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**A Study of Global Alfvén Waves Driven by Neutral
Beam Injection in D III-D**

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Within the framework of the global Alfvén eigenmode theory[1] in a cylindrical background plasma, I examine the excitation of global Alfvén eigenmodes by intense neutral beam injection in the D III-D tokamak operating at General Atomics. I have considered two separate sets of experimental conditions, a "low power" set of cases [2] using 10MW of hydrogen beams, and a "high power" shot of 20MW of deuterium beams[3].

My results are particularly sensitive to the background density profile. For parabolic background density profiles, $n_0 \times (1 - (r/\tilde{a})^2)$, I have determined that the plasma is stable to all toroidal and poloidal mode numbers for both high and low power cases. For density profiles which are of the form $n_0 \times (1 - (r/\tilde{a})^2)^{\frac{1}{2}}$, for the same n_0 , my calculation indicates that the $m = -1, l = 0$ mode is unstable in each case. The high power case has a considerably higher growth rate at the baseline conditions, which motivated me to study this case more extensively.

The results are also sensitive to the beam source radial scalelength, L_s , and the electron temperature, T_e . By narrowing the source from the baseline 36 cm to 20 cm, the growth rate of the $(0, -1)$ actually decreases, but the $(0, -2)$ mode appears with a substantial growth rate. If the source could be made even narrower, $L_s \approx 10$ cm, the $(1, -1)$ mode would appear, also with a large growth rate.

The electron temperature is a parameter which provides some leverage in modifying the stability properties of D III-D. If the electron temperature could be increased by a factor of 2, the growth rates of the modes could be increased by a similar factor.

I believe that a reasonable benchmark of the theory could be obtained by performing further experiments with high power deuterium beams and determine the fluctuation spectrum in the frequency range of 48kHz to 160kHz, the range where theory predicts instability. This benchmarking procedure is important because it may demonstrate the predictive capability of the theory to larger devices such as the International Thermonuclear Experimental Reactor (ITER).

The impact of these modes on the plasma equilibrium and energy transport must be treated apart from the linear stability theory presented here. Specifically, an understanding of the frequency spectrum of these modes evolved non-linearly must be acquired. If the spectrum is peaked around the mode frequency, there may be minimal impact on the plasma transport. If, on the other hand, the spectrum is broad-band and the individual waves' phases are uncorrelated, diffusion in velocity and configuration space may result. The experimentally determined spectral data collected in a new set of shots could answer some of these questions.

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1. Introduction

The injection of high energy, high power neutral beams is considered to be one of the best methods to heat magnetically confined plasmas to thermonuclear conditions. The plasma states produced by beam injection, at least in present day experiments, are far from equilibrium, in both velocity space (non-Maxwellian, anisotropic) and configuration space (non-uniform). These plasma states contain sources of free energy from which, under certain conditions, waves can extract energy and grow. The presence of these free energy sources from gradients in both configuration and velocity space requires that machine designers examine the possibility that instabilities are present and may degrade performance.

The D III-D experiment at General Atomics is capable of examining the stability of Alfvén waves in a hot plasma under intense beam injection. The results of such experiments might be used to predict the current drive performance of larger proposed tokamaks such as ITER, in that they could serve as a means to benchmark the theoretical description of these instabilities. The positive ion based injectors on D III-D can deliver up to 20MW of deuterium or 10MW of hydrogen at a full energy of 75keV. The machine can operate at fields as low as 7kG, and densities in the range of $5 \times 10^{19} \text{ cm}^{-3}$. The electron temperature under these conditions is from 1.5 to 2.5 keV. These parameters make D III-D ideal for the study of Alfvén waves.

These issues for beam injection in the two-component-torus (TCT) were addressed originally by Berk, et. al. [4] in the mid-1970's who examined a wide variety of modes and frequency ranges using theory based on the local approximation. The main conclusion from that work was that the long wavelength velocity space driven shear Alfvén mode appeared to be a candidate mode for an instability, with the hot particle density gradient driven mode a definite possibility, although it was not pursued in that paper. Rosenbluth and Rutherford [5] investigated this drift mode in a subsequent paper. They concluded that the mode would be quasilinearly stabilized by a flattening of the hot particle density profile.

The velocity space driven modes in the local approximation were examined by Kerbel and myself[6] for ITER, using Berk's formalism. We found that the long wavelength shear mode also was the most important, although we realized that a correct calculation would include the details of the mode structure.

The need to treat the long-wavelength modes correctly was at least in part the motivation for this present paper, as well as the desire to compare the theoretical predictions with experimental data.

The global mode examined here is typically long wavelength compared to gradient scalelengths of wave fields, background plasma parameters, and particle gyroradii. The longest wavelength modes¹ $m = -1, l = 0$ are potentially the most dangerous, since the eigenmode can extend across the entire plasma cross section. For this case, it is more difficult for quasilinear flattening of the profile to be stabilizing since for non-zero hot density there must always be a gradient somewhere in the plasma where the wave fields exist. This global mode can be driven by either beams or alpha particles.

¹ m is the poloidal mode number, l the toroidal mode number.

Indeed, the work on which this present work is based [1] examined hot alpha particles in a hypothetical high temperature TFTR plasma.

It is logical to first consider these global modes in an existing experimental device such as D III-D because of the potential for experimental verification of the theoretical predictions. If validated, the theory could then be applied to ITER where the assessment of plasma stability has important implications for the performance of the machine, particularly in the current driven technology phases.

I consider D III-D first with hydrogen beams with 10MW of injected beam power, and then with 20MW of deuterium beams. These beams are at an energy of 75 keV with the power distributed equally between the tangential and perpendicular beams. For the hydrogen beam cases, I have chosen the background plasma parameters to match the recent set of experimental shots which had parameters in the range where originally an instability of this type was thought to have been observed[2]. Very recent results on D III-D with 20MW of deuterium beams at high density and low magnetic field [3] motivated the study of the global modes under high power, record β_t operation. It is this high power case which appears to have the most potential for creating the Alfvén instability.

2. Choice of Parameters

The background plasma parameters chosen for the baseline stability calculation correspond closely to those measured in a recent series of shots[2],[3]. The beams in both sets of cases are injected into a deuterium plasma with a Z_{eff} value of 3. The machine was run at relatively low field and high density in order to test machine operation at high toroidal beta, β_t . This parameter range also is appropriate for beam driven Alfvén wave studies, since the Alfvén velocity is comparable to the beam injection speed. The general parameters for the device are shown in Table 1 for the baseline shot we considered. Figures 1, 2, and 3 show the assumed radial profiles of the electron density, electron temperature and safety factor, respectively for the hydrogen beam shots [2]. Figures 4, 5, 6, 7, and 8 show typical calculated parameters for the same hydrogen shot associated with the beam population, specifically the beam source strength, hot density, hot "temperature", Alfvén energy, and critical energy, respectively. The slowing down time for both the deuterium and hydrogen beam case is shown in Fig. 9.

3. General Methodology

I use the global Alfvén eigenmode theory as described by Li, *et. al.* [1] to study the global waves present when a neutral beam is injected into a plasma. My application is general with respect to hot distribution function, so I can investigate the impact of various assumptions. I have used Maxwellians, as well as isotropic and anisotropic slowing down-type distributions in the stability analysis.

The global modes owe their existence to the presence of finite shear (i. e. $\frac{d\theta}{dr} \neq 0$) and finite background ion gyroradius, with the shear effect being the strongest. These modes are characterized by frequencies just below the minimum Alfvén frequency,

Figure 1: Typical profiles of electron density, n_e , as a function of radius, r .

Figure 2: Typical profiles of electron temperature, T_e , as a function of radius, r .

Figure 3: Typical profiles of safety factor, $q(r)$, as a function of radius, r .

Figure 4: Typical profiles of hot particle source strength, $S(r)$, as a function of radius, r .

Figure 5: Typical profiles of hot particle density, $n_{hot}(r)$ as a function of radius, r .

Figure 6: Typical profiles of hot particle “temperature” $T_{hot} = \frac{2}{3}E_{hot}$ as a function of radius, r .

Figure 7: Typical profiles of Alfvén energy $E_A = \frac{1}{2}m_{\text{hot}}V_A^2$ as a function of radius, r .

Figure 8: Typical profiles of critical energy $E_c = \frac{1}{2}m_{hot}v_c^2$ as a function of radius, r .

Figure 9: Typical profiles of hot particle slowing down time, τ_D , as a function of radius, r .

$\omega_{A,\min}$, defined by $\omega_A = k_{\parallel} V_A$. The Alfvén velocity is given by $V_A = B/\sqrt{4\pi m_i n_i}$. This minimum will occur at a radius $r_0 \neq 0$, a typical profile of $\omega_A = k_{\parallel} V_A$, where $k_{\parallel} = (lq(r) - m)/q(r)R_0$, is shown in Figure 10 for the $m = -1$, $l = 0$ in D III-D. When the frequency of the mode is below $\omega_{A,\min}$ the wave equations represent an eigenvalue problem with discrete eigenvalues, since the coefficients of the highest order derivative are non-zero everywhere radially. The other case with $\omega > \omega_{A,\min}$ admits a continuum of eigenfrequencies, and the wave fields result from the solution of a singular fourth-order differential equation.

The modes I investigate in this paper exist in a cylindrical plasma, without any hot particles present, and are stable due to damping on thermal electrons. Toroidal corrections to the background modes, as well as toroidally induced modes (Toroidal Alfvén Eigenmodes) have been studied by Fu, *et. al.* [7], and will not be discussed further here. I believe that all the qualitative features of the modes are contained in the cylindrical approximation, and therefore the study of the toroidal effects for D III-D and ITER will be left to future work. I believe that before additional toroidal coupling be introduced into the mode equations, the non-circular cross-section should be treated. Strong non-circularity as in D III-D should introduce additional terms which would modify the stability picture as much as the lowest order toroidal effects on the background modes.

The instability I investigate here is due to the coupling between the wave motion and the vertical drift motion ($v_{D,\perp}$) of the hot particles. The vertical drift is due to the toroidal gradients and curvature of B . The sign of the energy derivative of the distribution function at resonant $v_{\parallel}$ values and the ordering of the diamagnetic drift frequency, $\omega_{*,h}$, of the hot particles relative to the mode frequency, ω_r , determines whether this coupling results in the growth or damping of the wave. When $\frac{\partial f_{hot}}{\partial v_{\parallel}}|_{v_{\parallel}} > 0$, an instability is possible, as well as the case when $\omega_{*,h} > \omega$. The condition on the drift frequency gives rise to the primary mode of instability.

The steps of the calculation involve first writing down the second order eigenmode equation, which is an approximate potential representation of the full fourth order equation. By solving this equation for the eigenfunction, φ , dispersion relations for the eigenfrequency, ω , and forms for the eigenfunction, φ , can be obtained. Separate cases for $|m| = 1$ and $|m| \neq 1$ are necessary because the mode is localized near r_0 for $|m| \neq 1$, and extends across the entire plasma for $|m| = 1$.

The drive for the instability is obtained from the parallel component of Ampère's law combined with quasineutrality. The perturbed parallel current is obtained from a moment of the drift kinetic equation (DKE). The use of the DKE which has been averaged over the fast gyro-motion precludes the direct treatment of finite hot particle gyroradius effects. This is not a serious omission for low m modes in which are of interest here, since the scalelength of the eigenfunction variation is long compared to ρ_{hot} . For higher m modes, the accurate calculation of the stability requires the use of a gyrokinetic equation.

Further, because the theory neglects $\mathbf{k} \cdot \mathbf{v}_{D,\perp}$ compared to ω in the wave-particles resonance condition $\omega - \mathbf{k} \cdot \mathbf{v}_{D,\perp} - k_{\parallel} v_{\parallel} = 0$, the trapped hot particles (where $v_{\parallel}$ can

²True for a Maxwellian hot particle distribution. The slowing down distributions yield instability at lower ratios of $\omega_{*,h}/\omega_r$.

Figure 10: Profile of Alfvén frequency, $k_{\parallel} V_A$, as a function of radius for the $m = -1, l = 0$ mode.

vanish) cannot be considered. The hot particles produced by the tangentially injected beam remain primarily in passing velocity space, whereas a large fraction of the hot ions originating from the perpendicular beam could be in trapped space.

I have included the response of the electrons to the wave in the perturbed current density in addition to the hot particle response. The electrons in this capacity provide a damping mechanism which is typically several times higher than the standard Landau damping usually included[1]. This fact has been recognized only recently[7].

Once the parallel current density in the presence of the hot species and electrons is calculated, the eigenmode equations including the kinetic effects can be formulated. At this point, the assumption is made that the kinetic effects are a small correction to the wave structure and frequency. A straightforward perturbation theory is then performed, which yields a growth rate of the mode in terms of the wave properties, plasma profiles, and hot particle properties.

4. Model Equations

The wave spatial structure is described by the wave equation eigenvalue problem:

$$\nabla \times \nabla \times \mathbf{E} = \frac{\omega^2}{c^2} \chi \cdot \mathbf{E} \quad (1)$$

where $\mathbf{E}$ is the wave electric field, and χ is the plasma susceptibility. The boundary conditions placed on this equation is that both the E_r and $E_\perp$ components of the electric field vanish at the wall. Later, $E_\parallel$ will be assumed to vanish in the infinite parallel conductivity assumption. The tensor χ contains the effect of the plasma on the wave propagation, and is typically comprised of two parts. The first part, χ_b for the background plasma is constructed from the uniform plasma susceptibility by invoking certain symmetry arguments. This term includes the response of the non-uniform plasma in the presence of sheared magnetic fields, the Landau damping on the background electrons and ion species, as well as first order gyroradius effects for the background ions. The solution of Eq. 1 with χ replaced by χ_b would correspond to the case where suprathermal particles are absent and would result in the damped, stable global Alfvén waves. The point of the stability analysis is to compute an additional term, χ_k , which includes the kinetic plasma response of the plasma due to the interaction of the wave with the vertical drift motion of the kinetic species of the plasma. This drift is due to the toroidal gradients and curvature of the toroidal magnetic field. Whether this term results in wave growth or damping depends upon specific gradients in the distribution function in both velocity and configuration space. The species which are important in χ_k are the hot injected particles and the electrons. The hot particles provide the growth for the mode under certain conditions, and the electrons are always dissipative.

The drift kinetic equation is used by Li, *et. al.* to compute χ_k . Implicit in this assumption is that the gyroradius of the species treated is small compared to the wavelength of the mode. For the global mode with low values of poloidal (m) and toroidal (l) mode numbers, this is a good assumption, even for the hot injected beam

particles. The linearized DKE for the perturbed distribution of species "s", $f_{1,s}$, can be written³ as[1]:

$$\begin{aligned} & (-i\omega + v_{\parallel} \nabla_{\parallel} + v_{D,s} \cdot \nabla) f_{1,s} \\ &= -e_s v_{D,s} \cdot \mathbf{E} \frac{\partial f_{s,0}}{\partial \mathcal{E}} - \frac{c}{i\omega B} v_{D,s} \cdot \frac{1}{r} \frac{\partial}{\partial \theta} \mathbf{E} \frac{\partial f_{s,0}}{\partial r}, \end{aligned} \quad (2)$$

where $\mathcal{E} = \frac{1}{2} m_s v^2$, and $v_{D,s}$ is the vertical drift velocity. This equation is correct to first order in the inverse aspect ratio, r/R_0 even for anisotropic beam heated plasmas. For anisotropic distributions, there will be a term on the right hand side of the DKE[8] which is second order in r/R_0 for finite β . Within the formal inverse aspect ratio expansion, this term must be dropped. If this term were to be kept, the lowest order expansion in the poloidal angle, θ , used in the analysis would be invalid. Whether the term is indeed small for a strong degree of anisotropy remains to be investigated. Eq. 2 is solved by expanding $f_{1,s}$ in a Fourier series in poloidal angle, noting that the lowest order poloidal coupling is due to the θ dependence of the vertical drift velocity. Orderings in r/R_0 give the result that the "m"-th coefficients in the expansion do not contribute to lowest order, whereas the " $m \pm 1$ "th sideband coefficients give a set of two algebraic equations which can be used to solve for $f_{s,1}$. The solution of Eq. 2 is used in the following integral appearing in the equation for the perturbed current density:

$$\int_0^\infty d^3v v_{D,s} \cdot \nabla f_{1,s} \equiv \nabla_{\perp} \cdot \left[\left(\frac{\omega}{4\pi i} \right) \chi_k \cdot \mathbf{E} \right] \quad (3)$$

From Eq. 3 and the solution of Eq. 2, the kinetic part of χ can be isolated and used in Eq. 1 to solve for the fields and complex eigenfrequency. The wave equation, Eq. 1, with $E_{\parallel} \approx 0$, represents two coupled second order differential equations for the electric fields ($E_r, E_{\perp}$), and the complex frequency, $\omega = \omega_r + \gamma$. By casting the equations in terms of a potential, φ , and a parallel vector potential, $A_{\parallel}$, the eigenvalue problem can be written as a single second order equation, which has the kinetic terms from the toroidal drift coupling:

$$\begin{aligned} & \frac{1}{r^2} \frac{d}{dr} \left[r^3 \left(\frac{\omega^2}{V_A^2} - k_{\parallel}^2 - A_k \right) \frac{d\varphi}{dr} \right] - \frac{(m^2 - 1)}{r^2} \\ & \times \left(\frac{\omega^2}{V_A^2} - A_k \right) \varphi + \left(\frac{\omega^2}{V_A^2} \right)' \frac{\varphi}{r} - A_k' \frac{\varphi}{r} - B_k' \frac{m\varphi}{r} = 0, \end{aligned} \quad (4)$$

where the prime denotes a radial derivative. The functions $A_k = \sum A_k$ and $B_k = \sum B_k$, include the kinetic drive due to toroidal gradients and curvature, and are given by Eqs. 15 and 16 presented later on.

From the theory of differential equations, if the coefficient of the highest derivative does not vanish, Eq. 4 is regular and possesses discrete eigenvalues. If, on the other hand, the coefficient of this equation vanishes at some point radially, there is a continuous "spectrum" of eigenvalues⁴. In the plasma problem, the continuous spectrum is

³The $E_{\parallel}$ terms are neglected, as is the sloshing part of the perturbed distribution.

⁴Such problems also occur in the solution of the monoenergetic neutron transport equation

what is referred to as the "MHD continuum" and occurs for frequencies $\omega > (k_{\parallel} V_A)_{\min}$. I will be studying in this paper the lower frequency range, $\omega < (k_{\parallel} V_A)_{\min} \equiv \omega_{A,\min}$. Figure 10 shows the radial profile of the Alfvén frequency, $\omega_A(r)$, for the (0, -1) mode and the hydrogen beam baseline.

The mode equation, Eq. 4, is solved approximately for φ and ω . The equation is first solved without the kinetic terms A_k and B_k for the eigenfunction and the real part of the eigenfrequency, ω_r . The function φ has a qualitatively different structure, depending on whether $|m| = 1$ or $|m| \neq 1$. The expressions derived by Li, *et. al.* are:

$$\frac{\varphi}{r} = \exp \left\{ - \left[\sqrt{\epsilon} / (2\Delta^2) \right] (r - r_0)^2 \right\} \quad (5)$$

for $|m| \neq 1$, and for $|m| = 1$, $\frac{\varphi}{r}$ is given by:

$$\frac{\varphi}{r} = C \times \tan^{-1} [(r - r_0) / \Delta] + D, \quad (6)$$

where ϵ is given by:

$$\epsilon = g - \frac{1}{4} - \sqrt{g - \frac{1}{2}},$$

$$g \equiv g(r_0) = \frac{L^2}{r_0^2} \left(\frac{2sm}{m - lq} - m \frac{|\omega_A(r_0)|}{\omega_{e,i}} a_1^{-\frac{1}{2}} \frac{r_0}{L_n} \right), \quad (7)$$

the parameter s is a measure of the shear, $s = (r/q)(dq/dr)$, and L_n is the background density gradient scalelength, $L_n^{-1} = (1/n)(dn/dr)$. The functions C and D are given in terms of the functions a_1 , a_2 , and b defined below in Eq. 10, $C = -(a_2 - a_1 - b)^{-1}$, and $D = -C \times (a_2 - b)$. The if the mode equation is viewed as a Schrödinger equation, the parameter g is a measure of the depth of the potential well[9]. For the well to exist at all, that is, to assure bound-state solutions, $g > 0$. The theory is strictly valid for $g > 1$, although I have applied it for all positive g . For those instances when $0 < g < \frac{1}{2}$, I have set $g = \frac{1}{2}$. More work needs to be done in this area, because for D III-D some experimentally interesting parameter space corresponds to $0 < g < 1$.

Eq. 4 is also used to obtain the dispersion relations for each case of $|m|$. For $|m| \neq 1$, the real part of the frequency is given by:

$$\omega_0^2 = \frac{\omega_A^2(r_0) (1 - \epsilon/k_{\perp}^2 L^2)}{1 + \left[\omega_A^2(r_0) / \omega_{e,i}^2 \right] (1 - \epsilon/k_{\perp}^2 L^2)}, \quad (8)$$

where $k_{\perp} = \frac{\pi}{r}$ to lowest order. For $|m| = 1$, the dispersion relation is given by:

$$1 - \zeta^2 = \zeta^4 \left[\frac{L r_0 (a_2 - a_1 - b)}{2(a^2 - r_0^2)} \right]^2 \quad (9)$$

the variable $\zeta = \frac{\omega_r}{\omega_{A,0}}$, and $L^{-1} = \frac{1}{\omega_A} \frac{\partial \omega_A^2}{\partial r}$. The functions a_1 , a_2 , and b are given by:

$$\begin{aligned} a_1 &= \tan^{-1} (-r_0 / \Delta), \\ a_2 &= \tan^{-1} [(a - r_0) / \Delta], \\ b &= \Delta / \left\{ K [\Delta^2 + (a - r_0)^2] \right\}. \end{aligned} \quad (10)$$

The variable Δ is given by $\Delta = L^2(1 - \zeta^2)$, and the function $K = (B - 1)/a$, where B is a complicated function of the background and mode properties, and is given in Ref.[1].

The electron Landau damping $\gamma_{e,L}$ for $|m| \neq 1$ is given by:

$$\frac{\gamma_{e,L}}{\omega_r} = -k_{\perp}^2 \rho_s^2 \sqrt{\pi} (V_A/v_{th,e}) f(\epsilon), \quad (11)$$

where:

$$f(\epsilon) = \frac{1}{2} \left(2\epsilon + 2\sqrt{\epsilon} + \frac{3}{2} \right) / \left(\sqrt{\epsilon} + 2\epsilon \right),$$

ϵ is given by Eq. 7 and ρ_s is the ion gyroradius based on the sound speed. All quantities are evaluated at, r_0 , the radial point where the mode is localized. For $|m| = 1$, Li, *et. al.* point out that the detailed averaging of the electron Landau damping over the mode structure is necessary, and is best accomplished through a complete numerical code which solves the complete Eq. 1. To gain some qualitative insight in the strength of the electron Landau damping, however, I will use Eq. 11 for the $|m| = 1$ mode also. The damping due to the magnetic drift, $\gamma_{e,M}$ which is being calculated consistently turns out to be several times the Landau terms presented above for representative parameters. This suggests that errors in $\gamma_{e,L}$ will not result in serious errors in computing stability.

Knowing the eigenmode structure and mode frequency, the growth rate can be determined again by the mode equation, assuming that $\omega = \omega_r + \gamma$, with $\gamma/\omega_r \ll 1$. The result of this operation is the normalized growth rate of the mode:

$$\frac{\gamma_{tot}}{\omega_r} = \frac{V_{A,0}^2}{2\omega_r^2} \times \frac{\int_0^a dr \mathcal{N}(r)}{\int_0^a dr \mathcal{D}(r)} \quad (12)$$

where the functions $\mathcal{N}(r)$ and $\mathcal{D}(r)$ are products of the density profiles, eigenfunctions, and specific velocity space integrals of the distribution function weighted with the drift velocity. The function $\mathcal{N}(r)$ is given by:

$$\mathcal{N}(r) \equiv (m^2 - 1) r A_s (\varphi_0/r)^2 + r^3 A_s \left[(\varphi_0/r)' \right]^2 - A_s' r^2 (\varphi_0/r)^2 - B_s' r^2 m (\varphi_0/r)^2, \quad (13)$$

and the function $\mathcal{D}(r)$ takes the form:

$$\mathcal{D}(r) \equiv (m^2 - 1) r \frac{n(r)}{n_0} (\varphi_0/r)^2 + r^3 \frac{n(r)}{n_0} \left[(\varphi_0/r)' \right]^2 - r^2 \frac{n'(r)}{n_0} (\varphi_0/r)^2. \quad (14)$$

The energy input from species "s" into the waves is described by the terms A_s and B_s :

$$A_s = \frac{\omega_r^2}{c^2} (Q_{s,m-1} + Q_{s,m+1}), \quad (15)$$

and,

$$B_s = \frac{\omega_r^2}{c^2} (Q_{s,m-1} - Q_{s,m+1}). \quad (16)$$

The functions $Q_{s,m\pm 1}$ are expressed as:

$$Q_{s,m\pm 1} = \frac{\pi e_s^2}{\omega_r T_s} \int d^3 v f_{s,0} \left(\hat{T} - \frac{\omega_{s,m}}{\omega_r} \right) v_{d,s}^2 \frac{1}{\omega_r + \gamma - k_{\parallel,m\pm 1} v_{\parallel}}. \quad (17)$$

The side band $k_{\parallel,m\pm 1} = [lq(r) - (m \pm 1)]/q(r) R_0$. The drift velocity, $v_{d,s}$, the diamagnetic drift frequency, $\omega_{s,m}$, and related parameters are given by:

$$\begin{aligned} v_{d,s} &= \frac{m_s c}{e_s B_0 R_0} \left(\frac{1}{2} v_{\perp}^2 + v_{\parallel}^2 \right), \\ \omega_{s,m} &= \frac{m T_s c}{r e_s B_0} \frac{\partial \ln(n_s T_s)}{\partial r} \\ \hat{T} &= -T_s \frac{\partial \ln f_{s,0}}{\partial \mathcal{E}}, \\ T_s &= \frac{2}{3n_s} \int d^3 v \mathcal{E} f_{s,0}. \end{aligned}$$

The expression for $Q_{s,m\pm 1}$ can be simplified into a single integral in $v_{\perp}$ at the resonant $v_{\parallel} = \omega_r/k_{\parallel,m\pm 1}$ by recognizing that $\gamma/\omega_r \ll 1$ and use the Plemelj formula. This transformation breaks the integral into a real (reactive) part and an imaginary (dissipative) part, the dissipative part being proportional to a delta function: $\delta(\omega_r - k_{\parallel,m\pm 1} v_{\parallel})$, and is related to the growth or damping of the mode.

The lowest-order distribution function, $f_{s,0}$, used in this analysis can either be a Maxwellian:

$$f_{s,0}(v, r) = \frac{n_s(r)}{\pi^{3/2} v_{th,s}^3} \exp(-v^2/v_{th,s}^2), \quad (18)$$

or a slowing down-type distribution. For the beam slowing down distribution, we rely on the work by Gaffey [10]. The distribution is written here for an arbitrary number of beams at arbitrary angles with respect to the magnetic field. The source distribution for each beam component is a delta function in energy, and a Gaussian in $\mu = \cos \vartheta$, with width $\Delta\mu(r)$ determined from the spatial footprint of the beam. The finite width in ϑ yields a width in $v_{\perp}$ which facilitates the proper evaluation of the numerical integrals. The distribution for an individual beam is given by:

$$f_{s,0,s}(v, r) = \frac{S_s(r)}{4\pi} \frac{\tau_D}{v^3 + v_c^2} \sum_{l=0}^{\infty} \frac{2l+1}{2} P_l(\mu) C_{l,s} \left[\left(\frac{v^3}{v_{b,s}^3} \right) \left(\frac{v_{b,s}^3 + v_c^2}{v^3 + v_c^2} \right) \right]^{\frac{1}{2}l(l+1)Z_s}, \quad (19)$$

for $v < v_{b,s}$, and for $v > v_{b,s}$ the effects of electron upscattering must be included. Gaffey's work in the high v regime reduces $f_{s,0,s}(v, r)$, for my purposes, to:

$$f_{s,0,s}(v, r) = \frac{S_s(r)}{4\pi} \frac{\tau_D}{v_{b,s}^3 + v_c^2} \sum_{l=0}^{\infty} \frac{2l+1}{2} P_l(\mu) C_{l,s} \exp \left[- \left(\frac{v - v_{b,s}}{\Delta v_{b,s}} \right)^2 \right]. \quad (20)$$

In these equations, the coefficients $C_{l,s}$ are given by:

$$C_{l,s} = \frac{\int_{-1}^{+1} d\mu \exp \left\{ - \left[(\mu - \mu_s) / \Delta\mu_s \right]^2 \right\} \times P_l(\mu)}{\int_{-1}^{+1} d\mu \exp \left\{ - \left[(\mu - \mu_s) / \Delta\mu_s \right]^2 \right\}}.$$

The total distribution is the superposition of all $f_{s,0,g}$ resulting from all the beams "g". The width of the beam in speed, $\Delta v_{s,g}$, can be derived from Gaffey's work, and increases with electron temperature:

$$\Delta v_{s,g} = \frac{1}{2} \frac{1}{v_g^2 + v_s^2} \left(\frac{m_e}{m_s} v_{th,e}^2 v_s^2 + v_e^2 v_s Z_s \right)$$

The parameters $Z_{s,3}$ are products of charge, masses, densities and thermal speeds, summed over the ionic species [10].

The formulae presented in this section are general with respect to profiles of background density, temperature and safety factor. I have retained much of this generality by keeping free parameters in the following model profiles:

$$\begin{aligned} n_{e,i}(r) &= n_{e,i,0} \left[1 - \left(\frac{r}{a+d} \right)^2 \right]^{2\alpha_n}, \\ T_{e,i}(r) &= T_{e,i,0} \left[1 - \left(\frac{r}{a} \right)^2 \right]^{2\alpha_T}, \\ q(r) &= q_0 \left[1 + a_q \left(\frac{r}{a} \right)^2 + b_q \left(\frac{r}{a} \right)^4 + c_q \left(\frac{r}{a} \right)^6 \right]^{n_q} \end{aligned} \quad (21)$$

The beam source radial profile for the "g"th beam is Gaussian in radius, and is given by:

$$S_g(r) = S_g(0) \exp \left[- \left(\frac{r}{a} \right)^2 \right] \quad (22)$$

Using a simplified localized form for the growth rate Eq. 12, qualitative features of the instability drive can be obtained. This expression is useful for understanding some of the parametrics, however for quantitative results, the complete expressions averaged over the eigenmode and background plasma profiles are used.

$$\frac{\gamma_s}{\omega_r} = - \frac{\beta_s}{4k_{\parallel}^2 P_0^2} \left(\sum_{j=\pm 1} P_{s,m+j} - \sum_{j=\pm 1} R_{s,m+j} \frac{\omega_{s,s,m}}{\omega_r} \right) \quad (23)$$

the integrals P and R are related to Q function:

$$\begin{aligned} P_{s,m\pm 1} &= \frac{2\pi^2 \omega_r}{|k_{\parallel,m\pm 1}| v_s^4} \int_{0, v_{\parallel} = \frac{v_r}{k_{\parallel,m\pm 1}}}^{\infty} dv_{\perp} v_{\perp} \left[\frac{1}{2} v_{\perp}^2 + \left(\frac{\omega_r}{k_{\parallel,m\pm 1}} \right)^2 \right]^2 \hat{T}_s \hat{f}_{s,0} \\ R_{s,m\pm 1} &= \frac{2\pi^2 \omega_r}{|k_{\parallel,m\pm 1}| v_s^4} \int_{0, v_{\parallel} = \frac{v_r}{k_{\parallel,m\pm 1}}}^{\infty} dv_{\perp} v_{\perp} \left[\frac{1}{2} v_{\perp}^2 + \left(\frac{\omega_r}{k_{\parallel,m\pm 1}} \right)^2 \right]^2 \hat{f}_{s,0}. \end{aligned} \quad (24)$$

The distribution $\hat{f}_{s,0}$ is normalized to unity, and the species "s" can either be hot particles or electrons. The term containing P corresponds to the instability drive originating from velocity space gradients. For the alpha particle problem, this term is always stabilizing, because the derivatives at the resonant $v_{\parallel}$ are always negative. If the Maxwellian were used, $\hat{T}_s \approx 1$, making $P = R$ and always positive. For isotropic

slowing down distributions, typically $0 < P < R$, making instability possible at lower ratios of $\omega_{s,hot,m}/\omega_r$. For the anisotropic beam created distribution, P can actually be negative, and therefore destabilizing. The term containing R is the contribution to the drive from the hot particle density gradients. From Eq. 18, instability will be enhanced by any process which tends to increase the average energy of the distribution. From Eq. 23, the growth rate will tend to be larger for the modes with smaller $k_{\parallel}$, and larger β_{hot} . For the electron magnetic drift damping, the term which contains P is the only one that contributes, since $\omega_{s,e,m}/\omega_r$ is small and is of opposite sign for electrons.

The functions $P_{s,m\pm 1}$ and $R_{s,m\pm 1}$ vanish if $v_{\parallel,max} < \frac{\omega_r}{k_{\parallel,m\pm 1}} \equiv v_{res}$, where $v_{\parallel,max}$ is the parallel speed above which there are no particles. For a Maxwellian distribution of particles, this is not an issue, because there will always be resonant particles, provided $v_{res} \ll c$. For beam injected particles, however, this condition sets a minimum parallel injection speed below which the mode will not be driven. This $v_{\parallel,inj,min}$ is given by:

$$v_{\parallel,inj,min} = \frac{\omega_r}{k_{\parallel,m\pm 1}} \approx \frac{k_{\parallel,m} V_A}{k_{\parallel,m\pm 1}} = \frac{lq - m}{lq - (m \pm 1)} \times V_A.$$

For kinetic-type modes, large m and l , the familiar condition that the beams be super-Alfvénic emerges. However for the global modes, especially the $(0, -1)$ mode, instability can persist even when $v_{\parallel,inj,max} \approx \frac{1}{2} V_A$.

5. Results

The equations presented in the previous section have been incorporated into a code running under the BASIS environment on the CRAYs. The code has been benchmarked for a Maxwellian distribution against Reference [1], and the damping rates and eigenfrequencies agree well for TFTR and TEXT parameters. I have also reproduced the trends in the growth rate as the hot particle density scale length and hot particle energy are varied. My results, while having the same shape as Reference [1], differ in magnitude by up to a factor of 2, my results are consistently larger. This factor can vary from 2, for the $(0, -1)$ mode to less than 20% for shorter wavelengths. I have been in contact with Li⁵ and we are working on resolving the discrepancy.

I now apply the equations presented in the previous section to study the stability of the beam-driven global modes for D III-D parameters. After looking at the instability drive for the baseline experimental conditions for both hydrogen and deuterium beams, described in Table 1, I examine how the drive terms for each of the modes change in different parameter ranges and with different assumptions. These parametrics may be able to provide guidance in defining new experiments which probe deeper into the unstable regime, therefore testing the predictive capability of the theory for larger machines such as ITER.

⁵Y.M. Li, GTE Syllvania, private communication, August 1989.

5.1. Baseline Results

5.1.1. Hydrogen Beams

Table 2 shows the results of the analysis of the baseline case with 10MW of hydrogen beams and parabolic density profiles ($\propto (1 - (r/\tilde{a})^2)$) for several different poloidal and toroidal mode numbers. These mode numbers are typically the most unstable for the global modes. Notice that the modes are all stable for this density profile. Table 3 shows the same information, using square root of parabolic density profiles ($\propto (1 - (r/\tilde{a})^2)^{\frac{1}{2}}$). For the flatter profile, the (0, -1) mode is unstable, since the hot particle growth rate γ_{hot} exceeds the sum of the damping due to electron Landau, $\gamma_{e,L}$, and electron "magnetic drift", $\gamma_{e,M}$, damping. The magnetic drift damping is the electron analog of the same drive as the hot particles provide to the wave. The reason the flatter profile is more unstable is: 1) the average density the mode sees is higher and hence the Alfvén speed lower, and 2) the $k_{||}$ of the mode is smaller for the flatter density; the smaller the $k_{||}$ value, the more unstable the mode.

Even for the flatter profile and hydrogen beams, the degree of instability expected for D III-D is not substantial, using the hydrogen beams. The theory on which these values are based probably has a larger uncertainty than the difference between the growth and damping rates suggesting that the real system could be stable. In order to test the predictive capability of the theory, it is desirable to define experimental conditions which yield waves which have higher growth rates relative to damping.

5.1.2. Deuterium Beams

One way to increase the drive is to use deuterium beams. The original motivation for the use of hydrogen beams was to achieve the highest beam speed relative to the Alfvén speed for a given energy. The speed ordering is important for the velocity space driven modes, where in general $v_{||} > V_A$ for the instability to exist. This requirement is no longer necessary for the global modes which derive their free energy source from hot particle density gradients instead of velocity space gradients. This was demonstrated by the cutoff behavior of the P and R functions above.

There are several advantages of using deuterium. First, up to 20MW of neutral beams has been injected into D III-D with deuterium, whereas only 10MW is typically injected with hydrogen. Another advantage of using deuterium is that the slowing down time is longer ($\propto m_{hot} T_e^{\frac{3}{2}}$), and therefore for the same source strength the hot density and energy will be higher, thereby increasing the drive for the instability. With the background plasma also of deuterium, the issue of contamination of the background with the introduction of hydrogen is irrelevant.

I have attempted to model a very recent shot[3] in which 20MW of deuterium beams were injected into a plasma at low toroidal field and high density. Table 4 shows the parameters for this shot which differ from those tabulated in Table 1.

Table 5 shows the results of the analysis of the baseline case with deuterium beams and parabolic density profiles ($\propto (1 - (r/\tilde{a})^2)$) for several different poloidal and toroidal mode numbers. As was the case with the hydrogen beams, the modes are all stable for

this density profile. Table 6 shows the same information, using square root of parabolic density profiles ($\propto (1 - (r/a)^2)^{1/2}$). For the flatter profile and deuterium beams, the $(0, -1)$ mode is unstable. The plasma heated by deuterium beams is substantially more unstable than a plasma heated by hydrogen beams which showed only weak instability. This is due to the fact that the combination of heavier beam particles and higher electron temperature due to the high input power result in a longer slowing down time. Because the hot particles remain hot longer, the distribution has both a higher energy and density, each contributing to an instability.

The result of the stability analysis suggests that fluctuation activity in the range of 48-160kHz should have been observed in the experiment with deuterium beams. Unfortunately, diagnostic data of fluctuations in this frequency range are unavailable for this shot. Perhaps these kinds of discharges could be reproduced in subsequent experiments, and diagnosed in this frequency range.

I now will present some parametrics in which beam and background parameters are varied to determine those which effect the stability picture to the greatest degree. Some of these parametrics will be performed about the 10MW hydrogen beam baseline case, others around the recent 20MW deuterium beam case.

5.2. Parametrics

5.2.1. Hot Particle Source Strength Scalelength Scaling

The radial scalelength, L_r , for the beam source in the model calculations which I perform is obtained from the ACCOME[11] code, rather than from direct experimental observation. Processes which are present in the real plasma could produce a profile either wider or narrower than the computed profile. It would therefore be desirable to examine the stability of the plasma to the global modes as L_r is changed. The cases I will examine here survey a range of L_r between 8 and 70 cm, for the $(0, -1)$, $(0, -2)$, $(0, -3)$, $(1, -1)$, $(1, -2)$, and $(1, -3)$ modes. The results of this study for the high power deuterium beam case are shown in Figure 11, where I have plotted the growth rates normalized to the real part of the mode frequency as a function of L_r [12].

This Figure is subdivided into two figures, with four sets of (l, m) shown in part a and two sets of (l, m) in part b. The reason for this subdivision is that there is a dramatic difference in the growth rates between the two figures. I suspect that the perturbation theory may break down in b. Notice first that the $(1, -3)$ and the $(1, -2)$ modes are stable throughout the range in L_r . Also note that except for a narrow band in L_r between 15 cm and 30 cm the $(0, -3)$ mode is also stable. The modes which seem to potentially have the highest growth rates are the $(0, -1)$, $(1, -1)$, and $(0, -2)$ modes. If the beam source scalelength was 20 cm rather than the baseline 36 cm, the $(0, -2)$ mode should be very unstable, if we can believe the perturbation theory in this case. This mode was also judged the most dangerous one by Reference[1] for the alpha particle problem, although at a much lower growth rate. If the source profile proved to be broader than that predicted by ACCOME, the $(0, -1)$ mode would persist and would be the mode of most concern.

Figure 11: Dependence of normalized growth rate on the hot particle source strength scalelength, L_s

5.2.2. Background Density Profile Scaling

The background density profile assumed makes a considerable difference in the stability results. This was illustrated in Section 5.1, when a flattening of the density profile from parabolic to square root of parabolic makes the difference between stability and instability. To illustrate this point further, I have plotted in Figure 12 the normalized growth rate γ_{hot}/ω_r as a function of the density profile exponent, α_n , for the $(0, -1)$ mode with for the hydrogen beam case. Also shown is the normalized damping rate γ_e/ω_r for comparison. The central density and the extrapolated boundary radius, $\tilde{a} = a + d$ remain fixed during this variation. This causes the edge density to rise as α_n is lowered. The growth rate is a strong function of α_n , whereas the damping rate is virtually independent of α_n . For the $(0, -1)$ mode, if the density profile could be flattened to $\alpha_n=0.3$, the growth rate of the mode could be increased a factor of 1.7 over the $\alpha_n=0.5$ case. I limited the flatness of the density to an α_n of 0.3, because for profiles much flatter, the minimum in ω_A disappears. I have also examined a similar parametric for the high power deuterium beam case, shown in Figure 13. In this parametric, the average density and edge density were held fixed. As can be seen, the trend is the same as the case where the central density is held fixed, although the increase in the growth rate is not as dramatic as the profile flattens. This parametric illustrates that one possible way to suppress the instability, is to create a peaked density profile. This information might be useful when establishing the operating point and designing the plasma fueling scheme for an advanced machine such as ITER.

5.2.3. Toroidal Magnetic Field Scaling

The effect of changing the toroidal magnetic field, B_0 , on the stability of the $(0, -1)$ for the deuterium beam case is shown on Figure 14. Lowering the magnetic field tends to increase the instability, since at lower field the field lines are more easily bendable, and therefore more susceptible to mode growth. The most important modes seem to be the $(0, -2)$, $(0, -1)$, $(0, -3)$ modes. The $(0, -2)$ mode results could in fact be invalid because the perturbation theory may have broken down at the lowest fields. The simplified growth rate expression tells the same story about stability, in that $\omega_{hot} \propto \frac{1}{B_0}$ as well as $\beta_{hot} \propto \frac{1}{B_0^2}$ both increase as B_0 decreases. Also illustrated from Fig. 14 is the cutoff behavior of the drive when V_A increases above the maximum $v_{||}$ in the distribution. The cutoff behavior is gradual, since the radial averaging takes into account that the maximum $v_{||}$ varies across the plasma.

5.2.4. Electron Temperature Scaling

The background electron temperature is an important parameter in determining the stability of the global modes. With some supplementary electron heating, perhaps the electron temperature could be raised in D III-D to test this hypothesis. Figure 15 shows the normalized growth rate, γ_{hot}/ω_r as a function of the electron temperature, in the range of 500 eV to 5 keV. For the mode which is unstable at the baseline source strength profile, $(0, -1)$, increasing the electron temperature tends to linearly increase the growth rate. For the other modes which are stable, increasing the electron

Figure 12: Dependence of normalized growth rate on the density profile exponent, α_n for hydrogen beams. For this parametric, the central density and $\tilde{a}$ were held fixed.

Figure 13: Dependence of normalized growth rate on the density profile exponent, α_n , for the deuterium beams. For this parametric, the average and edge densities were held fixed.

Figure 14: Dependence of normalized growth rate on the toroidal magnetic field, B_0 .

Figure 15: Dependence of normalized growth rate on the central electron temperature, $T_{0,e}$.

temperature does not create any further instability. The electron damping increases as $\sqrt{T_e}$, but the damping is so small that it does not reduce the instability to any great degree.

The physics behind the electron temperature scaling is quite simple, the purpose behind the formal stability analysis is to find out how much leverage is available by varying T_e . The mode is driven unstable when $\omega_{*,hot}/\omega_r$ is positive, and greater than⁶ $\langle \hat{T} \rangle$. Since $\omega_{*,hot} \propto E_{hot}$, any process which tends to increase the energy of the distribution will increase the drive. One way to do this for plasmas which the beam particle slowing down is on electrons is to increase the electron temperature. The higher T_e increases the slowing down time and hence the average energy and the hot particle density, for the same injection energy.

5.2.5. Beam Energy Scaling

The plasma can be made more unstable by raising the average energy of the distribution by increasing the injection energy. Figure 16 shows the dependence of the normalized growth rates as the full beam energy is increased from 40keV to 120 keV. The half and third energies are scaled to follow the full energy value. The $(0, -1)$ mode is unstable throughout the entire range in energy. For an energy just over 100 keV, the $(0, -2)$ mode goes unstable, actually becoming more unstable than the $(0, -1)$ mode at 120keV. The other modes shown remain stable over this particular energy range.

6. Conclusions and Additional Discussion

In this paper, I have examined the excitation of the global Alfvén eigenmodes by intense neutral beam injection in D III-D, within the framework of the global Alfvén eigenmode theory[1].

Stability is marginal in a series of shots[2] with 10MW of hydrogen beams, whereas in a very recent shot[3] with 20MW of deuterium beams a stronger instability should be present in the frequency range 49kHz to 160kHz, if the density profile exponent, $\alpha_n \leq 0.5$. If the source width, $L_s \geq 36$ cm, the $l = 0, m = -1$ mode is most important. However, if the source is narrower than 36 cm, the $l = 0, m = -2$ mode should also be present with a large growth rate. If the source distribution could be made very narrow, $L_s \leq 10$ cm, the $(1, -1)$ mode could appear.

By flattening the background density profile, higher growth rates of the $m = -1, l = 0$ mode can be obtained. By reducing the scalelength of the hot particle source from 36 cm to 20 cm, the $m = -2, l = 0$ mode can also be excited. The mode has a high growth rate for high power injection of deuterium beams. This parametric illustrates that to suppress the instability, the background density profile should become more peaked. This information might be useful when establishing the operating point and designing the plasma fueling scheme for an advanced machine such as ITER.

By increasing the electron temperature from the baseline 2.0 keV to 3.0 keV, the growth rate increases by a factor of 1.3 for the $m = -1, l = 0$ mode. By increasing the

⁶The brackets around $\hat{T}$ is a shorthand notation for the P terms in Eq. 23.

Figure 16: Dependence of normalized growth rate on neutral beam full energy, $E_{\text{inj},f}$.

injection energy of the beams from 75keV to 120keV, the growth rate of the $(0, -1)$ mode can be increased by a factor of 1.4. Above energies of about 100keV, the $(0, -2)$ mode becomes unstable, and in fact at 120keV is more unstable (higher growth rate) than the $(0, -1)$ mode by a factor of 2.

Based on the results of these parametric analyses, future experiments on D III-D might be designed which would place it deeper in the unstable regime of the global Alfvén wave, thereby testing the predictive capability of the theory for larger machines such as ITER. The two most important parameters which might be varied in the experiment are the density profile and the electron temperature.

I believe that a reasonable benchmark of the theory could be obtained by performing further experiments with high power deuterium beams and determine the fluctuation spectrum in the frequency range of 48kHz to 160kHz.

The impact of these modes on the plasma equilibrium and energy transport must be treated apart from the linear stability theory presented here. Specifically, an understanding of the frequency spectrum of these modes evolved non-linearly must be gained. If the spectrum is peaked around the mode frequency, there may be minimal impact on the plasma transport. If, on the other hand, the spectrum is broad-band and the individual waves' phases are uncorrelated, diffusion in velocity and configuration space may result. The experimentally determined spectral data collected in a new set of shots could answer some of these questions.

Parameter	Symbol (Units)	Value
Major Radius	R_0 (cm)	167.0
Minor Radius	a (cm)	67.0
Plasma Elongation	κ_{95}	2.8
Plasma Triangularity	δ_{95}	0.4
Toroidal Magnetic Field	B_0 (kG)	9.0
Plasma Current	I_p (kA)	600.0
Temperature Profile Exponent	α_T	1.0
Plasma Effective Charge	Z_{eff}	3.0
Central Electron Density	n_e (10^{13} cm $^{-3}$)	4.5
Central Temperatures, Electron (Ion)	$T_e(T_i)$ (keV)	1.53 (1.53)
Safety Factor at 95% flux surface	q_{95}	2.7
Safety Factor at plasma center	q_0	0.9
Beam Source Gaussian Width	L_s (cm)	36.0
Power Fractions (F/H/T)	$f_{p,f}/f_{p,h}/f_{p,t}$	0.53 / 0.3 / 0.17
Beam Energy (F/H/T)	E_{inj} (keV)	75.0 / 37.5 / 25.0
Beam Footprint	ΔR (cm)	20
Tangency Radius of Beam (T/P)	R_{tan} (cm)	114 / 76

Table 1: The baseline parameters for the D III-D case appropriate for Alfvén wave studies. Densities, temperatures, and magnetic fields are for the 10MW hydrogen beam case. See Table 4 for the 20MW deuterium beam case.

Parameter / (l, m)	(0,-1)	(0,-2)	(0,-3)	(1,-1)	(1,-2)	(1,-3)
r_0/a	0.76	0.76	0.76	0.49	0.60	0.65
$V_A(r_0)$ (10^8 cm/sec)	3.0	3.0	3.0	2.3	2.5	2.6
ω_r (10^8 sec $^{-1}$)	0.7	1.8	2.7	2.2	3.4	4.4
$k_{\parallel}(r_0)$ (10^{-3} cm $^{-1}$)	3.0	6.1	9.2	10.4	13.7	16.9
$-k_{\perp}(r_0)$ (10^{-2} cm $^{-1}$)	2.0	4.0	5.8	3.0	4.9	6.9
γ_{hot} (10^3 sec $^{-1}$)	-6.1	-9.6	-4.0	-13.0	-11.2	-11.0
$\gamma_{e,M}$ (10^3 sec $^{-1}$)	-0.40	-3.5	-1.9	-1.6	-1.0	-0.8
$\gamma_{e,L}$ (10^3 sec $^{-1}$)	-0.82	-0.77	-9.7	-2.0	-0.97	-2.3

Table 2: Mode Properties for D III-D, Hydrogen Beams, Baseline Parameters, for Parabolic Profiles.

Parameter / (l,m)	(0,-1)	(0,-2)	(0,-3)	(1,-1)	(1,-2)	(1,-3)
r_0/a	0.90	0.90	0.90	0.68	0.76	0.80
$V_A(r_0)$ (10^8 cm/sec)	2.83	2.83	2.83	2.37	2.47	2.56
ω_p (10^6 sec $^{-1}$)	0.41	1.0	1.9	1.8	3.0	3.7
$k_{ }(r_0)$ (10^{-3} cm $^{-1}$)	2.5	5.1	7.7	9.4	12.1	14.7
$-k_{\perp}(r_0)$ (10^{-2} cm $^{-1}$)	1.7	3.3	5.0	2.2	3.9	5.6
γ_{hot} (10^3 sec $^{-1}$)	0.55	-25.4	-7.6	-58.7	-5.3	-3.7
$\gamma_{e,M}$ (10^3 sec $^{-1}$)	-0.33	-5.3	-2.2	-1.6	-0.96	-0.73
$\gamma_{e,L}$ (10^3 sec $^{-1}$)	-0.119	-0.04	-0.18	-7.2	-0.55	-1.0

Table 3: Mode Properties for D III-D, Hydrogen Beams, Baseline Parameters, for Square Root of Parabolic Profiles.

Parameter	Symbol (Units)	Value
Toroidal Magnetic Field	B_0 (kG)	7.8
Plasma Current	I_p (kA)	1500.0
Central Electron Density	n_e (10^{13} cm $^{-3}$)	6.0
Central Temperatures, Electron (Ion)	$T_e(T_i)$ (keV)	2.0 (2.0)
Beam Injected Power	P_{nb} (MW)	20.0

Table 4: Some of the baseline parameters for the D III-D case for deuterium beams appropriate for Alfvén wave studies which differ from that of Table 1.

Parameter / (l,m)	(0,-1)	(0,-2)	(0,-3)	(1,-1)	(1,-2)	(1,-3)
r_0/a	0.76	0.76	0.76	0.49	0.60	0.65
$V_A(r_0)$ (10^8 cm/sec)	2.24	2.24	2.24	1.75	1.9	1.95
ω_p (10^6 sec $^{-1}$)	0.517	1.37	2.1	1.65	2.6	3.3
$k_{ }(r_0)$ (10^{-3} cm $^{-1}$)	3.1	6.1	9.2	10.4	13.7	16.9
$-k_{\perp}(r_0)$ (10^{-2} cm $^{-1}$)	2.0	4.0	5.8	3.0	4.9	6.9
γ_{hot} (10^3 sec $^{-1}$)	-6.6	-25.3	-13.1	-4.8	-33.3	-34.0
$\gamma_{e,M}$ (10^3 sec $^{-1}$)	-0.46	-3.85	-2.0	-1.85	-1.13	-0.9
$\gamma_{e,L}$ (10^3 sec $^{-1}$)	-0.69	-0.60	-5.2	-1.8	-0.88	-2.1

Table 5: Mode Properties for D III-D, Deuterium Beams, Baseline Parameters, for Parabolic Profiles.

Parameter / (l, m)	(0,-1)	(0,-2)	(0,-3)	(1,-1)	(1,-2)	(1,-3)
r_0/a	0.90	0.90	0.90	0.68	0.76	0.80
$V_A(r_0)$ (10^8 cm/sec)	2.1	2.1	2.1	1.8	1.9	1.9
ω_r (10^6 sec $^{-1}$)	0.31	0.78	1.4	1.4	2.2	2.8
$k_{ }(r_0)$ (10^{-3} cm $^{-1}$)	2.5	5.1	7.7	9.4	12.1	14.7
$-k_{\perp}(r_0)$ (10^{-2} cm $^{-1}$)	1.7	3.3	5.0	2.2	3.9	5.6
γ_{hot} (10^3 sec $^{-1}$)	8.8	-0.47	-19.9	-163.0	-16.8	-12.4
$\gamma_{e,M}$ (10^3 sec $^{-1}$)	-0.37	-4.9	-2.2	-1.8	-1.1	-0.81
$\gamma_{e,L}$ (10^3 sec $^{-1}$)	-0.10	-0.04	-0.15	-5.7	-0.44	-0.8

Table 6: Mode Properties for D III-D, Deuterium Beams, Baseline Parameters, for Square Root of Parabolic Profiles.

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- [12] This is in contrast to the work of Li, et. al. in which the scalelength of the density, was used as a parametric variable. In that work, the density was assumed a Gaussian. Since the hot density from the classical theory is proportional to $S(r) \times \tau_D(r)$, for a Gaussian source the density scalelength will not become large when L_s becomes large. This is because the shorter scalelength parameters comprising τ_D (i. e. n_e and T_e) will dominate the scalelength. This explains some of the qualitative differences between the scaling in this paper and the Li paper when L_s becomes comparable to a .