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EXPERIMENTAL AND ANALYTICAL REACTIVITY STUDIES
OF CLEAN CRITICAL STAINLESS STEEL CORES

A. H. Spano

June 16, 1961

**PHILLIPS
PETROLEUM
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ATOMIC ENERGY DIVISION

**NATIONAL REACTOR TESTING STATION
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OF CLEAN CRITICAL STAINLESS STEEL CORES

A. H. SPANO

PHILLIPS
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Atomic Energy Division

Contract AT(10-1)-205

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EXPERIMENTAL AND ANALYTICAL REACTIVITY STUDIES
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BY

A. H. SPANO

ABSTRACT

This report presents the results of critical water height measurements made on close-packed lattices of Spert III, highly-enriched, plate-type, stainless-steel-clad fuel elements. Experiments were conducted with cores containing no control rods and with cores containing a single, fully-inserted control rod. The "clean critical" data obtained in these experiments have been used to test the validity of various aspects of a four-group, diffusion theory analysis of the full scale Spert III reactor. The results of the analyses of the rod-free and single-rodded critical lattices show that for such stainless steel cores k_{eff} can be calculated to within 1% Δk and that the Spert III control rod worth is calculable to a few tenths % Δk .

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EXPERIMENTAL AND ANALYTICAL REACTIVITY STUDIES OF CLEAN CRITICAL STAINLESS STEEL CORES

I. Introduction

This report is concerned with the analysis of critical water height experiments on lattices comprised of the highly-enriched, plate-type, stainless-steel-clad fuel elements used in Spert III^(1,2,3). The object of these critical experiments was to obtain "clean" criticality data on geometrically-simple, close-packed arrays in order to test the validity of certain aspects of a previously performed four-group, neutron diffusion analysis of the full-scale Spert III reactor. In particular, an unambiguous experimental check was desired of the analytical method used for calculating control rod worth. To this end, critical water height and differential water worth measurements were made on (a) "rod-free" cores, i.e., close-packed, slab-shaped cores comprised of fuel elements only, with no control rods present, and (b) "rodded" cores, i.e., cores comprised of fuel elements plus a single, fully-inserted control rod. In all cases, reactivity control was exercised by variation of the water level in the reactor vessel. In this way, data were provided for checking the calculated eigenvalues of "clean" cores, both with and without the presence of a single control rod.

In the original four-group, multiregion analysis of the Spert III reactor^(4,5), flux-weighted, two-dimensional radial diffusion solutions were synthesized⁽⁶⁾ with a one-dimensional axial solution to give the eigenvalue and flux distributions over the core. For the Spert III two-dimensional IBM-704 PDQ⁽⁷⁾ calculations, Muft⁽⁸⁾ and Soffocate⁽⁹⁾ four-group reactor constants were developed for each composition of the mathematical model, which was specified in considerable detail to simulate the geometric complexity of the actual core section. The optical theory calculation made for the reactivity worth of the Spert III control rods was the same as that used in the present analysis of the experimental rodded cores, and is described below in Section III-D. In comparison with experimental results⁽¹⁰⁾, the results of these extensive reactivity calculations for the full-scale Spert III reactor at different operating temperatures showed the calculated k_{eff} to be systematically too high by 3 to 4% Δk . The application of similar analytical methods to Zr - H₂O systems, however, has yielded eigenvalue results which are in general good to about 1% of criticality⁽⁶⁾.

This discrepancy in precision of results obtained between stainless steel and zirconium core calculations was thought to be attributable to two factors: (a) the use of incorrect reactor constants arising from uncertainties in the stainless steel cross sections used in the Muft code, and (b) inadequacy of the control rod theory used. It was known that while the original Muft deck used took account of the inelastic scattering in the elements making up stainless steel, provision for resonance absorption in these same elements was apparently absent. The effect of this absence on the calculated reactivity of the core could not, however, be determined. The second possibility concerned the theoretical treatment of the box-shaped, B¹⁰-stainless steel control rods. The $1/v$ cross

section of the B^{10} in the thin poison region of the rod results in important epithermal absorption, which tends to impair the validity of a diffusion representation in that neutron energy region. Blackness theory was used for calculating effective diffusion constants for the thin poison region, but again, since the calculation for rod worth could not be isolated from the overall analysis, its individual success could not be determined. To obtain unambiguous data bearing separately on the question of control rod worth, reactivity experiments were then performed on cores of simple geometry, with and without the presence of a single control rod.

The body of the report is divided into two main parts. The first part, pertaining to the experimental work performed, describes the experimental arrangement, the various core configurations investigated, and the experimental results obtained. The second part describes the analytical work performed, including the calculation for control rod worth, the various parametric studies on the rod-free cores, and the final calculational results obtained.

II. Experimental Program

A. Description of Apparatus

In accordance with the limited objectives of the experiment, a simple experimental arrangement was used, which, while consistent with the requirements for reactor safety, allowed for flexibility in core configuration and for ease of assembly of the cores studied. To avoid ambiguity in the analytical interpretation of the criticality measurements, reactivity control was provided entirely by variation of the water height in the reactor tank, with no movable control rods used for control purposes. In the case of the rodded cores where a single poison rod was present, the rod remained stationary and fully inserted at all times. The critical experiments were conducted at ambient temperature in a small auxiliary tank located at the Spert I facility, with reactor control exercised remotely one-half mile from the experimental site.

1. Reactor Control - Cores were assembled in an open rectangular tank located adjacent to the Spert I shield vessel, into which water from the experimental rectangular tank could be dumped. The system for draining or filling the tank with water was remotely controlled. The water inlet, supplied by a 15 gpm positive-displacement pump, provided for two fill rates by means of an electric-pneumatic valve placed in parallel with a small, open by-pass line. With the pump on and the pneumatic valve open, a fill rate of approximately 1 in. of water height per minute was obtained. With the valve in the parallel line closed, the fill rate was reduced by a factor of seven. Use of the slow fill rate allowed for a more precise control of the water level in the region of the critical water height.

The drain system consisted of slow and fast drains and a scram water dump. For the slow drain, a 1.5 in.-diameter flexible hose was connected from the bottom of the tank to one of the Spert I control rod

drives to form a movable U-shaped overflow tube. The 20 in. maximum vertical travel of the Spert I rod drive limited the usable range of vertical movement of the overflow tube from the top of the experimental tank (approximately level with the top of the fuel assemblies) down to a level approximately 16 in. above the bottom of the fuel assemblies. In addition to the fine control on water level afforded by small movements of the rod drive, this overflow tube had the safety feature of effectively preventing the water level in the tank from rising above the height set by the rod drive. The fast drain consisted of a 2 in.-diameter pipe leading from the bottom of the experimental tank down to two 1.5 in. solenoid valves located at the bottom of the Spert I shield tank. For reactor safety, a fast acting and very effective water-dump was provided by a large 3.5 in.-diameter flexible hose. This hose was suspended from the center by an electromagnet to form an inverted U-shape tube, with apex above the level of the top of the experimental tank, with one end of the hose fitted to an outlet at the bottom of the experimental tank, and with the other end of the hose directed downward into the Spert I shield tank. This scram water-dump, while simple, did provide high water-dumping rates (equivalent to negative reactivity ramp rates of up to 50¢/sec), was dependable and had the virtue of being essentially foolproof. Finally, provision was made for an additional emergency reactor shutdown mechanism through the use of a 3-gal capacity, boron solution injection system.

2. Instrumentation - Nuclear instrumentation for the critical experiments was provided by the normal Spert I reactor instrumentation system. Neutron detectors consisted of two boron-lined pulse counters, used primarily for subcritical multiplication measurements, and two boron-lined, gamma-compensated ion chambers to give linear and log power traces for criticality and reactor period determinations. The neutron detectors were placed in water-tight metal tubes and positioned at least 6 in. from the sides of the critical assembly in order to minimize any perturbation by voids in the water reflector region.

The water level in the tank was monitored in two ways. For operational purposes, a coarse visual indication of the water height, good to about 1/8 in., was provided by means of a television camera viewing a yardstick mounted on the wall of the tank. For experimental measurements, a precision remote-reading water level indicator, based on a water float system and using synchros to transmit information on vertical movements of the float to the control center, gave differential water height measurements to better than one-hundredth of an inch⁽¹¹⁾. The effect of any water turbulence in the reactor tank on the stability of the water float was minimized by placing the float in a separate perforated container mounted in the reactor tank.

Water temperature was measured by a four-junction thermopile located in the reflector water. All critical experiments were performed in the temperature range between 55 and 60°F, with the temperature variation during the course of any critical experiment limited to less than 1°F. A small stirrer was operated intermittently to ensure uniform bulk water temperature throughout the core and reflector regions.

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Fig. 1 - Section Through the
Spert III Fuel Element

3. Fuel Elements - The Spert III fuel element used in these experiments is a plate-type, stainless steel unit, consisting of 38 fuel plates, 2 side plates, a center reinforcing plate, and upper and lower end-boxes. A section through the fuel element is shown in Fig. 1. Fuel element design parameters and atomic number densities pertinent to the analysis are given in Tables 1 and 2.

Cores were assembled with fuel elements spaced on 3 in. centers, a spacing identical to that used in the Spert III core. Positioning of the elements at the bottom was provided by a reinforced 19 x .34 x 0.5 in.-thick aluminum grid plate, having a 5 x 10 array of holes for the fuel element end-boxes. This grid supported the fuel elements approximately 0.5 in. above the floor of the tank. At the top of the core, aluminum angles, rigidly clamped to the sides of the

TABLE 1

SPERT III FUEL ELEMENT PARAMETERS

Nominal Active Length of Fuel Plate	36 in.
Full Length of Fuel Plate	37 in.
Nominal Active Width of Fuel Plate	1.045 in.
Full Width of Fuel Plate	1.445 in.
Meat Thickness	0.020 in.
Plate Thickness	0.030 in.
SS Clad Thickness	0.005 in.
Water Channel Thickness	0.128 in.
Number of Fuel Plates	38
Overall Width of Element	2.950 in.
Overall Thickness of Element	2.950 in.
Uranium Enrichment	93.2% U-235
Meat Composition:	
U-235	15.4 wt%
UO ₂	18.8 wt%
Type 304 SS	81.2 wt%
Total U-235 per Fuel Element	638 g
Cell Size	3 x 3 in.
Cell Metal-to-Water Ratio	0.298

TABLE 2

SPERT III FUEL ELEMENT NUMBER DENSITIES FOR A
3 x 3 in. CELL AREA AT 50°F

Element	Number Density (10^{24} atoms/cm ³)
H	0.0515
O	0.0264
U ²³⁵	0.000308
U ²³⁸	0.0000225
Fe	0.0128
Cr	0.00357
Ni	0.00141
Mn	0.000476

tank and to each other, formed a rectangular perimeter of appropriate dimensions surrounding the elements. To maintain the correct internal spacing between elements, aluminum shims were used near the top end-boxes. This arrangement for assembling cores minimized the time required for making changes in core configuration.

In connection with the analysis of the critical assemblies, it is noted that for close packing of the uniformly oriented fuel elements, in which the cell area assigned to each fuel element is 3 x 3 in., the thickness of the water channel formed between the last plate of one element and the first plate of the adjacent element is just equal to the water channel thickness within the elements. Thus, across the close-packed core in the direction normal to the orientation of the fuel plates, the 0.030 in.-thick fuel plate regions alternate uniformly with the 0.128 in.-thick water channel regions.

Fig. 2 - Section Through the
Spert III Control Rod

4. Control Rod - The stationary control rod used in these experiments was an exact duplicate of the poison section of the Spert III fuel-bearing control rod. The control rod assembly, occupying a 3 x 3 in. cell area, was basically a water-filled, square-shaped tube of enriched (93 wt% B¹⁰) boron alloyed with stainless steel, with the poison tube surrounded by a slightly larger, square-shaped, stainless steel rod-guide tube. The surface density of B¹⁰ in the poison alloy was 0.047 g B¹⁰/cm². A vertical section through the control rod is shown in Fig. 2.

B. Experimental Procedure

The critical water-height experiments were carried out with personnel excluded from the reactor site whenever water was present in the tank. The approach to critical for each core configuration was made by initially raising the water level in the tank in increments of 5 in. With a neutron source located in the water reflector adjacent to the core, the subcritical neutron multiplication was measured after each addition of water and a prediction of the critical level was obtained from a plot of the inverse multiplication vs water height. As the neutron multiplication increased, successive water level increments were decreased appropriately. Near criticality, the neutron source was moved away from the core and the water level was slowly adjusted to the exact critical height.

Following the determination of the critical water height h_c of a core, two measurements of the incremental water height reactivity worth, $d\rho/dh$, were made. For the latter measurements, incremental changes, Δh , in water height varied from about 0.2 to 0.6 in., yielding reactor periods in the range from several hundred seconds to about 50 sec. The corresponding reactivities $\Delta\rho$, in cents, were determined from the inhour relation, and the resulting calculated values for $d\rho/dh$ were associated with the mean water height $\bar{h}$ between h_c and $h_c + \Delta h$, i.e., $\bar{h} = h_c + \Delta h/2$.

C. Experimental Results

Fuel elements were arranged in slab arrays of 2, 3, 4, or 5 rows, either rod-free or with a single, fully-inserted control rod. Orientation of the elements was such that the fuel plates were always parallel to the row axis (i.e., the long dimension of the slab), with the single exception of case 5 (Table 3), where the fuel plates were oriented normal to the long dimension.

1. Two-Row Configuration - A two-row, nine-column (2 x 9) slab was found to be subcritical with the water height at 36 in., i.e., at the top of the active core. In addition, since no definite increase in neutron multiplication was observed for this long slab as the water height was raised to the top of the core, it is expected that the infinitely-long two-row slab is also subcritical.

2. Three-Row Configuration - In order to minimize the error in determining the axial buckling of a core by decreasing the sensitivity of the buckling to the error made in determining the axial reflector savings, several three-row configurations were studied to obtain a critical water height near the top of the fuel. The results of critical water height and incremental water height reactivity worth measurements made on three-row rod-free and three-row rodged slabs are summarized in Table 3.

3. Four-Row Configuration, - A 19 fuel element, 4 x 5 rodged array, with the control rod located at (3,3), was critical at a water height of 34.26 in. Measurements of incremental water worth yielded values of 14.4 cents per in. at $\bar{h} = 34.68$ in., and 13.0 cents per in. at $\bar{h} = 34.98$ in.

TABLE 3

EXPERIMENTAL RESULTS FOR THREE-ROW SLAB CORES

Case	Core Configuration ^(a)	Critical Water Height, h_c ^(b) (in.)	Incremental Water Worth $d\rho/dh$ (ϕ /in. at $\bar{h}$)
1	ooooo ooooo ooooo	Subcritical at 36 in.	
2	ooooo ooooo ooooo	34.04	15.5 at 34.30 in., 15.0 at 34.46 in.
3	ooooo ooooo ooooo	27.75	29.3 at 27.90 in., 28.7 at 28.03 in.
4	ooooo ooooo ooooo	23.49	47.2 at 23.61 in.
5(c)	ooooo ooooo ooooo	24.55	39.0 at 23.61 in.
6(d)	oooo oooo oooo oooo oooo oooo	28.33	28.9 at 28.54 in., 28.3 at 28.65 in.
7	oooooooo ooooXoooo oooooooo	Subcritical at 36 in.	
8	oooooooo ooooXooo oooooooo	31.41	20.9 at 31.62 in., 20.6 at 31.86 in.
9	oooooooo ooooXooo oooooooo	28.12	29.3 at 28.24 in., 29.2 at 28.41 in.
10	oooooooo ooooXooo oooooooo	23.64	45.1 at 23.76 in.
11	oooooooo ooooXooo oooooooo	31.26	21.9 at 31.52 in., 21.6 at 31.72 in.

(a) Circles represent fuel elements; the cross represents the control rod.

(b) The water height above the nominal bottom of the active core.

(c) Slab case in which the fuel plates were oriented normal to the rod axis (for comparison with case 4).

(d) Rod-free split-core case of two 3 x 4 slabs separated by a 3-in. water gap.

4. Five-Row Configuration, - A cylindrical core configuration was approximated by 20 fuel elements in a 5 x 5 rodded array, with the corners missing and the control rod in the center. Criticality was obtained at a water height of 29.37 in., and the incremental worth was measured to be 28.9 cents per in. at 29.72 in. An additional measurement was made with bare and cadmium-covered gold foils placed in the core and reflector regions to obtain a radial and a vertical flux traverse. The gold foils were 0.156 in. in diameter and 0.005 in. thick. With the foils placed in the reactor, the critical water level increased slightly to 29.56 in., while the measured incremental water worth decreased to 25.9 and 24.3 cents per in. at the respective mean water heights of 29.69 and 29.77 in.

Fig. 3 - Quarter Section Through the Five-Row Core Showing Location of Horizontal and Vertical Flux Traverse

In Fig. 3, a quarter section through the cylindrical core is illustrated, showing the positions where the flux traverses were taken. In the horizontal flux distribution shown in Fig. 4, the small peaks in the thermal flux distribution are seen to occur at the fuel-free inactive regions of the fuel plates near the center reinforcing plates and in the water channels regions between assemblies. Fig. 5 shows the vertical flux distribution obtained at the single radial position.

In Fig. 6, the incremental water height reactivity worth for rodded and rod-free, three-row slabs has been plotted as a function of the water height. The points appear to fit a fairly smooth curve, which may be integrated to give the dashed curve shown in this figure. The extension of the integral curve to the top of the core at 36 in. has been based on a reasonable extrapolation of the incremental curve from 34.5 to 36 in. The excess reactivity for each core (with water height at 36 in.) is derived from the integral curve.

Fig. 7 shows a plot of the water height as a function of $(dp/dh)^{-1/3}$. Except for the two points near the top of the core, where changes in reflector savings occur due to the proximity of the end-box stainless steel reflector region, a linear fit is indicated. Thus, for water heights in the range from 23 to 33 in., application may be made of the linear relationship

$$h_c = (2\pi^2 M^2)^{1/3} (dp/dh)^{-1/3} - s, \quad (1)$$

Fig. 4 - Horizontal Flux Distribution Through Five-Row Core

Fig. 5 - Vertical Flux Distribution Through Five-Row Core

Fig. 6 - Incremental Water Height Reactivity Worth vs Water Height for Three-Row Cores

Fig. 7 - Water Height vs Inverse Cube Root of Incremental Water Height Reactivity Worth for Three-Row Cores

to obtain the total axial reflector savings, S . In this equation, the quantity M^2 is given from two-group theory by

$$M^2 = \frac{\tau}{1 + \tau B^2} + \frac{L^2}{1 + L^2 B^2} \quad (2)$$

(In Eq. (2) a small correction term has been neglected, which takes into account the difference in the age τ between prompt and delayed neutrons.) A least-squares linear fit of the data in Fig. 7 yields a value of 3.7 in. for the reflector savings and a value of 477 β_{eff} in.² for M^2 .

III. Analysis

A. Summary of Analytical Program

The following briefly summarizes the analytical work performed on five of the thirteen cores that were investigated experimentally. These five, shown in Fig. 8, consisted of the rod-free cases 2, 4, and 6 (Table 3) and the rodded cases 12 and 13. The rod-free cores were analyzed first in order to test the calculational technique for stainless steel cores independent of any control rod calculations. In this analysis of the rod-free cores, initial four-group neutron diffusion calculations for the eigenvalues were carried out using the Muft and Soffocate constants originally developed for the Spert III reactor analysis. Parametric studies were then made to determine the effect on the calculated eigenvalue of such factors as the inclusion of resonance absorption by stainless steel in the calculation of the Muft constants, the heterogeneity in the fueled region of the core caused by the inactive regions at the edges of the fuel plates, the error in determining the axial reflector savings, etc. The eigenvalues obtained from a "best" analysis of the rod-free cores then served as a fiducial set for comparison with the eigenvalues obtained from a similar analysis of the rodded cores, the difference in eigenvalues between the rod-free and rodded cases being a measure of the success of the calculation for the control rod worth.

Fig. 8 - Schematic Diagrams of Rod-Free and Rodded Lattices Selected for Analysis (the Axial Savings is Included in H).

B. Determination of Axial Reflector Savings

In the two-dimensional, four-group neutron diffusion calculations of the rod-free cores (IBM-704 PDQ code), specification of the transverse (axial) buckling was required as an input quantity for the numerical computation. In calculating the axial buckling from the critical water height and reflector savings, estimates for the total axial reflector savings were determined (a) from the experimental data on incremental reactivity worth of the water height as a function of water height, (b) from the vertical flux distribution data obtained for rodged case 13, and (c) by calculation.

As shown in Fig. 7, the data indicate a linear fit between water height and the inverse cube root of the incremental water worth in the critical water height range between 23 and 33 in. Extrapolation of this straight line back to zero incremental water worth yields a total axial savings (due to the bottom end-box and water-reflector region and top fuel-air region) of 9.4 cm.

A more approximate value for the total axial savings for the 5 x 5 rodged core only (case 13) was determined by least squares fitting a cosine function to the experimental vertical flux distribution (Fig. 5) obtained at the single radial position. The reflector savings value obtained in this way was about 8 cm.

The reflector savings for each of the five cores analyzed was also obtained by calculation. This was given by first performing a one-dimensional, four-group, axial calculation for the eigenvalue of the reflected reactor model, consisting of the three axial regions of lower reflector, active core and upper reflector. Next, the height of the bare active core region was increased until the eigenvalue obtained for the lengthened bare core matched that obtained for the reflected core. The reflector savings was then given by the change in the height of the bare core required to give the eigenvalue of the reflected core. The calculated axial reflector savings are given in Table 4. In these one-dimensional calculations for the reflector savings, the four-group radial bucklings applied in each case were obtained from the output of PDQ calculations performed on the five cores (Section III-C).

TABLE 4

TOTAL AXIAL REFLECTOR SAVINGS

Core Case	Critical Water Height (in.)	Calculated Savings (cm)	Savings From Incremental Water Worth Data (cm)	Savings From Flux Plot Data (cm)
2	34.0	9.9	9.4	—
4	23.5	8.6	9.4	—
6	28.3	9.2	9.4	—
12	34.3	9.9	—	—
13	29.4	9.3	—	8

C. Analysis of Rod-Free Cores

1. Preliminary Reactivity Calculations - Two-dimensional, four-group, neutron diffusion calculations (PDQ code) for cases 2 and 4 were first carried out using the Muft 4 (P-1 approximation) and Soffocate (hardened) reactor constants obtained earlier for the Spert III calculation. These constants are given in the Appendix (Set 1).

In the analytical model the fuel region was considered effectively homogenized, with self-shielding factors applied to take account of the physical heterogeneity of fuel plate and water channel regions in the direction normal to the surface of the fuel plates. In calculating these self-shielding factors, the double P-1 approximation to the one-group, one-dimensional transport equation was solved in the direction normal to the fuel plates through the three regions of meat, cladding, and water channel. The results so obtained for the self-shielding factors to the thermal absorption and fission cross-sections for the fuel region were as follows:

$$\frac{\sum_a^{(\text{Homo})}}{\sum_a^{(\text{Het})}} = 0.939, \quad \frac{\sum_f^{(\text{Homo})}}{\sum_f^{(\text{Het})}} = 0.932,$$

corresponding to a reactivity effect (based on change in thermal utilization) of about 0.8%. In the direction parallel to the fuel plates, the heterogeneity effected by the fueled portion of the plate and the non-fueled edge portion of the plate was not taken into account.

For these initial PDQ eigenvalue calculations, the cores were considered effectively bare in the axial direction, with the value for the axial buckling based on the experimentally determined value of 9.4 cm for the axial reflector savings. When compared with experiment, the results of these initial calculations for the eigenvalues of the rod-free cores were again, as in the original Spert III core calculations, too high by approximately 3% in Δk . These results for rod-free cores suggested that the indicated systematic error of 3% might be due primarily to the calculation for the fast reactor constants (specifically, with respect to the stainless steel cross sections in the Muft code) and to a lesser extent to certain other aspects of the calculational method.

2. Reactivity Effect of Resonance Absorption in Stainless Steel

In the Muft 4 code originally used, the basic absorption cross section data for stainless steel took account of the $1/v$ portion of the absorption cross section of the elements making up stainless steel, but contained inadequate provision for resonance absorption. Using the data given by Macklin and Pomerance⁽¹²⁾ on the resonance integrals of Fe, Cr, and Ni, the Muft decks were modified to include the effects of resonance absorption in these elements. This was accomplished by making appropriate increases in the absorption cross section of the third fast group of the four-group scheme. The resulting modified set of fast constants is listed

as the second set of constants in the Appendix. The four-group calculations for cases 2 and 4 were then rerun using the new fast constants and a decrease of about 1.8% was obtained in the eigenvalue as a consequence of the inclusion of resonance absorption in stainless steel. The assignment of the entire resonance absorption effect to the third fast group only was somewhat arbitrary, and the effect of this choice on the calculational results remains somewhat uncertain.

3. Inactive-Edge-Plate Heterogeneity Effect - A tacit assumption made in the previous calculations involved the homogeneity of fuel assembly material in the direction parallel to the fuel plates. The active region of the fuel plate, however, only extends over two-thirds of the full width of the plate, leaving inactive regions at both edges of the fuel plate. Thus, in the direction parallel to the fuel plates, the "fueled" region of the core can be represented by alternate bands of active and inactive (source-free) diffusion regions, whose respective widths are in the ratio of about 2 to 1. The heterogeneity extends across the entire core, since all fuel assemblies in the core were oriented in the same direction. The reactivity effect of this edge-plate heterogeneity was determined analytically as follows:

(a) The fiducial case of the homogenized core was first considered. A four-group, one-dimensional diffusion theory calculation was performed, using Muft and Soffocate constants obtained for the homogenized mixture of active and inactive regions of the fuel plates. The problem was run in slab geometry, with the buckling adjusted to give an eigenvalue approximately equal to unity ($\lambda = 1.0001$).

(b) A DP-1 transport theory calculation (Flip code) was next run to obtain the thermal flux distribution in a two-region cell comprised of the active and inactive regions. For this calculation, the thermal source (the product of the removal cross section and fast flux) and appropriate transport constants were determined from supporting calculations. The thermal flux distribution was then used to determine flux-averaged fission and absorption thermal cross sections, which were then applied in a final four-group diffusion theory calculation similar to that for (a) above for the homogenized core. This yielded the eigenvalue $\lambda_1 = 0.979$, so that $\Delta\lambda_1 = \lambda_1 - \lambda_0 = -0.021$.

(c) An additional four-group diffusion theory calculation was performed, which was similar to (a) except that the active and inactive regions were specified explicitly. Appropriate Muft and Soffocate constants were determined for each region. This calculation yielded the eigenvalue $\lambda_2 = 0.983$, so that $\Delta\lambda_2 = \lambda_2 - \lambda_0 = -0.017$.

The results of these calculations indicate that the inactive-edge-plate heterogeneity effect leads to a reduction in the eigenvalue of the rod-free cores of about 1.7% on a diffusion theory basis and of about 2.1% on a combination transport-and-diffusion theory basis.

4. Additional Parametric Studies - Additional one- and two-dimensional studies were undertaken to determine the sensitivity of the eigenvalue to (a) the number of neutron energy groups used, (b) the buckling value used in the Muft calculations, (c) the coarseness of the mesh spacing used in the two-dimensional calculations, (d) the axial reflector savings value used, and (e) recent changes in basic cross section data for Muft 4 code. These studies yielded the following results:

(a) For calculations on stainless steel fuel, two-group theory is substantially inferior to four-group theory--the two-group scheme yielding eigenvalues which are in greater error than the four-group eigenvalues by about 2.5% Δk . In particular, one-dimensional calculations carried out using both the "old" fast constants (no resonance absorption in stainless steel) and the "new" fast constants (resonance absorption included) showed that the difference between two-group and four-group results decreased when resonance absorption was included in the third group of the four-group structure. Specifically, the results obtained are as follows:

TABLE 5

CHANGE IN EIGENVALUE BETWEEN TWO- AND
FOUR-GROUP THEORY CALCULATIONS

<u>Muft Constants</u>	<u>$[\lambda(2 \text{ Group}) - \lambda(4 \text{ Group})]$</u>
With Resonance Absorption in SS	0.020
Without Resonance Absorption in SS	0.029

(b) The input buckling value used in the Muft calculation is unimportant in the four-group calculation; and increase in the Muft buckling value from 1×10^{-6} to $1.5 \times 10^{-2} \text{ cm}^{-2}$ decreases the calculated k_{eff} of the rod-free slab by only 0.1%. In the two-group scheme the choice of the Muft buckling value is much more critical, the calculated k_{eff} of the slab decreasing by about 1.4% (for both the old or new Muft constants) for the same change in Muft buckling value given above.

(c) In the boundary regions where the thermal flux curvature is large, changing the PDQ mesh spacing from a "coarse" spacing of about 0.8 cm (coarse, in the sense that the spacing is larger than the transport mean free path of 0.66 cm for thermal neutrons) to a "fine" spacing of 0.4 cm increased the eigenvalue by only 0.2%.

(d) In determining the input value of the axial buckling for the two-dimensional calculations, an error in reflector savings of as much as 3 cm can affect the calculated k_{eff} for the slab core by only a few tenths of a per cent, the amount depending on the critical water height. Thus, as indicated in Fig. 9, a 3 cm error in reflector savings yields an error in k_{eff} of 0.5% for a core height of about 23 in. and an error of 0.2% if the core height is 34 in. While no errors have been assigned to the values determined for reflector savings, the general agreement found between the calculated and experimentally derived savings indicates that the error is probably less than 2 cm. Hence, the error in the calculated k_{eff} due to the value of reflector savings used in determining the axial buckling for the two-dimensional calculations is probably less than 0.3%.

Fig. 9 - Dependence of Calculated k_{eff} on Axial Reflector Savings

with respect to their effect on the calculated k_{eff} of the slab cores under consideration. The modifications in the Muft 4 code pertinent to enriched stainless steel cores consist primarily of changes in the inelastic scattering matrices for iron, the inclusion of new inelastic scattering matrices for U-235 and oxygen, and in U-235, the inclusion of absorption and fission resonance parameters rather than simply smooth capture and fission. Using Muft constants incorporating these modifications, one-dimensional reactivity calculations of the rod-free cores were carried out and the results for k_{eff} were compared with those obtained using the original Muft 4 constants. The results showed an overall decrease of about 0.3% in k_{eff} when the new Muft 4 constants were applied.

(e) Recent modifications⁽¹³⁾ to the Muft 4 code made at Bettis have been evaluated

5. Summary of Computational Results for Rod-Free Cores - The results of these studies indicate that with respect to the two-dimensional, four-group diffusion analysis of the rod-free cores, those factors which are recognized to affect the calculated eigenvalue by more than a few tenths of a per cent Δk are those of resonance absorption in steel and of the inactive-edge-plate heterogeneity. Inclusion of both effects leads to a reduction in eigenvalue of nearly 4%.

The final results of the two-dimensional, four-group reactivity calculations for the three rod-free cores studied are given in Table 6. These results are based on the calculated values given in Table 4 for the axial reflector savings and incorporate the effects of resonance absorption and edge-plate heterogeneity.

TABLE 6

CALCULATED k_{eff} FOR ROD-FREE CORES

Loading Pattern	Critical Water Height (in.)	Total Axial Reflector Savings (cm)	Calculated k_{eff}
ooooo ooooo ooooo	23.5	8.6	0.993
oooo ooooo oooo	34.0	9.9	0.994
oooo oooo oooo oooo oooo oooo	28.3	9.2	1.001

D. Analysis of Rodded Cores

In the two-dimensional PDQ calculations for the rodded cores, the 3 x 3 in. cross-sectional area occupied by the control rod was specified in the calculational model (Fig. 10) as three regions: the outer square annulus consisting of the stainless steel rod guide tube homogenized with the adjacent layers of water, the 0.188 in. thick square tube of B^{10} -stainless steel poison, and the interior region of homogenized stainless steel and water.

Fig. 10 - Quarter Section of Control Rod Model

For the inner and outer regions of the control rod, four-group constants were obtained using the Muft and Soffocate codes. For the thin poison tube, a transport theory black boundary condition was applied for the thermal group, i.e., the logarithmic derivative of the thermal flux was set equal to $[0.71 \lambda_{tr}]^{-1}$, where λ_{tr} is the transport mean free path. For the fast neutron groups, the $1/v$ absorption cross section of the B^{10} in the thin poison region results in important epithermal

absorption, which significantly affects the validity of a diffusion representation in that energy region. Following Voorhis and others^(14,15),

equivalent diffusion constants were obtained for the thin poison tube in terms of the energy dependent parameters $\alpha(E)$ and $\beta(E)$, which relate the current and flux at the surface of the poison as:

$$\alpha(E) = \frac{j^+(E) + j^-(E)}{\phi^+(E) + \phi^-(E)} \quad (3)$$

and

$$\beta(E) = \frac{j^+(E) - j^-(E)}{\phi^+(E) - \phi^-(E)} \quad (4)$$

where $j^\pm(E)$ represent the net neutron currents into the inner and outer surfaces of the poison tube and $\phi^\pm(E)$ represent the total scalar fluxes at the corresponding surfaces. The parameters $\alpha(E)$ and $\beta(E)$ are given in terms of the tabulated $E_3(x)$ and $E_4(x)$ functions by

$$\alpha(E) = \frac{1 - 2E_3(x)}{2[1 + 3E_4(x)]} \quad (5)$$

$$\beta(E) = \frac{1 + 2E_3(x)}{2[1 - 3E_4(x)]} \quad (6)$$

where $E_n(x)$ is defined by⁽¹⁶⁾

$$E_n(x) = \int_0^1 u^{n-2} e^{-x/u} du \quad (7)$$

and $x = t\Sigma_a(E)$ is the dimensionless product of the tube thickness t and absorption cross section Σ_a of the poison. Using (5), $\alpha(E)$ was calculated for each of 54 energy groups and then averaged over each of the three fast groups using

$$\langle \alpha \rangle_i = \frac{\sum_j \alpha_j(E) \phi_j(E) \Delta E_j}{\sum_j \phi_j(E) \Delta E_j} \quad (8)$$

where $j = 1, 2, \dots, 10$ for $i = 1$; $j = 11, 12, \dots, 25$ for $i = 2$; and $j = 26, 27, \dots, 54$ for $i = 3$. The 54 group fluxes $\phi_j(E)$ at the surface of the poison tube were calculated to a P-1 order of approximation in neutron angular distribution using the transport theory code PLMG-2⁽¹⁷⁾.

Fig. 11 - Cell Used
in PLMG Calculation

In this calculation, a symmetrical two-region cell was considered (Fig. 11). This consisted of the poison region and a region consisting of a homogenized composition of fuel, stainless steel, and water, with the composition of this homogenized region chosen to be equal to the average of the stainless steel-and-water mixture internal to the B¹⁰-SS tube and the rod guide-and-fuel assembly mixture external to the B¹⁰-SS poison tube.

On the basis that the poison is "gray" to epithermal neutrons, the product $\langle \alpha \rangle_i \langle \beta \rangle_i$, which lies between the value of 1/4 (for large optical thickness) and the value of 1/3 (for small optical thickness)⁽¹⁵⁾ was taken to be

equal to 1/3.5. In accordance with this choice, the equivalent diffusion parameters of absorption cross section and diffusion constant for the thin poison section were given by the approximate relations

$$\Sigma_{ai} = \frac{2\langle \alpha \rangle_i}{t} \left[1 + \frac{3.5}{3} \langle \alpha \rangle_i^2 + \dots \right] \quad (9)$$

and

$$D_i = \frac{1}{3.5 \Sigma_{ai}} \quad (10)$$

The calculated values for $\langle \alpha \rangle_i$, Σ_{ai} , D_3 , and the removal cross section Σ_{r_i} for the three fast groups are given in Table 7. The values for Σ_{r_i} were taken to be equal to those for stainless steel on the assumption that the scattering properties of the B¹⁰-stainless steel tube were identical to those for pure stainless steel. The values for the diffusion constant for the first two fast groups were also set equal to the values for stainless steel.

TABLE 7

EQUIVALENT DIFFUSION THEORY CONSTANTS FOR THE POISON TUBE

Fast Group	$\langle \alpha \rangle$	D cm	Σ_{a-1} cm ⁻¹	Σ_{r-1} cm ⁻¹
1	1×10^{-6}	1.68*	4×10^{-6}	0.060*
2	0.00208	0.935*	0.00872	0.011*
3	0.1174	0.570	0.501	0.0056*

*Same value as for pure stainless steel.

The calculated results for k_{eff} for the two rodDED cores, using the effective diffusion theory constants obtained for the control rod, are listed in Table 8. In these calculations, the fuel constants were the same as those developed for the rod-free calculations that pertain to Table 6. Comparison of the calculated k_{eff} given in Tables 6 and 8 for the rodDED and close-packed rod-free cores indicates that the presence of the control rod in the rodDED cores is correctly accounted for to about a few tenths % Δk .

TABLE 8

CALCULATED k_{eff} FOR RODDED CORES

Loading Pattern	Critical Water Height (in.)	Total Axial Reflector Savings (cm)	Calculated k_{eff}
ooooo ooooo ooxoo ooooo	34.3	9.9	0.993
ooo ooooo ooxoo ooooo ooo	29.4	9.3	0.992

IV. Summary of Calculation Results

Criticality measurements on rod-free and single-rodDED close-packed arrays of Spert III stainless steel fuel elements have yielded experimental data with which to check separately the calculation for control rod worth and the calculation for the reactivity of a core of given fuel and geometry. In particular, within the scope of four-group diffusion theory, the experimental data on cores of simple geometry have provided a basis for testing the efficacy of the reactor constants developed for highly-enriched, plate-type, stainless steel fuel assemblies and for estimating the reactivity effects due to fuel assembly heterogeneities. The results of parametric studies on the rod-free cores have been summarized as follows:

	Reactivity Effect
A. Muft Constants	
1. Inclusion of resonance absorption in stainless steel	-1.8%
2. Input buckling value to Muft calculations	-0.1%
3. Modifications in inelastic scattering matrices, etc.	-0.3%
B. Fuel Assembly Heterogeneities	
1. Fuel plate and water channel heterogeneity	-0.8%
2. Inactive-edge-plate heterogeneity	-2.1%
C. Two-Dimensional PDQ Calculations	
1. Changes in mesh spacing	<0.2%
2. Error in calculating transverse (axial) buckling	<0.5%
D. Two-Group Instead of Four-Group Theory	
With resonance absorption in stainless steel	+2.0%
Without resonance absorption in stainless steel	+2.9%

With the exception of the split core (case 6), for which neutron spectral and other effects due to the water gap were not evaluated, the calculated values for k_{eff} for the rod-free and rodded cores were all about 0.7% Δk too low. This means that a four-group, diffusion theory calculation, taking into account the resonance absorption in stainless steel and the fuel assembly heterogeneities, can be expected to predict the reactivity of close-packed rod-free cores to about 1% Δk and that the optical theory calculation for the worth of a central rod is correct to a few tenths % Δk .

V. Appendix

FOUR-GROUP REACTOR CONSTANTS (50°F)

Set *	Constant	Group 1	Group 2	Group 3	Group 4 **
1	D	2.0274	1.0260	0.5296	0.2229
	Σ_a	0.00151	0.000771	0.0144	0.1767
	Σ_r	0.0991	0.1175	0.1105	—
	$\nu\Sigma_f$	0.001143	0.001466	0.01915	0.2689
2	D	1.9870	1.0379	0.5326	0.2229
	Σ_a	0.00142	0.000871	0.01586	0.1767
	Σ_r	0.1006	0.1140	0.1071	—
	$\nu\Sigma_f$	0.001133	0.001453	0.01895	0.2689

* Set 1 derived from original Muft 4 deck (no resonance absorption in stainless steel elements).

Set 2 derived from modified Muft deck (resonance absorption effects added to stainless steel elements).

** Thermal constants Σ_a and Σ_f corrected for fuel plate and water channel self-shielding effects; the edge-plate heterogeneity effect is not included.

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