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MASTER

NONLINEAR DYNAMICS OF DOMAIN WALLS AND VORTICES IN TWO-DIMENSIONAL SYSTEMS

A. R. Bishop

We discuss analytic and numerical studies of experimentally accessible 2-dimensional condensed matter systems which support nonlinear domain wall and vortex structures. These systems include quasi-2-dimensional magnets and 2-dimensional arrays of Josephson junctions. We emphasize the importance of these systems for investigating: complex (frustrated) ground states, dynamics (including "glassy" dynamics), interactions between domain walls and vortices, and phase transitions at finite temperature accompanied by these excitations.

1. INTRODUCTION

Familiarity with concepts in strongly nonlinear systems has now matured to the point where *dynamics* in nontrivial condensed matter problems in *two* space dimensions can be addressed systematically. This opportunity and need is enhanced by the convergence of several additional circumstances.

First, there is an increasing emergence of well-characterized quasi-2-dimensional condensed matter materials. These include: magnetic materials, prompted by technical advances in fabricating artificially-structured, layered, magnetic lipid, and epitaxial surface layer materials; and arrays of Josephson junctions, stimulated by advances in lithographic techniques. *Second*, there is increasing attention to experimental techniques which can measure dynamic properties: in the case of the magnets, accurate inelastic neutron scattering data is becoming available at long wavelengths

and low frequencies [1]; for Josephson junction arrays, both complex impedance and large scale flow ("I-V" curve) measurements are made and may be relevant to "glassy" responses in the new high-temperature superconductors [2]. *Third*, large-scale computing can now handle sufficiently large systems to test phenomenological theories based on nonlinear excitations and their interactions. We limit ourselves here to classical simulation, although quantum simulations, even for dynamics, are now also practicable.

Our concern is to bring together these fortunate circumstances and begin to study specifically *domain walls* and *vortices*, their dynamic signatures and consequences. We will first summarize the theoretical projects now underway and then describe two of them in a little more detail.

The major computer simulation techniques we have used are: (i) MC-MD schemes, in which MC is used to age an initial configuration to equilibrium and then MD is used to follow subsequent dynamics. This is a microcanonical ensemble technique; and (ii) Langevin MD schemes, in which coupled Langevin equations are solved. This is a canonical ensemble technique. These approaches have been variously applied to the following 2-dimensional models:

(a) The anisotropic ferromagnetic Heisenberg model with exchange XY spin symmetry [3]. This is described by the Hamiltonian

$$H_0 = -J \sum_{(m,n)} [S_m^x S_n^x + S_m^y S_n^y + \lambda S_m^z S_n^z] \quad (J > 0) . \quad (1)$$

The limits $\lambda = 0$ and 1 are the planar XY and isotropic Heisenberg cases, respectively. For $\lambda = 1$ there are nonlinear *instanton* spin configurations. For $\lambda < 1$ there are two types (see below) of *vortex* spin configurations. The material K_2CuF_4 , for example, appears to correspond to $\lambda \simeq 0.99$ and evidence for an incipient Kosterlitz-Thouless phase (vortex-antivortex unbinding) (K-T) transition has been reported [1].

The classical spin dynamics for Hamiltonian (1) and corresponding spin Hamiltonians below are governed by the Landau-Lifshitz equations of motion:

$$\dot{\vec{S}}_i = [\vec{S}_i, H_0] , \quad (2)$$

where $\vec{S} = (S_x, S_y, S_z)$. In some case (Gilbert) damping terms have been added to study vortex relaxation and collisions [3].

(b) In many materials crystal field symmetry-breaking terms appear and modify Hamiltonian (1). For example, in graphite intercalated with staged magnetic layers

(e.g. GIC-CoCl₂), the neighboring graphite layers impart a 6-fold symmetry-breaking in the XY spin plane [1]. This leads to *two* distinct phase transitions as temperature is lowered [4] -- first a K-T topological transition and second a symmetry-breaking transition to domains ordered between the 6 degenerate ground state orientations. In such cases both vortex and domain wall nonlinear spin configurations are essential, and their dynamics and interactions are key concerns.

Another example of this type is the XY ferromagnet Rb₂CrCl₄, which has been extensively studied experimentally [1]. In this case there is essentially a 4-fold symmetry-breaking field arising from two sublattices A and B that add terms to the Hamiltonian $H_0(\lambda = 0)$ (eqn. (1)) of the form [5]

$$-G \sum_m [(S_{mA}^x)^2 + (S_{mB}^y)^2] + D \sum_m (S_m^z)^2 . \quad (3)$$

(c) *Antiferromagnetic* versions of (a) and (b) occur frequently in nature. These include the new layered superconductors, which are nearly isotropic Heisenberg antiferromagnets. Although these are $S = \frac{1}{2}$ and therefore strongly quantum, classical spin dynamics has been an important first step [6]. A second example for which relevant experimental data is available [7] is BaNi₂(PO₄)₂. This has XY symmetry plus weak 6-fold in plane symmetry-breaking terms. From a theoretical perspective, the antiferromagnets have some similarities but also some important differences as compared with ferromagnets. These will be discussed elsewhere [8].

(d) Magnetic fields applied in the XY plane amount to 1-fold symmetry-breaking, leading again to domain walls interacting with vortex spin textures. This case has been studied theoretically and by simulation [8], and recently experimentally -- via inelastic neutron scattering in GIC-CoCl₂ [9] and NMR in BaNi₂(PO₄)₂ [7].

(e) In the case of 2-dimensional arrays of Josephson junctions, various junction shapes have been used, with and without a *perpendicular* magnetic field. In addition, disordered and fractal (e.g. sierpinsky-gasket) lattices have been fabricated. Excellent pick-up coil techniques have been devised to measure real and imaginary components of the AC conductance [2].

The junction array has a total energy [2,10,11]

$$E_p = -E_J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j - A_{ij}) , \quad (4)$$

where $E_J = (\hbar/2e)\mathcal{L}$ is the Josephson coupling, θ_i is the order parameter phase in the i -th superconducting island, and A_{ij} lie along the links, with magnetic field $\vec{B}$

$= \text{curl } \vec{A}$. As discussed below, the case $B = 0$ corresponds to the planar XY spin model. With $B \neq 0$ the flux per plaquette is incomplete and leads to *frustration* of flux lattice ordering relative to the underlying symmetry of the junction lattice [2,10,11]. This competition results in a complex sequence of ground states as B is varied, characterized by ordered arrays of *domain*-like defects with respect to the $B = 0$ state. (Similar frustration can be produced in the real XY magnets, above, by introducing competing next neighbor spin interactions: appropriate layered magnets appear to exist [7]). If a current is applied to drive a voltage response, other types of domain walls can form within the inhomogeneous ground states (analogous to dislocations). Finite temperature creates vortex-antivortex excited states and eventually unbinding. Thus, yet again we are faced with a need to understand the dynamics and interaction of domain wall and vortex structures.

II. DYNAMICS OF UNBOUND VORTICES IN THE ANISOTROPIC HEISENBERG MODEL

Quasi-2D magnetic materials with easy-plane symmetry (e.g. Rb_2CrCl_4 or $\text{BaCo}_2(\text{AsO}_4)_2$) have been modeled [3-5] by a *phenomenological* theory for the dynamic correlations based upon a dilute gas of vortices produced at $T > T_c$, the K-T transition. For simplicity we summarize the situation for the Hamiltonian (1) with dynamics (2) [3].

MC simulations show a critical temperature $T_c(\lambda)$, where spin-spin correlations change from power law to exponential. Above T_c a fraction of the vortex-antivortex pairs unbind and the unbound vortices are in motion due to their interactions. Assuming that the positions are random locally, the velocity distribution is Gaussian [3]. These unbound vortices can be treated phenomenologically as an ideal gas, in the same spirit as the soliton-gas approach for 1-d magnets [12].

The correlations for the in-plane components S_x or S_y are quite distinct from those for the out-of-plane component S_z . We show below that the *velocity dependence* of the vortex structure is decisive for the out-of-plane correlations.

In-plane correlations. Here we use a continuum description and spherical coordinates for the spin configuration:

$$\vec{S}(\vec{r}, t) = S(\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta), \quad (5)$$

where $\vec{r} = (x, y)$. The equations of motion have *two* types of *static* vortex or antivortex solutions $\phi(\vec{r}) = \tan^{-1}(y/x)$. Molecular dynamics (MD) simulations have shown [3]

that for $0 \leq \lambda \lesssim 0.72$ only a planar solution $\theta(\vec{r}) \equiv \pi/2$ is stable, whereas for $\lambda \gtrsim 0.72$ only a solution which has an out-of-plane structure $\theta(\vec{r}) \neq \pi/2$ is stable. For brevity, only the former case is described here.

S_x and S_y are *not localized*, i.e. they have no spatial Fourier transforms. Thus the in-plane correlation function $S_{xx}(\vec{r}, t) = \langle S_x(\vec{r}, t) S_x(\vec{o}, 0) \rangle$ is only *globally* sensitive to the presence of vortices. Then the characteristic length is the average vortex-vortex separation 2ξ , where ξ is the K-T correlation length. When a planar vortex starts moving it develops an out-of-plane structure (see below). However, for $S_{xx}(\vec{r}, t)$ this is not important because the dominant effect of moving vortices is to act like "2-d sign functions" or "2-d kinks," i.e. every vortex that passes with its center between $\vec{o}$ and $\vec{r}$ in time t diminishes the correlations, changing $\cos \phi$ by a factor of (-1) , independent of the direction of movement and independent of the internal structure of the vortex.

The detailed calculation of $S_{xx}(\vec{r}, t)$ is published elsewhere [3] and gives a (squared) Lorentzian central peak for the dynamic form factor

$$S_{xx}(\vec{q}, \omega) = \frac{S^2}{2\pi^2} \frac{\gamma^2 \xi^2}{(\omega^2 + \gamma^2 [1 + (\xi q)^2])^2}, \quad (6)$$

with $\gamma = \sqrt{\pi} \bar{u} / (2\xi)$. Here $\bar{u}$ is the rms speed of the vortices which can be taken from Huber [13] who calculated the velocity auto-correlation function. The central peak (6) is in very good agreement with data obtained from combined MC-MD simulations [3] -- see Fig (1). Moreover there is a qualitative agreement with neutron scattering experiments [3].

Out-of-plane correlations. In this case $S_z(\vec{r}, t)$ is *localized* for a single vortex. Therefore correlations are sensitive to the vortex size and structure. We assume a dilute ($T \gtrsim T_c$) gas of N_v unbound vortices with positions $\vec{R}_i$ and velocities $\vec{u}_i$, and consider the incoherent superposition

$$S_z(\vec{r}, t) = S \sum_{i=1}^{N_v} \cos \theta(\vec{r} - \vec{R}_i - \vec{u}_i t). \quad (7)$$

The thermal average in $S_{zz}(\vec{r}, t) = \langle S_z(\vec{r}, t) S_z(\vec{o}, 0) \rangle$ is evaluated by integration over $\vec{R}$ and $\vec{u}$

$$S_{zz}(\vec{r}, t) = n_v S^2 \int \int d^2 R d^2 u P(\vec{u}) \cos \theta(\vec{r} - \vec{R} - \vec{u} t) \cos \theta(\vec{R}), \quad (8)$$

Fig. 1. Intensities (I_x, I_z) and widths (Γ_x, Γ_z) of central peaks at $T = 1.0J$ for the ferromagnetic XY model ($\lambda = 0$). Data points from MC-MD simulation and solid lines fits to phenomenological theory (see text and Ref. (3)).

where n_v is the vortex density and $P(\vec{u})$ is the velocity distribution. Introducing the vortex form factor $f(\vec{q})$ (the Fourier transform of $\cos \theta(\vec{r})$), we find

$$S_{zz}(\vec{q}, t) = \frac{S^2}{(2\pi)^2} n_v \int d^2u |f(\vec{q})|^2 P(\vec{u}) e^{-i\vec{q} \cdot \vec{u}t} . \quad (9)$$

Expression (9) can be evaluated easily if the static vortex solutions are inserted [3]. However, for $\lambda \lesssim 0.72$ only the planar solution is stable [3] and S_{zz} would then

vanish, in contradiction to the MC-MD simulation. Therefore the *velocity dependence* of $\theta(\vec{r})$ must be taken into account. For $\lambda \lesssim 0.72$ and small velocity u the equations of motion yield [3] the asymptotic solution (in the moving frame, with time unit $\hbar/JS$)

$$\cos \theta = \frac{-1}{4\delta} \frac{\vec{u} \cdot \vec{e}_\varphi}{r} \quad , \quad r \rightarrow \infty \quad (\delta = 1 - \lambda) \quad , \quad (10)$$

which is in agreement with MD-simulations. Here $\vec{e}_\varphi$ is the azimuthal unit vector in the XY-plane. The solution for $r \rightarrow 0$ can also be obtained but we are interested here only in the correlations for small q where the asymptotic solution should be a good approximation. This leads to a velocity depended form factor and eventually to

$$S_{zz}(\vec{q}, \omega) = \frac{n_v}{32\sqrt{\pi}\delta^2} \frac{\bar{u}}{q^3} \exp\left(-\left(\frac{\omega}{\bar{u}q}\right)^2\right) \quad . \quad (11)$$

The result (11) describes a Gaussian central peak reflecting the velocity distribution. The width $\Gamma_z = \bar{u}q$ has a linear q -dependence, which is, well supported by the MC-MD data -- see Fig. (1). The integrated intensity is

$$I_z(q) = \frac{n_v}{32\epsilon^2} \frac{\bar{u}^2}{q^2} \quad . \quad (12)$$

The divergence in (12) for $q \rightarrow 0$ results from the infinite range of the structure (10). However, the actual radius of a vortex must be on the order of ξ , which can be taken into account, e.g., by an ad-hoc cut-off function $\exp(-\epsilon r/\xi)$ with ϵ a parameter [3]. This gives an extra factor of κ^2 in (12), with $\kappa = 1 - 1/W$ and $W = [1 + (\xi q/\epsilon)^2]^{1/2}$. The final result for $I_z(q)$ is consistent with MC-MD data for small q -- see Fig. (1). Note that *absolute* intensities are compared here; we have chosen ϵ such that I_z is smaller than the data because other effects can also contribute to the central peak, e.g. 2-magnon difference processes and vortex-magnon interactions.

III. I-V CHARACTERISTICS IN 2-DIMENSIONAL FRUSTRATED JOSEPHSON-JUNCTION ARRAYS (JJA)

As we have suggested above, the *dynamics* of 2D JJAs is much less well studied than the statics but provides an excellent controlled laboratory example with which to probe pattern formation and complex ("glassy") dynamics in competing interaction systems [2]. The dynamics of a JJA with magnetic field $B = 0$ can be derived from the dynamical theory of the K-T transition. At finite temperatures the nonlinear

behavior $V \simeq i^{(1+a(T))}$ is expected and consistent with recent MD simulations [14]. However, in the case of frustrated arrays, the role of domain walls in the dynamics is not well understood.

Here we will briefly summarize our Langevin MD results for a frustrated JJA [11]. Square lattices up to 36×36 have been used. The frustration studied was $f = \frac{1}{3}$ with

$$f = \frac{1}{2\pi} \sum_{\text{plaquette}} A_{ij} = \Phi/\Phi_0, \quad (13)$$

where Φ is the flux of the external magnetic field through an elementary plaquette and $\Phi_0 = \hbar c/2e$ is the flux quantum. Thus $f = \frac{1}{3}$ corresponds to a perpendicular magnetic field with $\frac{1}{3}$ of quantum flux per plaquette. With this value, the ground state is highly degenerate and anisotropic, so we can expect a rich variety of domain wall excitations. At zero temperature and with a spatially uniform driving current only extended defects (i.e. domain walls) can be excited. Due to the pinning effect of the lattice discreteness the ground state flux lattice “melts” by the field nucleation of a one dimensional domain wall lattice. Depending on the strength of the driving current, very different patterns of the flux lattice have been found and correlated with interesting dynamical behaviors. In particular, AC voltages are driven by DC currents. This AC effect is understood by the formation of solitons-like defects in the flux lattice. A dynamical transition between metastable configurations is also observed, as well as a “chaotic” (“noisy”) regime near the flux flow threshold. Thus, the driven JJA is indeed seen to provide fascinating examples of space-time complexity and pattern formation.

At finite temperatures, local inhomogeneous defects are thermally excited (vortex-antivortex pairs) in addition to domain walls. Both of these excitations determine the nature of the I-V characteristics for low currents. We have observed that these local excitations provide the mechanism for the domain wall motion at low currents and decrease the critical current at finite temperatures. A scenario for the critical behavior is discussed below. To briefly summarize, we find evidence for: (a) discrete lattice pinning of domain walls in the flux lattice background potential; (b) “chaotic” dynamics associated with the motion of domain walls in a potential which varies according to individual local environments of lattice discreteness and domain walls; (c) transverse instabilities on moving domain walls nucleated thermally as vortex-antivortex pairs; (d) two characteristic temperatures corresponding to domain-wall melting and vortex-antivortex unbinding; and (e) critical driving currents corresponding to transitions between patterns in moving domain wall lattices. This rich variety of phenomena leads to distinct regions in the temperature-driving current parameter space dominated by either thermal- or field-nucleation.

Dynamics is introduced [11] using a resistance-capacitance shunted junction (RSJ) model. Various forms of capacitance matrix (C_{ij}) can be considered. Here we simplify to the diagonal case, $C_{ij} = C_0 \delta_{ij}$, which is appropriate if only the capacitance between superconducting island and background is relevant. In this case the normalized Langevin equations which we have solved numerically can be written as

$$\begin{aligned}\dot{\theta}_i &= \tilde{p}_i \\ \dot{\tilde{p}}_i &= J \sum_{\delta} \sin(\theta_{i+\delta} - \theta_i - A_{i,i+\delta}) - \tilde{\eta} \sum_{\delta} (\tilde{p}_i - \tilde{p}_{i+\delta}) + \tilde{\lambda}_i(t),\end{aligned}\quad (14)$$

with

$$\langle \tilde{\lambda}_i(t) \rangle = 0, \text{ and } \langle \tilde{\lambda}_i(t) \tilde{\lambda}_j(t') \rangle = 2\tilde{\eta} G_{ij}^{-1} \tilde{T} \delta(t - t').$$

Here $\tilde{p} = p/\hbar$, $J = E_J C_0/\alpha$, $\tilde{\eta} = \hbar/2e^2 R$, $\tilde{T} = TC_0/\alpha$, $\alpha = 4e^2/\hbar^2$, and $G_{ij} = C_1 C_{ij}^{-1}$ with C_1 the capacitance between neighboring islands. We are particularly interested in the overdamped regime ($\eta \gg 1$: $\eta = 5$ was used in the following). To simulate non-equilibrium dynamics, an external current I was injected uniformly along an edge of the array and removed at the opposite edge by adding to the Hamiltonian a term

$$H_1 = -J_1 \sum_j (\theta_{i=1,j} - \theta_{i=N,j}), \quad (15)$$

where $J_1 = (\hbar/2e)I$. Periodic and free boundary conditions were used in the y and x directions, respectively. Graphical displays of phase and "fractional charge" configurations (e.g. Fig. 3) allow the dynamics of individual vortices and domain walls to be followed. The fractional charge q is calculated [11] as the circulation of the gauge invariant phase around a plaquette:

$$q = \frac{1}{2\pi} \sum_{\text{plaquette}} (\theta_i - \theta_j - A_{ij}) = m - f. \quad (16)$$

Here $(\theta_i - \theta_j - A_{ij})$ is restricted to $(-\pi, \pi)$ and m takes values 1 or 0 with density 1/3 or 2/3, respectively. Using these charge variables, our system is equivalent to a charged Coulomb gas driven by a uniform electric field.

T = 0 Simulations. The first effect of a magnetic field is a critical current reduction. For $f = 0$ the array behaves like a single junction and obeys $V \simeq I^2$.

$I_c^2)^{1/2}$, with $L_c = 1$ in our normalized units. For $f = \frac{1}{3}$, however, $L_c \simeq 0.138$. Importantly, the depinning at $L_c(f)$ is fundamentally different for $f = 0$ and $f \neq 0$. If $f = 0$ the ground state is spatially constant and (in the absence of impurities, temperature, or an inhomogeneous driving current) the voltage response remains space-independent for $I > I_c(0)$. However, for $f \neq 0$, the ground state has spatial structure and depins at $L_c(f)$ via instabilities of finite wavevector modes which saturate as wall structures -- "dislocations" with respect to the complex ground state.

Fig. (2) shows examples of flux patterns in various flow regimes for which distinct space-time dynamical behaviors are observed [11], separated by sharp but hysteretic transition currents. We briefly summarize these behaviors:

For $I < I_c(\frac{1}{3})$ ($= 0.138$) no flux flow is possible. For $I_c < I \lesssim 0.2$ a broad spectrum of frequencies is observed in the power spectrum for the spatially-averaged voltage: For higher currents in this regime a broad spectrum appears with some prominent frequencies. As L_c is approached, the spectrum becomes noisier and more "chaotic." This behavior is understood in terms of domain wall textures as follows. In this regime each field-nucleated domain is nearly pinned by the periodic potential of the underlying ground state; this "Peierls-Nabarro" pinning is stronger than the wall-wall interaction and gives rise to an *irregular* lattice of walls. The complex spatial structure leads to the observed "noisy" dynamics because of the multiplicity of environments for the moving walls [11,15]. For $I \gtrsim 0.2$ the wall density is high enough that a uniform (superlattice) of walls forms. In this case an AC response occurs via the periodic injection of new domain wall-antiwall pairs through the open boundaries which annihilate inside the lattice. The density of these injected pairs again increases with increasing field until at a further critical field ($I \simeq 0.68$) melting to another pattern occurs. This final pattern (see Fig. (2)) is composed of a mixture of ground state flux configurations. At very large currents any of these structures are unimportant compared to the bulk flux flow, and ohmic linear response is approached.

Fig. 2. Flux lattice configurations at $T = 0$ for frustration $f = 1/3$. The ground state and examples of defects produced by current driving are shown: dots signify $m = 1$ (i.e. $q = 2/3$). See text and Ref. (11).

T > 0 Simulations. In view of our driving symmetry and boundary conditions, the above $T = 0$ results are effectively *one-dimensional*. The 2-D character, however, immediately becomes evident at finite temperature.

As could be expected, the $T = 0$ critical current is smeared due to *thermally* excited vortex-antivortex pairs and domain walls, and a voltage appears below I_c ($T = 0$). A finite density of walls is indeed observed at low currents and temperatures. Interestingly, at $I \approx I_c$ the curves cross—i.e. the voltage response *decreases* with increasing temperature. This finite T behavior is understood in a very fascinating way. Below I_c ($T = 0$) a rigid domain wall cannot readily overcome the pinning potential. However, thermal fluctuations cause the wall to lose its rigidity by nucleation of kink-antikink structures which propagate *transversely* to the domain wall motion [11,16]. The source of these kink-antikink pairs is thermally excited *vortex-antivortex* pairs. In this way the motion of domain walls occurs by propagation of transverse structures and is a result of wall-vortex interactions. The process is illustrated in Fig. (3).

Close to I_c domain walls can be induced both by thermal fluctuations and the current so that their density rapidly increases. The vortex-nucleated roughening of the walls can now result in non-negligible wall interactions and entanglement, *reducing* the voltage with increasing temperature (which increases the vortex density).

Finally, we note that *two* characteristic temperatures are found in equilibrium ($I = 0$). At $T \approx 0.2$ domain walls begin to appear. This critical temperature is in good

Fig. 3. Mechanism for domain wall motion at low driving currents and temperatures: vortex-antivortex nucleation of kink-antikink pair which separate transversely. See Ref. (11).

agreement with the result for the 2-D charged Coulomb gas [11]. At $T \simeq 0.28$ vortex-antivortex unbinding takes place. For $I \lesssim I_c$ ($f = \frac{1}{3}$) there is little T -dependence for $T > 0.2$, suggesting that internal wall structure plays little role at these high temperatures -- thermally-assisted rigid sliding of the flux texture dominates.

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REFERENCES

- [1] e.g. K. Hirakawa et al, J. Phys. Soc. Jpn. **51**, 2151 (1982); M. T. Hutchings et al, J. Phys. **C14**, 5327 (1981); L. P. Regnault and J. Rossat-Mignod, in "Magnetic Properties of Layered Transition Metal Compounds," eds. L. J. de Jongh and R. D. Willet (Reidel 1987); articles in "Magnetic Properties of Low-Dimensional Systems," eds. L. M. Falicov and J. L. Moran-Lopez (Springer 1986).
- [2] e.g., P. Martinoli et al, in "Nonlinearity in Condensed Matter," eds. A. Bishop, D. Campbell, P. Kumar and S. Trullinger (Springer 1988); and preprints.
- [3] C. Kawabata and A. R. Bishop, Solid State Comm. **60**, 169 (1986); F. G. Mertens et al, Phys. Rev. Lett. **59**, 117 (1987); F. G. Mertens et al, Phys. Rev. B **39**, 591 (1989); M. E. Gouvea et al, Phys. Rev. B **39**, 11840 (1989).
- [4] M. Elay and G. Dresselhaus, Phys. Rev. **30**, 7225 (1985); H. Zabel and S. M. Shapiro, Phys. Rev. B. **36**, 7292 (1987); C. Kawabata and A. R. Bishop, Z. Physik B **65**, 225 (1986).
- [5] M. E. Gouvea et al, J. Phys. C **1**, 4387 (1989).
- [6] S. Chakravarty, B. Halperin, D. Nelson, Phys. Rev. B **39**, 2344 (1989); S. Tye, B. Halperin, preprint; G. Wysin, A. R. Bishop, G. Reiter, in preparation.
- [7] L. P. Regnault et al, Physica **136B**, 329 (1986); J.-P. Boucher et al, in preparation (1988).
- [8] A. Völkel et al, in preparation.
- [9] D. G. Wiesler, H. Zabel and S. M. Shapiro, preprint; M. E. Gouvea et al, J. Phys. C (in press).
- [10] e.g., E. Granato and J. M. Kosterlitz, Phys. Rev. B **33**, 4767 (1986).
- [11] F. Falo, A. R. Bishop and P. S. Lomdahl, preprint (1989).
- [12] see, M. Steiner and A. R. Bishop in "Solitons," eds. S. Trullinger, V. Prokovsky and V. Zhakarov (North-Holland, 1986).

- [13] D. L. Huber, Phys. Lett. **76A**, 406 (1980); Phys. Rev. B **26**, 3758 (1982).
- [14] K. K. Mon and S. Teitel, Phys. Rev. Lett. **62**, 673 (1989).
- [15] S. Aubry and S. de Seze, Festkörperprobleme **xxv**, 85 (1985).
- [16] c.f. J. Pouget et al, Phys. Rev. B **39**, 9500 (1989); J. C. Ariyasu and A. R. Bishop, Phys. Rev. B **35**, 3207 (1987).