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FINITE ORDER POLYNOMIAL MOMENT SOLUTIONS OF THE
HOMOGENEOUS GRAD-SHAFRANOV EQUATION

By

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HOMOGENEOUS GRAD-SHAFRANOV EQUATION

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ABSTRACT

A method for generating the finite positive order polynomial moment solutions of the homogeneous Grad-Shafranov equation to arbitrary order and the explicit form of the first few moments are given. A criticism of the method is discussed, and several practical examples are given.

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The finite positive order polynomial moment solutions of the Grad-Shafranov equation, while not complete, form a sometimes convenient set of expansion functions for the expression of an applied poloidal magnetic field in a finite and singularity free region of a toroidally symmetric plasma confinement device. In contrast to other choices of expansion functions, they are easily visualized and physically interpreted, and may be rapidly calculated.

They are especially useful in machine design since the effect of each moment on the plasma shape is known [1,2], and one may optimize this shape without going into the details of the device coil structure, and in the interpretation of magnetic measurements, where the detailed locations and values of image currents may be unknown, yet easily expressed in terms of these moments. They are further useful in diagnostic measurements of the plasma current distribution [3,4], where they are used to define the moments of the toroidal current density.

A number of authors [1-7] have exhibited or derived and used partial sets of these moments. It is the purpose of the present note, since their use is so pervasive, to give a method for the derivation of these moments, both symmetric and antisymmetric about the midplane, to arbitrary order, together with a standard norm, and the algebraic form of the first few of these moments.

Here we use three coordinate systems, spherical (ρ, θ, ϕ) , cylindrical (R, ϕ, Z) and cartesian (X, Z) where $X = R - R_0$ and R_0 is a given arbitrary expansion point, typically the canonical major radius of a toroidal device (Fig. 1). The principle determining characteristic of these moments is that in the large aspect ratio limit $(X^2 + Z^2)/R_0^2 \ll 1$ they reduce to the form of simple cartesian moments $(X + iZ)^M/R_0^{(M-2)}$, $M > 0$, where M is the cartesian moment number, and the denominator is chosen to get convenient dimensions.

The poloidal magnetic field is defined as

$$\vec{B}_p = \frac{\vec{\nabla}\phi \times \vec{\nabla}\psi}{2\pi} \quad (1)$$

where ψ is the poloidal magnetic flux.

In cylindrical coordinates, assuming toroidal symmetry ($\partial/\partial\phi = 0$), $\vec{\nabla} \times \vec{B}_p = 0$ yields the familiar form of the homogeneous Grad - Shafranov equation

$$\Delta^* \psi = \left(R \frac{\partial}{\partial R} R \frac{1}{R} \frac{\partial}{\partial R} + \frac{\partial^2}{\partial z^2} \right) \psi = 0, \quad (2)$$

while in spherical coordinates, $\vec{\nabla} \times \vec{B}_p = 0$ yields (5) :

$$\rho^2 \frac{\partial^2 \psi}{\partial \rho^2} + \sin(\theta) \frac{\partial}{\partial \theta} \left(\frac{1}{\sin(\theta)} \frac{\partial \psi}{\partial \theta} \right) = 0, \quad (3)$$

whose solutions are

$$F_n = C_n \rho^n \sin(\theta) P_{n-1}^1(\cos(\theta)) \quad (4)$$

and

$$H_n = D_n \rho^{-n} \sin(\theta) P_n^1(\cos(\theta)) \quad (5)$$

where the P_n^1 are the associated Legendre functions, and C_n and D_n are arbitrary constants.

We do not include the Qn Legendre functions which diverge logarithmically for $\theta = 0$ and π , and we neglect for the present the solutions with negative powers of ρ . The incompleteness of the set of moments considered here is due to the neglect of the latter. The P_n are generated by [8]

$$P_\ell(\chi) = \frac{(-1)^\ell}{2^\ell \ell!} \frac{d^\ell}{d\chi^\ell} (1-\chi^2)^\ell, \quad (6)$$

$$P_\ell^m(\chi) = (-1)^m (1-\chi^2)^{m/2} \frac{d^m}{d\chi^m} P_\ell(\chi). \quad (7)$$

The F_n when expressed in cylindrical (R, Z) coordinates are finite positive integer order polynomials. We normalize the F_n such that the coefficient of the highest order term in R is equal to unity once the solution is expressed in cylindrical coordinates. That is

$$C_n = \frac{2^{n-1} \left(\frac{(n+1)/2}{(n-1)} \right)! \left(\frac{(n-3)/2}{(n+1)} \right)!}{(3n-1)/2} \quad \text{if } n \text{ is odd} \quad (8)$$

and,

$$C_n = \frac{2^{n-1} \left(\frac{n/2}{(3n-2)/2} \right)! \left(\frac{(n-2)/2}{n!} \right)!}{(-1)} \quad \text{if } n \text{ is even.} \quad (9)$$

The first few of these in cylindrical coordinates are:

$$\begin{aligned}
F_0 &= 1, \\
F_1 &= 0, \\
F_2 &= R^2, \\
F_3 &= R^2 Z, \\
F_4 &= R^2(R^2 - 4Z^2), \\
F_5 &= (R^2 Z(3R^2 - 4Z^2))/3.
\end{aligned} \tag{10}$$

In general series form the F_n are

$$\begin{aligned}
F_0 &= 1, \\
F_1 &= 0, \\
F_N(R, Z) &= \sum_{n=0}^{N-2} A_n^N R^{N-n} Z^n, \quad N \geq 2
\end{aligned} \tag{11}$$

where $A_0^N = 1$ and $A_1^N = 0$ if N is even, $A_0^N = 0$ and $A_1^N = 1$ if N is odd, and

$$A_n^N = - \frac{(N+2-n)(N-n)}{n(n-1)} A_{n-2}^N, \quad 2 \leq n \leq N-2. \tag{12}$$

The conjugate functions G_N which are needed [3,4] for calculation of moments of the toroidal current density and obey

$$\frac{\partial G_N}{\partial Z} = \frac{1}{R} \frac{\partial F_N}{\partial R}, \quad R \frac{\partial G_N}{\partial R} = - \frac{\partial F_N}{\partial Z} \tag{13}$$

are

$$G_0 = 0 ,$$

$$G_1 = 0 ,$$

$$G_N = \frac{Z}{R^2} \sum_{n=0}^{N-2} \frac{(N-n)}{(n+1)} A_n^N R^{N-n} Z^n - \left(\frac{R^{N-1}}{N-1} \right)_{N \text{ odd}} , \quad N \geq 2 \quad (14)$$

where the last term appears only if N is odd.

The F_n do not have the desired large aspect ratio expansion. The appropriate linear combination of the F_n which yields the cylindrical moments upon expansion is easily shown to be, allowing for an arbitrary expansion point $R = R_0$

$$\Psi_N = \left(\frac{2\pi}{M} \right) \cdot \frac{1}{2^M R_0^{N-2}} \sum_{\ell=0}^M (-1)^{M-\ell} R_0^{(N-2\ell)} \binom{M}{\ell} F_{2\ell}(R, Z) , \quad \text{if } N \text{ is even} , \quad (15)$$

$$\Psi_N = \left(\frac{2\pi}{M} \right) \cdot \frac{M}{2^{M-1} R_0^{N-2}} \sum_{\ell=1}^M (-1)^{M-\ell} R_0^{(N-2\ell-1)} \binom{M-1}{\ell-1} F_{2\ell+1}(R, Z) , \quad \text{if } N \text{ is odd} \quad (16)$$

where $\binom{M}{\ell}$ are the binomial coefficients and M is the cartesian moment number associated with N and is defined as

$$M = N/2, \quad \text{if } N \text{ is even,}$$

and

$$M = (N - 1)/2, \quad \text{if } N \text{ is odd.} \quad (17)$$

The large aspect ratio expansion of Eqs. (15) and (16) yields

$$\Psi_N + i\Psi_{N+1} = \left(\frac{2\pi}{M}\right) (R + iZ)^M / R_0^{M-2} \quad (N \text{ even}) . \quad (18)$$

The leading factor of $(2\pi/M)$ is justified as follows:

The first ($N=2$) of the nontrivial moments yields a uniform vertical field of strength B_V if we take

$$\Psi = \pi B_V (R^2 - R_0^2) . \quad (19)$$

We therefore define the ($n=2$) moment as

$$\Psi_2 = 2\pi \frac{(R^2 - R_0^2)}{2} . \quad (20)$$

The $(2\pi/M)$ factor, when multiplied by the large aspect ratio expansion of the ($N > 2$) moments, makes the norm of B_p equal to 1 at $(X^2 + Z^2)^{1/2} = R_0$.

Equations (15) and (16) generalize the low order moments given in Refs. [1-7], to all orders, and both even and odd symmetry about $Z = 0$.

The first few of the moments are given in the table at the end of this article. They were generated and checked using the REDUCE algebraic manipulations package on DEC10 and Cray computers. The Reduce input file, which allows one to generate the algebraic form of the moments to arbitrary order, and Fortran subroutines for evaluating these moments are obtainable from the authors.

As previously stated, the incompleteness of these polynomial moments does

not arise from our not including the Q_n functions but from the neglect of the negative power of ρ solutions. The symmetric Greens function for the Grad-Shafranov operator which obeys

$$\Delta_1^* G(R_1, Z_1, R_2, Z_2) = 2\pi R_1 \delta(R_1 - R_2) \delta(Z_1 - Z_2) \quad (21)$$

and in terms of elliptic integrals in cylindrical coordinates is

$$G(R_1, Z_1, R_2, Z_2) = - \frac{(R_1 R_2)^{1/2}}{k} \left[(2-k^2) K(k) - 2E(k) \right]$$

where

(22)

$$k^2 = 4R_1 R_2 / (R_1 + R_2)^2 + (Z_1 - Z_2)^2$$

can be shown to have the expansion in spherical coordinates

$$G(\rho_1, \theta_1, \rho_2, \theta_2) = -\pi \rho_1 \sin(\theta_1) \rho_2 \sin(\theta_2) \sum_{\ell=1}^{\infty} \frac{\rho_{<}^{\ell}}{\rho_{>}^{\ell+1}} \frac{P_{\ell}^1(\cos(\theta_1)) P_{\ell}^1(\cos(\theta_2))}{\ell(\ell+1)}$$

(23)

where $\rho_{>}$ is the larger of ρ_1 and ρ_2 and $\rho_{<}$ is the smaller of ρ_1 and ρ_2 .

This expansion does not involve the Q_n but does depend on both positive and negative powers of ρ . Clearly any distribution of toroidal current can be reduced to a moments-like expansion by integrating this expanded greens function times the form of the distribution and truncating the resultant

series. For most practical coil systems whose coil locations do not lie outside some region of interest bounded by spheres, the expansion derived directly in this manner is inconvenient.

One can show from the above Greens function expansion that neglecting the L th term in a moments expansion produces an error on the order of $(\rho</\rho>)^L$ times the field strength where $\rho<$ is the lesser of ρ coil and ρ , etc. Clearly the expansion will not in general converge rapidly unless $(\rho</\rho>) \ll 1$.

One further difficulty is that coils which lie on spherical radii whose values are less than the spherical radius of the region of interest may not be well represented by a moments expansion in only the positive powers of ρ , unless their combined effect on the region of interest approximates that of a long solenoid. The obvious way out of this latter difficulty is to include the negative power of ρ solutions in the moments expansion. In cylindrical coordinates they are of the form

$$H_n \propto F_{n+1}(R,Z)/(R^2 + Z^2)^{n+1/2} \quad (24)$$

where the $F_n(R,Z)$ are as previously defined and n is the power of $(1/\rho)$.

This generalized approach will be treated in a future paper.

In practice one can ignore both of the above limitations only on a case by case basis.

The application of the moments expansion technique is illustrated in what follows by some examples taken from PDX. In each case we numerically generated the exact poloidal flux in a finite subregion of the PDX device due to a given coil system and then did a least squares fit to this flux to obtain the coefficients (B_n) of a moments expansion of the form

$$\Psi(R,Z) = \sum_{n=0,2}^N B_n \psi_n(R,Z,R_0) \quad (25)$$

We arbitrarily truncated the series at $N = 11$ (decapole), and picked as our expansion point $R_0 = 1.4$ meters, the nominal center of the device.

As might have been expected, the vertical field (EF) coil system is well represented by this expansion to a root mean square error of less than one percent and is of a dominantly even dipole character.

Figure 2 shows the leakage field due to the ohmic heating solenoid. The subregion considered is the box in the center of the plot. The solid lines are contours at equal flux increments of the exact flux, while the dotted lines are the contours of the moments fit at the same values. The relatively large displacement of the moments expansion contours from their exact counterparts is due to the weak field gradient because the rms error is small ($5.0e-04$) and the field is well represented. The B_n for the ohmic leakage field are of approximately equivalent strength up to and including decapole because our chosen expansion point is far from the natural null ($R = 1.7$ meters) of this field.

Figure 3 shows an equivalent plot for the PDX divertor coil system (DF+CF), the rms error is large (30%), and the field is dominantly octopole but poorly represented because our subregion lies too near the coils which produce this field. The error could be reduced by shrinking the subregion, but the plasmas of interest would soon not fit into the subregion.

Figure 4 illustrates the moments expansion as applied to the eddy current fields arising in PDX from currents flowing in the vacuum cans which surround the divertor coils and the vacuum vessel early on ($t = 50$ ms) in a PDX shot. The rms error is small ($5.0e-03$), and the field is not a simple dipole due to

the vacuum cans. A generally useful way to express eddy current fields is to develop the transfer function (step function response) for each moment of the eddy field when excited by a given coil group by making appropriate measurements. The field which results from some arbitrary combination of coil currents is then easily calculated without resort to a detailed and often inaccurate model of the structures in which eddy currents flow.

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TABLE OF MOMENTS

Name	Symmetry @ $z = 0$	Cylindrical Moment Number
$\Psi_n =$	Factor to get $ \vec{B}_p = 1 @ R = R_0$	$\times$ Formula
Asymptotic Formula ($R = R_0 + X^2, X^2 + Z^2 \ll R_0^2$)		
Monopole	Even	0
Ψ_0	$= (2 * \pi) * R_0^2$ $= (2 * \pi) * R_0^2$	
Monopole	Odd	0
Ψ_1	$= 0$ $= 0$	
Dipole	Even	1
Ψ_2	$= (2 * \pi) * (R^2 - R_0^2) / 2$ $= (2 * \pi) * R_0 * X$	
Dipole	Odd	1
Ψ_3	$= (2 * \pi) * (R^2 * Z) / R_0$ $= (2 * \pi) * R_0 * Z$	
Quadrupole	Even	2
Ψ_4	$= (2 * \pi / 2) * (R^2 - 4 * R^2 * Z^2 - 2 * R^2 * R_0^2 + R_0^4) / (4 * R_0^2)$ $= (2 * \pi / 2) * (X^2 - Z^2)$	
Quadrupole	Odd	2
Ψ_5	$= (2 * \pi / 2) * (R^2 * Z * (3 * R^2 - 4 * Z^2 - 3 * R_0^2)) / (3 * R_0^3)$ $= (2 * \pi / 2) * 2 * X * Z$	

Hexapole

Even

3

$$\begin{aligned}\Psi_6 &= (2\pi/3) * (R^6 - 12R^4 * Z^2 - 3R^4 * R_0^2 + 8R^2 * Z^4 + 12R^2 * Z^2 * R_0^2 + 3R^2 * R_0^4 \\ &\quad - R_0^6)/(8R_0^4) \\ &= (2\pi/3) * (X^3 - 3 * X * Z^2)/R_0\end{aligned}$$

Hexapole

Odd

3

$$\begin{aligned}\Psi_7 &= (2\pi/3) * (R^2 * Z * (15R^4 - 60R^2 * Z^2 - 30R^2 * R_0^2 + 24Z^4 + 40Z^2 * R_0^2 + 15R_0^4)) / (20R_0^5) \\ &= (2\pi/3) * (3 * X^2 * Z - Z^3)/R_0\end{aligned}$$

Octupole

Even

4

$$\begin{aligned}\Psi_8 &= (2\pi/4) * (5R^8 - 120R^6 * Z^2 - 20R^6 * R_0^2 + 240R^4 * Z^4 + 240R^4 * Z^2 * R_0^2 \\ &\quad + 30R^4 * R_0^4 - 64R^2 * Z^6 - 160R^2 * Z^4 * R_0^2 - 120R^2 * Z^2 * R_0^4 - 20R^2 * R_0^6 \\ &\quad + 5R_0^8)/(80R_0^6) \\ &= (2\pi/4) * (X^4 - 6 * X^2 * Z^2 + Z^4)/R_0^2\end{aligned}$$

Octupole

Odd

4

$$\begin{aligned}\Psi_9 &= (2\pi/4) * (R^2 * Z * (35R^6 - 280R^4 * Z^2 - 105R^4 * R_0^2 + 336R^2 * Z^4 + 420R^2 * Z^2 * R_0^2 \\ &\quad + 105R^2 * R_0^4 - 64Z^6 - 168Z^4 * R_0^2 - 140Z^2 * R_0^4 - 35R_0^6)) / (70R_0^7) \\ &= (2\pi/4) * (4 * X^3 * Z - 4 * X * Z^3)/R_0^2\end{aligned}$$

Decapole

Even

5

$$\begin{aligned}\Psi_{10} &= (2\pi/5) * (7R^{10} - 280R^8 * Z^2 - 35R^8 * R_0^2 + 1120R^6 * Z^4 + 840R^6 * Z^2 * R_0^2 \\ &\quad + 70R^6 * R_0^4 - 896R^4 * Z^6 - 1680R^4 * Z^4 * R_0^2 - 840R^4 * Z^2 * R_0^4 \\ &\quad - 70R^4 * R_0^6 + 128R^2 * Z^8 + 448R^2 * Z^6 * R_0^2 + 560R^2 * Z^4 * R_0^4 \\ &\quad + 280R^2 * Z^2 * R_0^6 + 35R^2 * R_0^8 - 7R_0^{10}) / (224R_0^8) \\ &= (2\pi/5) * (X^5 - 10 * X^3 * Z^2 + 5 * X * Z^4)/R_0^3\end{aligned}$$

Decapole

Odd

5

$$\begin{aligned}
 \Psi_{11} &= (2\pi/5) * (R^2 * Z * (315 * R^8 - 4200 * R^6 * Z^2 - 1260 * R^6 * R_0^2 + 10080 * R^4 * Z^4 \\
 &\quad + 10080 * R^4 * Z^2 * R_0^2 + 1890 * R^4 * R_0^2 - 5760 * R^2 * Z^6 - 12096 * R^2 \\
 &\quad * Z^4 * R_0^2 - 7560 * R^2 * Z^2 * R_0^4 - 1260 * R^2 * R_0^6 + 640 * Z^8 \\
 &\quad + 2304 * Z^6 * R_0^2 + 3024 * Z^4 * R_0^4 + 1680 * Z^2 * R_0^6 + 315 * R_0^8)) / \\
 &\quad (1008 * R_0^9) \\
 &= (2\pi/5) * (5 * X^4 * Z - 10 * X^2 * Z^3 + Z^5) / R_0^3
 \end{aligned}$$

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FIGURE CAPTIONS

FIG. 1 Coordinate Systems.

FIG. 2 Ohmic Solenoid Leakage Field.

FIG. 3 Divertor Field.

FIG. 4 Eddy Current Field.

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Fig. 1

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Fig. 2

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Fig. 3

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Fig. 4

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