BROADBAND ELECTROMAGNETIC TESTING METHODS
PART IV MULTIPARAMETER TEST PRINCIPLES
JANUARY 1, 1969

AEC RESEARCH & DEVELOPMENT REPORT

BATTELLE NORTHWEST
BATTLE MEMORIAL INSTITUTE PACIFIC NORTHWEST LABORATORY
BATTLE BOULEVARD, P.O. BOX 999, RICHLAND, WASHINGTON 99352
LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

PACIFIC NORTHWEST LABORATORY
RICHLAND, WASHINGTON
operated by
BATTELLE MEMORIAL INSTITUTE
for the
UNITED STATES ATOMIC ENERGY COMMISSION UNDER CONTRACT AT(45-1)-1830
BROADBAND ELECTROMAGNETIC TESTING METHODS

PART IV MULTIPARAMETER TEST PRINCIPLES*

by

H. L. Libby

Battelle Memorial Institute
Pacific Northwest Laboratory
Richland, Washington

January 1, 1969

*This paper is based on work performed under United States Atomic Energy Commission Contract AT(45-1)-1830.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF FIGURES</td>
<td>ii</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>SUMMARY AND CONCLUSIONS</td>
<td>2</td>
</tr>
<tr>
<td>THREE EXPLANATIONS OF THE MULTIPARAMETER PRINCIPLE</td>
<td>3</td>
</tr>
<tr>
<td>Generalization of the Phase Discrimination Technique</td>
<td>4</td>
</tr>
<tr>
<td>Algebraic Representation</td>
<td>15</td>
</tr>
<tr>
<td>Vector Space Representation</td>
<td>18</td>
</tr>
<tr>
<td>SEPARATION OF PARAMETERS</td>
<td>21</td>
</tr>
<tr>
<td>Number of Test Frequencies Required</td>
<td>23</td>
</tr>
<tr>
<td>Pulse Methods</td>
<td>32</td>
</tr>
<tr>
<td>INSTRUMENTATION</td>
<td>32</td>
</tr>
<tr>
<td>Excitation</td>
<td>33</td>
</tr>
<tr>
<td>Detection</td>
<td>35</td>
</tr>
<tr>
<td>Transformation of Separation Circuits</td>
<td>35</td>
</tr>
<tr>
<td>TEST RESULTS</td>
<td>41</td>
</tr>
<tr>
<td>Two Frequency Tubing Tester</td>
<td>41</td>
</tr>
<tr>
<td>Four Frequency Thickness Test System</td>
<td>46</td>
</tr>
<tr>
<td>Tubing Tester with Cross Section Display Feature</td>
<td>50</td>
</tr>
<tr>
<td>FUTURE POTENTIAL</td>
<td>58</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>59</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>60</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

1. Phasor (Phase Vector) Diagram Showing Lift-Off and Subsurface Defect Signals - Probe Type Test Coil. 6
2. Eddy Current Test Signals - Single Frequency Phase Discrimination Technique - One or Two Parameter Separation. 8
3. Phasor Diagrams Showing Lift-Off and Subsurface Defect Signals for Two Frequencies. 11
4. Eddy Current Test Signals - Two Frequency System $\omega_2 > \omega_1$. 12
5. Three Dimensional Phasor Diagram of Signal Constructed From Phasors in Figure 4. 13
6. Separation of Parameters - Algebraic Concept. 17
7. Line Diagram Showing Separation of Parameters. 19
8. Parameter Space - Signal Space Transformations. 20
9. Idealized Eddy Current Test Represented by Electric Transmission Lines. 25
10. Instrumentation Functions of the Multiparameter Test. 34
11. Functional Diagram of Transformation Section. 37
12. Transformation Circuit for Successive Elimination of Parameters. 40
13. Multichannel (Multiparameter) Eddy Current Tubing Tester Block Diagram. 42
14. Test Specimen Used for Figure 14 Test. 44
15. Test Results for Tube in Figure 14 Showing Comparison of Single Frequency and Two Frequency Modes of Operation of a Two Frequency Multiparameter Eddy Current Tester. 45
16. Block Diagram of a Prototype Tester. 47
17. Test Probe - Test Specimen Arrangement Multiparameter Thickness Test. 49
18. Parameter Separation When Four Test Signals Are Used To Separate Three Parameters - ($p_1$ Variable). 51
19. Parameter Separation When Four Test Signals Are Used to Separate Three Parameters - ($p_2$ Variable). 52
20. Parameter Separation When Four Test Signals Are Used to Separate Three Parameters - ($p_3$ Variable). 53
21. Eddy Current Test With Tubing Cross Section Display 54
22. Dynamic Display of Tube Cross Section Two Frequency Multi-parameter Test 55
Table I 30
Multiparameter or multivariable eddy current principles and techniques are described whose application gives more information about test specimens than can be obtained using conventional eddy current nondestructive test methods. The test coil assembly is driven by multifrequency or pulsed currents. Its output, having been modulated by the test parameters or variables is amplified, applied to spectrum filters and demodulated. The demodulated signal components are then recombined in selected linear combinations to give the read-out of individual test parameters or separate output channels. The principles are explained by a generalization of the single frequency phase discrimination technique, by an algebraic method and by a geometrical interpretation involving vector spaces. Selected test results are given for two and four frequency test systems. A two frequency tubing cross-section display device is described which displays fabricated defects on the outer and inner walls of a tube in their relative locations.
INTRODUCTION

Serviceability requirements of materials and parts become more strict and specialized problems arise as metals technology advances. To meet new nondestructive testing needs, it is becoming more important to treat the tests in ways which lead to better use of available test information. The main objective of the study of broadband electromagnetic testing methods is to develop techniques to obtain needed information about test specimens which cannot be obtained through conventional techniques. The development of the multiparameter eddy current test has been one result of this study.\(^1\) It has been shown\(^3\)\(^,\)\(^4\) that the application of multiparameter principles yields test performance not possible by other techniques.

The purpose of this report is to explain the principles of the multiparameter eddy current test method from several points of view, to show its relation to single frequency tests, to report recent developments in the theory of the test, to show some selected previously published test results, and to describe a previously unreported eddy current tube tester which produces a dynamic tubing cross section display.
SUMMARY AND CONCLUSIONS

Three concepts of the multiparameter eddy current nondestructive test are described:

1. A generalization of the single frequency phase discrimination technique.
2. An explanation based on the algebraic solution of a set of simultaneous equations.
3. A geometrical interpretation involving vector spaces.

The principles of parameter separation are discussed and it is shown that this separation is attained without obtaining a complete solution of the system equations. Complete solution does give separation, or decoupling, but separation only satisfies the need in the multiparameter test and is less demanding than a complete solution.

It is shown that for small defect detection the number of parameters which can be identified (separated) is equal to two times the number of test frequencies used. Increasing the number of test frequencies and parameters compounds the difficulty of separating the parameters and increased "cross talk" or undesired coupling between the output channels can be expected.

Large defects produce large signals which do not vary linearly with defect magnitude and result in increased "cross talk" between output channels.

The generality of the method is stressed. Either multiple frequency or pulsed test coil excitation currents can be used with the multiparameter technique.

Typical simplified diagrams of circuits used in multiparameter eddy current testers are given and the roles of coil excitation, filtering, signal detection and separation circuits are discussed.
Test results for both two and four frequency test systems are shown. A tubing cross section display device is described and its cathode ray tube read-out shown. Fabricated defects on the outer and inner walls of a tubing section are shown on the display in their relative locations.

THREE EXPLANATIONS OF THE MULTIPARAMETER PRINCIPLE

The essence of the multiparameter eddy current test is that multiple frequency or pulsed currents drive the test coil assembly whose output signals are analyzed making use of their multivariable aspects. These test coil output signals, having been modulated by the test specimen parameters (or test variables), are amplified, applied to spectrum filters and demodulated. The demodulated signal components are then recombined in selected linear combinations giving the read-out of individual test parameters on separate output channels.

The principles of the multiparameter test may be explained in many different ways. As an aid to better understand the basic principles of the multiparameter test, three explanations follow. The first explanation relates the new technique to a generalization of the phase discrimination method\(^{(5,6)}\) used in single frequency eddy current tests. This explanation is given because of the appeal it has by virtue of the wide use of the phase discrimination technique. The second explanation is based on elementary algebra and at first appears simpler and less abstract than the others. Matrix and vector space concepts characterize the third explanation. The most general and abstract of the three, these vector concepts result in compact notation. As might be expected, each of the explanations has a common underlying basis - that of separating or decoupling variables in a multivariable system.
The test was developed based on concepts of the third explanation - that of matrix algebra and vector space theory. The vector space approach is very abstract and thus more difficult to visualize than other, less general explanations. However, it is the most powerful.

The explanations are made with the assumption of small signal conditions but are valid in principle for large signal conditions. Large signal conditions bring special problems and are discussed later.

**GENERALIZATION OF THE PHASE DISCRIMINATION TECHNIQUE**

The multiparameter test technique may be presented as a generalization of the phase discrimination technique, extending it to operate simultaneously at two or more frequencies. The explanation will be made by first showing the principle of the phase discrimination technique as used with one test frequency. It will be shown how two variables or parameters can be separated at a single frequency.

**Extension to More Parameters**

By adding a second test frequency and showing the resulting three dimensional signal, the method will then be extended to three parameters. Extension to four or more parameters cannot easily be shown graphically, but it can readily be inferred as a logical result of increasing the number of variables and dimensions over that already shown for 2 and 3 dimensions.

The electrical impedance or voltage output of an eddy current probe type test coil assembly for several test conditions at a test frequency $\omega_1$ is shown in the phasor (phase vector) diagram in Figure 1.

**Phasor Diagrams**

It is helpful in the interpretation of the phasor diagram to recall that it is a compact way of showing the relative phase angles and amplitudes of
ac sinusoidal quantities. Each phasor diagram constructed usually depicts conditions for a specific frequency. The length and direction of a straight line drawn between any two points on the diagram are proportional to the amplitude and describe the relative phase angle, respectively, of a sinusoidal quantity at the specific frequency. Thus, the phasor diagram, itself a rather static looking thing, gives a condensed display of dynamic quantities.

Effect of Parameter Variations

In Figure 1 the values are assumed for purposes of explanation and are in approximate agreement with some measured quantities. The phasor OA represents the coil output signal for the unloaded condition (no test specimen adjacent to coil). Consider a probe coil with its axis normal to a test specimen, moving toward that specimen and finally coming to rest on the surface. During the motion of the probe, the test coil output follows some locus such as ABC, ADE or AFG depending upon the electrical conductivity of the test specimen. These are often called "lift-off loci". The locus ACEG is the specimen conductivity locus for the particular test coil in its position adjacent to the nonferrous test specimen. Consider the point E associated with the locus ADE. The vector OE represents the coil output signal when the coil is adjacent to a particular specimen which we assume for the moment to be without defects within the coil's zone of sensitivity. Now consider the effect on the coil signal of the variation of three following described test parameters:

1. $p_1$, a lifting of the probe a small distance away from the surface;
2. $p_2$, the presence of a defect within the metal a short distance below the surface;
3. $p_3$, a defect within the metal at a greater distance below the surface.
FIGURE 1
Phasor (Phase Vector) Diagram showing Lift-Off and Sub-Surface Defect Signals - Probe-Type Test Coil
Lifting the probe from the surface will result in a signal $S_1$ lying along the lift-off locus ADE. Defects beneath the surface interfere with eddy currents flowing in the specimen. Due to the skin effect, these currents flow at increasingly greater lagging phase angles with increasing depth below the surface. Thus, a defect at a fairly shallow depth might give a signal $S_2$ while a defect at greater depth might give a signal $S_3$. Note that the phase angle of $S_2$ is lagging with respect to the lift-off signal $S_1$. Similarly, the signal $S_3$ lags with respect to signal $S_2$. This is the situation for the test frequency $\omega_1$. A little later we will discuss the effect on the relative phase angle of these test signals as some different, higher test frequency. The main effect will be seen to be an increase in lagging phase angle of the defect signals. This occurs due to the increased phase angle lag of the current at a given depth as the frequency increases. However, before we consider increasing the test frequency we will first free ourselves of unneeded portions of Figure 1 and discuss the phase discrimination technique, so often used in the single frequency technique.

The phasor signal $OE$ can be compensated by adding a signal $-OE$ to it, effectively removing the reference zero from point O to point E. Figure 2 depicts the resulting phasor arrangement. Here the origin is labeled $OE$ to remind us that it is still directly related to point E in Figure 1.

**Phase Discrimination in the Two Parameter Case**

For our immediate purpose of discussing the phase discrimination technique we show signals $S_{11}$ and $S_{21}$ only at only two parameters can be separated using this method. The amplitude-phase detector used in a single frequency eddy current test is so connected that its output gives a signal proportional to the component of the test signal in phase with a reference or switching signal. For example, let $\theta_1$ in Figure 2 be the reference signal phase angle and let it
FIGURE 2
Eddy Current Test Signals Single Frequency Phase Discrimination Technique One or Two Parameter Separation
be adjusted so that the reference signal is in quadrature with respect to the lift-off signal $S_{ll}$. Axis $A_1$ is called the axis of Phase Detector 1.

The phase Detector 1 output is proportional to the projection of $S_{ll}$ upon Axis $A_1$. With Axis $A_1$ adjusted to be in quadrature with respect to the lift-off signal, $S_{ll}$, its projection on $A_1$ is very small or zero. Thus, this adjustment discriminates against the lift-off signal. In contrast, signal $S_{2l}$, having projection component $O_{Ep}$ along $A_1$, will give indications of the Phase Detector 1 output. The foregoing describes the phase discrimination technique applied to discriminate against a signal $(S_{ll})$ having a particular phase angle and reading out a component of a signal $(S_{2l})$ having a different phase angle. This component is proportional to both the signal amplitude and the cosine of the phase angle between the signal and the reference signal.

Similarly the signal $S_{2l}$ may be discriminated against and a component of signal $S_{ll}$ read out by using a second phase detector with reference signal phase angle $\theta_2$ and phase detector axis $A_2$. In this case the read-out component of $S_{ll}$ is proportional to $O_{EQ}$.

Thus, operating with one test frequency, we have read one signal with one phase detector, and a second signal with a second phase detector. This is done even when the two signals may occur simultaneously.

Effect of a Third Parameter

If we add a third signal we cannot read it out separately with this system. This is because we are operating in two dimensions and are asking for separation of three variables. This theory will be discussed in detail later.

Extension of Principle to Higher Dimensions

The multiparameter test is based on extending the phase discrimination technique, so far described, into the third, fourth, and higher dimensions. This is done by increasing the number of applied test frequencies so that the
number of dimensions required to describe the signal increases thus permitting the handling of a greater number of parameters. For example, four parameter separation requires the use of two frequencies. However, for simplicity consider the separation of three parameters using two test frequencies. By using only three signal components and three parameters, the graphical construction can be made in the more familiar three space.

Figure 3 shows the additional signal information needed for the analysis. Part 3(a) reproduces a portion of the phasor diagram in Figure 1 for frequency \( \omega_1 \) and permits ready comparison with the new phasor diagram, 3(b), for a higher test frequency, \( \omega_2 \).

In Figure 3(b) the defect signal phasors \( S_{22} \) and \( S_{32} \) have been rotated clockwise (lagging phase) to show the effect of the lesser skin depth at the higher frequency.

The test signal information of Figure 3 has been transferred to Figure 4. The signal patterns have been rotated counterclockwise so that the phasors \( S_{11} \) and \( S_{12} \) are aligned with the axis of abscissas. Exact scale values for these phasors (signals) have been established to facilitate later comparisons and explanatory analyses. The scale values are in approximate agreement with some experimental values.

Note that the projections of the various signals on the \( C_1 \) and \( C_2 \) axes could be obtained from two phase detectors operating at frequency \( \omega_1 \) having quadrature switching signals adjusted to the required reference phase angle. Likewise, the signal projections on the \( C_3 \) and \( C_4 \) axes could be obtained from phase detectors operating at frequency \( \omega_2 \).

**Three Dimensional Case**

The signal components along the \( C_1, C_2, \) and \( C_3 \) axes of Figure 4 are now used to make the diagram of Figure 5 which shows the signal phasors in three dimensions. \( S_1, S_2, \) and \( S_3 \) identify the three signals of interest using
FIGURE 3
Phasor Diagrams Showing Lift-Off and Sub-Surface Defect Signals for Two Frequencies
FIGURE 4
Eddy Current Test Signals - Two Frequency System \( \omega_2 > \omega_1 \)
FIGURE 5
Three Dimensional Phasor Diagram of Signals Constructed from Phasors in Figure 4
parameter rather than frequency designating suffixes. The components along the
$C_4$ axis of Figure 4(b) are not used in this three dimensional construction,
although they would be needed for separation of four parameters. The algebraic
sign of the components along the $C_1$ and $C_3$ axes of Figure 4 have been reversed
for constructing Figure 5. This makes the figure easier to visualize with no
loss in generality. Axis $A_{21}$ is constructed normal to the plane containing
$S_1$ and $S_2$. Next, axes $A_{32}$ and $A_{31}$ are constructed normal to the planes
containing the signal pairs $S_3, S_2$ and $S_3, S_1$, respectively.

Phase Discrimination in Three Dimensions

Finally, a generalized explanation of the phase discrimination principle
to more than two dimensions can now be made. Suppose we wish to read an
output proportional to the signal $S_3$ independently of the signals $S_1$ and $S_2$. The signal $S_3$ has a component $O_a$ along axis $A_{21}$. However, signals $S_1$ and $S_2$ have no component in this direction because axis $A_{21}$ was constructed to be normal to both $S_1$ and $S_2$. Thus, by reading the signal along axis $A_{21}$ an
output proportional only to signal $S_3$ is obtained.

Similarly, it can be shown that signals $S_1$ and $S_2$ have components along
only axes $A_{32}$ and $A_{31}$, respectively. Thus, outputs proportional to $S_1, S_2,$
and $S_3$ can be obtained separately by reading signal projections (components)
along the specially selected axes $A_{32}, A_{31}$ and $A_{21}$, respectively. The
principle can be extended to higher dimensions by analogy.

We will defer the discussion of actual instrumental methods of reading
the signal projections along the chosen axes. Means to accomplish this should
become clear as we proceed with the descriptions of the algebraic and vector
space representations.
ALGEBRAIC REPRESENTATION

The principles underlying the multiparameter eddy current tests can also be explained by showing that the method involves the solution of a set of simultaneous algebraic equations. Referring to Figure 4, and recalling that the signal projections on the axes $C_1$, $C_2$, $C_3$ and $C_4$ are the outputs of the amplitude-phase detectors of an eddy current instrument, equations for these outputs can be written. As linear (small signal) conditions have been assumed, the phase detector output for each axis (or channel) is equal to the sum of the signal projections upon that axis. Assigning a coefficient $a_{ij}$ to each signal relates the value of the signal projection to the particular parameter value $p_i$. For example, the projection of the signal $S_{12}$ on the $C_2$ axis, disregarding the other signal contributions, becomes

$$a_{12}p_2 = c_2$$  \hspace{1cm} (1)

where

$$a_{12} = 1$$

Taking the other signals into consideration, a set of simultaneous equations may be written. Note that the phase detector outputs, $C_1$, $C_2$, $C_3$, and $C_4$ are each given by the sum of the signal projection upon the respective axes, i.e.,

$$a_{11}p_1 + a_{12}p_2 + a_{13}p_3 = c_1$$

$$a_{21}p_1 + a_{22}p_2 + a_{23}p_3 = c_2$$

$$a_{31}p_1 + a_{32}p_2 + a_{33}p_3 = c_3$$  \hspace{1cm} (2)

The values of the coefficients $a_{ij}$ for the three parameter example given in Figure 4 may be read from the scale values
The set of equations, Equation (2), has three unknown quantities \( p_1 \), \( p_2 \), and \( p_3 \). In a given test it is possible, in principle, to determine that coefficients \( a_{ij} \) and the numbers \( c_1 \), \( c_2 \), and \( c_3 \). In fact, the \( c_i \) values represent the various phase detector outputs of the instrument. The \( a_{ij} \) factors can be found by observing the effect on the \( c_i \) values by each parameter as they are caused to vary individually.

**Separation of Parameters**

The effects of the various parameters may be separated by applying the method of Gauss to successively eliminate variables in a set of simultaneous equations. This can be done by selecting the proper linear combinations of the \( c_i \) to eliminate the effect of \( p_1 \), \( p_2 \), and \( p_3 \). This is illustrated in the line diagram in Figure 6. The phase detector outputs \( c_1 \), \( c_2 \), and \( c_3 \) are used as inputs. \( Y_i \) are summing points where signals fed on the incoming lines are summed in the required linear combinations to eliminate parameters. For example at \( Y_1 \) combinations of \( c_1 \) and \( c_2 \) are selected to eliminate the effect of parameters \( p_3 \). Similarly at \( Y_2 \) combinations of \( c_1 \) and \( c_3 \) are selected to eliminate \( p_3 \). The next two summing points \( Y_3 \) and \( Y_4 \) are used to eliminate \( p_2 \). As the elimination (separation) of parameters proceeds the coefficients related to the various parameters (variables) change. The new coefficients depend upon the factors applied to \( c_1 \), \( c_2 \), and \( c_3 \) and the necessary summing operations for eliminating (separating) the parameters. For example, the coefficients \( b_{11} \) and \( b_{12} \) depend on the factors applied to \( c_1 \) and \( c_2 \) and the summing operation at \( Y_1 \) for the elimination of parameter \( p_3 \).
\[ C_1 = a_{11}p_1 + a_{12}p_2 + a_{13}p_3 \]

\[ C_2 = a_{21}p_1 + a_{22}p_2 + a_{23}p_3 \]

\[ C_3 = a_{31}p_1 + a_{32}p_2 + a_{33}p_3 \]
The parameters \( p_1 \) and \( p_2 \) are separated at summing points \( Y_5 \) and \( Y_6 \), and \( p_3 \) is separated at summing point \( Y_7 \). The final outputs, \( I_{p_1} \), \( J_{p_2} \) and \( K_{p_3} \) are proportional to the parameters \( p_1 \), \( p_2 \), and \( p_3 \). Other arrangements of line connections may be desirable depending upon the functions \( c_1 \), \( c_2 \), and \( c_3 \).

**Complete Solution Not Required**

A complete solution of the set of equations (Equation 2) is not required as our objective is to read out the effects of the various parameters on separate lines. Thus, it is the decoupling or separation of the parameters which is of interest here. A discussion follows in the section Separation Versus Solution.

Figure 7 shows the separation of parameters for the example given in Figure 4 using the algebraic representation. Here the multiplying factors are given in parentheses on the appropriate line just ahead of each summing point.

**VECTOR SPACE REPRESENTATION**

The multiparameter eddy current test may also be given a very general geometrical interpretation involving vector spaces. Figure 8 depicts the concept in simplified form. Generator A excites Test Coil Assembly B with a multifrequency or pulsed signal which is modulated by the test specimen parameters having generalized parameter \( P \) with components \( p_1, p_2, \ldots, p_n \). The output of Receiver D produces a generalized signal \( C \) related to the input parameter \( P \) by the equation

\[
C_j = [A] P
\]  

(4)

where \([A]\) is a modulation matrix which takes into account the test conditions, the test specimen, test coil and receiver characteristics.
FIGURE 7
Line Diagram Showing Separation of Variables
FIGURE 8
Parameter Space - Signal Space Transformations
The signal C has components \( c_1, c_2, c_3, \ldots c_n \) which are the coefficients of the generalized Fourier series expansion. The signal is next transformed by Transformation Section E into a function, Q, proportional to the original input parameter P. The output, Q, has components \( q_1, q_2, q_3, \ldots q_n \) which are proportional to their input parameter counterparts \( p_1, p_2, p_3, \ldots p_n \). The equation

\[ p] = [A]^{-1} c \]  

(5)

describes the transformation of C into output parameter signals.

Equation (4) is simply the matrix form of Equation (2) which describes the algebraic representation. Multiplying the signal matrix by the inverse of the modulation-receiver matrix \([A]\) produces the solution (separation) described in Equation (5). Conceptually a simple, direct set of equations, these are quite useful, although the complete solution represented is not required.

A more realistic nomenclature shows the final output being proportional to the original parameters. By denoting the output space as Q parameter space, Equation (5) can be modified to read:

\[ Q] = [P] = [A]^{-1} c \]  

(6)

The proportionality factor can be different for each component of P. Detailed discussion appears in the section Separation Versus Solution.

SEPARATION OF PARAMETERS

Separation of parameters refers to the adjustments or equipment functions which permit the signal response at specific instrument output terminals to be identified with specific parameters. Identification results from the variation of specific test parameters during calibration.
A parameter is defined as a test variable. The test specimen parameters of greater interest include electrical conductivity, other electrical properties, the presence of defects, location of defects and dimensions. Also of interest are the coupling between the test coil and specimen, variation in coil impedance due to environmental conditions, and instrument drift. It is desirable to select independent parameters.

Theoretically, independence refers to the parameters and components of the generalized parameter, assumed to be mutually orthogonal in parameter space.

**Separation versus Solution**

As previously stated in the sections Algebraic Representation and Vector Space Representation, a complete solution of the basic equations, Equation (2) and Equation (4) is not required. This is because the essence of the multiparameter test is the separation of parameters. Once the signal effects are processed to produce read-outs for different parameters at specific output terminals, magnitude adjustments can be made separately. The sensitivity of each output channel may be set as desired using a calibration procedure in which test parameters are caused to vary by known amounts.

The difference between solution and separation can be shown by solving Equation (4) in a way to give separation without complete solution. Equation (4) transforms the parameter vector $P$ into the signal vector $C$:

$$ C = [A] P \tag{4} $$

To solve for $P$, multiply the equation by $A^{-1}$, giving the complete solution

$$ P = [A]^{-1} C = [A]^{-1} [A] P = P \tag{7} $$

Now, the output we need for separation only is a set of components $q_1, q_2, \ldots, q_n$, ordered in the same way as the $P$ components, but with
arbitrary amplitudes. This arbitrary amplitude may be represented by multiplying \( P \) with a diagonal matrix \( D_a \) having arbitrary values on its diagonal:

\[
[D_a] \ P = [D_a] \ [A]^{-1} \ C
\]

Using the identity:

\[
A^{-1} = \frac{1}{|A|} \ \text{Ad}_A
\]

\(|A| = \text{Determinant of } A\)

\(\text{Ad}_A = \text{Adjoint of } A\)

Equation 7 may then be written

\[
|A| [D_a] \ P = [D_a] \ [\text{Ad}_A] \ C
\]

Since \(|A|\), the determinant of \( A \), is a scalar and \( D_a \) has arbitrary components, \(|A|\) may be absorbed into \( D_a \). However, the equation in its present form shows directly the effect of \( A \) becoming singular, that is, as \(|A|\) approaches zero. This equation shows that the separating or decoupling capability is obtained through the operation of the adjoint. The role of the arbitrary or calibrating factor \( D \) is also shown, and indicates that the requirements for separation are less restrictive than for complete solution.

Equation (10) is basic in the design and development of transformation circuits for separating parameters.

**NUMBER OF TEST FREQUENCIES REQUIRED**

In general, test parameter variations in the eddy current test cause curved signal loci in two space or in higher ordered spaces. This curvature, or nonlinearity, is a natural result of the spatial distribution of the electromagnetic fields and currents and the mutual interaction of the currents.
For purposes of simplifying the analysis an analytical approach involving linear approximations is used.

Small variations in parameters result in small signal changes which vary approximately linearly as the parameters. This makes it feasible to apply the powerful methods of linear algebra. Fortunately, many eddy current tests involve the detection of small flaws, thus the application of the principle of linear approximation is of practical value. The number of test frequencies required for separating a given number of parameters in the linear case will be discussed first, followed by a few comments on the nonlinear case.

Linear Approximation

The analysis of an idealized eddy current test shows that the number of parameters which can be identified equals twice the number of test frequencies used. The idealized eddy current test is represented in Figure 9 by a lossy transmission line having infinite length with a load \( Z_F \) representing a defect at distance (depth) \( X \) from the sending end. The effect of coupling circuits between generator and line (test specimen) is not represented, not being pertinent to the immediate problem.

It is assumed that fixed ac currents, \( I_n \), at various frequencies are applied to the sending end of the transmission line. The instrument reads the signal

\[
E(\omega) = I_n(\omega) Z_n(\omega)
\]

Thus, the signal observed is proportional to the line input impedance, \( Z(\omega) \). By standard methods, the input impedance \( Z_{IN} \) of a transmission line having a characteristic impedance \( Z_0 \), a propagation constant \( \gamma \), and length \( x \) is
I(ω)+---~
/                   /
E(ω)                E(ω)
Z(ω)IN              ZF
\                   /
Length = X          Length = ∞

Characteristic Impedance, Both Sections: \( Z_0 \)

FIGURE 9
Idealized Eddy Current Test Represented by Electric Transmission Lines
As we are simulating the metal of the test specimen with the transmission line, we will let

\[ Z = \text{series impedance} \]
\[ Y = \text{shunt admittance} \]
\[ \mu = \text{magnetic permeability of free space} \]
\[ \sigma = \text{metal conductivity} \]
\[ \delta = \text{depth of penetration, plane wave case, } \frac{1}{\sqrt{\omega \mu \sigma}} \]

Equation (12) may be written

\[ Z_{IN} = Z_0 \left[ 1 + \frac{Z_R - Z_0}{Z_R + Z_0} e^{-2\gamma x} \right] \] (12)

For small values of \( y \) (\( y < 1 \)) Equation (14) may be written

\[ Z_{IN} = Z_0 \left[ 1 + 2 \left( y + y^2 + y^3 + \ldots \right) \right] \] (16)

The higher order terms may be omitted with less than 2% error for values of \( y < 0.1 \). Hence,

\[ Z_{IN} \approx Z_0 (1 + 2y) = Z_0 + 2Z_0 y \] (17)

which gives, in terms of a new signal \( Z_{IN} - Z_0 \),

\[ Z_{IN} - Z_0 \approx 2Z_0 y \] (18)

Referring to Figure 9, it can be seen that the load \( Z_F \), which represents the defect at depth \( x \), is connected in parallel with the input impedance, \( Z_0 \).
of the line section (having infinite length) beyond $Z_F$. The quantity $Z_R$ in Equation (12) is then equal to

$$Z_F = \frac{Z_F Z_o}{Z_F + Z_o}$$  \hspace{1cm} (19)

A simple manipulation results in

$$\frac{Z_R - Z_o}{Z_R + Z_o} = \frac{Z_F Z_o}{Z_F + Z_o} - \frac{Z_o}{Z_F + Z_o} = -\frac{Z_o}{2Z_F + Z_o}$$  \hspace{1cm} (20)

Substituting Equation (20) into Equation (15) and that result into Equation (18) gives

$$Z_{IN} - Z_o \approx -\frac{2Z_o^2}{Z_o + 2Z_F} e^{-2\gamma x}$$  \hspace{1cm} (21)

Further, substituting Equation (13) into Equation (21) results in

$$Z_{IN} - Z_o \approx -\frac{2Z_o^2}{Z_o + 2Z_F} e^{-\frac{2x}{\delta}} e^{-j\frac{2x}{\delta}}$$  \hspace{1cm} (22)

This reduces to the following for small defect conditions, $(Z_F >> Z_o)$

$$Z_{IN} - Z_o \approx \frac{1}{Z_F} Z_o^2 e^{-\frac{2x}{\delta}} e^{-j\frac{2x}{\delta}}$$  \hspace{1cm} (23)

The impedance difference Equation, (23), is now converted to a signal voltage equation by multiplying it by a fixed current $I_n(\omega)$ as in Equation (11).

$I(\omega)$ and $Z_o^2/Z_F$ are absorbed in a new factor, $P_n e^{j\omega k t}$, where the subscripts $n$ and $k$ identify a particular defect and a particular test frequency. The resulting equation is:

$$E_{nk} = P_n e^{j\omega k t} e^{-\frac{2x_n}{\delta_k}} e^{-j\frac{2x_n}{\delta_k}}$$  \hspace{1cm} (24)
where
\[ E_{nk} \] is the signal caused by defect n at frequency \( \omega_k \).
\[ \omega_k \] is the test frequency in radians per second
\[ x_n \] is the depth of the defect from the surface adjacent to the test coil
\[ \delta_k \] is the depth of penetration of eddy currents in the metal at test frequency \( \omega_k \).
\[ p_n \] is a factor proportional to the value or "strength" of the defect.

Simplifying Assumptions

It is emphasized that Equation (24) is approximate. Its formation takes into account only the first reflection from the defect under assumed plane wave conditions. Also, the factor \( p_n \) - which appears as such a neat linear factor in Equation (24) - has its origin in the complex Equation (20). Actually, \( p_n \) can also vary with the test frequency. However, it is necessary to ignore these important details in the interest of simplicity. This is justified by noting that we limit our discussion to the effects of small defect signals which we assume can be treated using linear algebra. These simplifying assumptions should not affect the validity of the conclusions since a more accurate formulation should increase the complexity of the different \( E_{nk} \), making the new \( E_{nk} \) even more independent rather than less.

Fourier Series Expansion

In the multiparameter test the signals \( E_{nk} \) are expanded into their Fourier series components \( a_{nk} \sin \omega_k t \) and \( b_{nk} \cos \omega_k t \) by applying the signals \( E_{nk} \) to phase detectors. The outputs of the phase detectors are proportional to the Fourier coefficients \( a_{nk} \) and \( b_{nk} \). Thus, there are available two
coefficients at each test frequency. These are, from Equation (24),

\[ p_n e^{-\frac{2\pi n}{\delta_k}} \sin \frac{2\pi n}{\delta_k} \]  
\[ \text{(25)} \]

and

\[ p_n e^{-\frac{2\pi n}{\delta_k}} \cos \frac{2\pi n}{\delta_k} \]  
\[ \text{(26)} \]

For simplicity, miscellaneous phase shift angles at the different test frequencies are omitted without loss of generalization.

In the general case many defects or other test conditions contribute signals, yet only two Fourier coefficients are available at each test frequency. It is convenient to handle the resulting array of equations by use of matrix algebra. The phase detector output signals may now be assembled in the matrix equation

\[ [a_{mn}] p_n = y_n \]  
\[ \text{(27)} \]

where

1. \([a_{mn}]\) is a square matrix whose elements are the outputs of the various phase detectors for unit parameter value at each frequency. The new subscript \(m\) is required to identify the phase detector output signals as there are two of these for each frequency or \(k\) subscript.
2. \(p_n\) is a column matrix whose elements are proportional to the parameters, \(p_1, p_2, p_3, \ldots p_n\).
3. \(y_n\) is a column matrix whose elements are proportional to the outputs of the phase detectors for the existing (multiparameter valued) condition.

The matrix \([a_{mn}]\) can now be written and examined in detail to determine how many independent parameters, or variables, may be determined for a given number of test frequencies:
MATRICES ELEMENTS OF $A_{mn}$. IDEALIZED CASE. SUBSURFACE FLAWS REPRESENTED BY DISCONTINUITIES IN AN ELECTRIC TRANSMISSION LINE.

<table>
<thead>
<tr>
<th>k</th>
<th>m-1</th>
<th>m-2</th>
<th>m-3</th>
<th>m-4</th>
<th>n-1</th>
<th>n-2</th>
<th>n-3</th>
<th>n-4</th>
<th>...</th>
<th>n_n</th>
</tr>
</thead>
<tbody>
<tr>
<td>m-1</td>
<td>$\exp[-2\pi x_1 \delta_1] \sin \frac{2x_1}{\delta_1}$</td>
<td>$\exp[-2\pi x_2 \delta_1] \sin \frac{2x_2}{\delta_1}$</td>
<td>$\exp[-2\pi x_3 \delta_1] \sin \frac{2x_3}{\delta_1}$</td>
<td>$\exp[-2\pi x_4 \delta_1] \sin \frac{2x_4}{\delta_1}$</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>m-2</td>
<td>$\exp[-2\pi x_1 \delta_2] \sin \left(\frac{2x_1}{\delta_2} + \frac{\pi}{2}\right)$</td>
<td>$\exp[-2\pi x_2 \delta_2] \sin \left(\frac{2x_2}{\delta_2} + \frac{\pi}{2}\right)$</td>
<td>$\exp[-2\pi x_3 \delta_2] \sin \left(\frac{2x_3}{\delta_2} + \frac{\pi}{2}\right)$</td>
<td>$\exp[-2\pi x_4 \delta_2] \sin \left(\frac{2x_4}{\delta_2} + \frac{\pi}{2}\right)$</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>m-3</td>
<td>$\exp[-2\pi x_1 \delta_3] \sin \left(\frac{2x_1}{\delta_3} + \frac{\pi}{2}\right)$</td>
<td>$\exp[-2\pi x_2 \delta_3] \sin \left(\frac{2x_2}{\delta_3} + \frac{\pi}{2}\right)$</td>
<td>$\exp[-2\pi x_3 \delta_3] \sin \left(\frac{2x_3}{\delta_3} + \frac{\pi}{2}\right)$</td>
<td>$\exp[-2\pi x_4 \delta_3] \sin \left(\frac{2x_4}{\delta_3} + \frac{\pi}{2}\right)$</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>m-4</td>
<td>$\exp[-2\pi x_1 \delta_4] \sin \left(\frac{2x_1}{\delta_4} + \frac{\pi}{2}\right)$</td>
<td>$\exp[-2\pi x_2 \delta_4] \sin \left(\frac{2x_2}{\delta_4} + \frac{\pi}{2}\right)$</td>
<td>$\exp[-2\pi x_3 \delta_4] \sin \left(\frac{2x_3}{\delta_4} + \frac{\pi}{2}\right)$</td>
<td>$\exp[-2\pi x_4 \delta_4] \sin \left(\frac{2x_4}{\delta_4} + \frac{\pi}{2}\right)$</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

$a_{mn} = \exp\left[\frac{-2\pi x_n}{\delta_k \left(\frac{m}{2} + \frac{1}{4} (1 - (-1)^m)\right)}\right] \sin \left[\frac{2x_n}{\delta_k \left(\frac{m}{2} + \frac{1}{4} (1 - (-1)^m)\right)} + \frac{\pi}{4} (1 + (-1)^m)\right]$  

where

- $k$ = Frequency Number  
- $m$ = Row Number  
  (also phase detector output number)  
- $n$ = Column Number  
  (also parameter number)  
- $\delta_k$ = Skin Depth or Penetration Depth at Frequency $k$  
- $x_n$ = Distance from Surface to $n^{th}$ Defect  
- $a_{mn}$ = Matrix Element in $m^{th}$ Row and $n^{th}$ Column

**TABLE I**
To determine the number of parameters, \( p_n \), which may be separated (identified) per test frequency, it is necessary to show the number of independent equations represented by the matrix \( [a_{mn}] \) as a function of frequency. It is also necessary to determine the conditions under which the set of equations represented by the matrix \( [a_{mn}] \) has a solution. The matrix \( [a_{mn}] \) is a square matrix; thus, for \( n \) columns it will have an equal number of rows. As there are two different rows of the matrix resulting from each set of two phase detectors operating at each test frequency, the number of variables, \( n \), represented by the square matrix, equals twice the number of test frequencies. Examination for the individual matrix elements shows sufficient independence to solve for the \( n \) parameters. We will examine the case for \( n = 2 \). For this two parameter case, selecting arbitrary defect depths \( x_1 \) and \( x_2 \) will result in a matrix \( [a_{mn}] \) having 4 elements. It is found that this matrix is nonsingular. Thus, the equations it represents can be solved for the two variables. For example, assuming \( x_1/\delta_1 = 0.5 \) and \( x_2/\delta_1 = 0.7 \), and matrix \( [a_{mn}] \) is

\[
[a_{mn}] = \begin{bmatrix} 0.31 & 0.24 \\ 0.20 & 0.04 \end{bmatrix}
\]

(29)

with a determinant \( |A| \) equal to \(-0.0356\). Thus, the system of equations

\[
\begin{align*}
y_1 &= \begin{bmatrix} 0.31 & 0.24 \end{bmatrix} p_1 \\
y_2 &= \begin{bmatrix} 0.20 & 0.04 \end{bmatrix} p_2
\end{align*}
\]

(30)

has a solution.

If only one phase detector is used for each test frequency instead of two, the number of rows in the matrix \( [a_{mn}] \) will be halved. With only half the rows available, the number of parameters which can be separated or identified equals the number of test frequencies.
Nonlinear General Case

The more general case involving the curved signal loci in signal space is all inclusive. Even the small signals, treated in this paper by linear approximation, give curved loci in signal space, however small the curvature. Thus, there is no definite delineation between the "linear" and "nonlinear" regions. However, the effects resulting from nonlinear signals are very apparent in the operation of the multiparameter testers. As the signals extend further into the nonlinear region, separation of parameters becomes more difficult and cross channel interference occurs. For equivalent separation, fewer parameters may be separated for a given number of test frequencies than for the case of linear signals.

PULSE METHODS

Applications of the pulse techniques for multiple parameter extraction are not as far advanced as the multifrequency methods and are not treated in detail here. However, much theoretical work is available from other related fields. For example, the fields of signal analysis, system identification and pattern recognition are rich with theory and techniques pertinent to multiparameter problems. Signal analysis theory and techniques as presented by Huggins also apply. Some of his work has been reviewed with regard to its applicability in eddy current nondestructive testing. Litman and Huggins have investigated the use of growing exponentials in the identification of multiparameter systems.

INSTRUMENTATION

Instrumentation for the multiparameter test differs from that of the conventional eddy current test mainly as a result of simultaneous operation at several or many test frequencies, the demodulation of the resulting
multiplicity of test signals, and the summation of the demodulated components to give the separation of parameters.

The main functions required of eddy current multiparameter test instrumentation are shown in Figure 10. These are:

1. Excite the test coil assembly with a multidimensional driving function. This may be a multifrequency or pulsed current.
2. Provide means to bring the test coil assembly output signal to the desired state of ac null. This is done by compensating circuits or an ac bridge system.
3. Amplify, filter (as needed) and demodulate the test signal. Demodulation is performed by synchronous detectors. This expands the signal on a selected basis giving the signal vector components $c_k$. In the general case, the $c_k$ are proportional to the coefficients of the terms in a generalized Fourier expansion of the signal on the selected basis. When the analysis is performed in the frequency domain and the test coil is excited by multifrequency currents, the basis is a group of sinusoidal functions and the $c_k$ are proportional to the coefficients of the Fourier series expansion of the test signal.
4. Extract individual test parameters by transformation of the signal vector into parameter vector components. This is accomplished by constructing linear combinations of the signal vector components $c_k$.

**Excitation**

The principle of superposition applies if the test specimen is nonferrous. Thus, test coil excitation currents containing many frequency components can
2. Provide AC Null or AC Compensation of Test Coil Output Signals.

3. Expand Test Signal on a Selected Basis, Giving Signal Vector Components $C_K$

4. Extract Individual Test Specimen Parameters by Transformation of Response Vector into Parameter Vector Components.

FIGURE 10
Instrumentation Functions of the Multiparameter Test
be applied simultaneously without cross modulation effects. A great latitude exists in the choice of the test coil driving functions. However, the coil driving function does influence the design of the receiver equipment and choice of demodulation circuits. Synchronized or non-synchronized carriers can be used. The synchronized carriers can be generated by repetitive pulse generation circuits or by summation of a set of individual carriers which have been generated in synchronism. Single pulse excitation can also be used, as in principle all the necessary information is contained in the response of the test arrangement to an impulsive excitation.

Detection

The instrument designer may choose to design the equipment to perform the signal analysis in either the frequency domain or time domain. The former seems more appropriate for use in systems using excitation at several discrete carrier frequencies. In this case, the various carriers can be separated using wave filters and can then be applied to separate amplitude-phase detectors or other circuits which provide the Fourier series coefficients of the signal. In the case of repetitive pulse excitation, analysis can be made either in the frequency domain or in the time domain using pulse sampling techniques. The pulse signals may also be analyzed using an orthogonal spectrum analyzer. (2,8,10)

Transformation or Separation Circuits

The required transformation function is obtained by constructing linear combinations of the detector outputs $c_k$. The basic process is the same whether one follows either the algebraic solution concept or the signal space concept. There are, however, differences in circuit connections and adjustment techniques may vary widely. As the combinations are linear, the essential transformation circuit requirements must provide:
1. Sign changing
2. Amplitude adjustment
3. Summing

The portion of the circuit here called transformation circuit also has been called separation circuit, analyzer circuit, or computer circuit.

In practical circuits, it is convenient to use operational amplifiers to provide isolation, sign changing and summing functions. Potentiometers in the summing amplifier input circuits provide amplitude adjustment of the summed signals.

Two types of transformation circuits will be described. One based on the matrix solution and a second based on an algebraic solution.

**General Transformation Circuit**

Figure 11 depicts this circuit, based on the solution of the matrix (Equations (4) or (10)). This configuration provides separation of up to four parameters. It accepts the outputs of four phase detectors and delivers the selected linear combinations of these inputs to four output channels. The proper adjustment of the potentiometers U is of critical importance in the operation of the devices. Gain controls (not shown) are usually included in the output circuits for adjustment of the individual parameter channel gains.

**Adjustment Techniques**

Two main techniques used to adjust the overall transformation circuit will be discussed next. In one of these, the first step is to determine the matrix elements of the modulation matrix \([A]\), Equation (4). This is done by causing each parameter to vary individually and by measuring the the resulting change in each of the \(c_k\). For a four parameter system, an array of 16 matrix elements results. Next, one computes \([A]^{-1}\).
FIGURE 11
Functional Diagram of Transformation Section
The matrix element values of $[A]$ inverse then determine the settings of the coefficient potentiometers $U$. Doing this, one usually finds that due to unavoidable errors and some nonlinearity of signals, optimum separation of parameters is not obtained. To obtain better separation, this leads to trial and error adjustment of the potentiometers $U$, a time consuming process.

The other main overall transformation circuit adjustment technique eliminates computing the inverse elements of the modulation matrix and separates the parameters by direct adjustment of the potentiometers $U$. For example, in a four parameter system, one of the group of four potentiometers $U$ can be set at an arbitrary value. The other three are then adjusted to simultaneously eliminate the effect of three parameters on the output of the channel fed by these potentiometers and the associated summing amplifier. Although not a simple adjustment to carry out, it is flexible, thus permitting partial compensation of nonlinear effects associated with large signals. These four adjustments produce read-out, on that particular output channel, of the signal caused by only one parameter. Repeating the process for the next group of four potentiometers results in separation of the second parameter. The process is then repeated for the remaining two groups of four potentiometers.

**Aids in Potentiometer Adjustment**

The search for the desired adjustment in either the trimming operation or search separation method may be facilitated by convenient arrangement of the potentiometer knobs so that several of them may be simultaneously adjusted with one hand.
Transformation Circuit Based on an Algebraic Solution

Figure 12 depicts the second transformation circuit to be described. This circuit, one associated with the algebraic solution concept, applies the principle of successive elimination of parameters (variables). It follows the principles previously discussed and illustrated in Figures 6 and 7. This particular circuit is arranged to read out two parameters and eliminate or minimize the effects of two other parameters. Four parameter readout could be obtained by adding additional components and two more output channels.

The outputs $c_1$, $c_2$, $c_3$, and $c_4$ of phase detectors are inputs to the transformation section, or analyzer, although the detector outputs are not necessarily always connected to the correspondingly labeled input terminals of the analyzer section. The summing circuits are arranged in Figure 12 in three columns corresponding to the positions of the summing potentiometer controls on the instrument panel. Each summing potentiometer $R$ is driven by two operational amplifiers, not shown in Figure 12, so that both positive and negative signals are available as desired for the summing operation. The effects of four selected parameters, $p_1$, $p_2$, $p_3$, and $p_4$ to be separated are generally available in the signals $c_1$, $c_2$, $c_3$, and $c_4$ as indicated by the $p_1$, $p_2$, $p_3$, and $p_4$ designation on the signal leads in the diagram. Since this instrument reads out two parameters, for example $p_3$ and $p_4$, the effects of parameters $p_1$ and $p_2$ are eliminated or minimized. The effects of of $p_1$, probe wobble for example, are minimized by sequential adjustment of the potentiometers in the first column, so that signals $d_1$, $d_2$, $d_3$, in sequence, show a minimum variation when $p_1$ is varied. The effects of the second parameter $p_2$ are then minimized by adjustment of potentiometers in the second column so that a minimum effect of varying parameter 2 can be
ANALYZER SECTION
FUNCTIONAL DIAGRAM

LOCATION ON PANEL
FIRST COLUMN

SECOND COLUMN

THIRD COLUMN

FOURTH COLUMN

(R - SUMMING CIRCUITS
R₁, R₂ - OUTPUT GAIN CONTROLS

FIGURE 12
Transformation Circuit for Successive Elimination of Parameters
observed in signals $e_1$ and $e_2$. The last two parameters are similarly separated by adjustment of potentiometers in the third column to successively eliminate or minimize the effects of $p_4$ in signal $f_1$ and $p_3$ in signal $f_2$, thus separating the effects of parameters $p_3$ and $p_4$.

The two potentiometers in the fourth column are gain controls in the two output channels $f_1$ and $f_2$. The advantage of this circuit arrangement is that it eliminates the effect of each parameter in succession rather than almost simultaneously as in the general transformation circuit. The successive elimination technique is carried out faster with much more convenience in use. This is especially noted when the parameters and test specimens are such that all parameters cannot readily be presented in the rapid succession required by the technique used in adjusting the general transformation circuit.

**TEST RESULTS**

This section demonstrates some of the potential of the multiparameter eddy current test method. Three test systems, a two frequency tubing test system, a four frequency thickness test system, and a two frequency system with a tubing cross section display feature are described and test results given.

**TWO FREQUENCY TUBING TESTER**

A diagram of a two frequency multiparameter tester called a multichannel tubing tester\(^{(4)}\) is shown in Figure 13. It uses two harmonically related test frequencies, 100 kHz and 300 kHz. Synchronous phase detectors produce the Fourier series coefficients $c_1$, $c_2$, $c_3$, and $c_4$. The transformation circuit used here is the one discussed in the preceding section and shown in Figure 12.
FIGURE 13
Multichannel (Multiparameter) Eddy Current Tubing Tester Block Diagram
This eddy current tubing tester reduces the effect of test probe wobble signals on both output channels. It is flexible in operation and can be operated using either of its two test frequencies singly or both simultaneously. When operated in its single frequency modes, two test parameters may be separated, for example, probe wobble, and the presence of defects giving signals having components in quadrature with signals caused by probe wobble. When operated in its two frequency mode the effects of one test parameter can be minimized and the effects of two other test parameters may be read.

Comparative results were obtained operating the multichannel tubing tester in its two single frequency modes and its single two frequency mode using the test specimen whose cross section sketch is shown in Figure 14. This is type 304 stainless steel tubing having 0.050 in. wall and 1/2 in. inside diameter. Irregularities of interest in this specimen, from right to left (in the direction of movement of the test coil assembly) are a 1/8 in. long notch, approximately 0.015 in. deep by .003 in. wide opening on the inner surface, a kink or dent in the tubing, a suspected region of intergranular corrosion on the inner wall of the tubing, and a 1/8 in. long notch approximately .023 in. deep by .003 in. wide opening onto the outer surface of the tubing. The results of runs made using this specimen and an inside test coil assembly having two differentially connected test coils are shown in Figure 15.

**Single Frequency Modes**

For the single frequency runs, 100 kHz, Figure 15(a) and 300 kHz, Figure 15(b), the instrument was adjusted to minimize probe wobble signal on channel 1, the lower trace of each chart record. The test coil assembly was pulled nearly through the specimen tube, resulting in signal indications
Notes on Tube Defects:

A. Outside Longitudinal Notch 1/8 in. Long x 0.023 in. Deep
B. Suspected Region of Intergranular Corrosion on Inside of Tube
C. Dent
D. Inside Longitudinal Notch 1/8 in. Long x 0.015 in. Deep

FIGURE 14
Test Specimen used for Figure 15 Test
Test Results for Tube in Figure 14 Showing Comparison of Single Frequency and Two Frequency Modes of Operation of a Two Frequency Multiparameter Eddy Current Tester
due to the tubing irregularities. Near the end of the tubing, the test coil assembly was caused to wobble severely, resulting in the large signals near the left end of the upper trace of the chart records shown in Figures 15(a) and 15(b). In these two single frequency runs, the division of the defect signals between the two channels is very much predetermined by the phase relationship between the defect and probe wobble signals. In the conventional eddy current test this division is not under control of the operator.

Two-Frequency Mode

In contrast, performance of the new instrument operated in its two frequency mode is shown in Figure 15(c). Here the instrument has been adjusted to minimize the effect of probe wobble on both output channels, and to place the signal due to the inside notch mainly on channel 1 and the signal due to the outside notch mainly on channel 2. The probe wobble signal has been greatly reduced on channel 2 making signals of interest more useable than in the case of the single frequency test. Some loss of signal-to-noise ratio has occurred on channel 1.

FOUR FREQUENCY THICKNESS TEST SYSTEM

A prototype eddy current tester(3) using five test frequencies, 6 kHz, 22 kHz, 70 kHz, 250 kHz, and 3 MHz is shown in the block diagram of Figure 16. The 3 MHz test frequency was included for use in a probe lift-off compensating system but was not used in the tests described here.

These five excitation carriers were generated by crystal oscillators, not harmonically related, and thus were not phase locked. A probe type test coil was used for the flat metal sheet test specimens.
FIGURE 16
Block Diagram of a Prototype Tester
Separate ac nulling circuits were used to bring the input signals at the frequency separating section to a desired degree of compensation. Off-null or incomplete compensation was required because amplitude detectors were used instead of the more common amplitude-phase or synchronous detector.

Amplitude detectors when fed with the sum of a large fixed signal and a small varying signal will give an output whose variations are approximately proportional to that component of the small signal in phase with the large fixed signal. Band pass filters were used ahead of the detectors to separate the different carrier frequencies. The transformation section shown in Figure 11 was used and the output read on dc current meters. However, an oscilloscope or strip chart could also be used. Because of the type of detection circuit used, a large dc component appeared in the output. Use of balanced synchronous detectors would eliminate this disadvantage, or the dc component could be compensated by adjustable dc signals. As only one signal component per test frequency is used in this instrument it has a theoretical four parameter separating capability. However, the approximate nature or "quasi" status of the phase detection method used probably limits the operation.

Test Probe and Test Specimen Arrangement

The test probe and test specimen arrangement used for the tests to be discussed is shown in Figure 17. The test specimen comprised two metal sheets, a top sheet of brass and a lower sheet of carbon steel. This gave a three parameter system. These parameters were varied over the following listed ranges:

\[ p_1 \] - Probe spacing 0.0 to 0.030 in.
\[ p_2 \] - Brass thickness 0.001 to 0.006 in.
\[ p_3 \] - Carbon steel thickness 0.001 to 0.006 in.
Parameter $P_1$ - Probe to Surface Spacing

Parameter $P_2$ - Brass Thickness

Parameter $P_3$ - Carbon Steel Thickness

FIGURE 17
Test Probe Test Specimen Arrangement Multiparameter Thickness Test
For the tests to be described, three input signals \( (c_1, c_2, c_3) \) to the transformation section were formed by adding pairs of the outputs of four detectors as follows:

\[
\begin{align*}
    c_1 & \text{ - } 6 \text{ kHz and } 22 \text{ kHz} \\
    c_2 & \text{ - } 22 \text{ kHz and } 70 \text{ kHz} \\
    c_3 & \text{ - } 70 \text{ kHz and } 250 \text{ kHz}
\end{align*}
\]

This made it possible to use information from four test frequencies to be used in the separation of three parameters.

Matrix elements of the modulation matrix (Equation (4)) were determined by measurement, varying each parameter in turn. The functions were curved, so average slopes were used. Calculation of the inverse matrix of the modulation matrix permitted the setting of the transformation section potentiometers. Trimming these adjustments gives optimum results.

**Results of Test**

The results of this test are shown in Figures 18, 19, and 20. Good parameter separation is obtained over the following ranges:

\[
\begin{align*}
    p_1 & \text{ Prove spacing } \quad 0.005 \text{ to } 0.030 \text{ in.} \\
    p_2 & \text{ Brass thickness } \quad 0.002 \text{ to } 0.006 \text{ in.} \\
    p_3 & \text{ Carbon steel thickness } \quad 0.001 \text{ to } 0.003 \text{ in.}
\end{align*}
\]

**TUBING TESTER WITH CROSS SECTION DISPLAY FEATURE**

Figure 21, describing the test system, and Figure 22, showing typical test results, demonstrate further potential of the multiparameter test. Here a section of tubing having gross fabricated defects is inspected using a two frequency multiparameter eddy current tester. The output of the multiparameter tester, together with tubing radial position signals obtained from a resolver, produce a simulated pattern of the tubing being inspected. The oscilloscope
FIGURE 18
Parameter Separation when Four Test Signals are used to Separate Three Parameters ($P_1$ Variable)
Figure 19
Parameter Separation when Four Test Signals are used to Separate Three Parameters ($P_2$ Variable)

$P_1 = 0.000$ in. Probe Spacing
$P_2 = $ Brass Thickness
$P_3 = (0.003$ in.) Steel Thickness
Parameter Separation when Four Test Signals are used to Separate Three Parameters ($P_3$ Variable)
FIGURE 21
Eddy Current Test with Tubing Cross Section Display
Test Specimen: Type 304 Stainless Steel 2-3/4 in. O.D., 1/8 in. Wall
Artificial Flaws: 1/8 in. Diameter Holes in Depths Shown in Inches

FIGURE 22
Dynamic Display of Tube Cross Section Two Frequency Multiparameter Eddy Current Test
display shows the wall thickness or relative location and severity of flaws opening on the outer or inner walls.

**Formation of Display Pattern**

This pattern is generated by applying rotating radial sweep deflection voltages into the oscilloscope vertical and horizontal inputs, and by unblanking the cathode ray beam with defect or thickness signals derived from the multiparameter tester. The general system for obtaining the rotating radial sweep is patterned after the plan position indicator used in radar display systems.

**Description of Circuit Functions**

Referring to Figure 21, the system comprises a multiparameter eddy current tester A, test coil B, a section of tubing C being rotated by motor D, cathode ray tube display and other associated electronic circuits. The multiparameter tester operates at two frequencies, 70 kHz and 250 kHz. Demodulation of the eddy current signals is done by synchronous detectors (amplitude-phase detectors). A 1/2 inch diameter external tangent test coil is used.

Resolver E, also rotated by motor D and excited from the 5 kHz oscillator F, drives Phase Detectors G which provide the Modulator-Sweep Generator H with tubing angular position signals. The Modulator-Sweep Generator H produces a series of vertical and horizontal deflection voltages modulated with the sine-cosine tubing angular position information (c,d) to produce a rotating radial sweep of the cathode ray beam. The Amplitude-to-Pulse Position Modulators, I, convert the parameter information from the multiparameter tester into a multiplexed pulse position signal to unblank the cathode ray tube beam. These modulators are synchronized with a signal from Sweep Generator H to assure the proper orientation of flaw signals on the display. Amplitude-
phase detectors were used making available four demodulated signals for inputs to the transformation section. Figure 11 describes the transformation system used.

Dynamic Display Results

The dynamic display of the cross section of a piece of type 304 stainless steel tubing obtained with this system is shown in Figure 22. Similar orientation of the test specimen showing the fabricated defects and the photograph of the resulting dynamic display permit easy comparison. The tube diameter was 2-3/4 in. with 1/8-in. wall and had artificial defects made by drilling 1/8-in. diameter holes to the depths shown ranging from 0.012 to 0.062 in. The transformation section of the tester was adjusted to give read-outs of outer surface defects and inner surface defects essentially independent of probe wobble on separate channels. In this test, probe wobble information was not used. A tangent type test coil was used on the rotating tube. The rotating radial sweep was generated as previously described with tube position information being obtained from a sine-cosine resolver.

The inner and outer surface defect signals from the multiparameter tester were fed to the dynamic display circuitry on the parameter \( p_1 \) and parameter \( p_2 \) input circuits, respectively, shown in Figure 21. These signals are dc coupled from the detector circuits to the display; thus, the pattern shown is obtained at very slow to medium sample rotation speeds. Also, the dc connection permits wall thickness variations to be shown, even when they occur over large areas.

The simultaneous detection (and display) of the 0.012 in. deep outer surface flaw and the 0.040 in. deep inner surface flaw at location A in Figure 22 are of special significance. This demonstrates the capability of
the multiparameter tester to identify the signals due to these two types of flaws even when both flaws are present in the field of the coil at the same time. This type of performance was predicted in the description of the theory and original tests of the multiparameter test principle.

The adjustment of the summing networks in the transformation section of the multiparameter tester to give the desired read-out for particular test specimen standards is equivalent to a sort of instrument calibration for those standards. If, after such an adjustment, the tester is presented with a test specimen which has a new and different "uncalibrated for" parameter, such parameters will give an indication on more than one channel and should thus be readily identified as something new.

FUTURE POTENTIAL

It has been shown that application of present research study results gives test information in a form not available using conventional techniques. Thus, there is great potential just in the application of the presently demonstrated principles and in the refinement of instrumentation to implement them.

However, the greatest potential lies in the extension of the method, in a practical way, to handle an increased number of parameters and in the application of adaptive control techniques for automatic adjustment of the test equipment.

It is not expected that this new approach to eddy current test problems will replace present single frequency tests. The equipment is complex and adjustment is complicated in its present stage of development. It is expected that it will be in the area of special problems, not solvable with conventional tests, where using the more general approaches will be justified.

The multiparameter research has already given a better understanding of the single frequency tests and promises to give further clarification to aspects of conventional tests as the study is extended to more parameters or variables.
ACKNOWLEDGMENTS

This paper reports on work performed under Contract AT(45-1)-1830 between the U.S. Atomic Energy Commission and Battelle Memorial Institute and on earlier phases of the work performed under Contract AT(45-1)-1350 between the U.S. Atomic Energy Commission and the General Electric Company.

The contributions of Professor C. W. Cox and Dr. K. W. Atwood in early parts of the work involving theory measurements and instrument design are acknowledged. The author is also grateful for many stimulating discussions with Dr. W. H. Huggins and Dr. S. Litman in the area of system identification and signal analysis.

The assistance of C. R. Wandling in the development of the cross section display device and the two frequency multichannel tubing tester is also acknowledged.
REFERENCES


DISTRIBUTION LIST

<table>
<thead>
<tr>
<th>No. of Copies</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>C. L. Robinson</td>
</tr>
<tr>
<td>1</td>
<td>Technical Library</td>
</tr>
<tr>
<td>1</td>
<td>G. H. Lee</td>
</tr>
<tr>
<td>1</td>
<td>R. K. Sharp</td>
</tr>
<tr>
<td>278</td>
<td>For UC-37 Distribution</td>
</tr>
<tr>
<td>5</td>
<td>Technical Information</td>
</tr>
<tr>
<td>2</td>
<td>Technical Publications</td>
</tr>
<tr>
<td>1</td>
<td>G. J. Dau</td>
</tr>
<tr>
<td>25</td>
<td>H. L. Libby</td>
</tr>
<tr>
<td>1</td>
<td>V. I. Neeley</td>
</tr>
<tr>
<td>15</td>
<td>H. N. Pedersen</td>
</tr>
<tr>
<td>1</td>
<td>N. S. Porter</td>
</tr>
<tr>
<td>1</td>
<td>J. C. Spanner</td>
</tr>
<tr>
<td>1</td>
<td>D. C. Worlton</td>
</tr>
<tr>
<td>1</td>
<td>W. G. Spear</td>
</tr>
<tr>
<td>1</td>
<td>G. E. Driver</td>
</tr>
<tr>
<td>1</td>
<td>C. D. Swanson</td>
</tr>
<tr>
<td>1</td>
<td>R. N. Ord</td>
</tr>
<tr>
<td>1</td>
<td>C. R. Wandling</td>
</tr>
<tr>
<td>1</td>
<td>J. D. Jensen</td>
</tr>
<tr>
<td>1</td>
<td>L. T. Lamb</td>
</tr>
<tr>
<td>1</td>
<td>D. R. Green</td>
</tr>
<tr>
<td>1</td>
<td>R. L. Brown</td>
</tr>
</tbody>
</table>

Division of Reactor Development & Technology
Fuels and Materials Branch

1             | J. M. Simmons      |
1             | K. E. Horton       |