THEORETICAL INVESTIGATION OF MOMENTUM AND
ENERGY TRANSFER FREQUENCIES IN TWO TEMPERATURE PLASMAS¹

W. F. Walters and W. L. Everett

Nuclear Engineering Department
The University of New Mexico, Albuquerque, New Mexico

MASTER

Introduction

For fully ionized plasma systems within which the electron temperature is larger than the ion temperature, the long-range collective effects become important, which necessitates the use of the Lenard-Balescu equation to describe the behavior of the system. Investigators [1,2] have studied "test particle" transfer frequencies for systems very close to equilibrium, using the Fokker-Planck equation or its equivalent the Landau equation; however, "test particle" transfer frequency studies appropriate to two temperature systems have apparently not previously been carried out.

A general discussion of the "test particle" method of computing momentum and energy transfer frequencies is given in the following section. Using the Lenard-Balescu kinetic equation to represent particle interactions the "test particle" method is employed to compute like-particle and unlike-particle energy and momentum transfer frequencies over the range of T_e/T_i from 1 to 100. Each frequency is exhibited to be the sum of two terms. The first term takes account of the collision interactions within a Debye sphere and hence agrees with the results of Montgomery and Tidman [2] who compute some of these times using the Fokker-Planck equation with a Debye cutoff potential. The second term takes account of long-range collective effects as anticipated since the Lenard-Balescu equation differs from the Fokker-Planck equation in that long-range collective effects are accounted for through the inclusion of the dielectric function $[\epsilon^+(\vec{l}, U)]$. The long-range collective term ranges in magnitude up to 40 percent of the usual collisional term.

Method of Calculation

A tenuous stream of "test particles" is assumed to coexist with a fully ionized hydrogen plasma in which the electron and ion distribution functions are assumed to be Maxwellian with temperatures T_e and T_i , respectively, at time $t = 0$. The "test particle" stream distribution function at $t = 0$ is represented by

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$$f_r(\vec{c}) = n_r \delta(\vec{c} - \vec{c}_r) \quad \begin{array}{l} r \rightarrow e \text{ (electron stream)} \\ r \rightarrow i \text{ (ion stream)} \end{array}$$

The momentum and energy transfer frequencies are defined by

$$\nu_{Mrs} \equiv \left| \frac{\partial C_r^2}{\partial t} \right|_{t=0}^s \quad \begin{array}{l} s \rightarrow e \text{ (background electrons)} \\ s \rightarrow i \text{ (background ions)} \end{array}$$

$$\nu_{Ers} \equiv \left| - \frac{\partial C_r^2}{\partial t} \right|_{t=0}^s$$

Linearizing the Lenard-Balescu equation in $f_r(\vec{c})$ and taking the appropriate velocity moments, the frequencies defined above are calculated.

The collisional contributions to the frequencies are evaluated analytically and found to agree with those of Montgomery and Tidman. Numerical integrations necessary for the evaluation of the collective contributions were performed using a 32-point Gaussian quadrature formula.

Discussion of Results

Although the collisional terms and the long-range collective terms of the total transfer frequencies have been computed for like-particle energy and momentum and unlike-particle energy and momentum, i.e., 16 terms in all, for the range $1 \leq T_e/T_i \leq 100$, only those transfer frequencies which exhibit large long-range collective corrections are exhibited.

It may be noted from Figures 1 through 4 that the long-range collective correction for electron-electron momentum transfer is the most significant, ranging in value up to 40 percent. The corrections for the four frequencies not exhibited are found to be 4 percent or less.

Wu, et al. [3] have computed electron-electron, ion-ion, and electron-ion temperature "relaxation" times. On comparison it is interesting to note that $(\nu_{Eei})_{\text{coll.}}/(\nu_{Eei})_c$ exhibited here for the "test particle" problem agrees exactly with the corresponding ratio obtained using the values shown in Wu, et al.

Since these frequencies are computed using the linearized form of the Lenard-Balescu equation, they can be used as approximations to the interaction frequencies of the Lenard-Balescu equation which must be used to describe the behavior of two temperature plasmas. These

Figure 1. Ratio of the collective electron-electron momentum transfer frequency to the collisional electron-electron momentum transfer frequency

Figure 3. Ratio of the collective electron-ion energy transfer frequency to the collisional electron-ion energy transfer frequency

Figure 2. Ratio of the collective ion-ion energy transfer frequency to the collisional ion-ion energy transfer frequency

Figure 4. Ratio of the collective electron-electron energy transfer frequency to the collisional electron-electron energy transfer frequency

frequencies are utilized to develop a modified Chapman-Enskog solution for this important class of plasma systems in a companion paper being presented at this Conference.

References

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