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A STUDY OF GAMMA-RAY STRENGTH FUNCTIONS

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STRENGTH FUNCTIONS*

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ABSTRACT

Here we discuss the use of gamma-ray strength function systematics to calculate neutron capture cross sections and capture gamma-ray spectra. The ratio of the average capture width, $\bar{\Gamma}_\gamma$, to the average level spacing, D_{obs} , both at the neutron separation energy, can be derived from such systematics with much less uncertainty than from separate systematics for values of $\bar{\Gamma}_\gamma$ and D_{obs} . In particular, the E1 gamma-ray strength function is defined in terms of the giant dipole resonance (GDR). The GDR line shape is modeled with the usual Lorentzian function and also with a new energy-dependent, Breit-Wigner (EDBW) function. This latter form is further parameterized in terms of two overlapping resonances, even for nuclei where photonuclear measurements do not resolve two peaks. In the mass ranges studied, such modeling is successful for all nuclei away from the $N = 50$ closed neutron shell. Near the $N = 50$ shell, a one-peak EDBW appears to be more appropriate. Examples of calculated neutron capture excitation functions and capture gamma-ray spectra using the EDBW form are given for target nuclei in the mass - 90 region and also in the Ta - Au mass region.

I. INTRODUCTION

There is a well-established need for neutron capture and gamma-ray production information in such areas as reactor shielding, materials damage estimates, and nuclear weapons design. Recently Young¹ has reviewed those needs and some of the experimental and calculational efforts underway to satisfy them. The latter are used not only to interpolate between experimental measurements, but now, more increasingly, sophisticated statistical model calculations are used as a basis to choose between discrepant experimental data sets and to generate data where experimental information is lacking.

The purpose of this paper is to discuss a quantity which is fundamental to neutron capture and particularly to capture gamma-ray spectral calculations--the E1 gamma-ray strength function. We will limit our discussion to medium weight and heavy nuclei, where the strength function can be related to the giant dipole resonance (GDR). Different functional forms for the GDR line shape will be considered: the Lorentz form, which we have investigated extensively in the mass - 90 region, and an energy dependent, Breit-Wigner (EDBW) function. In addition, we have parameterized the EDBW model in terms of two overlapping resonances for all nuclei away from the $N = 50$ closed neutron shell, even when the photonuclear measurements do not resolve two peaks. Examples of calculated neutron capture excitation functions and capture gamma-ray spectra using the EDBW form will be given for target nuclei in the mass - 90 region and also in the Ta - Au mass region.

According to the usual statistical model formulation, the neutron capture cross section, in the region of overlapping resonances, may be expressed as

$$\sigma_{\text{cap}} = \pi \lambda^2 \sum_{J\pi} \frac{(2J+1)}{(2I+1)} \frac{\langle \Gamma_{\text{no}} \rangle^{J\pi}}{D^{J\pi}} \frac{\langle \Gamma_{\text{cap}} \rangle^{J\pi}}{\langle \Gamma \rangle^{J\pi}} S^{J\pi} \quad (1)$$

where I is the spin of the target state, $\langle \Gamma \rangle^{J\pi}$ is the average total width of the compound states with spin J and parity π , and $D^{J\pi}$ is the average spacing of such states. The average width for neutron emission to the target state is $\langle \Gamma_{\text{no}} \rangle^{J\pi}$, while $\langle \Gamma_{\text{cap}} \rangle^{J\pi}$ and $S^{J\pi}$ are the average width for neutron capture and the width fluctuation correction factor,² respectively.

We distinguish here the neutron capture width $\langle \Gamma_{\text{cap}} \rangle^{J\pi}$ from the average radiation width $\langle \Gamma_{\gamma} \rangle^{J\pi}$ to account for the possibility that the γ -ray cascade in the compound nucleus may be terminated by particle emission or fission. Thus,

$$\langle \Gamma_{\text{cap}} \rangle^{J\pi} = \langle \Gamma_{\gamma} \rangle^{J\pi} - \langle \Gamma_{\gamma n} \rangle^{J\pi} - \langle \Gamma_{\gamma p} \rangle^{J\pi} \dots \quad (2)$$

At sufficiently low incident neutron energies, the average capture width and the average s-wave radiation width, $\bar{\Gamma}_{\gamma}$, become essentially equal for most target nuclei, and it is $\bar{\Gamma}_{\gamma}$ at the neutron separation energy, S_n , that is often used to normalize γ -ray transmission coefficients. Recalling the relationship between transmission coefficients and widths, the s-wave radiation width at the separation energy may be obtained by integrating the γ -ray transmission coefficients over the density of the final states that may be reached in the first step of the γ -ray cascade. Suppressing spin and parity notation and considering only dipole transitions,

$$\theta_Y = \frac{2\pi\Gamma_Y}{D}$$

$$= \int_0^{S_n} (\Gamma_Y^{E1} + \Gamma_Y^{M1}) \rho(S_n - E_Y) dE_Y \quad (3)$$

The level density $\rho(U)$ is the reciprocal of the level spacing $D(U)$ and Γ_Y^{E1} and Γ_Y^{M1} are the γ -ray transmission coefficients. The normalization of these transmission coefficients may be accomplished by performing the integration indicated in Eq. 3 for each compound spin state produced by s-wave neutron capture, and then equating the results to the quotient of the measured quantities $\bar{\Gamma}_Y$ and D_{obs} , the observed s-wave resonance spacing,

$$\Gamma_Y/D_{obs} = (2\pi)^{-1} \sum_J \theta_Y^J \quad (4)$$

In order to utilize Eq. 3, one must know a) the energy dependence of the γ -ray transmission coefficients, b) the energy dependence of the nuclear level density, and c) values for $\bar{\Gamma}_Y$ and D_{obs} , or the quotient $\bar{\Gamma}_Y/D_{obs}$ at the neutron separation energy.

The simplest model for the γ -ray transmission coefficients is the single-particle model of Blatt and Weisskopf,³ where the energy dependence for a given multipole λ and magnetic or electric type x is

$$\Gamma_Y^{x\lambda}(E_Y) = a_{x\lambda} E_Y^{2\lambda+1} \quad (5)$$

The predicted absolute values for the coefficients $a_{x\lambda}$ are usually unreliable, but it is common to use the relative values for the various multipole types, and normalize them to some fraction of the total radiation

width. In the present work we have assumed that roughly 15% of $\bar{\Gamma}_\gamma$ is single particle in character, and primarily M1 in nature; very small amounts of higher multipole transitions were used in calculating γ -ray cascades merely to help depopulate high spin states that could not reach the discrete levels via dipole transitions. In the medium to heavy mass region of concern here, we have assumed the E1 transitions are related primarily to the low-energy tail of the GDR. This will be described in more detail in later sections.

For the nuclear level density calculation we use the formalism of Gilbert and Cameron,⁴ and the updated parameter values of Cook.⁵ Here the level density is taken as the product of three functions

$$\rho^{J\pi}(E) = F_\pi(\pi, E) \cdot F_J(J, E) \cdot \rho(E) \quad . \quad (6)$$

The first term of the product is the energy dependence of the parity distribution. We consider both positive and negative parities to be equally likely and independent of excitation energy E, therefore,

$$F_\pi(\pi, E) = 1/2 \quad . \quad (7)$$

The angular momentum dependence is given by

$$F_J(J, E) = \frac{(2J+1)}{2\sigma^2} \exp \left[- (J + 1/2)^2 / 2\sigma^2 \right] \quad (8)$$

where $\sigma = \sigma(E)$ is the energy dependent spin cut-off factor. We also include an yrast cut-off, but this is rarely important at low energies.

The remaining term in the level density expression, $\rho(E)$, is defined to be of a constant temperature form below a boundary energy E_x , and of a Fermi gas form for all energies $\geq E_x$.

$$\rho(E) = \frac{1}{T} \exp [(E - E_0)/T] \quad ; \quad E < E_x \quad (9)$$

$$\rho(E) = \frac{1}{12 \sqrt{2\sigma^2}} \frac{\exp [2 \sqrt{aU}]}{a^{1/4} U^{5/4}} \quad ; \quad E \geq E_x$$

We use $U = E - \delta$, where δ is the effective pairing energy. Details regarding the parameters T , E_0 , a , δ , σ , and E_x may be found in references 4 and 5.

The reliability of the level density expression is put to a severe test in the mass - 90 region because of the number of closed shells and subshells that occur here. A feeling for the uncertainty involved may be obtained from Fig. 1. Here we plot the ratio of calculated to experimental values for D_{obs} as a function of mass number. The data for these 33 nuclei in the range $31 \leq Z \leq 44$ were taken from the Livermore nuclear data library.⁶ A scatter of $\pm 50\%$ is common, while discrepancies of factors of 3 to 6 are observed. The figure does not include the estimated experimental errors, and these can be quite large in many cases, as evidenced when the "experimental" values in the report BNL-325⁷ are compared with those in Ref. 6. Other level density sets, such as those of Schmittroth,⁸ Baba⁹ or Dilg¹⁰ do not offer much improvement over the Cook set, and expectations of calculating unmeasured level spacings at the neutron separation energy with an accuracy of much better than 50% may not be reasonable, at least in this mass region. In many cases a 10% change in the level density parameter, a , can result in a factor-of-2 change in the level density.

In our work we modify E_0 , when necessary, to force agreement with the first 25 or so levels in each nucleus,¹¹ and when experimental values for

Fig. 1. The ratio of calculated to experimental values for the average level spacing, D_{obs} , as a function of the target mass number A .

D_{obs} are available they are used to better specify the parameter a . Then the boundary energy E_x is adjusted so that the two functions and their first derivatives are continuous at E_x .

Regarding the average s-wave radiation width $\bar{\Gamma}_Y$, we have evaluated a number of semi-empirical expressions and have had the most success with that of Stolovy and Harvey,¹²

$$\bar{\Gamma}_Y = KA^{2/3} U^\alpha [D_{ps}(S_n)]^\beta \quad (10)$$

Here, U is the neutron separation energy, S_n , corrected for pairing effects, $D_{ps}(S_n)$ is the level spacing per spin state at the neutron separation energy, and K , α , and β are constants. Average values of these constants have been reported.¹³

The parameter values given in Ref. 13 for the Stolovy-Harvey expression were determined about nine years ago by fitting all of the $\bar{\Gamma}_Y$ and level spacing data available at that time. The mass range covered was from 60 to 90. We have now refit the Stolovy-Harvey expression in a mass region around $A = 90$ with the data available in Ref. 6. We considered 33 nuclei in the range $31 \leq Z \leq 44$, and arrived at the following parameters for Eq. 10:

$$K = 2.65 \times 10^{-3}, \alpha = 1.185, \beta = 0.1113$$

Here $\bar{\Gamma}_Y$ is in eV units while U and D_{ps} are in MeV units.

Figure 2 shows the ratio of calculated to experimental values for $\bar{\Gamma}_Y$ versus the mass number of the target. While some improvement was noted over the fit given by the original parameters, the scatter is still about $\pm 50\%$.

Fig. 2. The ratio of calculated to experimental values for the average radiation width, Γ_{γ} , as a function of the target mass number A .

From a consideration of the typical errors involved in estimating separately, $\bar{T}_Y$ and D_{obs} , it appears that it is difficult to compute their ratio to within a factor of 2. Depending on the size of $\bar{T}_Y$ relative to the widths of other open channels, this might correspond to a 50% uncertainty in the capture cross section at low energies, and an increasingly larger uncertainty as the incident neutron energy goes up. An improvement was made by Arthur,¹⁴ who developed systematics for the quotient $\bar{T}_Y/D_{\text{obs}}$ in the mass-90 region, but the results were not entirely satisfactory because the quotient still exhibited a precipitous change in value around closed shells.

II. THE GAMMA-RAY STRENGTH FUNCTION

An alternate form of Eq. 3 is obtained when the gamma-ray transmission coefficients are expressed in terms of gamma-ray strength functions, $f_{x\lambda}(E_Y)$. With the same restrictions as before, we have

$$\frac{\bar{T}_Y}{D} = \int_0^{S_n} [f_{E1}(E_Y) + f_{M1}(E_Y)] E_Y^3 \rho(S_n - E_Y) dE_Y \quad (11)$$

We use the definition of the strength function as given by Bartholomew,¹⁵ and restrict our attention to E1 transitions,

$$f_{i\lambda E1}^{J\pi}(E_Y) = \frac{\bar{T}_{Yi\lambda E1}^{J\pi}}{E_Y^3} \rho_{J\pi}(E_\lambda) \quad (12)$$

The transition at an energy E_Y is from an upper state at E_λ to a lower state at E_i . The gamma-ray partial width for E1 transitions is averaged

over states λ of a given J and π . The average density of such states is $\rho_{J\pi}(E_\lambda)$. The energy denominator is E_γ^3 for dipole transitions, and thus if the E1 transition is single-particle in character, the strength function is independent of energy.

The strength function can be related to the photoabsorption cross section, $\bar{\sigma}_{\gamma a}$ in mb, by¹⁶

$$f_{E1}(E_\gamma) = 2.6 \times 10^{-7} \frac{\bar{\sigma}_{\gamma a}^J(E_\gamma)}{g^J E_\gamma} \quad (13)$$

where g^J is the statistical factor $(2J+1)/(2I+1)$; $\bar{\sigma}_{\gamma a}^J(E_\gamma)$ is the absorption cross section for the nucleus in ground state I to populate levels with spin J at $E_\lambda = E_\gamma$; E_γ is in MeV; and f is in MeV^{-3} . For E1 transitions one assumes¹⁷ that

$$\bar{\sigma}_{\gamma a}^J = \frac{g^J}{3} \bar{\sigma}_{\gamma a} \quad (14)$$

Combining Eqs. 13 and 14, we find

$$f_{E1}(E_\gamma) = 8.7 \times 10^{-8} \frac{\bar{\sigma}_{\gamma a}(E_\gamma)}{E_\gamma} \quad (15)$$

At this point we assume that the shape of the GDR can be adequately described by one or the sum of two functions (except for light nuclei), as indicated by photoabsorption measurements,¹⁸

$$\bar{\sigma}_{\gamma a}(E_\gamma) = \sum_{i=1}^2 \sigma_{mi} \cdot G_{\lambda i}(E_\gamma, E_{Ri}, \Gamma_{Ri}) \quad (16)$$

In some cases a sum of three functions has been used to fit photoneutron measurements,¹⁹ but since we will be dealing with a wide mass range of nuclei this refinement is not warranted here.

In Eq. 16 the parameter σ_{mi} represents the maximum cross-section value of the i^{th} peak function G_{xi} , while E_{Ri} and Γ_{Ri} represent the usual peak energy and width of the GDR. Defined this way, the function G_{xi} has a peak value of unity. Two functional forms that have been used extensively are the Lorentz and the Breit-Wigner forms:

Lorentz

$$G_L(E_Y, E_R, \Gamma_R) = \frac{1}{1 + \frac{(E_Y^2 - E_R^2)^2}{(E_Y \Gamma_R)^2}} \quad (17)$$

Breit-Wigner*

$$G_{BW}(E_Y, E_R, \Gamma_R) = \frac{1}{1 + \left(\frac{2}{\Gamma_R}\right)^2 (E_Y - E_R)^2} \quad (18)$$

In both cases the area under a given peak function is $(\pi/2) \sigma_m \Gamma_R$, which may be related to the classical dipole sum rule value,

$$\frac{\pi}{2} \sigma_m \Gamma_R = 60 \frac{NZ}{A} F_{SR} \text{ (mb-MeV)} \quad (19)$$

We combine into the parameter F_{SR} the contribution of exchange terms and the extent to which the sum rule is exhausted. In the situation where only one unresolved peak is present, or when the two peak cross sections may be taken as equal, then Eqs. 15, 16 and 19 may be combined into a single form:

$$f_{E^+}(E_Y) = 3.3 \times 10^{-6} \frac{NZ}{A} \frac{F_{SR}}{E_Y} \frac{\sum_i G_{xi}(E_Y, E_{Ri}, \Gamma_{Ri})}{\sum_i \Gamma_{Ri}} \quad (20)$$

While it has been more traditional to fit photoabsorption cross sections with one or the sum of two Lorentz curves, other forms have been used in the

*See note added in proof on page 57.

past, including the Breit-Wigner form.²⁵⁻³⁰ Dover, et. al., discuss assumptions under which either the Breit-Wigner or the Lorentz form may be derived within the framework of the Green's function approach to many-body theory.³¹ They assume that the width of the GDR arises from the damping of the particles and holes in a microscopic description of the resonance, and conclude that the Tamm-Dancoff approximation (TDA) for the GDR leads to a Breit-Wigner form, whereas the random-phase approximation (RPA) leads to a Lorentzian. While we follow the reasoning leading to this Breit-Wigner shape, we do not accept the conclusion that the RPA leads to a Lorentzian, because of an extreme approximation in which a part of the damped particle-hole propagator is given an abnormally large imaginary part (eq. 3.37b of Ref. 31). Although the authors of Ref. 31 argue rightly that the effect of this approximation is small at energies near the resonance (p. 484 of their paper), the same arguments show that at very low energies (midway between the poles at $E_Y \approx \pm E_R$ in the Lorentzian expression) the error may become comparable in magnitude to the cross section. The authors of Ref. 31 also point out that the derivation of a Lorentzian lineshape from another microscopic description of the damping (Ref. 27) is incorrect, although the reasoning leading to a Breit-Wigner form in that work is valid.

In an investigation related to the present one,³² a form for the photoabsorption through a single resonance has been derived in the form:

$$\sigma_{Ya}(E_Y) \propto \text{Im} \left\{ \frac{1}{E_Y - E_R + \frac{1}{2} \Gamma_{nuc} + \frac{1}{2} \Gamma_Y + iI} + \frac{1}{-E_Y - E_R + S_{nuc} + \frac{1}{2} \Gamma_Y + iI} \right\} \quad (21)$$

in which Γ_γ and Γ_{nuc} are the electromagnetic and nuclear parts of the resonant width, and I is an energy interval over which compound-nuclear fine structure is averaged with a Breit-Wigner profile. S_{nuc} is a real quantity, corresponding to an energy shift in the second term. In the case of no nuclear width or energy averaging ($\Gamma_{nuc} = S_{nuc} = I = 0$) the above form is very close to the Lorentzian. In the cases of interest in the present work, $I \ll \Gamma_{nuc}$ and $\Gamma_\gamma \ll \Gamma_{nuc}$; only the first term has a significant imaginary part at physically attainable energies, resulting in the Breit-Wigner form.

It is our conclusion that the Breit-Wigner is more appropriate as a starting point for parameterizing the strength functions at low energies rather than the Lorentzian. The Breit-Wigner form will be used in a parameterization of the E1 strength function in Section V, in which the width is treated as an energy-dependent quantity. In the past there have been attempts to utilize an energy-dependent width in studies involving the GDR,^{26,28,31} but, to our knowledge, not fully in conjunction with the Breit-Wigner shape.

III. ASSUMPTIONS CONCERNING THE GIANT RESONANCES

It has been observed for many years¹⁸ that the GDR parameters of peak energy E_R and width Γ_R follow relatively simple trends. The peak energy decreases with increasing mass number A , the dependence being somewhere between $A^{-1/6}$ and $A^{-1/3}$, while the width tends to follow nuclear structure, in the sense of being small around closed shells and somewhat larger between shells. For example, Holmes and co-workers²⁰ have studied a very wide range of nuclei, and give the following empirical expressions:

$$E_R = 35A^{-1/6} \quad (22a)$$

$$\Gamma_R = 33 kA^{-1/3} \quad (22b)$$

The parameter k takes on the value 0.6 at either a closed proton or neutron shell, the value 0.8 when either Z or N is within ± 1 or 2 units of a closed shell, and the value 1.2 for certain specified "deformed" nuclei.

When the present study began, we wished to have the most accurate representation for Γ_R in a restricted mass region around $A = 90$. From an examination of the data³³ from As to Cd we found^{22,23} that Γ_R , for single, unresolved peaks, could be parameterized satisfactorily in terms of the nuclear deformation parameter β_2 ,³⁴ or its square β_2^2 , with the latter being preferred:

$$\Gamma_R = \frac{A^{1/3}}{1.227} (1.00 + 12.78 \beta_2^2). \quad (23)$$

The peak energy in this mass range could be given by:

$$E_R = 35.4 A^{-1/6}. \quad (24)$$

Our expression for E_R was essentially that of Holmes,²⁰ but our results for Γ_R differed appreciably for certain nuclei and, of course, varied with mass in a somewhat different manner.

More recently we have determined an alternate expression for Γ_R , for single peak resonances in the full mass range from Cu to Bi,

$$\Gamma_R = 59.94 \beta_2^2 + 3.859 \quad (25)$$

How well this expression fits the data³³ is shown in Fig. 3. The agreement is surprisingly good, considering that there is no dependence on mass. This suggests that the GDR widths for all spherical nuclei are about the same, regardless of mass. In a later section we will show that Eq. 25 is an approximation to a more complicated expression for Γ_R which does, in fact, include a mass dependence.

We now address a problem which casts doubt on the appropriateness (for the present purposes) of using a single-peak GDR for any except those nuclei very near a closed shell. Occasionally the same set of photonuclear data has been analyzed with both a single-peak and a double-peak GDR line shape, usually of the Lorentz form. While the two fits in the region of the peak may each be quite similar, as much as a factor of 2 difference in magnitude may be observed in their low energy tails. A recent example of this involves

Fig. 3. The GDR width, Γ_R (in MeV), versus the nuclear deformation parameter squared, β_2^2 . The solid circles were obtained from single-peak data in the mass range from Cu to Bi,³³ the straight line is a fit given by Eq. 25.

isotopes of Os,¹⁹ whose GDR had originally been fit with single-peak functions³³ and for which the fits were not very satisfactory well below (≈ 8 MeV) the GDR peak. Since the E1 gamma-ray strength function, which is derived from the low energy tail of the GDR, cannot depend on someone's preference for a single or a double peak, there appears to be a basic difficulty involved in the assumption of a single-peak resonance. This is particularly true and well known for those nuclei which exhibit a static deformation and have two clearly separated GDR peaks.

To avoid this problem we have studied the usefulness of a model for the GDR that consists always of two overlapping peaks, even in the regions of low β_2 values near closed shells. In developing this model we have taken into consideration all of the resonance parameters listed in Berman's compilation,³³ for 80 nuclei from V to Bi. These encompass some 99 sets of parameters; no attempt was made to eliminate questionable or poor data sets.

In our model we have made some simplifying assumptions to reduce the number of parameters:

- 1) All nuclei, in the mass range considered, behave like prolate spheroids to some degree, and we take it that

$$\Gamma_{R1} \sigma_{m1} = 1/2 \Gamma_{R2} \sigma_{m2} \quad .$$

- 2) Both peak cross sections, σ_{m1} and σ_{m2} , are equal. Therefore

$$\Gamma_{R1} = 1/2 \Gamma_{R2} \quad .$$

- 3) The integrated photonuclear absorption cross section is always 1.25 times the sum rule value. This corresponds to $F_{SR} = 1.25$ in Eq.

19. Figure 4 presents integrated cross sections, obtained from Ref. 33, for V through Bi, expressed in sum rule units. Data from both single- and double-peak interpretations are included; connecting

vertical lines indicate a range of data for the same nucleus. One sees that over most of the mass range the relation 1.25 times the sum rule is reasonable. Around $A = 90$ and below the value appears to be less than 1.25, which is expected since the data refer to photoneutron measurements and charged-particle emission is not included.

- 4) The two widths, Γ_{R1} and Γ_{R2} , are independent of the deformation parameter β_2 , but do vary with mass. As a first attempt, we used the $A^{-1/3}$ dependence suggested by Holmes.²⁰
- 5) The average energy of the two peaks is that of a prolate spheroid, i.e.,

$$E_R = (E_{R1} + 2 E_{R2})/3. \quad (26)$$

The mass dependence of E_R may be taken as¹⁸

$$E_R = 31.2 A^{-1/3} + 20.6 A^{-1/6}. \quad (27)$$

This results in E_R values that are within ± 5 percent of those calculated from Eq. 22a for the mass range of interest here. The ratio of the two peak energies is related to the ratio of the major to minor axes of the prolate spheroid, and thus to β_2 . We find

$$\frac{E_{R2}}{E_{R1}} = F(\beta_2) = 0.95 \left[\frac{2.126 + 0.822\beta_2}{2.126 - \beta_2} \right]. \quad (28)$$

Fig. 4. Total, integrated photoneutron cross sections³³ expressed in sum rule units versus mass number A

See Appendix A for details of the derivation. After the term in brackets was derived and compared with the data in Ref. 33, it appeared that a reduction of 5 percent was in order. This gave rise to the factor 0.95.

- 6) The apparent width of two overlapping peaks of equal height, and with widths differing by a factor of 2, can satisfactorily be represented by a function involving the square of the difference of the peak energies, plus the average of the two widths. Thus

$$\Gamma_R = a(E_{R2} - E_{R1})^2 + 3/4 \Gamma_{R2}, \quad (29)$$

where a is a constant.

The above assumptions yield the following expression for Γ_R ,

$$\Gamma_R = a \left[\frac{3E_R (F(\beta_2) - 1)}{1 + 2 F(\beta_2)} \right]^2 + 3/4 \frac{b}{A^{1/3}}, \quad (30)$$

where again a and b are constants. The model was tested by the least squares fitting of Eq. 30 to the 63 parameter sets in Ref. 33 for the 42 nuclides from Cu to Bi that are treated as single-peak resonances. The results of the fit gave the parameter values: $a = 0.3974$ and $b = 29.70$.

Figure 5a shows the ratio of experimental to calculated single-peak widths versus A . The connected points represent two or more different data sets for the same nucleus, and thus give some feeling for the spread within the experimental data sets. For most nuclei the calculated width is within 10% of the measured value, the notable exceptions appearing in the mass-90 region. We will show in a later section that the two-peak assumption breaks

down at or very near the $N = 50$ closed neutron shell, but the remaining assumptions are valid.

If the two-peak model has any validity, it should be able to predict the widths of the two peaks in cases where they have been resolved. Covariance terms connect the six parameters in the two-peak data sets, because they come from a single fitting analysis; errors in one parameter may be compensated by errors in other parameters. Also, a few of the data sets are more representative of oblate rather than prolate spheroids. Nevertheless, the general trends should be predictable.

The analysis of the single-peak data yielded the result that the A -dependent term was $(3/4)(29.70 A^{-1/3})$. This we identify with the width of the second peak, Γ_{R2} . Thus

$$\begin{aligned}\Gamma_{R2} &= 29.70 A^{-1/3} \\ \Gamma_{R1} &= 1/2 \Gamma_{R2} \\ \Gamma_{R1} + \Gamma_{R2} &= 3/2 \Gamma_{R2}\end{aligned}\tag{31}$$

These predictions are compared in Fig. 5b with 36 data sets for 28 nuclides which were specified in terms of two peaks in Ref. 33. The solid lines are the predicted values, and the empty and filled circles are the measurements. Keeping in mind that none of these 36 data sets were used to fix the parameters in the double-peak model, it appears that there is substantial agreement with the predictions.

In the heavy mass region, however, the data suggest that the supposed $A^{-1/3}$ dependence of the width is not strong enough. The single-peak data

Fig. 5. a) The ratio of experimental widths for single-peak E1 resonances to widths calculated from Eq. 30 versus A . b) Predicted widths (solid lines) for double-peak resonances from Eqs. 31, compared with experimental values³³ versus A ; dashed line results from assumption of $A^{-1/2}$ dependence of the widths.

were refit using an $A^{-1/2}$ dependence for Γ_{R2} , which resulted in $\Gamma_{R2} = 66.08 A^{-1/2}$, and the dashed line in Fig. 5b. This improves the fit in the heavy mass region, but elsewhere the agreement is lessened. In particular, the fit to all of the single-peak data is worsened.

One further attempt was made to determine the A -dependence of the resolved widths. All of the data in Fig. 5b were averaged under the assumptions that $\Gamma_{R1} = 1/2 \Gamma_{R2}$, and that the widths were independent of the atomic number Z of the species. In this way the 108 points shown in Fig. 5b were collapsed to 26 averaged values. These are plotted in Fig. 6, together with the $A^{-1/3}$ and the $A^{-1/2}$ predictions. One might conclude from Fig. 6 that there is no simple, monotonic dependence of the resolved widths on the mass number. On the other hand, the analysis of the 63 unresolved single-peak data sets, under the $A^{-1/3}$ assumption, yielded quite an acceptable fit to these data. Until further theoretical guidance becomes available, we recommend the use of the $A^{-1/3}$ dependence for the Γ_{R1} and Γ_{R2} for mass numbers less than about 160. For higher masses we have obtained better results using the $A^{-1/2}$ dependence.

That Eq. 25 is actually an approximate form of Eq. 30 may be seen if $F(\beta_2)$ is roughly approximated and if Eq. 22a is used to estimate E_R . This results in the expression

$$\Gamma_R \approx (67 \beta_2^2 + 22) A^{-1/3}, \quad (32)$$

whereas Eq. 25 is independent of A . However, for large values of A , Eq. 25 will calculate values of Γ_R somewhat too large with respect to Eq. 32, and for low values of A the opposite result is found. Hence Eq. 25 represents an average over A of the results predicted by Eq. 30.

Fig. 6. Values of Γ (in MeV), obtained by averaging all the experimental data indicated in Fig. 5b, versus A ; compared with the predictions of an $A^{-1/3}$ and $A^{-1/2}$ dependence.

IV. EVALUATION OF THE LORENTZ LINE SHAPE

We began our study of the functional form of the E1 gamma-ray strength function by calculating neutron capture cross sections and capture gamma-ray spectra for 12 target nuclei from ^{75}As to ^{103}Rh , and then comparing the results²¹⁻²⁴ with the available experimental data. The statistical model calculations were carried out using the latest versions of the STAPRE³⁵ and COMNUC³⁶ nuclear reaction codes. A number of assumptions were made:

- a) The E1 γ -ray strength function was single peaked, with the Lorentz line shape.
- b) The GDR parameters E_R and Γ_R were computed from Eqs. 23 and 24.
- c) The M1 γ -ray strength function was single particle in character (independent of energy), and contributed about 15 percent to the total s-wave radiation width $\bar{\Gamma}_\gamma$.
- d) The strength functions were independent of the excitation energy of the nucleus (Brink-Axel assumption).
- e) The neutron transmission coefficients were calculated either from the optical model parameters of Moldauer,³⁷ or from those of Lagrange,³⁸ when available.
- f) The level density expression was that of Gilbert and Cameron,⁴ with parameters modified as described previously.

Since the γ -ray strength function is an averaged quantity, it is appropriate to extract it from capture cross sections in the overlapping resonance region. Our approach in each case was to adjust the value of $\bar{\Gamma}_\gamma/D_{\text{obs}}$, used in the normalization the E1 and M1 γ -ray transmission coefficients, until agreement with the experimental data was achieved; then from this ratio

calculate the effective E1 strength function. This, in turn, was compared with the prediction from Eq. 20, in order to obtain a value for the parameter F_{SR} . It was hoped that the values of F_{SR} so extracted would vary in a systematic way, so that capture calculations could be made with some confidence for cases where experimental data were lacking.

To illustrate the method, we show in Fig. 7 the results for target nuclei $^{85,87}\text{Rb}$. The points are the data^{39,40} and the lines are our adjusted calculations. The inset shows the single extracted E1 strength function that represents both isotopes. Here we can see the power of the strength function approach--while the capture cross sections for the two isotopes differ by a factor of 10 and the $\sigma_{\gamma}/D_{\text{obs}}$ values differ by a factor of 27, the same strength function is applicable. The reason is that the neutron separation energy in compound nucleus ^{86}Rb exceeds that for ^{88}Rb by 2.5 MeV. Hence the larger cross section for the target ^{85}Rb is due primarily to the circumstance that in Eq. 11 one is integrating 2.5 MeV further up the strength function curve. In addition, the density of accessible states, $\rho(S_n - E_{\gamma})$, for each gamma-ray energy is much greater in ^{86}Rb than it is in ^{88}Rb .

The results of our extraction of F_{SR} values are given in Fig. 8. We plot, in addition, the total integrated photoneutron cross section³³ expressed in sum rule units for a large class of nuclei. In general, the two agree fairly well. The value for F_{SR} appears to be somewhat mass dependent in this region--being about 0.75 for compound nuclei with $A = 90$, and about 1.1 for $A = 94$. The two marked exceptions were the compound nuclei ^{76}As and ^{104}Rh . To get around this situation one might argue for more M1 strength.

Fig. 7. The $^{85,87}\text{Rb}(n,\gamma)$ cross sections versus incident neutron energy, E_n . The circles are experimental data^{39,40} and the lines are calculational fits that yield the $2\pi\Gamma_\gamma/D$ values indicated. The inset shows the single extracted E1 strength function that represents both isotopes, versus E_γ .

Fig. 8. The solid circles represent the ratio of the f_{E1} values extracted from experiment to those calculated by Eq. 20 versus A ; the open circles are total integrated photoneutron cross sections³³ expressed in sum rule units versus A .

In the ^{104}Rh case, the M1 contribution would have to be raised from the standard 15 percent of $\bar{\Gamma}_\gamma$ up to 32 percent in order to repair the situation. However, the ^{76}As case would require that the M1 strength be 60 percent of $\bar{\Gamma}_\gamma$, which is unrealistically large and not readily explainable on a shell model basis. The discrepancies for these nuclei cannot be readily understood as long as the Lorentz form is taken to represent the GDR tail.

Another problem arose when the calculated s-wave capture γ -ray spectra were compared with the thermal capture data of Orphan.⁴¹ The experimental spectra were harder than those calculated by the statistical model codes. This was particularly apparent for the monoisotopic targets ^{75}As , ^{93}Nb , ^{103}Rh . It has been well known for many years that in quite a few instances capture γ -ray spectra have shown deviations from the Lorentzian predictions, consisting of an excess of gamma rays in the 5-8 MeV region relative to those at lower energies.⁴² In order to reproduce this effect in their calculations various groups have added either a "pigmy" resonance (E1 or M1)¹⁵ or a step-like decrease,²⁴ at an energy around 5 or 6 MeV, in the tail of the γ -ray strength function. Figure 9 illustrates this for the γ -ray spectrum for thermal neutron capture on ^{103}Rh , where a resonance in the M1 strength function is employed.

In fact, the capture gamma-ray spectra for the targets ^{75}As , ^{93}Nb , and ^{103}Rh could be fit equally well by either a "pigmy" resonance or a step decrease or any number of mixtures of the two. The problem is that, because the neutron separation energy for most stable nuclei is only about 6 to 8 MeV, it is in general not possible to sample a large enough energy range of the γ -ray strength function to definitely distinguish between the effects of a resonance or a step decrease. When we try to improve the situation by

Fig. 9. The gamma-ray spectrum for thermal neutron capture on ^{103}Rh . The histogram is the experimental data of Orphan,⁴¹ the triangles indicate a calculation using a Weisskopf M1 strength of 32%, the crosses indicate a calculation using a Weisskopf M1 strength of 8% and a pigmy resonance M1 strength of 24%. Inset shows corresponding E1 and M1 strength functions.

increasing the energy of the incident neutron, and thus sample a greater energy range of primary capture γ -rays, the inelastic scattering cross section rapidly becomes so large that the remaining capture cross section is difficult to measure with accuracy. This is well illustrated in Fig. 10, for 3.49-4.00 MeV neutrons on ^{93}Nb . The data were taken by the Oak Ridge group,⁴³ and it can be seen that even at this energy the error bars on the high energy end of the spectrum are sufficiently large to accommodate almost any calculated spectral shape. Another type of approach that holds more promise for accurately sampling the γ -ray strength function over a wider energy range is the measurement of proton capture γ -ray spectra.²⁴ The three primary advantages are the following. Firstly, the (p,p') inelastic scattering does not compete as strongly with the (p,γ) reaction as the (n,n') inelastic scattering does with the (n,γ) reaction because of the Coulomb barrier. Thus a wider range of incident particle energies may be utilized. Secondly, monoenergetic protons with small energy spreads are obviously easier to obtain than are such neutrons. Thirdly, for light and medium weight nuclei, particularly around closed neutron shells, there are a number of targets with large, negative (p,n) Q-values. This allows the (p,γ) cross section and its associated γ -ray strength function to be studied over a wide energy range without the (p,n) reaction competing. A disadvantage with protons is the Coulomb barrier, which limits this approach to light and medium weight targets (probably $Z \leq 50$).

To summarize this section, we may say that it is possible to parameterize the GDR's width and energy, and its peak cross section as defined in terms of the classical dipole sum rule, the parameter F_{SR} , and the assumption of a Lorentz line shape. This permits one to calculate capture cross sections with

Fig. 10. The gamma-ray spectrum for incident neutrons in the energy range 3.49-4.0 MeV on ^{93}Nb . The histogram is the experimental data of Dickens, et al.,⁴³ the dashed curves are calculated results in this energy range using a Lorentzian E1 strength function with a step decrease in the tail.

some confidence in the mass-90 region. However, there are exceptions, such as targets ^{75}As and ^{103}Rh , where the predictions were shown to be unreliable. For neutron capture calculations up to several hundred keV, the excitation function is very insensitive to the shape of the γ -ray strength function. It is only when the neutrons reach the MeV energy region that it becomes possible to distinguish between different strength function shapes, but by now the capture cross section is usually so small that experimental errors limit the usefulness of the measurements.

The capture γ -ray spectrum, on the other hand, is sensitive to the strength function shape even for thermal neutrons. Now it becomes necessary to introduce arbitrary bumps or step decreases in the strength function's tail in order to fit the measurements. While it might be possible to parameterize such bumps or steps we have chosen not to do so. The need for such modifications seems to be more wide spread than has been thought in the past, being required in every case we have studied. Furthermore, if a step decrease is used in order to achieve a harder γ -ray spectrum, then the remainder of the strength function must be shifted upwards in magnitude, so that the integral in Eq. 11 still yields the proper $\bar{\Gamma}_{\gamma}/D_{\text{obs}}$ value. Then the strength function extrapolation, based on the Lorentz form, no longer connects up with the photonuclear measurements.

What we wish to do in the next section is to redefine the functional form of the GDR, such that a step will automatically appear with the following properties: a) the low energy portion will extrapolate upward properly to agree with the photonuclear measurements, b) the correct $\bar{\Gamma}_{\gamma}/D_{\text{obs}}$ will be obtained for agreement with the neutron capture cross-section data, and c) the observed hardness in the capture γ -ray spectra will be reproduced. Some plausible arguments for the new line shape parameterization will also be given.

V. THE ENERGY-DEPENDENT BREIT-WIGNER FUNCTION

As we have mentioned previously, various alternatives to the Lorentz form of the GDR have been used in the past. For example, the Breit-Wigner function with a constant, energy independent width was given in Eq. 18. Around the peak of the GDR there is little difference between the Lorentz and the Breit-Wigner forms, but as we extrapolate the tail down to low energies the Breit-Wigner function has a much less desirable shape. The tail is not only greater in magnitude than the Lorentz shape, for the same E_R and Γ_R values, but it is also much flatter. Thus the Breit-Wigner function without modification produces a softer γ -ray spectrum than the Lorentz function, much closer to the single-particle (energy independent) result, and this is just opposite from what the experimental data require.

On the other hand, reaction theories show that the width parameter in a resonance expression (either Breit-Wigner or Lorentzian) is, in principle, energy dependent. In fact, comparison of the Lorentzian form (Eq. 17), used in fitting photonuclear data, with expressions derived from general reaction theories show that the energy-independent parameter Γ_R appearing in Eq. 17 is not the width in the sense used in reaction theories, but that the true width is actually $\Gamma_R(E_Y/E_R)$. Thus the width in the conventional Lorentzian expression is actually strongly energy dependent ($\propto E_Y$). The parameter Γ_R appearing in the Breit-Wigner expression (Eq. 18) is, in contrast, the true width. To compare the Lorentzian and Breit-Wigner expressions with the same assumption about the energy dependence of the width, Γ_R in Eq. 18, should be replaced by $\Gamma_R(E_Y/E_R)$, and then the constant Γ_R will have the same significance in both expressions. If this

is done, it will be seen that the two forms are identical at the peak, $E_Y = E_R$, but that as E_Y approaches zero, the Breit-Wigner expression actually falls below the Lorentzian, by a factor approaching 1/4 at the lowest energies. The actual energy dependence of the width should depend on nuclear dynamics, and there is no a priori reason to favor the particular energy dependence implicit in the Lorentzian as conventionally used. Below we make a first attempt at a reparameterization of the width which takes into account nuclear damping mechanisms. This reparameterization will be made in conjunction with the Breit-Wigner shape, since we believe that the Breit-Wigner form is more likely to be the correct starting point than the Lorentzian. As a cautionary note, it should be pointed out that all the E1 electromagnetic strength is assumed to originate from one or two giant-resonance states at high excitation, and that this strength is spread to lower energies by damping. Strength originating at lower excitation, such as is sometimes discussed in connection with various types of "pygmy" resonances, is not included in the present strength-function model.

We will proceed by making the assumption that $\Gamma_R(E_Y)$ is primarily a damping width, of one particle--one hole states into states of increasingly larger exciton number. As a rough guide we can look at the rate expressions for damping from precompound evaporation model treatments. For example, Williams⁴⁴ gives an expression for the rate for increasing the exciton number n (in steps of 2 units):

$$\lambda_+(n) = \frac{2\pi}{h} |M|^2 \frac{g(gE - C_{p+1,h+1})^2}{p+h+1} \quad (33)$$

Here $n = p+h$ (particle + hole), g is the single-particle level density at the Fermi level, $|M|$ is the effective matrix element for residual interactions,

and $C_{p,h}$ is a correction due to the Pauli principle. The energy of the system is E , which we take to be E_γ , corresponding to γ -ray absorption from the ground state.

Systematics⁴⁵ for $|M|^2$ from medium-energy precompound reaction studies have shown that, to a fair approximation

$$|M|^2 \propto (g^3 E)^{-1}. \quad (34)$$

This has an unrealistic energy dependence at low energies (below the range of concern of the precompound studies), but it leads to a nearly linear dependence of $\Gamma_R(E_\gamma)$ on E_γ at high energies. Instead, we choose to use the form

$$|M|^2 = \frac{k_1'}{k_2' + g^3 E_\gamma}, \quad (35)$$

where k_1' and k_2' are adjustable parameters to be obtained from fitting data. If we take it that $\Gamma_R(E_\gamma)$ is proportional to λ_+ , and that the correction term $C_{p,h}$ can be neglected, then

$$\Gamma_R(E_\gamma) = \frac{k_1' g^3 E_\gamma^2}{k_2' + g^3 E_\gamma}. \quad (36)$$

This parameterization of the damping width of a giant resonance is suggested by the behavior of the damping of single-particle configurations in nuclear matter calculations, in which the damping width at low energies is quadratically dependent on the excitation energy of the particle (or hole) above the Fermi surface. It is convenient to reexpress Eq. 36 as

$$\Gamma_R(E_Y) = \Gamma_R \left(\frac{C + E_x}{C + E_Y} \right) \left(\frac{E_Y}{E_x} \right)^2 \left(\frac{2}{E_x + E_R} \right) . \quad (37)$$

We retain here the usual meanings for E_R and Γ_R (i.e., as energy-independent quantities), and have converted the two constants k_1 and k_2 into two new constants, E_x and C . Because we are accustomed to viewing strength functions in relation to the Lorentz extrapolation, we have expressed k_1 in terms of E_x , the energy at which point the function combining the energy-dependent damping width with the Breit-Wigner form (abbreviated here EDBW) will intersect the Lorentz at or below the peak energy E_R . Details of this reparameterization will be found in Appendix B. If the parameterization of Eqs. 36 and 37 is maintained at high energies, the integral over the EDBW function diverges, which violates the sum rule. We have therefore allowed $\Gamma_R(E_Y)$ to increase until it equals Γ_R , at which point it is held constant (and the EDBW function becomes the normal Breit-Wigner expression). In this way the area under the EDBW curve approximates the value of $(\pi/2)\Gamma_R\sigma_m$ fairly closely. We see from Eq. 37 that $\Gamma_R(E_Y)$ approaches a linear dependence on E_Y when E_Y becomes large compared to C . Our studies so far indicate that C is small, perhaps around 1 to 5 MeV, and that E_x is about 5 or 6 MeV. Allowing $\Gamma_R(E_Y)$ to remain constant above some energy appears somewhat less arbitrary if $\lambda_+(n)$, and therefore $\Gamma_R(E_Y)$, is related to the imaginary part of the optical model potential.⁴⁶ Recent studies by Lagrange³⁸ have shown precisely this kind of behavior for the imaginary potential for neutron scattering in the mass-90 region. In further studies of this type, investigation of a smoother transition between the low- and high-energy regions would be in order.

As so constructed, the EDBW function lies below the Lorentz function at low energies, and crosses the Lorentz at the energy E_x . The slope of this low energy portion of the EDBW curve, and the amount of overshoot above the Lorentz curve depends on the parameter C --the larger the value of C the steeper the curve and the greater the overshoot. At some energy below E_R the width in Eq. 37 becomes equal to Γ_R , and the curve then continues on to higher energies along the normal Breit-Wigner function. The general situation is shown in Fig. 11, for an arbitrary set of parameter values. It is interesting to note that the overshoot gives something of the appearance of a broad resonance with a peak around 8 or 9 MeV riding on the Lorentz curve. Then, going to lower energies, the step decrease occurs. Although the behavior of the deviation from the Lorentzian near 8-9 MeV may be accentuated by the very simple model chosen for the transition between low and high energies, the same general features (of a deficiency relative to the Lorentzian at very low energies and an excess at higher energies) could easily be retained in a model with a smoother dependence of the width on energy. At the same time, we must remember that in cases for which the gamma-ray strength function is believed to be significantly altered by "pygmy" electromagnetic strength not originating in the GDR, it should be necessary to include an additional resonance to obtain a physically sensible parameterization. Effects of shell structure on the damping width are also not included in the present parameterization, other than through their influence on the two parameters of the expression for the GDR width.

Fig. 11. The GOR line shape calculated with an arbitrary set of parameters and represented by various functional forms. The solid curve results from the EDBW model, parameterized as in Eq. 37, with the C and E_x values indicated; the solid circles show results from a Lorentzian model; and the dashed curve indicates the results of an unmodified Breit-Wigner model.

VI. EVALUATION OF THE EDBW LINE SHAPE

VI.A. The Mass - 90 Region

Using the same level density and optical model parameters, and operating under the same assumptions as during our study of the Lorentz line shape, we have reanalyzed this mass region using the FDBW line shape and the double-peak treatment of the GDR as described above.^{47,48} We wished to develop global systematics for the parameters C and E_x in the energy-dependent GDR width expression, and then see if these parameter values were suitable in other mass regions.

We have found that a value of $E_x = 5$ MeV and $C = 1$ to 5 MeV, together with the double-peak GDR formulation, reproduced well the neutron capture cross sections and capture γ -ray spectra for all of the target nuclei studied -- with the exceptions of ^{89}Y and ^{90}Zr . The sensitivity of the calculated results to a range of C values has not yet been fully studied. In the case of ^{93}Nb , for neutron capture cross section calculations, the sensitivity to different C values in the range of 1 to 5 MeV was found to be less than or equal to the experimental error in the data; the capture gamma-ray spectrum calculations were more sensitive, with a C value equal to 5 giving better results. However, all other calculations in the mass 90 region described here were carried out with $C = 1$. Figs. 12-14 show some of our results for the targets ^{75}As and ^{103}Rh , which were the deviant cases in the first study. Although the cross-section data for both targets contain large uncertainties, the predicted results are reasonably satisfactory. The agreement with the thermal capture data of Orphan shows the desired amount of

Fig. 12. The gamma-ray spectrum for thermal neutron capture on ^{75}As . The histogram is the experimental data of Orphan.⁴¹ Calculated spectra using various functional forms of the E1 strength function are: solid circles resulting from the double-peak, EDBW model; open circles resulting from a Lorentzian model; and open, inverted triangles obtained by a Weisskopf model.

Fig. 13. The (p, γ) cross section on ^{75}As : the solid symbols are recent measurements⁴⁹⁻⁵³ and the curve results from a double-peak, EDBW model calculation.

Fig. 14. a) The (n,γ) cross section on ^{103}Rh : the solid triangles are the data of Macklin et al.⁵⁴ and the solid squares that of Joly et al.⁵⁵ the curve is a double-peak, EDBW model calculation. b) The gamma-ray spectrum for thermal neutron capture on ^{103}Rh . The histogram is the data of Orphan;⁴¹ the solid hexagons result from a double-peak, EDBW model calculation.

hardness in the calculated spectra, though the structure that often appears in the high-energy end of the measured spectra is not well reproduced. It is possible that this reflects either a shortage in the discrete level information used in the calculation or, more likely, nonstatistical processes frequently observed in thermal or resonance region capture measurements.

There is another possibility, however, that we have not as yet thoroughly evaluated. Our pseudo "thermal" code calculations are made at the lowest energy that is convenient for STAPRE, a code in which the energy of the system is divided into energy bins. The lowest incident energy is the midpoint energy of a bin, which is usually in the 10's of keV range. The code then merely sets to zero all populations in the compound nucleus that cannot be produced by s-wave neutrons. Thus absolute cross sections are not available, but the relative results of photons/100 captures are unaffected. However, if two spin states can be produced by s-wave neutrons, they are both included in the calculation in the proportion of $(2J + 1)$, whereas the actual thermal capture data will emphasize one spin state over the other. This could affect the high-energy end of the spectrum, since this is produced primarily in the first step of the γ -ray cascade, as well as the relative population of isomeric states that may be produced in the cascade.

Although successful elsewhere, it appears that for nuclei at or near the closed neutron shell, $N = 50$, the assumption breaks down that these nuclei can still be treated to some extent as prolate spheroids and that their E1 strength functions can be described by two, superimposed giant dipole peaks. Our studies so far, for nuclei with $N = 50$ and 51 , indicate that the double-peak EDBW model overestimates the E1 strength function by a factor of two or more at some energies. This leads to calculated neutron capture cross sections that are much too large.

In these cases, reasonable results are obtained if all assumptions and systematics given above are used, but the EDBW E1 strength function is computed in terms of a one-resonance model. This one-peak model is described by: E_R , the average energy from the two-peak model (Eq. 26), Γ_1 , the width of the first, narrower peak from the two-peak model; and the corresponding peak cross section, $\sigma_1 \Gamma_1 = 1.25(\pi/2) 60(NZ/A)$. In Fig. 15 is shown the single-peak EDBW E1 strength function for ^{90}Zr compared with information inferred from experiments.^{33,56,57} Using the one-resonance model to calculate the strength functions for the compound nuclei ^{90}Y and ^{91}Zr , the results shown in Fig. 16a and 16b were obtained. The calculated cross sections are in good agreement with the data.^{39,54,58-62} In both cases the energy-dependent widths for the strength functions were calculated with the parameter values $E_x = 5$ MeV and $C = 1$ MeV.

VI.B. The Ta-Au Mass Region

We have applied the EDBW model with the double-peak GOR to calculate capture cross sections and capture γ -ray spectra for the targets ^{181}Ta , $^{186,187,188}\text{Os}$, ^{187}Re , and ^{197}Au . Good results were obtained for all except ^{197}Au , where the calculated spectra still lacked sufficient high-energy γ -rays.⁴⁸ As before we are only interested in global trends in the calculated results. No attempt is made to fine-tune parameters for any specific mass region. In each case the width parameter values were fixed at $E_x = 5$ MeV and $C = 5$ MeV. The spectra were only slightly softer with the parameter value of $C = 1$ MeV.

As an example, consider the results for target nucleus ^{181}Ta , presented in Fig. 17. The predicted cross sections agree fairly well with the measurements of Lindner⁶³ and Poenitz.⁶⁴ The data for the capture γ -ray spectrum⁶⁵ were for neutrons in the energy range of 1-2 MeV. The

Fig. 15. Comparison of the single-peak, EOBW E1 strength function for ^{90}Zr versus E_γ (solid curve) with: measurements of Axel et al.⁵⁶ (solid circles) and of Szeftlinska et al.⁵⁷ (open circles); Lorentz form with parameter sets from Ref. 33.

Fig. 16. The calculated (n, γ) cross sections for ^{89}Y and ^{90}Zr (solid curves) compared with measurements.^{39,54,58-62}

Fig. 17. Comparison of the measured capture gamma-ray spectrum for ^{181}Ta with neutrons between 1 and 2 MeV (X) from Ref. 65 with calculations (solid lines) at these two energies. Insert compares the calculated (n,γ) cross section with measurements by Lindner⁶³ (circles) and Poenitz⁶⁴ (triangles).

calculations did not include the γ -rays produced by inelastically scattered neutrons, and so the curves are not shown below 2 MeV. An equally good fit was achieved when the γ -ray spectrum from s-wave neutrons was compared with the measurements of Orphan.⁴¹

We show as solid lines our calculated neutron capture cross sections for ^{187}Os in Fig. 18, together with the double-peak EDBW strength function computed from our model. Also shown are the Lorentz extrapolation of the single-peak and double-peak fits to the ^{188}Os photoneutron data of Berman.^{19,33} These produce the larger cross sections indicated in the figure. The data were those of Macklin and Winters,⁶⁶ who also measured the neutron capture cross sections for the isotopes $^{186,188}\text{Os}$. Our calculations agree equally well for all three nuclides. This figure also illustrates the effect referred to earlier in this paper, that the same photoneutron data fitted with a single or a double GDR can produce significantly different γ -ray strength functions, and therefore different calculated capture cross sections.

The last case we will discuss in this mass region is the target nuclide ^{197}Au . The calculated neutron capture cross section and two sets of measurements^{55,63} are given in Fig. 19. The EDBW strength function is shown in the insert, compared with the Lorentz extrapolation of the peak parameters measured at Saclay.³³ While the calculated cross section is adequate, the capture γ -ray spectrum is not. We have computed this spectrum at a variety of neutron energies, and with several sets of values for the parameters E_x and C . A typical result appears in Fig. 20, where some Oak Ridge data⁶⁷ are given. We have not been successful in producing the degree of hardness in the spectra that the Oak Ridge data as well as the thermal

capture data of Orphan all seem to exhibit. The reason is clear when our calculated strength function is compared with those extracted from various types of experimental measurements, as presented, for example, in a report by Earle, et.al.⁶⁸ While our strength function compares well at γ -ray energies of 4 and 10 MeV, in between the data require an abrupt loss of strength near 8 MeV which our calculations do not reproduce. Perhaps the extracted strength function is actually the sum of a flat M1 function and a true E1 function that falls off even more steeply than that shown in Fig. 19, but we have not pursued this possibility any further.

Fig. 18. Comparison of the calculated (solid line) (n,γ) cross section for ^{187}Os with the data of Macklin and Winters.⁶⁶ Insert shows the E1 strength functions used. The short dash (1) and long dash (2) strength functions represent, respectively, extrapolations of single- and double-peak fits of Berman,^{19,33} which yield the indicated cross sections.

Fig. 19. Comparison of the calculated (solid line) (n,γ) cross section for ^{197}Au with the data of Lindner⁶³ (squares) and Joly⁵⁵ (triangles). Insert shows the $E1$ strength function used, compared with the extrapolation of the Sacloy results³³ (dashed line).

Fig. 20. Comparison of the measured capture gamma-ray spectrum of ^{197}Au with neutrons from 0.2 to 0.6 MeV 67 (circles) with the calculations (solid lines) at these two energies.

VII. SUMMARY AND CONCLUSIONS

Reliable neutron capture excitation functions can be computed by the use of gamma-ray strength function systematics. The work here shows that the E1 strength function exhibits a slow and predictable behavior over the mass ranges investigated. Such behavior is advantageous since the ratio of Γ_γ to D_{obs} can be derived from the E1 strength function with much less uncertainty than that associated with the separate systematics for these quantities. This is particularly important in calculations for targets around closed shells and for unstable targets where there is a lack of experimental data.

The new parameterization of the E1 strength function, in terms of a two-peak Breit-Wigner function with an energy dependent width, yields both satisfactory neutron capture cross sections and capture gamma-ray spectra in the mass regions studied; except near the N=50 closed shell where a one-peak model appears more appropriate. The use of an energy-dependent Breit-Wigner function to describe the giant dipole resonance is appealing both from the point of view of reaction theory and from its application in the calculation of capture gamma-ray spectra. Such a function automatically provides a loss of strength (relative to the Lorentzian) at low gamma-ray energies, allowing one to reproduce the observed hardness in a spectrum. The low-energy portion of the function can still be extrapolated upward to agree with photonuclear data and be of the correct magnitude to yield capture cross sections compatible with experimental measurements.

Further investigations are now underway to obtain more information about the E1 strength function in the mass 90 region from (p, γ) measurements. The use of incident protons allows the gamma-ray strength function to be studied over a wide energy range and should, therefore, provide more knowledge of the strength function's behavior between low- and high-energy regions. Other mass ranges will be studied in the future, perhaps shedding light on the apparent failure of the two-peak, EDBW model near the N=50 shell. Further "fine-tuning" of parameters in each specific mass range will be addressed to yield accurate capture cross section and capture gamma-ray spectra results.

NOTE ADDED IN PROOF

After the present work was completed, R. Chrien of Brookhaven National Laboratory reminded one of us (DGG) that the usual expression for the Breit-Wigner line shape employed a product of a constant width Γ_R and an energy-dependent width $\Gamma_R(E_Y)$, such as given in Eq. 1.5 of Ref. 31. To a good approximation, Eq. 18 of the present work may be rewritten as

$$G_{BW}(E_Y, E_R, \Gamma_R) = \frac{1}{1 + \left(\frac{4}{\Gamma_R \Gamma_R(E_Y)} \right) (E_Y - E_R)^2} \quad (38)$$

If we wish to retain the present general form for the parameterization of the E1 gamma-ray strength function, we may do so by following the procedure in Appendix B but with the new line shape specified above. The end result is a new expression for $\Gamma_R(E_Y)$, similar to that originally derived, with the constant C being replaced by a new constant C^1 .

$$\text{new} \quad \Gamma_R(E_Y) = \Gamma_R \left(\frac{C^1 + E_X}{C^1 + E_Y} \right) \left(\frac{E_Y^2}{1} \right) \left(\frac{2}{E_X + E_R} \right)^2 \quad (39)$$

$$\text{original} \quad \Gamma_R(E_Y) = \Gamma_R \left(\frac{C + E_X}{C + E_Y} \right) \left(\frac{E_Y^2}{E_X} \right) \left(\frac{2}{E_X + E_R} \right) \quad (37)$$

A preliminary evaluation of Eq. 39 with Eq. 38 showed that E1 gamma-ray strength functions very similar to those shown in the text can be produced if the new constant C^1 is allowed to assume rather large values, say in the neighborhood of E_R , whereas C was usually in the range 1 to 5 MeV.

The reason that the two different forms for the Breit-Wigner line shape, as given by Eqs. 18 and 38, accommodate quite similar expressions for the energy-dependent width $\Gamma_R(E_Y)$ is due to the flexibility of the form we have chosen to use for the matrix elements $|M|^2$, given in Eq. 35. By choosing the size of k_2^1 appropriately, $|M|^2$ can be made to be essentially independent of E_Y or to vary roughly as E_Y^{-1} . Hence $\Gamma_R(E_Y)$ will then vary as E_Y^2 or as E_Y , and so either form of the Breit-Wigner line shape can be made to produce essentially the same energy dependence of the E1 gamma-ray strength function that is required to fit the experimental data.

We are grateful to Dr. Chrien for his comments and suggestions and we plan to reanalyze in the near future the experimental data presented here with the aid of Eqs. 38 and 39.

APPENDIX A

Relation of the Two GDR Energy Peaks to the Nuclear Deformation Parameter, β_2 .

The derivation of Eq. 28, which relates the ratio of the two GDR energy peaks, E_{R2}/E_{R1} , to the nuclear deformation parameter, β_2 , is given here. For statically deformed, prolate nuclei, where the giant resonance is split into two components, the ratio of the two peak energies is given by ¹⁸

$$E_2/E_1 = 0.911 \eta + 0.089. \quad (A.1)$$

Here, the deformation parameter η is the ratio of the major axis, a , to the minor axis, b ¹⁸

$$\eta = \frac{a}{b}. \quad (A.2)$$

Two other quantities should be defined:
the nuclear eccentricity, ϵ ¹⁸

$$\epsilon = \frac{(a^2 - b^2)}{R^2} \quad (A.3)$$

where R is the radius of a sphere of equal volume, and the nuclear deformation parameter, β_2 ³⁴

$$\beta_2 = \frac{1.06 (a - b)}{R} \quad (A.4)$$

We wish to express the ratio E_2/E_1 in terms of β_2 .

From Eq. A.4, we have

$$(a - b) = \frac{\beta_2 R}{1.06} \quad (A.5)$$

Eq. A.2 can be reexpressed as

$$\eta = 1 + \frac{(a - b)}{b} \quad (A.6)$$

and Eq. A.3 reexpressed as

$$\begin{aligned} R^2 \epsilon &= a^2 - b^2 \\ &= (a - b)(a + b) \\ &= (a - b)(2b + (a - b)) \\ &= 2b(a - b) + (a - b)^2 \end{aligned} \quad (A.7)$$

From the above

$$b = \frac{R^2 \epsilon - (a - b)^2}{2(a - b)} \quad (A.8)$$

Substituting Eq. A.5 and Eq. A.8 into Eq. A.6 and simplifying, we obtain

$$\eta = \frac{1.124\epsilon + \beta_2^2}{1.124\epsilon - \beta_2^2} \quad (A.9)$$

The relation of ϵ to β_2 may be derived from two definitions of the intrinsic quadrupole moment, Q_0 ,¹⁸

$$Q_0 = \frac{2}{5} Z R^2 \epsilon \quad (\text{A.10})$$

and

$$Q_0^2 = \frac{16\pi}{5} B(E2) \quad (\text{A.11})$$

where $B(E2)$ is the reduced electric-quadrupole transition probability between the 0^+ ground state and the first 2^+ state in even-even nuclei and is related to β_2 by the following³⁴

$$B(E2) = \frac{9 Z^2 R^4 \beta_2^2}{16\pi^2} \quad (\text{A.12})$$

Substituting Eq. A.12 into Eq. A.11, equating Eq. A.11 to Eq. A.10, and simplifying leads to

$$\beta_2 = \frac{2}{3} \left(\frac{\pi}{5} \right)^{1/2} \epsilon \quad (\text{A.13})$$

or

$$\epsilon = 1.892 \beta_2 \quad (\text{A.14})$$

Substituting Eq. A.14 into Eq. A.9 results in

$$\eta = \frac{2.126 + \beta_2}{2.126 - \beta_2} \quad (\text{A.15})$$

Using this definition of η in Eq. A.1 and simplifying leads to

$$E_2/E_1 = \frac{2.126 + 0.8228\beta_2}{2.126 - \beta_2} \quad (\text{A.16})$$

APPENDIX B

The Parameterization of the EDBW

Expression for the Width

As given in Eq. 36, the energy dependent width can initially be expressed as

$$\Gamma_R(E_Y) = \frac{k_1 g^3 E_Y^2}{k_2 + g^3 E_Y} \quad (B.1)$$

We will reexpress the constants k_1 and k_2 in the following way. Firstly, let us assume that $k_2 = Cg^3$, where C is another constant. Eq. B.1 then becomes

$$\Gamma_R(E_Y) = \frac{k_1 E_Y^2}{C + E_Y} \quad (B.2)$$

Secondly, consider the case when $E_Y = E_X$, where $E_X \leq E_R$, and such that $\sigma_{EDBW}(E_X) = \sigma_{LORENTZ}(E_X)$.

Recalling that

$$\sigma_{EDBW}(E_X) = \frac{\sigma_m}{1 + \left(\frac{E_X^2}{\Gamma_R(E_X)} \right)^2 (E_X - E_R)^2} \quad (B.3)$$

and that

$$\sigma_{LORENTZ}(E_X) = \frac{\sigma_m}{1 + \left(\frac{E_X^2 - E_R^2}{E_X \Gamma_R} \right)^2} \quad (B.4)$$

then if $\sigma_{EDBW}(E_X) = \sigma_{LORENTZ}(E_X)$, the following terms are equivalent

$$\frac{2}{\Gamma_R(E_X)} (E_X - E_R) = \frac{E_X^2 - E_R^2}{E_X \Gamma_R} . \quad (B.5)$$

Now $\Gamma_R(E_X)$, as defined by Eq. B.2, is

$$\Gamma_R(E_X) = \frac{k_1 E_X^2}{C + E_X} . \quad (B.6)$$

Substituting this expression into Eq. B.5, we have

$$\frac{2 (C + E_X) (E_X - E_R)}{k_1 E_X^2} = \frac{E_X^2 - E_R^2}{E_X \Gamma_R} , \quad (B.7)$$

and k_1 can be defined, after simplification, as

$$k_1 = \frac{2 \Gamma_R (C + E_X)}{E_X (E_X + E_R)} . \quad (B.8)$$

Substituting this definition of k_1 into Eq. B.2, we have

$$\Gamma_R(E_Y) = \Gamma_R \left(\frac{C + E_X}{C + E_Y} \right) \left(\frac{E_Y^2}{E_X} \right) \left(\frac{2}{E_X + E_R} \right) . \quad (B.9)$$

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