraction of 0.016 is in the gap between targets.)

Figure 3.2 Results of the acceptance scan. The dotted line is for the 1/4 target, the dashed line is the 3/4 target, and the solid line is the full target.

A survival fraction correction is necessary because pions decay in flight between the scattering target and the rear focal plane. For exponential decay, $dN/dt = -\lambda N$. Integrating this gives the survival fraction as $SF = N/N_0 = e^{-\lambda t}$. The following definitions are then used: $\lambda = 1/\tau$ where τ is the mean lifetime, $t = t_\pi = t_l/\gamma$ where t_π is time in the pion rest frame and t_l is time in the lab frame, $t_l = D/v$ where D is the distance the pion travels in the lab frame and $v = \beta c$ is its velocity, and $\gamma\beta = p/m$. Putting these together the survival fraction is:

$$SF = \exp\left[-\frac{D \cdot m}{c\tau \cdot p}\right].$$

For pions at EPICS, D is the distance from the scattering chamber to the S3 scintillator and depends on the pion momentum as $D = (12.28 - 0.035\delta_{part})$ in meters, $m_\pi = 139.567$ MeV, and $c\tau = 7.804$ m. For the range of momentum in this experiment, $SF = 0.44$ for the elastic peak at $\delta = +6\%$ and 0.42 for a 13 MeV excited state at $\delta = 0\%$.

Both the acceptance correction and the survival fraction correction vary with momentum across the focal plane; therefore, each bin in the missing mass histogram must be corrected separately. A program to do this correction was run on the Q histograms. The absolute magnitude of this correction is unimportant as long as the same correction is used for input to both data and normalization runs. The correction could therefore be made such as to leave the adjusted histogram with approximately the same number of counts as the original histogram. Thus the percent uncertainty in the peak areas found during peak fitting (based on the number of counts in a peak) would be calculated correctly.

Normalization

Neither the solid angle of the spectrometer nor the $\#pions/IC1$ are known absolutely, therefore both are included in the factor $norm$. The $\#pions/IC1$ is dependent on the incident pion energy and charge because the relative number of pions, muons, and electrons changes with energy and charge [See 81]. Incident pion energy also affects $\#pions/IC1$ because the $IC1$ current depends on the amount of energy lost by a particle in the ion chamber. The energy remained constant throughout this experiment, however; therefore $norm$ should depend only on the pion charge.

The factor $norm$ was calculated by using the elastic ${}^1H(\pi, \pi)$ reaction and the elastic ${}^{12}C(\pi, \pi)$ reaction, both of which have previously determined experimental cross sections. For the ${}^1H(\pi, \pi)$ reaction, CH_2 and C were used as targets at each angle and pion sign and 500 to 5000 counts were accumulated in the elastic hydrogen peak. The background was variously taken care of by a linear fit and by subtracting carbon from CH_2 . Both methods of background subtraction gave similar results. For the ${}^{12}C(\pi, \pi)$ reaction, the summed carbon calibration spectra at each angle and pion sign were used. The known elastic π -p cross sections were taken from Arndt's SM86 phase shifts [Arn 86] and the elastic ${}^{12}C$ pion scattering cross sections from Piffaretti et al. [Pif 77]. Using the area under the elastic peak and the known elastic cross section, equation (3.1) was inverted to find $norm$. (See Brack et al. [Bra 86] and references therein for a discussion about the accuracy of the elastic π -p cross sections used here for normalization.)

The resulting $norm$ for each pion sign and angle is given in Table 3.2. There is no consistent $norm(\pi^-)/norm(\pi^+)$ ratio from π -p and

^{12}C ; therefore, one *norm* is used for all angles and both pion charges. The average value for *norm* (in arbitrary units) was found to be 8.19 ± 0.31 using the π -p cross sections, 8.14 ± 0.69 using the elastic ^{12}C cross sections, and 8.17 ± 0.38 using a combination. In the following calculations, the average of all ten values is used: $\text{norm} = 8.2 \pm 5\%$.

Table 3.2 Normalization factors are shown as a function of angle and pion sign. Normalizations are done both for ^1H and ^{12}C elastic scattering. The units are arbitrary.

angle	$^1\text{H}(\pi, \pi)$		$^{12}\text{C}(\pi, \pi)$	
	π^-	π^+	π^-	π^+
65°	9.32	8.66	6.16	9.99
80°	7.72	7.75	6.64	8.30
90°		7.48		9.63

Peak Fitting, Adding Runs, and Binning Spectra

Peak fitting was done on histograms in SPECTR after they had been transferred from Q missing mass histograms. The elastic and strong 2^+ and 3^- peaks were fit in each of the carbon calibration runs and the nickel data runs. The line shape used for fitting was an asymmetric Gaussian with the width, skew, and centroid separately adjustable for each peak. A skew was necessary because there is a tail on the high excitation energy side of the peak. The final resolution (peak width) was 190 ± 10 keV.

To determine the location of the 0 MeV channel and thus the offset needed in adding individual runs, a least squares fit was done on the centroids of the three low-lying peaks in each run. After adding spectra from the individual runs, there were five nickel data spectra and five carbon calibration spectra. The five nickel spectra are shown in Figure 3.3. The five

Figure 3.3 The ^{60}Ni spectra for each angle and pion sign. Each channel is 50 keV wide.

nickel spectra were added together into one spectrum, shown in Figure 3.4, to facilitate finding the stretched states. Carbon runs for both signs at each angle were added together to give the three spectra shown in Figure 3.5.

To optimize the search for stretched states, the channel spectra were rebinned so that a peak would be 4 channels wide. With a resolution of about 200 keV and an original binning of 10 keV/channel, this involved a rebinning factor of 5. The sets of 5 channels binned (0-4, 1-5, 2-6, etc.) were optimized to enhance the stretched states.

The centroids (excitation energies) of the stretched states, including possible pure isoscalar states that would not have been found in electron scattering, were determined from the one summed nickel spectrum. Then in the five individual nickel spectra, the stretched state peaks were fit using the width and skew of the elastic peak, the centroid from the summed spectrum, and a linear background. (In practice, the fixed centroids to use for the stretched states were determined from a weighted average of the excitation energies found in electron scattering, proton stripping, and this pion scattering experiment.)

Energy Calibration of ^{60}Ni Spectra

After determining the centroids of the low lying states in the ^{60}Ni and ^{12}C spectra, it was found that the histograms did not have an exact conversion of 100 channels to 1 MeV. This may be because the ray tracing to determine the `POL.DAT` file was not done using 162 MeV incident pions. As a result the `POL.DAT` file was not optimized for the energy of this experiment, and it was necessary to determine a channel/MeV conversion.

Figure 3.4 The ^{60}Ni summed spectra over all angles and signs. Shown are the low-lying states, the 8^- stretched states from electron scattering, and several other prominent states.

Figure 3.5 The spectra used for calibration. Top is ^{60}Ni summed over all angles and signs. The other three are C spectra replayed using ^{60}Ni kinematics at 65°, 80°, and 90°.

The nickel spectrum summed over scattering angles and pion signs and the three carbon spectra summed over signs, as shown in Figure 3.5, were used to determine the calibration. The carbon had been replayed using nickel kinematics, so the equivalent nickel excitation energies for the carbon peaks were found by running a kinematics program. The elastic, 2^+ , and 3^- peaks were used: 0.0, 1.333, and 4.040 MeV for ^{60}Ni [Aub 79] and 0.0, 4.439, and 9.641 MeV for ^{12}C [Ajz 75]. A least squares fit was done to these 12 data points assuming a second order polynomial. The importance of each point was weighted by the uncertainty from fitting. Actually, the final result had an effective dependence on only 8 input values, because the offset between each of the 4 spectra was varied to get the best least squares fit.

The final result was:

$$\text{channel} = A(\text{exc. eng.})^2 + B(\text{exc. eng.}) + C ,$$

where $A = 0.075 \pm 0.006$, $B = 98.08 \pm 0.08$, and C is the centroid channel of the elastic peak. Thus the channel to MeV conversion is:

$$\frac{\Delta \text{channel}}{\Delta \text{MeV}} = 0.15(\text{exc. eng.}) + 98.08$$

This results in 100 channels/MeV at an excitation energy of 12.8 MeV where $\delta = 0\%$, as expected from the setup.

The resulting energies and uncertainties for the states of interest in ^{60}Ni are shown in Table 3.3 along with energies measured previously using other reactions.

Table 3.3 Measured excitation energies (in MeV) and uncertainties for the states found in ^{60}Ni pion scattering. These are compared to the energies and uncertainties found in previous electron scattering [Lin 81b; Pet 87] and proton stripping reactions [Pet 87].

(π, π')	(e, e')	(α, t)
5.11 ± 0.02		
5.37 ± 0.04		
5.70 ± 0.04		
6.22 ± 0.02		
6.91 ± 0.02		
7.08 ± 0.02		
7.57 ± 0.02	7.522 ± 0.015	7.550 ± 0.008
	8.433 ± 0.021	8.445 ± 0.011
8.96 ± 0.03	8.959 ± 0.042	8.994 ± 0.010
9.21 ± 0.03	9.172 ± 0.030	9.208 ± 0.010
	12.333 ± 0.028	12.305 ± 0.020
	12.505 ± 0.039	12.515 ± 0.016
13.91 ± 0.04	13.908 ± 0.041	13.883 ± 0.016
	14.840 ± 0.040	14.817 ± 0.010
15.54 ± 0.05	15.499 ± 0.040	15.483 ± 0.019
	16.080 ± 0.040	16.110 ± 0.023

Results and Error Analysis

The cross sections measured in this study are given in Table 3.4. The cross sections for the low lying states in ^{60}Ni are new, but similar to the corresponding cross sections in ^{58}Ni [Olm 80]. Cross sections are listed for six new states between 5 and 7 MeV that were investigated in case they were pure isoscalar 8^- stretched states not seen in electron scattering. The two peaks at 5 MeV and the two peaks at 7 MeV are both doublets. Finally, cross sections are given for the ten states between 7.5 and 16.1 MeV identified as stretched states in electron scattering. The 7.5, 9.0, and 9.2 MeV $T_<$ states and the 13.9 and 15.5 MeV $T_>$ state were found unambiguously in this pion scattering experiment. The other five states were possibly found.

Table 3.4 Center-of-mass cross sections ($\mu\text{b}/\text{sr}$) for ^{60}Ni from this experiment. The 65.28° , 80.31° , and 90.31° center-of-mass angles are 65° , 80° , and 90° in the lab frame. The listed uncertainties are only statistical; absolute uncertainties can be found by adding 7% in quadrature to the statistical uncertainties. Cross sections denoted by a * are questionable as to whether the fit was to a peak or a background fluctuation and whether whatever peak is there has the right energy centroid.

J^π	Energy	65.28°	80.30°	90.31°
π^-				
0^+	0.00	3481 ± 23	84.9 ± 2.9	
2^+	1.33	36.5 ± 3.5	156.5 ± 3.9	
3^-	4.04	278.1 ± 6.9	34.1 ± 1.9	
$8^- ?$	5.11	52.5 ± 4.6	32.5 ± 2.5	
	5.37	21.3 ± 3.7	<3	
	5.70	33.5 ± 3.8	11.6 ± 1.5	
	6.22	26.0 ± 3.8	17.5 ± 1.8	
	6.91	19.0 ± 4.0	8.7 ± 1.8	
	7.08	14.7 ± 3.8	3.5 ± 1.6	
	7.55	<6	$*7.6 \pm 1.6$	
8^-	8.44	$*10.6 \pm 4.0$	<6	
8^-	8.98	<5	<5	
8^-	9.20	$*10.4 \pm 4.0$	4.7 ± 2.0	
8^-	12.32	$*11.8 \pm 4.0$	<3	
8^-	12.51	<4	<7	
8^-	13.88	17.9 ± 4.1	7.3 ± 2.5	
8^-	14.82	<7	$*7.8 \pm 2.5$	
8^-	15.49	$*13.5 \pm 4.0$	<7	
8^-	16.10	$*11.8 \pm 4.3$	$*6.1 \pm 2.5$	
π^+				
0^+	0.00	1550 ± 7	234.5 ± 2.5	110.8 ± 1.8
2^+	1.33	92.0 ± 1.8	101.9 ± 1.7	17.9 ± 0.8
3^-	4.04	182.5 ± 2.5	19.5 ± 0.8	35.1 ± 1.0
$8^- ?$	5.11	37.5 ± 1.5	31.1 ± 1.2	21.7 ± 1.0
	5.37	12.2 ± 1.1	<3	5.1 ± 0.8
	5.70	20.5 ± 1.3	<4	6.5 ± 0.8
	6.22	22.8 ± 1.4	8.6 ± 0.9	6.4 ± 0.9
	6.91	16.6 ± 1.6	5.5 ± 0.7	7.1 ± 1.0
	7.08	7.0 ± 1.4	<2	<2
	7.55	6.3 ± 1.5	9.6 ± 0.9	5.2 ± 1.0
8^-	8.44	<2	<2	$*1.6 \pm 0.8$
8^-	8.98	5.6 ± 1.5	5.8 ± 1.1	5.1 ± 1.0
8^-	9.20	7.8 ± 1.4	3.3 ± 0.9	6.1 ± 1.0
8^-	12.32	<3	<3	<3
8^-	12.51	$*6.0 \pm 1.6$	<2	<3
8^-	13.88	$*2.9 \pm 1.6$	4.2 ± 1.1	2.4 ± 1.1
8^-	14.82	<3	<4	$*2.9 \pm 1.2$
8^-	15.49	5.0 ± 1.7	$*5.5 \pm 1.3$	$*3.2 \pm 1.2$
8^-	16.10	$*6.4 \pm 1.8$	<2	<3

but it is questionable whether real peaks were being fit or only fluctuations in the background.

The uncertainties for the scattering cross sections are separated into statistical uncertainties and peak-independent uncertainties. Each individual peak area has an uncertainty associated with it, due to uncertainty in the background fit (in general $\pm 5\%$) and due to the statistics and best χ^2 fit for the peak. These are the uncertainties included in Table 3.4.

The peak-independent uncertainties are as follows. The focal plane acceptance scan was taken with 3-4% statistics, so an uncertainty of $\pm 2\%$ is included. The survival fraction correction introduces a systematic uncertainty of 2% because the path length in the spectrometer is not accurately known. Possible errors in measuring chamber efficiencies and in beam monitoring are estimated to be $\pm 3\%$ each, based on how much the values varied over the 2 week experiment. A $\pm 5\%$ uncertainty is used for *norm*, based on its variation between ^1H and ^{12}C and on its variation for different scattering angles and pion sign. An uncertainty of $\pm 2\%$ in the target density, as explained in the previous chapter, is included. Adding these errors in quadrature gives 7%.

When comparing π^+ and π^- cross sections the uncertainties cancel for the focal plane acceptance, the survival fraction, and the target density. The resulting relative uncertainty, irrespective of peaks, is 6%.

Summary

This concludes the experimental section of the thesis. The cross sections listed in this chapter will next be compared to theoretical predic-

tions. Chapter IV describes the theoretical calculations and Chapter V compares the data and theory.

CHAPTER IV

REACTION THEORY

This chapter is designed to explain the basic concepts of scattering theory. The formalism and some general approximations are given first. These are then applied to pion scattering where multiple-scattering theory is used for pion-nucleus reactions. Descriptions are given of the inelastic scattering potential used for stretched states and of the optical potential used for distorting the waves. The use of computer codes in doing these pion scattering calculations is explained. A short section describes electron scattering (which uses the plane wave Born approximation). The final section describes how pion and electron scattering data will be compared in Chapter V.

Basic Scattering Theory

This section gives a brief summary of scattering theory based on Cohen, McCarthy, Merzbacher, and Satchler [Coh 71; McC 70; Mer 70; Sat 80]. Quantum mechanical scattering theory is based on finding a solution of the Schrödinger equation, $H\Psi = E\Psi$, where the scattering potential is treated as a perturbation of a plane wave. The Hamiltonian ($H = H_0 + V$) for the system is made up of kinetic ($H_0 = -\hbar^2\nabla^2/2m$) and potential (V) energies. The Schrödinger equation can then be rewritten as

$$(\nabla^2 + k^2)\Psi = \frac{2m}{\hbar^2}V\Psi, \quad (4.1)$$

where $k^2 = 2mE/\hbar^2$. Here it is assumed that the incident and emitted particles have no other interactions with the nucleus and they may be represented by plane waves before and after the reaction. Note, however, that the complete wave function should be made up of the relative motion of the projectile and target, $e^{i\mathbf{k}\cdot\mathbf{r}}$, plus the wave functions for the internal structure of the projectile and target, ψ .

The formal solution. First, equation (4.1) is solved formally for some general potential, V . When $V = 0$, equation (4.1) becomes $(\nabla^2 + k^2)\Psi = 0$ for a free particle of energy E . This has as a solution the plane wave $\Psi = e^{i\mathbf{k}\cdot\mathbf{r}}$. The presence of the potential V introduces scattered waves in addition to the plane wave. The $V\Psi$ on the right side of equation (4.1) can be treated as the source term that generates these scattered waves. The solution of equation (4.1) then consists of the plane waves plus the scattered waves as:

$$\Psi(\mathbf{k}, \mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} - \frac{2m}{4\pi\hbar^2} \int \frac{\exp(ik|\mathbf{r} - \mathbf{r}'|)}{|\mathbf{r} - \mathbf{r}'|} V(\mathbf{r}') \Psi(\mathbf{k}, \mathbf{r}') d\mathbf{r}' . \quad (4.2)$$

Scattered particles are detected experimentally at a relatively large distance r , therefore $\Psi(\mathbf{r})$ for large r can be used. Now $V(\mathbf{r}')$ vanishes for large r' so that for large r , $1/|\mathbf{r} - \mathbf{r}'| \rightarrow 1/r$ and $k|\mathbf{r} - \mathbf{r}'| \approx kr - \mathbf{k}' \cdot \mathbf{r}'$. Equation (4.2) then becomes:

$$\Psi \rightarrow e^{i\mathbf{k}\cdot\mathbf{r}} - \frac{m}{2\pi\hbar^2} \frac{e^{ikr}}{r} \int \exp(-i\mathbf{k}' \cdot \mathbf{r}') V(\mathbf{r}') \Psi(\mathbf{k}, \mathbf{r}') d\mathbf{r}' . \quad (4.3)$$

Using operator notation, equation (4.1) can be written as $(E - H_0)\Psi = V\Psi$ and then converted into an integral equation. It is necessary to recognize, though, that the inverse operator, $(E - H_0)^{-1}$, is singular

when acting upon an eigenstate of H_0 with the eigenvalue E . In scattering theory this is avoided by including an infinitesimal imaginary quantity $i\epsilon$ with ϵ positive (a boundary condition of outgoing waves at infinity)

$$\frac{1}{E - H_0} \Rightarrow \lim_{\epsilon \rightarrow 0} \frac{1}{E - H_0 \pm i\epsilon} \equiv G_0 , \quad (4.4)$$

where $G_0(\mathbf{r}, \mathbf{r}')$ is the Green's function operator. In the momentum representation this would be called a propagator. Using this technique, the formal solution for Ψ can be written as

$$\Psi = \phi - G_0 V \Psi , \quad (4.5)$$

with $\phi = e^{i\mathbf{k} \cdot \mathbf{r}}$. This can be compared to equation (4.2), with $G_0(\mathbf{r}, \mathbf{r}') = \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}$.

Equations (4.3) and (4.5) are two ways of writing the formal solution of the Schrödinger equation. It consists of a non-scattered plane wave, plus a scattered spherical wave and can be written as

$$\Psi = e^{i\mathbf{k} \cdot \mathbf{r}} + f(\theta) \frac{e^{ikr}}{r} . \quad (4.6)$$

The scattering amplitude $f(\theta)$ is

$$\begin{aligned} f(\theta) &= -\frac{m}{2\pi\hbar^2} \int \exp(-i\mathbf{k}' \cdot \mathbf{r}') V(\mathbf{r}') \Psi(\mathbf{k}, \mathbf{r}') d\mathbf{r}' \\ &= \langle \phi_f | V | \Psi_i \rangle \\ &= \langle \phi_f | T | \phi_i \rangle , \end{aligned} \quad (4.7)$$

where i and f refer to initial and final states with momentum $\mathbf{k}$ and $\mathbf{k}'$, respectively. (From here on the factor $-m/(2\pi\hbar^2)$ in the scattering amplitude will be dropped to simplify notation.) The scattering operator or

T -matrix is defined by $T\phi = V\Psi$. It is the transition operator between the initial and final states of the system. Using this with equation (4.5) gives the Lippmann-Schwinger equation

$$T = V - VG_0T. \quad (4.8)$$

The differential cross section is given by the square of the scattering amplitude

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2. \quad (4.9)$$

The partial wave solution. Equation (4.3) is now solved in a partial wave analysis with some central potential, V . Use is made of partial wave functions of definite l , since the relative angular momentum $l\hbar$ of two colliding particles is quantized in units of $\hbar$. This is important for studying stretched states because these reactions select a particular angular momentum from the beam. The wave function Ψ in equation (4.3) is expanded in terms of the separable solutions of the central force problem, which are assumed to constitute a complete set of orthonormal eigenfunctions of H , L^2 , and L_z . Thus

$$\Psi = \sum_{l=0}^{\infty} a_l(k) Y_l^m(\theta, \phi) \frac{u_{l,k}(r)}{r}. \quad (4.10)$$

When using a spherically symmetric central potential in equation (4.3), it is to be expected that the solutions, representing an incident and a scattered wave, should exhibit cylindrical symmetry about the direction of incidence. Hence, it is assumed without loss of generality that the incident momentum $\mathbf{k}$ points in the positive z -direction, and that, for a given value of k , Ψ is a function of only r and the scattering angle θ . For cylindrical

symmetry about the incident direction, $Y_l^m = \delta_{m0} \sqrt{(2l+1)/4\pi} P_l(\cos \theta)$.

The radial function $u_{l,k}(r)$ satisfies the differential equation

$$\left[-\frac{d^2}{dr^2} + \frac{2m}{\hbar^2} V(r) + \frac{l(l+1)}{r^2} - k^2 \right] u_{l,k}(r) = 0 . \quad (4.11)$$

Ignoring Coulomb effects, the general solution of this equation is a linear combination of the regular and irregular solutions and has the form

$$\frac{u_{l,k}(r)}{r} = A_l j_l(kr) + B_l n_l(kr) \xrightarrow{r \rightarrow \infty} \frac{\sin(kr - l\pi/2 + \delta_l)}{kr} . \quad (4.12)$$

The last step uses the asymptotic approximation for large kr and the definition $B_l/A_l = -\tan \delta_l$, where δ_l measures the amount by which the phase of the radial function for angular momentum l differs from the no-scattering case ($\delta_l = 0$).

The expansion coefficients $a_l(k)$ in equation (4.10) can be determined by comparing equation (4.6) and equation (4.10) for large distances r . The expansion for the plane waves of equation (4.6) is

$$\begin{aligned} e^{i\mathbf{k} \cdot \mathbf{r}} &= e^{ikz} = e^{ikr \cos \theta} \\ &= \sum_{l=0}^{\infty} i^l (2l+1) j_l(kr) P_l(\cos \theta) \\ &\rightarrow \sum_{l=0}^{\infty} (2l+1) i^l \frac{\sin(kr - l\pi/2)}{kr} P_l(\cos \theta) . \end{aligned} \quad (4.13)$$

By substituting equation (4.13) into equation (4.6) and equation (4.12) into equation (4.10) the expansion coefficients $a_l(k)$ can be found. Equation (4.10) then becomes

$$\Psi = \sum_{l=0}^{\infty} (2l+1) i^l e^{i\delta_l} \frac{\sin(kr - l\pi/2 + \delta_l)}{kr} P_l(\cos \theta) \quad (4.14)$$

which differs from a plane wave by the presence of δ_l and is called a distorted plane wave. Comparing the coefficients of e^{ikr}/r in equation (4.14) and equation (4.6) and using equation (4.13) to represent the plane wave in the latter equation, gives the scattering amplitude

$$f(\theta) = \sum_{l=0}^{\infty} (2l+1) \alpha_{2T,2J}^l P_l(\cos \theta) . \quad (4.15)$$

The phase shifts are

$$\alpha_{2T,2J}^l = \frac{\exp(i\delta_{2T,2J}^l) \sin(\delta_{2T,2J}^l)}{k} , \quad (4.16)$$

and each phase shift is a function of the energy.

The plane wave Born approximation. Equation (4.7) is the formal definition of the scattering amplitude $f(\theta)$; however, it is not an explicit expression because it contains Ψ in the integral. Approximations are made so that the definition is usable for practical purposes. In the plane wave Born approximation (PWBA), it is assumed that the potential V is weak enough in equation (4.7) so that Ψ can be approximated by just the plane wave $\phi = e^{i\mathbf{k} \cdot \mathbf{r}}$. The scattering amplitude in the (first) Born approximation is

$$\begin{aligned} f(\theta) &= \int \exp(-i\mathbf{k}' \cdot \mathbf{r}') V(\mathbf{r}') \exp(i\mathbf{k} \cdot \mathbf{r}') d\mathbf{r}' \\ &= \int \exp(i\mathbf{q} \cdot \mathbf{r}') V(\mathbf{r}') d\mathbf{r}' \\ &= \langle \phi_f | V | \phi_i \rangle , \end{aligned} \quad (4.17)$$

where $\mathbf{q} = \mathbf{k} - \mathbf{k}'$.

This first Born approximation corresponds to a first iteration of equation (4.5), where the plane wave is substituted for Ψ in the integral. If

this expression is used as an improved approximation and substituted into equation (4.5), the result is the second Born approximation. By iterating the integral equation in this way a series expansion of $\Psi(\mathbf{r})$ called the Born series is developed. This Born series for the scattering amplitude (4.7) is written in operator notation as

$$f(\theta) = \langle \phi_f | V - VG_0V + VG_0VG_0V - \dots | \phi_i \rangle . \quad (4.18)$$

The operator in this expression is the Born expansion of the T -matrix. Combining this with equation (4.8) gives the Born series for T

$$T = V - VG_0V + VG_0VG_0V - \dots . \quad (4.19)$$

The physical interpretation of this series is that the scattering can be expressed as a sum of terms in which each successive action of the potential is followed by periods of free propagation. This series will converge rapidly if V is weak. In that case terms of high order in V can be neglected.

In the simplest approximation V can be taken to be a zero-range potential that acts only at a single radius R ; thus $V = V_0\delta(\mathbf{r} - \mathbf{r}')\delta(r - R)$. Using this approximation for V in equation (4.17) and including the nuclear wave functions, ψ_i and ψ_f , gives

$$f(\theta) = V_0 \int \psi_f^* e^{i\mathbf{q} \cdot \mathbf{R}} \psi_i d\mathbf{r}' . \quad (4.20)$$

The mathematical expansion for $e^{i\mathbf{q} \cdot \mathbf{R}}$ in equation (4.13) is now used. When it is inserted into (4.20), the result is

$$f(\theta) \propto \langle \psi_f | j_l(qR) | \psi_i \rangle , \quad (4.21)$$

where $j_l(qR)$ are the spherical Bessel functions.

In some calculations, the $\delta(\mathbf{r} - \mathbf{r}')$ is replaced by some finite range of interaction. In more extensive calculations, the velocity dependence in the interaction V (which arises from the velocity dependence of the nucleon-nucleon force) is taken into account. This is referred to as the use of nonlocal interactions.

The distorted wave Born approximation. A more detailed reaction theory which is often successful in predicting spectroscopic factors and at least the first peak in angular distributions is the distorted wave Born approximation (DWBA). This theory replaces the plane waves, $\phi = e^{i\mathbf{k}\cdot\mathbf{r}}$, describing the probability amplitude of finding an unbound particle, by optical model wave functions, $\chi^{(\pm)}(\mathbf{k}, \mathbf{r})$, for the elastic scattering of the particle at the appropriate energy.

The scattering potential can be written as $V = V_{el} + V_{non}$, which is made up of an optical potential describing the elastic scattering plus the interaction which describes the non-elastic transition. It is then assumed that elastic scattering is the predominant reaction and non-elastic transitions can be treated as a perturbation. Equation (4.1) is then written as

$$(\nabla^2 + k^2 - V_{el})\Psi = \frac{2m}{\hbar^2} V_{non}\Psi . \quad (4.22)$$

When $V_{non} = 0$, the result is just equation (4.1) with $V = V_{el}$. The solution, usually called χ , is then a wave distorted by the optical potential V_{el} . Equation (4.3) is then used as the solution for the "distorted wave,"

$$\chi \rightarrow e^{i\mathbf{k}\cdot\mathbf{r}} - \frac{m}{2\pi\hbar^2} \frac{e^{ikr}}{r} \int \exp(-i\mathbf{k}' \cdot \mathbf{r}') V_{el}(\mathbf{r}') \chi(\mathbf{k}, \mathbf{r}') d\mathbf{r}' .$$

Two types of solutions can then be distinguished. One consists of a plane wave plus an outgoing scattered wave denoted by χ^+ and the other consists of a plane wave plus an incoming scattered wave denoted by χ^- . The V_{non} is now included in equation (4.22) and treated as a perturbation. The result is:

$$\Psi = \chi^+ - \frac{m}{2\pi\hbar^2} \frac{e^{ikr}}{r} \int \chi^-(\mathbf{k}', \mathbf{r}') V_{non}(\mathbf{r}') \Psi(\mathbf{k}, \mathbf{r}') d\mathbf{r}' . \quad (4.23)$$

This solution for Ψ still has Ψ in the integrand. To remove this the DWBA is used and $\Psi(\mathbf{k}, \mathbf{r}')$ in the integrand of equation (4.23) is approximated by the first term in the equation, i.e., the distorted wave χ^+ . By using this approximation, the scattering amplitude for the non-elastic transition in the DWBA is

$$\begin{aligned} f(\theta) &= \int \chi_f^-(\mathbf{k}', \mathbf{r}') V_{non}(\mathbf{r}') \chi_i^+(\mathbf{k}, \mathbf{r}') d\mathbf{r}' \\ &= \langle \chi_f | V_{non} | \chi_i \rangle . \end{aligned} \quad (4.24)$$

The DWBA includes all the reactions in which the incident particle interacts with the nucleus through an optical model potential $V_{el} = V_i$ before the non-elastic transition and the emerging particle interacts through $V_{el} = V_f$ after the non-elastic transition. The χ_i and χ_f , respectively, are the wave distorted by these optical potentials. The non-elastic potential acts only once in the reaction, hence the name (first) Born approximation. Since DWBA calculations require the use of optical model potentials for all particles involved in nuclear reactions, these potentials have been developed by measuring elastic scattering of these particles from nuclei.

Pion Scattering

Now that the general scattering formalism has been developed, it will be applied to pion-nucleus (π -A) scattering. To derive the transition amplitude for π -A inelastic scattering, the reaction mechanism is assumed to consist of 1) those scattering steps which serve only to distort the pion wave function (and in which the nucleus remains in the ground state) plus 2) a one-step interaction which causes the nuclear excitation (and is described by the π -A T -matrix). The π -A T -matrix is usually expressed in terms of the free pion-nucleon (π -N) t -matrix using the impulse approximation of multiple-scattering theory. This is done because the free π -N t -matrix is well parameterized from experimental data. The brief discussion of multiple-scattering theory given here is based on Satchler's [Sat 80] description.

Multiple-scattering theory. For the scattering of a pion by a nucleus, the interaction is a sum of two-body interactions, $V = \sum_i^A v_i$, where v_i is the effective in-medium interaction between the incident pion and the i^{th} nucleon. The Born series for T , using equation (4.19) is then

$$T = \sum_{j=1}^A v_j - \sum_{j=1}^A v_j G_0 \sum_{k=1}^A v_k + \dots \quad (4.25)$$

It is convenient to group all the terms that involve successive scatterings from the same nucleon together so that the entire scattering of the incoming pion with the k^{th} nucleon is accounted for by a single transition operator, τ_i , which differs from that for scattering from a free nucleon because the scattering occurs imbedded in a medium of other nucleons. These other

nucleons exert an influence through the Pauli Exclusion Principle and because they generate a potential in which the scattering pair are moving. This operator satisfies the equation, $\tau_i = v_i - v_i G_0 \tau_i$. In terms of τ_i , the Born series for T becomes the Watson series [Wat 57] or multiple-scattering expansion

$$T = \sum_j \tau_j - \sum_j \tau_j G_0 \sum_{k \neq j} \tau_k + \dots \quad (4.26)$$

This expansion describes the π -A T -matrix in terms of single, double, triple, and higher order multiple scatterings of the pion with individual nucleons such that no two successive scatterings are from the same nucleon.

The operator τ_i that describes the in-medium scattering of the pion and the i^{th} nucleon is unknown due to the presence of the nuclear Hamiltonian in the Lippmann-Schwinger equation for τ_i , therefore some approximations must be used. Retaining only the first term of this multiple-scattering series gives a single-scattering approximation. Further, if the relative kinetic energy is very high, two particles inside a nucleus scatter as if they were in free space and the presence of the other nucleons has little effect. Then the operator τ_i is replaced with the free π -N operator, t_i . This then is the impulse approximation:

$$T = \sum (t_i = t_{\pi N}) \quad (4.27)$$

The impulse approximation assumes that scattering from two or more nucleons is unimportant and that the projectile bombarding energy is much greater than the Fermi energy of the nucleons in the nucleus. This is not a Born approximation, however, because the exact transition operator t_i is used for the single scattering.

The scattering amplitude. The π -N t -matrix can be determined by studying the scattering of the projectile from a free nucleon. The amplitude of the wave scattered from the whole nucleus is then simply the sum of the wavelets scattered from each target nucleon. If the target nucleons were rigidly fixed in position and the pion transfers momentum $\mathbf{q} = \mathbf{k} - \mathbf{k}'$, then the total scattering amplitude would be simply $f = \sum_j t_j e^{i\mathbf{q} \cdot \mathbf{r}_j}$. However, the target nucleons are not rigidly located but are distributed over the nuclear volume with a probability and momentum distribution given by their wave functions. Let ψ_A and ψ_B describe the probability distribution of a single target nucleon excited from a state A to a state B . Then corresponding momentum distributions are given by the Fourier transform $\varphi(\mathbf{k}) = (2\pi)^{-3/2} \int \psi(\mathbf{r}) e^{-i\mathbf{k} \cdot \mathbf{r}} d\mathbf{r}$. The transition amplitude for the projectile with momentum $\mathbf{k}$ scattering to momentum $\mathbf{k}'$ is then

$$f = \int \varphi_B^*(\mathbf{k}') \langle \mathbf{k}' | t_{\pi N} | \mathbf{k} \rangle \varphi_A(\mathbf{k}) d\mathbf{k} d\mathbf{k}' . \quad (4.28)$$

Using the Fourier transform of $\varphi(\mathbf{k})$ in equation (4.28) gives

$$f = t_{\pi N} F(q) , \quad F(q) = \int \psi_B^*(\mathbf{r}) \psi_A(\mathbf{r}) e^{i\mathbf{q} \cdot \mathbf{r}} d\mathbf{r} . \quad (4.29)$$

For elastic scattering $\psi_B = \psi_A$ and the summation of $|\psi_A|^2$ over all the target nucleons is just the target density distribution, $\rho(r)$. This gives $F_{el}(q) = \int \rho(r) e^{i\mathbf{q} \cdot \mathbf{r}} d\mathbf{r}$, where $F_{el}(q)$ is a form factor, or the Fourier transform of the ground-state mass distribution. For inelastic scattering an inelastic form factor, $F_{non}(q)$, is used and the quantity $\psi_B^*(r)\psi_A(r)$ is called a transition density, where ψ_B and ψ_A are the particle and the hole state of the nucleus.

Summary. For convenience, the π -A scattering formulas are summarized here for a reaction of the form $A(a, b)B$. In this reaction, A and B are the initial and final states of the nucleus and a and b are the initial and final states of the pion projectile. The nuclear transition density is then $\rho_{tr} \propto \psi_A \psi_B$, for which the Fourier transform is $F_{non}(q) = \int \rho_{tr}(r) e^{i\mathbf{q} \cdot \mathbf{r}} d\mathbf{r}$. The π -A inelastic scattering potential is taken as the sum of π -N scattering interactions between initial and final nuclear states in the impulse approximation:

$$V_{non} = U_{\pi A} = \langle \psi_B | \Sigma t_{\pi N} | \psi_A \rangle = \int \psi_B^* \Sigma t_{\pi N} \psi_A d\mathbf{r} \Rightarrow t_{\pi N} F_{non}(q) . \quad (4.30)$$

The cross section for inelastic scattering is then

$$\frac{d\sigma}{d\Omega} = |f|^2 = |\langle \chi_b | U_{\pi A} | \chi_a \rangle|^2 , \quad (4.31)$$

where the χ are pion waves distorted by the optical potential, $V_{el} = U_{opt}$. The inelastic scattering potential (especially for stretched states) is discussed in the next section and the optical potential in the section after that.

Pion Inelastic Scattering

Carr et al. [Car 83; Hol 85] present a simple single-scattering model for describing the excitation of unnatural parity levels in nuclei via inelastic pion scattering in the region of the $\Delta_{3,3}$ -resonance. The model is based on the use of the short-range limit, or p-wave form, of the spin-orbit operator [Pet 81] to represent the spin-dependent part of the effective π -A interaction. With this assumption, the π -A interaction is characterized by a single isospin and energy dependent strength parameter, t^{LS} .

This strength parameter is obtained by forming the matrix element of the spin-orbit operator $t_{\pi N}$ in the π -N center-of-mass. The spin-dependent part of the π -N interaction is then given in momentum space by

$$t_{\pi N} = \langle \mathbf{k}' | t_{\pi N}(\mathbf{r}) | \mathbf{k} \rangle = -it^{LS} \sigma_N \cdot \mathbf{k} \times \mathbf{k}' , \quad (4.32)$$

where σ_N is the nucleon Pauli spin operator. This form is minimally non-local and its use is analogous to the Kisslinger prescription [Kis 55] for treating the spin-independent part of the π -N interaction.

Using $t = -(2\pi\hbar^2/\omega_N)f$, and making use of standard representations of f [Lee 71; Row 78], gives:

$$t^{LS} = t_0^{LS} + t_1^{LS} \vec{t}_\pi \cdot \vec{\tau}_N , \quad (4.33a)$$

$$t_0^{LS} k_N^2 = \frac{2\pi\hbar^2}{\omega_N} \frac{1}{3} [-\alpha_{11} - 2\alpha_{31} + \alpha_{13} + 2\alpha_{33}] , \quad (4.33b)$$

$$t_1^{LS} k_N^2 = \frac{2\pi\hbar^2}{\omega_N} \frac{1}{3} [\alpha_{11} - \alpha_{31} - \alpha_{13} + \alpha_{33}] , \quad (4.33c)$$

where k_N and ω_N are the wave number and reduced energy in the π -N c.m., $\vec{t}_\pi$ is the pion isospin operator, and $\vec{\tau}_N$ is twice the nucleon isospin operator. The strength parameters are fixed from the free π -N phase shifts [Row 78] as defined in equation (4.16).

Now the description of inelastic π -A scattering uses the impulse approximation [Lee 71] and the π -N interaction of equation (4.32). A matrix element is formed between the initial and final states of the target nucleus to obtain the scattering potential

$$U_{\pi A} = \gamma \langle \psi_A | \sum_N t_{\pi N}(\mathbf{r}) | \psi_A \rangle . \quad (4.34)$$

In this equation $\gamma \approx k\omega_N/k_N\omega$ is the factor required for transformation from the π -N c.m. to the π -A c.m., with k and ω representing the wave number and reduced energy in the π -A c.m. frame.

The final result of the integration in equation (4.34) for a $0^+ \rightarrow J^\pi$ unnatural parity transition [where $\pi = (-1)^{J+1}$] is

$$U_{\pi A} = \bar{U}_J(r_\pi)[Y_J(\hat{r}_\pi) \times \nabla_\pi]^J, \quad (4.35)$$

where $\bar{U}_J(r_\pi)$ is the reduced radial transition potential which is essentially the form factor required for a distorted wave calculation. This is given by

$$\bar{U}_J(r_\pi) = \alpha_\pi \frac{2}{\pi} \sum_{n,T} k_n^2 \Delta k_n j_J(k_n r_\pi) \epsilon^T t_T^{LS} k_n^2 \rho_J^{sT}(k_n), \quad (4.36)$$

where k_n and Δk_n are the momentum eigenvalues on a finite sphere and associated differences, j_J is a spherical Bessel function, $\alpha_\pi \approx k_N/k$ results from the decomposition of $\vec{p} = \alpha_\pi \vec{p}_\pi - \alpha_N \vec{p}_N$ in the π -A c.m., and for $\pi^\pm$ scattering ϵ^T is 1 for $T = 0$ and ∓ 1 for $T = 1$ (assuming $t_z = +\frac{1}{2}$ for n and $+1$ for π^-). The $\bar{U}_J$ is evaluated using the discretized momentum technique with $k_n = n(\pi/R)$, $R = 12$ fm, and $n = 1, 2, \dots, 40$. The $\rho_J^{sT}(k_n)$ in equation (4.36) is the transverse spin transition density defined by

$$\rho_J^{sT}(k_n) = \left[\frac{J+1}{2J+1} \right]^{1/2} \rho_{J,J-1}^{sT}(k_n) - \left[\frac{J}{2J+1} \right]^{1/2} \rho_{J,J+1}^{sT}(k_n), \quad (4.37a)$$

$$\rho_{JL}^{sT}(k_n) \propto \langle f || \sum_N j_L(k_n r_N) [Y_L(\hat{r}_N) \times \sigma_N]^J \tau_N^T || i \rangle, \quad (4.37b)$$

$$\tau_N^0 = 1, \quad \tau_N^1 = \vec{\tau}_N, \quad (4.37c)$$

Note that for states of unnatural parity, in general there exists both a spin transition density, ρ^s , and a current transition density, ρ^l [for example, see Appendix D, equation (2) for electron scattering].

Absolute pion scattering cross sections for unnatural parity states are in general difficult to calculate because of Coulomb and nuclear distortion, coupling to Δ -hole components, and coupling to current and spin-current transition densities through the central and spin-orbit components of the π -A interaction. However, transitions to states of stretched configuration are a special class of unnatural parity transitions where the coupling to Δ -hole components is negligible [Suz 81], the current transition densities ρ^I vanish, and the spin-current transition densities $\rho_{J,J+1}^{sT}$ in equation (4.37a) vanish (due to angular momentum restrictions). The result is that (e, e') and (π, π') reactions for stretched states are related through a common spin transition density $\rho_{J,J-1}^{sT}$ [Pet 80; Pet 81; Lin 79]. The direct relationship between these two reactions makes interpretation of the data more straightforward than is usual. It should be noted that this is strictly true only if it is assumed that proton and neutron wave functions have the same radial dependence and that only $1\hbar\omega$ excitations are allowed.

In the plane wave Born approximation the pion scattering differential cross section for the excitation of unnatural parity levels is [Pet 81]

$$\frac{d\sigma}{d\Omega} = 4\pi \left(\frac{\omega}{2\pi\hbar^2} \right)^2 (2J+1) \frac{k^4}{2q^4} \sin^2 \theta \left| \sum_T t_T^{LS}(q) \rho_J^{sT}(q) \right|^2, \quad (4.38)$$

where ω and k are the reduced energy and wave number of the incident pion as defined before. Separating reaction dynamics (in the matrix elements, M^π) from nuclear structure effects (in the Z coefficients), the cross section for pion scattering can be written as [Gee 84]:

$$\frac{d\sigma^\pm}{d\Omega} = (M_1^{\pi^\pm})_{thr}^2 \left[\frac{M_6^\pi}{M_1^\pi} Z_0 \mp Z_1 \right]^2 N^\pm. \quad (4.39)$$

The Z coefficients introduced here are part of ρ_{JL}^{sT} in equation (4.37b). The empirical normalization factor $N^\pm$ is also included in the above equation because theoretical and experimental cross sections do not usually agree. The ratio of π^+ to π^- scattering cross sections for incident pion energies near the $\Delta_{3,3}$ -resonance is derived in Appendix A. At the resonance, the simple relationship $M_0^\pi/M_1^\pi = t_0^{LS}/t_1^{LS} = \mp 2$ holds for $\pi^\pm$ scattering.

Distorted Wave Calculations for Pion Scattering

The distorted wave approximation for pion scattering needs an optical potential in order to calculate the incoming and outgoing distorted waves, $\chi^\pm$, resulting from elastic scattering. The simple optical potential has been modified in many ways to try to account for the complexities of the nucleus. One sophisticated form is described here along with two forms of nuclear matter distributions used in the optical model.

The optical potential. Satchler [Sat 80] gives a general description of the simple optical model. The interaction of a pion with a nucleus is a complicated many-body problem. For the purpose of describing elastic scattering, the optical model attempts to replace this problem by the much simpler problem of two structureless bodies interacting by a simple potential that represents some smoothed average of the actual interaction. This model potential is usually assumed to depend only upon the distance r between the centers-of-mass of the projectile and target.

The radial dependence can be understood from the fact that at resonance energies pion elastic scattering is highly diffractive (thus the term optical model). The elastic cross section then has an angular distribution

that is proportional to $J_1(kR\theta)^2/(kR\theta)^2$; therefore the minima in the angular distribution at a fixed incident energy are only dependent upon some characteristic radius of the ground state matter distributions, ρ_n and ρ_p .

The first order potential for elastic scattering is similar to that for inelastic scattering in equation (4.30), $U_{opt}^1 = \langle \psi_A | \sum_i t_i | \psi_B \rangle$. If it is assumed that forward scattering dominates, and thus the momentum transfer goes to zero, $t(\mathbf{q} \rightarrow 0)$, the configuration space optical potential becomes $U_{opt}^1(r) = tA\rho(r)$. This result is only approximate, but it should be adequate for medium energy pion scattering.

Whenever inelastic scattering is possible, there is a loss of flux from the elastic channel, so the optical potential must be absorptive. This is accomplished by making the potential complex. In the simplest form, the resulting optical model potential is

$$U(r) = -V_r\rho_r - iV_i\rho_i - iW_i\rho' \quad (4.40)$$

where ρ_r is some matter distribution such as the Woods-Saxon form, $\rho_r = [e^{(r-R)/a} + 1]^{-1}$, ρ_i has the same shape but may have a different R and a value, and ρ' is the derivative of ρ and is peaked at the nuclear surface.

This first order treatment ignores modifications of the pion propagation due to the presence of the nuclear medium. It is possible to expand the optical potential to higher orders in the density and get terms that are proportional to ρ^2 , ρ^3 , etc. This is often done in an attempt to account for other effects present in π -A scattering.

Examples of optical potentials. For π -N scattering, the optical potential arises from a scattering amplitude of the form $f = b_0 +$

$b_1 \vec{t} \cdot \vec{\tau} + (c_0 + c_1 \vec{t} \cdot \vec{\tau}) \mathbf{k} \cdot \mathbf{k}'$. This optical potential can be written to first order in the multiple-scattering series using the Kisslinger form [Kis 55]. In coordinate space this is

$$U_{opt}^1(r) = [bk_\pi^2 \rho(r) - c \nabla \cdot \rho(r) \nabla] . \quad (4.41)$$

Here $\rho(r)$ is the radial dependence of the nuclear matter density and the complex constants b and c are related by kinematic factors to the $l = 0$ and $l = 1$ π -N phase shifts, $\delta_{2T,2J}^l$.

A potential which more closely models the nucleus is known as the modified Kisslinger potential. It has the form

$$U_{opt}^1(r) = b_0 k_\pi^2 \rho_0(r) + c_0 \nabla \cdot \rho_0(r) \nabla - \epsilon_\pi [b_1 k_\pi^2 \rho_1(r) + c_1 \nabla \cdot \rho_1(r) \nabla] , \quad (4.42)$$

where ϵ_π is the sign of the pion and the density is defined in terms of isoscalar and isovector parts

$$\rho_0(r) = \frac{1}{A} [N \rho_n(r) + Z \rho_p(r)] ,$$

and

$$\rho_1(r) = \frac{1}{A} [N \rho_n(r) - Z \rho_p(r)] .$$

It is usually assumed that $\rho_n(r) = \rho_p(r) = \rho(r)$. In that case $\rho_0 = \rho$ and

$$\rho_1 = \delta \rho(r) = \frac{(N - Z)}{A} \rho(r) . \quad (4.43)$$

Stricker, McManus, and Carr [Str 79; Str 80] describe a more sophisticated potential that includes absorptive terms and the Ericson-Ericson factor. More recent extensions of this work have been done by Siciliano et al. [Sic 86].

In this case the wave equation used to generate the distorted waves for inelastic scattering is not the Schrödinger equation (4.1), but comes from the Klein-Gordon equation [Eis 74] for bosons

$$(p^2 + m^2)\Psi = E^2\Psi , \quad (4.44)$$

where $\hbar = c = 1$. Using $p = -i\nabla$ and $E^2 \rightarrow (E - U_{opt})^2 \approx E^2 - 2EU_{opt}$, and then using $k^2 = E^2 - m^2$ gives

$$(\nabla^2 + k^2 - 2\bar{\omega}U_{opt})\Psi = 0 . \quad (4.45)$$

Because of frame transformations the following definitions have been used: k and ω are the wave number and total energy of the pion in the π -A c.m. system; $k_L = k[1 + \omega_L/(MA)]$ is the wave number of the pion in the lab; $\bar{\omega} = \omega/[1 + \omega/(MA)]$ is the reduced energy; and $M = 931$ MeV. The complete optical potential used for pion scattering is of the form

$$\begin{aligned} 2\bar{\omega}U_{opt} = & -4\pi \left[b(r) + p_2 B_0 \rho^2(r) + \frac{(p_1 - 1)}{2} \nabla^2 c(r) + \frac{C_0(p_2 - 1)}{2p_2} \nabla^2 \rho^2(r) \right] \\ & + 4\pi \left[\vec{\nabla} \cdot L(r)c(r)\vec{\nabla} + \frac{C_0}{p_2} \vec{\nabla} \cdot \rho^2(r)\vec{\nabla} \right] \\ & + 2\bar{\omega}V_c(r) , \end{aligned} \quad (4.45)$$

where

$$\begin{aligned} b(r) &= p_1 [\bar{b}_0 \rho(r) - \epsilon_\pi b_1 \delta \rho(r)] , \\ c(r) &= p_1^{-1} [c_0 \rho(r) - \epsilon_\pi c_1 \delta \rho(r)] , \\ L(r) &= \left[1 + \frac{4\pi\lambda}{3} \frac{(A-1)}{A} c(r) \right]^{-1} , \end{aligned}$$

and $\epsilon_\pi = \pm 1$ gives the pion charge. The kinematical factors are

$$p_1 = \frac{1 + \epsilon}{1 + \epsilon/A} \quad \text{and} \quad p_2 = \frac{1 + \epsilon/2}{1 + \epsilon/A} ,$$

where $\epsilon = \omega/M$.

The complex parameters, B_0 and C_0 , describe the absorption in the second order terms. The quantities b_0 , b_1 , c_0 , and c_1 obtained from the pion-nucleon scattering amplitudes are the single-nucleon parameters. The parameter b_0 has been replaced by $\bar{b}_0$, the effective s-wave scattering length in a nucleus, where $\bar{b}_0 \approx b_0 - (3k_f/2\pi)(b_0^2 + 2b_1^2)$ and k_f is the Fermi momentum, taken to be 1.4 fm^{-1} .

The terms that are quadratic in the nuclear densities reflect processes involving more than one nucleon, such as true absorption and various correlation effects. The medium modifications take a form that is completely analogous to the first order potential. The term $L(r)$ describes p-wave correlation effects. It is intended to account for p-wave Pauli correlations and the Lorentz-Lorenz Ericson-Ericson effect [Eri 66]. It extends the potential to second order in the multiple-scattering series and has the effect of weakening the p-wave attraction to an extent determined by the parameter λ . If inhibition is complete, $\lambda = 1$.

Matter distributions. The ground state matter distributions appear in the optical potential used for generating the distorted waves. Two different forms were used for this distribution. The harmonic Gaussian distribution was used for the lighter nuclei

$$\rho(r) = \rho_0 \left[1 + \alpha(r/w)^2 \right] e^{-(r/w)^2}, \quad (4.47)$$

where $\rho_0 = 2/[(2 + 3\alpha)(\sqrt{\pi}w)^3]$, w is the radius parameter, and α is a second parameter. A three-parameter Fermi distribution was used for the

heavier nuclei

$$\rho(r) = \rho_0 \frac{[1 + w(r/c)^2]}{[1 + \exp((r - c)/t)]} , \quad (4.48)$$

where $\rho_0 = \frac{3}{4\pi c^3} \{1 + (\pi t/c)^2 + w[3 + 10(\pi t/c)^2 + 7(\pi t/c)^4]/5\}^{-1}$, c is the radius parameter, $t = a/(4 \ln 3)$, a is the 90-10% "skin thickness," and w is a third parameter.

Usually the nuclear density is determined by elastic electron scattering. This determines the mean square radius of the charge distribution ($\langle r^2 \rangle_{chg}$). The nuclear densities used for the optical potential, however, should be the mean square radius of the point distribution ($\langle r^2 \rangle_{pt}$) because the effects of the nucleon form factor are included, to some extent, in the free π -N t -matrix. This is found by unfolding the mean square radius of the proton ($\langle r^2 \rangle_{pro}$), taken to be 0.8 fm; thus

$$\langle r^2 \rangle_{pt} = \langle r^2 \rangle_{chg} - \langle r^2 \rangle_{pro} . \quad (4.49)$$

Because electrons interact electromagnetically, electron scattering is mostly sensitive to the distribution of the protons. However, it is usually assumed that the mean square radius for the neutrons and protons is the same.

The Computer Codes Used for Pion Scattering

The theoretical calculations were made with two readily available computer codes: the general inelastic scattering potential code ALLWRLD [Car 81a] which generates the reduced transition potential defined in equation (4.36) and the pion distorted wave code MSUDWPI [Car 81b] which can handle the derivative operator in equation (4.35). Sample input for these computer codes along with an explanation is found in Appendix C.

The code MSUDWPI is based on codes originally written by Eisenstein and Miller [Eis 74; Eis 76]. The elastic scattering for distorted waves in MSUDWPI is calculated by solving the Klein-Gordon equation (4.44). This program integrates the Klein-Gordon equation in coordinate space for each partial wave out to a match point, where the internal logarithmic derivatives are compared to the external logarithmic derivatives to obtain the phase shifts used to calculate the differential cross sections.

The optical potential in MSUDWPI is based on the one described by Stricker, McManus, and Carr [Str 79]. It is the same as defined in equation (4.46) except that a term involving C_2 is included. The parameters are taken from the phase shifts of Rowe, Salomon, and Landau [Row 78]. These parameters were calculated using subroutines from DWPIES [Sic 84].

The MSUDWPI code had to be modified slightly to be able to use it for the high angular momentum 8^- states in ^{60}Ni . In the subroutine PIWAVE (described by Eisenstein and Miller [Eis 74]) the starting values for u_l and u'_l were smaller than the acceptable minimum of $0.29\text{E}-38$ for a FORTRAN program. An extra factor of $10.\text{E}+10$ was therefore included in the definitions of these starting values:

$$\text{FAC} = 10.0 * *(-L) \quad * \quad 10.\text{E} + 10 .$$

The general outline of the calculations follows the pion scattering summary given in a previous section. The initial and final state nuclear wave functions ψ_A and ψ_B are found by running DWUCK4. The transition density, $\rho_{tr} \propto \psi_B \psi_A$, is then used as input to ALLWRLD. This code calculates the pion form factors, $t_{\pi N} F_{non}(q) = V_{non}$, where $F_{non}(q) = \int \rho_{tr} e^{i\mathbf{q} \cdot \mathbf{r}} d\mathbf{r}$. These pion form factors are then used as input to MSUDWPI which calculates

the cross sections, $d\sigma/d\Omega = |f|^2 = |\langle \chi_b | V_{non} | \chi_a \rangle|^2$. The wave functions χ are the pion wave functions distorted by the optical potential, $V_{el} = U_{opt}$.

Electron Scattering

The pion scattering data in this thesis are compared in Chapter V to electron scattering data from previous experiments. Details of the theory of electron scattering to stretched states are given in Appendix D and references therein. A brief summary of the important points is presented here.

The inelastic electron scattering differential cross section for magnetic transitions is given by

$$\frac{d\sigma}{d\Omega} \propto |F_T(q)|^2, \quad (4.50)$$

where F_T is a transverse or magnetic form factor. The transverse form factor squared can be written as

$$|F|^2 = (M_0^e Z_0 + M_1^e Z_1)^2 = \left[\frac{M_0^e}{M_1^e} \frac{Z_0}{Z_1} + 1 \right]^2 Z_1^2 (M_1^e)_{thr}^2, \quad (4.51)$$

to emphasize the separate dependence on the reaction dynamics in M_0 and M_1 and on the nuclear structure effects in Z_0 and Z_1 . The isovector matrix element, M_1^e , is proportional to $g_1^s I_1$. The center-of-mass correction is included by $f_{cm}(q) = \exp[(bq/2)^2/A]$ [Tas 58]. The M_0^e/M_1^e ratio is

$$\frac{M_0^e}{M_1^e} = \frac{g_0^s I_0}{g_1^s I_1}, \quad (4.52)$$

where $g_0^s = \mu_n + \mu_p$ and $g_1^s = \mu_n - \mu_p$. When using harmonic oscillator (HO) wave functions, $I_0 = I_1$, and the ratio $g_0^s/g_1^s = -0.187$. The radial integral is $I(q) = \int \psi_A(r) j_{J-1}(qr) \psi_B(r) r^2 dr$, as shown in equation (4.21).

These electron scattering calculations involve plane waves. For a distorted wave calculation (necessary for high Z targets), a simple local-momentum approximation [Plu 85] is used. In the local-momentum approximation, the plane wave momentum transfer q is replaced by an effective momentum transfer q_{eff} . As the incident electron approaches the nucleus, it feels an attractive force and is accelerated toward the nucleus. It is as if an electron of higher energy were being scattered, and the effect of the correction is to increase the momentum transferred to the nucleus. If it is assumed that the electron is in the presence of a spherically symmetric static potential $V(r)$ of radius R , the result is

$$q_{\text{eff}} = q [1 + 3Ze^2/(2E_i R)] . \quad (4.53)$$

Comparing Pion and Electron Scattering

Equation (4.39) gives pion scattering cross sections in terms of Z coefficients and pion matrix elements. Equation (4.51) gives electron scattering form factors in terms of Z coefficients and electron matrix elements. The matrix elements describe the reaction dynamics and the structure effects are included in the Z coefficients.

For magnetic electron scattering, M_0^e/M_1^e is small (-0.187 if HO wave functions are used) and thus probes mainly the isovector transitions. For pion scattering M_0^π/M_1^π is approximately 2 near the $\Delta_{3,3}$ -resonance and thus probes mainly isoscalar transitions.

Once both pion and electron scattering have been done to the stretched states in a given nucleus, the two equations can be solved simultaneously to determine the two Z coefficients. These Z coefficients are thus

experimental fitting parameters found for each stretched state. The Z coefficients are then compared to theoretical predictions, either from nuclear structure calculations or more general sum rules.

The structure effects embodied in Z_0 and Z_1 are difficult to predict for individual states, although calculations have been done for ^{14}C and ^{54}Fe [Plu 85; Gee 84]. Upper limits for the Z coefficients can, however, be determined from simple nuclear models. These sum rules can be found for $T_<$ states (where the magnetic transition, $T_i \rightarrow T_f$, is for nuclear states of isospin $T_0 \rightarrow T_0$), for $T_>$ states (where the transition is for $T_0 \rightarrow T_0 + 1$), and for the sum of the two.

A method given by Holtkamp et al. [Hol 85] is used here. Looking at a $T_>$ stretched transition of multipolarity J induced by an isovector one-body operator, the square of the isovector Z coefficient summed over all observed transitions is

$$\sum_i (Z_1^i)^2 = \frac{1}{2J} \left[\frac{2}{T_0 + 1} n_p \right], \quad \text{for } T_>, \quad (4.54)$$

where $J = 2j_a + 1$ for stretched states. The sum rule is also valid when the ground state (or excited state) contains admixtures of other orbital configurations such as j_c^n so long as $j_c + j_a < J$. The maximum value of the sum occurs when n is evaluated in the extreme single-particle shell model as above.

For $T_<$ stretched magnetic transitions, Holtkamp et al. [Hol 85] give the maximum isovector strength as:

$$\sum_i (Z_1^i)^2 = \frac{1}{2J} \left[n_n + \left(\frac{T_0 - 1}{T_0 + 1} \right) n_p \right], \quad \text{for } T_<. \quad (4.55)$$

The sum of the two gives:

$$\sum_i (Z_1^i)^2 = \frac{1}{2J} [n_n + n_p] = \frac{n}{2J}, \text{ for } T_{tot}. \quad (4.56)$$

Note that in general $n = 2J$ for a full orbital. Equation (4.56) is also the sum rule for the isoscalar Z coefficient, $\sum_i (Z_0^i)^2$.

The next chapter will describe the theoretical calculations for pion scattering reactions. The matrix elements from these calculations will then be compared to data from pion and electron scattering reactions for several well-studied cases, especially ^{60}Ni , to determine the Z coefficients. These Z coefficients will then be compared to the sum rules to determine how much, if any, quenching has occurred.

CHAPTER V

COMPARISON OF DATA WITH THEORY

This chapter describes the results of using the computer codes ALLWRLD and MSUDWPI, introduced in the previous chapter, to calculate theoretical cross sections for pion scattering to stretched states in ^{60}Ni as well as five other nuclei. First, the input values to the codes and the results of varying these values are discussed. This includes the Woods-Saxon wave functions from DWUCK4 used as input to ALLWRLD. A description of calculating Z coefficients and normalization factors is given. Finally, the theoretical calculations are compared to the experimental data.

Input Values for the Computer Codes

Appendix C explains all of the input values used in running the computer codes, ALLWRLD and MSUDWPI, and gives sample input files used for calculations on ^{60}Ni . The parameters in these sample input files remained constant in all calculations except as explained here. Table 5.1 lists the standard values used for the radial parameters and the optical potential. Table 5.1 also gives the variations in these parameters used to check the sensitivity of the results to the input parameters. Table 5.2 gives the theoretical matrix elements calculated from the different input values. These sample calculations were done for the 11.7 MeV state in ^{14}C . This nucleus was chosen because input values from Plum [Plu 85] were available as a comparison.

Table 5.1 Values and variations used for the radial parameters and the optical potential. Values denoted by an * are the standard values used. Results of sample calculations using the other values are given in Table 5.2. The Rowe, Salomon, and Landau (RSL) values shown here were calculated for the 11.7 MeV state in ^{14}C . The values from Plum are also for ^{14}C . The ES is the energy shift in MeV.

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radial parameters (fm) — for ALLWRLD and MSUDWPI

[deJ 74]*	$a = 1.73$	$\alpha = 1.38$
[Plu 85]	$a = 1.716$	$\alpha = 1.067$
check	$a = 1.65$	$\alpha = 1.38$

spin-orbit force (fm^3) — for ALLWRLD

RSL*	$S_0 = +0.2451+0.2332i$	$S_1 = +0.1338+0.1162i$
[Plu 85]	$S_0 = +0.2100+0.4800i$	$S_1 = +0.1300+0.2400i$

first-order parameters (b 's in fm; c 's in fm^3) — for MSUDWPI

RSL (ES=30)*	in	$\bar{b}_0 = -0.0695+0.0309i$	$b_1 = -0.0948+0.0036i$
		$c_0 = +0.4958+0.5983i$	$c_1 = +0.2764+0.2982i$
	out	$\bar{b}_0 = -0.0701+0.0306i$	$b_1 = -0.1001+0.0026i$
		$c_0 = +0.5606+0.5222i$	$c_1 = +0.3135+0.2602i$
RSL (ES=0)	in	$\bar{b}_0 = -0.0695+0.0309i$	$b_1 = -0.0948+0.0036i$
		$c_0 = +0.3638+0.9768i$	$c_1 = +0.2104+0.4874i$
	out	$\bar{b}_0 = -0.0701+0.0306i$	$b_1 = -0.1001+0.0026i$
		$c_0 = +0.5441+0.9166i$	$c_1 = +0.3053+0.4574i$
[Plu 85]	in	$\bar{b}_0 = -0.0840+0.0290i$	$b_1 = -0.1250+0.0040i$
		$c_0 = +0.3300+0.7000i$	$c_1 = +0.1900+0.3500i$
	out	$\bar{b}_0 = -0.0800+0.0260i$	$b_1 = -0.1260+0.0020i$
		$c_0 = +0.5600+0.6200i$	$c_1 = +0.3000+0.3100i$

second-order parameters (B 's in fm^4 ; C 's in fm^6) — for MSUDWPI

0*	in	$B_0 = 0.0000+0.0000i$	$C_0 = 0.0000+0.0000i$
			$C_2 = 0.0000+0.0000i$
	out	$B_0 = 0.0000+0.0000i$	$C_0 = 0.0000+0.0000i$
			$C_2 = 0.0000+0.0000i$
[Cha 79]	in	$B_0 = -0.0380+0.0670i$	$C_0 = +0.1800+0.4000i$
			$C_2 = 0.0000+0.0000i$
	out	$B_0 = -0.0380+0.0670i$	$C_0 = +0.1800+0.4000i$
			$C_2 = 0.0000+0.0000i$
[Plu 85]	in	$B_0 = -0.1500+0.2850i$	$C_0 = 0.0000+0.0000i$
			$C_2 = +1.3000+3.0000i$
	out	$B_0 = -0.1300+0.2600i$	$C_0 = 0.0000+0.0000i$
			$C_2 = 1.2000+2.6000i$

Table 5.2 Resulting squared matrix elements (in $\mu\text{b}/\text{sr}$) when using the different input parameters of Table 5.1 in ALLWRLD and MSUDWPI. These sample calculations were done for the 11.7 MeV 4^- state in ^{14}C using HO wave functions. Values denoted by an * are the standard values used. The ES is the energy shift in MeV.

parameters changed	$(M_1^{\pi^-})_{thr}^2$	$(M_1^{\pi^+})_{thr}^2$
radial parameters — for ALLWRLD and MSUDWPI		
[deJ 74]*	71	82
[Plu 85]	72	83
check	72	82
spin-orbit force — for ALLWRLD		
RSL*	71	82
[Plu 85]	169	196
first-order parameters — for MSUDWPI		
RSL (ES=30)*	71	82
RSL (ES=0)	44	53
[Plu 85]	55	63
second-order parameters — for MSUDWPI		
0*	71	82
[Cha 79]	61	69
[Plu 85]	37	40

The radial parameters for both codes were taken from de Jager et al. [deJ 74]. An updated list of slightly different radial parameters (not available when the calculations were being done) is given by De Vries et al. [DeV 87]. The parameter they list as c is the charge density. To find the point density to use in ALLWRLD and MSUDWPI, the size of the proton had to be unfolded as explained in the previous chapter. These parameters were varied by using a different α [Plu 85] (his definitions for a and α are reversed) and by using a different a . These variations had little effect on the theoretical matrix elements.

The ALLWRLD values for the spin-orbit force (S_0 , S_1) which induces the nuclear transition were calculated from the phase shifts of Rowe, Salomon, and Landau (RSL) [Row 78]. The equations used are

$$S_0 = (3k_c^2)^{-1} (-\alpha_{11} - 2\alpha_{31} + \alpha_{13} + \alpha_{33})$$

$$S_1 = (3k_c^2)^{-1} (\alpha_{11} - \alpha_{31} - \alpha_{13} + \alpha_{33}) .$$

These come from combining equations (4.33b) and (4.33c) with $S_T = t_T^{LS}(\omega_c/2\pi\hbar^2)$ from Plum [Plu 85]. Values from Plum for the spin-orbit force were used as a variation in making these calculations. The values from Plum gave much larger theoretical matrix elements than when using the standard values.

The parameters for the first-order optical potential (b_0 , b_1 , c_0 , c_1) used in MSUDWPI were calculated using the RSL phase shifts. This calculation is done, for example, in a subroutine of the code DWPIES [Sic 84]. An energy shift (ES) of 30 MeV [Cot 80] and an s-wave Fermi wave number of $SFM=0$ were used in the calculations. If a value of $SFM=1.4 \text{ fm}^{-1}$ is used, the calculated cross sections change by less than 1%. The first-order parameters were varied by using an energy shift of 0 MeV and by using the values listed by Plum [Plu 85]. As seen in Table 5.2, both variations gave smaller theoretical matrix elements than did the standard values.

The parameters for the second-order optical potential (B_0 , C_0 , C_2) used in MSUDWPI were set to zero for the calculations. Variations in these parameters gave an appreciable change in the theoretical cross sections. These variations were taken from Chai and Riska [Cha 79] and from Plum [Plu 85]. The definitions of B_0 and C_0 in Chai and Riska are different

than in MSUDWPI, thus B_0 and C_0 from Chai and Riska had to be transformed from the π -N frame to the π -A frame. Both variations gave smaller theoretical matrix elements than when using the standard values.

The Wave Functions

The computer codes were run using two different types of wave functions. The HO wave functions were generated internally in ALLWRLD and the WS wave functions were generated by the computer code DWUCK4 [Kun 87]. The method of generating these wave functions and their use is discussed here.

Generating the wave functions. The HO wave function is given by the equation:

$$\psi_{HO}(r) = \sqrt{\frac{2k!}{(k+l+1/2)! b^3}} e^{-r^2/2b^2} (r/b)^l L_k^{l+1/2}(r^2/b^2), \quad (5.1)$$

where b is the HO parameter (fm^{-1}), $k = (n - l)/2$, n is the number of phonons ($0, 1, 2, \dots$), l is the angular momentum quantum number, and the $L_k^{l+1/2}$ are the associated Laguerre polynomials. The HO parameter used for generating HO wave functions for each nucleus is listed in Table 5.3. The input parameter for ALLWRLD is $\alpha = 1/b$. Usually a center-of-mass correction is included in the HO analysis of pion scattering by using $b' = b[(A - 1)/A]^{1/2}$ in the calculations, and then the final theoretical cross section is adjusted by the center-of-mass (cm) correction squared, f_{cm}^2 , where

$$f_{cm} = \left(\frac{A}{A - 1} \right)^{L/2} \quad (5.2)$$

Table 5.3 Radial parameters used in calculating wave functions. The oscillator length b is used in HO calculations. For WS calculations the nuclear radius is $R = r_0 \times A^{1/3}$ and the nuclear diffuseness is a . Where λ is listed for the spin-orbit strength, this represents the strength multiplying the usual Thomas form. A Coulomb radius of $r_c = 1.25 \times A^{1/3}$ fm was used for all nuclei.

		$b(\text{fm})$	$r_0(\text{fm})$	$a(\text{fm})$	λ/V_{so} (MeV)	Ref.
^{12}C	HO	1.53				[Hic 84]
	WSl		1.20	0.65	$V_{so} = 24$	[Don 84]
	WSs		1.0	0.65	$V_{so} = 24$	[Cla 87]
^{14}C	HO	1.49				[Plu 85]
	WSl		1.25	0.5	$V_{so} = 24$	[Plu 85]
	WSs		1.05	0.5	$\lambda = 25$	[Cla 87]
^{16}O	HO	1.77				[Hyd 87]
	WSl		1.232	0.6	$V_{so} = 24$	[Don 84]
	WSs		0.9	0.6	$V_{so} = 24$	[Cla 87]
^{28}Si	HO	1.80				[Yen 80]
	WSl		1.25	0.65	$\lambda = 25$	[Cia 84]
	WSs		1.2	0.6	$\lambda = 25$	[Cla 87]
^{54}Fe	HO	1.90				[Lin 81a]
	WSl		1.232	0.65	$V_{so} = 28$	[Don 84]
	WSs		1.05	0.65	$V_{so} = 28$	[Cla 87]
^{60}Ni	HO	1.98				[Lin 81b]
	WSl		1.173	0.65	$V_{so} = 28$	[Pla 82]
	WSs		1.0	0.6	$V_{so} = 28$	[Cla 87]

[Com 81; Plu 85]. In the WS calculation this correction cannot be made so simply. Therefore, for consistency in the calculations, the center-of-mass correction is not included for either type of wave function. When the center-of-mass corrections are included for HO wave function calculations for the 7.5 MeV state in ^{60}Ni , the theoretical cross section decreases by 4% and the angular maximum changes by less than a degree.

Clausen et al. [Cla 87] describe the method of generating the WS wave functions for use in analyzing electron scattering to stretched states. That paper closely parallels this thesis' description of using WS wave functions for analyzing pion scattering to stretched states. This section only briefly discusses the binding energies and potential parameters used to calculate hole-state and particle-state wave functions. More details can be found in a copy of the paper given in Appendix D.

The binding energy of the hole-state is assumed to be the separation energy of the nucleon in that state. However, for ^{60}Ni and several other nuclei considered here, the ground state of the (A-1) nucleus does not have the spin of the hole state. In that case, the excitation energy of the lowest state in the (A-1) nucleus with the proper spin should be added to the separation energy. Also, for ^{60}Ni where $T_0 \neq 0$, the ground state of the neutron-hole nucleus need not be the sole isospin parent for the stretched states. The neutron hole-state wave function should then be a weighted sum of $T_0 \pm 1/2$ wave functions, where the binding energies for each are known from neutron pickup reactions. However, there is very little sensitivity to the binding energy of the hole-state. The $T = 3/2$ $f_{7/2}$ neutron hole-state in ^{59}Ni has a binding energy of -15.29 MeV ($S_n = -11.389$ MeV plus $E_x = (-)3.9$ MeV [Zim 78]), while the $T = 5/2$ $f_{7/2}$ hole-state lies at

-18.74 MeV ($S_n = -11.389$ MeV plus $E_x = (-)7.35$ MeV [Ike 78]). As shown in Appendix D, electron scattering M8 squared form factors computed with these two extremes differ by less than 1%. For simplicity, then, only the lower isospin neutron hole-state is used in the calculations. These stretched state levels for ^{60}Ni are shown in Figure 5.1.

The binding energy of the particle-state is that for the hole-state (as outlined above) plus the excitation energy of the stretched state; thus a different binding energy is used for each particle-state. Here the lower isospin neutron hole-state cannot always be used for simplicity. For all $T_<$ stretched states the decay to the lower isospin neutron hole-state is used, since this will dominate the decay probability. Likewise, for $T_> (= T_0 + 1)$ stretched states in $T_0 = 0$ nuclei, the decay to the lower isospin neutron hole-state can be used. However, for $T_>$ stretched states in $T_0 \neq 0$ nuclei such as ^{60}Ni , the decay to the upper $T_0 + 1/2$ isospin neutron hole-state is used, since this is the only allowed decay if the isospin rules are strictly obeyed. In the $T_0 \neq 0$ nuclei, ^{54}Fe and ^{60}Ni , the resulting $T_>$ stretched states are neutron bound and proton unbound. Such $T_>$ stretched states may, however, break the isospin rules and decay to the lower isospin state of the daughter. In this case, the resulting $T_>$ stretched states in the two nuclei are all unbound, or nearly so. The binding energies for sample particle-states in each of the six nuclei studied here are shown in Table 5.4.

The radial parameters used in generating the WS wave functions are listed in Table 5.3. These are some of the parameter sets used in the calculations of Clausen et al. [Cla 87] as listed in Appendix D. For each nucleus, the parameter set with the larger r_0 comes from fits to elastic electron scattering [Pla 82; Don 84], (α, t) reactions [Cia 84], or (d, p) reactions

Figure 5.1 Stretched state levels for ^{60}Ni .

Table 5.4 Particle-state binding energies. All energies are in units of MeV. The separation energies are from the Atomic Data and Nuclear Data Tables. The excitation energies are for the hole-state with the correct spin (j) and isospin (T). The binding energies are for the stretched state with the given (underlined) excitation energy. A sample $T_<$ and a sample $T_>$ state are given for each nucleus except ^{14}C where no pion scattering data are available for the $T_>$ state.

T_0		Sep. Energy	Excitation j	Energy T	Binding $T_{<}$	Energy $T_{>}$
^{12}C	0					<u>19.5</u>
	n	18.72	0.00	0.00		+0.8
	p	15.96	0.00	0.00		+3.5
^{14}C	1				<u>11.67</u>	† <u>17.28</u>
	n	8.18	3.68	15.11	-0.2	+5.4
	p	20.83	0.00	0.00	-9.2	-3.5
^{16}O	0				<u>17.79</u>	<u>18.98</u>
	n	15.66	6.18	0.00	-4.0	-2.9
	p	12.13	6.32	0.00	-0.7	+0.5
^{28}Si	0				<u>11.58</u>	<u>14.36</u>
	n	17.17	0.00	0.00	-5.6	-2.8
	p	11.58	0.00	0.00	0.0	+2.8
^{54}Fe	1				<u>10.68</u>	<u>13.26</u>
	n	13.38	0.00	4.26	-2.7	-4.4
	p	8.85	0.00	0.00	+1.8	+4.4
^{60}Ni	2				<u>7.55</u>	<u>13.88</u>
	n	11.39	3.9	7.35	-7.7	-8.8
	p	9.53	0.00	0.00	-2.0	+4.3

† $T_<$ state instead of $T_>$ state

[Pet 84]. The parameter set with the smaller r_0 comes from fits (with no MEC effects included) for electron scattering to stretched states [Cla 87]. The WS wave functions using the larger r_0 will be designated WSl, and the wave functions using the smaller r_0 will be designated WSs.

Using the wave functions. A computer program was written to calculate transition densities to be used as input to ALLWRLD. The transition densities were calculated using either the initial and final state WS wave functions from DWUCK4 or internally generated HO wave functions. The equation used in calculating the transition density was:

$$\rho_{tr} = -(-1)^{J/2} \sqrt{\frac{J+2}{2\pi}} \langle j_h j_p \frac{1}{2} - \frac{1}{2} | J0 \rangle \frac{[Z_n \psi_f^n \psi_i^n \pm Z_p \psi_f^p \psi_i^p]}{\sqrt{2}}, \quad (5.3)$$

where J , j_h , and j_p are defined in Chapter I. The “+” in the last factor is for the isoscalar transition density and the “-” is for the isovector transition density. When using HO wave function $\psi^n = \psi^p = \psi$ and the last factor reduces to $Z_0 \psi_f \psi_i$ for the “+” and $Z_1 \psi_f \psi_i$ for the “-”. The relations between Z coefficients are given in Chapter I. The program was checked to see if it was working correctly by having it generate HO wave functions to use in the transition density. When the HO transition density was input to ALLWRLD, the results were the same as when ALLWRLD internally generated its own HO transition density.

The matrix elements calculated by ALLWRLD and MSUDWPI using the HO and WS wave functions discussed above are given in Table 5.5 for each stretched state in the six nuclei studied here. In most cases the matrix elements are larger when using WS wave functions, and thus a smaller normalization to theory will be needed. It is also seen that for a

Table 5.5 Pion matrix elements, $(M_1^\pi)_{thr}^2$, as given in equation (4.39) are calculated by ALLWRLD and MSUDWPI using various wave functions. These matrix elements are at the peak of the angular distribution. The parameters used in the calculations are given in Table 5.3. The matrix elements are in units of $\mu\text{b/sr}$.

J^π	Energy	HO		WSI		WSs	
		π^-	π^+	π^-	π^+	π^-	π^+
^{12}C 4^-	19.5	103.0	96.0	48.0	104.0	49.5	85.4
^{14}C 4^-	11.7	75.7	86.5	106.0	168.0	82.7	105.0
	15.2	75.3	85.4	121.0	156.0	72.8	108.0
	17.3	75.0	84.8	305.0	104.0	133.0	87.6
	24.4	73.8	83.7	127.0	78.8	96.9	46.1
^{16}O 4^-	17.8	85.2	78.7	112.0	96.3	81.0	72.2
	17.9	85.2	78.7	112.0	95.8	80.8	71.9
	18.6	84.8	78.4	116.0	73.6	87.1	50.1
	19.0	84.5	78.3	115.0	71.5	86.3	49.4
	19.8	84.1	78.0	112.0	65.6	84.2	47.1
	20.5	83.7	77.7	107.0	64.1	81.3	46.1
^{28}Si 6^-	11.6	63.7	57.9	72.2	65.6	66.5	61.4
	14.4	62.8	57.2	76.6	44.9	71.2	41.0
^{54}Fe 8^-	8.3	33.4	30.6	90.8	81.9	51.4	46.5
	9.0	33.3	30.4	98.8	59.1	57.5	30.5
	9.8	33.2	30.3	97.3	59.2	57.1	30.9
	10.0	33.1	30.3	97.0	59.3	57.0	31.0
	10.7	33.0	30.1	95.6	59.3	56.6	31.3
	11.7	32.9	30.0	93.6	59.3	56.0	31.6
	13.3	32.6	29.9	94.8	57.3	55.3	32.0
^{60}Ni 8^-	5.0	30.4	29.4	69.0	66.7	34.4	34.5
	7.5	30.0	29.3	68.1	65.4	34.6	34.5
	8.4	29.9	29.2	67.7	64.9	34.6	34.5
	9.0	29.9	29.2	67.5	64.6	34.7	34.5
	9.2	29.9	29.1	67.4	64.4	34.7	34.5
	12.3	29.8	28.9	72.1	46.7	37.5	23.6
	12.5	29.8	28.9	72.0	46.7	37.5	23.6
	13.9	29.7	28.8	71.5	46.1	37.5	24.0
	14.8	29.6	28.7	71.2	45.4	37.5	24.1
	15.5	29.6	28.6	71.0	44.8	37.5	24.2
	16.1	29.5	28.5	70.8	44.3	37.5	24.2

given state the theoretical matrix elements are about the same for π^- and π^+ when using HO wave functions, but can be quite different when using WS wave functions. This effect is discussed next.

Wave function asymmetries. The theoretical matrix elements shown in Table 5.5 are about the same for π^- and π^+ when using HO wave functions, since the same wave function is used for neutrons and protons. The differences of about 10% or less are a result of the distorted wave calculations in the code MSUDWPI. However, when using WS wave functions, these π^- and π^+ matrix elements can be quite different, since neutron and proton wave functions are used that extend to different radii. As expected, this is mainly true for unbound states at high excitation energies, where large asymmetries are seen in Table 5.4 for the binding of neutrons and protons. So, for pure isovector $T_>$ states, HO wave functions predict π^- and π^+ cross sections that are equal, and WS wave functions predict cross sections that can be quite different.

Looking at the $T_>$ experimental data in Table 5.6, it is seen that the 13.9 MeV state in ^{60}Ni particularly, but also all the other $T_>$ states, have a much larger π^- cross section than π^+ cross section. However, the 19.0 MeV state in ^{16}O and the 13.3 MeV state in ^{54}Fe have equal experimental cross sections for π^- and π^+ , and the 14.4 MeV state in ^{28}Si has a smaller cross section for π^- than for π^+ . The asymmetries seen in the ^{60}Ni data and predicted by the WS wave functions have been previously demonstrated for giant quadrupole resonances [Pet 85], but shown to be slight for the nearly-bound stretched state of isosymmetric ^{12}C [Sic 80].

Table 5.6 Experimental pion scattering cross sections (at the peak of the angular distribution) to be used in calculating the Z coefficients. The energy is in MeV and the cross sections are in units of $\mu\text{b/sr}$. The ^{60}Ni values are from the results of Table 3.4. The values for the other nuclei were found from the references as follows: ^{12}C —combining numbers from several references; ^{14}C —a list; ^{16}O and ^{28}Si —plots; ^{54}Fe —plots or from calculations using the Z coefficients given. Note that pion scattering on ^{14}C found two stretched states at 19.25 MeV and 19.65 MeV; whereas, electron scattering found only one state at 19.5 MeV. The two pion states are therefore combined to give a total cross section.

J^π	T	Energy	σ_{exp}^-	σ_{exp}^+	Ref.
^{12}C	4^-				[Moo 78; Hal 81; Cot 87]
		(19.25)	37 $\pm$ 13	167 $\pm$ 29	
		(19.65)	121 $\pm$ 27	21 $\pm$ 7	
		19.5	158 $\pm$ 30	188 $\pm$ 30	
^{14}C	4^-				[Hol 85]
		< 11.7	67 $\pm$ 4	< 4	
		< 15.2	17 $\pm$ 5	33 $\pm$ 4	
		< 17.3	< 10	110 $\pm$ 8	
^{16}O	4^-				[Hol 80]
		< 17.8	43 $\pm$ 5	67 $\pm$ 5	
		> 19.0	51 $\pm$ 5	53 $\pm$ 5	
		< 19.8	74 $\pm$ 5	45 $\pm$ 5	
^{28}Si	6^-				[Olm 79]
		< 11.6	28 $\pm$ 6	36 $\pm$ 3	
		> 14.4	16 $\pm$ 5	25 $\pm$ 3	
^{54}Fe	8^-				[Gee 84]
		< 8.3	10.6 $\pm$ 1	0.1 $\pm$ 3	
		< 8.9	3.0 $\pm$ 1	2.0 $\pm$ 3	
		< 9.8	6.5 $\pm$ 1	5.0 $\pm$ 3	
		< 10.0	0.2 $\pm$ 1	7.7 $\pm$ 3	
		< 10.7	6.3 $\pm$ 1	19.0 $\pm$ 2.6	
		< 11.6	5.4 $\pm$ 1	1.4 $\pm$ 3	
		> 13.3	14.2 $\pm$ 1	13.7 $\pm$ 1.7	
^{60}Ni	8^-				this thesis
		< 5.1	53 $\pm$ 4	41 $\pm$ 5	
		< 7.5	< 6	9.7 $\pm$ 1	
		< 8.4	< 9	< 2	
		< 9.0	< 5	6.9 $\pm$ 1	
		< 9.2	8.5 $\pm$ 3.3	7.8 $\pm$ 3.5	
		> 12.3	< 6	< 3	
		> 12.5	< 5	< 3	
		> 13.9	15.0 $\pm$ 5	4.1 $\pm$ 1	
		> 14.8	< 7	< 4	
		> 15.5	11.7 $\pm$ 3	6.4 $\pm$ 1	
		> 16.1	10.5 $\pm$ 3	< 4	

The symmetries seen in the ^{16}O and ^{54}Fe data, however, are as predicted by the HO wave functions.

The Z Coefficients and Normalization Factors

The calculation of Z coefficients here follows a procedure similar to that outlined by Holtkamp et al. [Hol 85].

Steps in the calculations. Two equations were given in Chapter IV that must be solved simultaneously for Z_0 and Z_1 : equation (4.39) for pion scattering and equation (4.51) for electron scattering. The experimental cross sections (and form factor) used in these equations are from the peak of the angular distributions. Now by defining several ratios, the equations are easier to work with:

$$R^e = F_{exp}^2 / (M_1^e)^2_{thr} ,$$

$$R^+ = \sigma_{exp}^+ / (M_1^{\pi^+})^2_{thr} ,$$

$$R^- = \sigma_{exp}^- / (M_1^{\pi^-})^2_{thr} ,$$

and

$$R = R^+ / R^- .$$

The result is an equation for electron scattering:

$$R^e = \left[\frac{M_0^e}{M_1^e} \frac{Z_0}{Z_1} + 1 \right]^2 (Z_1)^2 , \quad (5.4)$$

and one for pion scattering:

$$R^\pm = \left[\frac{M_0^\pi}{M_1^\pi} \frac{Z_0}{Z_1} \mp 1 \right]^2 (Z_1)^2 [N^\pm f_{cm}^2] . \quad (5.5)$$

If the center-of-mass correction is used, f_{cm} is given by equation (5.2).

The values needed in calculating the Z coefficients are the three experimental cross sections [F_{exp}^2 and $\sigma_{exp}^\pm$], the three theoretical matrix elements [$(M_1^e)_{thr}^2$ and $(M_1^{\pi^\pm})_{thr}^2$], and the two ratios [M_0^e/M_1^e and M_0^π/M_1^π]. When using HO wave functions, the M^e ratio is just $g_0/g_1 = -0.187$. When using WS wave functions however, the neutron and proton wave functions are not the same and the ratio varies as seen in Appendix D [Cla 87]. The M^π ratio is approximately equal to 2 at the $\Delta_{3,3}$ -resonance as shown in Appendix A. From running ALLWRLD and MSUDWPI this ratio was found to be 1.91 for all nuclei at the 162 MeV beam energy. (See also [Hol 85].)

If all the above values are known (case #1), the derivation of the Z coefficients proceeds in several steps. The calculations carry uncertainties along with them in each step. *STEP #1*: Using the definition for R , R^+ is divided by R^- and then solved for the Z_0/Z_1 ratio. Assuming that the normalization factors for π^- and π^+ are the same, $N^- = N^+ = N$, allows these factors to cancel. There are then two possible solutions:

$$\frac{Z_0}{Z_1} = \frac{1}{M_0^\pi/M_1^\pi} \frac{1 \mp \sqrt{R}}{1 \pm \sqrt{R}}. \quad (5.6)$$

STEP #2: Solving equation (5.4) for Z_1 , gives

$$Z_1 = \pm \frac{\sqrt{R^e}}{\left[\frac{M_0^e}{M_1^e} \frac{Z_0}{Z_1} + 1 \right]}. \quad (5.7)$$

The Z_1 is arbitrarily taken to be the positive solution. The Z_0 is found by using this Z_1 and Z_0/Z_1 from equation (5.6). Since there are two solutions

for Z_0/Z_1 , there will be two sets of Z coefficients. *STEP #3*: Now equation (5.5) is solved for the normalization factor:

$$N^{\pm} = \frac{R^{\pm}}{\left[\left(\frac{M_0^{\pi}}{M_1^{\pi}} \frac{Z_0}{Z_1} \mp 1 \right) Z_1 f_{cm} \right]^2} . \quad (5.8)$$

This calculation results in four values—two for each Z_0/Z_1 solution. However, since it is assumed that $N^+ = N^-$, the calculations result in only two nonequal values—one for each Z_0/Z_1 solution.

STEP #4: Steps #1 to #3 are carried out for all stretched states of the nucleus. The normalization factors for all states and Z_0/Z_1 solutions are then compared. In most cases the correct Z_0/Z_1 solution is easily determined as the solution that gives a consistent normalization factor from state to state. An N_{avg} is then found as an average weighted by the uncertainties of “ N ” for each state.

STEP #5: Now equation (5.5) is solved for Z_0 :

$$Z_0 = \frac{\sqrt{R^{\pm}/(N_{avg} f_{cm})}}{\left[\frac{M_0^{\pi}}{M_1^{\pi}} \mp \frac{1}{Z_0/Z_1} \right]} . \quad (5.9)$$

This can give four solutions, but two are eliminated because the correct Z_0/Z_1 ratio has already been determined. The other two solutions should be equal because it has been assumed that $N^+ = N^-$. *STEP #6*: Lastly, equation (5.4) is solved for Z_1 :

$$Z_1 = \sqrt{R^e} - \frac{M_0^e}{M_1^e} Z_0 . \quad (5.10)$$

The Z_0 and Z_1 of steps #5 and #6 are the final results.

Special cases. If not all the experimental cross sections are known, then a different procedure must be followed. If the electron scattering cross section is not known for some state (case #2), steps #2 and #3 are deleted and the average normalization from other stretched states is used. If only an upper limit can be put on one of the pion scattering cross sections for a stretched state (cases #3 and #4), then R is bounded on only one side. Each of the two solutions for Z_0/Z_1 calculated from R in step #1 has both an upper and lower bound, however. (See Holtkamp et al. [Hol 85] for a more complete description.) A value for Z_0/Z_1 can be found with an uncertainty based on these upper and lower bounds, and then steps #2 through #6 can be executed. If no pion scattering cross sections are known for a state (case #5), the full calculation cannot be done. If a state has been found in electron scattering, but not in pion scattering, it is mainly an isovector state and Z_0 is small. Then $Z_1 \approx +\sqrt{R^e}$ from equation (5.7) is used. Errors in this assumption are discussed in [Cla 87] (Appendix D).

In looking at the above steps it can be seen that Z_1 is mainly determined by electron scattering, since $(M_0^e/M_1^e)^2 \approx 1/29$. This Z_1 is very dependent on the wave functions used to calculate the electron scattering matrix element for R^e in equation (5.10). It is also seen that the Z_0/Z_1 ratio in equation (5.6) and thus Z_0 is mainly determined from the ratio between π^+ and π^- cross sections for each state. Thus Z_0 is not very dependent on the wave functions used in equation (5.6) to calculate the pion scattering matrix element for R . These observations will be verified in the next section where the calculations are carried out.

A comment on the pion normalization factor should be included here. The Z coefficients are chosen so that experimental electron scattering

results agree with the theoretical matrix elements, $(M^e)_{thr}$. Using these Z coefficients, the experimental pion scattering data are compared with theoretical pion calculations. Usually a normalization factor is necessary to get agreement between pion theory and experiment. A normalization is used for pion scattering rather than electron scattering because the electromagnetic interaction that governs electron scattering is understood much better than the strong interaction that governs pion scattering.

A small computer code was written to do the calculations described in this section. The values used in the calculations are from various sources. The pion theoretical matrix elements are listed in Table 5.5, the pion data are given in Table 5.6, and 1.91 was used for the pion isospin ratio. The electron scattering theory, data, and ratios are found in Appendix D [Cla 87]. Since no procedure for including MEC effects in pion scattering has yet been developed, MEC effects were not included in the electron scattering theory either. When using HO wave functions, -0.187 was used for the isospin ratio for electron scattering.

A sample output from the program for calculating Z coefficients is given in Figure 5.2 for several states in ^{60}Ni .

Resulting Z Coefficients. Table 5.7 lists the calculated Z coefficients using both HO and WS wave functions. The WS wave functions use the radial parameters with the larger r_0 . Using the radial parameter with the smaller r_0 changes these Z coefficients by less than their uncertainties. Table 5.8 lists the sums of the experimental Z coefficients squared, the theoretical sum rules determined from equations (4.54) to (4.56), and the ratio of the two (defined here to be $S^2 = \sum Z_{exp}^2 / \sum Z_{thr}^2$). All the

NOTE: for (e,e) - exp. and thr. values are xE-45
for (pi,pi) - exp. and thr. values are in ub/sr
A=60.
J=8

ENG= 5.1 case=2		electron	pi+	pi-	+/-	
0/1		-0.20	1.91			
exp		0.00+-0.00	41.00+-5.00	53.00+-4.00	0.77+-0.11	
thr		8.30	66.70	69.00	0.97	
exp/thr		0.00+-0.00	0.61+-0.07	0.77+-0.06	0.80+-0.11	
sign soln.	Z0	Z1	Z0/Z1	N (pi+)	N (pi-)	NP01
-1. a	0.01+-0.01	0.19+-0.41	0.03+-0.02	2.93+-0.64	2.93+-0.64	1
-1. b	0.25+-0.03	0.03+-0.02	9.41+-5.73	2.93+-0.64	2.93+-0.64	0
+1. a	0.01+-0.01	0.19+-0.43	0.03+-0.02	2.93+-0.64	2.93+-0.64	1
+1. b	0.25+-0.03	0.03+-0.02	9.41+-5.73	2.93+-0.64	2.93+-0.64	0
ENG= 7.5 case=3		electron	pi+	pi-	+/-	
0/1		-0.20	1.91			
exp		0.34+-0.20	9.70+-1.00	3.00+-3.00	3.23+-3.25	
thr		8.23	65.40	68.10	0.96	
exp/thr		0.04+-0.02	0.15+-0.02	0.02+-0.02	3.37+-3.38	
sign soln.	Z0	Z1	Z0/Z1	N (pi+)	N (pi-)	NP01
+0. a	-0.06+-0.04	0.19+-0.06	-0.34+-0.18	1.49+-0.77	4.82+-7.88	1
+0. b	-0.19+-0.12	0.17+-0.05	-1.15+-0.63	0.52+-0.36	0.55+-0.91	p
-1. a	-0.08+-0.14	0.19+-0.07	-0.43+-0.72	2.93+-0.64	2.93+-0.64	p
-1. b	-0.08+-0.06	0.19+-0.06	-0.50+-0.38	2.93+-0.64	2.93+-0.64	p
+1. a	-0.05+-0.02	0.20+-0.06	-0.24+-0.11	2.93+-0.64	2.93+-0.64	1
+1. b	-0.08+-0.02	0.19+-0.06	-0.49+-0.18	2.93+-0.64	2.93+-0.64	p
ENG= 9.0 case=3		electron	pi+	pi-	+/-	
0/1		-0.20	1.91			
exp		0.20+-0.01	6.90+-1.20	2.50+-2.50	2.76+-2.80	
thr		8.12	64.60	67.50	0.96	
exp/thr		0.03+-0.00	0.11+-0.02	0.02+-0.02	2.88+-2.93	
sign soln.	Z0	Z1	Z0/Z1	N (pi+)	N (pi-)	NP01
+0. a	-0.05+-0.03	0.15+-0.01	-0.33+-0.19	1.83+-0.64	6.12+-9.69	1
+0. b	-0.16+-0.10	0.13+-0.02	-1.27+-0.75	0.57+-0.36	0.57+-0.90	p
-1. a	-0.07+-0.12	0.14+-0.02	-0.48+-0.80	2.93+-0.64	2.93+-0.64	p
-1. b	-0.07+-0.05	0.14+-0.01	-0.56+-0.37	2.93+-0.64	2.93+-0.64	p
+1. a	-0.04+-0.01	0.15+-0.00	-0.26+-0.10	2.93+-0.64	2.93+-0.64	1
+1. b	-0.07+-0.02	0.14+-0.00	-0.56+-0.14	2.93+-0.64	2.93+-0.64	p
ENG= 9.2 case=1		electron	pi+	pi-	+/-	
0/1		-0.20	1.91			
exp		0.19+-0.01	7.80+-3.50	8.50+-3.30	0.92+-0.54	
thr		8.10	64.40	67.40	0.96	
exp/thr		0.02+-0.00	0.12+-0.05	0.13+-0.05	0.96+-0.57	
sign soln.	Z0	Z1	Z0/Z1	N (pi+)	N (pi-)	NP01
+0. a	0.00+-0.01	0.15+-0.00	0.01+-0.08	5.40+-2.07	5.40+-1.87	1
+0. b	0.84+-****	0.02+-0.26	51.83+-****	0.05+-1.48	0.05+-1.46	0
-1. a	0.00+-0.02	0.15+-0.00	0.01+-0.10	2.93+-0.64	2.93+-0.64	1
-1. b	0.11+-0.03	0.17+-0.01	6.63+-****	2.93+-0.64	2.93+-0.64	n
+1. a	0.00+-0.02	0.15+-0.00	0.01+-0.11	2.93+-0.64	2.93+-0.64	1
+1. b	0.11+-0.03	0.17+-0.01	6.63+-****	2.93+-0.64	2.93+-0.64	n
ENG=12.3 case=5		electron	pi+	pi-	+/-	
0/1		-0.12	1.91			
exp		0.12+-0.01	1.50+-1.50	3.00+-3.00	0.50+-0.71	
thr		6.94	46.70	72.10	0.65	
exp/thr		0.02+-0.00	0.03+-0.03	0.04+-0.04	0.77+-1.09	
sign soln.	Z0	Z1	Z0/Z1	N (pi+)	N (pi-)	NP01
+0. a	0.00+-0.00	0.13+-0.00	0.00+-3.08	1.89+-1.34	2.45+-1.73	1
+0. b	0.00+-0.00	0.13+-0.00	0.00+-****	1.89+-1.34	2.45+-1.73	1
ENG=13.9 case=1		electron	pi+	pi-	+/-	
0/1		-0.12	1.91			
exp		0.34+-0.02	4.10+-0.40	15.00+-4.70	0.27+-0.09	
thr		6.87	46.10	71.50	0.64	
exp/thr		0.05+-0.00	0.09+-0.01	0.21+-0.07	0.42+-0.14	
sign soln.	Z0	Z1	Z0/Z1	N (pi+)	N (pi-)	NP01
+0. a	0.02+-0.01	0.23+-0.01	0.11+-0.03	2.81+-0.40	2.81+-0.67	1
+0. b	0.78+-0.26	0.31+-0.04	2.48+-J.78	0.06+-0.04	0.06+-0.03	0
-1. a	0.02+-0.01	0.23+-0.01	0.11+-0.03	2.93+-0.64	2.93+-0.64	1
-1. b	0.12+-0.02	0.24+-0.01	0.37+-0.09	2.93+-0.64	2.93+-0.64	n
+1. a	0.02+-0.01	0.23+-0.01	0.11+-0.04	2.93+-0.64	2.93+-0.64	1
+1. b	0.12+-0.02	0.24+-0.01	0.37+-0.07	2.93+-0.64	2.93+-0.64	n

Figure 5.2 Sample output from the program for calculating Z coefficients. The results are for several states in ^{60}Ni .

Table 5.7 The Z coefficients for the six nuclides on which both electron and pion scattering to the stretched states have been done. The Z_1 coefficients have arbitrarily all been chosen as the positive solution. No MEC effects or center-of-mass corrections have been included. In most cases the HO results are in agreement with those presented previously for ^{12}C [Hic 84], ^{14}C [Hol 85; Plu 85], ^{16}O [Car 83; Hyd 87], ^{28}Si [Car 83], and ^{54}Fe . The exceptions are: 1) Hicks et al. did not include cross terms in calculating Z_0 for ^{12}C and 2) the signs here for ^{16}O agree with Carr et al., but not with Hyde-Wright et al.

	Eng.	T	HO		WS	
			Z_0	Z_1	Z_0	Z_1
^{12}C	19.5		-0.02 ± 0.02	0.64 ± 0.02	0.08 ± 0.03	0.98 ± 0.02
^{14}C	11.7	<	0.14 ± 0.02	0.32 ± 0.01	0.15 ± 0.02	0.37 ± 0.01
	15.2	<	-0.18 ± 0.03	0.05 ± 0.03	-0.17 ± 0.03	0.03 ± 0.03
	17.3	<	-0.21 ± 0.03	0.29 ± 0.01	-0.22 ± 0.03	0.32 ± 0.01
	24.4	>	0.00 ± 0.00	0.49 ± 0.01	0.00 ± 0.00	0.62 ± 0.02
^{16}O	17.8	<	-0.34 ± 0.05	0.06 ± 0.01	-0.37 ± 0.06	0.07 ± 0.01
	17.9	>	0.00 ± 0.00	0.15 ± 0.00	0.00 ± 0.00	0.16 ± 0.00
	18.6	>	0.00 ± 0.00	0.18 ± 0.00	0.00 ± 0.00	0.21 ± 0.00
	19.0	>	-0.01 ± 0.01	0.65 ± 0.00	-0.05 ± 0.02	0.76 ± 0.00
	19.8	<	0.36 ± 0.06	0.07 ± 0.02	-0.42 ± 0.06	0.01 ± 0.03
	20.5	>	0.00 ± 0.00	0.23 ± 0.00	0.00 ± 0.00	0.29 ± 0.01
^{28}Si	11.6	<	-0.34 ± 0.05	0.06 ± 0.04	-0.36 ± 0.05	0.06 ± 0.04
	14.4	>	-0.04 ± 0.02	0.52 ± 0.01	-0.08 ± 0.03	0.61 ± 0.02
^{54}Fe	8.3	<	0.09 ± 0.01	0.21 ± 0.00	0.09 ± 0.01	0.20 ± 0.00
	8.9	<	0.01 ± 0.02	0.18 ± 0.01	0.00 ± 0.02	0.18 ± 0.00
	9.8	<	0.15 ± 0.03	0.01 ± 0.03	-0.15 ± 0.03	0.02 ± 0.03
	10.0	<	-0.08 ± 0.01	0.18 ± 0.01	-0.09 ± 0.01	0.19 ± 0.01
	10.7	<	-0.22 ± 0.02	0.13 ± 0.01	-0.22 ± 0.03	0.15 ± 0.00
	11.6	<	0.11 ± 0.02	0.06 ± 0.03	0.11 ± 0.03	0.04 ± 0.03
	13.3	>	0.00 ± 0.01	0.44 ± 0.01	-0.03 ± 0.01	0.45 ± 0.01
^{60}Ni	5.1	<	0.27 ± 0.03	0.03 ± 0.02	0.25 ± 0.03	0.03 ± 0.02
	7.5	<	-0.05 ± 0.02	0.21 ± 0.07	-0.05 ± 0.02	0.20 ± 0.06
	8.4	<	0.00 ± 0.00	0.15 ± 0.00	0.00 ± 0.00	0.13 ± 0.00
	9.0	<	-0.04 ± 0.02	0.16 ± 0.01	-0.04 ± 0.01	0.15 ± 0.00
	9.2	<	0.00 ± 0.02	0.16 ± 0.01	0.00 ± 0.02	0.15 ± 0.00
	12.3	>	0.00 ± 0.00	0.13 ± 0.00	0.00 ± 0.00	0.13 ± 0.00
	12.5	>	0.00 ± 0.00	0.17 ± 0.00	0.00 ± 0.00	0.17 ± 0.00
	13.9	>	0.04 ± 0.01	0.23 ± 0.01	0.02 ± 0.01	0.23 ± 0.01
	14.8	>	0.00 ± 0.00	0.18 ± 0.00	0.00 ± 0.00	0.18 ± 0.00
	15.5	>	0.02 ± 0.01	0.16 ± 0.01	0.00 ± 0.01	0.16 ± 0.01
	16.1	>	0.05 ± 0.02	0.15 ± 0.01	0.05 ± 0.02	0.15 ± 0.01

Table 5.8 Sums of Z coefficients squared. The experimental values are sums for WS wave functions from Table 5.7. The theoretical values are calculated from the sum rules of equations (4.54) to (4.56). The ratio of experiment to theory is defined as $S^2 = \Sigma Z_{exp}^2 / \Sigma Z_{thr}^2$.

	T	experimental		theoretical		S_0^2	S_1^2	S_0^2/S_1^2
		ΣZ_0^2	ΣZ_1^2	ΣZ_0^2	ΣZ_1^2			
^{12}C	<				0			
	>				1			
	total	0.01	0.96	1	1	0.01	0.96	0.01
^{14}C	<	0.10	0.24		1/2		0.48	
	>	0.00	0.38		1/2		0.77	
	total	0.10	0.62	1	1	0.10	0.62	0.16
^{16}O	<	0.31	0.01		0			
	>	0.00	0.73		1		0.73	
	total	0.32	0.74	1	1	0.32	0.74	0.43
^{28}Si	<	0.13	0.00		0			
	>	0.01	0.37		1		0.37	
	total	0.14	0.38	1	1	0.14	0.38	0.36
^{54}Fe	<	0.10	0.13		1/2		0.27	
	>	0.00	0.20		3/8		0.54	
	total	0.10	0.34	7/8	7/8	0.11	0.38	0.30
^{60}Ni	<	0.07	0.10		2/3		0.15	
	>	0.00	0.18		1/3		0.54	
	total	0.07	0.28	1	1	0.07	0.28	0.25

values in Table 5.8 use WS wave functions. If MEC effects had been included for electron scattering, the matrix elements would increase by 15% to 20%, thus decreasing the percentage of the sum rule exhausted by the same amount [Cla 87] (Appendix D).

The ^{12}C nucleus is a unique case because electron scattering found one stretched state at 19.5 MeV [Hic 84], but pion scattering found two stretched states: a proton-like state at 19.25 MeV and a neutron-like state at 19.65 MeV [Mor 79]. These two states were combined to calculate the Z coefficients.

It is seen that in general the Z coefficients are quenched significantly from that predicted by the sum rule ($S^2 < 1$), with the isoscalar transitions quenched even more than the isovector. These results are shown graphically in Figure 5.3. The same results are described and plotted by Geesaman et al. [Gee 84]. Electron scattering preferentially excites isovector states and pion scattering at the $\Delta_{3,3}$ -resonance preferentially excites isoscalar states. Since the pion flux at LAMPF is much smaller than the flux of electron beams in general, one would expect much poorer statistics for pion cross sections. One would thus expect to find isovector states more easily than isoscalar states. These experimental details could at least partially explain the observed larger quenching of isoscalar than of isovector transitions.

Resulting normalization factors. The weighted " N_{avg} " found as described in step #4 above is given in Table 5.9. The normalization factors listed here for HO wave functions are in agreement with the factors found previously. Carr et al. [Car 83] found it necessary to

Figure 5.3 At the top is a graphical display of the quenching of isovector (open bar) and isoscalar (solid bar) strengths for the six nuclei where both electron and pion scattering have been done to the stretched states. No quenching corresponds to 1. The values plotted are total S_0^2 and total S_1^2 from Table 5.8. At the bottom is a display of the ratio of isoscalar to isovector quenching, where S_0^2/S_1^2 is from Table 5.8. Equal quenching corresponds to 1.

use normalization factors of 1.3 for ^{16}O and 1-15 for ^{28}Si ; Holtkamp et al. [Hol 85] found that a normalization factor of 1.8 ± 0.3 for ^{14}C was necessary. It should be noted that the normalization factor is in general closer to 1 when WS wave functions are used.

Table 5.9 Normalization factors, " N_{avg} ", necessary to arrive at agreement between theory and experiment for pion scattering. The first column includes the center-of-mass correction for comparison.

	HO (cm)	HO	WSI	WSs
^{12}C	3.4 ± 0.5	4.3 ± 0.6	2.6 ± 0.4	2.8 ± 0.5
^{14}C	2.1 ± 0.3	2.5 ± 0.3	1.7 ± 0.4	1.9 ± 0.3
^{16}O	1.3 ± 0.4	1.5 ± 0.5	1.0 ± 0.3	1.4 ± 0.4
^{28}Si	1.1 ± 0.2	1.2 ± 0.2	1.0 ± 0.2	1.1 ± 0.2
^{54}Fe	2.0 ± 0.3	2.2 ± 0.3	0.9 ± 0.1	1.7 ± 0.2
^{60}Ni	5.3 ± 1.1	5.8 ± 1.2	2.9 ± 0.6	5.6 ± 1.2

The normalization factor is very dependent on the theory to which the data are compared—the wave functions used and the optical potential parameters used. The wave functions that give the larger matrix elements in Table 5.5 result in the smaller normalization factors in Table 5.9. Using different optical potential parameters from Table 5.1 as input could change the normalization factors drastically.

Experimental and Theoretical Angular Distributions

Figures 5.4 through 5.7 compare the experimental data to the $J = 8$ theoretical angular distributions from MSUDWPI. The theoretical curves have been normalized by the factor, N , found in STEP #3 for calculating Z coefficients. In most cases this is close to the N_{avg} listed in Table 5.9.

Figure 5.7 shows the data for four of the six states between 5 and 7 MeV that were looked at in a search for isoscalar 8^- stretched states

Figure 5.4 The MSUDWPI angular distributions compared to sample stretched states in ^{12}C [Moo 78], ^{14}C [Hol 85], and ^{16}O [Hol 80]. The solid curves use HO wave functions; the single-dot dashed curves use WSs; the double-dot dashed curves use WSl. Open circles are π^- data; closed circles are π^+ data. The π^- data are multiplied by 0.1.

Figure 5.5 The MSUDWPI angular distributions compared to sample stretched states in ^{28}Si [Olm 79] and ^{54}Fe [Gee 84]. Same legend as in Figure 5.4. The π^- data are multiplied by 0.1.

Figure 5.6 The MSUDWPI angular distributions compared to 8^- stretched states in ^{60}Ni . Same legend as in Figure 5.4. The π^- data as well as the 7.5 MeV π^+ data are multiplied by 0.1.

Figure 5.7 The MSUDWPI angular distributions compared to data to check for 8^- stretched states in ^{60}Ni . Same legend as in Figure 5.4. The π^- data are multiplied by 0.1.

not found in electron scattering. Many excited states have been found in ^{60}Ni in this energy range [Aub 79], but none with $J = 8$. Unfortunately, the angular distributions in this experiment are not complete enough to be very definitive, but the 5.1 MeV state looks promising. A more complete angular distribution may be forthcoming due to a recent experiment at EPICS on ^{60}Ni taken at forward angles. The 5.1 MeV state is tentatively included as an 8^- state in this thesis. If in fact, it is not an 8^- state, then the entries for ^{60}Ni in Table 5.8 are $\Sigma Z_0^2 = 0.01$ and $S_0^2/S_1^2 = 0.02$.

In ^{16}O , ^{28}Si , and ^{54}Fe the angular distributions for π^- and π^+ data peak at about the same angle, whereas in ^{14}C and ^{60}Ni , the angular distribution for the π^- data peaks at a smaller angle than for the π^+ data. The theoretical curves using either WS wave functions or HO wave functions are able to reproduce these features about equally well, except in ^{14}C where the WS wave functions do better.

The two WS wave functions used in the calculations are seen to result in the curves peaking at different angles—smaller angles for larger r_0 . The theoretical curves calculated from WS wave functions fit the data quite well in all cases except ^{12}C . WSl fits better than WSs for ^{60}Ni , but in the other cases WSs give the better fit. In other words the parameters from elastic electron scattering fit the pion scattering ^{60}Ni data better, whereas, the parameters from electron scattering to stretched states fit all the other pion scattering data better. It should be noted for ^{60}Ni specifically that the data peaks at a smaller angle than the HO curves, but reasonably close to the WSl curves.

Figure 5.8 compares the experimental data for the 7.5 MeV state in ^{60}Ni to the theoretical angular distributions from MSUDWPI for several

Figure 5.8 Theoretical angular distributions for several angular momentum transfers (all with negative parity) compared to the π^+ data for the 7.5 MeV state in ^{60}Ni . The HO wave functions were used. Each set of data is offset by a factor of ten from the set above it.

angular momentum transfers. The theoretical curve for $J = 8$ has been normalized as in Figure 5.6. The other angular distributions have been normalized arbitrarily. For simplicity, HO wave functions were used in the calculations and only negative parity transitions were included. It was found that positive parity transitions resulted in similar angular distributions. The $J = 8$ curve fits the 7.5 MeV data best, although the $J = 4$ curve is also a possibility. The same thing could be said about the rest of the data in Figure 5.6 (except the 9.2 MeV state) and the 5.1 MeV data in Figure 5.7. The 9.2 MeV state is an exception and may not be the stretched state found in electron scattering.

Sample Transition Densities

The theoretical location of the peak of the angular distribution, the theoretical equality (or non-equality) of the π^- and π^+ cross sections, and the normalization factor are all related to what wave functions are used in the calculations. These three variables can be understood better by looking at plots of the transition density, $\rho_{tr} \propto \psi_f \psi_i$. Plots are given in Figure 5.9 for four sample stretched states. The transition densities were calculated using the absolute value of equation (5.3).

Since the pion is strongly absorbed, pion scattering is particularly sensitive to the nuclear surface, and the pion matrix element is related to the size of the wave function at the surface. With the surface around 3 fm for ^{14}C and ^{16}O , 4 fm for ^{28}Si , and 5 fm for ^{60}Ni , it is found that the transition density is largest when using WS1 wave function and smallest when using HO wave function. When the transition density is larger at the surface, the resulting matrix element is larger; thus the normalization

Figure 5.9 Sample transition densities, ρ_{tr} , showing the effect of using different wave functions: HO (solid line), WSs (dot-dashed line), and WSI (double dot-dashed line).

factor does not need to be as large. The resulting normalization factors are seen in Table 5.9.

Another piece of information found from the transition density is the angle where the reaction peaks. When the transition density peaks at a larger radius, the reaction peaks at a smaller momentum transfer and thus at a smaller angle. In all cases in Figure 5.9 the transition density peaks at the largest radius using the WSI wave function and at a smaller radius using HO and WSs wave functions. This results in the reaction peaking at a smaller angle using the WSI wave function and at a larger angle using the other wave functions, as seen in the theoretical curves of Figures 5.4 through 5.7.

It is also useful to look at the transition density for neutrons and protons separately. Figure 5.10 shows transition densities for the same states as in Figure 5.9, but with WS neutron and proton transition densities separated.

Table 5.4 shows that ^{14}C is a unique case where the neutron is less tightly bound than the proton. Since the neutron wave function peaks at a larger radius than the proton wave function, it is expected that π^- scattering will peak at a smaller angle than π^+ scattering. Also, the theoretical π^- cross section will be larger than the π^+ cross section, since the neutron wave function is larger than the proton wave function at the surface. The use of HO wave functions, on the other hand, treats neutrons and protons the same and predicts little difference between π^- and π^+ cross sections.

For the other three nuclei represented in Figure 5.10, the proton is less tightly bound than the neutron. In these cases the proton wave function extends to larger radii, but the neutron wave function is still larger than

Figure 5.10 Sample transition densities, ρ_{tr} , showing the results of neutron (dotted line) and proton (dashed line) WS wave functions separately. The composite transition densities use: WSs (dot-dashed line) and WSl (double dot-dashed line) for comparison.

or equal to the proton wave function at the surface. This results in the π^- cross sections being larger than π^+ cross sections as seen in Table 5.5. Little can be determined about the relative location of the peak for π^- and π^+ angular distributions. The π^- angular distributions peak at smaller angles than for π^+ using either HO or WS wave functions. This is not dependent on the wave functions used, but on the distortions introduced in the distorted wave calculations.

Summarizing: information about the transition densities can be used to make generalizations concerning 1) the absolute magnitude of the angular distributions, 2) the location of the peak of the angular distribution, and 3) the relative π^- to π^+ magnitude. However, it is seen that generalizations about the relative size of the transition density at the surface cannot be made directly from knowing what nucleons are bound.

Discussion of the ^{60}Ni Results

The ^{60}Ni nucleus has turned out to be an important nucleus on which to test the use of WS wave functions. The use of WS wave functions in the theoretical calculations gives much better fits to the experimental data than HO wave functions. The WSI wave functions resulted in a normalization factor much closer to 1 and an angular distribution peaking at smaller angles in accordance with the data. It was also found that the $\sigma_{\epsilon xp}^-/\sigma_{\epsilon xp}^+$ ratio for the $T_>$ states (all of which are proton unbound) was much larger than 1 as predicted by WS wave functions, but not by HO wave functions.

It should also be noted that ^{60}Ni is unique in that it required the use of the WS wave functions with the larger r_0 (WSI) to fit the angular distributions, unlike the other nuclei where WSs was used. This can perhaps

be explained by the fact that ^{60}Ni is structurally different than the other nuclei. In ^{60}Ni there are four neutrons outside the closed major shell that includes the $1f_{7/2}$ orbital from which the nucleons for the stretched states are taken. Perhaps, it is thus necessary to use a larger radius for the ^{60}Ni nucleus as a whole.

Finally, the isoscalar strength in ^{60}Ni was found to be only 25% of the isovector strength. This is the same percentage found in ^{54}Fe . However, if the questionable new isoscalar state at 5.1 MeV (not found in electron scattering) is not included, the isoscalar strength would only be 2% of the isovector strength. Geesaman et al. [Gee 84] also found new isoscalar states in their pion scattering experiment on ^{54}Fe , but their new states are also somewhat questionable.

CHAPTER VI

SUMMARY AND CONCLUSIONS

This thesis has described the study of pion scattering to stretched states in ^{60}Ni . Of the few nuclei where such exemplary single-particle states have been studied using pion scattering, ^{60}Ni is unique in being the only one with neutrons outside the closed shell containing the hole-state from which the nucleon is promoted. This thesis also presents the first use of Woods-Saxon wave functions in the consistent comparison of several nuclei where pion scattering to stretched states has been done. Significant departures from isospin symmetry seen for states in ^{60}Ni are successfully explained by this procedure.

The pion scattering experiment performed on the EPICS spectrometer at LAMPF studied stretched states using π^+ and π^- at scattering angles of 65° , 80° , and 90° . The replay of the data used the standard EPICS software package. The calibration of the spectra included a new nonlinearity in the channel-to-excitation-energy conversion.

This chapter summarizes and discusses five points: 1) the ^{60}Ni stretched states found by pion scattering, 2) the location of the angular distribution peak for π^- and π^+ data, 3) the $T_>$ stretched state cross sections for π^- and π^+ , 4) the normalization factors necessary to arrive at agreement of theory with data, and 5) the isoscalar and isovector transition strengths.

Summary

First, a summary of the ^{60}Ni data is given. Of the ten stretched states found in electron scattering to ^{60}Ni , five were found in pion scattering at 7.5, 9.0, 9.2, 13.9, and 15.5 MeV. The other five states were possibly found, but are quite uncertain. Of these ten states most were determined to be primarily isovector states, but the two at 7.5 MeV and 9.0 MeV are proton-like excitations. Six possible isoscalar stretched states, not found in electron scattering, were also looked at. One of these six states at 5.1 MeV may be an isoscalar 8^- stretched state, but the angular distribution was not complete enough to be definitive.

For the ^{60}Ni data the peak of the angular distribution for the π^- data was at a much smaller angle than for the π^+ data. Also, the π^- cross sections for the $T_>$ stretched states (identified as such from electron scattering) were appreciably larger than the π^+ cross sections.

Second, a summary of the theoretical calculations for ^{60}Ni is given. This involved the use of both harmonic oscillator (HO) and Woods-Saxon (WS) wave functions. To explain the difference in $T_>$ experimental cross sections for π^- and π^+ data in ^{60}Ni , it was necessary to use WS wave functions where the proton and neutron wave functions are not treated the same. Two other features of the ^{60}Ni data favored the use of WS wave functions (with radial parameter, $r_0 = 1.17$). The normalization factor necessary to arrive at agreement between theory and experiment was closer to unity when using WS wave functions ($N = 2.9$) than when using HO wave functions ($N = 5.8$). The angular distribution of the data peaked at an angle smaller than the HO curves, but close to the WS curves.

Total isoscalar and isovector 8^- strengths were determined for the stretched states in ^{60}Ni . The isovector strength was only 25% and the isoscalar strength only 7% (1% if the new isoscalar state is not included) of that predicted by the extreme single-particle sum rules. Using the different wave functions did not result in an appreciable difference in the calculated strengths.

Third, the results for ^{60}Ni are compared to five other nuclei on which both pion and electron scattering have been done. 1) This experiment identified a new isoscalar state in ^{60}Ni not found in electron scattering, just as previous pion scattering experiments found new isoscalar states in four of the other five nuclei. 2) The peak of the angular distribution was at a smaller angle for the π^- data than for the π^+ data in ^{60}Ni and ^{14}C , whereas, it was equal in the other nuclei. 3) The π^- and π^+ experimental cross sections were unequal for the $T_{>}$ stretched states in ^{60}Ni , but equal for $T_{>}$ stretched states in the other nuclei. 4) The WS normalization factor for ^{60}Ni and ^{12}C was close to $N = 3$, whereas, for the other nuclei it was close to unity. This factor was significantly increased in both cases if HO wave functions were used. 5) Quenching of the isovector strength and an even larger quenching of the isoscalar strength was found in ^{60}Ni , as has been found previously.

Discussion

The use of WS wave functions in the theory calculations adequately fits the experimental angular distributions. Two different sets of radii were used in these calculations. The set with the larger radius parameter r_0 was found to be necessary for ^{60}Ni , whereas a smaller radius

parameter was used in the other nuclei. The uniqueness of ^{60}Ni in having nucleons outside a major closed shell may explain the need for this larger factor. Also, the peaking of the π^- angular distributions at a smaller angle than the π^+ angular distributions in ^{60}Ni as well as in ^{14}C can be theoretically explained by the excess of neutrons in both nuclei.

Using WS wave functions results in unequal π^- and π^+ theoretical cross sections for stretched states in all nuclei where the proton is unbound. This explains the fact that the $T_{\geq}$ experimental cross sections for π^- and π^+ are unequal in ^{60}Ni . However, this is contradicted by the $T_{\geq}$ experimental cross sections that are equal in the other nuclei even when the proton is unbound.

For previous pion scattering calculations done on other nuclei, normalization factors between 1 and 2 have been used to obtain agreement between data and theory based on HO wave functions. When WS wave functions are used, the factors are nearer to unity, reflecting a more realistic model. Although the input quantities used in the theoretical calculations were not unique and the resulting cross sections are quite sensitive to their variation, these input quantities were kept constant when comparing the results using HO and WS wave functions.

The much larger quenching of isoscalar strengths than isovector strengths, found for all six nuclei, may be just a result of experimental realities. Pion fluxes at experimental facilities are in general much smaller than electron fluxes; thus states that are predominately isoscalar and excited preferentially by pion scattering are much more difficult to find than states that are predominately isovector and excited preferentially by electron scattering.

Some of the above points, particularly 3), are dependent on a correct assignment of isospin to the 13.9 and 15.5 MeV stretched states. These assignments will therefore need to be looked at more carefully in the future.

Conclusions

Following are the most important results of this study: 1) To explain adequately pion scattering data for the ^{60}Ni stretched states, Woods-Saxon wave functions need to be used rather than the commonly used harmonic oscillator wave functions. In particular, the isospin asymmetries implicit in using WS wave functions were necessary in explaining the non-equality of pure isospin cross sections. 2) The factor necessary to arrive at agreement between theory and data comes closer to unity when using WS wave functions than HO wave functions, but is still large for ^{60}Ni . 3) The ^{60}Ni nucleus continues the trend found in other nuclei of finding isoscalar strengths quenched more than isovector strengths and provides the heaviest example. 4) The ^{60}Ni nucleus is about the heaviest nucleus where stretched states can be found by pion scattering with a reasonable amount of beam time.

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APPENDIX A

THE PION-NUCLEON SYSTEM

There is overwhelming evidence for the hypothesis that the strong interaction is charge independent. In particular this includes nuclear forces and the pion-nucleon (π -N) interaction. In addition to the strong interaction there is a Coulomb interaction between charged pions and charged nucleons, but in general this is so small compared to the strong interaction that it is at most a small perturbation and often can be ignored at "intermediate energies."

The hypothesis of charge independence means that the proton and neutron can be treated as two states of the same particle, the nucleon, and likewise the pion can be treated as one particle with three states. To exploit this, an isotopic spin (isospin) formalism has been developed in analogy with ordinary spin. The formalism developed in quantum mechanics for angular momentum can then be carried over unchanged for the description of isospin. The outline here of this formalism is from Cence [Cen 69] and from Lock and Measday [Loc 70].

For the nucleon there are two possible states; therefore, it is reasonable to regard the nucleon as having an isospin of $\vec{t} = 1/2$ and two projections of $t_z = +1/2$ for the proton and $t_z = -1/2$ for the neutron. This can be written as $p = |1/2, 1/2\rangle$ and $n = |1/2, -1/2\rangle$. Similarly, for pions one can write $\pi^+ = |1, 1\rangle$, $\pi^0 = |1, 0\rangle$, and $\pi^- = |1, -1\rangle$.

The hypothesis of charge independence in the strong interaction implies that: 1) the total isospin ($\vec{T}$) in the π -N interaction is conserved and 2) the interaction is independent of the third component of isospin (T_z) which describes the charge.

In describing a π -N interaction the total isospin ($\vec{T}$) is the vector sum of the isospins of the two particles ($\vec{t}_1$ and $\vec{t}_2$). The rules for addition are the same as those for addition of angular momenta: $\vec{T} = \vec{t}_1 + \vec{t}_2$ and $|t_2 - t_1| \leq T \leq t_1 + t_2$. The addition of $\vec{t}_1 = 1/2$ and $\vec{t}_2 = 1$ can result in a total isospin of either $\vec{T} = 1/2$ or $\vec{T} = 3/2$. In general a given π -N state, will not be in just one of these isospin states but will be a linear combination of the two. The coefficients of the two states will be the Clebsch-Gordan coefficients for the addition of angular momentum by way of the Wigner-Eckart theorem. The π -N state functions ($|TT_z\rangle$) can be written in terms of the isospin state functions ($|tt_z\rangle$) of the two particles as:

$$|TT_z\rangle = \sum_{t_{1z} t_{2z}} \langle t_1 t_2 t_{1z} t_{2z} | TT_z \rangle |t_1 t_{1z}\rangle |t_2 t_{2z}\rangle .$$

Writing this out gives the π -N system in six different states:

$$\begin{aligned} |3/2 \ 3/2\rangle &= |1 \ 1\rangle |1/2 \ 1/2\rangle \\ |3/2 \ 1/2\rangle &= \sqrt{2/3} |1 \ 0\rangle |1/2 \ 1/2\rangle + \sqrt{1/3} |1 \ 1\rangle |1/2 \ -1/2\rangle \\ |3/2 \ -1/2\rangle &= \sqrt{1/3} |1 \ -1\rangle |1/2 \ 1/2\rangle + \sqrt{2/3} |1 \ 0\rangle |1/2 \ -1/2\rangle \\ |3/2 \ -3/2\rangle &= |1 \ -1\rangle |1/2 \ -1/2\rangle \\ |1/2 \ 1/2\rangle &= -\sqrt{1/3} |1 \ 0\rangle |1/2 \ 1/2\rangle + \sqrt{2/3} |1 \ 1\rangle |1/2 \ -1/2\rangle \\ |1/2 \ -1/2\rangle &= -\sqrt{2/3} |1 \ -1\rangle |1/2 \ 1/2\rangle + \sqrt{1/3} |1 \ 0\rangle |1/2 \ -1/2\rangle . \end{aligned}$$

These can be inverted to give:

$$|t_1 t_{1z}\rangle |t_2 t_{2z}\rangle = \sum_T \langle t_1 t_2 t_{1z} t_{2z} | T (t_{1z} + t_{2z}) \rangle |T (t_{1z} + t_{2z})\rangle .$$

Written out this is:

$$\begin{aligned} |\pi^+ p\rangle &= |1 \ 1\rangle |1/2 \ 1/2\rangle = |3/2 \ 3/2\rangle \\ |\pi^0 p\rangle &= |1 \ 0\rangle |1/2 \ 1/2\rangle = \sqrt{2/3} |3/2 \ 1/2\rangle + \sqrt{1/3} |1/2 \ 1/2\rangle \\ |\pi^+ n\rangle &= |1 \ 1\rangle |1/2 \ -1/2\rangle = \sqrt{1/3} |3/2 \ 1/2\rangle - \sqrt{2/3} |1/2 \ 1/2\rangle \\ |\pi^- p\rangle &= |1 \ -1\rangle |1/2 \ 1/2\rangle = \sqrt{1/3} |3/2 \ -1/2\rangle + \sqrt{2/3} |1/2 \ -1/2\rangle \\ |\pi^0 n\rangle &= |1 \ 0\rangle |1/2 \ -1/2\rangle = \sqrt{2/3} |3/2 \ -1/2\rangle - \sqrt{1/3} |1/2 \ -1/2\rangle \\ |\pi^- n\rangle &= |1 \ -1\rangle |1/2 \ -1/2\rangle = |3/2 \ -3/2\rangle \end{aligned}$$

There are 10 possible transitions between these states:

$$\begin{array}{ll}
 \pi^+ p \rightarrow \pi^+ p & \pi^- n \rightarrow \pi^- n \\
 \pi^- p \rightarrow \pi^- p & \pi^+ n \rightarrow \pi^+ n \\
 \pi^- p \rightarrow \pi^0 n & \pi^+ n \rightarrow \pi^0 p \\
 \pi^0 p \rightarrow \pi^0 p & \pi^0 n \rightarrow \pi^0 n \\
 \pi^0 p \rightarrow \pi^+ n & \pi^0 n \rightarrow \pi^- p .
 \end{array}$$

The amplitudes for these reactions will be equal to the matrix element of the operator connecting the initial and final states, $A = \langle (\pi N)_f | M | (\pi N)_i \rangle$. By the hypothesis of charge independence, T is conserved in a reaction and the matrix element is independent of T_z . Thus, all ten of the above reactions depend on only two amplitudes or reduced matrix elements, $A_1 = \langle 1/2 || M || 1/2 \rangle$ and $A_3 = \langle 3/2 || M || 3/2 \rangle$ where T_z has been suppressed and M is a scattering operator. Of the ten reactions, 4 are difficult to observe experimentally because the π^0 has too short a half-life to be used as a projectile. The amplitudes for the remaining 6 reactions are:

$$\begin{array}{ll}
 A_{+p} (\pi^+ p \rightarrow \pi^+ p) & \propto A_3 \\
 A_{-p} (\pi^- p \rightarrow \pi^- p) & \propto 1/3 A_3 + 2/3 A_1 \\
 A_{scx} (\pi^- p \rightarrow \pi^0 n) & \propto \sqrt{2}/3 A_3 - \sqrt{2}/3 A_1 \\
 A_{-n} (\pi^- n \rightarrow \pi^- n) & \propto A_3 \\
 A_{+n} (\pi^+ n \rightarrow \pi^+ n) & \propto 1/3 A_3 + 2/3 A_1 \\
 A_{scx} (\pi^+ n \rightarrow \pi^0 p) & \propto \sqrt{2}/3 A_3 - \sqrt{2}/3 A_1 .
 \end{array}$$

The important quantum numbers for the π -N system are: the isospin (T) which can be $1/2$ or $3/2$, the spin (s) which is $1/2$ for the nucleon and 0 for the pion, the relative orbital angular momentum (l) where the strongest states are $l = 0$ (S) and $l = 1$ (P), and the total angular momentum ($J = s_\pi + s_N + l_{rel}$) which can then be $1/2$ or $3/2$. The possible combinations in terms of $l_{2T,2J}$ are then S_{11} , S_{31} , P_{11} , P_{31} , P_{13} , P_{33} . The most important combination is the P_{33} resonance or the $\Delta_{3,3}$ -resonance. In this combination the reaction is dominated by the A_3 amplitude. The amplitudes at this resonance are then:

$$\begin{aligned}
A_{+p} &\propto A_3 \\
A_{-p} &\propto 1/3 A_3 \\
A_{scx} &\propto \sqrt{2}/3 A_3 \\
A_{-n} &\propto A_3 \\
A_{+n} &\propto 1/3 A_3 \\
A_{scx} &\propto \sqrt{2}/3 A_3 .
\end{aligned}$$

The cross sections for these reactions will be equal to the square of the amplitude. The ratio of cross sections for neutron and proton transitions is then 9:1:2 for $\sigma(\pi^+p) : \sigma(\pi^-p) : \sigma(\text{SCX})$ and $\sigma(\pi^-n) : \sigma(\pi^+n) : \sigma(\text{SCX})$.

Looking at pion-nucleus (π -A) reactions and assuming that the pion interacts with all nucleons equally, the π^+ to π^- ratio of cross sections should be

$$R = \frac{\sigma(\pi^+)}{\sigma(\pi^-)} = \left| \frac{A_{+p} + A_{+n}}{A_{-p} + A_{-n}} \right|^2 = \left| \frac{3Z + N}{Z + 3N} \right|^2 ,$$

where Z is the number of protons and N is the number of neutrons in the nucleus. In general this assumption is not correct, however. In that case, the ratio

$$R = \frac{\sigma(\pi^+)}{\sigma(\pi^-)} = \left| \frac{3A_p + A_n}{A_p + 3A_n} \right|^2 \quad (\text{A.1})$$

can be inverted to find A_n/A_p . The result,

$$\frac{A_n}{A_p} = \frac{3 \mp \sqrt{R}}{-1 \pm 3\sqrt{R}}, \quad (\text{A.2})$$

has two solutions.

The amplitudes for pure isoscalar and isovector transitions are:

$$\begin{aligned}
\text{isoscalar} - \pi^+(p+n) : A_{+0} = A_{+p} + A_{+n} &\propto 1 + 1/3 = 4/3 \\
&\pi^-(p+n) : A_{-0} = A_{-p} + A_{-n} &\propto 1/3 + 1 = 4/3 \\
\text{isovector} - \pi^+(p-n) : A_{+1} = A_{+p} - A_{+n} &\propto 1 - 1/3 = 2/3 \\
&\pi^-(p-n) : A_{-1} = A_{-p} - A_{-n} &\propto 1/3 - 1 = -2/3 .
\end{aligned}$$

The ratio of amplitudes is then $|A_0/A_1| = 2$ and the ratio of cross sections is 4.

Equation (A.1) can be rewritten in terms of isoscalar and isovector amplitudes as:

$$R = \frac{\sigma(\pi^+)}{\sigma(\pi^-)} = \left| \frac{2A_0 - A_1}{2A_0 + A_1} \right|^2, \quad (\text{A.3})$$

where $A_0 = A_n + A_p$ and $A_1 = A_n - A_p$. The charge asymmetry can also be written in terms of isoscalar and isovector amplitudes:

$$\frac{\sigma(\pi^-) - \sigma(\pi^+)}{\sigma(\pi^-) + \sigma(\pi^+)} = \frac{4A_0 A_1}{4A_0^2 + A_1^2}. \quad (\text{A.4})$$

APPENDIX B

SAMPLE Q FILES

Data Word Title File

```
; partial BLK001.TXT
; *
; *****
001, IFLAG0, , DUMMY
002, IDCRW , (6,17,*), COINC REG #1
; * BIT 0 S4 MUON
; * BIT 1 S5
; * BIT 2 S6
; * BIT 3 S7
; * BIT 4 S8
; * BIT 5 S9

. . .
023, LRX1 , (6,13,00), (L-R) CX1
024, LRX2 , (6,13,01), (L-R) CX2
025, LRY1 , (6,13,02), (L-R) CY1
026, LRY2 , (6,13,03), (L-R) CY2
027, LRX5 , (6,13,04), (L-R) CX5
028, LRY5 , (6,13,05), (L-R) CY5
029, LRX6 , (6,13,06), (L-R) CX6
030, LRY6 , (6,13,07), (L-R) CY6

. . .
071, LRX9 , (6,13,08), (L-R) CX9
072, LRY9 , (6,13,09), (L-R) CY9
073, LRX10 , (6,13,10), (L-R) CX10
074, LRY10 , (6,13,11), (L-R) CY10
; *
; *****

. . .
121, DRFTMX(1), CMBR 1 DRIFT TIME X PLANE
122, DRFTMX(2), CMBR 2 DRIFT TIME X PLANE
123, DRFTMX(3), CMBR 3 DRIFT TIME X PLANE
124, DRFTMX(4), CMBR 4 DRIFT TIME X PLANE
125, DRFTMX(5), CMBR 5 DRIFT TIME X PLANE
126, DRFTMX(6), CMBR 6 DRIFT TIME X PLANE
127, DRFTMX(7), CMBR 7 DRIFT TIME X PLANE
128, DRFTMX(8), CMBR 8 DRIFT TIME X PLANE
129, DRFTMX(9), CMBR 9 DRIFT TIME X PLANE
```

```

130, DRFTMX(10), CMBR 10 DRIFT TIME X PLANE
. . .
181, DRFTMY(1), CMBR 1 DRIFT TIME Y PLANE
182, DRFTMY(2), CMBR 2 DRIFT TIME Y PLANE
183, DRFTMY(3), CMBR 3 DRIFT TIME Y PLANE
184, DRFTMY(4), CMBR 4 DRIFT TIME Y PLANE
185, DRFTMY(5), CMBR 5 DRIFT TIME Y PLANE
186, DRFTMY(6), CMBR 6 DRIFT TIME Y PLANE
187, DRFTMY(7), CMBR 7 DRIFT TIME Y PLANE
188, DRFTMY(8), CMBR 8 DRIFT TIME Y PLANE
189, DRFTMY(9), CMBR 9 DRIFT TIME Y PLANE
190, DRFTMY(10), CMBR 10 DRIFT TIME Y PLANE
. . .
221, XFRON, X FRONT
222, THTFR, THETA FRONT
223, YFRON, Y FRONT
224, PHIFR, PHI FRONT
225, XREAR, X REAR
226, THTRE, THETA REAR
227, YREAR, Y REAR
228, PHIRE, PHI REAR
. . .
233, XTGT, X TARGET(UNROT)
234, THTTGT, THETA TARGET
235, YTGT, Y TARGET(UNROT)
236, PHITGT, PHI TARGET
237, THCHK, THETA CHECK
238, PHCHK, PHI CHECK
239, THTA1, THETA AT A1 TGT
240, DELTPA, DELTA OF PART A TO BEAM
241, DELTPC, DELTA OF PART C TO SPECT
242, QOPT, QEXACT OPTIMIZED
243, CRUDEL, CRUDE DELTA
244, XROTG, X AT REAL ROTATED TGT
245, YROTG, Y AT REAL ROTATED TGT
246, QEXACT, MISSING MASS FROM CALKIN
247, THTSCT, SCATTERING ANGLE FROM CALKIN
248, PTRANS, MOMENTUM XFER FROM CALKIN
249, XDEP, DEP VAR FROM XXDEP
250, THTBM
; *****
; *
; THESE QUANTITIES HAVE A MULT OF 1.0, BUT OFFSET OF 0
. . .
253, DES23, GEOMETRIC MEAN S2 AND S3
. . .
260, S23TOF, S23 TIM OF FLIGHT
; *
; *****
. . .
272, DSQXF, DIF12X**2 + DIF34X**2

```

```

273, DSQYF, DIF12Y**2 + DIF34Y**2
274, DSQXR, DIF56X**2 + DIF910X**2
275, DSQYR, DIF56Y**2 + DIF910Y**2
. . .

```

Scaler Title File

```

; partial SLRTTL.TXT
;
1, 701-00 (R*BG) EVENT
2, 701-01 (R*BG) EVENT*NOT BUSY
3, 701-02 (R*BG) FAST CLEAR
4, 701-03 (R*BG) FRONT
5, 701-04 (R*BG) S1*(S2*S3)
6, 701-05 (R*BG) S2*S3
. . .
60, 709-11 (R*BG) (10MHz pulser*BG)
61, 711-00 (RUN) IC1
62, 711-01 (RUN) BOT
63, 711-02 (RUN) 1ACM02
64, 711-03 (RUN) 2ACM03
65, 711-04 (RUN) BG
. . .

```

PRM Integer Array Title File

```

; partial ITTL.TXT
;
001, RUNNUM, RUN NUMBER
002, QMODE, Q MODE (0=REPLAY, -1=DATA ACQ.)
003, RGATEC, RUN GATE CAMAC CRATE
004, RGATEN, RUN GATE CAMAC STATION
005, RGATED, RUN GATE CAMAC DATA WORD
. . .
084, ITSTAB TEST NO. FOR STABILIZATION EVENT.
085, ITXDEP 1ST TEST NUMBER FOR XXDEP GATES
. . .
; *****
;
101, IFLAG(1) - =-12345 FOR IMMEDIATE RETURN
102, IFLAG(2) - .NE.-12345 =>RETRIEVE EVENT DATA
103, IFLAG(3) - .NE.-12345 =>SHUFFLE DATA
104, IFLAG(4) - =-12345 TO LIST RAW DATA
105, IFLAG(5) - .NE.-12345 =>INITIALIZE RAW QUANTITIES
106, IFLAG(6) - .NE.-12345 =>CALCULATE TIMES AND PULSEHEIGHTS
107, IFLAG(7) - .NE.-12345 =>DO FIRST MULTIS CALC.
108, IFLAG(8) - .NE.-12345 =>INTEGERIZE TOFS AND PH'S
109, IFLAG(9) - .NE.-12345 =>DO FIRST TEST BLOCK
110, IFLAG(10) - TEST # WHICH MUST BE .T. TO DO REST OF CALC

```

```

111, IFLAG(11) - .NE.-12345 =>CALCULATE DRIFT REFERENCE TIMES
112, IFLAG(12) - .NE.-12345 =>CALL DCHMBR
113, IFLAG(13) - .NE.-12345 =>CALC DRIFT CORR. POSITIONS
114, IFLAG(14) - .NE.-12345 =>CALCULATE FRNT AND RR POSITIONS
115, IFLAG(15) - .NE.-12345 =>DO SECOND MULTIS LOOP
116, IFLAG(16) - .NE.-12345 =>CALCULATE ROTATED TGT POSITIONS
117, IFLAG(17) - .NE.-12345 =>DO KINEMATICS
118, IFLAG(18) - .NE.-12345 =>DO THIRD MULTIS CALCS
119, IFLAG(19) - .NE.-12345 =>CALC MOUT AND FLANGE STUFF
120, IFLAG(20) - .NE.-12345 =>INTEGERIZE EVERYTHING
121, IFLAG(21) - .NE.-12345 =>DO SECOND BLOCK OF TESTS
122, IFLAG(22) - .NE.-12345 =>GET XXDEP VALUE
123, IFLAG(23) - .NE.-12345 =>WRITE OUT RAYS
124, IFLAG(24) - .NE.-12345 =>DO HISTOGRAMS
125, IFLAG(25) - .NE.-12345 =>DO DOTPLOTS
126, IFLAG(26) - ==-12345 ==> LIST COMPUTED R*4 DATA
. . .
135, ICNTRL(5) DCHMBR * CHMBR TYP (0=OFF,1=EPICS,2=HRS,3=L/R)
136, ICNTRL(6) DCHMBR * CHMBR TYP (0=OFF,1=EPICS,2=HRS,3=L/R)
137, ICNTRL(7) DCHMBR * CHMBR TYP (0=OFF,1=EPICS,2=HRS,3=L/R)
138, ICNTRL(8) DCHMBR * CHMBR TYP (0=OFF,1=EPICS,2=HRS,3=L/R)
139, ICNTRL(9) DCHMBR * CHMBR TYP (0=OFF,1=EPICS,2=HRS,3=L/R)
140, ICNTRL(10) DCHMBR * CHMBR TYP (0=OFF,1=EPICS,2=HRS,3=L/R)
. . .

```

PRM Real Array Title File

```

; partial RTTL.TXT
;

```

```

. . .
R26, THR * SPECTROMETER ANGLE (RADIANs)
R27, PSP * SPECTROMETER MOMENTUM (MEV/C)
R28, TA * BEAM ENERGY (MEV)
R29, EX * EXCITATION ENERGY (MEV)
. . .
R36, AMA * INCOMING MASS (AMU)
R37, AMB * TARGET MASS (AMU)
R38, AMC * OUTGOING MASS (AMU)
R39, AMD * RESIDUAL MASS (AMU)
R40, ZA * INCOMING CHARGE
R41, ZC * OUTGOING CHARGE
R42, QGND * GROUND STATE Q-VALUE (MEV)
R43, DQ * ERROR IN QGND (MEV)
R44, THO * ABSOLUTE VALUE OF THR (RADIANs)
R45, TGTD * TARGET ANGLE (DEG)
R46, CTHDIF * COS(THR-TGTD)
R47, STHDIF * SIN(THR-TGTD)
R48, P1 * BEAM MOMENTUM (MEV/C)
R49, P3 * OUTGOING PARTICLE MOMENTUM (MEV/C)
R50, CHAN * CHAN B FIELD/MOM. (MEV/C/GAUSS)

```

```

R51, SPECT * SPECT B FIELD/MOM. (MEV/C/GAUSS)
R52, AMA*MEV * INCOMING MASS (MEV)
R54, AMB*MEV * TARGET MASS (MEV)
R56, AMC*MEV * OUTGOING MASS (MEV)
R58, AMD*MEV * RECOIL MASS (MEV)
R60, DM*MEV * (AMB*MEV - AMD*MEV) (MEV)
R62, AMDI * INVERSE OF AMD
R63, PLA * ROTATED MOM LOSS OF INCIDENT PART (MEV/C)
R64, PLC * ROTATED MOM LOSS OF EXIT PART (MEV/C)

. . .
R76, PROC7 * FRONT TO REAR TOF (NS AT V=C)
R77, PROC7 * UNITS CONV (NS TO DRIFT)
R78, PROC7 * DISPERSION EPICS CHANNEL
R81, QSTD PROC7 * VALUE OF Q STABILIZED TO
R82, OFFSCL PROC7 * SCALE FACTOR FOR STABIL (TYP. .01)
R83, OFFSET PROC7 * PRESENT VALUE OF OFFSET.
. . .

```

Test File

```

;[mp10rep.exp984.misc]EXP984REP.TST
;
/TE:150/BL:3/IG:1/IB:1
;
BLOCK,1
;
;;;; MUON REJ TESTS
1,BI,2,- ;MUON REJ
;
;;;; LEFT RIGHT TESTS
2,GAT,23,___,1200 ;C1XL-R
3,GAT,24,___,1200 ;C2XL-R
4,GAT,27,550,1200 ;C5XL-R
5,GAT,29,483,1200 ;C6XL-R
6,GAT,71,418,1200 ;C9XL-R
7,GAT,73,512,1200 ;C10XL-R
8,GAT,25,___,1200 ;C1YL-R
9,GAT,26,___,1200 ;C2YL-R
10,GAT,28,425,1200 ;C5YL-R
11,GAT,30,515,1200 ;C6YL-R
12,GAT,72,502,1200 ;C9YL-R
13,GAT,74,433,1200 ;C10YL-R
;
;;;; PID
15,IBOX,1 ;BOX 1 ==> PID
;
16,IOR,1,-1 ;LOOP 1 COUNTER
;
BLOCK,2
;

```



```

;;;;; CHAMBER DRIFT TIME TESTS
17,GAT,121,-2,42      ;DRFTMX1
18,GAT,122,-2,42      ;DRFTMX2
21,GAT,125,-2,42      ;DRFTMX5
22,GAT,126,-2,42      ;DRFTMX6
23,GAT,129,-2,42      ;DRFTMX9
24,GAT,130,-2,42      ;DRFTMX10
25,GAT,181,-2,42      ;DRFTMY1
26,GAT,182,-2,42      ;DRFTMY2
29,GAT,185,-2,42      ;DRFTMY5
30,GAT,186,-2,42      ;DRFTMY6
31,GAT,189,-2,42      ;DRFTMY9
32,GAT,190,-2,42      ;DRFTMY10
;
;;;;; CHAMBER FIRED TESTS
40,AND,15,17,18        ;PID * FRNT X CHAMBERS FIRED
41,AND,15,21,22,23,24  ;PID * REAR X CHAMBERS FIRED
42,AND,15,25,26        ;PID * FRNT Y CHAMBERS FIRED
43,AND,15,29,30,31,32  ;PID * REAR Y CHAMBERS FIRED
44,AND,40,42           ;PID * FRNT FIRED
45,AND,41,43           ;PID * REAR FIRED
46,AND,44,45           ;PID * ALL FIRED ==> CHMBRS FIRED
;
;;;;; CHAMBER DRIFT DIFFERENCE TESTS
50,GAT,222,-1500,1500  ;THFRNT reasonable
51,GAT,224,-800,800    ;PHFRNT reasonable
52,GAT,272,-9,16       ;FRNT DRIFT DIFF PLANE X
53,GAT,273,-9,16       ;FRNT DRIFT DIFF PLANE Y
54,GAT,274,-9,16       ;REAR DRIFT DIFF PLANE X
55,GAT,275,-9,16       ;REAR DRIFT DIFF PLANE Y
56,AND,15,50,51        ;PID * FRNT ANGLES reasonable
57,AND,15,52,53        ;PID * FRNT DRIFT DIFF OK (not used)
58,AND,15,54,55        ;PID * REAR DRIFT DIFF OK
59,AND,56,58           ;ALL OK ==> DRFDIF OK
;
;;;;; TARGET TESTS AND CHECKS
60,GAT,233,-1040,980   ;XTGT
61,GAT,234,-900,850    ;THTGT
62,GAT,235,-420,420    ;YTGT
63,GAT,236,-520,420    ;PHITGT
;
65,GAT,237,-170,130    ;THTCHK
66,GAT,238,-170,130    ;PHICHK
;
68,AND,60,61,62,63     ;TGT TST
69,AND,65,66           ;ANG CHK
;
;;;;; EFFICIENCY TESTS
71,AND,41,42,43,18     ;ALL BUT X1 (#17)
72,AND,41,42,43,17     ;ALL BUT X2 (#18)
73,AND,40,42,43,22,23,24 ;ALL BUT X5 (#21)

```

```

74,AND,40,42,43,21,23,24 ;ALL BUT X6 (#22)
75,AND,40,42,43,21,22,24 ;ALL BUT X9 (#23)
76,AND,40,42,43,21,22,23 ;ALL BUT X10 (#24)
81,AND,40,41,43,26 ;ALL BUT Y1 (#25)
82,AND,40,41,43,25 ;ALL BUT Y2 (#26)
83,AND,40,41,42,30,31,32 ;ALL BUT Y5 (#29)
84,AND,40,41,42,29,31,32 ;ALL BUT Y6 (#30)
85,AND,40,41,42,29,30,32 ;ALL BUT Y9 (#31)
86,AND,40,41,42,29,30,31 ;ALL BUT Y10 (#32)
;
;;;;; GOOD EVENT TESTS
90,AND,46,59 ;CHMBRS FIRED * DRFDIF OK ==> GD CHMBRS
91,AND,68,90 ;TGT TST * GD CHMBRS
92,AND,69,90 ;ANG CHK * GD CHMBRS
93,AND,68,69,90 ;TGT TST * ANG CHK * GD CHMBRS ==> GD EVNT
;
100,AND,93,-1 ;GD EVNT * PION ==> PION GOOD EVENT
101,AND,93,1 ;GD EVNT * MUON ==> MUON GOOD EVENT
;
;;;;; GATES FOR STRIP TARGETS
111,GAT,233,-1040,-500 ;GATE 1 (CH2 GATE)
112,GAT,233,-470,980 ;GATE 2 (60Ni GATE)
113,GAT,233,-1040,980 ;GATE 3 (FULL GATE)
;
;;;;; ANGLE CUTS FOR RESOLUTION
121,GAT,222,-1200,400 ;THFRNT
122,GAT,224,-500,500 ;PHFRNT
129,AND,121,122 ;ANGCUT
;
;;;;; STRIP TARGET TESTS
131,AND,93,111 ;GD EVNT * GATE1 (CH2)
132,AND,93,112 ;GD EVNT * GATE2 (60Ni)
133,AND,93,113 ;GD EVNT * GATE3 (FULL)
134,AND,93,111,-1 ;GD EVNT * GATE1 (CH2) * PION
135,AND,93,112,-1 ;GD EVNT * GATE2 (60Ni) * PION
136,AND,93,113,-1 ;GD EVNT * GATE3 (FULL) * PION
137,AND,93,111,-1,129 ;GD EVNT * GATE1 (CH2) * PION * ANGCUT
138,AND,93,112,-1,129 ;GD EVNT * GATE2 (60Ni) * PION * ANGCUT
139,AND,93,113,-1,129 ;GD EVNT * GATE3 (FULL) * PION * ANGCUT
;
;;;;; TEST FOR OPTIMIZATION ROUTINE
140,GAT,246,---,--- ;gate on stabilization peak
145,AND,140,--- ;(stab. peak) * (strip target test)
;
150,IOR,1,-1 ;LOOP 2 COUNTER

```

Histogram Setup File

```

;[mp10rep.exp984.misc]EXP984REP.HST
;

```

```

/VE:NONE
/DE:ALL
/BL:1/OV:-1/TE:0/DF
;
;;;;;;;;;; LR histograms ;;;;;;;;;;
/X:0:1000/BI:5/TE:44/DF
LRX1/IN:23
LRX2/IN:24
LRY1/IN:25
LRY2/IN:26
/X:0:1000/BI:5/TE:45/DF
LRX5/IN:27
LRX6/IN:29
LRX9/IN:71
LRX10/IN:73
LRY5/IN:28
LRY6/IN:30
LRY9/IN:72
LRY10/IN:74
;
;;;;;;;;;; TGT histograms ;;;;;;;;;;
/TE:90/DF
XTGT/XP:233:-1500:1500:10
THTTGT/XP:234:-1500:1500:10
YTGT/XP:235:-800:800:10
PHITGT/XP:236:-800:800:10
;
;;;;;;;;;; FRNT and CHK histograms ;;;;;;;;;;
/TE:91/DF
THFRNT/XP:222:-1500:1500:10
PHFRNT/XP:224:-800:800:10
THTCHK/XP:237:-300:300:10
PHICLK/XP:238:-300:300:10
;
;;;;;;;;;; strip target histograms ;;;;;;;;;;
;MMC/TE:131/XP:246:-500:2000:2
;MMN/TE:132/XP:246:-500:2000:2
;MMF/TE:133/XP:246:-500:2000:2
MMCP/TE:134/XP:246:-500:2000:2
MMNP/TE:135/XP:246:-500:2000:2
MMFP/TE:136/XP:246:-500:2000:2
MMCPA/TE:137/XP:246:-500:2000:2
MMNPA/TE:138/XP:246:-500:2000:2
MMFPA/TE:139/XP:246:-500:2000:2
;
;;;;;;;;;; include stabilization data word ;;;;;;;;;;
;MMCS/TE:131/XP:242:-500:2000:2
;MMNS/TE:132/XP:242:-500:2000:2
;MMFS/TE:133/XP:242:-500:2000:2
MMCP/TE:134/XP:242:-500:2000:2
MMNPS/TE:135/XP:242:-500:2000:2

```

```
MMFPS/TE:136/XP:242:-500:2000:2
MMCPAS/TE:137/XP:242:-500:2000:1
MMNPAS/TE:138/XP:242:-500:2000:1
MMFPAS/TE:139/XP:242:-500:2000:1
```

Building an EPICS Analyzer

```
$ !files needed to build an analyzer for EPICS
$ !
$ !   to define Q data base and buffers - AQDB.MAC
$ !   to compile AQDB.MAC - AQDBCMP.COM
$ !   to initialize MP10 analyzer tasks - AINIT1.FLX
$ !   to handle scaler event data - PROC8.FLX
$ !   to analyze events - PROC6
$ !       LR calcs. using odd-even signals - PROC685JD.FLX
$ !       LR calcs. using offset MWPC pairs - PROC6BLC.FLX
$ !       subroutines to calc. positions in drift chambers -
$ !       DCHMBRNEW.FLX , DRF85JDZ.FLX , DRF85.FLX ,
$ !       TANGLE.FLX , XDRIFT.FLX , XXDEP.FLX
$ !       subroutine to calculate kinematics - CALKIN.FLX
$ !   other subroutines -
$ !       MULTIS.ALX , FITPOL.FLX , SHUFFL.FLX
$ !   include files -
$ !       DRGCMN.INC , EV6EPX.INC , EV8.INC , PRMEPXL.R.INC
$ !
$ !   to tskbld analyzer for events where all MWPC are good
$ LINK/EXE=QANL984gd.EXE/MAP=QANL984gd.MAP -
  USER$Q:QLIB/INCLUDE=(AQMAIN),-
  AQDB,AINIT1,PROC8,-
  PROC685JD,DCHMBRNEW,DRF85JDZ,-
  CALKIN,XXDEP,-
  MULTIS,FITPOL,SHUFFL,-
  USER$Q:QLIB/LIBRARY,-
  USER$Q:REGION/OPTIONS
$
$ !   to tskbld analyzer for events where some MWPC are bad
$ LINK/EXE=QANL984bd.EXE/MAP=QANL984bd.MAP -
  USER$Q:QLIB/INCLUDE=(AQMAIN),-
  AQDB,AINIT1,PROC8,-
  PROC6BLC,DCHMBRNEW,DRF85,-
  TANGLE,XDRIFT,CALKIN,XXDEP,-
  MULTIS,FITPOL,SHUFFL,-
  USER$Q:QLIB/LIBRARY,-
  USER$Q:REGION/OPTIONS
```

APPENDIX C

INPUT FILES FOR ALLWRLD AND MSUDWPI

A Description of Input for ALLWRLD

The computer code ALLWRLD is designed to predict electron, proton, and pion scattering cross sections based on input of shell model configurations.

The program reads shell model configuration and Z coefficients and calculates the radial transition density (which may also be read in), and its Fourier transform. The ground state density is generated (or read in) for use in calculating the density dependence of the interaction or consistent elastic scattering optical potentials. The electron scattering form factors are calculated along with the electromagnetic decay rates.

When proton or pion forces are read in, the program will also evaluate the scattering cross section in Born approximation, and/or calculate and output the form factors required by the distorted waves codes. The use of either the free *t*-matrix or a density-dependent *g*-matrix is allowed for both projectiles.

These are the important line by line descriptions as used in this thesis.

Files used:

- 2 - program control input
- 3 - printout
- 7 - transition density input
- 9 - pion form factor output

File 2 - sample input for ALLWRLD

```
0.50854 28 60 120
1 -1 2 1 0 0 0
0
162.00000
-1.00000 5
60NI 0+ T=2 (G.S.)
0 4
60NI 8- T=2
16 4
15 10 8 0.021212 0.233330 0
-1
2 4.07216 2.54000
1., 1., 1., 1., 1., 1., 1., 1., 1., 1.
0., 0., 0., 0., 0., 0.
```

0., 0., 0., 0., 0., 0., 0.
 0.24087 0.22915 0.13154 0.11421
 0.,0

File 2 - input file description

1 - nuclear interaction

ALPHA = $\alpha = 1/b$ of harmonic oscillator wave functions
 = 0 to stop program
 IZ = Z of target
 IA = A of target
 NR = number of integration points in the distorted wave code
 = RMAX $\times$ 10 (usually 120 or 160)

2 - control parameters

KSCAT = 1 for inelastic scattering
 IDPI = product of initial and final parity = $\pi_i \times \pi_f$
 KTRL = 0 for Born only
 = 1 for fold only
 = 2 for Born then fold
 KFNFF = 0 for no change in electron scattering
 = 1 to add finite proton size
 = 2 for 4-pole fit for proton size
 KCOM = 0 for no changes in electron scattering
 = 1 to add center-of-mass correction
 KTM = 0 for transition magnitude to exclude $4\pi/Z^2$
 = 1 for transition magnitude to include $4\pi/Z^2$
 ICOUL = 0 for no Coulomb excitation
 = 1 to include Coulomb excitation

3 - proton energy line

EP = 0 to skip proton calculations and skip to line 5

5 - pion energy

EPI = pion beam energy

6 - pion force control

PICHG = pion charge
 IPOT = 1 for FSU kinematics, internal phase shift
 = 3 for FSU kinematics, read the force
 = 4 for MSU kinematics, read the force
 = 5 for MSU kinematics, read the force
 = 6 for MSU kinematics, read the force

Notes:

1,2,4,6 use Laplacian model
 5 does central in coordinate space
 4 uses $\rho(q) = 1/7$ for density dependence
 6 uses $\rho(q)$ for density dependence
 negative drops ∇^2 terms from MSU kinematics

7 - initial title card

ITITLE (initial)

8 - initial state

II = 2 × ground state spin

ITI = 2 × ground state isospin

9 - final title card

ITITLE (final)

10 - final state

IF = 2 × final state spin

ITF = 2 × final state isospin

11 - shell model description of transition

ID2 = identifier for particle in configuration

ID1 = identifier for hole in configuration

J = total J of coupled configuration

Z0 = first Z coefficient (isoscalar/proton amplitude)

Z1 = second Z coefficient (isovector/neutron amplitude)

KTC = 0 for isospin (T=0, T=1)

= 1 for nucleon (p,n)

Note: line 11 is read until ID2 = -1

Note: if no configurations given, density read from File 7

Note: the first 15 configurations for ID1 and ID2 are

$1s_{1/2}$, $1p_{1/2}$, $1p_{3/2}$, $2s_{1/2}$, $1d_{3/2}$, $1d_{5/2}$

$2p_{1/2}$, $2p_{3/2}$, $1f_{5/2}$, $1f_{7/2}$

$3s_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, $1g_{7/2}$, $1g_{9/2}$

12 - ground state density (see line 7 of MSUDWPI input)

NDEN = 0 for Woods-Saxon with $AR = 1.127 \times A^{1/2}$

and $TR = 2.373$

= 1 for harmonic Gaussian [eqn. (4.47)]

= 2 for two parameter Fermi or Woods-Saxon

[eqn. (4.48)]

= 3 for square well of radius AR

= 4 for reading density from File 6

AR = radius parameter

TR = second parameter (90-10 surface thickness for 2-p Fermi
or the harmonic parameter for Gaussian)

13 - density control parameters

DENKTRL (1-11) = parameters multiply the given momentum
space density whenever it is used to calculate an observable

⇒ skip to line 1 if IPOT=1,3

14 - s-wave pion force

Re(b0), Im(b0), Re(b1), Im(b1), Re(B0), Im(B0)

Note: B0 has meaning only if IPOT= 4,5,6

Note: not used for calculations in this thesis

15 - p-wave pion force

$Re(c0)$, $Im(c0)$, $Re(c1)$, $Im(c1)$, $Re(C0)$, $Im(C0)$, λ

Note: $C0$ and λ only have meaning if $IPO T=4,5,6$

Note: not used for calculations in this thesis

16 - spin-orbit pion force

$Re(s0)$, $Im(s0)$, $Re(s1)$, $Im(s1)$

1 $\Rightarrow$ second pass

File 7 - transition density

NR+1 values (in free format) giving the radial transition densities from 0 to R_{MAX} inclusive; these values are calculated from equation (5.3); the program was adjusted to read only RSJLL for abnormal parity transitions (RSJLU, RLJLL, RLJLU are zero)

A Description of Input for MSUDWPI

MSUDWPI is a program that calculates pion elastic and inelastic scattering in the distorted wave Born approximation. It has a large degree of flexibility in the choice of form factors and optical potentials used for the calculation, including the option to read in the form factor produced by another program such as ALLWRLD.

These are the important line by line descriptions as used in this thesis. As explained in Chapter IV of the text, this program had to be slightly modified to run the 8^- stretched states in ^{60}Ni .

Files used:

5 - program control input

6 - printout

9 - pion form factor input from ALLWRLD

File 5 - sample input for MSUDWPI

```
-2
-0.0682  0.0304 -0.0932  0.0035  0.4957  0.5979  0.2763  0.2980
 1.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
60NI 8-
162.000  0.100  120  16 -1  1.000  1  0  0
60.000 28.000  0.000  8.000 13.908  1.000  1.000  1.000
 2  4.07  2.54  0.00  4  4.15
-1  0.00  0.00  0  2
-0.0688  0.0301 -0.0995  0.0024  0.5701  0.5068  0.3192  0.2525
 1.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
2.0 , 178.0
0
0
9
```


File 5 - input file description

- 1 - chooses form of optical potential
  - NPOT = 0 - use previous value and skip next 2 card
  - = 1 - standard (all terms operational)
  - = 2 - standard, but read outgoing optical potential
  - = 3 - standard, but read force for inelastic calc.
  - = 4 - standard, but read both (same as 2 plus 3)
  - = 9 - stop

Note: if NPOT is positive, $\text{Im}(b_0)$, b_1 , c_0 , c_1 come from phase shifts (and multiply by Q)

Note: if NPOT is negative, these values are read in and the Pauli factor is not used
- 2 - first order optical potential (see description in Chapter IV)
  - b_0 - isoscalar s-wave term (complex); actually $\bar{b}_0$
  - b_1 - isovector s-wave term (complex)
  - c_0 - isoscalar p-wave term (complex)
  - c_1 - isovector p-wave term (complex)
- 3 - second order optical potential (see description in Chapter IV)
  - λ - LLEE effect parameter (real)
  - B_0 - s-wave absorption (complex)
  - C_0 - p-wave absorption outside LLEE effect (complex)
  - C_2 - p-wave absorption inside LLEE effect (complex)
- 4 - title card
  - alphanumeric title for output of this run
- 5 - calculation parameters
  - E = pion lab kinetic energy
  - DR = step size (0.1 or 0.125 normally)
  - LDX = number of steps
  - $LMAXI$ = number of partial waves (between 6 and 20)
  - $JCHAR$ = pion charge
  - Q = Pauli factor (between 0 and 1)
  - $IPRNT$ = print options (1 for T-matrix; 2 for TDR)
- 6 - nucleus and state description
  - AN = A of target
  - ZZ = Z of target
  - AI = ground state spin of target (maximum of 5/2)
  - AIP = spin of excited state (maximum ΔL is 14)
  - EXE = excitation energy
  - $BETAL$ = β_L for collective model
  - TEE = ΔT for transition (1 for isovector)
  - ESS = ΔS for transition (1 for spin flip)
- 7 - elastic scattering densities
  - NDEN = 0 - square well (only uses A)
  - = 1 - harmonic Gaussian [eqn. (4.47)]

= 2 - three parameter Fermi or Woods-Saxon
[eqn. (4.48)]

A = radius parameter [w (Gaussian); c (3-p Fermi)]

T = second parameter [α (Gaussian); a (3-p Fermi)]

TZ = third parameter [0 (Gaussian); w (3-p Fermi)]

NCOUL = 1 - uniform model with V_C^2 term included

= 2 - uniform model with only linear term

= 3 - exact model (uses AZ and T) with V_C^2 term

= 4 - exact model with only linear term

AZ = first parameter (radius for Coulomb potential)

this is the value listed as c or a in [deJ 74]

$$(AZ)^2 = A^2 + (0.8)^2$$

8 - inelastic transition options

NTD = -1 - read in ALLWRD form factor (line 10b)

CTR

ATR

ICE

IDEF = 2 - full force used for inelastic calculation

11A - outgoing optical potential

same as line 2

11B - second order outgoing potential

same as line 3

xx - an addition for plotting data

THETMN

THETMX

12 - elastic data for printer plot

NED = 0 (number of elastic data points)

14 - inelastic data for printer plot

NID = 0 (number of inelastic data points)

1 $\Rightarrow$ second pass

APPENDIX D

ELECTRON SCATTERING PAPER

September 10, 1987

NPL-1031

An analysis of electron scattering to magnetic stretched states using Woods-Saxon wave functions

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Form factors for magnetic electron scattering to stretched states are computed using radial wave functions from realistic nuclear potentials, including the unbound nature of final states above particle decay thresholds. The calculated form factors are compared to data for 4^- states in ^{12}C , ^{14}C , and ^{16}O , 6^- states in ^{24}Mg , ^{26}Mg , and ^{28}Si , 8^- states in ^{48}Ca , ^{54}Fe , ^{58}Ni , and ^{60}Ni , the 10^- state in ^{90}Zr , and the 14^- state in ^{208}Pb . We assess the fraction of the single-particle sum rule strengths using these realistic nuclear potentials in place of the standard theoretical results using harmonic oscillator wave functions. Appreciably greater fractions are obtained for low mass nuclei in the present work, totalling 80% of the sum strength for ^{12}C and 70% for ^{26}Mg . Much less damping of the magnetic strength is thus observed than was necessary when oscillator wave functions are used for comparison. Effects of meson exchange currents are also included in this analysis.

PACS 25.30.Dh, 21.10.Jx, 23.20.Js
Prepared for Physical Review C

I. INTRODUCTION

The systematic failure of theory to account for the observed quenching of the transition strength for magnetic nuclear excitations is an important unresolved problem in nuclear physics.¹ The loss of magnetic strength at low multipoles such as observed for 1^+ states in Gamow-Teller (p,n) reactions² and in electron scattering^{3,4} has been partially accounted for by including core polarization, ground state correlations, and mixing with the delta-particle nucleon-hole states in the nucleus. Recently,⁵⁻⁷ discrepancies between theory and experiment for magnetic excitations were related to the role of the occupation number distributions in the nuclear ground states. For high-spin magnetic multipole excitations, it was expected that fewer difficulties stand in the way of more completely understanding the magnetic strength to stretched excitations because mixing with the subnucleon degrees of freedom is negligible and nuclear configuration mixing may be minimal due to the uniqueness of the maximally stretched particle-hole configuration.⁸ This has not been the case. Systematic shell model studies, which include configuration mixing in the ground and excited states, predict too much cross section by at least a factor of two for many of the observed transitions.⁹ Including meson exchange currents in the analysis of the electron scattering data to stretched states results in yet further disagreement between experiment and theory.¹⁰

The principal means of investigating high-spin particle-hole states has been by inelastic electron scattering. The fraction of the single-particle strength exhausted by the data has been evaluated most commonly by comparing F_{Texp}^2 [the experimental transverse (magnetic) form factor] to F_{Tthr}^2 [the form factor calculated in the extreme (isovector) single-particle shell model]. For simplicity, and to provide systematics over a wide range of nuclei, strengths have been previously extracted using harmonic oscillator radial wave functions.^{11,12} For example, see the $(f_{7/2}d_{5/2}^{-1})$ 6^- states in the sd shell¹² and the $(g_{9/2}f_{7/2}^{-1})$ 8^- states in the fp shell.¹¹

The states of interest lie at fairly high excitation energies, sometimes above the proton and/or neutron separation energies, and thus the

radial wave functions for the final states may extend greatly beyond those computed for a harmonic oscillator (HO). Even the implicit isospin symmetry assumed in HO calculations is in doubt, since neutron and proton wave functions may differ substantially due to their differing decay probabilities. These effects are accounted for in bound state Woods-Saxon (WS) well calculations, extended in this work to states unbound to particle decay. Consequently, overlap integrals used in computing the form factors will differ for protons and neutrons, which could significantly affect the total strength extracted from the data.

In this report we calculate the neutron and proton transition densities to 52 stretched states in twelve different nuclei using radial wave functions computed for nucleons in a WS potential by the computer code DWUCK4.¹³ For cases where the nucleon is unbound, the excited nucleon is treated as a resonance by a method well-established for single-nucleon stripping reactions.¹⁴ Comparisons are made to results from HO potentials. We also include the effects meson exchange currents (MEC).¹⁵

II. (e, e') SCATTERING THEORY

The inelastic electron scattering differential cross section for magnetic transitions is given by¹

$$\frac{d\sigma}{d\Omega} = \frac{Z^2 \sigma_M}{1 + 2(E_0/M_T) \sin^2(\theta/2)} \left[\frac{1}{2} \frac{q_\mu^2}{q^2} + \tan^2(\theta/2) \right] |F_T(q)|^2, \quad (1)$$

where Z denotes the nuclear charge, σ_M the Mott cross section, E_0 the incident electron energy, M_T the target mass, θ the scattering angle, q_μ the four-momentum transferred from the electron to the nucleus, and F_T the transverse or magnetic form factor. The three-momentum transfer q is defined from $q_\mu^2 = q^2 - \omega^2$, where ω is the difference between the incoming and outgoing electron energy. The relation between the cross section and the form factor in equation (1) has also been defined without the Z^2 ,¹⁶ and with Z^2 replaced by 4π .^{12,17,18}

The transverse form factor squared can be written as^{1,19}

$$|F_T(q)|^2 = \frac{4\pi}{Z^2} (2J+1) \left| \frac{q\hbar}{2Mc} \sum_\tau \left\{ \frac{1}{2} g_\tau^* \rho_{J\tau}^*(q) - 2g_\tau^l \rho_{J\tau}^l(q) \right\} \right|^2, \quad (2)$$

where M is the nucleon mass, τ is an isospin index equal to 0 or 1, g_τ^s and g_τ^l are the spin and orbital g-factors, $\rho_{J\tau}^l$ is the diagonal current transition density, and $\rho_{J\tau}^s(q)$ is the transverse spin density written as¹

$$\rho_{J\tau}^s(q) = \left[\frac{J+1}{2J+1} \right]^{1/2} \rho_{JJ-1}^{s\tau} - \left[\frac{J}{2J+1} \right]^{1/2} \rho_{JJ+1}^{s\tau} . \quad (3)$$

In a shell model picture, the $\rho_{JL}^{s\tau}$ transition density can be written as the sum of products of one-body density matrix elements and single-particle matrix elements as:¹

$$\rho_{JL}^{s\tau}(q) = \sum_{j_a j_b} Z_{J\tau}(j_a j_b) \sqrt{2} \hat{j}_a \hat{j}^{-1} \langle j_a \| j_L(qr) [Y_L(\hat{r}) \times \vec{\sigma}]^J \| j_b \rangle , \quad (4)$$

where $\hat{j} = \sqrt{2j+1}$, j_a is for the hole-state, and j_b is for the particle-state. $Z_{J\tau}$ is the "Z coefficient" given by the one-body matrix element²⁰

$$Z_{J\tau}(j_a j_b) = \langle J_f T_f \| [a_{j_a}^\dagger \times a_{j_b}]^{J,\tau} \| J_i = 0 \ T_i \rangle , \quad (5)$$

which has the maximum value of unity for an isoscalar or isovector pure particle-hole excitation from a closed shell.

In this work we are interested in describing $1\hbar\omega$ stretched states of the type $(j_b j_a^{-1})_{J_{max}}$ where $J_{max} = j_a + j_b$, $j_a = l_a + 1/2$, $j_b = l_b + 1/2$, $l_b = l_a + 1$, and j_a and j_b are the largest angular momenta found in the last filled shell and first open shell, respectively. This configuration is unique in a shell model space excluding 1p-1h excitations with $E \geq 3\hbar\omega$, so there is no mixing with other 1p-1h configurations. Of course, the stretched configurations can mix with multi-nucleon-multi-hole configurations within the same shell. Such mixing produces physical ground and excited states that are not pure closed shell and particle-hole wave functions, respectively; however, the additional components in the wave functions will not be connected by the one body spin and orbital current operators. The only effect is then a reduction of the transition strength to each state with the cross sections remaining proportional to a single-particle matrix element corresponding to the stretched configuration.

Under the assumption that only the stretched particle-hole configuration contributes, a further simplification is that the convection current

transition density $\rho_{J\tau}^I$ and also $\rho_{JJ+1}^{\sigma\tau}$ vanish as a result of angular momentum restrictions on the single-particle matrix element. Combining Eqs. (2) through (4), evaluating the reduced matrix element for the stretched particle-hole configuration, including the center-of-mass and nucleon finite size corrections, and using either $\alpha = 0, 1$ or $\alpha = n, p$ we get¹

$$|F_T(q)|^2 = \left| f_{cm} f_{fs} \frac{1}{Z} \left(\frac{q\hbar}{2Mc} \right) \sqrt{\frac{J+1}{2J}} \hat{j}_a \hat{j}_b \langle j_a j_b \frac{1}{2} - \frac{1}{2} | J0 \rangle \frac{1}{\sqrt{2}} \right. \\ \left. \times \sum_{\alpha} [C_{\alpha} Z_{\alpha} g_{\alpha}^{\sigma} I_{\alpha}] \right|^2 . \quad (6)$$

This uses:

$$f_{cm}(q) = \exp[(bq/2)^2/A] ,$$

$$f_{fs}(q) = \exp[-(0.8q)^2/6] ,$$

$$C_{\alpha} = \sqrt{2} \text{ for } \alpha = 0, 1 \quad \text{and} \quad C_{\alpha} = 1 \text{ for } \alpha = n, p ,$$

$$Z_n = (Z_0 + Z_1)/\sqrt{2} \quad \text{and} \quad Z_p = (Z_0 - Z_1)/\sqrt{2} \quad (\text{Ref. 21}) ,$$

$$g_0^{\sigma} = \mu_n + \mu_p , \quad g_1^{\sigma} = \mu_n - \mu_p , \quad g_n^{\sigma} = 2\mu_n , \quad \text{and} \quad g_p^{\sigma} = 2\mu_p .$$

A 4-pole expansion²² can be used to do the finite size correction more accurately, but the results are the same as when using the above expression within 1% out to $q = 2 \text{ fm}^{-1}$ and within 10% out to $q = 3 \text{ fm}^{-1}$. The radial integral is

$$I_{\alpha}(q) = \int_0^{\infty} \psi_{a\alpha}(r) j_{J-1}(qr) \psi_{b\alpha}(r) r^2 dr . \quad (7)$$

Using the fact that $\sum_{\alpha} [C_{\alpha} Z_{\alpha} g_{\alpha}^{\sigma} I_{\alpha}]$ is the same for $\alpha = 0, 1$ and for $\alpha = n, p$, we can write the isoscalar and isovector integrals in terms of neutron and proton integrals (since we will deal with neutron and proton states):

$$g_{\tau}^{\sigma} I_{\tau}(q) = [g_n^{\sigma} I_n + (-1)^{\tau} g_p^{\sigma} I_p] / 2 . \quad (8)$$

The radial integral I_{α} depends on the radial form of the single-particle and single-hole nucleon wave function, ψ_{α} . For a HO potential well the proton and neutron wave functions with the same radial quantum numbers are

taken to have exactly the same form. In that case $I_p = I_n = I_\alpha$ and I_α has the simple form:¹

$$I_\alpha = \frac{(bq)^{J-1} \exp[-(bq/2)^2]}{2^{(J-1)/2} \sqrt{(J+1)!!(J-1)!!}}.$$

More realistically, the proton and neutron radial wave functions can be quite different as a result of Coulomb forces on the proton. Consequently, we expect to see differences in $F_T(q)$ and, therefore the form factor will take on different shapes and values depending on the specific choice of radial wave functions.

Equation (6) can be written as

$$|F|^2 = (M_0^e Z_0 + M_1^e Z_1)^2$$

or

$$|F|^2/M_1^2 = \left[\frac{M_0^e}{M_1^e} Z_0 + Z_1 \right]^2 = \left[\frac{M_0^e}{M_1^e} \frac{Z_0}{Z_1} + 1 \right]^2 Z_1^2 \quad (9)$$

to emphasize the separate dependence on the reaction dynamics in M_0 and M_1 and on the nuclear structure effects in Z_0 and Z_1 . A similar equation can be written for pion scattering:²³

$$\frac{d\sigma^\pm}{d\Omega} = (M_1^\pi)^2 \left[\frac{M_0^\pi}{M_1^\pi} Z_0 \mp Z_1 \right]^2. \quad (10)$$

The isovector M_1^e matrix element and the M_0^e/M_1^e ratio become

$$M_1^e = f_{cm} f_{fs} \frac{1}{Z} \left(\frac{q\hbar}{2Mc} \right) \sqrt{\frac{J+1}{2J}} \hat{j}_a \hat{j}_b \langle j_a j_b \frac{1}{2} - \frac{1}{2} | J0 \rangle g_1^e I_1 \quad (11)$$

and

$$\frac{M_0^e}{M_1^e} = \frac{g_0^e I_0}{g_1^e I_1} = \frac{g_n^e I_n + g_p^e I_p}{g_n^e I_n - g_p^e I_p}. \quad (12)$$

Note that for $I_0 = I_1$, as when using HO wave functions, we have the ratio $g_0^e/g_1^e = -0.187$.

The structure effects embodied in Z_0 and Z_1 are difficult to predict for individual states, although calculations have been done for ^{14}C , ^{26}Mg , and ^{54}Fe (Refs. 22,23). Upper limits for the Z coefficients can, however, be

determined from simple nuclear models. These sum rules can be found for $T_<$ states (where the magnetic transition, $T_i \rightarrow T_f$, is for nuclear states of isospin $T_0 \rightarrow T_0$), for $T_>$ states (where the transition is for $T_0 \rightarrow T_0 + 1$), and for the sum of the two.

One method of including the structure effects is to compute the form factors for the extreme single-particle total transition strengths using coefficients of fractional parentage (cfp's)²⁴ and standard shell model algebra.^{22,25} We use n_n and n_p as the number of neutrons and protons in the orbital j_a , and n as the total number of nucleons in that orbital. Then $T_0 = (N - Z)/2$ and $n = n_n + n_p$. For a system of n nucleons in the partially filled shell j_a and none in j_b , these maximum strengths are

$$\begin{aligned} |F_<|^2 &= \frac{1}{2J} \left\{ nM_0^2 + 4T_0M_0M_1 + \frac{T_0(n+2)}{T_0+1}M_1^2 \right\}, \\ |F_>|^2 &= \frac{1}{2J} \left\{ \frac{n-2T_0}{T_0+1}M_1^2 \right\}, \\ |F_{tot}|^2 &= \frac{1}{2J} \left\{ nM_0^2 + 4T_0M_0M_1 + nM_1^2 \right\}. \end{aligned} \quad (13)$$

If instead the shell j_a is completely full, and again no nucleons are in shell j_b , the maximum strengths are

$$\begin{aligned} |F_<|^2 &= M_0^2 + \frac{T_0}{T_0+1}M_1^2, \\ |F_>|^2 &= \frac{1}{T_0+1}M_1^2, \\ |F_{tot}|^2 &= M_0^2 + M_1^2. \end{aligned} \quad (14)$$

The observed fraction of these maximum strengths gives the quenching of magnetic multipole strength.

We here, however, use a different method given by Holtkamp et al.²¹ First we look at a $T_>$ stretched transition of multipolarity J induced by an isovector one-body operator to a state which has the form $|CORE\rangle_{0+} \times |j_a^{n-1}j_b\rangle_J$, where $J = j_a + j_b = 2j_a + 1$ for a stretched state. Then the

square of the isovector Z coefficient summed over all observed transitions is given by²¹

$$\sum_i (Z_1^i)^2 = \frac{1}{2J} \left[\frac{2}{T_0 + 1} n_p \right], \quad \text{for } T_>. \quad (15)$$

The sum rule is also valid when the ground state (or excited state) contains admixtures of other orbital configurations such as j_c^n so long as $j_c + j_a < J$. The maximum value of the sum occurs when n is evaluated in the extreme single-particle shell model as above.

For $T_<$ stretched magnetic transitions, the maximum isovector strength is given by²¹

$$\sum_i (Z_1^i)^2 = \frac{1}{2J} \left[n_n + \left(\frac{T_0 - 1}{T_0 + 1} \right) n_p \right], \quad \text{for } T_<. \quad (16)$$

The sum of the two gives:

$$\sum_i (Z_{0,1}^i)^2 = \frac{1}{2J} [n_n + n_p] = \frac{n}{2J}, \quad \text{for } T_{tot}. \quad (17)$$

For nuclei where $(N - Z) = (n_n - n_p)$, equations (15) to (17) yield the coefficients on the M_1^2 term in equations (13) and (14), and equation (17) yields the coefficients on the M_0^2 term in equations (13) and (14). Note that in general $2n_n = 2n_p = n = 2J$ for a full orbital.

For comparison to theory, we can determine Z_0 and Z_1 from experiment as fitting parameters for each stretched state. If only electron scattering is available for a nucleus, the two Z coefficients cannot both be determined. For magnetic electron scattering, M_0^e/M_1^e is small (-0.187 if HO wave functions are used) and thus this reaction probes mainly the isovector transitions; we thus neglect the expected small isoscalar contribution to the cross section and from equation (9) we use $|F|^2/M_1^2 \approx (Z_1)^2$. For $T_>$ states, only the isovector amplitude contributes, so this assumption is correct. For $T_<$ states our analysis will incorrectly yield too small a Z_1 if the Z_0/Z_1 ratio is positive and too large a Z_1 if the ratio is negative. If these constructive and destructive interference effects cancel over all states, however, the $\Sigma(Z_1)^2$ will be correct. If pion scattering has also been done, the Z_0/Z_1 ratio can be found by comparing π^- and π^+ cross sections. For

pion scattering M_0^π/M_1^π is approximately 2 near the delta resonance and thus probes mainly isoscalar transitions.

III. CALCULATIONS

We will treat M4 excitations in ^{12}C ,²⁶ ^{14}C ,²⁷ and ^{16}O ,²⁸ M6 excitations in ^{24}Mg ,¹⁸ ^{26}Mg ,²² and ^{28}Si ,¹² M8 excitations in ^{48}Ca ,¹⁷ ^{54}Fe ,²⁵ ^{58}Ni ,²⁹ and ^{60}Ni ,¹¹ one M10 excitation in ^{90}Zr ,³⁰ and one M14 excitation in ^{208}Pb .¹⁶ A range of both bound and unbound excitations is treated for each nucleus, matching those states examined by electron scattering.

Previously, form factors have been calculated using HO wave functions. Fits to experimental results were obtained by varying the length parameter b to match the dependence of each transition on the momentum transfer q ; Table I lists the value of b averaged over all stretched transitions for each target.

In this paper we use WS wave functions, in which case there are more input variables to adjust. Here we discuss the binding energies and potential parameters used to calculate hole-state and particle-state wave functions and the sensitivity of $|F|^2$ to these choices.

The binding energy of the hole-state is assumed to be the separation energy of the nucleon in that state. However, for many of the nuclei, the ground state of the $(A-1)$ nucleus does not have the proper spin, j_a . In that case, the excitation energy of the lowest state in the $(A-1)$ nucleus with the proper spin should be added to the separation energy. Also, for nuclei where $T_0 \neq 0$, the ground state of the neutron-hole nucleus need not be the sole isospin parent for the stretched states. The neutron hole-state wave function should then be a weighted sum of $T_0 \pm 1/2$ wave functions, where the binding energies for each are known from neutron pickup reactions. However, there is very little sensitivity to the binding energy of the hole-state. For example, the $T = 3/2$ $f_{7/2}$ neutron hole-state in ^{59}Ni has a binding energy of -15.29 MeV ($S_n = -11.389$ MeV plus $E_x = (-)3.9$ MeV³¹), while the $T = 5/2$ $f_{7/2}$ hole-state lies at -18.74 MeV ($S_n = -11.389$ MeV plus $E_x = (-)7.35$ MeV³²). Electron scattering M8 squared form factors computed with these two extremes differ by less than 1%. For simplicity, we therefore use the lower isospin neutron hole-state for our calculations.

The binding energy of the particle-state is that for the hole-state (as outlined above) plus the excitation energy of the stretched state; thus a different binding energy is used for each particle-state. Here, however, we cannot always use the lower isospin neutron hole-state for simplicity. For all $T_<$ stretched states and for $T_> (= T_0 + 1)$ stretched states in $T_0 = 0$ nuclei, we do use the decay to the lower isospin neutron hole-state, since this will dominate the decay probability. For $T_0 + 1$ stretched states in $T_0 \neq 0$ nuclei, however, we use the decay to the upper $T_0 + 1/2$ isospin neutron hole-state, since this is the only allowed decay if the isospin rules are strictly obeyed. In the five nuclei where this applies, the resulting stretched states are all bound except the one in ^{14}C . Such $T_>$ stretched states may, however, break the isospin rules and decay to the lower isospin state of the daughter. In this case, the resulting stretched states in the five nuclei are all unbound except the one in ^{54}Fe . This ambiguity leads to a small uncertainty in $|F|^2$, but in general the calculated $|F|^2$ is smaller by no more than 20% (for any individual state) when we assume isospin rules are broken than when we assume them to be obeyed. For the 24.4 MeV 4^- state in ^{14}C , however, our method failed to converge when the low isospin state of the daughter was used.

For unbound states we used the option in the DWUCK4¹³ computer code which treats the unbound level as a resonance. The unbound wave function is normalized by forcing it and its derivative to be zero at the radius picked for the limit of integration.¹⁴

The radial integrations were performed with a step size of 0.1 fm and a maximum outer radius of 40 fm. This was found to be adequate to obtain stability in the predicted decay-particle widths. For example, the 11.7 MeV state in ^{14}C had a calculated single-particle width of 2.4 MeV when a 22 fm radius was used and 1.9 MeV when a 40 fm radius was used. For the 14.4 MeV $T=1$ state in ^{28}Si this method yields a single-particle proton width of 15.4 keV which compares well with the 10.8 to 17.4 keV width found from more sophisticated calculations.³³ For the 24.4 MeV $T=2$ state in ^{14}C this method yields a single-particle neutron width of 0.54 MeV when the correct isospin decay is used. This state has an observed width

of 1.7 MeV,²¹ which is the broadening expected if some decay to the lower isospin daughter is present.

The shape parameters for the potentials may be determined by elastic magnetic electron scattering from the valence nucleon; these parameters have been determined in several cases.^{34,35} It is also advantageous to use the same potential well to generate wave functions for stripping reactions and for magnetic electron scattering;³⁶ thus, we include several examples of stripping parameters.³⁷⁻⁴⁰ This should yield a more consistent comparison between the two classes of reactions. We also include shape parameters determined for scattering to stretched states in three cases.^{26,28,41} Since the parameters that describe the potential for the final state cannot be determined by any independent means, some freedom exists and, consequently, some ambiguity in the predictions is encountered. We have chosen to keep these parameters the same for initial and final states.

Table I lists radial parameters and spin-orbit strengths for nuclei close to those studied here. In most cases, use of these parameters resulted in a poor fit to the data for electron scattering to stretched states. Use of a smaller radius (r_0), however, gave very good fits to the data. These r_0 used in our calculations (without MEC effects) are listed in Table I. Fig. 1 shows examples of the fits for two states each in ^{16}O and ^{60}Ni . The HO curve is shown along with two WS curves. All calculations shown in Fig. 1 have been normalized to the magnitude of the data.

For ^{24}Mg , ^{26}Mg , and ^{28}Si the r_0 from the literature gave a fairly good fit to the data. In all other cases a much smaller r_0 near 1.05 fm was needed. For the heavier nuclei (^{48}Ca to ^{208}Pb), using a smaller r_0 changed the predicted $|F|^2$ results by less than 8% (usually 1% to 3%). For ^{12}C , ^{14}C , and ^{16}O , however, decreasing r_0 to near 1.05 fm increased $|F|^2$ by 15% to 20% and thus decreased the fraction of the sum strength the data exhaust by this same percent.

There have been other references suggesting the need for using smaller radii. Although the ground state of ^{13}C is not the hole state for stretched excitations in ^{14}C , it is interesting to note that a value of $r_0 = 0.9$ fm for ^{13}C is given by Donnelly and Sick³⁵ as the best fit to the magnetic elastic data. In a recent reference that calculates the mean field for negative

and positive energy neutrons in ^{208}Pb , the radius of the real potential was found to decrease for more positive (unbound) single-particle energies, and for more negative (deeply bound) single-hole energies.⁴² This decrease in radius amounted to about 3% at ± 10 MeV. This is much less than the decrease in radius (from the radii found for ground state distributions) detailed in Table I of about 10%, even for states close to the nucleon binding energy.

To understand better how the calculated form factors are dependent on the wave functions used, we have plotted in Fig. 2 the integrand of the radial integral [see equation (7)] for several states. These were evaluated at a value of q where the form factor is a maximum. The neutron component is seen to extend beyond the proton component due to the higher decay probability. This is particularly true for the final states at higher energies, as seen in the lower part of Fig. 2. For the more unbound final states, the overlap of the particle and hole states decreases, yielding smaller calculated form factors. The results in Table II are the clear result of the amount of overlap seen in Fig. 2.

The effect of meson exchange currents (MEC) was also computed, and the influence of WS and HO wave functions was compared. The expressions of Dehesa et al.^{15,43} were evaluated with both sets of wave functions for the isovector mode. The entire ensemble of initial states (not just that for the hole state) was computed using the WS potential. In general, using MEC effects leaves $|F|^2$ at values of q below the maximum unchanged and moves $|F|^2$ beyond the maximum to larger q by about 0.1 fm^{-1} . Fig. 3 shows this general result for ^{26}Mg and includes the curves for the seagull and pionic terms which enter with opposite signs. The effect is the same to within 3% (usually 1%) whether HO or WS wave functions are used. The ratio of $|F|^2$ with and without MEC effects is given in Table II for all the stretched states considered. This ratio is taken at the peak of the $|F|^2$ curve, where inclusion of MEC effects increases the predictions between 14% and 22%. Note that these effects will decrease yet further the fractions of single-particle strength obtained by comparing data to calculations, as shown previously.¹⁰ All results to be shown here will include the appropriate MEC terms.

IV. RESULTS

In Fig. 4 some representative electron scattering data are compared to the theoretical $|F|^2$ calculated from both HO and WS wave functions. Also included are the WS form factors calculated separately for neutrons and protons. In this case the matrix elements were calculated using only neutron or only proton radial integrals instead of a fraction of both. The theoretical form factor curves were adjusted downwards to fit the data. Neutron and proton WS curves were adjusted by the same factor as was the combined WS curve. The $^{208}\text{Pb } 14^-$ transition is interpreted as having purely neutron content.

From Fig. 4 we notice the isospin asymmetry of the proton and neutron form factors. The WS form factors include this asymmetry in the calculations, whereas the HO calculations assume symmetry. As expected, $|F|^2$ peaks at smaller q for neutrons than for protons, since neutrons are less bound and thus the radius is larger. In general the difference in $|F|^2$ between neutron and proton curves increases with increasing excitation energy.

Using HO wave functions with a constant b results in $|F|^2$ peaking at the same q for all stretched states. However, for stretched states at higher excitation energies, the wave functions extend further out and $|F|^2$ should peak at smaller q . When using WS wave functions, the change in q_{max} between the lowest and highest stretched state is in general 0.03 fm^{-1} or less. For ^{26}Mg , however, the 18.0 MeV state peaks at 0.07 fm^{-1} lower than the 7.5 MeV state. This shift can be seen in the data for ^{26}Mg in Fig. 4.

Table II gives the results of the analysis using WS wave functions and compares these to results using HO wave functions. The HO form factors do not take into account the excitation energy of the state, whereas the WS form factors do. As a result, the HO form factor curves are the same for all states for the constant value of b used for each target. For M_1^2 the ratio between HO and WS calculations shows that the theoretical $|F|^2$ for HO increases with respect to WS as the excitation energy increases. The ratio abruptly increases at energies where a nucleon becomes unbound.

This is as expected since the overlap between initial and final wave functions decreases; this effect was also pointed out by Siciliano and Weiss.⁴⁴ For the M_0/M_1 ratio in many cases the WS results are much different than the -0.187 for HO calculations. Again, the ratio changes abruptly at the nucleon separation energies. In some cases the ratio is much larger than -0.187 , making neglect of the isoscalar term a poor approximation. The ratios M_0^e/M_1^e and F_{exp}^2/M_1^2 can be used in conjunction with pion scattering data to calculate Z_0 and Z_1 by solving equations (9) and (10) simultaneously.

The sums of F_{exp}^2/M_1^2 from Table II are given in Table III for $T_<$ states, $T_>$ states, and the total. The isospin assignments in the literature are assumed to be correct, although untested in most cases. The F_{thr}^2/M_1^2 in Table III are calculated using equations (15) to (17). The use of equations (13) and (14) gives similar results to within 10%, except for ^{48}Ca , ^{90}Zr , and ^{208}Pb , where T_0 is large and the M_0M_1 cross term is the same order of magnitude as the M_1^2 term. Experimental data are compared to theoretical predictions for $T_<$ and $T_>$ states separately, as well as the total. Using WS wave functions, the ^{12}C data exhaust close to 80% of the sum expected and close to 70% is exhausted for ^{26}Mg . These would be close to 100% if MEC effects had not been included. In general the data exhaust a large fraction of the sum rule for light nuclei, but exhaust a decreasing amount for heavier nuclei. Also listed are the comparisons between HO and WS calculations for each nucleus. The WS calculations enhance the amount exhausted for light nuclei, but decrease the amount exhausted for heavier nuclei, compared to HO calculations.

V. DISCUSSION

The distinction between results with HO and WS wave functions is most dramatic for the light nuclei, diminishing for heavier nuclei. This can possibly be understood in terms of the increasing role of the centrifugal potential term retarding the extension of the unbound radial wave function. Using $R = 1.1 \times A^{1/3}$ fm and l for the particle state, this potential

$$V_l = \frac{l_b(l_b + 1)\hbar^2}{2MR^2}$$

is 18 MeV, 22 MeV, 24 MeV, 26 MeV, and 27 MeV for ^{14}C , ^{28}Si , ^{54}Fe , ^{90}Zr , and ^{208}Pb respectively. The particle angular momentum increases faster than does the nuclear size, decreasing the decay probability and decreasing the binding energy effects for heavier nuclei.

Several comments can be made about the fraction of the sum rule strength exhausted by the data in Table III. Leaving out MEC effects increases all fractions by 15% to 20%. Use of the ground state isospin to calculate $T_>$ state wave functions would increase the fraction of the sum rule, since decay probabilities would increase and theoretical form factors decrease. Using the larger r_0 (near 1.2 fm), as listed in the literature for ^{12}C , ^{14}C , and ^{16}O , would increase the fraction of the sum rule by close to 20%. For nuclei on which pion scattering has been done, including isoscalar transition effects in equation (9) changes the sum rule fraction in several cases. The Z_0/Z_1 ratio can be calculated directly from the pion scattering σ^+/σ^- ratio with no need to assume any particular wave function. Using the Z_0/Z_1 ratios from the literature, the fraction of the total sum strength changes by less than 2% for ^{14}C ,^{21,22} ^{28}Si ,⁴⁵ and ^{54}Fe .²³ For the 19.5 MeV state in ^{12}C , the fraction increases from 77% to 147% if the ratio, $|Z_0/Z_1| = 1.09$,²⁶ is positive as for a neutron state, but the fraction decreases to 47% if the ratio is negative as for a proton state. If isoscalar effects are included for the 17.8 MeV state in ^{16}O , the total sum rule fraction for ^{16}O increases from 63% to 86% if the $|Z_0/Z_1|$ ratio is positive,²⁸ but remains unchanged if the ratio is negative.⁴⁵

An analysis of pion scattering to stretched states using WS wave functions (especially for unbound states) should be even more revealing than this analysis of electron scattering. The electromagnetic probe interacts with the whole nuclear volume, but the strongly absorbed pion is particularly sensitive to the nuclear surface. In neutron excess nuclei such as ^{14}C the neutron component of the transition density extends beyond the proton component and enhances negative pion scattering. This effect has been demonstrated for giant quadrupole resonances,⁴⁶ but shown to be slight for the nearly-bound stretched state of isosymmetric ^{12}C .⁴⁴ An analysis using WS wave functions is now in progress on new $^{60}\text{Ni}(\pi, \pi')$ data as well as previously published data on other nuclei.⁴⁷

VI. CONCLUSIONS

The result of this work is a substantial increase in the fraction of the expected stretched magnetic transition strength for light nuclei when existing data are compared to new theoretical predictions. For ^{12}C , the M4 strength fraction is increased by a factor of 2.2. For ^{26}Mg , one of the best cases examined experimentally,²² we find 70% of the total expected M6 strength. The increase is less dramatic for heavier nuclei, and in fact for ^{90}Zr and ^{208}Pb there is a marked decrease. It is not unreasonable to expect that unobserved weak states in these nuclei could account for the difference from 100%, or that a lack of closure of the hole-orbital hampers the excitation.

The present analysis treats all known cases of magnetic electron scattering to stretched states in doubly-even nuclei, and it spans a wide range of hole states ($l_a = 1, 2, 3, 4, 6$) and particle states ($l_b = 2, 3, 4, 5, 7$). Realistic radial wave functions and meson exchange currents have been included, and we have demonstrated that our conclusions as to the magnitudes of our calculations are quite insensitive to the parameters selected. For light nuclei, where experimental results are most complete, we conclude that there is no significant quenching of isovector stretched magnetic strength from the simple single-particle sum rules, and invocation of exotic effects does not appear to be appropriate.

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TABLE I. Parameters are listed for the WS potential binding the initial and final nucleons for the present studies. A Coulomb radius of $r_c = 1.25 \times A^{1/3}$ fm was used for all nuclei. The oscillator length b is used for calculating HO wave functions for comparisons. The nuclear radius is $R = r_0 \times A^{1/3}$ and the nuclear diffuseness is a . Where λ is listed for the spin-orbit strength, this represents the strength multiplying the usual Thomas form. The "fit" values (made without including MEC effects) were used for Fig. 4 and the results listed in Table II. The final column shows how much the theoretical $|F|^2$ changes (average over all states) if r_0 from the literature is used instead of the r_0 from a fit.

Nucleus	b(fm)	r_0 (fm)	a (fm)	λ/V_{so} (MeV)	Ref.	$ F ^2/ F _{fit}^2$
^{12}C		1.20	0.65	$V_{so} = 24$	35	0.84
		1.25	0.5	$V_{so} = 24$	26	0.94
	1.53	1.0	0.65	$V_{so} = 24$	fit	1.
^{14}C		1.25	0.5	$V_{so} = 24$	22	0.84
		1.16	0.78	$\lambda = 25$	38*	0.93
	1.49	1.05	0.5	$\lambda = 25$	fit	1.
^{16}O		1.232	0.6	$V_{so} = 24$	35	0.82
		1.19	0.6		28	0.84
	1.58	0.9	0.6	$V_{so} = 24$	fit	1.
^{24}Mg		1.25	0.65	$\lambda = 25$	39	1.
	1.69	1.25	0.65	$\lambda = 25$	fit	1.
^{26}Mg		1.417	0.5	$V_{so} = 24$	22	1.03
		1.25	0.65	$\lambda = 25$	39	1.
	1.77	1.25	0.65	$\lambda = 25$	fit	1.
^{28}Si		1.25	0.65	$\lambda = 25$	40	0.95
		1.2	0.65		41	0.97
	1.8	1.2	0.60	$\lambda = 25$	fit	1.
^{48}Ca		1.17	0.65	$V_{so} = 28$	34	1.02
	1.85	1.0	0.65	$V_{so} = 28$	fit	1.
^{54}Fe		1.232	0.65	$V_{so} = 28$	34	0.98
	1.9	1.05	0.65	$V_{so} = 28$	fit	1.
^{58}Ni		1.11	0.65	$V_{so} = 24$	35	1.02
	1.95	1.0	0.65	$V_{so} = 24$	fit	1.
^{60}Ni		1.173	0.65	$V_{so} = 28$	34,37	0.98
		1.11	0.65	$V_{so} = 24$	35	0.98
	1.983	1.0	0.60	$V_{so} = 28$	fit	1.
^{90}Zr		1.265	0.65	$V_{so} = 28$	34	1.02
	2.08	1.07	0.65	$V_{so} = 28$	fit	1.
^{208}Pb		1.199	0.65	$V_{so} = 28$	34	1.08
	2.22	1.0	0.65	$V_{so} = 28$	fit	1.

* Binding parameters were misprinted.

TABLE II. Listed here are the results of the present analysis giving fractions of the single-particle stretched magnetic sum rule strengths as defined in the text. For each nuclide, nucleon separation energies are listed for comparison and the stretched states are listed with the assigned isospin. Note that the 19.5 MeV state in ^{12}C has mixed isospin and the ^{48}Ca , ^{90}Zr , and ^{208}Pb states are pure neutron transitions. The first row for each nuclide gives the HO results which are constant for all states, to compare to the WS values. F_{exp}^2 is from the peak of the data curve (with data adjusted by $4\pi/Z^2$ for ^{24}Mg , ^{28}Si , and ^{48}Ca). M_1^2 and M_0/M_1 are calculated using equations (11) and (12). $F_{exp}^2/M_1^2 \approx Z_1^2$ is found from $F_{exp}^2 \div [M_1^2(\text{no MEC}) \times F^2(\text{MEC})/F^2(\text{no MEC})]$. Sums of F_{exp}^2/M_1^2 for each nuclide and comparison with theory are given in Table III. The energies in the table have units of MeV; all other quantities are unitless.

T	State (MeV)	F_{exp}^2 $\times 10^{-5}$	M_1^2 (no MEC) $\times 10^{-5}$ ($\frac{M_0}{M_1}$)	$\frac{F^2(\text{MEC})}{F^2(\text{no MEC})}$	$\frac{M_0^c}{M_1^c}$	F_{exp}^2/M_1^2 (HO)	F_{exp}^2/M_1^2 (WS)
^{12}C	$J^\pi = 4^-$ ($j_a = \frac{3}{2}^-, j_b = \frac{5}{2}^+$)			$S_p = 16.0, S_n = 18.7$		Ref. 26	
HO			268.	1.17	-.187		
	19.5	110.	120.	2.23	1.19	-.254	.352 .769
^{14}C	$J^\pi = 4^-$ ($j_a = \frac{3}{2}^-, j_b = \frac{5}{2}^+$)			$S_n = 8.2, S_p = 20.8$		Ref. 27	
HO			261.	1.21	-.187		
<	11.7	23.2	214.	1.22	1.21	-.278	.0735 .0896
<	15.2	0.63	187.	1.40	1.21	-.326	.0020 .0028
<	17.3	28.6	210.	1.24	1.21	-.216	.0906 .112
>	24.4	62.3	160.	1.63	1.21	-.085	.197 .321
^{16}O	$J^\pi = 4^-$ ($j_a = \frac{3}{2}^-, j_b = \frac{5}{2}^+$)			$S_p = 12.1, S_n = 15.7$		Ref. 28	
HO			136.	1.19	-.187		
<	17.8	2.2	115.	1.18	1.20	-.175	.0135 .0159
>	17.9	3.1	115.	1.18	1.20	-.175	.0190 .0224
>	18.6	4.5	98.1	1.39	1.21	-.119	.0277 .0379
>	19.0	57.0	96.5	1.41	1.21	-.118	.351 .486
<	19.8	0.058	92.1	1.48	1.22	-.114	.0003 .0005
>	20.5	7.30	88.6	1.53	1.22	-.115	.0449 .0675

^{24}Mg	$J^\pi = 6^-$	$(j_a = \frac{5}{2}^+, j_h = \frac{7}{2}^-)$	$S_p = 11.7, S_n = 16.5$	Ref. 18
HO		49.3	1.14	-.187
> 15.0	8.73	35.0	1.41	1.15
				-.112
				.155
				.217

^{26}Mg	$J^\pi = 6^-$	$(j_a = \frac{5}{2}^+, j_h = \frac{7}{2}^-)$	$S_n = 11.1, S_p = 14.1$	Ref. 22
HO		48.4	1.16	-.187
< 7.5	0.217	45.3	1.07	1.15
				-.214
				.0039
				.0042
< 9.2	1.62	43.8	1.11	1.15
				-.221
				.0288
				.0320
< 12.5	2.14	36.4	1.33	1.16
				-.298
				.0383
				.0510
< 12.9	0.948	35.8	1.35	1.16
				-.301
				.0169
				.0228
< 13.0	0.646	35.9	1.35	1.16
				-.298
				.0115
				.0155
< 14.0	0.871	34.6	1.40	1.16
				-.302
				.0155
				.0217
< 14.5	1.19	29.5	1.64	1.16
				-.243
				.0212
				.0346
< 15.4	1.29	29.5	1.64	1.16
				-.230
				.0231
				.0377
< 15.5	2.35	29.8	1.62	1.16
				-.221
				.0420
				.0679
< 16.5	3.20	29.6	1.64	1.17
				-.200
				.0571
				.0928
> 18.0	7.37	32.7	1.48	1.17
				-.113
				.121
				.193

^{28}Si	$J^\pi = 6^-$	$(j_a = \frac{5}{2}^+, j_h = \frac{7}{2}^-)$	$S_p = 11.6, S_n = 17.2$	Ref. 12
HO		34.7	1.15	-.187
< 11.6	0.12	31.3	1.11	1.16
				-.173
				.0030
				.0033
> 14.4	9.68	25.4	1.37	1.17
				-.113
				.241
				.326

^{48}Ca	$J^\pi = 8^-$	$(j_a = \frac{7}{2}^-, j_h = \frac{9}{2}^+)$	$S_n = 9.9, S_p = 15.8$	Ref. 17
HO		14.2	1.18	-.187
9.1	0.314	15.3	0.93	1.16
				-.215
				.0187
				.0176
9.3	0.942	15.3	0.93	1.15
				-.216
				.0561
				.0532
10.0	0.289	13.7	1.04	1.15
				-.285
				.0172
				.0185

^{54}Fe	$J^\pi = 8^-$	$(j_a = \frac{7}{2}^-, j_h = \frac{9}{2}^+)$	$S_p = 8.9, S_n = 13.4$	Ref. 25
HO		8.22	1.17	-.187
< 8.3	0.30	9.24	0.89	1.16
				-.192
				.0311
				.0279
< 8.9	0.26	7.80	1.05	1.16
				-.124
				.0270
				.0287
< 9.8	0.0001	7.76	1.06	1.16
				-.127
				.0000
				.0000
< 10.0	0.32	7.74	1.06	1.16
				-.128
				.0332
				.0356
< 10.7	0.25	7.71	1.07	1.16
				-.130
				.0259
				.0279
< 11.6	0.002	7.62	1.08	1.16
				-.134
				.0002
				.0003
> 13.3	1.56	7.71	1.07	1.16
				-.120
				.161
				.173

^{58}Ni	$J^\pi = 8^- (j_a = \frac{7}{2}^-, j_b = \frac{9}{2}^+)$				$S_p = 7.0, S_n = 8.6$		Ref. 29	
HO		6.97			1.17	-.187		
< 7.9	0.278	6.56	1.06	1.18	-.127	.0340	.0359	
< 8.8	0.280	6.53	1.07	1.18	-.131	.0342	.0364	
> 10.2	0.170	6.69	1.04	1.18	-.114	.0208	.0215	
> 11.2	0.285	6.64	1.05	1.18	-.115	.0348	.0362	
> 12.5	0.510	6.59	1.06	1.18	-.116	.0622	.0653	

^{60}Ni	$J^\pi = 8^- (j_a = \frac{7}{2}^-, j_b = \frac{9}{2}^+)$				$S_p = 9.5, S_n = 11.4$		Ref. 11	
HO		6.89		1.18	-.187			
< 7.5	0.344	8.23	0.84	1.18	-.197	.0423	.0355	
< 8.4	0.146	8.15	0.85	1.17	-.198	.0179	.0152	
< 9.0	0.203	8.12	0.85	1.17	-.199	.0250	.0215	
< 9.2	0.185	8.10	0.85	1.17	-.199	.0227	.0194	
> 12.3	0.118	6.94	0.99	1.17	-.116	.0145	.0144	
> 12.5	0.206	6.94	0.99	1.18	-.116	.0252	.0252	
> 13.9	0.341	6.87	1.00	1.18	-.117	.0418	.0421	
> 14.8	0.223	6.81	1.01	1.18	-.117	.0273	.0278	
> 15.5	0.177	6.76	1.02	1.18	-.117	.0218	.0222	
> 16.1	0.134	6.69	1.03	1.18	-.117	.0165	.0170	

^{90}Zr	$J^\pi = 10^- (j_a = \frac{9}{2}^+, j_b = \frac{11}{2}^-)$				$S_p = 8.4, S_n = 12.0$		Ref. 30,1	
HO		2.94		1.18	-.187			
7.4	0.80	3.61	0.81	1.15	-.187	.231	.192	

^{208}Pb	$J^\pi = 14^- (j_a = \frac{13}{2}^+, j_b = \frac{15}{2}^-)$				$S_n = 7.4, S_p = 8.0$		Ref. 16	
HO		0.548		1.21	-.187			
6.7	0.12	0.740	0.74	1.18	-.187	.180	.137	

TABLE III. Sums for each nuclide from Table II are listed as F_{exp}^2/M_1^2 . Theory calculations use equations (15), (16), and (17). MEC effects are included throughout. Note that ^{48}Ca , ^{90}Zr , and ^{208}Pb are pure neutron transitions. The HO results are in close agreement with those presented previously where comparisons to previous extreme single-particle model calculations could be made.

Nuclide, J^π T			$\Sigma[F_{exp}^2/M_1^2]$		F_{thr}^2/M_1^2	F_{exp}^2/F_{thr}^2		
			(HO)	(WS)		(HO)	(WS)	($\frac{WS}{HO}$)
^{12}C	4^-	<			0			
		>	.352	.769	1	35%	77%	
		total	.352	.769	1	35%	77%	2.18
^{14}C	4^-	<	.166	.204	1/2	33%	41%	
		>	.197	.321	1/2	39%	64%	
		total	.363	.526	1	36%	53%	1.45
^{16}O	4^-	<	.014	.016	0	—	—	
		>	.442	.614	1	44%	61%	
		total	.456	.630	1	46%	63%	1.38
^{24}Mg	6^-	<			1/3	0%	0%	
		>	.155	.217	1/3	47%	65%	
		total	.155	.217	2/3	23%	33%	1.40
^{26}Mg	6^-	<	.258	.380	1/2	52%	76%	
		>	.131	.193	1/3	39%	58%	
		total	.390	.573	5/6	47%	69%	1.47
^{28}Si	6^-	<	.003	.003	0	—	—	
		>	.241	.326	1	24%	33%	
		total	.244	.330	1	24%	33%	1.35
^{48}Ca	8^-	<	.092	.089	1/2	18%	18%	
		total	.092	.089	1/2	18%	18%	0.97
^{54}Fe	8^-	<	.117	.120	1/2	23%	24%	
		>	.161	.173	3/8	43%	46%	
		total	.279	.294	7/8	32%	34%	1.05
^{58}Ni	8^-	<	.068	.072	1/2	14%	14%	
		>	.118	.123	1/2	24%	25%	
		total	.186	.195	1	19%	19%	1.05
^{60}Ni	8^-	<	.108	.092	2/3	16%	14%	
		>	.147	.149	1/3	44%	45%	
		total	.255	.241	1	26%	24%	0.95
^{90}Zr	10^-	<	.231	.192	1/2	46%	38%	
		total	.231	.192	1/2	46%	38%	0.83
^{208}Pb	14^-	<	.180	.137	1/2	36%	27%	
		total	.180	.137	1/2	36%	27%	0.76

FIGURE CAPTIONS

- FIG. 1. The results of using different potential well parameters to fit the data are shown for two states in ^{16}O and two states in ^{60}Ni . The solid curves shows the HO fit and the dot-dashed curves show the WS fit. The dot-dashed curves for ^{16}O are from $r_0 = 1.232$ fm (left curves) and $r_0 = 0.9$ fm (right curves), and for ^{60}Ni they are from $r_0 = 1.11$ fm (left curves) and $r_0 = 1.0$ fm (right curves).
- FIG. 2. The integrand of the radial integral at the maximum on the form factor curve for two states each in ^{14}C ($q_{\text{max}} = 1.74 \text{ fm}^{-1}$), ^{26}Mg ($q_{\text{max}} = 1.83 \text{ fm}^{-1}$), and ^{60}Ni ($q_{\text{max}} = 1.99 \text{ fm}^{-1}$). Solid curves use HO wave functions, while dotted (neutron) and dashed (proton) curves use WS wave functions (with the “fit” parameter from Table I). The states plotted in each nucleus are the two most extreme stretched states for which there are good data for comparison.
- FIG. 3. The results of including meson exchange current (MEC) effects are shown for the 9.17 MeV state in ^{26}Mg . The solid circles show the data compared to the solid line which includes MEC effects. These effects are made up of the non-MEC term (dot-dashed line), the seagull term (dashed line), and the pionic term (dotted line). The curves are very similar whether HO or WS wave functions are used.
- FIG. 4. Electron scattering data compared to predictions for: ^{14}C (data from Refs. 22 and 27), ^{26}Mg (data from Ref. 22), ^{60}Ni (data from Ref. 11), and ^{208}Pb (data from Ref. 16). Solid curves use HO wave functions, while dotted (neutron), dashed (proton), and dot-dashed (total) curves use WS wave functions. The magnitudes of the theoretical curves have been adjusted to fit the data. These plots do not include MEC effects.

Figure D.1

Figure D.2

Figure D.3

Figure D.4