*A Numerical Study
of Rayleigh-Taylor Instability
in Aluminum and Steel Plates*

Bart J. Daly

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Los Alamos Los Alamos National Laboratory
Los Alamos, New Mexico 87545

A NUMERICAL STUDY OF RAYLEIGH-TAYLOR INSTABILITY IN ALUMINUM AND STEEL PLATES

by

Bart J. Daly

ABSTRACT

The SCRAM code is applied to the study of Rayleigh-Taylor instability in metal plates, and comparisons of these computer results are made with experimental data for 1100-O aluminum, 6061-T6 aluminum, and 304 stainless steel. Various models for the pressure, temperature, and strain-rate dependencies of the flow stress are compared in the computer calculations. The coefficients that are required in these models to give good agreement with the experimental results are generally close to values that were determined from previous experimental comparisons. The sensitivity of the computed results to modeling parameters, to variations in the hardening modulus, and to the amplitude and wavelength of the perturbations in the plate surface is examined. Very little growth in amplitude occurs if either the initial amplitude or the wavelength is sufficiently small. The growth rate increases monotonically with increasing initial amplitude. There appears to exist a wavelength of maximum growth, such that the growth rate increases rapidly with wavelength up to this wavelength, but then decreases slowly as the wavelength is further increased.

I. INTRODUCTION

Blewett¹ and Barnes et al.² describe a series of experiments that were performed to determine the growth rate of surface waves in plates composed of 1100-O aluminum, 6061-T6 aluminum, and 304 stainless steel. In these tests the plates were accelerated by high explosives (H.E.). The entire apparatus was enclosed in a vacuum chamber, and a vacuum was maintained between the H.E. and the plate (Fig. 1) in order to provide a shockless acceleration of the plate. Table I lists the conditions and results of these experiments. The aluminum plates were 0.254 cm thick, while the stainless steel plates were 0.190 cm thick. In all but one test the wavelength of the surface perturbations was 0.508 cm and the amplitude of the

Fig. 1. Setup of the aluminum experiments. In the stainless steel experiments the gap between the H.E. and the plate is 0.5 in. and the plate thickness is 0.075 in.

TABLE I
EXPERIMENTAL CONDITIONS

Shot	Material	Plate Thickness (cm)	Wave-length (cm)	Initial Amplitude (cm)	Amplitude at Time (cm)	(μ s)
1	1100-O Al	0.254	0.508	0.0203	0.1515	8.00
2	1100-O Al	0.254	0.254	0.0102	0.0165	8.01
3	1100-O Al	0.254	0.508	0.0203	0.0880	6.41
4	1100-O Al	0.254	0.508	0.0203	0.1168	7.32
5	6061-T6 Al	0.254	0.508	0.0203	0.1127	8.09
6	304 S.S.	0.190	0.508	0.0203	0.0476	5.54
7	304 S.S.	0.190	0.508	0.0203	0.0725	6.50

perturbations was 0.0203 cm. All of these tests showed sizable increases in the amplitude of the perturbations for times ranging from 5.5 μ s to 8.1 μ s after the onset of H.E. blowoff. The one remaining experiment in 1100-O aluminum had a perturbation wavelength and amplitude that were each one half of the standard value. This experiment showed essentially no growth in perturbation amplitude.

References 1 and 2 describe results of numerical calculations and their comparison with experimental data. One-dimensional calculations of the high explosive burn and blowoff of the H.E. products were used to calculate the pressure loading of the plates. The stainless steel experiments had a 1.27-cm gap between the H.E. and the plate, while the gap in the aluminum experiments was 2.54 cm. Consequently the pressure loading in the stainless steel tests was calculated to be approximately twice that of the aluminum experiments. These pressure histories, which are shown in Figs. 2 and 3, were tested by comparing the computed velocity of the unperturbed plates with experimental measurements. These comparisons were sufficiently good to warrant the use of the calculated pressure histories in two-dimensional plate acceleration studies. Two-dimensional calculations of the motion of the perturbed plates were carried out with the MAGEE code³, which incorporated a Mie-Gruneisen equation of state and an elastic-perfectly plastic strength model. The flow stress, which was assumed to be constant, was varied until agreement with the experimental data was obtained. A value of 3.25 kb gave good agreement with the 1100-O aluminum data and reasonable agreement with the 6061-T6 aluminum datum point. A single value of the flow stress could not correlate the stainless steel data, possibly because of a lack of reproducibility of the experimental data in that case. A yield strength of 9-10 kb resulted in a growth rate that was close to that of the experiments.

In this paper we reexamine the comparison of numerical calculations of plate stability with these measurements. The computer code used in this study is the SCRAM code, which was developed by John Dienes. SCRAM combines the SALE hydrodynamics code⁴, which utilizes the Arbitrary-Lagrangian-Eulerian method, and the statistical crack mechanics code SCM⁵⁻⁷. The Lagrangian differencing option of SALE is used in all of the calculations described in this study. The SCM code incorporates an elastic-plastic material strength model, with kinematic hardening and a finite deformation capability. The code also accounts for the motion of open and shear cracks. The strength models used in this study make use of data from property measurements reported in the literature.

In Sec. II we describe the various models that we have used to account for the pressure, temperature, and strain rate sensitivity of the flow stress in SCRAM. In

Fig. 2. History of the driving pressure on the aluminum plates. From Blewett¹.

Fig. 3. History of the driving pressure on the stainless steel plates. From Blewett¹.

Sec. III we present results obtained when these models are applied for the conditions of the aluminum and stainless steel experiments described in References 1 and 2. Some conclusions regarding the usefulness of the constitutive relations used in this study are given in Sec. IV.

II. FLOW STRESS MODELING

The flow stress Y in SCRAM is related to the stress deviator by

$$2Y^2 = S_{ij} S_{ij} \quad (1)$$

where

$$S_{ij} = \sigma_{ij} - \bar{\sigma} \delta_{ij} \quad (2)$$

and

$$\bar{\sigma} = \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33}) . \quad (3)$$

Here S_{ij} is the stress deviator, σ_{ij} is the stress tensor, $\bar{\sigma}$ is the negative of the hydrostatic pressure, and δ_{ij} is the Kronecker delta. In pure shear $\bar{\sigma} = 0$, so $S_{ij} = \sigma_{ij}$. Also, by symmetry, $S_{12} = S_{21}$, so

$$2Y^2 = 2S_{12}S_{12}$$

and

$$Y = |S_{12}| . \quad (4)$$

However, most of the stress-strain data in the literature has been obtained in tension or compression, for which

$$[\sigma_{ij}] = \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} , \quad (5)$$

$$[S_{ij}] = \begin{bmatrix} 2/3\sigma_{11} & 0 & 0 \\ 0 & -1/3\sigma_{11} & 0 \\ 0 & 0 & -1/3\sigma_{11} \end{bmatrix} , \quad (6)$$

and

$$2Y^2 = S_{11}S_{11} + S_{22}S_{22} + S_{33}S_{33} = 2/3 \sigma_{11}\sigma_{11} .$$

Thus Y is related to the tensile yield strength by

$$Y = |\sigma_{11}| / \sqrt{3} . \quad (7)$$

The SCRAM code incorporates a kinematic hardening model⁶. The hardening modulus used in the calculations of this study is assumed to be constant and is measured from available stress-strain data for the aluminum and stainless steel materials at strains of about 10-12%. If these data are obtained from tension or compression tests, the measured hardening modulus should be divided by $\sqrt{3}$.

A. Pressure Dependence of the Flow Stress

In torsional experiments at pressures up to 500 kb Vereshchagin and Shapochkin⁸ demonstrated that the yield strength varies approximately linearly with pressure for a large number of elements and steels:

$$Y = A + Bp . \quad (8)$$

They did not examine aluminum or 304 stainless steel, which are the materials of interest in this study. However, Spitzig and Richmond⁹ did examine 1100-O aluminum, brass, and several steels in tension and compression tests, which they performed at pressures up to 8.3 kb to determine the pressure dependence of yielding. They found that

$$\sigma = \frac{c}{1 \pm a} \left(1 + \frac{3ap}{c} \right) , \quad (9)$$

where the + sign gives the results from tension tests, and the – sign the result for compression. The coefficients a and c vary with material and with strain, but Spitzig and Richmond⁹ found that the ratio a/c was constant for the material. In 1100-O aluminum this ratio was 56 TPa^{-1} and in iron crystal and steel it varied from 13 to 23 TPa^{-1} , with an average value of 20 TPa^{-1} . As discussed above, these tension/compression data should be divided by $\sqrt{3}$ to obtain the yield strength in shear

$$Y = \frac{c/\sqrt{3}}{1 \pm \alpha} \left(1 + \frac{3ap}{c/\sqrt{3}} \right). \quad (10)$$

Spitzig and Richmond⁹ compare their experimental measurements in aluminum and iron-based materials with theoretical models from Shmatov¹⁰, Jung¹¹, and Ashby and Verrall¹², which account for the effects of pressure on dislocation motion. These various models for the shear stress can be written:

$$\tau = \tau_o \left(1 + \frac{p}{G_p} \right) \left(1 + \frac{p}{G_o} \frac{\partial G}{\partial p} \right) \text{ Shmatov}^{10}, \quad (11)$$

$$\tau = \tau_o \left(1 - \eta_o p \right) \left(1 + \frac{2p}{G_o} \frac{\partial G}{\partial p} \right) \text{ Jung}^{11}, \quad (12)$$

and

$$\tau = \tau_o \left(1 + \frac{p}{G_o} \frac{\partial G}{\partial p} \right) \text{ Ashby and Verrall}^{12}. \quad (13)$$

Here τ is the shear stress, G is the shear modulus, and η is the compressibility. The subscripts o and p indicate conditions at one atmosphere and at pressure, respectively. Jung's model is in best agreement with the data, but the term $1 - \eta_o p$ is not appropriate at high pressures. The method of Shmatov is in fairly good agreement with experiment and can be brought into good agreement by the modification shown below

$$\tau = \tau_o \left(1 + 1.5 \frac{p}{G_p} \right) \left(1 + \frac{p}{G_o} \frac{\partial G}{\partial p} \right) \text{ Modified Shmatov}^{10}. \quad (14)$$

B. Temperature and Strain Rate Dependence of the Flow Stress

Zhurkov and Sanfirova¹³ have shown for a large number of materials that the time to fracture and the strain rate depend on the temperature and stress through a Boltzmann equation. Their experiments on aluminum and platinum, as well as alloys of aluminum with zinc, aluminum with magnesium, and nickel with cobalt, were carried out over 8 to 10 orders of magnitude variation in the strain rate. The resulting expressions for the strain rate and time to fracture are:

$$\dot{\epsilon} = \dot{\epsilon}_0 e^{(-Q_0 + \alpha \sigma) / kT} \quad (15)$$

and

$$t = t_0 e^{(Q_0 - \alpha \sigma) / kT} . \quad (16)$$

Here $\dot{\epsilon}$ is the strain rate, t is the time to fracture, Q_0 is the activation energy barrier at zero stress, σ is the tension stress, k is the Boltzmann constant, and T is the temperature. Zhurkov and Sanfirova show that Q varies linearly with stress, with slope α . The dependence of the flow stress on temperature and strain rate in the SCRAM code is obtained by inverting Eq. (15) for the strain rate to obtain

$$\sigma = \frac{Q_0}{\alpha} \left(1 + \frac{kT}{Q_0} \ln \frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right) . \quad (17)$$

Zhurkov and Sanfirova¹³ show that the activation energy Q is very nearly the same as the sublimation energy, so if we define Q to be the sublimation energy per gram, Eq.(17) can be written

$$\sigma \propto \left(1 - \frac{c_v T}{3Q} \ln \frac{\dot{\epsilon}_0}{\dot{\epsilon}} \right) , \quad (18)$$

where c_v is the specific heat.

The reference strain rate $\dot{\epsilon}_0$, as measured in Reference 13, varies from 10^8 to 10^{12} s^{-1} . We have used a value 10^{10} s^{-1} in the SCRAM calculations. Lindholm, Yeakley, and Bessey¹⁴ provide stress-strain curves in tension and compression for 1100-O aluminum at temperatures ranging from 294 K to 672 K and at strain rates from about 10^{-3} to 10^3 s^{-1} . Figure 4 shows a plot of the stress as a function of strain rate obtained by back extrapolating the compression data at 294 K to zero strain. Also shown in Fig. 4 are two fits to these data. A linear fit of Eq. (18) to the stress and strain-rate data gives

$$\sigma = 40.73(0.3093 + 0.01 \ln \dot{\epsilon}) \text{ KSI} . \quad (19)$$

As can be seen in Fig. 4, Eq. (19) is not in good agreement with the experimental results except at the extreme strain-rate values, and does not accurately predict the trend at high strain rates. Better agreement can be obtained if one assumes a quadratic variation with $\ln \dot{\epsilon}$, as given by

Fig. 4. Stress variation with strain rate, where the datum points show values obtained by back extrapolating compressive stress-strain curves at 294 K to zero strain (Reference 14). The straight line is a plot of Eq. (19) and the curved line is a plot of Eq. (20).

$$\sigma = 33.01 \{ 0.3382 + 0.01 [\ln \dot{\epsilon} + 0.1 (\ln \dot{\epsilon})^2] \} \text{ KSI} . \quad (20)$$

Equation (20) is in excellent agreement with the measurements and has the correct slope at the higher strain rates. Converting Eq. (20) to the form of Eq. (18) gives

$$\sigma = 1.30 \left\{ 1 - \frac{T}{0.58 T_r} \left[\ln \left(\frac{\dot{\epsilon}_0}{\dot{\epsilon}} \right) - 0.1 \left(\ln \left(\frac{\dot{\epsilon}}{B} \right) \right)^2 \right] \right\} kb \quad (21)$$

where

$$T_r = \frac{3Q}{c_v}$$

and $B = 1 \text{ s}^{-1}$. The quadratic term in Eq. (21) is limited in magnitude to its value at the largest strain rate shown in Fig. 4.

Figure 5 shows a comparison of the measured stress variation with temperature and that computed from Eq. (21). While the computed stress shows a greater sensitivity to temperature than the measurements, the slopes of the curves are similar. These comparisons are for 8% strain.

Fig. 5. Stress variation with temperature. Curves A and B show data from Reference 14 at 8% strain and at strain rates $4 \times 10^{-3} \text{ s}^{-1}$ and $2.6 \times 10^3 \text{ s}^{-1}$, respectively. Curves 1 and 2 show the calculated values at the same strain rates.

Klopp, Clifton, and Shawki¹⁵ have examined the sensitivity of shear stress to strain rate for 1100-O aluminum, 6061-T6 aluminum, a high-purity alpha iron, and martensitic 4340 VAR steel. They find that the rate sensitivity of the pure metals is much greater than that of the alloys. In particular, they point out that the flow stress in 6061-T6 aluminum at a strain rate of 10^5 s^{-1} is about 3 kb, not far above the 2 kb estimated value for the quasi-static value at 0 K. In contrast, 1100-O aluminum demonstrates a very rapid increase in shear stress for strain rates greater than 10^4 s^{-1} , increasing from less than 1 kb at 10^4 s^{-1} to about 3 kb at $4 \times 10^6 \text{ s}^{-1}$. A similar rapid growth in shear stress with strain rate is seen in the high purity iron, while the steel shows only a modest rate sensitivity. However, these authors advise caution regarding the flow stresses measured at strain rates greater than 10^6 s^{-1} , which were obtained using vapor-deposited specimens of the materials. They point out that the grain sizes in these vapor-deposited specimens are significantly smaller than those of the bulk specimens.

In finite-difference simulations of these experimental tests, Shawki¹⁶ calculated shear stresses in the 1100-O aluminum that were in good agreement with the experiments using an empirical constitutive relation

$$\tau = 1.25 \left(\frac{295}{T} \right)^{0.4} \left(\frac{\gamma}{0.05} \right)^{0.04} \left(\frac{\dot{\gamma}}{1.53 \times 10^5} \right)^{0.254}, \quad (kb) \quad (22)$$

where γ is the shear strain. In Sec. III we test this flow-stress model, neglecting the small strain dependence, in the plate stability study.

C. Combined Effects of Pressure, Temperature, and Strain Rate on Flow Stress

When experimentalists examine the pressure dependence of the flow stress they generally ignore or minimize variations in temperature or strain rate. Likewise, temperature and strain-rate variations are usually made near atmospheric pressure. Thus, when one is constructing constitutive models, it is not clear how these separate dependencies of the flow stress should be combined. In SCRAM we have made the simple, but so far unjustified, assumption that these effects are multiplicative. For example, if we were using the modified Shmatov model, Eq. (14), for the pressure dependence and the Zhurkov and Sanfirova model¹³, Eq. (18), for the temperature and strain rate dependencies, then the flow stress would be expressed

$$Y = Y_o \left(1 + 1.5 \frac{p}{G_p} \right) \left(1 + \frac{p}{G_o} \frac{\partial G}{\partial p} \right) \left(1 - \frac{T}{T_r} \ln \frac{\dot{\epsilon}_o}{\dot{\epsilon}} \right), \quad (23)$$

where,

$$T_r = \frac{3E_s}{c_v},$$

and E_s is the sublimation energy per gram.

III. NUMERICAL CALCULATIONS

All of the calculations reported here were performed with the SCRAM computational procedure. SCRAM combines two codes: SALE⁴, which is an Arbitrary Lagrangian-Eulerian finite-difference code, and SCM⁵⁻⁷, which incorporates the material strength model. The latter is an elastic-plastic model that includes kinematic hardening, finite-deformation theory, and crack growth and coalescence. A fundamental consideration in SCM is the superposition of strain rates, extending the theory of Reuss¹⁷, who expressed the strain rate as the sum of elastic and plastic contributions, to include the effects of fragmentation and porosity. As discussed in Sec. II, the effects of pressure and strain-rate hardening and thermal softening are in-

cluded in the flow stress model in SCM. A Mie-Gruneisen equation of state is used in all of the calculations reported in this paper.

Typical finite-difference mesh plots, showing the initial amplitude perturbation and the spike-bubble displacement after 8 μ s, are presented in Fig. 6. Calculations are carried out for one half of a sinusoidal wave, with symmetry boundaries on the left and right edges of the mesh. The top of the mesh is a free surface at which a zero-pressure boundary condition is applied. The time-varying driving pressures from the aluminum and stainless steel experiments, as shown in Figs. 2-3, are applied uniformly along the bottom of the mesh.

As a first test of the computer code we have compared a calculation of the motion of the back surface of an unperturbed plate with experimental measurements that were described and presented by Blewett¹. In Fig. 7 we show this comparison with steel data obtained using the HAT method, the ASM method, and radiography. Blewett describes the HAT method as a technique for measuring the plate motion by

Fig. 6. Finite-difference mesh showing the perturbation and the spike-bubble displacement after 8 μ s of the calculation. One half of a sinusoidal wave is resolved in these calculations and reflective boundary conditions are applied at the left and right edges of the mesh. The bottom surface is an applied-pressure boundary, while the top is a zero-pressure boundary. The x indicates the cell in which transient variations in flow stress are presented below.

Fig. 7. Motion of an unperturbed stainless steel plate, where $\square$ shows computed values, * indicates experimental measurements using the HAT method, + gives the radiographic measurements, and the curve shows data obtained using the ASM method. The experimental data are from Blewett¹.

imprinting a polar coordinate grid on its rear surface and then taking a series of pictures of the plate as it moves toward the camera. The ASM method involves the use of an axially symmetric magnetic probe. The calculated results are in good agreement with these experimental data.

A. 1100-O Aluminum

The kinematic hardening modulus, b , for 1100-O aluminum, obtained from the tension and compression data of Lindholm, Yeakley, and Bessey¹⁴, is approximately 1.6 kb. Dividing this value by $\sqrt{3}$ to convert to pure shear, as discussed in Sec. II, gives $b=0.92$ kb. This value is used in all of the 1100-O aluminum calculations, except as noted below.

Figure 8 shows a comparison of the 1100-O aluminum data with the time-varying interface displacement computed using SCRAM, when the flow stress is based upon the Spitzig and Richmond⁹ pressure dependence, Eqs. (9) and (10), and the temperature and strain rate dependence shown in Eq. (21)

$$Y = 1.15 \left(1 + \frac{\sqrt{3}ap}{c} \right) \left\{ 1 - \frac{T}{0.58T_r} \left[\ln \left(\frac{\dot{\epsilon}_v}{\dot{\epsilon}} \right) - 0.1 \left(\ln \left(\frac{\dot{\epsilon}}{B} \right) \right)^2 \right] \right\} kb. \quad (24)$$

Fig. 8. Spike-bubble displacement for 1100-O aluminum. The datum points show the experimental measurements^{1,2} and the curve shows the calculated results when the flow stress incorporates the Spitzig and Richmond⁹ pressure dependence, Eq. (24).

The coefficient of 1.15 kb in Eq. (24) was chosen to give good agreement with the measured displacements for 1100-O aluminum from References 1 and 2, as shown in Fig. 8. This value may be compared with the coefficient 0.75 kb ($1.30/\sqrt{3}$) from Eq. (21), which was extracted from the Lindholm, Yeakley, and Bessey¹⁴ results at 294 K. The coefficient in Eq. (24) is in good agreement with the shear-stress data of Li for 1100-O aluminum at strain rates about 10^5 s^{-1} , as reported by Klopp, Clifton, and Shawki¹⁵.

The interface displacement shown in Fig. 8 is the crest to trough displacement, and therefore is twice the conventionally defined amplitude. The plate acceleration begins at $t=2 \text{ } \mu\text{s}$, when the H.E. blowoff gases reach the plate, as indicated by the driving pressure plot in Fig. 2. The mean velocity of the plate is shown in Fig. 9. Figure 10 shows the flow stress in the mesh cell at the center of the bubble, as indicated in Fig. 6. Prior to plate acceleration at $t=2 \text{ } \mu\text{s}$, the flow stress is constant at 0.6 kb. The increase in flow stress after that time closely parallels the pressure plot, indicating the dominant influence of the pressure term in Eq. 24. However, after the pressure peak the decrease in flow stress is not as rapid as the decrease in pressure, as a result of strain-rate hardening. The maximum temperature in the system increases from an initial value of 300 K to about 370 K at $t=4 \text{ } \mu\text{s}$, then decreases to

Fig. 9. Mean velocity of the 1100-O aluminum plate. The plate acceleration begins at about 2.4 μ s.

Fig. 10. Transient variation of the flow stress [Eq. (24), 1100-O aluminum] in a calculation cell at the center of the bubble as indicated in Fig. 6. Note that the flow stress is sensitive to the pressure, Fig. 2, during the loading phase, but relatively insensitive during unloading. Strain-rate hardening partially accounts for the maintenance of relatively high flow stress at late times.

about 350 K, and finally increases again as the plate distortion becomes large. A contour plot of temperature at $t=8 \mu\text{s}$ is shown in Fig. 11. As indicated in this figure, the maximum temperature is located in the bubble region, while another region of high temperature occurs in the vicinity of the spike. The lowest temperatures are at the top of the plate. The maximum strain rate is approximately 10^5 s^{-1} until about $6 \mu\text{s}$ when it begins to increase due to plate distortion. A contour plot of the strain rate at $t=8 \mu\text{s}$ is shown in Fig. 12. The maximum strain rate at this time is $4.6 \times 10^5 \text{ s}^{-1}$.

Figure 13 shows the interface displacement when the flow stress is the same as in Eq. 24, except that the pressure dependency follows the modified Shmatov model, Eq. 14

$$Y = 1.07 \left(1 + 1.5 \frac{p}{G_p} \right) \left(1 + \frac{p}{G_o} \frac{\partial G}{\partial p} \right) \left[1 - \frac{T}{0.58 T_r} \left| \ln \left(\frac{\dot{\epsilon}_o}{\dot{\epsilon}} \right) - 0.1 \left(\ln \left(\frac{\dot{\epsilon}}{B} \right) \right)^2 \right| \right]. \quad (25)$$

The coefficient in Eq. 25 is slightly smaller than was used with the Spitzig and Richmond⁹ pressure model, Eq. 24. The flow stress, temperature, and strain rate

Fig. 11. Contour plot of temperature at $8.0 \mu\text{s}$ for the 1100-O aluminum calculation when the flow stress is given by Eq. (24). The maximum temperature is 109 K, the H and L contours indicate 398 K and 311 K, respectively, and the contour interval is 10.9 K. Note that the highest temperature is located at the center of the bubble.

Fig. 12. Contour plot of strain rate at $8.0 \mu\text{s}$ for the 1100-O aluminum calculation when the flow stress is given by Eq. (24). The maximum strain rate is $4.6 \times 10^5 \text{ s}^{-1}$, the H and L contours are $4.2 \times 10^5 \text{ s}^{-1}$ and $4.6 \times 10^4 \text{ s}^{-1}$, respectively, and the contour interval is $4.65 \times 10^4 \text{ s}^{-1}$.

Fig. 13. Spike-bubble displacement for 1100-O aluminum. The datum points show the measurements^{1,2} and the curve shows the calculated results when the flow stress incorporates the modified Shmatov¹⁰ model, Eq. (25).

evolutions are very similar to those discussed above. The formulation for G_p and estimates of values for G_0 and $\partial G/\partial p$ are obtained from Steinberg, Cochran, and Guinan¹⁸, using quoted values for 6061-T6 aluminum since data for 1100-O aluminum is not provided.

If we retain this variation of flow stress with pressure but make use of the temperature and strain rate modeling of Shawki¹⁶ from Eq. 22, then the flow stress expression becomes

$$Y = 1.28 \left(1 + 1.5 \frac{p}{G_p} \right) \left(1 + \frac{p}{G_0} \frac{\partial G}{\partial p} \right) \left(\frac{295}{T} \right)^{0.4} \left(\frac{\dot{\gamma}}{1.53 \times 10^5} \right)^{0.254}, \quad (26)$$

and the resulting interface displacement is shown in Fig. 14. The coefficient in Eq. 26 differs only slightly from that of Shawki in Eq. 22. The time variation of the flow stress for this calculation is shown in Fig. 15. This transient plot differs markedly from that in Fig. 10, and from that which resulted from the flow-stress modeling in Eq. 25. As seen in Fig. 15 the flow stress increases more rapidly than in Fig. 10 and does not decrease monotonically as in that figure. The magnitude of the strain rate remains considerably smaller in this calculation than in the previous ones, but, apparently, this strain rate is sufficient to provide the hardening seen in Fig. 15 at late

Fig. 14. Spike-bubble displacement for 1100-O aluminum. The datum points show the measurements^{1,2} and the curve shows the calculated results when the flow stress incorporates the power law temperature and strain rate modeling of Shawki¹⁶, Eq. (26).

times. Indeed, this late-time hardening appears to compensate for early-time strain-rate softening with this power law model, which accounts for the different flow-stress history seen in Fig. 15 as compared to Fig. 10.

B. Sensitivity of Results to Parameter Variations for 1100-O Aluminum

In this section we examine the effects of varying the parameters that enter into the flow stress modeling for 1100-O aluminum, as well as variations in the hardening modulus and the initial conditions for the calculations. In all cases the flow-stress model will be that shown in Eq. 25, which incorporates the modified Shmatov model for the pressure dependence and the temperature and strain rate term that was extracted from the data of Lindholm, Yeakley, and Bessey¹⁴. The standard values for these various terms are listed in Table II.

The effect of a 10% variation in the leading coefficient in Eq. 25 results in approximately a 50% difference in the final interface displacement at 8 μ s as shown in Fig. 16. Figure 17 shows the effect of a 50% variation in the sublimation energy, which enters into the reference-temperature expression. Increasing the sublimation energy results in a decrease in the growth rate of the instability, since it results in less temperature softening. An order-of-magnitude change in the reference strain rate $\dot{\epsilon}_0$ in Eq. 25 gives rise to the different growth rates shown in Fig. 18. Increasing

Fig. 15. Transient variation of the flow stress [Eq. (26)] in a calculation cell at the center of the bubble as indicated in Fig. 6. Note that the flow stress differs markedly from that in Fig. 10, particularly in the late-time magnitude.

TABLE II
PARAMETER VALUES FOR 1100-O ALUMINUM CALCULATIONS

Flow Stress Coefficient	1.07 kb
Sublimation Energy	8×10^{10} erg/g
Reference Strain Rate	10^{10} s ⁻¹
Strain Hardening Modulus	0.92 kb
Initial Perturbation Amplitude	0.0204 cm
Perturbation Wavelength	0.508 cm
Plate Thickness	0.254 cm

this quantity leads to a greater growth rate because of the resulting decrease in strain-rate hardening. However, because of the logarithmic dependence of the flow stress on strain rate, this order-of-magnitude variation of $\dot{\epsilon}_0$ has a relatively minor effect on growth rate. The strain-hardening modulus was varied by an order of magnitude from the standard value, resulting in the different growth rates shown in Fig. 19. Decreasing this value has only a small effect on growth, but an order-of-magnitude increase has the effect of limiting the total displacement of the interface to a relative small value.

We turn now to the effects of varying the initial perturbation amplitude and wavelength. Both of these quantities were reduced in one of the experiments², resulting in a greatly reduced growth rate for the instability. In Reference 2 this decrease in growth rate was attributed to a strength-inhibiting effect at shorter wavelengths. However, Drucker¹⁹ disputes this, claiming that the growth rate depends on the quantity $\rho a h_0$, where ρ is density, a is the plate acceleration, and h_0 is the initial perturbation amplitude. If this quantity is greater than a threshold value $\rho h_0(\text{Th})/H$, large interface displacement will occur, and if the value is below this threshold, the change in amplitude will be slight. Here p is the shockless pressure that results in the acceleration a , and H is the plate thickness. To test Drucker's con-

Fig. 16. Spike-bubble displacement curves for 1100-O aluminum when the flow stress is based on Eq. (25), and the leading coefficient in that model has values: 1: 0.97 kb, 2: 1.07 kb, and 3: 1.17 kb.

Fig. 17. Spike-bubble displacement curves for 1100-O aluminum when the flow stress is based on Eq. (25), and the sublimation energy has values: 1: 4×10^{10} , 2: 8×10^{10} , and 3: 12×10^{10} ergs/g.

Fig. 18. Spike-bubble displacement curves for 1100-O aluminum when the flow stress is based on Eq. (25), and the reference strain rate has values: 1: 10^9 , 2: 10^{10} , and 3: 10^{11} s⁻¹.

Fig. 19. Spike-bubble displacement curves for 1100-O aluminum when the flow stress is based on Eq. (25) and the strain-hardening modulus has values: 1: 0.092, 2: 0.92, and 3: 9.2 kb.

clusion that the growth rate depends on the initial amplitude rather than the wavelength, Barnes et al.²⁰ conducted two experiments. One of these repeated the experiment that had the short wavelength and the small initial amplitude, and the other was the same except that the wavelength of the perturbations was approximately twice as large. All three experiments exhibited a small final amplitude, from which Barnes et al. concluded that Drucker's assertion, that the growth depended on initial amplitude rather than wavelength, was correct.

We have performed a series of calculations in which we separately vary the initial amplitude and wavelength. Figure 20 shows the effects of variations in the initial amplitude. This plot shows that doubling the initial amplitude of the perturbations results in a much more rapid increase in the growth rate, while halving the initial amplitude suppresses growth almost entirely. This would seem to confirm Drucker's conclusion. However, Fig. 21 indicates that variations in the perturbation wavelength have similar effects. Reducing the wavelength by one half results in virtually no growth of the instability, but doubling the wavelength leads to approximately the same interface displacement. Thus, both the initial perturbation amplitude and the wavelength strongly affect the growth rate of the instability. In other calculations of Rayleigh-Taylor instability in metals we have found that there is a

Fig. 20. Spike-bubble displacement curves for 1100-O aluminum when the flow stress is based on Eq. (25) and the initial displacement has values: 1: 0.104, 2: 0.208, and 3: 0.416 mm.

Fig. 21. Spike-bubble displacement curves for 1100-O aluminum when the flow stress is based on Eq. (25) and the perturbation wavelength has values: 1: 0.254, 2: 0.508, and 3: 1.016 cm.

wavelength for which maximum growth occurs, and that this wavelength is approximately twice the plate thickness. There is a large increase in growth rate with wavelength for wavelengths less than this value, but the growth rate decreases very slowly with wavelength for larger wavelengths. Since the wavelength in the aluminum experiments is twice the plate thickness, it is not surprising that doubling this wavelength results in little change in growth rate in Fig. 21.

C. 6061-T6 Aluminum

Klopp, Clifton, and Shawki¹⁵ indicate that, contrary to 1100-O aluminum, the flow stress in 6061-T6 aluminum at a strain rate of 10^5 s^{-1} is not much greater than the estimated quasi-static flow stress at 0 K, which they indicate is about 2 kb. From the *Aerospace Structural Metals Handbook*,²¹ vol. 3, we estimate that the static yield stress at room temperature in tension and compression is approximately 2.4 kb. Dividing this value by $\sqrt{3}$ to convert to simple shear gives 1.4 kb, which we use for the coefficient Y_0 in Eq. 23. This flow-stress equation uses the modified Shmatov model for the pressure dependence and the Zhurkov and Sanfirova¹³ model for the temperature and strain rate. Also from Reference 21 we estimate the strain-hardening modulus to be 4.5 kb, which converts to 2.6 kb in simple shear. With these exceptions, the parameter values for 6061-T6 aluminum are the same as those listed for 1100-O aluminum in Table II.

A comparison of the calculated interface displacement and the single experimental datum point is shown in Fig. 22. The calculated value at 8 μs is slightly low compared to experiment. In Fig. 23 we show the evolution of the flow stress in the mesh cell at the center of the bubble, as indicated in Fig. 6. This flow stress variation is very similar to that shown in Fig. 10 for 1100-O aluminum, except that the peak value is slightly higher in this case because of the larger value of Y_0 . As indicated for 1100-O aluminum, the initial increase in flow stress is primarily due to pressure hardening, which peaks at 4 μs . The maximum temperature in the system increases from 300 K to about 365 K during this time period and the maximum strain rate is approximately 10^5 s^{-1} . After 4 μs the flow stress decreases less rapidly than the pressure because of strain rate hardening. After 6.5 μs the strain rate increases from about 10^5 s^{-1} to about $4.3 \times 10^5 \text{ s}^{-1}$ at 8 μs . A contour plot showing the strain rate at 8 μs is presented in Fig. 24. This plot shows that the peak value of strain rate at this time occurs in the bubble region, the point of maximum deformation. Figure 25 is a contour plot of temperature at 8 μs . The highest temperatures occur in a band stretching from the bubble to the spike region.

Fig. 22. Spike-bubble displacement for 6061-T6 aluminum. The datum point shows the experimental measurement^{1,2} and the curve shows the calculated results when the flow stress, Eq. (23), incorporates the modified Shmatov¹⁰ model for the pressure dependence and the Zhurkov and Sanfirova¹³ model for temperature and strain-rate effects.

Fig. 23. Transient variation of the flow stress [Eq. (23), 6061-T6 aluminum] in a calculation cell at the center of the bubble as indicated in Fig. 6. The flow stress variation is similar to that in Fig. 10 for 1100-O aluminum.

Fig. 24. Contour plot of strain rate at $8.0 \mu\text{s}$ for the 6061-T6 aluminum calculation when the flow stress is given by Eq. (23). The maximum strain rate is $1.2 \times 10^5 \text{ s}^{-1}$, the H and L contours are $1.1 \times 10^5 \text{ s}^{-1}$ and $1.2 \times 10^4 \text{ s}^{-1}$, respectively, and the contour interval is $1.20 \times 10^4 \text{ s}^{-1}$.

Fig. 25. Contour plot of temperature at $8.0 \mu\text{s}$ for the 6061-T6 aluminum calculation when the flow stress is given by Eq. (23). The maximum temperature is 357 K, the H and L contours indicate 351 K and 306 K, respectively, and the contour interval is 5.7 K. Note that the highest temperature is located at the center of the bubble.

D. 304 Stainless Steel

The published stress-strain data for 304 stainless steel show a wide range of variation. The *Aerospace Structural Metals Handbook*,²¹ vol. 1, indicates tensile yield-stress values that vary from about 45 KSI to about 80 KSI at room temperature for the annealed alloy. McClintock and Argon²² report a yield stress of 30 KSI for the annealed metal and 75 KSI when it is cold worked. The yield stress that gives the best fit to the data^{1,2} in the calculations for 304 stainless steel is 3.5 kb in simple shear, which translates to 89 KSI in tension. The fact that this value is high compared to the published data might be explained by work hardening through the machining of the perturbations in the surface of the metal or by the rolling of the plate. From the stress-strain data of Ref. 21, vol. 1, we estimate the strain-hardening modulus to be 17 kb tensile, or 9.8 kb in simple shear. The standard values for the 304 stainless steel calculations are listed in Table III, and the flow stress model is based on the modified Shmatov model for pressure and the Zhurkov and Sanfirova model¹³ for the temperature and strain rate, Eq. 23.

$$Y = 3.5 \left(1 + 1.5 \frac{p}{G_p} \right) \left(1 + \frac{p}{G_o} \frac{\partial G}{\partial p} \right) \left(1 - \frac{T}{T_r} \ln \frac{\dot{\epsilon}_o}{\dot{\epsilon}} \right) kb . \quad (27)$$

As shown in Fig. 3 the peak driving pressure for the 304 stainless-steel experiments was twice as large as that for the aluminum experiments, Fig. 2. Also, the plate thickness in the stainless-steel tests was three quarters that of the aluminum plates. These modifications offset the density difference between aluminum and steel, so that the peak acceleration in all of the experiments was nearly the same. Therefore, the differences in the instability growth rate can mostly be attributed to differences in material strengths. As can be seen in Fig. 26, the measured displacements in the stainless-steel experiments are considerably less than those of the aluminum tests, indicating greater strength in the steel as expected. The stainless-steel results show less self-consistency than the 1100-O aluminum data. The calculated growth rate obtained with the flow-stress formulation of Eq. 27 lies between the experimental points.

The time variation of the flow stress at the center of the bubble is shown in Fig. 27. The peak flow stress is much larger than in the aluminum calculations because of the larger flow stress coefficient Y_o and because of the greater pressure hardening. Strain-rate hardening is less important in this calculation than in the aluminum calculations, since the strain rates tend to be about an order of magnitude smaller because of the reduced deformation. Also, there is no increase in temperature late in the calculation as there was in the aluminum calculations due to the reduced deformation.

TABLE III
PARAMETER VALUES FOR 304 STAINLESS STEEL CALCULATIONS

Flow Stress Coefficient	3.5 kb
Sublimation Energy	6.28×10^{10} erg/g
Reference Strain Rate	10^{10} s ⁻¹
Strain-Hardening Modulus	9.8 kb
Initial Perturbation Amplitude	0.0204 cm
Perturbation Wavelength	0.508 cm
Plate Thickness	0.1905 cm

Fig. 26. Spike-bubble displacement for 304 stainless steel. The datum points show the measurements^{1,2} and the curve shows the calculated results when the flow stress, Eq. (27), incorporates the modified Shmatov¹⁰ model for the pressure dependence and the Zhurkov and Sanfirova¹³ model for the temperature and strain rate.

Fig. 27. Transient variation of the flow stress [Eq. (27), 304 stainless steel] in a calculation cell at the center of the bubble as indicated in Fig. 6. The magnitude of the flow stress is greater than in the aluminum calculations and is more sensitive to pressure during the unloading phase.

IV. CONCLUSIONS

The constitutive relations that have been used in the SCRAM calculations described in this report are based on phenomenological models and data from the open literature, most of which were developed by varying one parameter and keeping all other parameters approximately constant. We have assumed that these various models could be combined in a multiplicative fashion. The validity of this assumption could be tested only by comparing calculations that utilize these constitutive relations with experimental data obtained under conditions much different from those on which the constitutive modeling is based.

Most of the model testing is done for 1100-O aluminum because this is the material for which the most consistent data are available in References 1 and 2. The temperature and strain-rate dependencies of the flow stress make use of extensive data from Lindholm, Yeakley, and Bessey¹⁴ to extend the Zhurkov and Sanfirova¹³ model, while for the pressure dependence we use both the Spitzig and Richmond⁹ and the modified Shmatov¹⁰ models. Each of these combined models gives good agreement with the Barnes et al.² data using modeling parameters that are entirely consistent with the previous data. We also compare calculated results using a power-law temperature and strain-rate model that we obtained from Shawki¹⁶, combined with the modified Shmatov¹⁰ model, with the 1100-O aluminum data from Ref. 2. Again, the agreement is good when we use model coefficients that are very close to those of Shawki.

A further test of the accuracy of the constitutive relation that combines the modified Shmatov¹⁰ model for pressure with the Lindholm, Yeakley, and Bessey data¹⁴ for temperature and strain rate is obtained by varying the parameters in this combined model. We also examine the sensitivity of the results to variations in the strain-hardening coefficient and the perturbation amplitude and wavelength. The results are sensitive to variations in the flow-stress coefficient and the strain-hardening modulus, and less sensitive to variations in the reference strain rate and the sublimation energy, which enter into the temperature expression. However, it should be noted that the temperature range examined in these calculations is not large, so a greater sensitivity of results to variations in the reference temperature and strain rate might result at larger temperatures.

Previous studies^{19,20} had encouraged the belief that the stability of metal plates was governed by the amplitude of the initial perturbations but not by the wavelength of those perturbations. We show that the growth rate is governed by both quantities. If either the initial amplitude or the wavelength is sufficiently

small, there will be no Rayleigh-Taylor instability. In comparing calculations with the data for 6061-T6 aluminum and 304 stainless steel in References 1 and 2, we have used only the modified Shmatov¹⁰ model and the Zhurkov and Sanfirova¹³ model. These are preferred because it is relatively easy to find data in the literature with which to fit these models for a wide variety of materials. The agreement of the calculated instability with experiment is good for the 6061-T6 aluminum when appropriate values of the modeling coefficients are used. However, the initial flow stress coefficient that is required to give good agreement with the data for 304 stainless steel is large compared to published values for the yield strength for that material. We suggest as a possible explanation for this discrepancy the existence of a prestrain in these steel plates as a result of the machining of the perturbations or as a result of the rolling process in forming the sheet.

ACKNOWLEDGMENT

I would like to thank John Dienes for his invaluable help in the course of this study and for his assistance in reviewing the manuscript.

REFERENCES

1. P. J. Blewett, "Perturbation Growth in Aluminum and Steel Plates Accelerated by High Explosives - Interim Report," unpublished report, Los Alamos National Laboratory (1973).
2. J. F. Barnes, P. J. Blewett, R. G. McQueen, K. A. Meyer, and D. Venable, "Taylor Instability in Solids," *J. Appl. Phys.* **45**, 727 (1974).
3. P. J. Blewett, "Stress Calculations in MAGEE Difference Form," Los Alamos Scientific Laboratory report LA-4601-MS (1971).
4. A. A. Amsden, H. M. Ruppel, and C. W. Hirt, "SALE: A Simplified ALE Computer Program for Fluid Flow at All Speeds," Los Alamos Scientific Laboratory report LA-8095 (1980).
5. J. K. Dienes, "Statistical Crack Mechanics," Colloque International du CNRS No. 351, Failure Criteria of Structured Media, Villard-de-Lans, France (1983).
6. J. K. Dienes, "Fracture, Failure, and Fragmentation," Symposium on Advances and Trends in Structures and Dynamics, Washington University, Washington, D.C. (1984).
7. J. K. Dienes, "A Statistical Theory of Fragmentation Processes," *Mech. Mat.* **4**, 325 (1985).
8. L. F. Vereshchagin and V. A. Shapochkin, "The Effect of Hydrostatic Pressure on Resistance to Shear in Solids," *Fiz. Metal. Metall.* **9**, 258 (1960).

9. W. A. Spitzig and O. Richmond, "The Effect of Pressure on the Flow Stress of Metals," *Acta Metall.* **32**, 457 (1984).
10. V. T. Shmatov, "Lattice Resistance to Dislocation Motion under Hydrostatic Pressure," *Fiz. Metal. Metall.* **35**, 277 (1973).
11. J. Jung, "A Note on the Influence of Hydrostatic Pressure on Dislocations," *Phil. Mag.* **43**, 1057 (1981).
12. M. F. Ashby and R. A. Verrall, "Micromechanisms of Flow and Fracture, and their Relevance to the Rheology of the Upper Mantle," *Phil. Trans. R. Soc. Lond.* **A288**, 59 (1977).
13. S. N. Zhurkov and T. P. Sanfirova, "Relation between Strength and Creep of Metals and Alloys," *Zhur. Tekh. Fiz.* **28**, 1719, *Soviet Physics-Technical Physics* **3**, 1586 (1958).
14. U. S. Lindholm, L. M. Yeakley, and R. L. Bessey, "An Investigation of the Behavior of Materials under High Rates of Deformation," Air Force Materials Laboratory report AFML-TR-68-194 (1968).
15. R. W. Klopp, R. J. Clifton, and T. G. Shawki, "Pressure-Shear Impact and the Dynamic Viscoplastic Response of Metals," *Mech. Mat.* **4**, 375 (1985).
16. T. G. Shawki, "Analysis of Shear Band Formation at High Strain Rates and the Visco-plastic Response of Polycrystals," Ph.D. thesis, Brown University, Providence, RI (1985).
17. A. Reuss, "Berucksichtigung der elastischen Formanderung in der Plastizitats Theorie," *Zeits. Ang. Math. Mech.* **10**, 266 (1930).
18. D. J. Steinberg, S. G. Cochran, and M. W. Guinan, "A Constitutive Model for Metals Applicable at High-Strain Rate," *J. Appl. Phys.* **51**, 1498 (1980).
19. D. C. Drucker, "'Taylor Instability' of the Surface of an Elastic-Plastic Plate," *Mechanics Today* **5**, 37 (1980).
20. J. F. Barnes, D. H. Janney, R. K. London, K. A. Meyer, and D. H. Sharp, "Further Experimentation on Taylor Instability in Solids," *J. Appl. Phys.* **51**, 4678 (1980).
21. Metals and Ceramics Information Center, "Aerospace Structural Metals Handbook," Battelle Columbus Laboratories, Columbus, Ohio (1985).
22. F. A. McClintock and A. S. Argon, Eds., *Mechanical Behavior of Materials*, Addison-Wesley Publ. Co., Reading, MA (1966).