THE NUCLEAR MATTER EQUATION OF STATE
FROM RELATIVISTIC HEAVY IONS TO SUPERNOVAE

John W. Harris

Nuclear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

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INTRODUCTION

From a few microseconds after the beginning of the universe, when quarks became confined in nuclear particles, until the explosive end of the nuclear lifecycle of large stars, the nuclear matter equation of state plays an important role in governing processes that are fundamental to understanding the world around us. Until recently the study of the nuclear equation of state was confined to temperatures and densities near those of ground state nuclei. In relativistic collisions of nuclei high temperatures and densities are attained providing a unique opportunity to study, in a somewhat controlled manner, nuclear matter under extreme conditions.

In this presentation the relationship between relativistic nucleus-nucleus collisions and the nuclear equation of state will be discussed. The connection between observables measured in the experiments and thermodynamic variables used to describe the system will be made. Through this connection a semi-empirical nuclear equation of state is extracted from the data. The resulting equation of state will be discussed in terms of nuclear matter calculations, neutron star stability and supernova collapse.

RELATIVISTIC NUCLEUS-NUCLEUS COLLISIONS, EXPERIMENTAL OBSERVABLES AND THERMODYNAMIC VARIABLES

In collisions of two heavy nuclei at the Bevalac where velocities approach the speed of light, three distinct stages of the reaction are predicted - compression, high density and expansion. On the microscopic level binary nucleon-nucleon collisions initially dominate as the two nuclei collide. As the nuclei interpenetrate, successive nucleon-nucleon collisions occur in the interaction region. In macroscopic terms this results in conversion of part of the energy of relative motion into internal degrees of freedom while also increasing the temperature and density of the interaction region. This first stage of the collision is the compression stage. The dynamics of the collision is easily seen in predictions of the intranuclear cascade model shown in Fig.1 for central (small impact parameter) collisions of La + La at 1.0 GeV/n. Plotted logarithmically are the a) baryon density, b) number of baryon-baryon collisions per unit time and c) number of produced particles (pions and delta resonances) in the system as a function of time in the collision. The number of created particles represents part of the energy that is transformed from relative motion into other degrees of freedom. All three variables in Fig.1 are observed to increase rapidly during the compression stage. At a somewhat later time, in the middle of the collision process, the baryon density reaches a maximum before rapidly decreasing exponentially. This is the high density stage of the collision.
Fig. 1. Time dependence of the baryon density, number of baryon-baryon collisions per fm/c and number c. pions + deltas from the intranuclear cascade.\textsuperscript{1,3}

Fig. 2. Time dependence of the baryon density, pion multiplicity and flow angle from VUU calculations.\textsuperscript{5}
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Here the baryon-baryon collision rate peaks at approximately 90 collisions per fm/c and the number of created particles also reaches a maximum. The exponential decrease in density, the rapid decrease in the collision rate, and a saturation of the abundance of produced particles characterizes the third stage, namely expansion.

To understand the behavior of nuclear matter at high temperatures and densities, it is important to study central collisions. From rate estimates it has been shown\textsuperscript{3} that both thermal and chemical equilibrium of pions, nucleons and deltas should be attained in the high density stage of the collision. Thermal equilibrium continues late into the expansion stage while chemical equilibrium, which determines the number of deltas and pions, cannot be maintained in the expansion. The final number of pions in the system is determined at the time of high density and should not change with expansion. This is observed in Fig.1 as the constancy of the pion multiplicity from the time of highest density onwards. A prediction for the pion multiplicity from a chemical equilibrium model\textsuperscript{4} is also shown and agrees with the cascade result. Intuitively, the large number of baryon-baryon collisions in the high density stage strongly suggests a large amount of "mixing" which leads to the predicted equilibrium conditions.

Results of another microscopic model Vlasov-Uehling-Uhlenbeck (VUU)\textsuperscript{5} which also includes potential effects in the form of a mean field are displayed in Fig.2 for the system Nb + Nb at incident energy $E_{\text{lab}}=1.05 \text{ GeV/n}$ and $3 \text{ fm}$ impact parameter. Again there is a high density region after which time the total pion multiplicity decreases slightly then remains constant. In addition, the flow angle is established at this time. The flow angle refers to the direction of maximum energy flow in an event and like the pion multiplicity provides information on the high density stage of the reaction. In addition to the pion multiplicity and flow angle, the entropy has been shown\textsuperscript{6} to remain constant from the beginning of the expansion onward. The entropy can best be extracted from a determination of the nuclear cluster concentrations,\textsuperscript{7} however neither comprehensive measurements of the physical observables (ratios of p, d, t, $^3\text{He}$, $^4\text{He}$, ... ) nor a consistent method of interpretation in terms of the thermodynamic variable (entropy) have yet been made. In the following, only results from the pion production and flow of matter will be presented.

PION PRODUCTION

The sensitivity of pion production to the nuclear equation of state was initially pointed out by several authors.\textsuperscript{8,9} The first quantitative results\textsuperscript{10} were obtained by using hydrodynamics and are shown in Fig.3. The pion multiplicity is significantly lower for the stiff ($K_0 = 300 \text{ MeV}$) equation of state, which has higher compressional energy at a given baryon density, than for the softer one ($K_0 = 100 \text{ MeV}$). Displayed in Fig.4 are the experimentally observed ratios of pions to participant nucleons as a function of incident energy taken in the Streamer Chamber\textsuperscript{11} at the Bevalac. These ratios rise monotonically with energy and are identical for both the Ar + KCl and La + La systems. Also shown are the prediction of the chemical model and predictions from two intranuclear
Fig. 3. Relative pion multiplicity as a function of incident laboratory energy assuming relatively a) stiff and b) soft equations of state.\textsuperscript{10}

Fig. 4. Ratio of pions to participant nucleons as a function of incident energy.\textsuperscript{3,13}
cascade models,\(^1,\(^{12}\) all of which are similar and overpredict the observed pion/participant ratios. These models do not incorporate potential degrees of freedom as manifested in the equation of state. In a simple approximation\(^1\(^\text{3}\) the total energy available in the system can be partitioned into the kinetic energy and the potential energy degrees of freedom. The kinetic energy is available for particle production while the potential energy is not. Thus, the noninclusion of potential degrees of freedom in the purely thermal models (chemical and cascade) lead to all the energy being available for pion production, thus the overprediction of the pion multiplicity. With only kinetic degrees of freedom, the results of VUU calculations without an equation of state (not shown) are very similar to those of the thermal models. However, when potential degrees of freedom are included, in the form of a stiff equation of state, the VUU predicts the observed pion multiplicities as seen in Fig.4.

COLLECTIVE FLOW OF MATTER

Recent observations of sidewards flow of matter have provided further evidence for the necessity to include potential degrees of freedom, in descriptions of relativistic nucleus-nucleus collisions. Both the Plastic Ball\(^1\(^\text{5}\) and Streamer Chamber\(^1\(^\text{6}\) groups have observed sidewards flow. A systematic study\(^1\(^\text{7}\) of the energy and mass dependence of sidewards flow has just been completed using the Plastic Ball. The mass and multiplicity dependence of the data are displayed in Fig.5. Plotted as a function of fractional multiplicity, which is inversely related to the impact parameter, are the distributions of flow angles for three systems Ca + Ca, Nb + Nb and Au + Au at fixed incident energy. The flow angle is defined as the angle of the major axis of an event ellipsoid with respect to the beam axis. This angle is determined from a sphericity analysis where the ellipsoid represents the shape of an event in a (weighted) momentum or velocity space. The flow angle is observed to increase with the mass of the incident system and the fractional charged-particle multiplicity, i.e. the centrality of the collision. The observation of finite flow angles is a strong indication for collective sidewards flow in these events. Models which do not include potential degrees of freedom, such as the intranuclear cascade, do not predict the large flow angles observed.

A more sensitive global event analysis technique is the transverse momentum \((p_t)\) analysis.\(^1\(^\text{8}\) Displayed in Fig.6a are the mean \(p_t\) per particle projected onto the event reaction plane as a function of the particle’s rapidity for Streamer Chamber data. As observed in the sphericity analysis for the heavier systems, a definite sidewards flow is observed for the lighter Ar + KCl system from this analysis. Using this technique, the sidewards flow appears in the form of a mean transverse momentum boost which is opposite in the forward and backward hemispheres of the c.m. system. As displayed in Fig.6b, the intranuclear cascade predicts much lower values for the mean \(p_t\) projected onto the reaction plane. VUU with the stiff equation of state, used to describe the pion multiplicities, predicts fairly well the mean \(p_t\) in the reaction plane as shown in Fig.6c. When a soft equation of state is incorporated into the VUU code the mean \(p_t\) is underpredicted as seen in Fig.6d. The actual forms of these
Fig. 5. Flow angular distributions for three systems at fixed incident energy as a function of fractional multiplicity.\textsuperscript{17}

Fig. 6. Transverse momentum per nucleon projected into the reaction plane\textsuperscript{18} as a function of rapidity for 1.8 GeV/n Ar + KCl a) data, b) cascade and VUU model with c) stiff and d) soft equations of state.\textsuperscript{5}
equations of state will be presented below. Together the pion multiplicity data and the flow data support a stiff nuclear equation of state in relativistic nucleus-nucleus collisions.

THE NUCLEAR MATTER EQUATION OF STATE

A semi-empirical equation of state has been extracted from the pion multiplicity data. If the total available c.m. energy is partitioned into thermal and potential energies, the thermal energy can easily be found using the “thermal” models (chemical and cascade) in Fig.4. The potential energy is the difference between the total c.m. energy of the experiment and the thermal energy which is the energy necessary in the “thermal” models to predict the observed pion multiplicity. These potential energies as a function of c.m. energy are represented by arrows in Fig.4. The relationship between the potential energy per nucleon and the nuclear density will be referred to as the nuclear equation of state. To derive an equation of state from the potential energies of Fig.4 for the chemical model, the densities were found by assuming one dimensional shock compression. The resulting equation of state incorporating the density dependent Fermi energy is shown in Fig. 7. A previous analysis using the intranuclear cascade densities determines an equation of state which lies within the error bars of the chemical approach. Also shown in Fig.7 are the FP, BCK and VUU equations of state. The VUU curve is the stiff equation of state which successfully predicts both the pion multiplicities and the flow data. The FP equation of state has been used successfully in nuclear matter calculations and BCK in supernova simulations. Both are considerably softer than that determined from the data and the stiff equation of state used in VUU calculations. In fact, the soft equation of state used in the VUU calculations of Fig.6d above is similar to the FP curve. The discrepancy between the various equations of state that are used for nuclear matter, supernova explosions and relativistic nucleus-nucleus interaction calculations may be understood by investigating the regions of temperature and density that the calculations address. Nuclear matter calculations are sensitive to the nuclear equation of state at zero temperature near the saturation density of nuclear matter \( \rho_0 \). Neutron star structure and supernova explosions are governed by the equation of state at low temperatures, \( T \sim 15 \text{ MeV} \), and densities \( \rho \sim 2 \rho_0 \). On the other hand, relativistic nucleus-nucleus collisions occur at high temperatures, \( T \sim 60 - 100 \text{ MeV} \), and densities \( \rho \sim 3-4 \rho_0 \). The behavior of the nuclear equation of state under these drastically different conditions must still be understood. The equation of state is expected to be stiffer at higher temperatures due to a decrease in the effective mass of the nucleons at high \( T \) and a decrease in the strength of the attractive sigma-interaction coupled with an increase in strength of the repulsive omega-interaction.
Equation of State of Nuclear Matter

Fig. 7. Internal energy per baryon as a function of nuclear density extracted from pion multiplicity data (points). Also displayed are the "stiff" VUU, FP\textsuperscript{20} and BCK\textsuperscript{21} equations of state. See text for details.
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