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SOLID STATE NEUTRON
DETECTORS

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SOLID STATE NEUTRON DETECTORS

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Lieutenant Commander, USN

May, 1961

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ABSTRACT

Two methods of fabricating neutron detectors have been developed based upon the principles of the solid state ionized particle detector. The (n, α) reactions of Li^6 and B^{10} are used for slow neutron detection. The lithium and boron are used as diffusants, for p- and n-type silicon respectively, to create p-n junction diode detectors with efficiencies of the order of 1%. The fission of U^{235} is used for the detection of both slow and fast neutrons. The U^{235} is vacuum-coated onto surface barrier detectors. Efficiencies of 0.004% have been obtained from these units in a thermal beam from the Livermore 2Mw pool-type reactor.

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I. INTRODUCTION

The development of ionized particle detectors has received considerable attention during the past few years.¹⁻⁶ At the present time the Chalk River Laboratory,⁷ the Lewis Research Center,⁸ and the Westinghouse Electric Corporation⁹ are all experimenting with ionized particle detectors which have been coated with neutron-sensitive elements. This paper is concerned with the use of Li^6 and B^{10} as diffusants to directly form silicon p-n junction neutron detectors, and the use of uranium to form surface-barrier neutron detectors.

Naturally occurring lithium, an interstitial donor, has recently been used by Elliott⁶ to form a p-n junction and to extend the depletion region with an ion drift process. A similar process for the separated isotope Li^6 is discussed in the present paper. In this case neutron detection is accomplished with a $\text{Li}^6(n, \alpha)\text{T}$ reaction, converting captured neutrons into charged particles which in turn are detected within the depletion region. The p-n junctions were also directly formed by a shallow diffusion of B^{10} (90-92% enriched) in n-type silicon. Here the $\text{B}^{10}(n, \alpha)\text{Li}^7$ reaction formed ionized particles for neutron detection.

Additionally, neutron detection was investigated by means of the fission process in U^{238} and U^{235} for the conversion of fast and slow neutrons. In this case the restricted range of the highly ionized fission fragments required the use of a surface-barrier junction to detect the product charged particles. This was achieved by vacuum-coating uranium onto preformed gold surface-barrier silicon diodes. Attempts were also made to form uranium surface barriers directly on silicon.

II. GENERAL THEORY

1. Crystalline structure

All solids may be classified as crystals or non-crystals. A crystal is a structure which is produced by the repetition of a basic unit due to translation, and is characterized by extreme regularity. If this regularity or

periodicity extends throughout the entire piece of material the piece is said to be a "single crystal." If the piece of material is an agglomerate of smaller crystallites, it is said to be "polycrystalline." In a non-crystal the atoms or atom groups are present in an irregular or random fashion. Examples of non-crystals are supercooled liquids, glasses and plastics (plastics sometimes exhibit a repetition of a basic unit, but it is of spiral type and not translatable).

Crystals may be divided into seven crystal systems, fourteen space lattices, two hundred and thirty space groups and an infinite number of crystal structures. A complete discussion of these systems may be found in any basic solid state physics text.¹⁰

Crystalline silicon is intrinsically an insulator with a valence of four. Like germanium, it has a diamond structure in which each cell is composed of 8 atoms and each atom has 4 nearest neighbors. Strong bonds between the atoms result from a sharing or mutual exchange of electrons with its nearest neighbors. This type of binding is particularly stable and is called "covalent binding." It is of an electrostatic nature, but can be explained only on a quantum-mechanical basis in terms of exchange interaction.

2. Band structure

The theory of band structure is contained in any standard solid state physics text. The electron energy levels are determined by the quantum numbers. In a collection of atoms of the same type, the electron energy levels will separate into bands of energy levels. These bands have energy widths characteristic of the solid, but the number of levels within the band is determined only by the number of atoms present. N atoms provide N levels within each band. However, each level can accommodate two electrons because of spin. Therefore, a collection of N atoms leads to $2N$ states within each band. In silicon and certain other diamond structures, there is a quantum-mechanically forbidden zone between the valence and conduction bands. (The conduction band is the lowest unfilled band at 0°K . The valence band is the next lower band.) Metals contain an overlapping of bands so that no forbidden zone exists at these higher energy levels. In silicon this forbidden zone or gap occurs between the 3s and 3p shells and has an energy span of 1.11 ev. In germanium the gap is 0.72 ev and occurs between the 4s and 4p shells. Diamond on the other hand has a comparatively large gap 6 ev, located between the 2s and 2p shells.

3. Fermi levels

The location of the Fermi level determines the type of silicon (i. e. , intrinsic (i), donor (n), or acceptor (p)).

Intrinsic silicon contains no impurities. At 0°K there will be no electrons in the conduction band, while the valence band will be full. At any elevated temperature some states in the conduction band will be occupied by electrons while an equal number of vacant states will exist in the valence band. It can be shown that since the number of electrons in the conduction band and the number of vacant states in the valence band are a function of the Fermi probability function, there is a virtual Fermi face midway between the two bands.¹¹

If a group V contaminant is introduced, the atoms of which are substituted for silicon atoms, there will exist one very loosely bound electron for each atom substituted. Only four of the contaminant's five valence electrons are required to complete the bonding with the silicon. To preserve charge neutrality the fifth electron must orbit the parent nucleus. The radius of the orbit is extended by the dielectric constant of the silicon. Since this fifth electron is loosely bound, it is easily excited into the conduction band of the silicon, leaving the parent nucleus ionized. This process provides an excess number of electrons in the conduction band; thus the Fermi probability function and consequently the Fermi face are shifted in the direction of the conduction band. Silicon containing a group V contaminant is called "donor" or "n" type.

If a group III contaminant is introduced substitutionally into silicon, there is a deficit of one bonding electron for each atom substituted. The vacant bond or hole can be thought of as an orbiting positive charge. Thermal excitation will fill this hole with a silicon valence electron, thus transferring the hole to a silicon neighbor. The process of filling and transferring gives mobility to the hole. The group III contaminant thus provides more holes in the valence band than there are electrons in the conduction band. The Fermi probability function and consequently the Fermi face resultantly shift in the direction of the valence band. In taking on the extra electron the contaminant takes on a negative charge. Silicon containing a group III contaminant is called "acceptor" or "p" type.

4. Surface barrier junction

A metal can be thought of as a structure formed by positively charged metal ions embedded in a sea of mobile free electrons. In spite of the fact that the electrons move freely from atom to atom, charge neutrality is preserved. The Fermi level lies at the upper edge of the electron distribution within the partially filled valence band. If by some mechanism additional electrons are introduced into the valence band, the Fermi level will be raised and the metal will take on a net negative charge. This charge will appear as an infinitesimally thin sheet of charge on the surface of the metal. If, on the other hand, electrons are somehow drained from the valence band, the Fermi level is lowered and the metal takes on a net positive charge. This charge also appears as a thin sheet on the surface. In a semiconductor the situation is somewhat similar. In n-type material the electrons in the conduction band are present as a result of excitations from the valence band and from ionization of the donor impurities. If the electrons are somehow drained from the conduction band, the Fermi level will be lowered and the semiconductor will take on a net positive charge. Unlike in the case of the metal, the charge in this case will be a distributed positive charge made up of the ionized donor atoms. Figure 1 shows a metal and an n-type semiconductor side by side, but not touching. In this case the work function (ϕ_m) of the metal is larger than the work function (ϕ_s) of the semiconductor. When the two are brought into contact, Fig. 2, electrons will drain from the conduction band of the semiconductor and will be injected into the valence band of the metal until the two Fermi levels are at the same energy. The metal now has a thin negative surface charge while the semiconductor has a distributed positive charge or "space charge region." An electric field is set up which is located entirely within the space charge region of the semiconductor. The height of the resulting potential barrier corresponds to the difference of the two work functions.

The current flow across the barrier is described in detail in the literature.^{10,11} The total current is made up of four components, the electron flow from the metal to the semiconductor (I_a), the electron flow from the semiconductor to the metal (I_b), the hole flow from the metal to the semiconductor (I_c), and the hole flow from the semiconductor to the metal (I_d). See Fig. 2. In equilibrium, an energy $q(\phi_m - \phi_s)$ must be supplied to the electrons to enable them to cross the potential barrier. Then

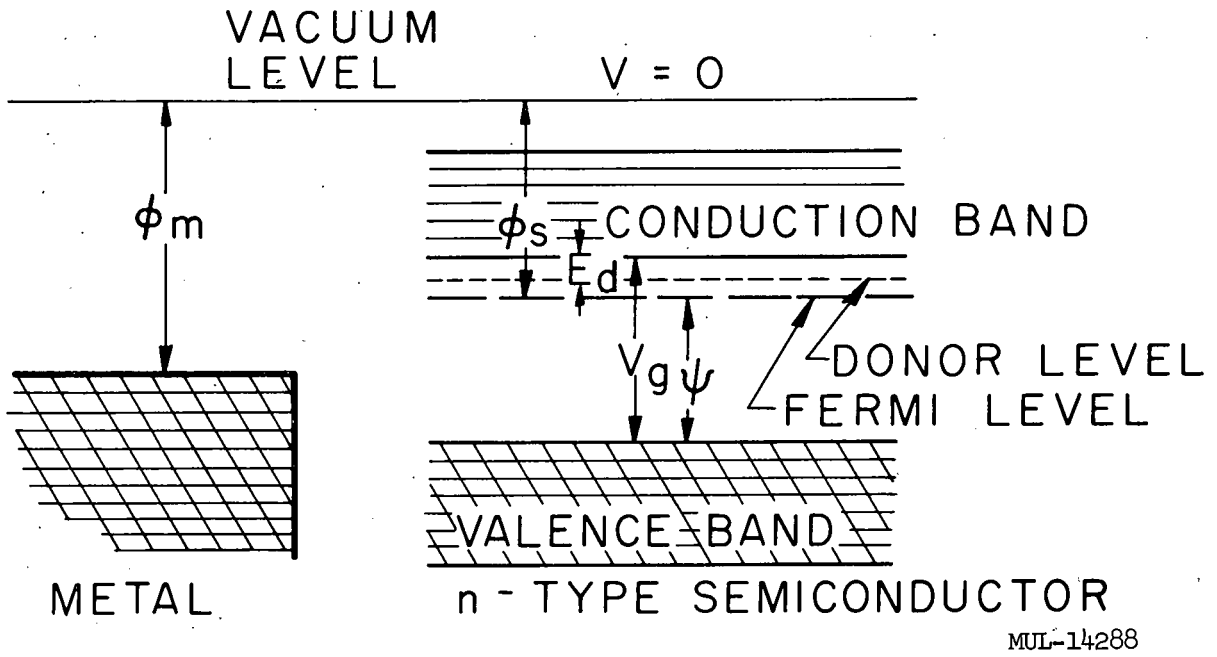


Fig. 1. Metal and n-type semiconductor showing the relative position of the Fermi levels for the case of $\phi_m > \phi_s$.

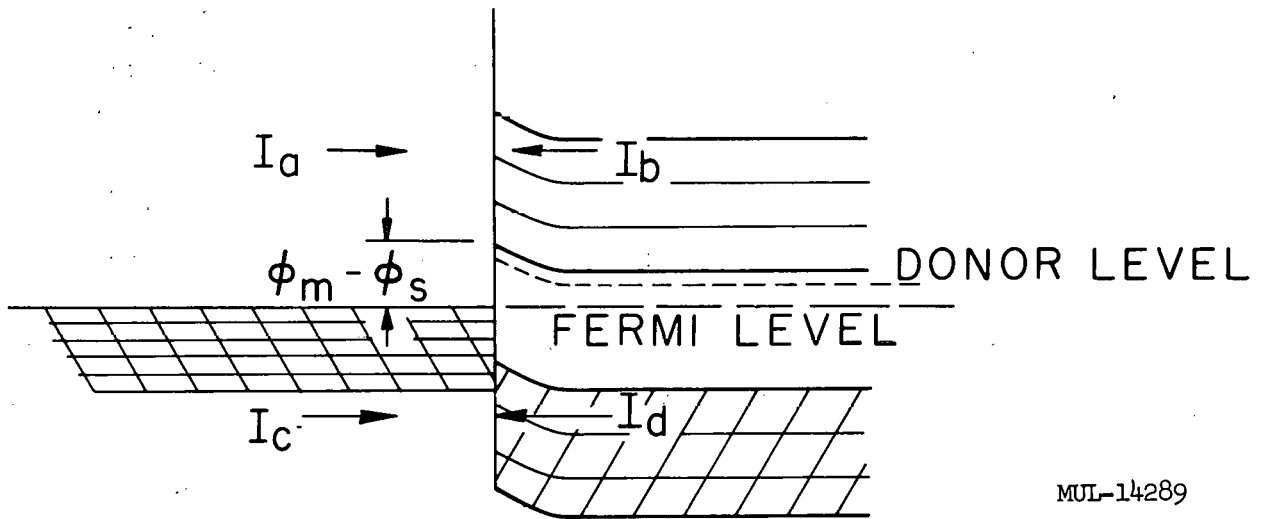


Fig. 2. Metal and n-type semiconductor joined to produce a barrier of height $\phi_m - \phi_s$.

$I_a = -I_b = A \exp[-q(\phi_m - \phi_s)/kT]$. The holes must be supplied energy in the amount of $q\psi$ in order to cross the barrier. Then $I_c = -I_d = B e^{-q\psi/kT}$

Under small forward or reverse voltage bias, the current components I_a and I_d will remain the same as for equilibrium conditions since the potential barrier they confront does not change. However, the potential barrier as seen by components I_b and I_c does change. This arises from the fact that when a bias voltage (V_b) is applied, the Fermi level in the semiconductor is raised or lowered depending on whether the bias is forward or reverse. Under forward bias, the amount of energy necessary to be supplied to I_b electrons and I_c holes, to cross the barrier, is reduced by the amount qV_b from the equilibrium condition. Under reverse bias the opposite is true.

When the Fermi level is raised under forward bias, it means that fewer electrons will be drained from the conduction band, hence fewer donors will be ionized and the space charge region (Eq. (A-10)) will be narrower. Conversely the space charge region will be increased under a reverse bias. This property gives rise to the effective capacity of the junction. In Eq. (A-12) it is noted that the expression for capacitance is the same as for a parallel plate capacitor. In a parallel plate capacitor the capacitance varies directly with the area of the plates and inversely with the separation between them, while the charge appears as a thin sheet of charge on each plate. In a metal-to-semiconductor contact there is a thin sheet of charge on the metal but on the other plate which is the semiconductor there is a distributed space charge. Applying a reverse bias increases the width of the space charge region, thus increasing the effective plate separation and decreasing the capacitance. The only other method of decreasing the capacitance is to physically reduce the area of the contact. The reduction of capacitance is important in detectors since the pulse height of the output pulse is inversely proportional to the capacitance.

By evaporating a thin coating of a metal onto n-type silicon, assuming ϕ_m is greater than ϕ_s , a rectifying contact is made which under reverse bias becomes a detector for ionized particles.

When applying leads to a semiconductor device care must be taken or a rectifying junction will occur where it is not desired. To obtain an ohmic (non-rectifying) junction between a metal and a semiconductor, the metal must be chosen such that its work function is smaller than that of the n-type semiconductor. In this case the Fermi levels must once again line up. To

accomplish this, electrons are drained from the valence band of the metal, lowering its Fermi level. These electrons are injected into the conduction band of the semiconductor raising its Fermi level. Since there are many unoccupied states in the conduction band of the semiconductor there is no opposition to the flow of electrons in either direction, and no potential barrier is set up. (Fig. 3.)

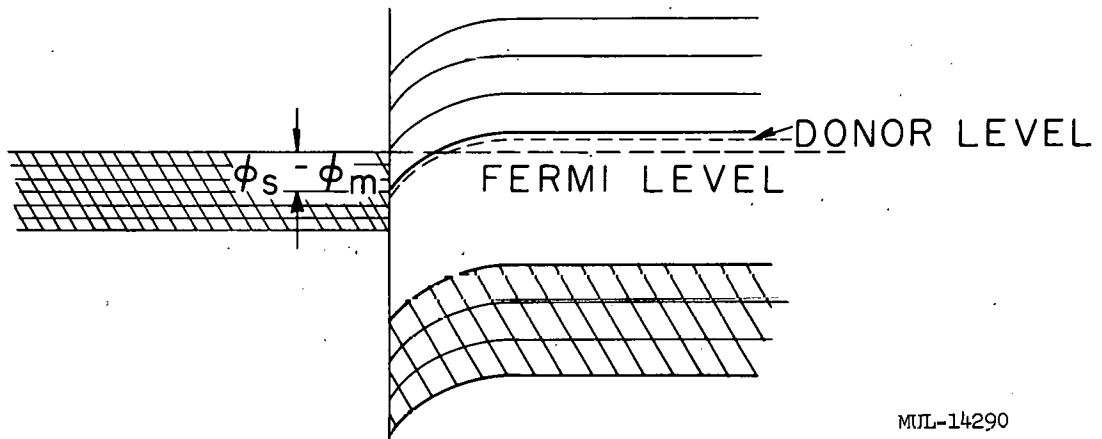
The analysis of the metal-to-p-type-semiconductor contact is similar to the one just carried out for the metal-to-n-type semiconductor. To obtain a rectifying contact between a metal and p-type semiconductor the metal must be chosen such that its work function is smaller than that of the semiconductor. To obtain an ohmic contact the work function of the metal must be greater than that of the semiconductor.

The solution of Poisson's equation for the case of the metal-to-semiconductor contact is contained in Appendix A. The charge distribution arises from the fact that there exists an abrupt change from low-resistivity material to high-resistivity material.

5. p-n junctions

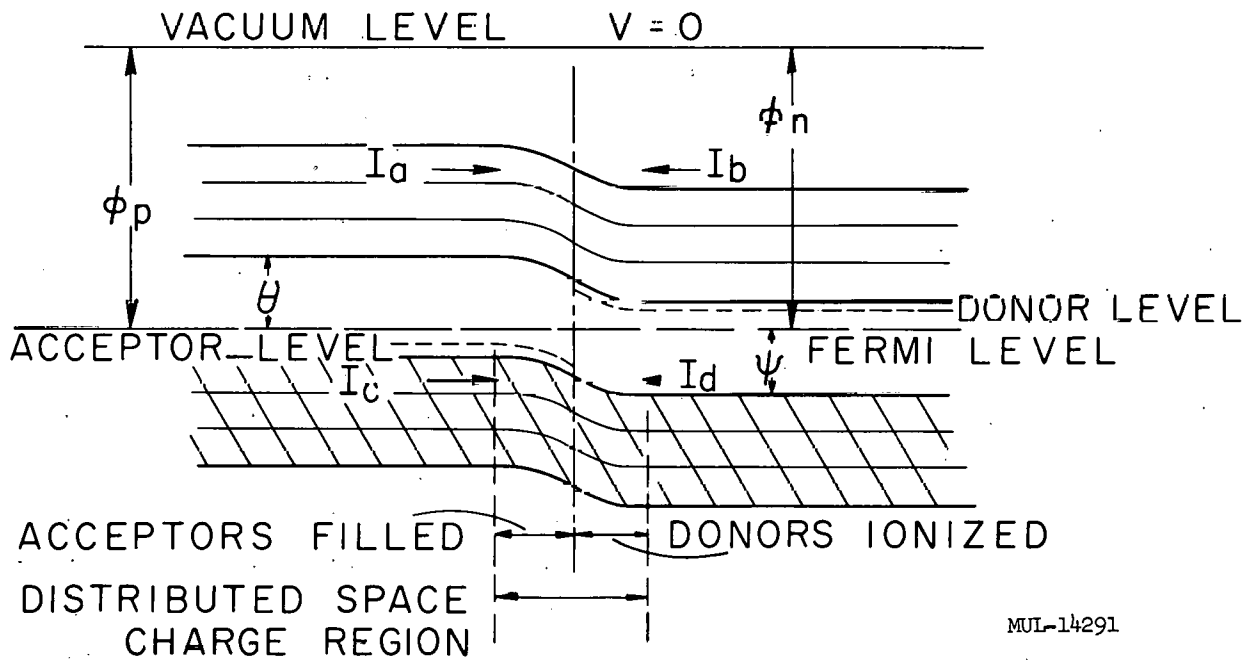
The contact between two semiconductors of opposite type behaves similarly to that between a metal and a semiconductor. The Fermi level of the two semiconductors must again line up. To accomplish this, electrons will drain from the n-type semiconductor and be injected into the conduction band of the p-type material. Holes will drain from the valence band of the p-type semiconductor and be injected into the valence band of the n-type material. This process equalizes the Fermi levels and creates a space charge region on both sides of the contact. The space charge region is composed of ionized donors and negatively charged acceptors in the n and p material respectively (Fig. 4).

The current is again composed of four components and is described in detail in the literature.^{12,13} I_a is the flow of electrons from the p- to the n-type semiconductor. I_b is the flow of electrons from the n- to the p-type semiconductor. I_c is the flow of holes from the p- to the n-type semiconductor and I_d is the flow of holes from the n- to the p-type semiconductor. In equilibrium, $I_a = -I_b = Ae^{-q\psi/kT}$, and $I_c = -I_d = Be^{-q\psi/kT}$. A and B are constants only for a constant temperature and they can be determined by solving the continuity equation and applying Fermi statistics.¹¹



MIL-14290

Fig. 3. Metal to n-type semiconductor ohmic contact.



MIL-14291

Fig. 4. p-n junction.

$$A = \frac{qD_n}{\sqrt{D_n t_n}} \cdot \frac{2(2\pi m k T)^{3/2}}{(h^2)}$$

$$B = \frac{qD_p}{\sqrt{D_p t_p}} \cdot \frac{2(2\pi m^* k T)^{3/2}}{(h^2)}$$

where

- q = electronic charge
- D_n = diffusion constant for electrons
- D_p = diffusion constant for holes
- t_n = minority carrier lifetime for electrons in p type material
- t_p = minority carrier lifetime for holes in n type material
- m = mass of the electron
- m^* = effective mass of the holes
- k = Boltzmann's constant
- h = Planck's constant
- T = temperature in degrees Kelvin.

Under reverse bias I_b and I_c reduce to almost zero. The total current flow then is the sum of I_a and I_d , which is called the "saturation current." The sum of the widths of the two space charge regions, called the "depletion region," is increased under reverse bias, Eq. (A-10) and the capacitance is decreased, Eq. (A-12). For the analysis of the abrupt p-n junction, see Appendix A.

The charge distribution for a diffused junction may be approximated by a straight line. For the solution of Poisson's equation for this graded type of junction, see Appendix B. Under certain circumstances, an extremely high concentration of dopant is located on one side of a diffused junction. As a result, the electric field and the depletion region would be located mostly within the lightly doped side.

6. Detection of ionized particles

Energetic charged particles passing through a solid produce hole-electron pairs through interactions with the solid's electrons. These electrons, even from deep-lying occupied bands, may be excited into unoccupied higher energy bands. The maximum energy that may be transferred to these electrons is $E_{\max} = \frac{4mME}{(m+M)^2} \approx \frac{4mE}{M}$, where m is the mass of the electron and

M and E are the mass and energy of the charged particles. For a 4-Mev alpha or a 1-Mev proton this is approximately 2 kev of energy. On the average only 3.5 ev of energy is expended to produce an electron-hole pair in silicon.^{2,13} It takes on the order of 10^{-12} sec for the excited electrons to seek the lowest-lying unoccupied band (conduction band) and for the holes to seek the highest occupied band (valence band). The hole and electron will then combine to complete the de-excitation with a characteristic time, known as the "carrier lifetime." If these electron-hole pairs are produced within the depletion region they are rapidly swept aside and collected owing to the action of the electric field therein. If the electron-hole pairs are produced outside the depletion region but within a diffusion length (L), they will probably diffuse into the depletion region and be collected. Diffusion length (L) is defined as $L = \sqrt{Dt}$, where D is the diffusion constant for holes or electrons, as the case may be, and t is the carrier lifetime. Electron-hole pairs created more than a diffusion length from the depletion region will recombine in their characteristic lifetime and not contribute to the output pulse.

Within the depletion region the clouds of electrons and holes pass each other so rapidly, owing to the action of the electric field, that very little recombination can take place. If, however, certain impurities or crystal defects are present, trapping or recombination centers may be set up. A trapping center is a center in which a carrier may be temporarily trapped and then re-emitted. Carriers re-emitted from a trapping center cause a tail to form on the current pulse. A recombination center is a center in which a carrier may also be temporarily trapped but where prior to re-emission a carrier of the opposite type is also trapped, in which case the two carriers recombine. Recombination centers cause a degradation of the current pulse since fewer charges will be collected. The contribution of each carrier to the total charge is derived as follows:

$$m \frac{dv}{dt} = q\mathcal{E} \quad \text{or} \quad mv \frac{dv}{dt} = \frac{dE}{dt} = qv\mathcal{E}.$$

$\frac{dE}{dt}$ is the rate at which the electric field gives energy to the carrier, and q, v, and m are charge, velocity and mass of the carrier. This energy comes from the junction capacitor which has $\frac{Q^2}{2C}$ of energy stored. Thus

$$\frac{dE}{dt} = \frac{d(Q^2)}{dt(2C)} = \frac{Qi}{C} = V_c i = qv\mathcal{E}.$$

V_c is the total voltage across the capacitor. Then

$$i = \frac{dQ}{dt} = \frac{qv\zeta}{V_c} = \frac{q}{V_c} \frac{dx}{dt} \frac{dV}{dx} = \frac{q}{V_c} \frac{dV}{dt}$$

$\frac{dV}{dt}$ is the rate of change of the potential at the location of the carrier due to the carrier drift in the electric field. Finally,

$$\int dQ = Q_{\text{eff}} = \frac{q}{V_c} \int dV = \frac{q\Delta V}{V_c},$$

where Q_{eff} is the effective charge at the electrodes due to the motion of one carrier through the fraction $\Delta V/V_c$ of the total potential on the capacitor.

Thus it can be seen that one carrier will contribute one charge q on making a complete passage across the capacitor. An electron-hole pair created between the plates of the capacitor will still only contribute one charge q , since each carrier will contribute a fraction of the charge. The total charge (Q) is equal to $Q_{\text{eff}}N$, where N is the number of hole-electron pairs liberated by a particle whose energy is E . If E is in Mev then

$$Q = \frac{Q_{\text{eff}}E}{3.5} \times 10^6 \text{ coulombs.} \quad (1)$$

The charge collected is some fraction of the total charge available, because some of the hole-electron pairs are created more than a diffusion length from the depletion region, while others are lost due to recombination within recombination centers.

The current pulse produced by the collection of the hole-electron pairs will have a rise time which is the carrier transit time (t_t) across the depletion region.

$$t_t = \frac{d}{v_d} = \frac{d}{\mu\mathcal{E}},$$

where d is the width of the depletion region, v_d is the carrier drift velocity and μ is the carrier mobility. These carrier transit times are typically of the order of nanoseconds. The carrier mobilities are functions of the impurity concentration and temperature. Typical values for mobility at room temperature for high resistivity silicon is $\mu_n = 450 \text{ cm}^2/\text{volt-sec}$ and $\mu_p = 1200 \text{ cm}^2/\text{volt-sec}$. Curves of electron and hole mobilities as functions of the impurity concentration may be found in reference (14).

The ideal solid state detector should have rapid charge collection with an accompanying low loss of carriers. Rapid charge collection is attained through use of high-resistivity material. High-resistivity material has high carrier mobilities and in addition is capable of supporting large electric fields. The low loss of carriers is attained by having wide depletion regions which originate at the surface, and by absence of recombination centers.

7. Neutron detection

Neutrons, possessing no charge, can be detected only through secondary elastic or inelastic reactions.

a. Elastic reactions

Elastic collisions between fast neutrons and a hydrogenous material produce recoil protons. These protons can then be detected in the manner described in the previous section. Fast-neutron detection devices might thus be fabricated merely by encapsulating the detector in plastic. No investigations were made using recoil proton counters in this study.

b. Inelastic reactions

Vacuum-coating p-n or surface barrier detectors with fissionable material presents a means of detection of both slow and fast neutrons. The fragments resulting from fission are highly ionized and possess high kinetic energies. These fragments are detectable as described in Section II-6, producing very large pulses.

Uranium was used as the fissionable material in this investigation. It was desired to check previous work where uranium was coated on preformed p-n junction detectors^{7,8} and to compare these units with uranium-coated surface barrier detectors.

As a first approximation the energy of the incident neutron may be neglected, since the reaction energy for uranium is approximately 200 Mev. In this case the laboratory coordinate system will coincide with the center-of-mass coordinate system. The average mass of the two fission fragments of U^{235} are 97 and 139 amu.¹⁶

$$T_L = \frac{M_H}{M_L + M_H} R = \frac{139}{236} 200 = 118 \text{ Mev,}$$

$$T_H = \frac{M_L}{M_L + M_H} R = 82 \text{ Mev,}$$

where T_L and M_L are the kinetic energy and mass of the light fragment and T_H and M_H are the kinetic energy and mass of the heavy fragment.

Many fissionable materials will not fission with slow neutrons. The neutron energy at which they commence to fission is called the "threshold energy." Some examples are:¹⁷

Isotope	Threshold energy (Mev)	Cross section at 2 Mev (barns)
$^{90}\text{Th}^{232}$	1.3	0.11
$^{91}\text{Pa}^{231}$	0.5	1.05
$^{92}\text{U}^{235}$	0	1.32
$^{92}\text{U}^{236}$	1.0	0.8
$^{92}\text{U}^{238}$	1.0	0.53
$^{94}\text{Pu}^{239}$	0	2.0

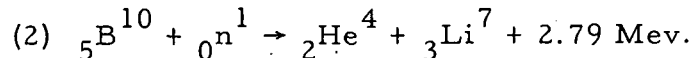
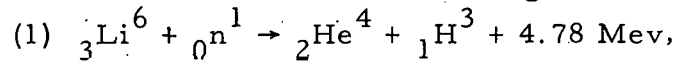
If it is desired to detect only fast neutrons, one of the isotopes with a high threshold energy might be used for the detector coating. One difficulty arises from the fact that isotope separation is difficult for these high-mass elements. Thus a cadmium absorber would be required to selectively filter out the slow neutrons, which might otherwise strongly react with the thermally fissionable impurities.

Slow neutrons may be detected by using (n, α) or (n, p) reactions. In general these are exoergic reactions. To determine the total charge which may be obtained, Eq. (1) must be modified to include the reaction energy (R).

$$Q_T = \frac{Q_{\text{eff}} (E_n + R) \times 10^6}{3.5} \text{ coulombs,} \quad (2)$$

where R is in Mev.

The two reactions used in this investigation are:



Both Li^6 and B^{10} can be used as impurity diffusants to fabricate solid state neutron detectors. Lithium is a group I element. It acts as a donor since it diffuses interstitially. Boron is a group III acceptor.

The average thermal neutron energy (E_n) is 0.025 ev. As a first approximation E_n can be neglected, since $E_n \ll R$.

$$Q_T \approx \frac{Q_{\text{eff}} R \times 10^6}{3.5} \text{ coulombs.} \quad (3)$$

Since E_n is neglected the laboratory coordinate system again coincides with the center-of-mass coordinate system. The kinetic energies of the reaction particles are:

$$T_1 = \frac{M_2}{M_1 + M_2} R, \quad (4)$$

$$T_2 = \frac{M_1}{M_1 + M_2} R. \quad (5)$$

T_1 and M_1 are the kinetic energy and mass of the first reaction particle. T_2 and M_2 are the kinetic energy and mass of the second reaction particle. For reactions (1) and (2) the reaction particle energies are

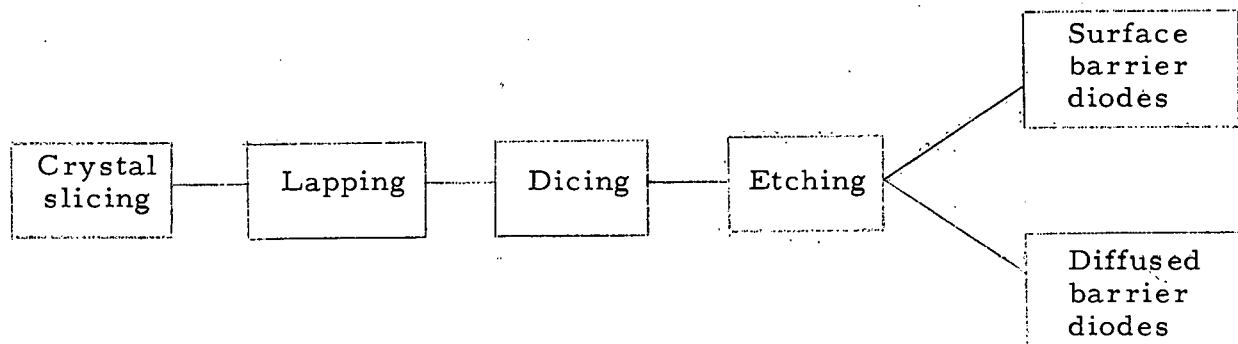
<u>Reaction</u>	<u>Particle</u>	<u>Kinetic energy (Mev)</u>
(1)	${}_2\text{He}^4$	2.05
(1)	${}_1\text{He}^3$	2.73
(2)	${}_2\text{He}^4$	1.78
(2)	${}_3\text{Li}^7$	1.01

The total charge (Q_T) is independent of the angle of incidence of the neutron since the kinetic energy of the reaction particles is due almost entirely to the reaction energy (R).

III. PREPARATION OF DETECTORS

1. Crystals

Diode preparation requires several mechanical operations common to both surface-barrier and p-n types, as follows:

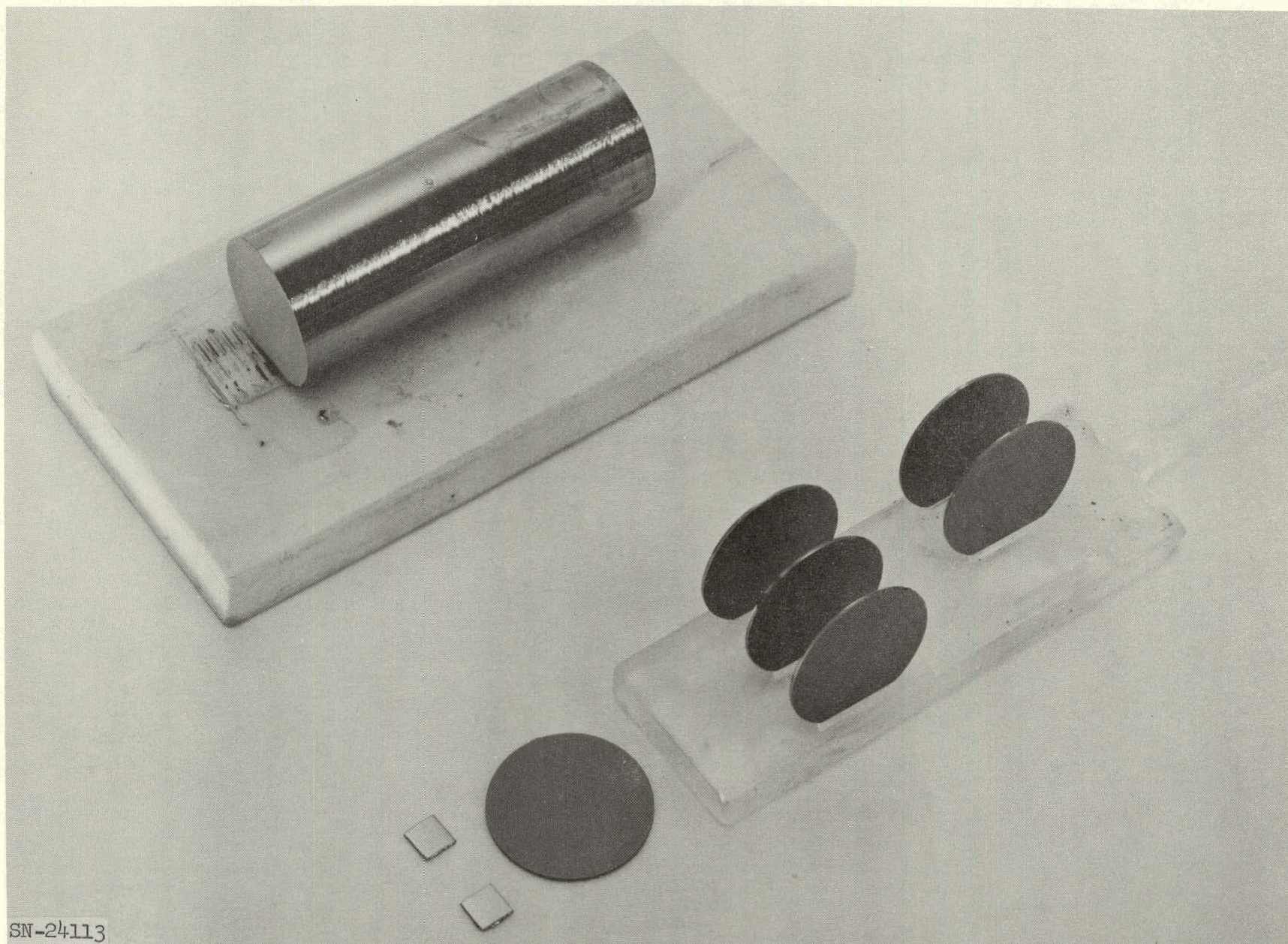


In all operations the utmost cleanliness must be observed because any contamination may ruin the detector. Between each operation the silicon must be thoroughly cleaned with ultra-high-purity chemicals. A degreaser such as trichlorethylene should be used, followed by an alcohol rinse plus several rinses in distilled and deionized water. The silicon should be handled with clean tweezers to eliminate possible contamination by the hands. Several silicon single crystals were purchased from the Merck Chemical Company. They are cylindrical in shape with diameters of approximately 20 mm, and of various lengths between three and four inches. These crystals are of high purity and thus high resistivity, with resistivities ranging between 100 and 4000 ohm-cm for both n and p types.

Silicon is hard, brittle and chips easily. For this reason a high-speed diamond saw must be used for slicing the crystal into circular discs (Fig. 5). The thickness of the disc is largely dependent upon the ultimate use of the detector; a dE/dx detector must be thin so that very little of the detected particle's energy is lost in passing through the silicon. If it is desired to use the detector in a spectrometer, all of the detected particle's energy must be expended within the silicon and thus a fairly thick disc must be used. If it is desired merely to detect, almost any thickness may be used. A thickness of 0.030 inch was arbitrarily settled upon. Approximately 0.015 inch will be removed in subsequent operations.

During the slicing operation the silicon crystal must be held perfectly rigid. To do this the crystal is glued to a rectangular slab of porcelain using a resin pitch. The resin pitch was selected because it has good adhesive qualities, melts at a low temperature and is easily removed with alcohol (Fig. 5). The porcelain slab is then clamped in a metal holder and placed in a magnetic vice on the saw bench. A saw blade thickness of 0.020 inch is used to minimize silicon loss.

The scarf marks of the saw blade must be removed or uneven etching will occur in the next operation. These scarf marks are removed in a lapping process similar to that used for grinding glass.¹⁹ A lucite mask containing six circular holes is placed between two stainless steel lapping blocks. The function of the mask is to contain the silicon slices, force the slices to slide over the lower lapping block and act as a limit so too much silicon will not be ground off. The upper block is driven by an eccentric mounted in the chuck of a slow-speed drill press and operated at 16 rpm. At this speed



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Fig. 5. Silicon single crystal, silicon slices and quartz holder.

approximately 0.002 inch of silicon are removed per hour. A mixture of John Crane 1900 or 1950 lapping compound and water is used for the grinding compound. Stainless steel was used as the material for the lapping blocks since the water in the grinding compound would rust cast iron. Both faces of the silicon are lapped at the same time. The upper block (5 in. in diameter), driven by the eccentric, rotates against a lip on the mask (6 in. in diameter) thus forcing the mask, containing the silicon, to slide over the surface of the lower block (7 in. in diameter), which is stationary. A mask thickness of 0.020 inch was used. When the silicon slices are ground to a thickness of 0.020 inch, the mask, being the same thickness and relatively soft, inhibits further grinding. In this manner, also, the two faces of the silicon discs are maintained very close to parallel.

Upon completion of lapping the silicon slices are diced to their final size. This size is determined by the amount of capacitance the ultimate system can tolerate and still get a good signal-to-noise ratio. Since the expression for capacitance is the same as that of a parallel plate capacitor, Eq. (A-12), the capacitance varies directly with the first power of the area. As a result the smaller the area the smaller the capacitance and the larger the voltage signal. Most of the slices were diced into squares 0.200 inch on a side. To do this the silicon slices are once again mounted on the porcelain slab with the resin pitch but this time they are mounted in horizontal stacks to facilitate cutting.

The dicing operation may be deferred until after the diffusion operation. If it is, there exists the possibility of damaging the diffused surface due to increased handling; but the sides will be free of the diffusant. If the dicing takes place at this point, any diffusion operation will also diffuse into the sides. This diffusant, however, may be removed by hand lapping or etching.

Etching is a chemical means of polishing the silicon surface. The purpose is to expose any possible dislocations or other crystalline imperfections, which would render the crystal useless. It also provides a smooth boundary from which diffusion can take place. To protect the reverse side during the etching process a mask of picene wax, dissolved in trichlorethylene, is applied. The volatile portions are then removed by placing the sample under an infrared lamp for 2 to 3 minutes. This leaves a hard wax coating which the acid will not attack and which is easily removed by again dissolving with trichlorethylene. The etch solution is a modified form of the standard CP-4 solution which contains:

	Parts by volume
Concentrated nitric acid (70%)	3
Hydrofluoric acid (48%)	1
Glacial acetic acid	1

The nitric acid oxidizes the exposed silicon surface. The oxide formed is then removed by the action of the hydrofluoric acid. This reaction is fast therefore a buffering agent, the glacial acetic acid, must be used to make the reaction controllable. Dearnaley and Whitehead⁴ also recommend cooling the etch solution in an icewater bath as a further means of controlling the reaction. During the etching process small bubbles form on the surface of the silicon. If they are not removed uneven etching will occur. These bubbles may be removed by agitating the silicon samples or by bubbling nitrogen gas through the etch solution. The latter is recommended. Upon completion of etching the silicon must not be immediately exposed to air as this produces a brown discoloration. Instead the etch solution must be diluted and decanted several times before removal of the silicon to a beaker of distilled and de-ionized water, where it is washed by diluting and decanting. The silicon is then dried by placing it in a stream of dry nitrogen gas. The silicon samples sometimes float to the surface of the etch solution where they become discolored. To avoid this the reverse side of the silicon should be attached to a piece of teflon or polyethylene, using the picene wax. The teflon or polyethylene will then act as an anchor to hold the samples below the surface. A fresh batch of etchant will produce a mirror finish in 4 to 5 minutes. An aged batch may take somewhat longer.

2. Surface barrier detectors

Surface barrier diodes are generally made from n-type silicon because the difference between the thermionic work functions of most metals and that of silicon is generally greater for n type than it is for p type. This means that by using n-type silicon for a base, a larger barrier will be produced resulting also in a wider space-charge region. Care must be taken to ensure that the work function of the metal is greater than that of the n-type silicon. If the work function of the metal is smaller than that of the n-type silicon, an ohmic contact is made. The opposite holds true for the case of p-type silicon.

For n-type silicon a surface oxide layer provides an excellent barrier. After the etching process, the silicon sample is placed in a wet atmosphere for several hours, preferably overnight, to form the barrier. After removal

of the mask and a thorough cleaning the diodes are mounted (see Fig. 6). Silver conducting paint is used to attach the base of the diode to the mounting and also serves as one electrode. The other electrode is a 0.008-inch-diameter stainless steel wire catwhisker.

Gold is generally evaporated onto the oxide surface to increase carrier collection efficiency. Dearnaley and Whitehead⁴ prefer to evaporate the gold onto the freshly etched surface and allow the oxidation to take place through the gold. They believe this procedure protects the silicon from contamination.

3. Diffused p-n detectors

Diffused devices are made in an open tube electric furnace (Fig. 7). The silicon samples are placed on a quartz holder and inserted into a quartz tube which passes through the furnace. The diffusant is passed through the tube by means of a carrier gas and deposited on the surface of the silicon samples, from whence diffusion takes place. The concentration of the diffusant is such that the amount deposited on the silicon provides a maximum surface concentration regardless of time. An alternate method is to paint the diffusant onto the surface of the silicon. In either case a shallow diffusion is desired in order to place the depletion region as close to the surface as possible. To do this a relatively low temperature and a short diffusion time are used. At the end of this time the furnace is turned off and allowed to cool slowly. Fast cooling or quenching sets up recombination centers within the silicon, with the result that the minority carrier lifetime is shortened and the output pulse degraded. The slow cooling, on the other hand, allows the diffusant atoms to properly substitute for the silicon atoms in the lattice and thus reform a perfect crystal.

After the silicon is removed from the furnace the reverse face and the four sides are hand-lapped or etched to remove all traces of the diffusant. The samples are then mounted as before.

The diffusion of impurities into the silicon is governed by Fick's second law which states that the rate of accumulation of the impurity at any point is proportional to the divergence of the gradient of the impurity. The constant of proportionality is the diffusion constant (D). Mathematically stated it is:

$$\frac{dN}{dt} = D \nabla^2 N.$$

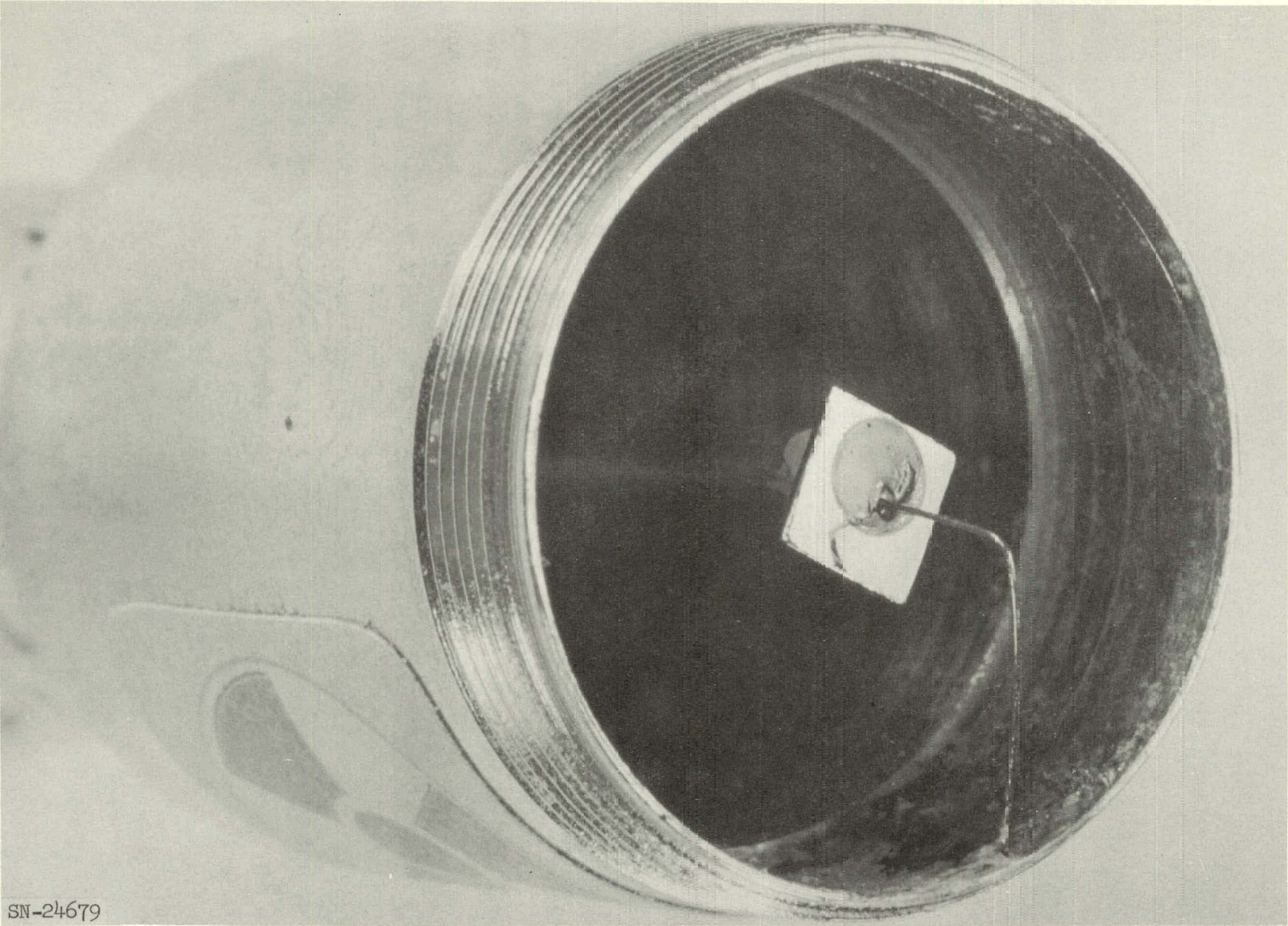


Fig. 6. Mounted silicon detector.

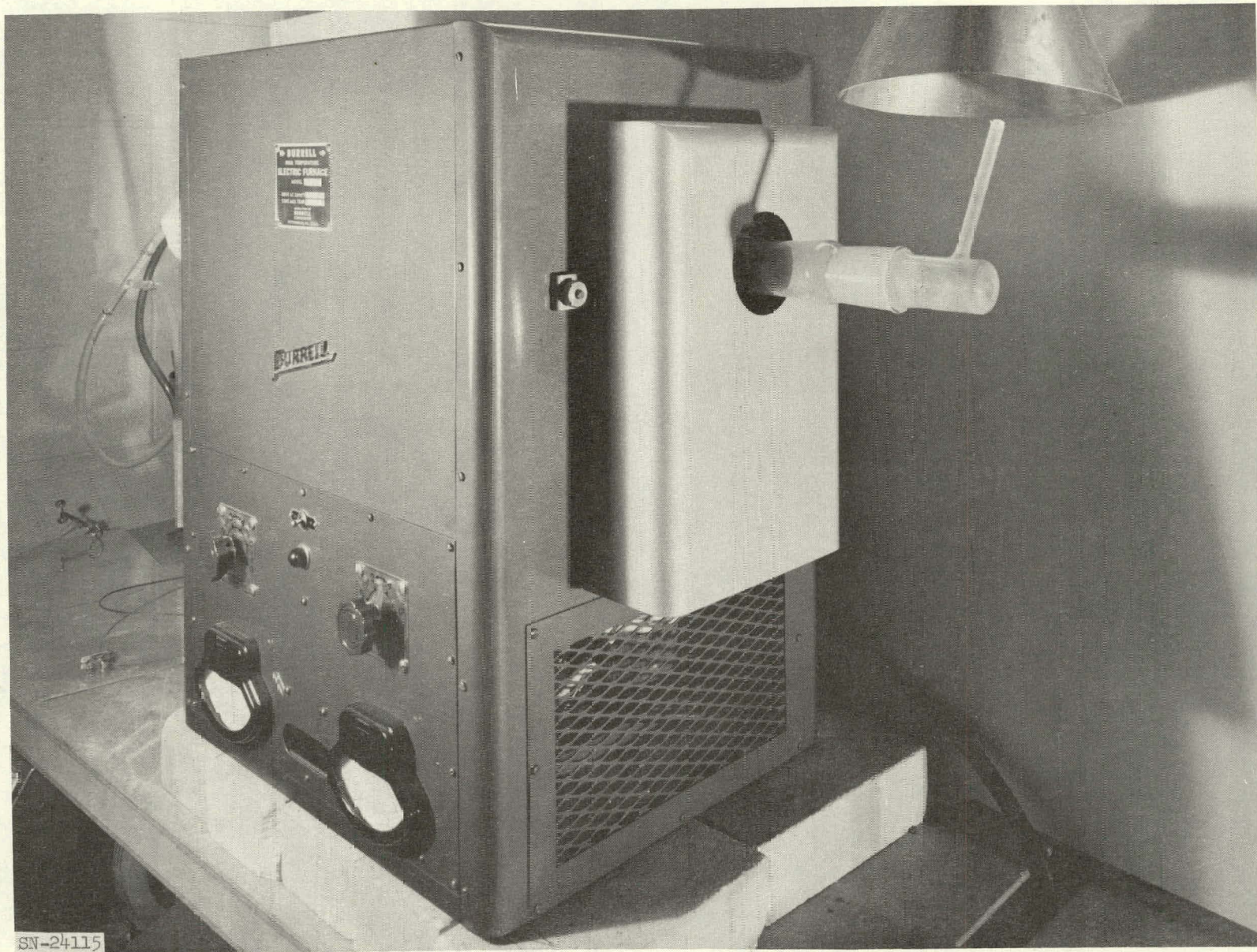


Fig. 7. Diffusion furnace.

For diffusion from a plane source the equation reduces to

$$\frac{dN_x}{dt} = D \frac{\partial^2 N}{\partial x^2}.$$

Two boundary conditions are necessary to solve the equation completely. The laboratory setup was designed to provide a constant surface source, and therefore $N(0, t) = N_s$, N_s is the surface concentration. The other boundary condition is a function of the distribution of the diffusant. In this case it is a step function.

$$N(x, 0) = f(x), \quad f(x) = \begin{cases} N_s & -\infty < x < 0 \\ 0 & 0 < x < \infty. \end{cases}$$

The solution of the equation is found in most texts on semiconductor technology.¹⁴ The solution is:

$$\begin{aligned} N(x) &= N_s \left[1 - \frac{2}{2\sqrt{\pi}} \int_0^{\frac{x}{2\sqrt{Dt}}} e^{-\lambda^2} d\lambda \right] \\ &= N_s \left[1 - \operatorname{erf} \frac{x}{2\sqrt{Dt}} \right] \\ &= N_s \operatorname{erfc} \frac{x}{2\sqrt{Dt}}. \end{aligned}$$

Group V elements are used as diffusants to form p-n junctions in p-type material. Specifically, the phosphorus pentoxide compound (P_2O_5) was used. Its probable chemical reaction is $P_2O_5 + Si = P + \text{phospho-silicate glass}$.¹⁵ The P_2O_5 is preheated to 225°C, at which temperature it sublimates. The P_2O_5 vapor is carried to the silicon in the furnace with a carrier gas of nitrogen. This nitrogen is obtained from the natural boil-off from a Dewar flask of liquid nitrogen.

Group III elements are used as diffusants to form p-n junctions in n-type silicon. Specifically two methods are used. In the first method, a suspension of B^{10} in mineral oil is painted on the surface of the silicon. The silicon is then heated to a temperature of about 250°C in a vacuum chamber to drive off the mineral oil. The boron-coated pieces of silicon are then placed in the diffusion oven for the 10-minute diffusion.

The second method is to drift boron trichloride gas (BCl_3) over the silicon in the diffusion furnace. The probable chemical reaction is $4BCl_3 + 3Si = 4B + 3SiCl_4$.

IV. EXPERIMENTAL UNITS AND RESULTS

1. Uranium detectors

a. Preparation of experimental units

Eight experimental detectors were prepared from 100 ohm-cm n-type silicon as described in Section III. Two detectors were given initial shallow-diffused p-n junctions (barrier depth 0.07 microns) using boron trichloride in the open tube electric furnace. Four units were given oxide coatings by exposure to a wet atmosphere overnight. Two of the oxide units were then coated with 5000 Å of gold. The final two units were freshly etched. All eight units were placed in the slotted holders (Fig. 8) and placed in the vacuum chamber (Fig. 9). Uranium-235 was evaporated from the spread tungsten filament (Fig. 10) in an initial vacuum of 5×10^{-7} mm of Hg. An area of 0.08 cm^2 was exposed on each unit. Upon completion of the uranium evaporation, the units were mounted (Fig. 6) and sprayed with an acrylic plastic.

Uranium is extremely difficult to evaporate. Preliminary investigations using tantalum, molybdenum and carbon vessels showed that uranium went into solution with these materials at elevated temperatures (about 1000°C). The only material that uranium would not go into solution with was tungsten. When the uranium melts it partially wets the tungsten, forming a ball of molten uranium around the filament. An oxide crust forms on the surface of the ball which inhibits evaporation until temperatures of about 2200°C are reached. Unfortunately this ball generally migrates to one end of the filament, thereby depositing coatings of different thickness on the silicon units. Since uranium is a metal, the molten ball forms a conducting path with subsequent local cooling. The adjacent portions of the filament must then be excessively heated to provide sufficient heat for the evaporation. Another problem encountered was filament breakage before evaporation was complete. Initially 0.3 gram of uranium was placed on the filament. From past experience it was expected that about a 1-micron-thick coating would be deposited on the silicon, giving a thermal neutron efficiency of 0.3%.

The diode characteristics (Figs. 13 to 16) were measured on a Tektronix 575 transistor curve tracer.

b. Results

The uranium-coated, boron-diffused units would detect neither the alpha decay of the U^{235} nor the fission fragments resulting from thermal

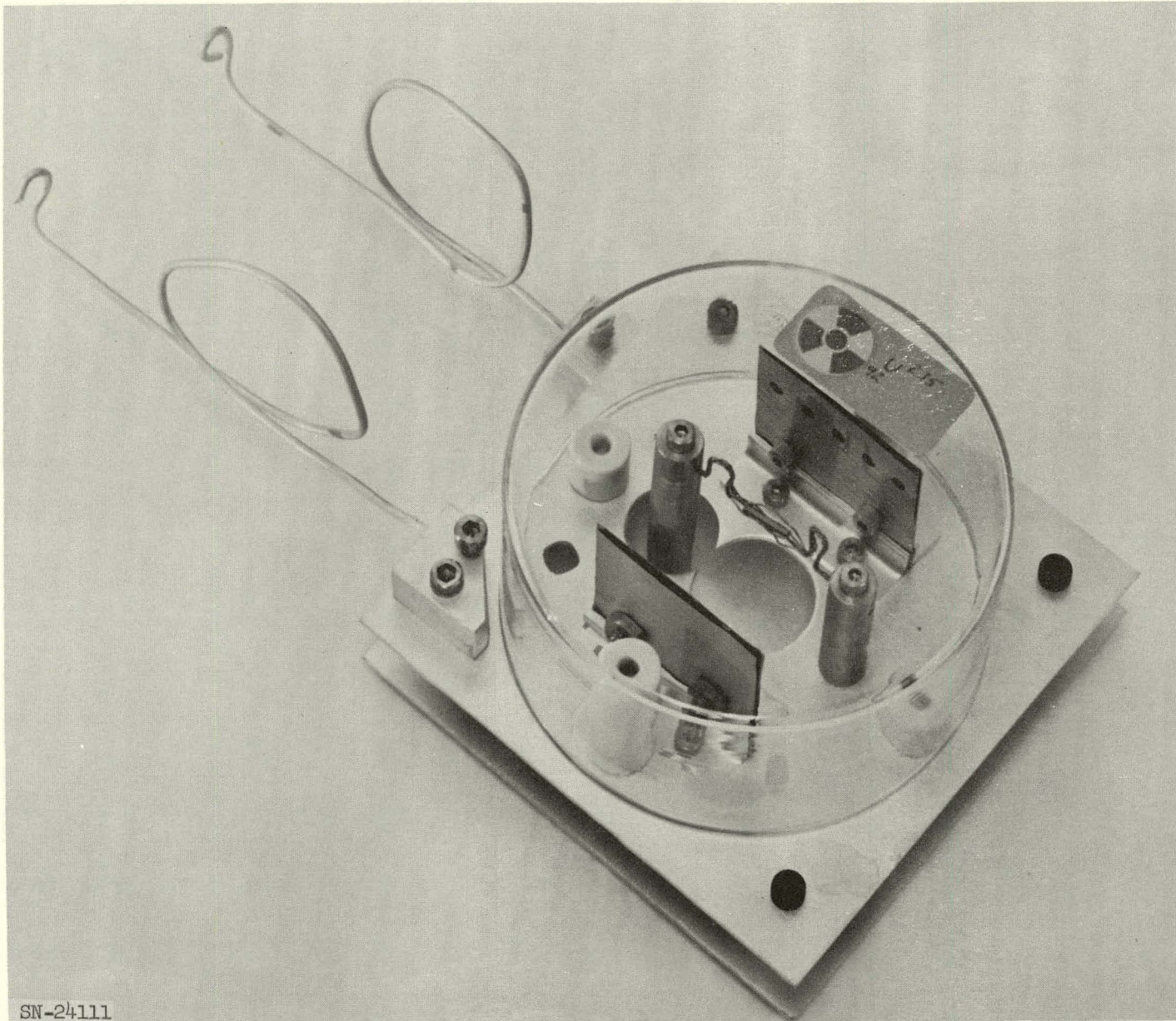
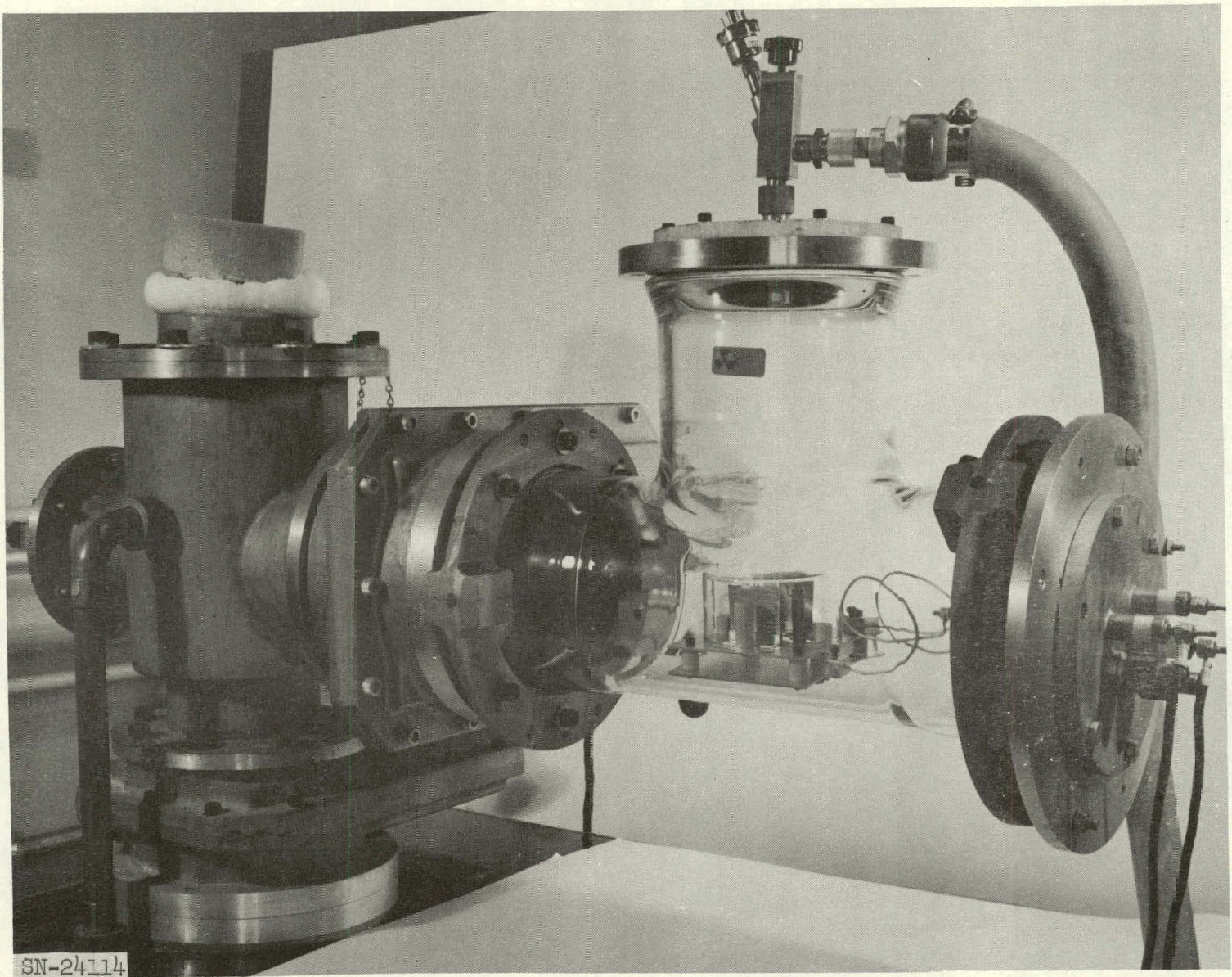


Fig. 8. Uranium evaporation jig.



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Fig. 9. Vacuum chamber.

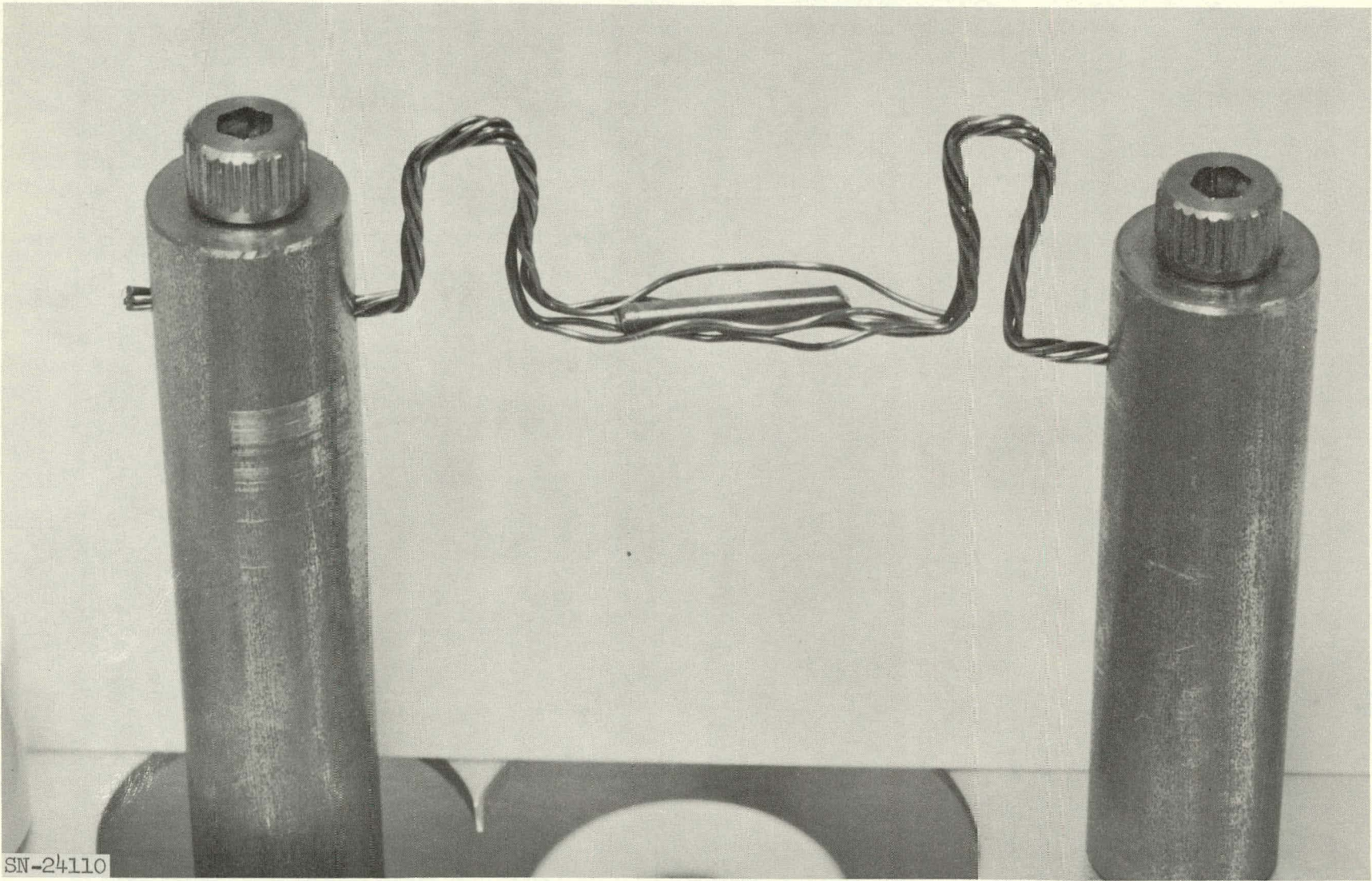


Fig. 10. Filament detail.

neutron irradiation. It is believed this lack of response was caused by the poor diode characteristics of these units.

The uranium-coated freshly etched units and surface oxide units would self-count, and also count fission fragments, but their counting rate was extremely poor. Their counting efficiency for thermal neutrons was of the order of 0.0005%.

The uranium-and-gold-coated surface oxide units were the best of the units tested. The efficiency of these units to thermal neutrons was of the order of 0.004%. Figure 11 shows the spectrum obtained from the alpha decay of the U^{235} . Figure 12 shows the spectrum of the energies of the fission fragments obtained as the result of thermal neutron irradiation. It is interesting to note that this technique offers a simple method of obtaining the mass distribution of the fission fragments. The energies shown are only approximate and were obtained by scaling from the 4.58-Mev self-count peak. The energies of the two peaks are approximately 40 Mev lower than was expected (Section II-7). This discrepancy can partially be explained by the fact that fission fragments are highly ionized and lose most of their energy in the initial

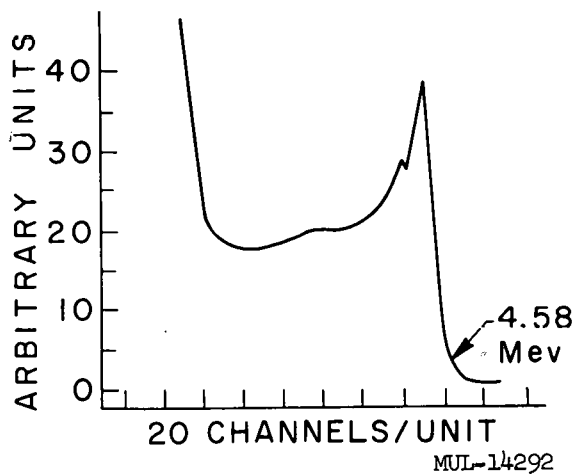


Fig. 11. Uranium-235 alpha spectrum.

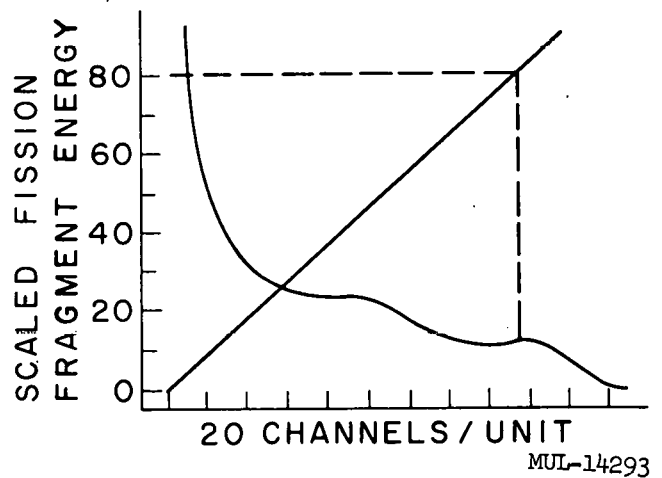


Fig. 12. Uranium-235 fission fragment spectrum.

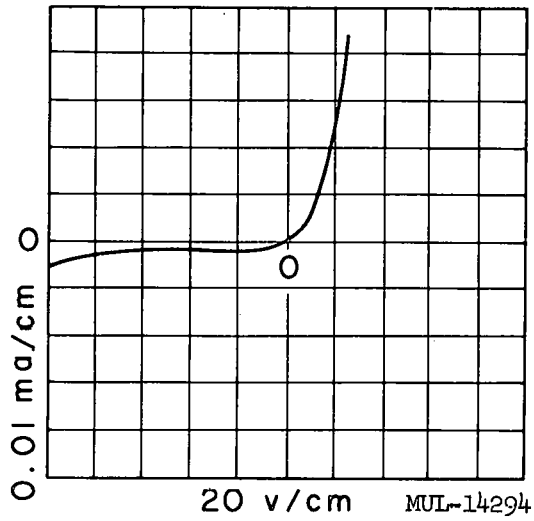


Fig. 13. Diode characteristics of the uranium coated, oxide surface barrier detector.

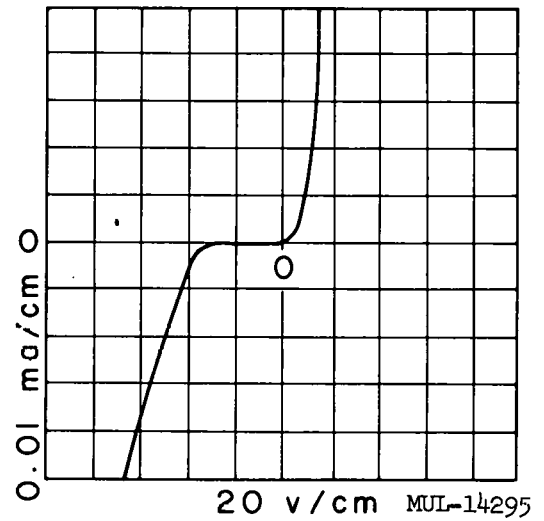


Fig. 14. Diode characteristics of the uranium coated, freshly etched surface barrier detector.

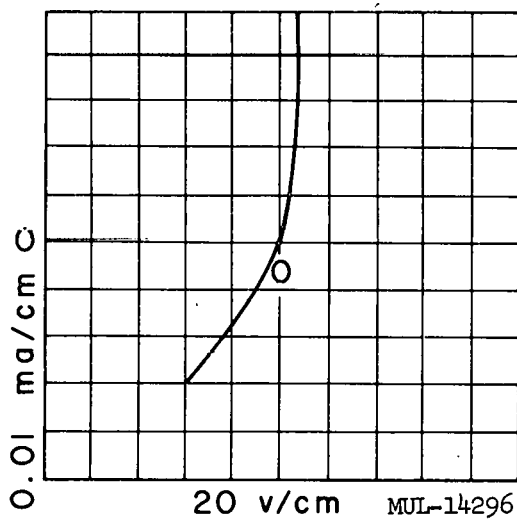


Fig. 15. Diode characteristics of the uranium coated, boron-diffused detector.

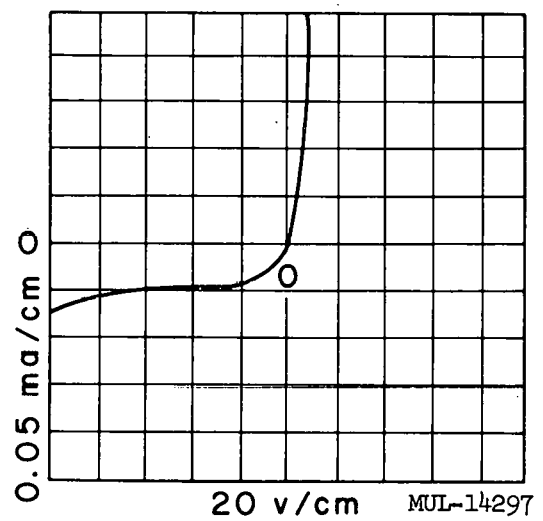


Fig. 16. Diode characteristics of the uranium and gold coated surface barrier detector.

part of their path, part of which is beyond a diffusion length from the depletion region. The alpha particles, however, lose most of their energy toward the end of their path, thus losing very little energy prior to entering the depletion region.

2. B¹⁰ detectors

a. Preparation of experimental units

Four experimental B¹⁰ detectors were prepared, using 2000 ohm-cm n-type silicon. Two units received an initial boron-diffused p-n junction (0.08 micron), with boron trichloride in the open-tube electric furnace. They were subsequently painted with 90-92% enriched B¹⁰ (amorphous in oil). The oil was driven off in a vacuum chamber by heating to 250°C for 10 minutes. The other two units were painted with the enriched B¹⁰ directly on the freshly etched surface. The oil was removed in the same manner. All four units were then placed in the furnace at 900°C for 10 minutes. The four units were then mounted the same as the uranium detectors and sprayed with plastic. The diode characteristics are as shown in Figs. 17 and 18.

b. Results

The two units having the preformed p-n junctions did not operate as well as those that were given only the B¹⁰. Figures 19, 20, and 21 show the reaction-particle energy spectra, to a 10⁶-neutrons/cm²/sec thermal neutron beam, for 0, 5, and 10 volts reverse bias. The efficiency to thermal neutrons of the best of the four units was about 1.3%.

3. Li⁶ detectors

a. Preparation of experimental units

Two experimental Li⁶ detectors were prepared from 1000 ohm-cm p-type silicon. Li⁶ (amorphous in oil) was painted on the freshly etched silicon surface. These units were then placed in a vacuum chamber and heated at 250°C for 10 minutes to drive off the oil. The temperature was then raised to 450°C for 90 seconds to diffuse the lithium into the silicon, producing a barrier depth of 154 microns. The units were then mounted.

b. Results

Figure 22 shows the reaction-particle energy spectrum produced by a 10⁶-neutron/cm²/sec thermal neutron beam. The maximum energy of the triton (2.73 Mev) is shown in the same figure. By scaling this energy to the small peak, the large peak indicates an energy of 2.05 Mev, which is the

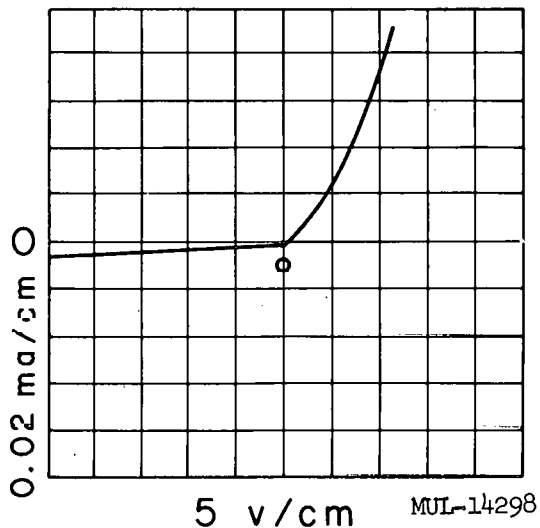


Fig. 17. Diode characteristics of a boron-10 painted, boron diffused detector.

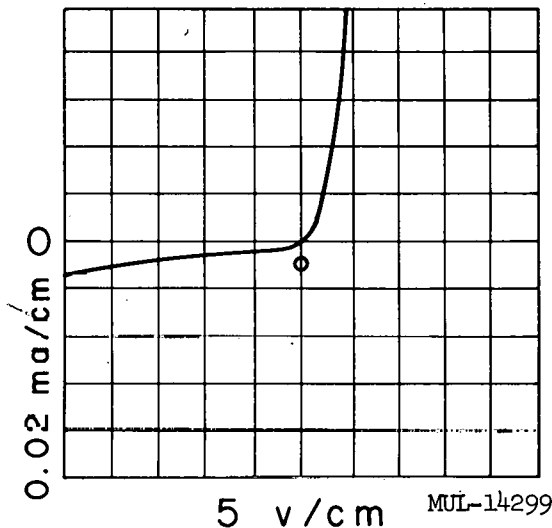


Fig. 18. Diode characteristics of a boron-10-diffused detector.

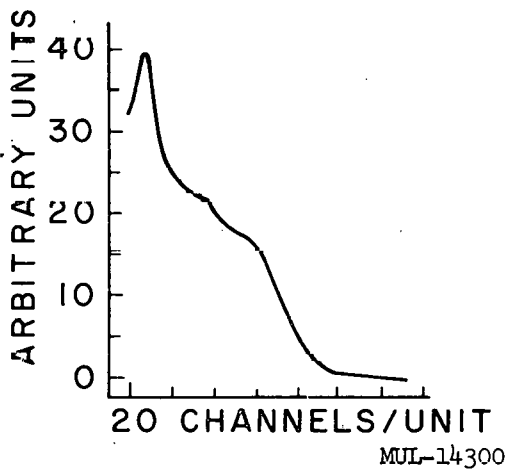


Fig. 19. Boron-10 reaction particle spectrum, 0 volts reverse bias.

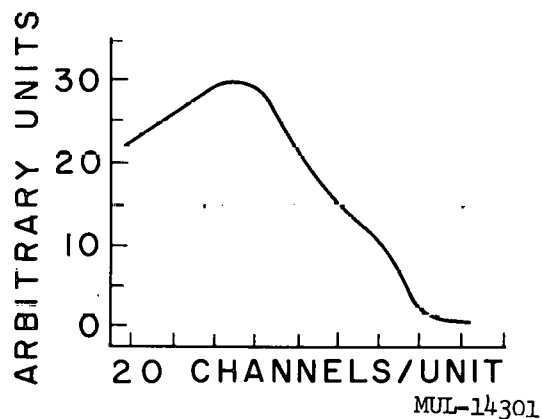


Fig. 20. Boron-10 reaction particle spectrum, 5 volts reverse bias.

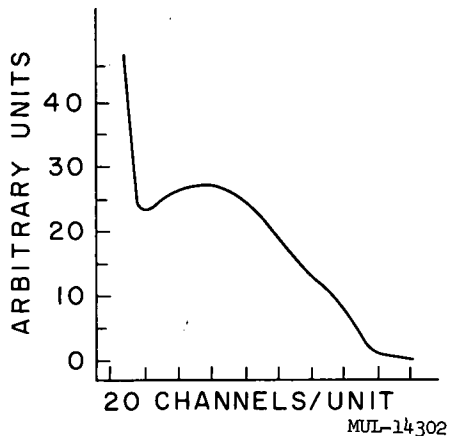


Fig. 21. Boron-10 reaction particle spectrum, 10 volts reverse bias.

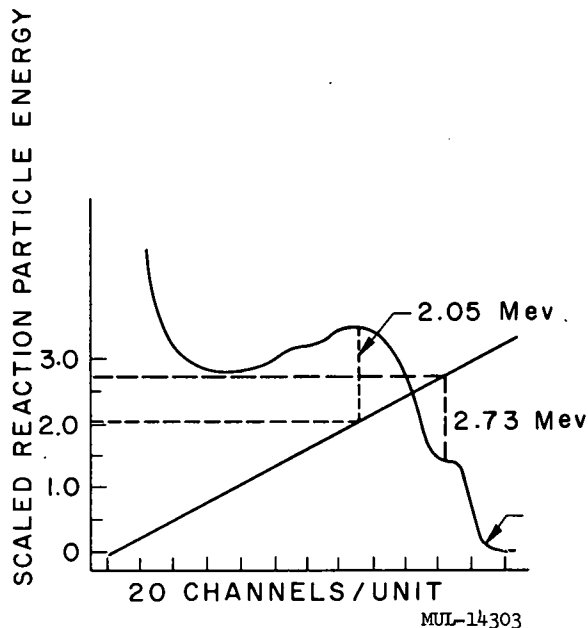
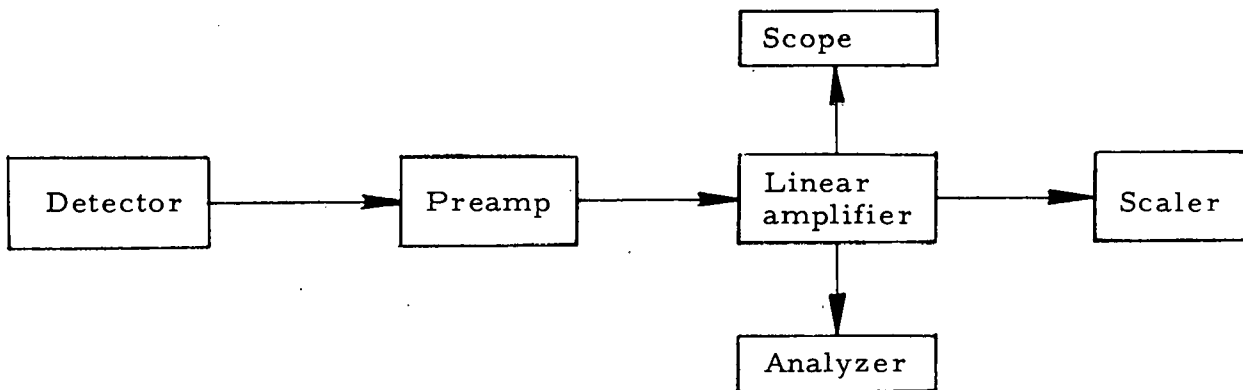


Fig. 22. Lithium-6 reaction particle spectrum, 20 volts reverse bias.

energy expected of the alpha particle. The efficiency of these detectors to thermal neutrons is about 0.4%.

V. ELECTRONICS

The electronics test equipment is diagrammed as follows:



The heart of the system is a low-noise preamplifier, designed by F. S. Goulding. A schematic diagram of an improved version of this preamplifier may be found in reference (5), Fig. 9. The 256-channel analyzer (Radiation Counting Laboratory) requires input pulses between 0 and 100 volts with 0.5-microsecond rise times. A laboratory-designed linear amplifier complements the analyzer.

The detector pulse has a rise time of the order of nanoseconds. The pulse is then integrated in both the preamplifier (rise time 0.4 microsecond) and the linear amplifier (rise time 0.5 microsecond) before going to the analyzer. A 0.55 microsecond clipping line was installed between the preamplifier and the linear amplifier to eliminate the low frequency noise. It was noted that the reverse current characteristics of the detectors was a direct indication of the noise to be expected.

VI. CONCLUSIONS

Additional research is necessary to improve the efficiency of these detectors. The B^{10} detectors appear to hold the most promise for the detection of slow neutrons, owing to their large cross section and ease of fabrication. The use of a guard ring⁵ on these detectors would greatly improve their diode characteristics and enable the use of larger reverse-bias voltages.

Possible uses of these devices include: Space probe detection units; reactor core flux mapping; neutron beam collimation and exploration; medical research; and neutron spectroscopy.

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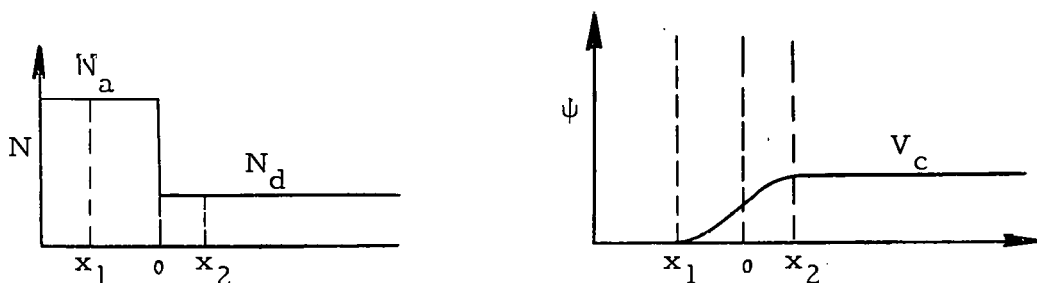
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APPENDIX A

Solution of Poisson's Equation for a Surface Barrier Junction

An abrupt junction or Schottky-type barrier occurs when the semiconductor dopant changes abruptly from n to p type or vice versa in the relatively short space of 1 to 2 atomic distances. This solution is applicable to surface-barrier alloyed diodes.



The above two figures illustrate the dopant concentration (N) and potential distribution in initially n-type silicon with a p-type surface concentration. For convenience the origin is considered at the point where the concentration changes.

N_a = acceptor surface concentration

N_d = donor concentration

$0-x_1$ = width of the depletion region in the p-type material

x_2-0 = width of the depletion region in the n-type material

V_c = voltage across the equivalent capacitor

$$V_c = V_B + \phi_a - \phi_d$$

V_B = reverse bias

ϕ_a = Fermi potential of the acceptor material

ϕ_d = Fermi potential of the donor material

The boundary conditions taken from the figures are:

$$\left. \frac{d\psi_a}{dx} \right|_{x=x_1} = 0$$

$$\left. \frac{d\psi_d}{dx} \right|_{x=x_2} = 0$$

$$\psi_a \Big|_{x=x_1} = 0$$

$$\psi_d \Big|_{x=x_2} = V_c$$

ψ and $\frac{d\psi}{dx}$ are continuous at $x = 0$.

$$\nabla \cdot D = \rho$$

$$D = \epsilon E$$

$$E = -\nabla\psi$$

$$\therefore \nabla^2 \psi = -\frac{\rho}{\epsilon}$$

$$\rho_a = -q(N_a + n-p)$$

n = electron concentration

p = hole concentration

Assume $N_a \gg (n-p)$

$$\therefore \rho_a = -qN_a$$

$$\text{also } \rho_d = +qN_d$$

q = electronic charge

All the y and z donations drop out
since the junction is in the yz plane.

then

$$\nabla^2 \psi_a = \frac{qN_a}{\epsilon}$$

$$\frac{d^2 \psi_a}{dx^2} = \frac{qN_a}{\epsilon}$$

$$\left. \frac{d\psi_a}{dx} \right|_{x_1}^x = \frac{qN_a}{\epsilon} x \Big|_{x_1}^x$$

$$\frac{d\psi_a}{dx} = \frac{qN_a}{\epsilon} (x - x_1) \tag{A-1}$$

$$\psi_a \Big|_{x_1}^x = \frac{qN_a}{\epsilon} \left(\frac{x^2}{2} - x_1 x \right) \Big|_{x_1}^x$$

$$\psi_a = \frac{qN_a}{2\epsilon} (x^2 - 2x_1 x + x_1^2)$$

$$\psi_a = \frac{qN_a}{2\epsilon} (x - x_1)^2 \tag{A-2}$$

The solution for the acceptor region follows the same method.

$$\frac{d\psi_a}{dx} = -\frac{qN_d}{\epsilon} (x - x_2) \tag{A-3}$$

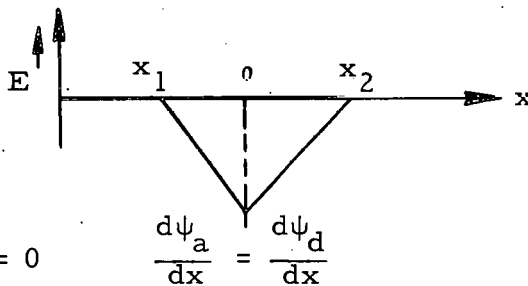
$$\psi_d \Big|_{x_2}^x = -\frac{qN_d}{2\epsilon} \left(x^2 - 2x_2 x \right) \Big|_{x_2}^x$$

$$\psi_d = V_c - \frac{qN_d}{2\epsilon} (x - x_2)^2 \quad (\text{A-4})$$

From Eqs. (A-1) and (A-3) the electric field is:

$$E_a = -\frac{qN_a}{\epsilon} (x - x_1) \quad (\text{A-5})$$

$$E_d = +\frac{qN_d}{\epsilon} (x - x_2) \quad (\text{A-6})$$



at $x = 0$

$$\frac{d\psi_a}{dx} = \frac{d\psi_d}{dx}$$

$$\begin{aligned} \frac{q}{\epsilon} N_a (0 - x_1) &= -\frac{q}{\epsilon} N_d (0 - x_2) \\ &= \frac{q}{\epsilon} N_d (x_2 - 0) \end{aligned}$$

$(0 - x_1) = d_a =$ depletion region in the p material

$(x_2 - 0) = d_d =$ depletion region in the n material

$$d_a = \frac{N_d}{N_a} d_d \quad (\text{A-7})$$

Equation (A-7) shows that for equal concentrations, the portions of the depletion region on each side of the barrier are equal. Realistically this is not the case. For high-resistivity material ($N_d \ll N_a$) the depletion region approaches zero on the acceptor side and hence the entire electric field is located on the donor side.

The total depletion region (d) is

$$d = d_a + d_d$$

$$= \frac{N_d}{N_a} d_d + d_d$$

$$d = \left(1 + \frac{N_d}{N_a} \right) d_d \quad (\text{A-8})$$

at $x = 0$ $\psi_a = \psi_d$

$$\begin{aligned} \frac{qN_a}{2\epsilon} (d_a)^2 &= V_c - \frac{qN_d}{2\epsilon} (-d_d)^2 \\ \frac{qN_a}{2\epsilon} \left(\frac{N_d}{N_a} d_d \right)^2 &= V_c - \frac{qN_d}{2\epsilon} d_d^2 \\ V_c &= \frac{qN_d^2 d_d^2}{2\epsilon N_a} + \frac{qN_d d_d^2}{2\epsilon} \\ &= d_d^2 \frac{qN_d}{2\epsilon} \left[1 + \frac{N_d}{N_a} \right] \\ d_d &= \left[\frac{2\epsilon V_c}{qN_d \left(1 + \frac{N_d}{N_a} \right)} \right]^{1/2} \end{aligned} \tag{A-9}$$

Combining (A-8) and (A-9)

$$d = \left[\frac{2\epsilon V_c \left(1 + \frac{N_d}{N_a} \right)}{qN_d} \right]^{1/2} \tag{A-10}$$

Since $(N_a \gg N_d)$, the term N_d/N_a approaches zero. The depletion region becomes wider as N_d becomes smaller. Thus by using higher-resistivity material the depletion region can be extended. Another means of extending it is by use of higher reverse-bias voltages ($V_c = V_B + \phi_a - \phi_d$).

Since charge must be conserved,

$$Q_a = Q_d$$

$$Q_d = qN_d d_d$$

$$d = \left[\frac{2\epsilon qN_d V_c}{\left(1 + \frac{N_d}{N_a} \right)} \right]^{1/2} \tag{A-11}$$

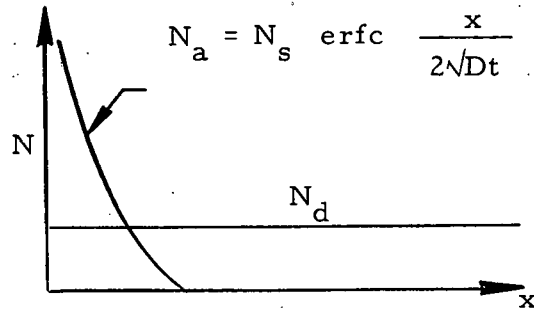
$$\begin{aligned}
 C &= \frac{dQ}{dV} = \frac{1}{2} \left[\frac{2\epsilon qN_d V_c}{1 + \frac{N_d}{N_a}} \right]^{-1/2} \frac{2\epsilon qN_d}{1 + \frac{N_d}{N_a}} \\
 &= \frac{\epsilon qN_d}{1 + \frac{N_d}{N_a}} \left[\frac{1 + \frac{N_d}{N_a}}{2\epsilon qN_d V_c} \right]^{1/2} \\
 C &= \left[\frac{\epsilon qN_d}{2V_c \left(1 + \frac{N_d}{N_a} \right)} \right]^{1/2} = \frac{\epsilon}{d} \tag{A-12}
 \end{aligned}$$

The expression for capacitance is recognizable as that for a parallel plate capacitor, and varies directly as the square root of the donor concentration and inversely with the square root of the reverse bias.

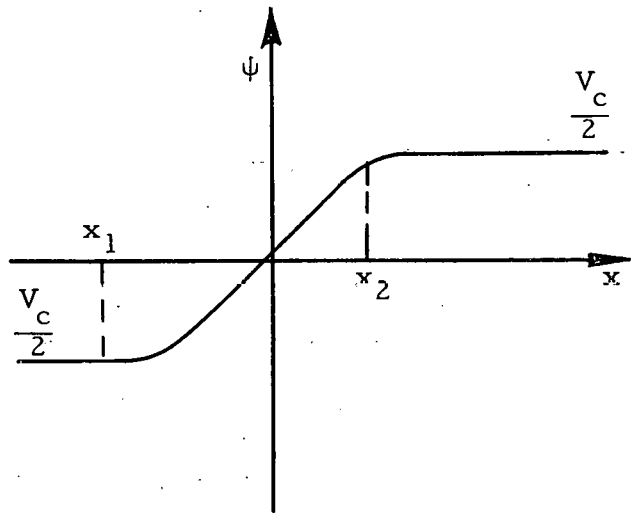
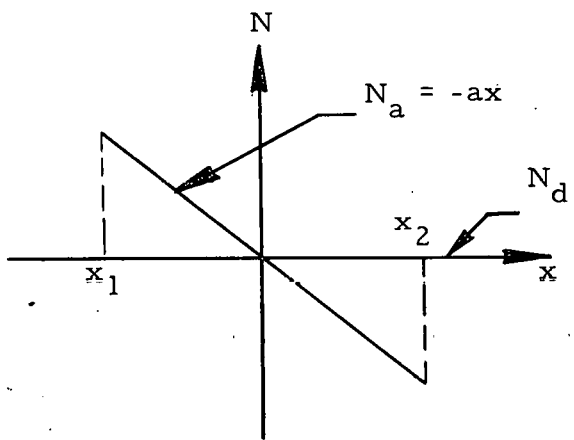
APPENDIX B

Solution of Poisson's Equation for a Graded Junction

The graded junction solution is applicable to the case of a grown p-n junction or a diffused p-n junction.



The equation for N_a is the solution of the diffusion equation for a constant surface source. As a first approximation it can be assumed that N_a is a straight line in the vicinity where it crosses N_d :



Boundary conditions

$$\left. \frac{d\psi_a}{dx} \right|_{x=x_1} = 0$$

$$\left. \frac{d\psi_d}{dx} \right|_{x=x_2} = 0$$

$$\psi_a \Big|_{x=x_1} = -\frac{V_c}{2} \qquad \psi_d \Big|_{x=x_2} = \frac{V_c}{2}$$

$$\psi_1 = \psi_2 \quad \text{at } x = 0$$

$$x_1 = -x_2$$

$$N_a = -ax$$

$$\rho_a = -qN_a = qax$$

$$\frac{d^2\psi_a}{dx^2} = -\frac{\rho_a}{\epsilon} = -\frac{qax}{\epsilon}$$

$$\frac{d\psi_a}{dx} \Big|_{x_1}^x = -\frac{qa}{\epsilon} \frac{x^2}{2} \Big|_{x_1}^x$$

$$\frac{d\psi_a}{dx} = -\frac{qa}{2\epsilon} (x^2 - x_1^2) \qquad \text{(B-1)}$$

$$a \Big|_{x_1}^{x_2} = -\frac{qa}{2\epsilon} \left(\frac{x^3}{3} - x_1^2 x \right) \Big|_{x_1}^{x_2} = -x_2$$

$$V_c = \frac{qa}{2\epsilon} \frac{4}{3} x_2^3$$

$$x_2 = \left[\frac{3\epsilon V_c}{2qa} \right]^{1/3}$$

$$d = 2x_2 = \left[\frac{12\epsilon V_c}{qa} \right]^{1/3} \qquad \text{(B-2)}$$

Equation (B-2) shows that the depletion region varies directly with the cube root of the applied reverse bias.

$$C = \frac{dQ}{dV}$$

$$dQ_a = -qN_a dx = qax dx$$

$$Q_a \Big|_0^{x=x_2} = \frac{qax^2}{2} \Big|_0^{x=x_2}$$

$$Q_a = \frac{qax^2}{2} = \frac{qa}{2} \left[\frac{3\epsilon V_c}{2qa} \right]^{2/3}$$

$$\frac{dQ_a}{dV} = C = \frac{\epsilon}{2 \left[\frac{3\epsilon V_c}{2qa} \right]^{1/3}} = \frac{\epsilon}{d} \quad (\text{B-3})$$

Equation (B-3) is the equation of a parallel plate capacitor and it shows that the capacitance varies inversely with the cube root of the applied reverse bias.

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