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**OPTIM: A MULTIDIMENSIONAL NONLINEAR
OPTIMIZATION CODE**

DECEMBER 1968

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OPTIM: A MULTIDIMENSIONAL NONLINEAR
OPTIMIZATION CODE

by

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D. R. Haffner

ABSTRACT

The optimization computer code, OPTIM, is designed to optimize nonlinear problems iteratively for ten or less independent variables by applying a differential approach to a quadratic function with interaction terms derived from a least square fitting technique to a known sample set of data.

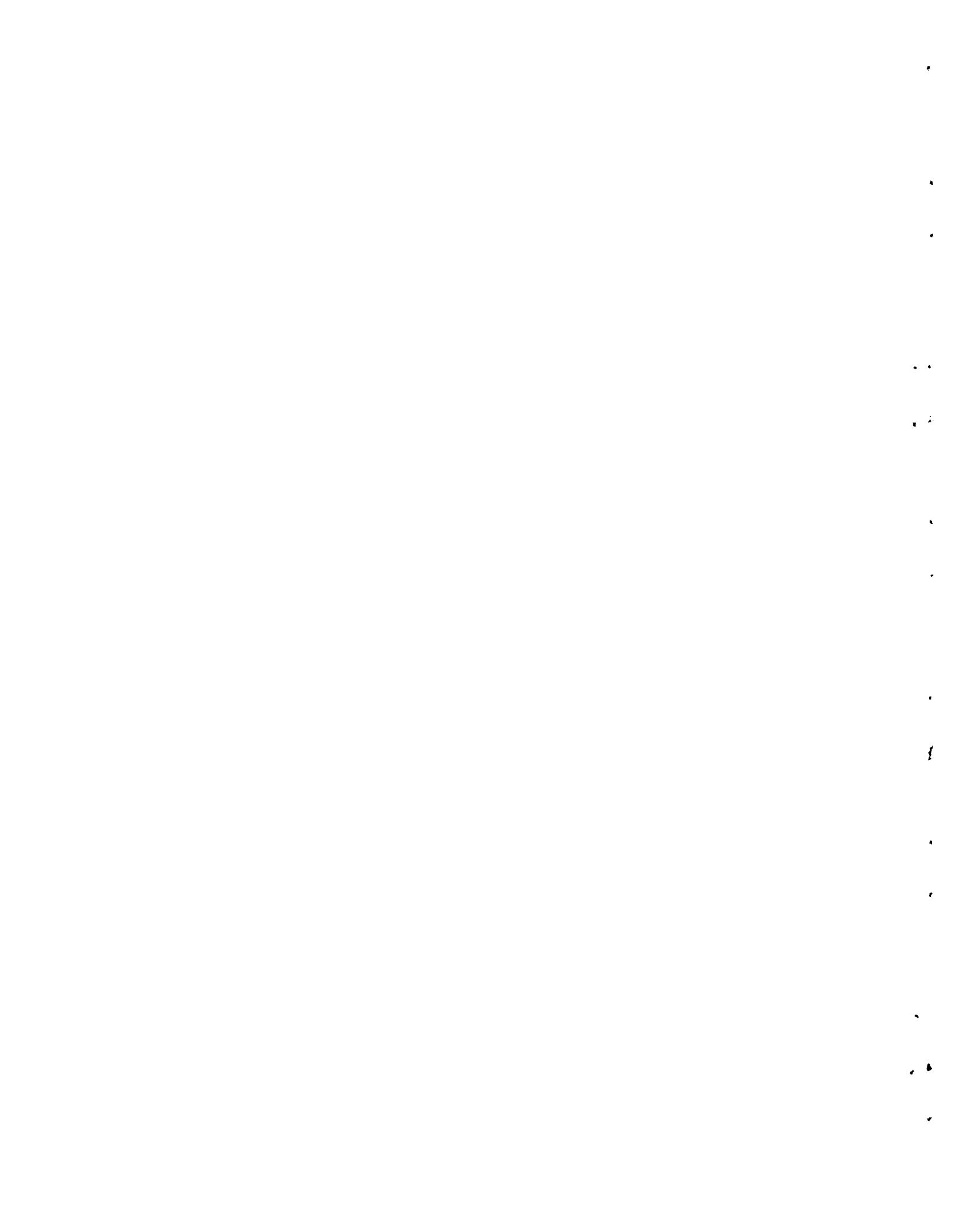
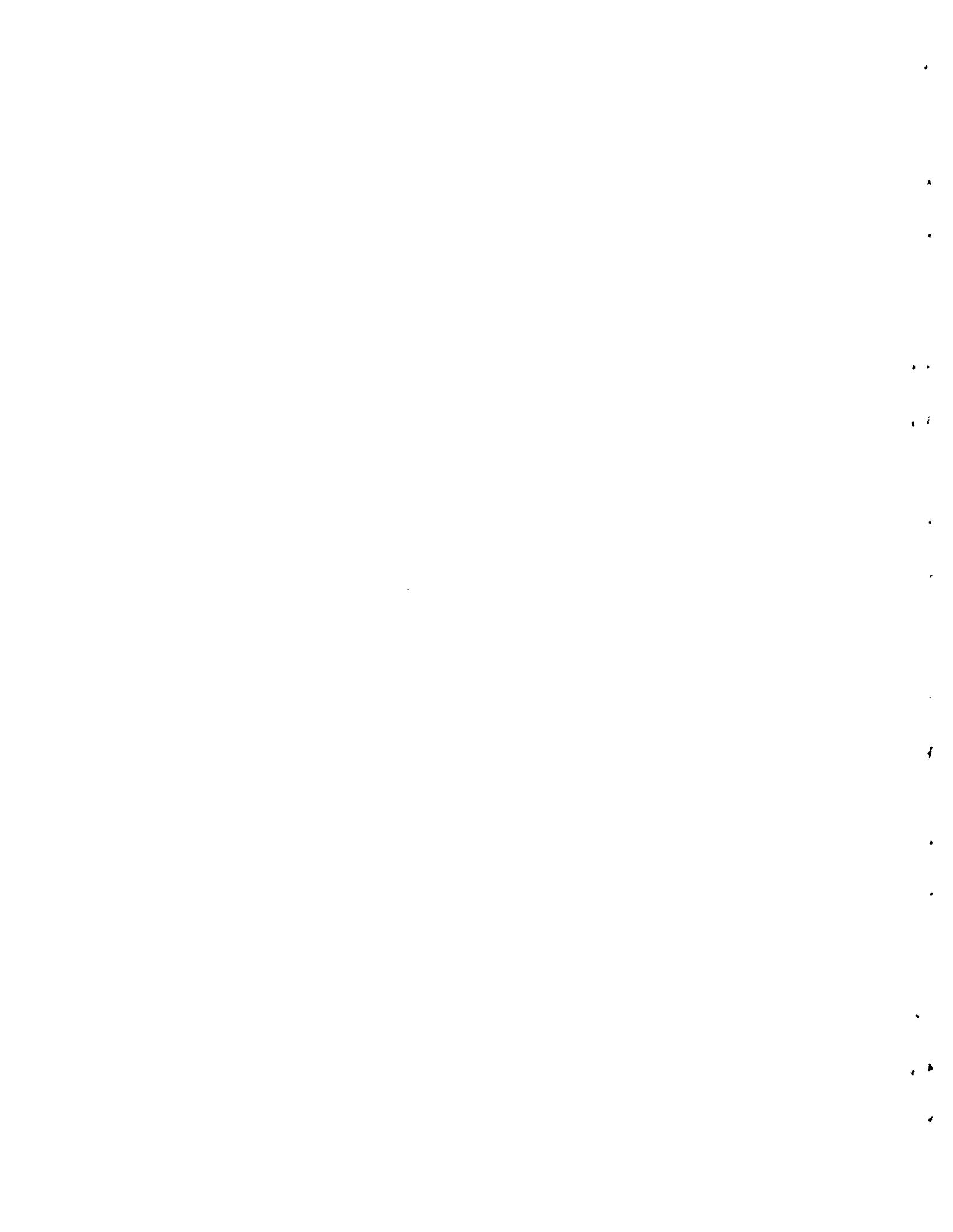


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INTRODUCTION

The need for a generalized optimization code was realized for various optimization problems encountered in reactor fueling analysis. The ultimate goal of this optimization code was its incorporation into the Fuel Cycle Analysis Chain of computer codes named FULCYC,⁽¹⁾ so that optimum fueling of Hanford reactors can be investigated for the MFC-12 program.

Many optimization codes have been developed with various levels of complexity. The requirements to be met by the optimization code, OPTIM, were:

1. Must be capable of handling nonlinear optimization problems,
2. Must require minimal calculation time per iteration within OPTIM (less than one minute),
3. Must be flexible to allow use with other computer codes,
4. Must be capable of optimizing a system having ten or less independent variables (it was felt that many optimization problems exist with ten or less independent variables), and
5. Must be capable of converging to an optimum solution in a reasonable number of iterations (less than 10 iterations).

After extensive research into optimization codes existing and in use at Hanford and many offsite sources, decisions were made to develop a quadratic regression analysis technique to fit a quadratic function to each

set of data points generated per iteration. After fitting the data by a Least Squares Method, a differential approach would then calculate a predicted optimum point which would then be used as the basis for the next iteration.

A version of the quadratic regression analysis technique used in an optimization code, LOLA,⁽²⁾ developed here at Hanford was adopted for OPTIM. LOLA was developed basically as a research tool and has both linear and quadratic regression analyses but lacked the iterative features required, was too slow for our purpose, and required extensive modification to become flexible for use with other computer codes.

To eliminate confusion as to the meaning of experiment link and experiment, the following are defined:

1. Experiment link refers to the analytical model used to compute the dependent variable for a given set of values of the independent variables. This link will change for each different optimization problem.
2. Experiment refers to a data point defined by one set of n independent variables.

The differential approach to optimization for OPTIM was chosen over a steepest gradient (maximum ascent or descent) method mainly because theoretically for well-behaved systems which can be described reasonably well with a quadratic function, the region of the optimum would be reached in much less time than by a steepest gradient method which must "step" its way to the optimum. Considerable computation time in the experiment link (FULCYC in the present version) is therefore saved by using the differential

approach. However, if the physical system is not well-behaved, OPTIM will revert to a method very similar to steepest gradient by "stepping" its way to the optimum.

Only minor program modifications are necessary for optimization problems requiring a different computer code than the present version coupled to the Fuel Cycle Analysis Code, FULCYC (Figure 1 below). The subroutine, GENRAT, must be converted to transform the set of requested experiments into input data for the experiment link used for each particular optimization problem (such as FULCYC in the present version). The present version of OPTIM has a limit of six independent variables because of core limitations imposed by FULCYC. This limitation may be raised simply by expanding the array dimensions within OPTIM.

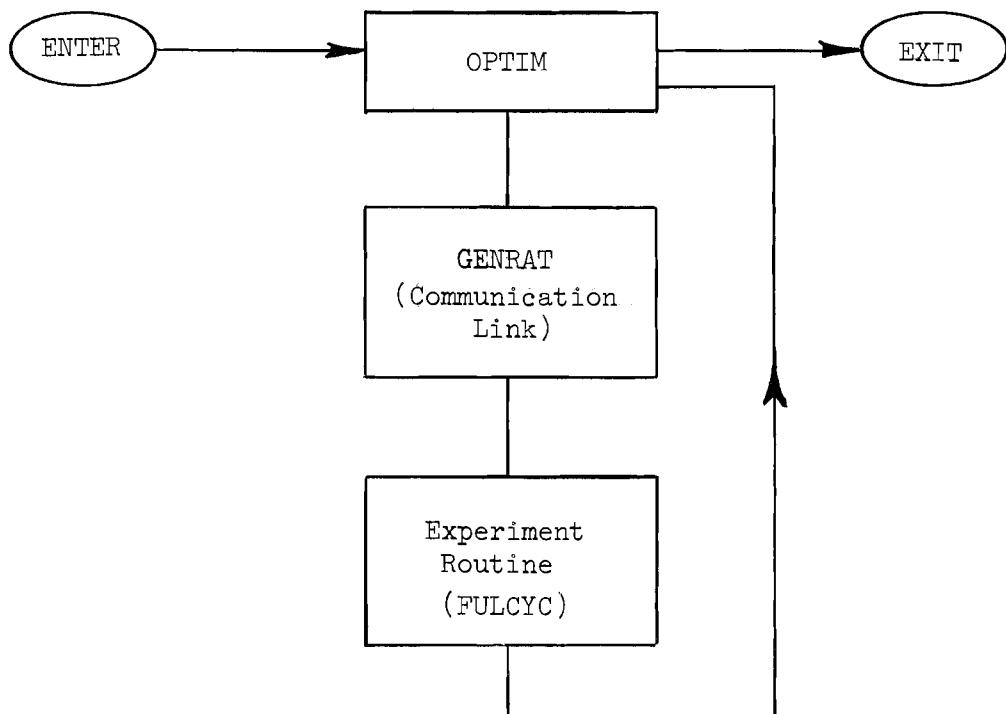


Figure 1. Linkage of OPTIM with an Experiment Routine

OPTIMIZATION TECHNIQUE IN OPTIM

The optimization technique used in OPTIM is based on the differential approach. Differential approach refers to the fact that the slope or the first derivative vanishes or equals zero at the optimum.

The system to be optimized having n independent variables and one dependent variable is assumed to be a function which can be represented approximately by a generalized quadratic equation of the form:

$$y = \beta_0 x_0 + \sum_{i=1}^n \beta_i x_i + \sum_{i=1}^n \beta_{ii} x_i^2 + \sum_{i \neq j=1}^n \beta_{ij} x_i x_j \quad (\text{Eq. 1})$$

where:

y = dependent variable or objective function

x_i, x_j = independent variables, $x_0 \geq 1$

n = number of independent variables

β_0 = constant term coefficient

β_i = linear term coefficients

β_{ii} = quadratic term coefficients

β_{ij} = interaction term coefficients

Essential to the differential approach is the assumption that the objective function is continuous and differentiable in the region bounded by the constraints of the system.

To arrive at a function describing the physical system, a sampling of experiments in a particular region is necessary. The sample set assumes a base point is established about which sampling is done by changing the

values of the independent variables, x_i , by prescribed deltas, δx_i . Sampling about the base point yields a set of data,

$$x_{11}, x_{21}, x_{31}, \dots x_{n1}, y_1$$

$$x_{12}, x_{22}, x_{32}, \dots x_{n2}, y_2$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$x_{1m}, x_{2m}, x_{3m}, \dots x_{nm}, y_m$$

where $m =$ the number of experiments performed and $m \geq$ the number of β coefficients = $1 + 2n + \frac{n(n-1)}{2}$.

By substituting this set of data into the generalized quadratic equation (Eq. 1) and rearranging, the x_i 's and y 's can be treated as knowns and we can now solve for the β coefficients which will approximate the physical system.

$$\beta_0 x_0 + \sum_{i=1}^n \beta_i x_i + \sum_{i=1}^n \beta_{ii} x_i^2 + \sum_{i \neq j=1}^n \beta_{ij} x_i x_j = y \quad (\text{Eq. 2})$$

If we define the following:

$$\begin{aligned} X_1 &\equiv x_0 \\ X_2 &\equiv x_i \\ X_3 &\equiv x_i^2 \\ X_4 &\equiv x_i x_j \end{aligned} \quad (\text{Eq. 3})$$

then

$$X_1 \beta_0 + \sum_{i=1}^n X_2 \beta_i + \sum_{i=1}^n X_3 \beta_{ii} + \sum_{i \neq j=1}^n X_4 \beta_{ij} = y. \quad (\text{Eq. 4})$$

To express this in matrix form we also define for the m experiments performed;

$$[X] = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & x_{m3} & x_{m4} \end{bmatrix}, \quad \text{(Eq. 5a)}$$

such that for the given set of data;

$$[X] \begin{bmatrix} \beta \end{bmatrix} = \begin{bmatrix} Y \end{bmatrix} \quad . \quad (\text{Eq. 6})$$

After inverting and solving for $\begin{bmatrix} \beta \end{bmatrix}$,

$$\begin{bmatrix} \beta \end{bmatrix} = [X]^{-1} \begin{bmatrix} Y \end{bmatrix} . \quad (\text{Eq. 7})$$

This method of inverting an $n \times m$ matrix where $n \neq m$ is referred to as a generalized matrix inverse.⁽³⁾ A definite advantage is gained by using this matrix inversion routine in that it applies a least squares fitting to the data simultaneously in the matrix inversion.

At this point we have determined a particular quadratic equation representing approximately our physical system which we are attempting to optimize.

Having the approximate function describing the system, we determine the point at which all slopes equal zero (the predicted optimum) by taking the first partial derivatives with respect to each independent variable and set these equal to zero as follows:

$$\left. \begin{array}{l} \frac{\partial y}{\partial x_1} = 0 \\ \frac{\partial y}{\partial x_2} = 0 \\ \vdots \\ \frac{\partial y}{\partial x_n} = 0 \end{array} \right\} \rightarrow \begin{bmatrix} 2\beta_{11} & \beta_{12} & \beta_{13} \dots \beta_{1n} \\ \beta_{12} & 2\beta_{22} & \beta_{23} \dots \beta_{2n} \\ \vdots & \vdots & \ddots \\ \vdots & \vdots & \vdots \\ \beta_{1n} & \beta_{2n} & \beta_{3n} & 2\beta_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} -\beta_1 \\ -\beta_2 \\ \vdots \\ -\beta_n \end{bmatrix} \quad (\text{Eq. 8})$$

or in simpler matrix form

$$[\beta^{\prime\prime}] \begin{bmatrix} x^* \end{bmatrix} = \begin{bmatrix} -\beta' \end{bmatrix} \quad (\text{Eq. 9})$$

where

$[-\beta']$ = first order coefficient vector

$[\beta^{\prime\prime}]$ = second order coefficient matrix, and

$[x^*]$ = optimum independent variable vector.

From Equation 9 the values of each independent variable at the optimum are determined,

$$\begin{bmatrix} x^* \\ \end{bmatrix} = [\beta'']^{-1} \begin{bmatrix} -\beta' \\ \end{bmatrix} . \quad (\text{Eq. 10})$$

These values are then considered for the next iteration to be the base point about which sampling will occur. This base point is then compared with the previous base point and when all independent variables have moved less than a specified tolerance, ϵ_{x_i} , the optimum is assumed to have been attained.

APPLICATION OF OPTIM TECHNIQUE

Constraints

Essentially all physical systems have constraints; i.e., each independent variable has a maximum value and a minimum value that physically are invalid, impractical or impossible. Therefore to constrain all independent variables within their specified bounds, whenever the derived quadratic function predicts an optimum outside the boundary, OPTIM moves the variable in violation to the nearest boundary. Although constraining one or more independent variables may result in a base point not exactly on the surface of the function, this problem is resolved in the next iteration.

Sampling Methods

The choice of the best sampling method about the base point depends mostly on the behavior of the physical system and in some cases on the computation times necessary for each experiment. If the system is relatively well-behaved and can be represented quite accurately by the derived quadratic function a full replicate⁽⁴⁾ or fractionated replicate design⁽⁵⁾ about the base point may result in rapid convergence to an optimum. However, if the quadratic function can only approximate trends of the system and/or the computation time per experiment is large, a minimal sampling set (only the experiments needed to measure each β coefficient) may be advisable. The flexibility of OPTIM allows for the use of any sampling method chosen.

Normalization to Reduce Errors

Because of the possibility of a large range of numbers between independent variables appreciable roundoff and truncation errors are likely to

occur in the process of optimization, especially in the inversion of matrices. Normalizing the independent variables will greatly reduce these errors.

Associated with each independent variable is a delta, δx_i , which in sampling defines the magnitude of the move away from the base point for the i^{th} variable. This δx_i is the factor of normalization for that particular independent variable. In other words all x_i 's are normalized to multiples of δx_i 's before being "digested" by the optimization routine.

Levelizing of Effects - Reduction of Deltas

An assumption is made in OPTIM -- this being that all independent variables should contribute to the total objective function. Although one independent variable may have more significance than another, "levelizing of effects" reduces the chance of completely "washing-out" a less-significant independent variable. Levelizing of effects is accomplished by adjusting the reduction of the individual δx_i 's in the following fashion:

1. The most-significant independent variable, x_i , is divided by the maximum reduction, r_{max} (normally 2.0)

$$\delta x'_i = \delta x_i / r_{max}, \quad (\text{Eq. 11})$$

2. Lesser-significant independent variables, x_j 's, are divided by a factor between r_{max} and 1.00 reflecting the ratio of the quadratic β coefficients, β_{ii}/β_{jj} .

$$\delta x'_j = \delta x_j [\min(r_{max}, \sqrt{\frac{|\beta_{ii}|}{|\beta_{jj}|}})] / r_{max} \quad (\text{Eq. 12})$$

Reduction of δx_i 's is also an important feature which is necessary as the optimum is being approached. In essence OPTIM is looking in finer detail in the region of the optimum.

Acceptance of Predicted Optimum

Several tests of the function and the predicted optimum are made before the predicted optimum is accepted as the base point for the next iteration.

The experiment yielding the most optimal of all samples is retained for future reference by OPTIM. If this pseudo-optimum from previous iterations is more optimal than any experiment of the present iteration, it is included as an additional experiment. Normally the pseudo-optimum will be replaced at each iteration, however, this retention of the pseudo-optimum will eliminate the possibility of diverging if the tendency occurs.

The following tests of the function and the predicted optimum are made:

1. Analysis of the coefficients of the quadratic equation reveals whether a saddle exists or the function indicates maximizing or minimizing. Analysis of the coefficients of independent variables in pairs of x_i , x_j for $i \neq j$ proceeds as follows:

$$t_{ij} = B^2 - AC \quad (\text{Eq. 13})$$

$$\text{where } B = \frac{\partial^2 y}{\partial x_i \partial x_j} = \beta_{ij} \quad (\text{Eq. 14a})$$

$$A = \frac{\partial^2 y}{\partial x_i^2} = 2\beta_{ii} \quad (\text{Eq. 14b})$$

$$C = \frac{\partial^2 y}{\partial x_j^2} = 2\beta_{jj} \quad (\text{Eq. 14c})$$

- a. If $t_{ij} > 0$, a saddle exists with respect to x_i and x_j , and the predicted optimum is ignored.

- b. If $t_{ij} = 0$, an indeterminate function exists, and the predicted optimum is ignored.
 - c. If $t_{ij} < 0$, and $A + C > 0$, the function indicates a minimum exists, or if $A + C < 0$, the function indicates a maximum exists.
If the extremum indicated by $A + C$ is contrary to the problem solution of minimizing or maximizing the predicted optimum is ignored.
2. If the average error-of-fitting is greater than a specified amount (currently 10%), the predicted optimum is ignored.
3. If the predicted optimum is less optimal than the pseudo-optimum retained from previous iterations, the predicted optimum is ignored.

Only when all of the above three tests are satisfied will the predicted optimum be accepted as the base point for the next iteration. If any test fails the pseudo-optimum is used.

Internal Alteration of the Minimal Sampling Set

When a minimal sampling set is used, additional analysis of the interaction coefficients of the quadratic function will allow if necessary the alteration of the sampling set to investigate more optimal regions.

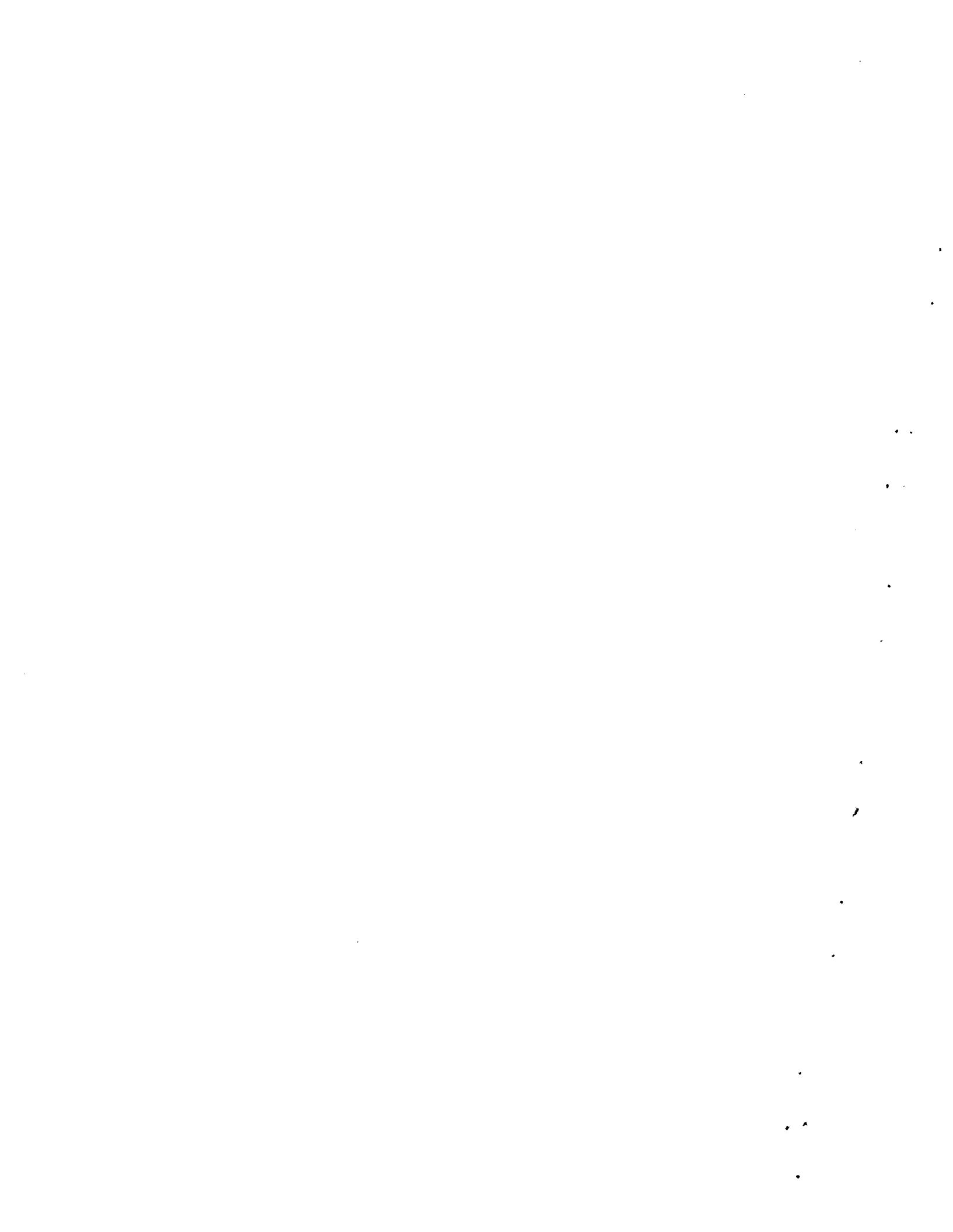
If the contribution to the objective function of any interaction term is contrary to the direction of optimization, the sample set measuring this interaction term is altered by inspecting each respective linear term such that the sampling will be performed in a more optimal region.

Partial Derivative Printout

Included in the OPTIM computer printout, (Appendix C) immediately after the tabulation of each experiment are the partial derivatives for that experiment. These may assist in determining optimizations trends, however they are not used internally by OPTIM.

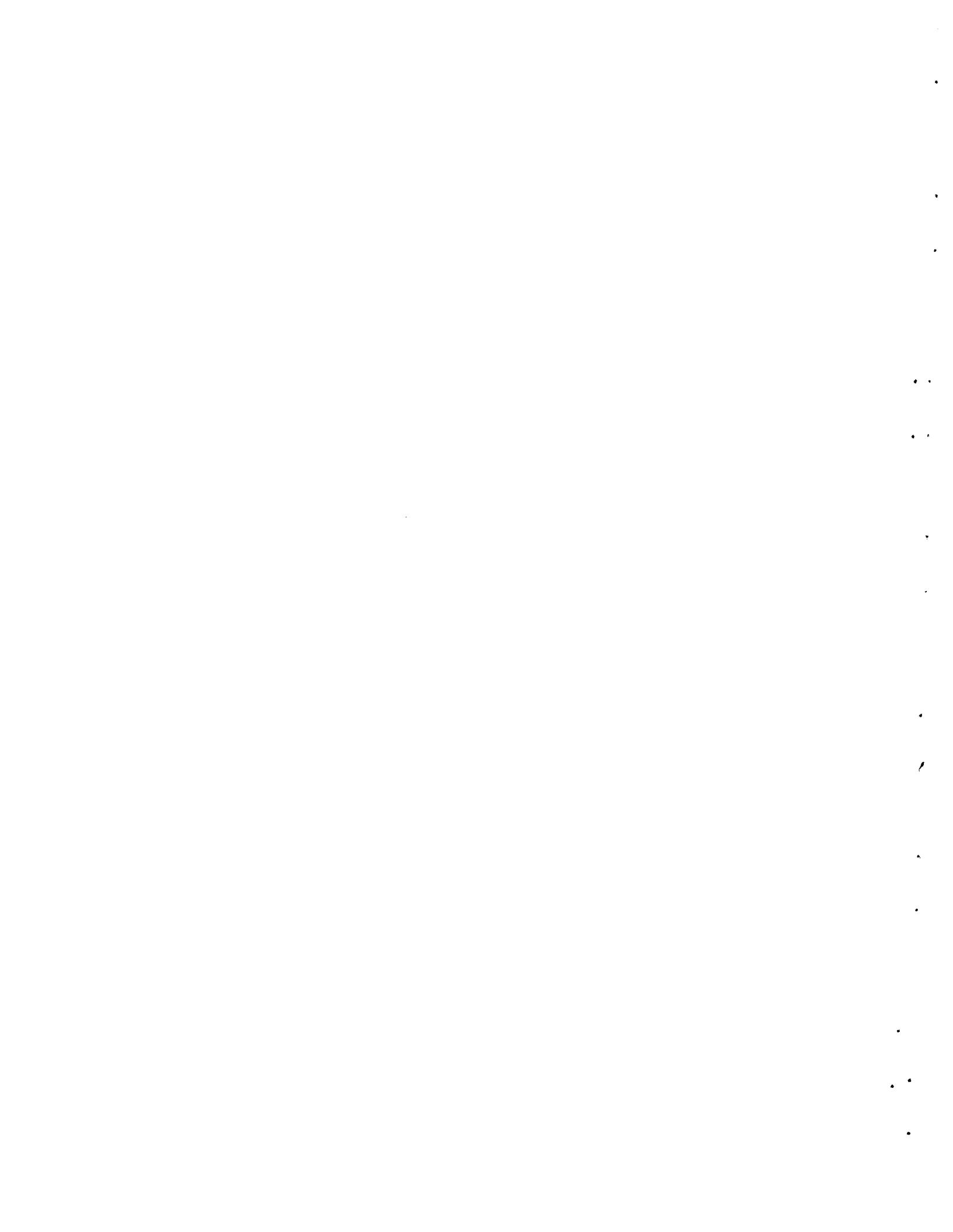
ACKNOWLEDGEMENTS

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APPENDIX A

INPUT REQUIREMENTS OF OPTIM

APPENDIX A

INPUT REQUIREMENTS OF OPTIM

Input to OPTIM consists of three types of input cards: (1) two control cards (Figures 2 & 3), (2) n independent variable cards (Figure 4), and (3) the sample set cards (Figure 5). Additional to these cards are the data cards necessary to the experiment routines which may vary according to the particular optimization problem. These may be divided into ten-or-less segments as defined by the first control card. A segment is defined as a set of basic input cards which remain unaltered for an optimization run. This implies that the changing independent variables are input via "change" cards generated by Subroutine GENRAT and inserted between the segments.

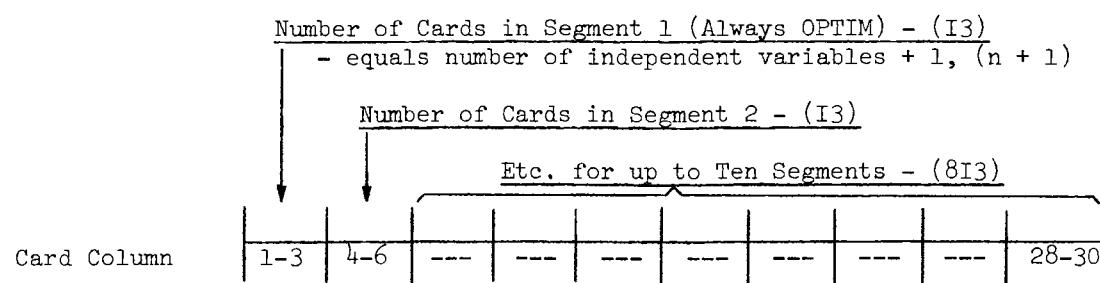


Figure 2. Control Card #1 Format

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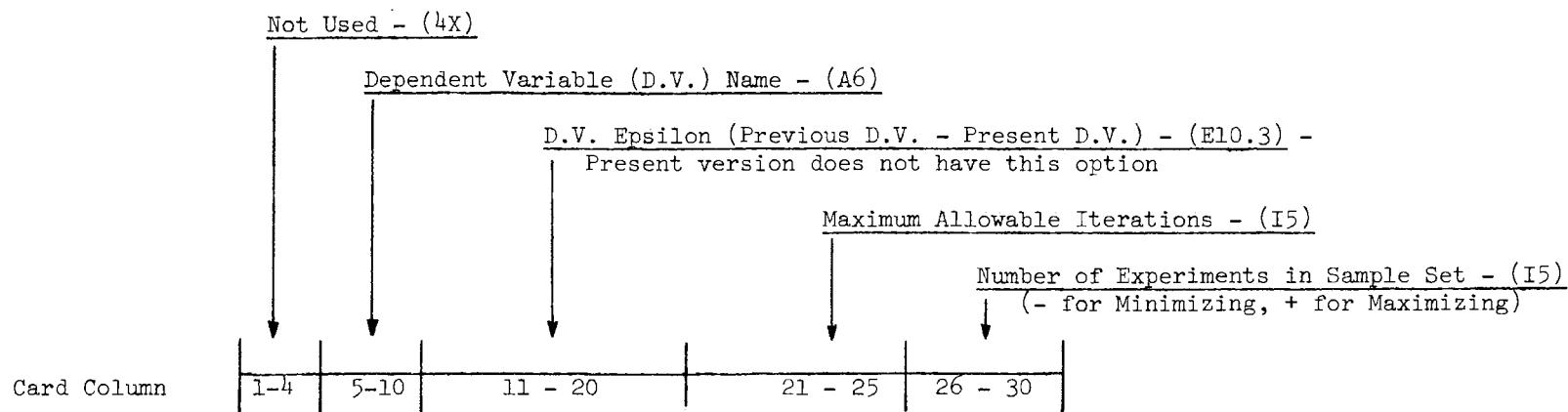


Figure 3. Control Card #2 Format

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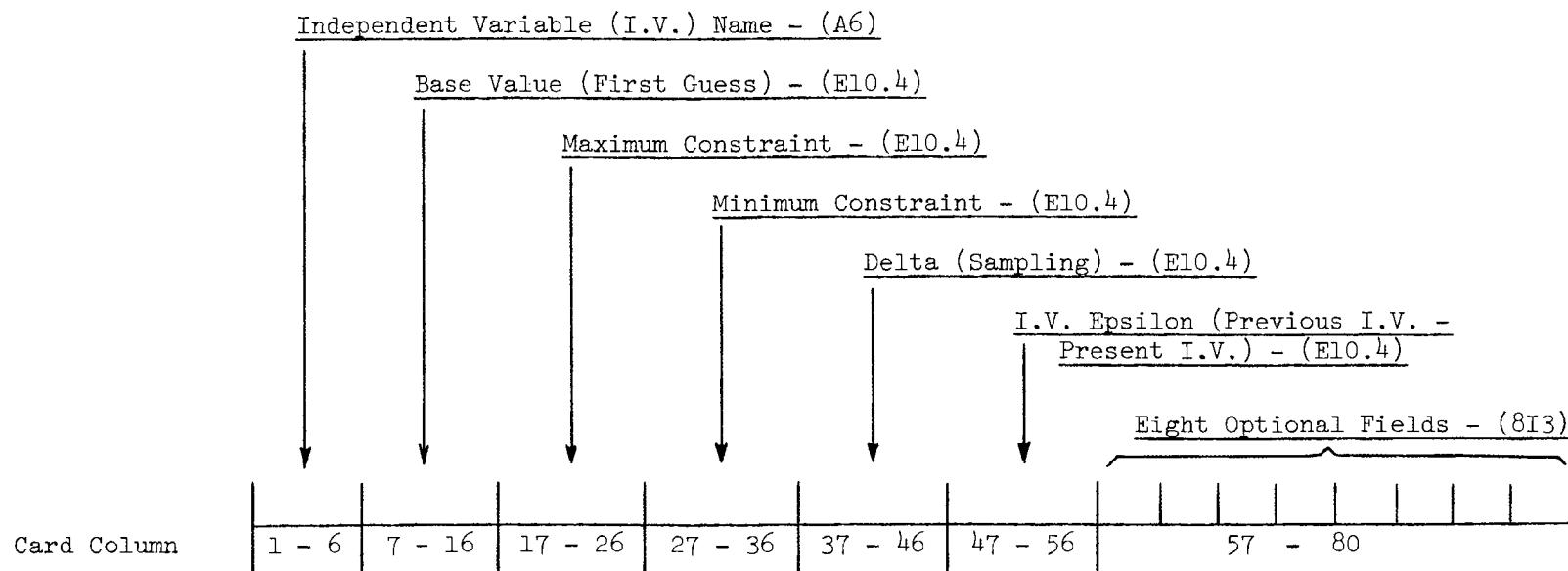


Figure 4. Independent Variable Card Format

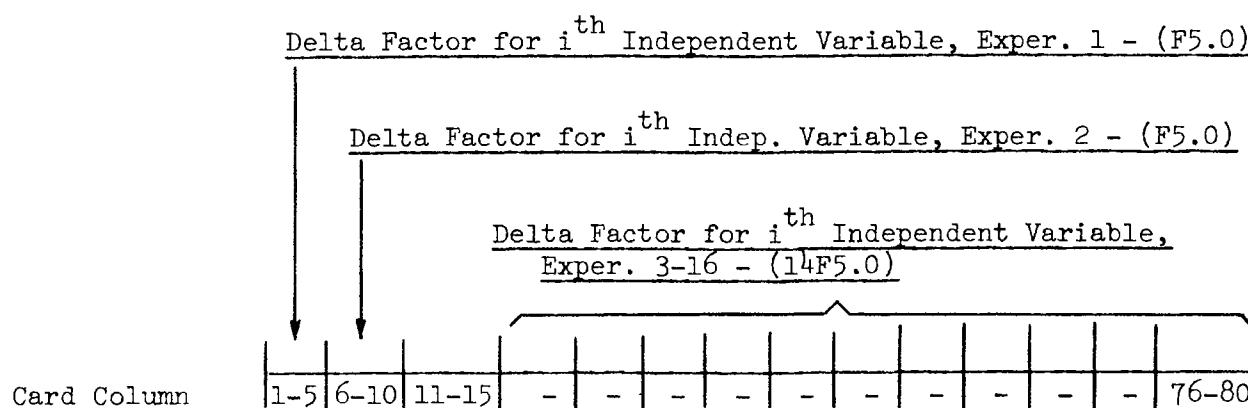


Figure 5. Sample Set Card Format

APPENDIX B

FORTRAN SOURCE PROGRAM LISTING

APPENDIX B

FORTRAN SOURCE PROGRAM LISTING

The following contains a source program listing of the present version of OPTIM which is linked to the Fuel Cycle Analysis Chain of codes, FULCYC. OPTIM is written in FORTRAN V language for the UNIVAC 1108 computer and makes use of several magnetic drum files established within the OPTIM subroutine.

A flow diagram of OPTIM is illustrated in Figure 6.

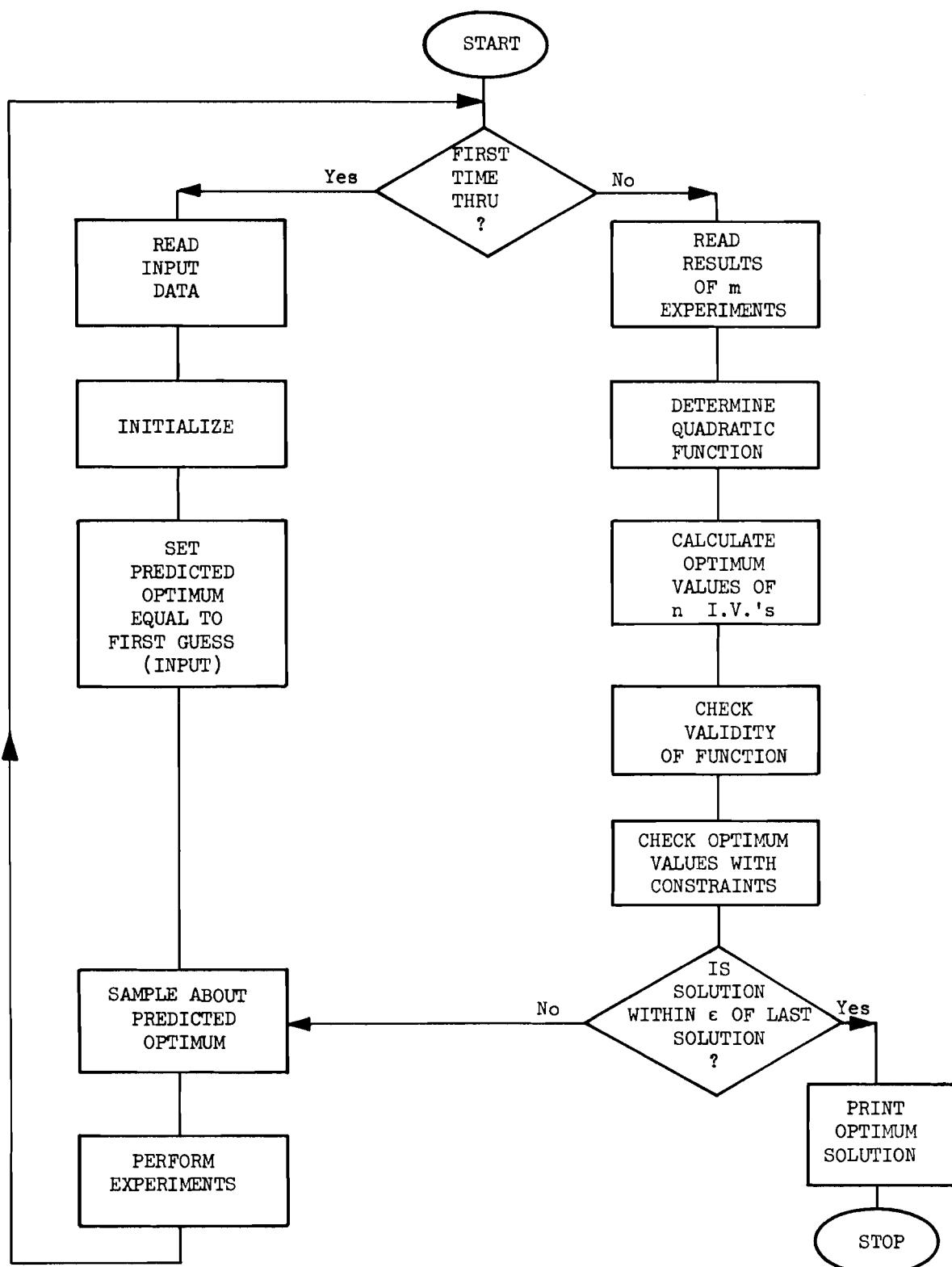


FIGURE 6. FLOW DIAGRAM OF OPTIM

Source Program Listing

```
1      COMO* FCOPY
2      COMMON IDUM(100)
3      COMMON DVNAM,EPSIY,NOIV,NOSAM,MAXITR,NEXCH,LASCH,ITRSW,NOITR
4      COMMON VARNAME(6),BASE(6),MAX(6),MIN(6),DELTA(6),EPSIX(6),IVF(6,8)
5      COMMON X(6,77),Y(77),XF(6),YF,X0(6),Y0,SMPSET(6,77),DELT(6)
6      COMMON NS(10),SC(14,100),MINMAX,JEPS,MM,MAXY,MINY,NBEST
7      COMMON XP(28,77),XINV(77,28),FACTOR(6),A(28,28),D(6,1),BETA(1,28)
8      REAL MAX,MIN
9      C
10     DIMENSION ICD(6),IFD(6),NCC(6),NSQ(6)
11     EQUIVALENCE (ICD(1),IVF(1,1)),(IFD(1),IVF(1,2)),(NCC(1),IVF(1,3))
12     EQUIVALENCE (NSQ(1),IVF(1,4))
13     EQUIVALENCE (NS0,NS(1)),(NS1,NS(2)),(NS2,NS(3)),(NS3,NS(4))
14     END
15     C
16     C
```

```

1*      SUBROUTINE OPTIM
2*      INCLUDE COMO,LIST
3*      C
3*      COMMON/ IDUM(100)
3*      COMMON/ DVNAM,EPSIY,NOIV,NOSAM,MAXITR,NEXCH,LASCH,ITRSW,NOITR
3*      COMMON/ VARNAM(6),BASE(6),MAX(6),MIN(6),DELTAT(6),EPSIX(6),IVF(6,8)
3*      COMMON/ X(6,77),Y(77),XF(6),YF,X0(6),Y0,SMPSET(6,77),DELTS(6)
3*      COMMON/ NS(10),SC(14,100),MINMAX,JEPS,MMT,MAXY,MINY,NBEST
3*      COMMON/ XP(28,77),XINV(77,28),FACTOR(6),A(28,28),D(6,1),BETA(1,28)
3*      REAL MAX,MIN
3*      C
3*      DIMENSION ICD(6),IFD(6),NCC(6),NSQ(6)
3*      EQUIVALENCE (ICD(1),IVF(1,1)),(IFD(1),IVF(1,2)),(NCC(1),IVF(1,3))
3*
3*      EQUIVALENCE (NSQ(1),IVF(1,4))
3*      EQUIVALENCE (NS0,NS(1)),(NS1,NS(2)),(NS2,NS(3)),(NS3,NS(4))
3*      END
3*      C
3*      C
4*      DIMENSION DIFV(6)
5*      DIMENSION PARTD(6),XFP(6)
6*      C
7*      COMMON /TABULZ/ T5(7),T7(7),T8(7),T9(7),T10(7),T29(7),T6(7)
8*      T17(7),T18(7),T19(7)
9*      C
10*     INTEGER T19
11*     C
12*     100 FORMAT (10I3)
13*     110 FORMAT (4X,A6,E10.3,2I5)
14*     120 FORMAT (A6,5E10.4,8I3)
15*     130 FORMAT (13A6,A2)
16*     131 FORMAT (73X,A1)
17*     132 FORMAT (I3,12A6,A5)

```

```
18* 140 FORMAT (//6H NUMBER OF INDEPENDENT VARIABLES SPECIFIED EXCEEDS THE
19* 1MAXIMUM. )
20* 150 FORMAT (16F5.5)
21* 160 FORMAT (1H1)
22* 200 FORMAT (1H1//10X,13HITERATION NO.,I3 /11H0EXPERIMENT,14X
23* 1,17HFUNCTION PERCENT /10H NUMBER ,5X,A6,5X
24* 2,15HAPPROX. ERROR ,4X,6{6X,A6}//)
25* 205 FORMAT (25H0 MAXIMUM OF EXPERIMENTS/I6,3X,1PE12.5,25X,1P6E12.5)
26* 206 FORMAT (25H0 MINIMUM OF EXPERIMENTS/I6,3X,1PE12.5,25X,1P6E12.5)
27* 220 FORMAT (I6,3X,1P2E12.5,0PF9.3,4X,1P6E12.5)
28* 222 FORMAT (46X,1P6E12.4)
29* 225 FORMAT (31H0 AVERAGE ERROR IN FUNCTION =,F11.3/)
30* 250 FORMAT (20H EFFECT OF VARYING ,A6,2H =,F7.3,F8.3)
31* 350 FORMAT (25H0 OPTIMUM OF FUNCTION /21X,1PE12.5,13X,1P6E12.5)
32* 360 FORMAT (/ 15X, 14HPREVIOUS VALUE, 17X,1P6E12.5)
33* 361 FORMAT ( 15X, 13HPRESENT VALUE, 18X,1P6E12.5)
34* 362 FORMAT (/ 15X, 19HABSOLUTE DIFFERENCE, 12X,1P6E12.5)
35* 363 FORMAT ( 15X, 17HEPSILON REQUESTED, 14X,1P6E12.5)
36* 400 FORMAT (58H0 THE ITERATION HAS CONVERGED TO THE FOLLOWING SOLUTION
37* 1N. /)
38* 450 FORMAT (34H0 SOLUTION HAS NOT BEEN REACHED IN I4,10H ATTEMPTS.)
39* IF (T19(2) .GT. T19(1) ) GO TO 1100
40* CALL SETDR (5,265720,150000,T5)
41* CALL SETDR (9,280720,40000,T9)
42* LU = 2
43* GO TO 1150
44* 1100 LU = 19
45* REWIND 19
46* 1150 CONTINUE
47* READ (LU,100) NEXCH,LASCH,ITRSW
```

```

48*      WRITE (3,100) NEXCH,LASCH,ITRSW
49*      IF (ITRSW .GT. 0) GO TO 2000
50*      C
51*      C      READ INPUT DATA
52*      C
53*      1300 READ (2,100) (NS(I),I=1,10)
54*      WRITE(3,100) (NS(I),I=1,10)
55*      NOIV = NS0 - 1
56*      READ (2,110) DVNAM,EPSIY,MAXITR,NOSAM
57*      WRITE(3,110) DVNAM,EPSIY,MAXITR,NOSAM
58*      MINMAX = NOSAM/IABS(NOSAM)
59*      NOSAM = IABS(NOSAM)
60*      DO 1400 I=1,NOIV
61*      READ (2,120) VARNAM(I),BASE(I),MAX(I),MIN(I),DELTAT(I),EPSIX(I),
62*      1,(IVF(I,J),J=1,8)
63*      WRITE(3,120) VARNAM(I),BASE(I),MAX(I),MIN(I),DELTAT(I),EPSIX(I),
64*      1,(IVF(I,J),J=1,8)
65*      X0(I) = BASE(I)
66*      IF (DELTAT(I)) 1350,1345,1350
67*      1345 DELTAT(I) = (MAX(I) - MIN(I))/10.0
68*      1350 DELTS(I) = DELTA(I)
69*      1400 CONTINUE
70*      JEPS = 2
71*      IF (EPSIY) 1490,1500,1490
72*      1490 JEPS = 1
73*      1500 CONTINUE
74*      DO 1600 I=1,NOIV
75*      READ (2,150) (SMPSET(I,J),J=1,NOSAM)
76*      WRITE(3,150) (SMPSET(I,J),J=1,NOSAM)
77*      1600 CONTINUE
78*      IF (NOSAM .LE. 77) GO TO 1700
79*      WRITE ( 3,140)
80*      CALL CHAIN (0,4)
81*      1700 CONTINUE
82*      IC1 = 1
83*      LC1 = NS1
84*      READ (2,130) ((SC(I,J),I=1,14),J=IC1,LC1)
85*      WRITE(3,130) ((SC(I,J),I=1,14),J=IC1,LC1)
86*      IC2 = LC1 + 1
87*      LC2 = LC1 + NS2
88*      DO 1730 J=IC2,LC2
89*      IF (J .EQ. (IC2+4)) GO TO 1725
90*      READ (2,130) (SC(I,J),I=1,14)
91*      WRITE (3,130) (SC(I,J),I=1,14)
92*      GO TO 1730
93*      1725 READ (2,132) N41,(SC(I,J),I=1,13)
94*      WRITE (3,132) N41,(SC(I,J),I=1,13)
95*      1730 CONTINUE
96*      IC3 = LC2 + 1

```

```

97*      LC3 = LC2 + NS3
98*      READ (2,130) ((SC(I,J), I=1,14), J=IC3,LC3)
99*      WRITE(3,130) ((SC(I,J), I=1,14), J=IC3,LC3)
100*     CALL SETDR (17,257000, 8000,T17)
101*     CALL SETDR (18,265000, 698,T18)
102*     C----- UNIT 4 IS WRITTEN TO IN SUBROUTINE ITR WHICH
103*     C----- IS REFERRED TO AS UNIT 18 IN OPTIM
104*     CALL SETDR ( 4,265000, 698,T18)
105*     CALL SETDR (19,246000,11000,T19)
106*     CALL SETDR ( 2,246000,11000,T19)
107*     GO TO 4200
108*     C
109*     C----- READ COMMON AND DATA
110*     C
111*     2000 REWIND 17
112*     CALL NTRAN (17,2,8000, IDUM, IST)
113*     2001 IF (IST + 1) 2002,2001,2003
114*     2002 CALL EXIT
115*     2003 CONTINUE
116*     REWIND 18
117*     MINY = 1
118*     MAXY = 1
119*     DO 2200 I=1,NOSAM
120*     READ (18) Y(I)
121*     IF (Y(I) .LT. Y(MINY))   MINY = I
122*     IF (Y(I) .GT. Y(MAXY))  MAXY = I
123*     2200 CONTINUE
124*     REWIND 18
125*     NOSAMS = NOSAM
126*     NBEST = MAXY
127*     IF (MINMAX .LT. 0)  NBEST = MINY
128*     IF (NOITR .LT. 1)  GO TO 2400
129*     IF (Y(77)*MINMAX .LT. Y(NBEST)*MINMAX)  GO TO 2400
130*     NOSAM = NOSAM + 1
131*     Y(NOSAM) = Y(77)
132*     DO 2250 I=1,NOIV
133*     X(I,NOSAM) = X(I,77)
134*     2250 CONTINUE
135*     IF (MINMAX .GT. 0)  MAXY = 77
136*     IF (MINMAX .LT. 0)  MINY = 77
137*     GO TO 2700
138*     2400 Y(77) = Y(NBEST)
139*     DO 2450 I=1,NOIV
140*     X(I,77) = X(I,NBEST)
141*     2450 CONTINUE
142*     2700 CALL OPTIMZ
143*     WRITE (3,200) NOITR,DVNAM,(VARNAM(I),I=1,NOIV)
144*     YERT = 0.0
145*     DO 2715 I=1,NOSAM

```

```
146*      DO 2710 J=1,NOIV
147*      XFP(J) = X(J,I)
148* 2710  XF(J) = X(J,I)
149*      CALL PART (XFP,PARTD)
150*      CALL FUNCT
151*      YER = (Y(I) - YF) / (Y(MAXY) - Y(MINY))*100.
152*      YERT = YERT + ABS(YER)
153*      WRITE (3,220) I,Y(I),YF,YER,(X(J,I),J=1,NOIV)
154*      WRITE (3,222) (PARTD(IV),IV=1,NOIV)
155* 2715 CONTINUE
156*      YERT = YERT/NOSAM
157*      WRITE (3,225) YERT
158*      IX = 0
159*      DO 2717 I=1,NOIV
160*      PCON1 = (Y(IX+2*I) - Y(1))/(Y(MAXY) - Y(MINY))*100.
161*      PCON2 = (Y(IX+2*I+1) - Y(1))/(Y(MAXY) - Y(MINY))*100.
162*      WRITE (3,250) VARNAM(I) , PCON1,PCON2
163* 2717 CONTINUE
164*      IF (MINMAX .LT. 0) GO TO 2718
165*      WRITE (3,205) MAXY,Y(MAXY),(X(J,MAXY),J=1,NOIV)
166*      GO TO 2719
167* 2718 WRITE (3,206) MINY,Y(MINY),(X(J,MINY),J=1,NOIV)
168* 2719 CONTINUE
169* C
170* C      CHECK FOR SADDLEPOINT OR INVALID FUNCTION
171* C
172*      CALL CHECK
173* C
174* C      CHECK CONSTRAINTS
175* C
176*      DO 3000 I=1,NOIV
177*      IF (BASE(I) .LT. MAX(I) - DELTA(I) ) GO TO 2900
178*      BASE(I) = MAX(I) - DELTA(I)
179*      GO TO 3000
180* 2900 IF (BASE(I) .GT. MIN(I) + DELTA(I) ) GO TO 3000
181*      BASE(I) = MIN(I) + DELTA(I)
182* 3000 XF(I) = BASE(I)
183*      CALL PART (XF,PARTD)
184*      CALL FUNCT
185*      WRITE (3,350) YF,(XF(I),I=1,NOIV)
186*      WRITE (3,222) (PARTD(I),I=1,NOIV)
187*      IF (MINMAX .LT. 0) GO TO 3050
188*      IF (YERT .LT. 10.0 .AND. Y(MAXY) .LT. YF ) GO TO 3200
189*      DO 3020 I=1,NOIV
190* 3020 XF(I) = X(I,MAXY)
191*      YF = Y(MAXY)
192*      IF (MAXY - 1) 3200,3400,3200
193* 3050 CONTINUE
194*      IF ((YERT. LT. 10.0) .AND. (Y(MINY). GT. YF)) GO TO 3200
```

```

195*      DO 3100 I=1, NOIV
196*      3100 XF(I) = X(I,MINY)
197*      YF = Y(MINY)
198*      IF (MINY .EQ. 1) GO TO 3400
199*      3200 IF (NOITR .EQ. 0) GO TO 3400
200*      C
201*      C      CHECK TOLERANCES OF INDEPENDENT VARIABLES
202*      C
203*      JJ = 0
204*      DO 3300 I = 1, NOIV
205*      DIFV(I) = ABS (XF(I) - X0(I))
206*      IF (DIFV(I) .GT. EPSIX(I)) JJ = I
207*      3300 CONTINUE
208*      WRITE (3,360) (X0(I), I = 1, NOIV)
209*      WRITE (3,361) (XF(I), I = 1, NOIV)
210*      WRITE (3,362) (DIFV(I), I = 1, NOIV)
211*      WRITE (3,363) (EPSIX(I), I = 1, NOIV)
212*      IF (JJ. EQ. 0) GO TO 9000
213*      3400 NOITR = NOITR + 1
214*      WRITE (3,160)
215*      C
216*      C      RESTORE NO. OF SAMPLES (NOSAM)
217*      C
218*      NOSAM = NOSAMS
219*      DO 3500 I = 1, NOIV
220*      3500 X0(I) = XF(I)
221*      Y0 = YF
222*      C
223*      C      ADJUST DELTAS
224*      C
225*      NPCM = 1
226*      BDM = 0.
227*      IX = NOIV + 1
228*      DO 3600 I=1,NOIV
229*      BD = ABS(BETA(1,IX+I))
230*      IF (BD .LT. BDM) GO TO 3600
231*      NPCM = I
232*      BDM = BD
233*      3600 CONTINUE
234*      WRITE (3,1) (DELTA(I), I=1,NOIV), BD, BDM
235*      1 FORMAT (6H0DELTA/10E12.5)
236*      DO 3700 I=1,NOIV
237*      R = SQRT(ABS(BETA(1,IX+NPCM)/BETA(1,IX+I) ) )
238*      IF (R .GT. 2.00) GO TO 3700
239*      DELTA(I) = DELTA(I)*R/2.0
240*      3700 CONTINUE
241*      WRITE (3,1) (DELTA(I), I=1,NOIV), R
242*      4110 IF (NOITR .EQ. MAXITR) 4200,4150,4150
243*      C

```

```
244* C      MAXIMUM ITERATIONS HAS BEEN EXCEEDED
245* C
246* 4150 WRITE (3,450) NOITR
247*     CALL CHAIN (0,4)
248* C
249* C      GENERATE CHANGE CARDS FOR NEXT ITERATION
250* C
251* 4200 CALL SAMPLE
252*     REWIND 19
253*     ITRSW = 1
254*     WRITE (19,100) NEXCH,LASCH,ITRSW
255*     WRITE (3,100) NEXCH,LASCH,ITRSW
256*     CALL GENRAT
257* GO TO 9100
258* C
259* C      SOLUTION HAS BEEN REACHED
260* C
261* 9000 WRITE (3,400)
262*     WRITE (3,350) YF,(XF(I),I=1,NOIV)
263*     WRITE (3,160)
264*     ITRSW = 2
265*     NEXCH = LASCH
266*     CALL CHAIN(NEXCH,NDUM)
267* C
268* C      PREPARE FOR NEXT ITERATION
269* C
270* 9100 CONTINUE
271*     END FILE 19
272*     REWIND 19
273*     READ (19,130) SKIP
274*     REWIND 17
275*     CALL NTRAN (17,1,8000,1DUM,IST)
276*     9200 IF (IST + 1) 9300,9200,9400
277*     9300 RETURN 0
278*     9400 CONTINUE
279*     REWIND 7
280*     REWIND 8
281*     REWIND 9
282*     CALL CHAIN(NEXCH,NDUM)
283*     RETURN
284*     END
```

```
1*      SUBROUTINE SAMPLE
2*      INCLUDE COM0,LIST
3*      C
3*      COMMON  IDUM(100)
3*      COMMON  DVHNM,EPSTY,NOIV,NOSAM,MAXITR,NEXCH,LASCH,IIRSW,NOITR
3*      COMMON  VARNAM(6),BASE(6),MAX(6),MIN(8),DELTA(6),EPSIX(6),IVF(6,8)
3*      COMMON  X(6,77),Y(77),XF(6),YF,X0(6),Y0,SMPSET(6,77),DELTS(6)
3*      COMMON  NS(10),SC(14,100),MINMAX,JEPS,MM,MAXY,MINY,NBEST
3*      COMMON  XP(28,77),XINV(77,28),FACTOR(6),A(28,28),D(6,1),BETA(1,28)
3*      REAL MAX,MIN
3*      C
3*      DIMENSION ICD(6),IFD(6),NCC(6),NSQ(6)
3*      EQUIVALENCE (ICD(1),IVF(1,1)),(IFD(1),IVF(1,2)),(NCC(1),IVF(1,3))
3*      EQUIVALENCE (NSQ(1),IVF(1,4))
3*      EQUIVALENCE (NS0,NS(1)),(NS1,NS(2)),(NS2,NS(3)),(NS3,NS(4))
3*      END
3*      C
3*      C
4*      DO 2000 J=1,NOSAM
5*      DO 1500 I=1,NOIV
6*      X(I,J) = X0(I) + DELTA(I)*SMPSET(I,J)
7*      1500 CONTINUE
8*      2000 CONTINUE
9*      RETURN
10*     END
```

```
1*      SUBROUTINE PART (XFP,PD)
2*      INCLUDE COMO,LIST
3*      C
4*      COMMON IDUM(100)
5*      COMMON DVNAM,EPSTY,NOIV,NOSAM,MAXITR,NEXCH,LASCH,ITRSW,NOITR
6*      COMMON VARNAM(6),BASE(6),MAX(6),MIN(6),DELTA(6),EPSIX(6),IVF(6,8)
7*      COMMON X(6,77),Y(77),XF(6),YF,X0(6),Y0,SMPSET(6,77),DELTS(6)
8*      COMMON NS(10),SC(14,100),MINMAX,JEPS,MM,MAXY,MINY,NBEST
9*      COMMON XP(28,77),XINV(77,28),FACTOR(6),A(28,28),D(6,1),BETA(1,28)
10*     REAL MAX,MIN
11*
12*      C
13*      DIMENSION ICD(6),IFD(6),NCC(6),NSQ(6)
14*      EQUIVALENCE (ICD(1),IVF(1,1)),(IFD(1),IVF(1,2)),(NCC(1),IVF(1,3))
15*      EQUIVALENCE (NSQ(1),IVF(1,4))
16*      EQUIVALENCE (NSQ,NS(1)),(NS1,NS(2)),(NS2,NS(3)),(NS3,NS(4))
17*      END
18*      C
19*      C
20*      DIMENSION PD(6),XFP(6)
21*      DIMENSION B(28,77)
22*      EQUIVALENCE (XP,B)
23*      C
24*      DO 200 I=1,NOIV
25*      PD(I)= -D(I,I)
26*      DO 100 J=1,NOIV
27*      PD(I)= PD(I) + B(J,I)*(XFP(J) - X(J,1))/DELTA(J)
28*      100 CONTINUE
29*      200 CONTINUE
30*      RETURN
31*      END
```

```
1*      SUBROUTINE GENRAT
2*      INCLUDE COMO,LIST
3*      C
4*      COMMON  IDUM(100)
5*      COMMON  DVNAM,EPSIY,NOIV,NOSAM,MAXITR,NEXCH,LASCH,ITRSW,NOITR
6*      COMMON  VARNAM(6),BASE(6),MAX(6),MIN(6),DELTAT(6),EPSIX(6),IVF(6,8)
7*      COMMON  X(6,77),Y(77),XF(6),YF,X0(6),Y0,SMPSET(6,77),DELTS(6)
8*      COMMON  NS(10),SC(14,100),MINMAX,JEPS,MM,MAXY,MINY,NBEST
9*      COMMON  XP(28,77),XINV(77,28),FACTOR(6),A(28,28),D(6,1),BETA(1,28)
10*     REAL MAX,MIN
11*    C
12*    DIMENSION ICD(6),IFD(6),NCC(6),NSQ(6)
13*    EQUIVALENCE (ICD(1),IVF(1,1)),(IFD(1),IVF(1,2)),(NCC(1),IVF(1,3))
14*    EQUIVALENCE (NSQ(1),IVF(1,4))
15*    EQUIVALENCE (NSQ,NS(1)),(NS1,NS(2)),(NS2,NS(3)),(NS3,NS(4))
16*    END
17*    C
18*    C
19*    100 FORMAT (13A6,A2)
20*    200 FORMAT (2I3,I14)
21*    300 FORMAT (2I3,E14.6)
22*    400 FORMAT (3I4,E12.5)
23*    500 FORMAT (80X)
24*    600 FORMAT (6H 4 1,1I4)
25*    700 FORMAT (80X/80X/80X)
26*    750 FORMAT (I3,12A6,A5)
27*    IC1 = 1
28*    LC1 = NS1
29*    WRITE (19,100) ((SC(I,J),I=1,14),J=IC1,LC1)
30*    WRITE (3,100) ((SC(I,J),I=1,14),J=IC1,LC1)
31*    JUSTG = 1HG
32*    DO 1400 J=IC1,LC1
33*    IF (FLD(6,6,SC(13,J)) .NE. FLD(0,6,JUSTG) ) GO TO 1400
34*    NOGC = J
35*    GO TO 1410
36*    1400 CONTINUE
37*    1410 CONTINUE
38*    IFLD = 1
39*    NTACC = NOSAM*2 - 1
40*    DO 1500 I=1,NOIV
41*    IF (NCC(I) .EQ. 0) NTACC = NTACC + NOSAM
42*    1500 CONTINUE
43*    WRITE (19,200) NOGC,IFLD,NTACC
44*    WRITE (3,200) NOGC,IFLD,NTACC
45*    WRITE (19,700)
46*    WRITE (3,700)
47*    NTLCC = 0
```

```
33*      DO 2000 J=1,NOSAM
34*      NLCC = 0
35*      DO 1800 I=1,NOIV
36*      IF (NCC(I) .EQ. 0) GO TO 1800
37*      NLCC = NLCC + NCC(I)
38*      1800 CONTINUE
39*      NTLCC = NTLCC + NLCC
40*      WRITE (19,600) NLCC
41*      WRITE ( 3,600) NLCC
42*      DO 1900 I=1,NOIV
43*      IF (NCC(I) .NE. 0) GO TO 1900
44*      WRITE (19,300) ICD(I),IFD(I),X(I,J)
45*      WRITE ( 3,300) ICD(I),IFD(I),X(I,J)
46*      1900 CONTINUE
47*      IF (J .EQ. NOSAM ) GO TO 2000
48*      WRITE (19,500)
49*      WRITE ( 3,500)
50*      2000 CONTINUE
51*      IC2 = LC1 + 1
52*      LC2 = LC1 + NS2
53*      DO 2400 J=IC2,LC2
54*      IF (J .NE. IC2+4) GO TO 2300
55*      WRITE (19,750) NTLCC,(SC(I,J),I=1,13)
56*      WRITE (-3,750) NTLCC,(SC(I,J),I=1,13)
57*      GO TO 2400
58*      2300 WRITE (19,100) (SC(I,J),I=1,14)
59*      WRITE ( 3,100) (SC(I,J),I=1,14)
60*      2400 CONTINUE
61*      DO 3000 J=1,NOSAM
62*      DO 2800 I=1,NOIV
63*      IF (NCC(I) .EQ. 0) GO TO 2800
64*      WRITE (19,400) ICD(I),IFD(I),NSQ(I),X(I,J)
65*      WRITE ( 3,400) ICD(I),IFD(I),NSQ(I),X(I,J)
66*      IF (NCC(I) .EQ. 1) GO TO 2800
67*      ICK = 16
68*      IF (ICD(I) .EQ. ICK) ICK = 17
69*      WRITE (19,400) ICK,IFD(I),NSQ(I),X(I,J)
70*      WRITE ( 3,400) ICK,IFD(I),NSQ(I),X(I,J)
71*      2800 CONTINUE
72*      3000 CONTINUE
73*      IC3 = LC2 + 1
74*      LC3 = LC2 + NS3
75*      WRITE (19,100) ((SC(I,J),I=1,14),J=IC3,LC3)
76*      WRITE (-3,100) ((SC(I,J),I=1,14),J=IC3,LC3)
77*      RETURN
78*      END
```

```
1*      SUBROUTINE FUNCT
2*      INCLUDE 'COMO,LIST'
3*      C
4*      COMMON  IDUM(100)
5*      COMMON  DVNAM,EPSIY,NOIV,NOSAM,MAXITR,NEXCH,LASCH,ITRSW,NOITR
6*      COMMON  VARNAM(6),BASE(6),MAX(6),MIN(6),DELTA(6),EPSIX(6),IVF(6,8)
7*      COMMON  X(6,77),Y(77),XF(6),YF,X0(6),Y0,SMPSET(6,77),DELTS(6)
8*      COMMON  NS(10),SC(14,100),MINMAX,JEPS,MM,MAXY,MINY,NBEST
9*      COMMON  XP(28,77),XINV(77,28),FACTOR(6),A(28,28),D(6,1),BETA(1,28)

10*      REAL MAX,MIN
11*      C
12*      DIMENSION ICD(6),IFD(6),NCC(6),NSQ(6)
13*      EQUIVALENCE (ICD(1),IVF(1,1)),(IFD(1),IVF(1,2)),(NCC(1),IVF(1,3))
14*      EQUIVALENCE (NSQ(1),IVF(1,4))
15*      EQUIVALENCE (NS0,NS(1)),(NS1,NS(2)),(NS2,NS(3)),(NS3,NS(4))
16*      END
17*      C
18*      C
19*      DIMENSION XFF(6)
20*      C
21*      DO 1000 I=1,NOIV
22* 1000 XFF(I) = (XF(I) - X(I,1))/DELTA(I)
23*      YF = BETA(1,1)
24*      DO 1500 J=1,NOIV
25*          YF = YF + BETA(1,J+1)*XFF(J) + BETA(1,NOIV+J+1)*XFF(J)**2
26* 1500 CONTINUE
27*      MM = 2*NOIV + 2
28*      NN = NOIV - 1
29*      DO 1700 K=1,NN
30*          JJ = K + 1
31*          DO 1700 KK=JJ,NOIV
32*              YF = YF + BETA(1,MM)*XFF(K)*XFF(KK)
33* 1700 MM = MM + 1
34*      RETURN
35*      END
```

```

1*      SUBROUTINE GENINV
2*      INCLUDE COMO,LIST
3*      C
4*      COMMON IDUM(100)
5*      COMMON DVNAM,EPSIY,NOIV,NOSAM,MAXITR,NEXCH,LASCH,ITRSW,NOITR
6*      COMMON VARNAM(6),BASE(6),MAX(6),MIN(6),DELTAT(6),EPSIXT(6),IVF(6,8)
7*      COMMON X(6,77),Y(77),XF(6),YF,X0(6),Y0,SMPSET(6,77),DELTS(6)
8*      COMMON NS(10),SC(14,100),MINMAX,JEPS,MM,MAXY,MINY,NBEST
9*      COMMON XP(28,77),XINV(77,28),FACTOR(6),A(28,28),D(6,1),BETA(1,28)
10*     REAL MAX,MIN
11*     C
12*     DIMENSION ICD(6),IFD(6),NCC(6),NSQ(6)
13*     EQUIVALENCE (ICD(1),IVF(1,1)),(IFD(1),IVF(1,2)),(NCC(1),IVF(1,3))
14*     EQUIVALENCE (NSQ(1),IVF(1,4))
15*     EQUIVALENCE (NS0,NS(1)),(NS1,NS(2)),(NS2,NS(3)),(NS3,NS(4))
16*     END
17*     C
18*     E
19*     DIMENSION Z(77,28),NORM(28),Q(77,28),C(77,28),E(77,28),YN(28,28)
20*     EQUIVALENCE (Z,I),(NOSAM,NOSt)
21*     REAL NORM
22*     NORM(1) = 0.0
23*     DO 150 I=1,NOS
24*     150 NORM(I) = NORM(I) + XP(1,I)**2
25*     DO 200 I=1,NOS
26*     200 Q(I,I) = XP(I,I)/SQRT(NORM(I))
27*     DO 400 J = 2,MM
28*     400 NORM(J) = 0.0
29*     L = J-1
30*     DO 250 I=1,MM
31*     250 M=1,MM
32*     A(I,M) = 0.0
33*     DO 250 K=1,NOS
34*     250 A(I,M) = A(I,M) + XP(I,K)*Q(K,M)
35*     DO 350 I = 1,NOS
36*     350 C(I,J) = XP(J,I)
37*     DO 300 K = 1,L
38*     300 C(I,J) = C(I,J) - A(J,K)*Q(I,K)
39*     350 NORM(J) = NORM(J) + C(I,J)**2
40*     DO 400 I = 1,NOS
41*     400 O(I,J) = C(I,J)/SQRT(NORM(J))
42*     WRITE(3,27)((Q(I,J),J=1,MM), I=1,NOS)
43*     2 FORMAT(2H00/(15E8.2))
44*     DO 450 I = 1,MM
45*     450 J = 1,MM
46*     E(I,J) = 0.0
47*     IF(I-J)450,440,450
48*     440 E(I,J) = E(I,J) + 1.0
49*     450 CONTINUE

```

```
35*      DO 500 I = 1,MM
36*      500 Z(I,I) = E(I,I)/SQRT(NORM(I))
37*      DO 600 J=2,MM
38*      L = J-1
39*      DO 600 I = 1,MM
40*      YN(I,J) = E(I,J)
41*      DO 550 K = 1,L
42*      550 YN(I,J) = YN(I,J) - A(J,K)*Z(K,I)
43*      600 Z(J,I) = YN(I,J)/SQRT(NORM(J))
44*      WRITE(3,4) ((Z(J,I), J=1,MM), I=1,MM)
45*      4 FORMAT(2H0Z/(15E8.2))
46*      DO 625 I=1,N05
47*      DO 625 J=1,MM
48*      XINV(I,J) = 0.0
49*      DO 625 K=1,MM
50*      625 XINV(I,J) = XINV(I,J) + Q(I,K)*Z(K,J)
51*      RETURN
52*      END
```

```

1*      SUBROUTINE CHECK
2*      C
3*      INCLUDE COMO,LIST
4*      C
5*      COMMON  IDUM(100)
6*      COMMON  DVNAM,EPSIY,NOIV,NOSAM,MAXITR,NEXCH,LASCH,ITRSW,NOITR
7*      COMMON  VARNAME(6),BASE(6),MAX(6),MIN(6),DETA(6),EPSIX(6),IVF(6,8)
8*      COMMON  X(6,77),Y(77),XF(6),YF,X0(6),Y0,SMPSET(6,77),DELT(6)
9*      COMMON  NS(10),SC(14,100),MINMAX,JEPS,MM,MAXY,MINY,NBEST
10*     COMMON  XP(28,77),XINV(77,28),FACTOR(6),A(28,28),D(6,1),BETA(1,28)
11*     REAL MAX,MIN
12*     C
13*     DIMENSION ICD(6),IFD(6),NCC(6),NSQ(6)
14*     EQUIVALENCE (ICD(1),IVF(1,1)),(IFD(1),IVF(1,2)),(NCC(1),IVF(1,3))
15*     EQUIVALENCE (NSQ(1),IVF(1,4))
16*     EQUIVALENCE (NSQ,NS(1)),(NS1,NS(2)),(NS2,NS(3)),(NS3,NS(4))
17*     END
18*     C
19*     C
20*     DIMENSION SAVXP(28,77), B(28,77)
21*     EQUIVALENCE (XP,B)
22*     C
23*     50 FORMAT (/10G13.6)
24*     100 FORMAT (45H01)DETERMINANT OPTIMUM POINT WITH RESPECT TO ,A6,5H AND
25*           1 ,A6)
26*     150 FORMAT (30H01SADDLE POINT WITH RESPECT TO ,A6,5H AND ,A6)
27*     200 FORMAT (34H01RELATIVE MAXIMUM WITH RESPECT TO ,A6,5H AND ,A6)
28*     250 FORMAT (34H01RELATIVE MINIMUM WITH RESPECT TO ,A6,5H AND ,A6)
29*     C
30*     IR = 0
31*     DO 1000 I = 1,NOIV
32*     IF (BETA(1,I+NOIV+1)*MINMAX .GT. 0.0) GO TO 1000
33*     IR = IR + 1
34*   1000 CONTINUE
35*     IF (IR .EQ. NOIV) GO TO 5000
36*     C
37*     ELIMINATE SADDLES FOR SINGLE INDEP. VAR.
38*     C
39*     DO 1100 I = 1,28
40*     DO 1100 J = 1,NOSAM
41*   1100 SAVXP(I,J) = XP(I,J)
42*     C
43*     GENERATE REDUCED PARTIAL DERIVATIVE MATRIX AND DATA VECTOR
44*     C
45*     DO 1190 I=1,NOIV

```

```
31* 1190 D(I,1) = 0.0
32* DO 1400 I=1,NOIV
33* IF (BETA(1,I+NOIV+1) .GT. 1200,1300,1300
34* 1200 D(I,I) = D(I,I) - BETA(1,I+1)
35* DO 1220 J=1,NOIV
36* IF (B(I,J) .GT. 99E8) GO TO 1220
37* B(I,J) = SAVXP(I,J)
38* 1220 CONTINUE
39* GO TO 1400
40* C
41* C      SET INVALID INDEP. VARIABLES TO WITHIN DELTA OF THE MINIMUM
42* C      OR MAXIMUM CONSTRAINT
43* C
44* 1300 BASE(I) = MAX(I) - DELTA(I)
45* IF (BETA(1,I+1)*MINMAX .LT. 0.0) BASE(I) = MIN(I) + DELTA(I)
46* DO 1310 J=1,NOIV
47* IF (D(J,1) .GT. 99E8) GO TO 1305
48* D(J,1) = D(J,1) - B(J,I)*(BASE(I) - X(I,1)) / DELTA(I)
49* 1305 IF (I .EQ. J) D(J,1) = 99E9
50* B(I,J) = 99E9
51* 1310 B(J,I) = 99E9
52* 1400 CONTINUE
53* C WRITE (3,50) ((B(I,J),I=1,NOIV),J=1,NOIV)
54* C WRITE (3,50) (D(I,1),I=1,NOIV)
55* IR = 0
56* DO 1500 I=1,NOIV
57* IF (D(I,1) .GT. 99E8) GO TO 1500
58* IR = IR + 1
59* D(IR,1) = D(I,1)
60* 1500 CONTINUE
61* IR = 0
62* DO 1600 I=1,NOIV
63* IR = IR + 1
64* IC = 0
65* DO 1550 J=1,NOIV
66* IF (B(I,J) .GT. 99E8) GO TO 1550
67* IC = IC + 1
68* BTIR,IC) = B(I,J)
69* 1550 CONTINUE
70* IF (IC .EQ. 0 ) IR = IR - 1
71* 1600 CONTINUE
72* C WRITE (3,50) T (B(I,J),I=1,IR),J=1,IR)
73* C WRITE (3,50) (D(I,1), I=1,IR)
74* C WRITE (3,50) (BASE(I),I=1,NOIV)
75* IF (IR .LE. 0) GO TO 2400
76* C
77* C      INVERT REDUCED PARTIAL DERIVATIVE MATRIX AND
78* C      SOLVE FOR VALID INDEP. VARIABLES
79* C
```

```
80*      NOS = NOSAM
81*      NOSAM = IR
82*      MM = IR
83*      CALL GEMINV
84*      NOSAM = NOS
85*      DO 2200 I = 1,IR
86*      A(I,1) = 0.0
87*      DO 2200 K=1,IR
88*      2200 A(I,1) = A(I,1) + XINV(I,K)*D(K,1)
89*      IR = 0
90*      DO 2300 I = 1,NOIV
91*      IF(BETA(1,I + NOIV + 1)*MINMAX .GT.0.0) GO TO 2300
92*      IR = IR + 1
93*      BASE(I) = A(IR,1)*DELTAT(I) + X(I,1)
94*      2300 CONTINUE
95*      2400 CONTINUE
96*      C      WRITE(3,50)(BASE(I),I=1,NOIV)
97*      C
98*      C      RESTORE XP MATRIX
99*      C
100*     DO 3000 I = 1,28
101*     DO 3000 J = I,NOSAM
102*     3000 XP(I,J) = SAVXP(I,J)
103*      C
104*      C      CHECK INTERACTIONS TERMS AND
105*      C      CHANGE SAMPLING IF NECESSARY.
106*      C
107*     5000 MM = I + 2 * NOIV
108*     NGOPT = 0
109*     DO 6000 I = 2,NOIV
110*     DO 5600 J = I,NOIV
111*     MM = MM + 1
112*     BB = BETA(1,MM)
113*     AA = 2.*BETA(1,NOIV + I)
114*     CC = 2.*BETA(1,NOIV + J + 1)
115*     TM = AA + CC
116*     TT = BB**2 - AA * CC
117*     IP = I - 1
118*     IF(TT .EQ.0.0) WRITE(3,100) VARNAM(IP),VARNAM(J)
119*     IF(TT .GT. 0.0) WRITE(3,150) VARNAM(IP),VARNAM(J)
120*     IF(TT.LT.0.0 .AND. TM .LT. 0.0) WRITE(3,200)VARNAM(IP),VARNAM(J),
121*     IF(TT.LT.0.0 .AND. TM .GT. 0.0) WRITE(3,250)VARNAM(IP),VARNAM(J)
122*      C      WRITE (3,50) BB,AA,CC,TM,TT
123*      IF (TT .LT. 0.0 .AND. TM*MINMAX .LT. 0.0) GO TO 5600
124*      NGOPT = 1
125*      C
126*      C      CHANGE SAMPLING
127*      C
128*      DO 5400 K = 1,NOSAM
```

```

129*      IF (SMPSET(IP,K) .EQ. 0.0 .OR. SMPSET(J,K) .EQ. 0.0) GO TO 5400
130*      SI = SMPSET(IP,K)/ABS(SMPSET(IP,K))
131*      SJ = SMPSET(J,K)/ABS(SMPSET(J,K))
132*      BBSISJ = BB * SI * SJ
133*      C      WRITE (3,50) BB,SI,SJ,BBSISJ
134*      IF (BBSISJ * MINMAX .GT. 0.0) GO TO 5600
135*      SMPSET(IP,K) = -SMPSET(IP,K)
136*      ST = -SI
137*      BISI = BETA(1,IP + 1) * SI
138*      BJSJ = BETA(1,J + 1) * SJ
139*      SUM = BISI + BJSJ
140*      C      WRITE (3,50) BISI,BJSJ,SUM
141*      IF (SUM * MINMAX .GT. 0.0) GO TO 5600
142*      SMPSET(IP,K) = -SMPSET(IP,K)
143*      SMPSET (J,K) = -SMPSET(J,K)
144*      5400 CONTINUE
145*      5600 CONTINUE
146*      6000 CONTINUE
147*      IF (NGOPT .EQ. 0) RETURN
148*      C      USE BEST EXPERIMENT VALUES OF INDEP. VAR. INSTEAD OF PREDICTED OPTIMUM
149*      C
150*      C
151*      DO 6200 I = 1,NOIV
152*      BASE -(I)- = X(I,NBEST)
153*      6200 CONTINUE
154*      RETURN
155*      END

```

```
1*      SUBROUTINE OPTIMZ
2*      INCLUDE COMO,LIST
3*      C
4*      COMMON  IDUM(100)
5*      COMMON  DVNAM,EPSIY,NOIV,NOSAM,MAXITR,NEXCH,LASCH,ITRSW,NOITR
6*      COMMON  VARNAM(6),BASE(6),MAX(6),MIN(6),DELT(A(6),EPSIX(6),IVF(6,8)
7*      COMMON  X(6,77),Y(77),XF(6),YF,X0(6),Y0,SMPSET(6,77),DELTS(6)
8*      COMMON  NS(10),SC(14,100),MINMAX,JEPS,MM,MAXY,MINY,NBEST
9*      COMMON  XP(28,77),XINV(77,28),FACTOR(6),A(28,28),D(6,1),BETA(1,28)
10*     REAL   MAX,MIN
11*     C
12*     DIMENSION ICD(6),IFD(6),NCC(6),NSQ(6)
13*     EQUIVALENCE (ICD(1),IVF(1,1)),(IFD(1),IVF(1,2)),(NCC(1),IVF(1,3))
14*     EQUIVALENCE (NSQ(1),IVF(1,4))
15*     EQUIVALENCE (NS0,NS(1)),(NS1,NS(2)),(NS2,NS(3)),(NS3,NS(4))
16*     END
17*     C
18*     C
19*     DIMENSION PARTD(6),XFP(6)
20*     DIMENSION B128,77)
21*     EQUIVALENCE (XP,B)
22*     C
23*     NOS = NOSAM
24*     DO 120 I=1,NOS
25*     WRITE (3,110) (X(J,I),J=1,NOIV)
26*     110 FORMAT (2H X,6E15.6)
27*     120 CONTINUE
28*     C
29*     DEVELOP SCALED XP MATRIX FROM X'S (INDEP. VAR.)
30*     C
31*     DO 300 I=1,NOS
32*     XP(1,I) = 1.
33*     MM = 1
34*     DO 200 J=1,2
35*     DO 200 K=1,NOIV
36*     MM = MM + 1
37*     200 XP(MM,I) = ((X(K,I) - X(K,1))/DELT(A(K))**J
38*     MM = NOIV + 1
39*     DO 300 J=1,NN
40*     JJ = J + 1
41*     DO 300 K=JJ,NOIV
42*     MM = MM + 1
43*     300 XP(MM,I) = (X(J,I) - X(J,1))/DELT(A(J)) * (X(K,I) - X(K,1))/DELT(A(K)
```

```

29*      WRITE (3,1) ((XP(I,J),J=1,NOS),I=1,MM)
30*      1 FORMAT (3H0XP/(16E8.2))
31*      C
32*      C      INVERT XP MATRIX
33*      C
34*      CALL GENINV
35*      C
36*      C      FORM IDENTITY MATRIX Y = XP*XINV
37*      C
38*      DO 400 I = 1,MM
39*      DO 400 J = 1,MM
40*      A(I,J) = 0.0
41*      DO 400 K = 1,NOS
42*      400 A(I,J) = A(I,J) + XP(I,K)*XINV(K,J)
43*      WRITE(3,12)((XINV(I,J), J=1,MM), I=1,NOS)
44*      12 FORMAT(3H0XI/(15E8.2))
45*      WRITE (3,14) ((A(I,J),J=1,MM), I=1,MM)
46*      14 FORMAT (2H0I/(13F10.7))
47*      WRITE(3,11)(Y(I), I=1,NOS)
48*      11 FORMAT(2H0Y/(16E8.2))
49*      C
50*      C      DETERMINE BETA COEFFICIENTS
51*      C
52*      DO 407 J=1,MM
53*      BETA(1,J) = 0.0
54*      DO 407 K=1,NOS
55*      407 BETA(1,J) = BETA(1,J) + Y(K)*XINV(K,J)
56*      WRITE (3,6) (BETA(1,J),J=1,MM)
57*      6 FORMAT (5H0BETA/(12E11.5))
58*      C
59*      C      FORM MATRIX OF PARTIAL DERIVATIVES
60*      C
61*      MM = 2
62*      DO 500 I=1,NOIV
63*      D(I,1) = -BETA(1,MM)
64*      B(I,1) = 2.*BETA(1,MM+NOIV)
65*      500 MM = MM + 1
66*      NN = NOIV - 1
67*      DO 600 I=1,NN
68*      N = I + 1
69*      DO 600 J=N,NOIV
70*      B(J,I) = BETAT(I,MM+NOIV)
71*      B(I,J) = B(J,I)
72*      600 MM = MM + 1
73*      WRITE (3,7) ((B(I,J),J=1,NOIV),I=1,NOIV)
74*      7 FORMAT (3H0AD/(5E18.8))
75*      WRITE (3,8) (D(J,1),J=1,NOIV)
76*      8 FORMAT (3H0BD/(E18.8))
77*      C

```

```
78* C INVERT PARTIAL DERIVATIVE MATRIX
79* C
80* MM = NOIV
81* NOSAM = NOIV
82* CALL GENINV
83* WRITE(3,9)(XINV(I,J),J=1,NOIV), I=1,NOIV)
84* 9 FORMAT(4H0AID/(5E18.8))
85* NOSAM = MOS
86* C
87* C CALCULATE OPTIMUM VALUE OF INDEPENDENT VARIABLES
88* C
89* DO 605 I=1,NOIV
90* A(I,1) = 0.0
91* DO 605 K=1,NOIV
92* 605 A(I,1) = A(I,1) + XINV(I,K)*D(K,1)
93* DO 900 I=1,NOIV
94* BASE(I) = A(I,1)*DELTA(I) + X(I,1)
95* XFP(I) = BASE(I)
96* 900 CONTINUE
97* WRITE(3,10) (BASE(I),I=1,NOIV)
98* 10 FORMAT(6H0OPT X/6X,6E18.8)
99* CALL PART-(XFP,PARTD)
100* WRITE(3,13) (PARTD(I),I=1,NOIV)
101* 13 FORMAT(6H0PARTD/ 6E18.8)
102* RETURN
103* END
```

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APPENDIX C

RESULTS OF A SPECIFIC OPTIMIZATION PROBLEM

APPENDIX C

RESULTS OF A SPECIFIC OPTIMIZATION PROBLEM

The sample optimization problem that follows is one in which the objective was to minimize the error between the known isotopic concentrations of U-235, Pu-239, Pu-240, Pu-241, and Pu-242 and the isotopic concentrations calculated by the Fuel Cycle Analysis Chain, FULCYC. The reactor configuration was that of the Yankee Reactor from which the fuel samples were discharged. The method used in FULCYC was to "burn" to the exposure assigned to the fuel samples analyzed having the known isotopic concentration. The independent variables were parameters used in FULCYC cross-section calculation of the various isotopes. Definition of the variables is as follows:

Dependent Variable,

$$\text{SSQERR} = \sum_{i=1}^5 \left(\frac{\bar{c}_i - c_i}{\bar{c}_i} \right)^2$$

where \bar{c}_i = known concentration of isotope i

c_i = calculated concentration of isotope i

Independent Variables,

1. RAYK2
2. Z-28
3. S1-41
4. Z-40

For the initial base point of the independent variables, the parameters used were the best values known at that time. A comparison of the individual percent deviations of the five isotopic concentrations before and after optimization as shown in Table I illustrates the results of optimization.

TABLE I. IMPROVEMENT BY OPTIM

<u>Isotope</u>	<u>Deviation of Isotopic Concentration (%)</u>	
	<u>First Guess</u>	<u>Optimum</u>
U-235	- 2.5	- 3.2
Pu-239	+13.8	+ 0.3
Pu-240	- 7.5	- 0.1
Pu-241	+13.7	- 0.5
Pu-242	- 2.3	- 1.8

Concerning the larger U-235 deviation at the optimum as compared to the "first guess"--this is likely due to the inaccuracies in determining the exposure of the analyzed fuel samples which is a function of Cs-137 content and Mev/fission constant used in FULCYC. Possibly to further improve the isotopic concentrations the Mev/fission constant should also be included as an independent variable or instead of "burning" to a given exposure, "burn" until the particular U-235 concentration is reached.

In addition to Table I, a more detailed look at the trends of the variables as OPTIM proceeds to the optimum is illustrated in Figure 7.

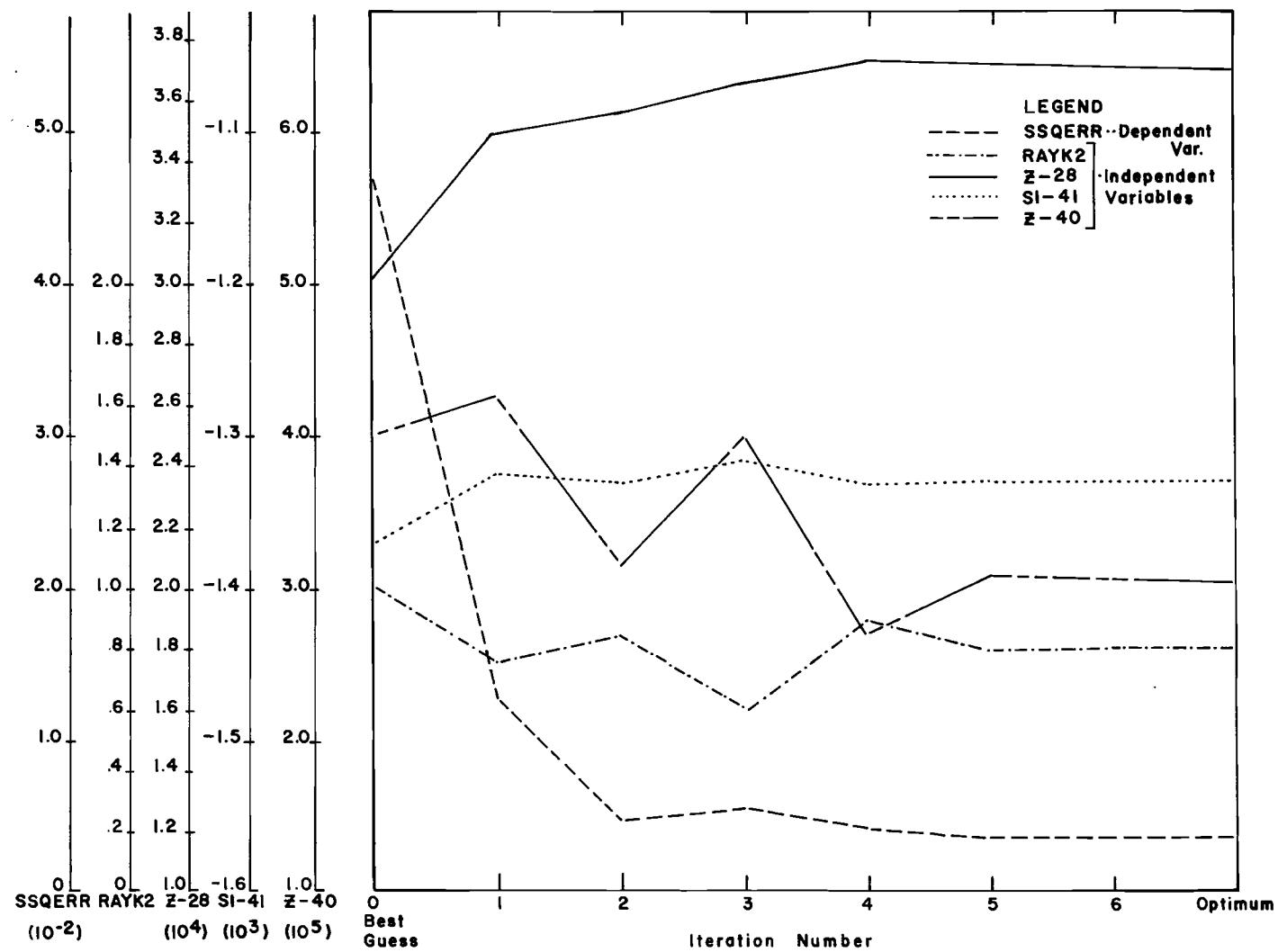


FIGURE 7 SAMPLE OPTIMIZATION PROBLEM

ITERATION NO. 0

EXPERIMENT NUMBER	SSQERR	FUNCTION APPROX.	PERCENT ERROR	RAYK2	Z-28	S1-41	Z-40
1	4.79038-02	4.79034-02	.000	1.00000+00 2.5480-02	3.00000+04 -7.1601-02	-1.37000+03 -1.5920-02	4.00000+05 8.3402-03
2	4.80935-01	4.37659-01	8.459	2.00000+00 3.6428-01	3.00000+04 -7.7162-02	-1.37000+03 2.1655-01	4.00000+05 9.0621-02
3	2.90461-01	3.35738-01	-8.850	0.00000 -3.1331-01	3.00000+04 -6.6039-02	-1.37000+03 -2.4839-01	4.00000+05 -7.3940-02
4	2.20089-02	2.44138-02	-.470	1.00000+00 1.9919-02	4.00000+04 4.8111-02	-1.37000+03 -6.9930-03	4.00000+05 5.4992-03
5	3.11220-01	3.10817-01	.079	1.00000+00 3.1042-02	2.00000+04 -1.9131-01	-1.37000+03 -2.4848-02	4.00000+05 1.1181-02
6	3.39387-01	2.95885-01	8.503	1.00000+00 2.5795-01	3.00000+04 -6.2673-02	-1.17000+03 2.6390-01	4.00000+05 5.7432-02
7	3.14063-01	3.59566-01	-8.894	1.00000+00 -2.0699-01	3.00000+04 -8.0528-02	-1.57000+03 -2.9574-01	4.00000+05 -4.0752-02
8	9.83077-02	8.73640-02	2.139	1.00000+00 1.0776-01	3.00000+04 -7.4442-02	-1.37000+03 3.3171-02	6.00000+05 3.1120-02
9	4.10582-02	5.40032-02	-2.530	1.00000+00 -5.6800-02	3.00000+04 -6.8760-02	-1.37000+03 -6.5012-02	2.00000+05 -1.4440-02
10	3.11687-01	3.66665-01	-10.746	1.50000+00 3.4947-01	3.50000+04 -1.1482-02	-1.27000+03 2.6924-01	5.00000+05 8.3996-02
11	2.11919-01	2.21453-01	-1.864	1.50000+00 2.6719-01	3.50000+04 -8.6415-03	-1.27000+03 2.2014-01	5.00000+05 6.1216-02
12	1.24642-01	1.08014-01	3.250	1.50000+00 1.1700-01	3.50000+04 -2.0410-02	-1.47000+03 -1.0585-02	5.00000+05 3.4904-02
13	6.79987-02	6.09857-02	1.371	1.50000+00 3.4720-02	3.50000+04 -1.7569-02	-1.47000+03 -5.9677-02	3.00000+05 1.2124-02
14	4.86191-01	5.09342-01	-4.525	1.50000+00 3.5504-01	2.50000+04 -1.3119-01	-1.27000+03 2.6031-01	5.00000+05 8.6837-02
15	3.59333-01	3.58448-01	.173	1.50000+00 2.7275-01	2.50000+04 -1.2835-01	-1.27000+03 2.1122-01	3.00000+05 6.4057-02
16	2.75825-01	2.68545-01	1.423	1.50000+00 1.2256-01	2.50000+04 -1.4012-01	-1.47000+03 -1.9513-02	5.00000+05 3.7745-02
17	1.87141-01	2.15835-01	-5.609	1.50000+00 4.0282-02	2.50000+04 -1.3728-01	-1.47000+03 -6.8605-02	3.00000+05 1.4965-02
18	3.26303-02	6.51280-03	5.105	5.00000-01 1.0678-02	3.50000+04 -5.9209-03	-1.27000+03 3.6764-02	5.00000+05 1.7155-03
19	2.31594-02	2.58620-02	-.528	5.00000-01 -7.1602-02	3.50000+04 -3.0799-03	-1.27000+03 -1.2328-02	5.00000+05 -2.1065-02
20	2.16500-01	2.12807-01	.722	5.00000-01 -2.2179-01	3.50000+04 -1.4848-02	-1.47000+03 -2.4306-01	5.00000+05 -4.7376-02
21	3.50914-01	3.30340-01	4.021	5.00000-01 -3.0407-01	3.50000+04 -1.2007-02	-1.47000+03 -2.9215-01	5.00000+05 -7.0156-02

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22	1.35631-01	1.38066-01	-.476	5.00000-01	2.50000+04	-1.27000+03	5.00000+05
23	1.32528-01	1.51733-01	-3.754	1.6240-02	-1.2563-01	2.7836-02	4.5565-03
				5.00000-01	2.50000+04	-1.27000+03	3.00000+05
				-6.6040-02	-1.2279-01	-2.1255-02	-1.8224-02
24	3.69174-01	3.62216-01	1.360	5.00000-01	2.50000+04	-1.47000+03	5.00000+05
				-2.1623-01	-1.3456-01	-2.5199-01	-4.4535-02
25	5.33622-01	4.74066-01	11.641	5.00000-01	2.50000+04	-1.47000+03	3.00000+05
				-2.9851-01	-1.3172-01	-3.0108-01	-6.7315-02

AVERAGE ERROR IN FUNCTION = 3.860

EFFECT OF VARYING RAYK2 = 84.640 47.410

EFFECT OF VARYING Z-28 = -5.061 51.468

EFFECT OF VARYING S1-41 = 56.973 52.023

EFFECT OF VARYING Z-40 = 9.852 -1.338

MINIMUM OF EXPERIMENTS

4 2.20089-02 1.00000+00 4.00000+04 -1.37000+03 4.00000+05

RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND Z-28

RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND S1-41

RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND Z-40

RELATIVE MINIMUM WITH RESPECT TO Z-28 AND S1-41

RELATIVE MINIMUM WITH RESPECT TO Z-28 AND Z-40

RELATIVE MINIMUM WITH RESPECT TO S1-41 AND Z-40

OPTIMUM OF FUNCTION

-1.38198-03 7.48856-01 3.50000+04 -1.32503+03 4.25582+05
4.0447-04 -8.7043-03 -6.4910-04 2.0659-04

ITERATION NO. 1

EXPERIMENT NUMBER	SSQERR	FUNCTION APPROX.	PERCENT ERROR	RAYK2	Z-28	S1-41	Z-40
1	1.25786-02	1.25785-02	.000	7.48856-01	3.50000+04	-1.32503+03	4.25582+05
2	1.00891-01	9.18979-02	6.258	1.24886+00	3.50000+04	-1.32503+03	4.25582+05
3	4.17664-02	5.00724-02	-5.780	6.8863-02	-9.4707-03	6.4547-02	3.9990-02
4	2.12474-02	2.35409-02	-1.596	2.48856-01	3.50000+04	-1.32503+03	4.25582+05
5	8.13256-02	7.83446-02	2.074	-4.7950-02	-1.7931-02	-4.6212-02	-2.3228-02
6	1.37998-01	1.30466-01	5.241	7.48856-01	4.34114+04	-1.32503+03	4.25582+05
7	8.69490-02	9.37942-02	-4.763	1.4687-02	2.4663-02	1.3789-02	7.6396-03
8	5.01579-02	5.01609-02	-.002	7.48856-01	2.65886+04	-1.32503+03	4.25582+05
9	1.73277-02	1.66373-02	.480	6.2262-03	-5.2065-02	4.5465-03	9.1222-03
10	1.29624-01	1.40557-01	-7.607	7.48856-01	3.50000+04	-1.21500+03	4.25582+05
11	6.88139-02	6.92497-02	-.303	6.5836-02	-9.0795-03	1.0872-01	3.2058-02
12	4.25267-02	3.85428-02	2.772	7.48856-01	3.50000+04	-1.43507+03	4.25582+05
13	1.52743-02	1.45901-02	.476	-4.4923-02	-1.8322-02	-9.0384-02	-1.5296-02
14	1.56292-01	1.59848-01	-2.474	7.48856-01	3.50000+04	-1.32503+03	4.25582+05
15	8.54567-02	8.70586-02	-1.115	4.2066-02	-1.4442-02	3.2845-02	2.9201-02
16	6.85912-02	6.70772-02	1.053	7.48856-01	3.92057+04	-1.38005+03	5.25582+05
17	3.33115-02	4.16419-02	-5.797	8.5269-02	9.5363-03	1.0078-01	4.6064-02
18	3.57983-02	2.84248-02	5.131	9.98856-01	3.92057+04	-1.27002+03	3.25582+05
19	1.90920-02	2.03365-02	-.866	9.98856-01	3.07943+04	-1.38005+03	5.25582+05
20	3.90414-02	3.71700-02	1.302	8.1039-02	-2.8828-02	9.6161-02	4.6805-02
21	7.90351-02	7.64359-02	1.809	9.98856-01	3.07943+04	-1.38005+03	3.25582+05
				-5.9498-03	-3.2708-02	-2.7067-02	2.3074-03
				4.9430-02	-2.8087-02	7.2484-02	2.5985-02
				2.5660-02	-3.3449-02	-3.3902-03	2.3128-02
				2.6863-02	5.3061-03	4.5403-02	1.4454-02
				4.98856-01	3.92057+04	-1.27002+03	3.25582+05
				-4.7467-03	6.0474-03	2.1726-02	-6.3661-03
				-2.8517-02	6.8469-04	-5.4148-02	-9.2227-03
				4.98856-01	3.92057+04	-1.38005+03	3.25582+05
				-6.0126-02	1.4260-03	-7.7825-02	-3.0043-02

22	5.57621-02	5.61768-02	-.289	4.98856-01	3.07943+04	-1.27002+03	5.25582+05
				2.2632-02	-3.3058-02	4.0782-02	1.5196-02
23	4.16650-02	4.66059-02	-3.438	4.98856-01	3.07943+04	-1.27002+03	3.25582+05
				-8.9770-03	-3.2317-02	1.7105-02	-5.6248-03
24	7.36437-02	7.41648-02	-.363	4.98856-01	3.07943+04	-1.38005+03	5.25582+05
				-3.2747-02	-3.7680-02	-5.8770-02	-8.4814-03
25	1.23150-01	1.11948-01	7.794	4.98856-01	3.07943+04	-1.38005+03	3.25582+05
				-6.4356-02	-3.6938-02	-8.2447-02	-2.9302-02

AVERAGE ERROR IN FUNCTION = 2.751

EFFECT OF VARYING RAYK2	= 61.451	20.310
EFFECT OF VARYING Z-28	= 6.032	47.836
EFFECT OF VARYING S1-41	= 87.271	51.749
EFFECT OF VARYING Z-40	= 26.149	3.305

MINIMUM OF EXPERIMENTS

1	1.25786-02	7.48856-01	3.50000+04	-1.32503+03	4.25582+05
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RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND Z-28

RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND S1-41

RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND Z-40

RELATIVE MINIMUM WITH RESPECT TO Z-28 AND S1-41

RELATIVE MINIMUM WITH RESPECT TO Z-28 AND Z-40

RELATIVE MINIMUM WITH RESPECT TO S1-41 AND Z-40

OPTIMUM OF FUNCTION

6.58715-03	8.31579-01	3.57943+04	-1.33213+03	3.11907+05
	-1.0205-03	-9.2553-03	-1.1149-03	1.7883-04

PREVIOUS VALUE	7.48856-01	3.50000+04	-1.32503+03	4.25582+05
PRESENT VALUE	8.31579-01	3.57943+04	-1.33213+03	3.11907+05

ABSOLUTE DIFFERENCE	8.27225-02	7.94281+02	7.10054+00	1.13674+05
EPSILON REQUESTED	5.00000-03	5.00000+02	2.00000+01	5.00000+03

ITERATION NO. 2

EXPERIMENT NUMBER	FUNCTION APPROX.	PERCENT ERROR	RAYK2	Z-28	S1-41	Z-40	
1	4.52130-03	4.52123-03	.000	8.31579-01	3.57943+04	-1.33213+03	3.11907+05
				-1.0189-03	-5.9720-03	-1.3385-03	-2.9714-03
2	3.38720-02	2.89300-02	6.464	1.15797+00	3.57943+04	-1.33213+03	3.11907+05
				2.5428-02	-2.5968-03	1.8703-02	2.4661-02
3	2.97396-02	3.30054-02	-4.271	5.05192-01	3.57943+04	-1.33213+03	3.11907+05
				-2.7465-02	-9.3471-03	-2.1380-02	-3.0604-02
4	1.29146-02	1.29768-02	-.081	8.31579-01	4.25691+04	-1.33213+03	3.11907+05
				2.3563-03	1.4428-02	4.9677-04	-2.4184-03
5	3.86029-02	3.68646-02	2.274	8.31579-01	2.90194+04	-1.33213+03	3.11907+05
				-4.3940-03	-2.6371-02	-3.1738-03	-3.5244-03
6	3.02309-02	2.69128-02	4.340	8.31579-01	3.57943+04	-1.27712+03	3.11907+05
				1.9023-02	-4.1367-03	2.3730-02	1.2383-02
7	3.06247-02	3.22668-02	-2.148	8.31579-01	3.57943+04	-1.38715+03	3.11907+05
				-2.1060-02	-7.8072-03	-2.6407-02	-1.8326-02
8	3.16839-02	3.84847-02	-8.895	8.31579-01	3.57943+04	-1.33213+03	5.11907+05
				2.6614-02	-5.4190-03	1.4016-02	3.6935-02
9	5.88474-02	5.03704-02	11.087	8.31579-01	3.57943+04	-1.33213+03	1.11907+05
				-2.8652-02	-6.5250-03	-1.6693-02	-4.2878-02
10	5.21354-02	5.55716-02	-4.494	9.94772-01	3.91817+04	-1.30463+03	4.11907+05
				3.7729-02	7.1095-03	2.9811-02	3.8752-02
11	1.64637-02	1.79744-02	-1.976	9.94772-01	3.91817+04	-1.30463+03	2.11907+05
				1.0096-02	6.5565-03	1.4457-02	-1.1546-03
12	2.30958-02	2.10176-02	2.718	9.94772-01	3.91817+04	-1.35964+03	4.11907+05
				1.7688-02	5.2742-03	4.7428-03	2.3397-02
13	1.21180-02	1.41291-02	-2.630	9.94772-01	3.91817+04	-1.35964+03	2.11907+05
				-9.9451-03	4.7212-03	-1.0612-02	-1.6509-02
14	6.22487-02	6.17522-02	.649	9.94772-01	3.24068+04	-1.30463+03	4.11907+05
				3.4354-02	-1.3290-02	2.7976-02	3.8199-02
15	2.14511-02	2.52609-02	-4.983	9.94772-01	3.24068+04	-1.30463+03	2.11907+05
				6.7211-03	-1.3843-02	1.2622-02	-1.7075-03
16	3.35891-02	3.08687-02	3.558	9.94772-01	3.24068+04	-1.35964+03	4.11907+05
				1.4312-02	-1.5125-02	2.9075-03	2.2844-02
17	1.89987-02	2.50861-02	-7.962	9.94772-01	3.24068+04	-1.35964+03	2.11907+05
				-1.3320-02	-1.5678-02	-1.2447-02	-1.7062-02
18	1.16020-02	6.56007-03	6.594	6.68385-01	3.91817+04	-1.30463+03	4.11907+05
				1.1283-02	3.7343-03	9.7699-03	1.1119-02
19	2.08773-02	2.42283-02	-4.383	6.68385-01	3.91817+04	-1.30463+03	2.11907+05
				-1.6350-02	3.1814-03	-5.5845-03	-2.8787-02
20	1.52680-02	1.20889-02	4.158	6.68385-01	3.91817+04	-1.35964+03	4.11907+05
				-8.7589-03	1.8991-03	-1.5299-02	-4.2353-03
21	5.89238-02	6.04658-02	-2.017	6.68385-01	3.91817+04	-1.35964+03	2.11907+05
				-3.6392-02	1.3461-03	-3.0653-02	-4.4142-02

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22	2.08713-02	1.94909-02	1.805	6.68385-01	3.24068+04	-1.30463+03	4.11907+05
23	3.51414-02	3.82651-02	-4.086	6.68385-01	3.24068+04	-1.30463+03	2.11907+05
24	2.91554-02	2.86902-02	.608	6.68385-01	3.24068+04	-1.35964+03	4.11907+05
25	8.09788-02	7.81731-02	3.670	6.68385-01	3.24068+04	-1.35964+03	2.11907+05
				-3.9767-02	-1.9053-02	-3.2488-02	-4.4695-02

AVERAGE ERROR IN FUNCTION = 3.834

EFFECT OF VARYING RAYK2 = 38.388 32.983
 EFFECT OF VARYING Z-28 = 10.978 44.576
 EFFECT OF VARYING S1-41 = 33.626 34.141
 EFFECT OF VARYING Z-40 = 35.526 71.054

MINIMUM OF EXPERIMENTS

1 4.52130-03 8.31579-01 3.57943+04 -1.33213+03 3.11907+05

RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND Z-28

RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND S1-41

RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND Z-40

RELATIVE MINIMUM WITH RESPECT TO Z-28 AND S1-41

RELATIVE MINIMUM WITH RESPECT TO Z-28 AND Z-40

RELATIVE MINIMUM WITH RESPECT TO S1-41 AND Z-40

OPTIMUM OF FUNCTION

2.55757-03	5.95392-01	3.66126+04	-1.31375+03	4.00272+05
	-8.4261-04	-5.0928-03	-4.5818-04	-1.3806-04

PREVIOUS VALUE	8.31579-01	3.57943+04	-1.33213+03	3.11907+05
PRESENT VALUE	5.95392-01	3.66126+04	-1.31375+03	4.00272+05

ABSOLUTE DIFFERENCE	2.36187-01	8.18286+02	1.83858+01	8.83647+04
EPSILON REQUESTED	5.00000-03	5.00000+02	2.00000+01	5.00000+03

ITERATION NO. 3

EXPERIMENT NUMBER	FUNCTION APPROX.	PERCENT ERROR	RAYK2	Z-28	S1-41	Z-40	
1	5.59616-03	5.67275-03	-.340	5.95392-01 -2.9558-04	3.66126+04 -2.5001-03	-1.31375+03 6.5011-04	4.00272+05 8.9708-04
2	1.43333-02	1.36236-02	3.154	7.95858-01 8.2465-03	3.66126+04 -7.1562-04	-1.31375+03 7.9154-03	4.00272+05 7.4239-03
3	1.42447-02	1.48060-02	-2.495	3.94927-01 -8.8377-03	3.66126+04 -4.2846-03	-1.31375+03 -6.6152-03	4.00272+05 -5.6298-03
4	9.48974-03	9.97396-03	-2.152	5.95392-01 1.4889-03	4.13504+04 6.8013-03	-1.31375+03 1.7883-03	4.00272+05 1.2655-03
5	2.05368-02	1.99745-02	2.499	5.95392-01 -2.0801-03	3.18747+04 -1.1802-02	-1.31375+03 -4.8803-04	4.00272+05 5.2868-04
6	1.84577-02	1.79950-02	2.057	5.95392-01 6.9697-03	3.66126+04 -1.3620-03	-1.27964+03 1.1672-02	4.00272+05 5.0616-03
7	1.50230-02	1.53946-02	-1.652	5.95392-01 -7.5609-03	3.66126+04 -3.6383-03	-1.34846+03 -1.0372-02	4.00272+05 -3.2674-03
8	1.34879-02	1.38004-02	-1.389	5.95392-01 6.2313-03	3.66126+04 -2.1317-03	-1.31375+03 4.8146-03	5.00272+05 7.2306-03
9	1.06416-02	1.02121-02	1.909	5.95392-01 -6.8224-03	3.66126+04 -2.8685-03	-1.31375+03 -3.5144-03	3.00272+05 -5.4364-03
10	2.31269-02	2.38478-02	-3.204	6.95625-01 1.1764-02	3.89815+04 3.7961-03	-1.29639+03 1.2445-02	4.50272+05 9.5937-03
11	1.10659-02	1.09939-02	.320	6.95625-01 5.2369-03	3.89815+04 3.4277-03	-1.29639+03 8.2806-03	3.50272+05 3.2602-03
12	1.03244-02	9.97970-03	1.532	6.95625-01 4.4985-03	3.89815+04 2.6580-03	-1.33110+03 1.4230-03	4.50272+05 5.4292-03
13	5.58580-03	5.45478-03	.582	6.95625-01 -2.0284-03	3.89815+04 2.2896-03	-1.33110+03 -2.7414-03	3.50272+05 -9.0429-04
14	2.54517-02	2.55570-02	-.468	6.95625-01 9.9793-03	3.42436+04 -5.5054-03	-1.29639+03 1.1307-02	4.50272+05 9.2253-03
15	1.30496-02	1.34400-02	-1.735	6.95625-01 3.4524-03	3.42436+04 -5.8737-03	-1.29639+03 7.1425-03	3.50272+05 2.8918-03
16	1.40594-02	1.39652-02	.419	6.95625-01 2.7140-03	3.42436+04 -6.6435-03	-1.33110+03 2.8490-04	4.50272+05 5.0608-03
17	9.38877-03	1.01771-02	-3.504	6.95625-01 -3.8129-03	3.42436+04 -7.0119-03	-1.33110+03 -3.8796-03	3.50272+05 -1.2727-03
18	9.65704-03	8.86235-03	3.532	4.95160-01 3.2217-03	3.89815+04 2.0116-03	-1.29639+03 5.1798-03	4.50272+05 3.0668-03
19	8.93254-03	9.06216-03	-.576	4.95160-01 -3.3051-03	3.89815+04 1.6432-03	-1.29639+03 1.0153-03	3.50272+05 -3.2667-03
20	9.90457-03	9.52480-03	1.688	4.95160-01 -4.0436-03	3.89815+04 8.7348-04	-1.33110+03 -5.8423-03	4.50272+05 -1.0976-03
21	1.80664-02	1.80536-02	.057	4.95160-01 -1.0570-02	3.89815+04 5.0508-04	-1.33110+03 -1.0007-02	3.50272+05 -7.4311-03

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22	1.40371-02	1.41406-02	-.460	4.95160-01	3.42436+04	-1.29639+03	4.50272+05
				1.4372-03	-7.2899-03	4.0417-03	2.6984-03
23	1.46455-02	1.50772-02	-1.919	4.95160-01	3.42436+04	-1.29639+03	3.50272+05
				-5.0897-03	-7.6583-03	-1.2281-04	-3.6351-03
24	1.69388-02	1.70793-02	-.625	4.95160-01	3.42436+04	-1.33110+03	4.50272+05
				-5.8281-03	-8.4280-03	-6.9804-03	-1.4660-03
25	2.70197-02	2.63449-02	2.999	4.95160-01	3.42436+04	-1.33110+03	3.50272+05
				-1.2355-02	-8.7964-03	-1.1145-02	-7.7995-03
26	4.52130-03	4.57308-03	-.230	8.31579-01	3.57943+04	-1.33213+03	3.11907+05
				-1.5571-04	-2.9326-03	-5.0527-04	7.2067-04

AVERAGE ERROR IN FUNCTION = 1.596

EFFECT OF VARYING RAYK2 = 38.835 38.441
 EFFECT OF VARYING Z-28 = 17.306 66.408
 EFFECT OF VARYING S1-41 = 57.167 41.900
 EFFECT OF VARYING Z-40 = 35.077 22.426

MINIMUM OF EXPERIMENTS

77	4.52130-03	8.31579-01	3.57943+04	-1.33213+03	3.11907+05
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RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND Z-28

RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND S1-41

RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND Z-40

RELATIVE MINIMUM WITH RESPECT TO Z-28 AND S1-41

RELATIVE MINIMUM WITH RESPECT TO Z-28 AND Z-40

RELATIVE MINIMUM WITH RESPECT TO S1-41 AND Z-40

OPTIMUM OF FUNCTION

3.40923-03	8.98919-01	3.70865+04	-1.33324+03	2.66422+05
	-5.9372-09	-1.0914-09	5.2387-10	-6.7521-09

PREVIOUS VALUE

5.95392-01 3.66126+04 -1.31375+03 4.00272+05

PRESENT VALUE

8.98919-01 3.70865+04 -1.33324+03 2.66422+05

ABSOLUTE DIFFERENCE

3.03527-01 4.73962+02 1.94925+01 1.33851+05

EPSILON REQUESTED

5.00000-03 5.00000+02 2.00000+01 5.00000+03

ITERATION NO. 4

EXPERIMENT NUMBER	SSQERR	FUNCTION APPROX.	PERCENT ERROR	RAYK2	Z-28	S1-41	Z-40
1	4.01454-03	4.01453-03	.000	8.98919-01 2.9312-05	3.70865+04 1.0201-04	-1.33324+03 -4.5834-05	2.66422+05 -5.6687-04
2	7.57380-03	7.38274-03	2.483	1.01278+00 3.3389-03	3.70865+04 4.5736-04	-1.33324+03 2.1090-03	2.66422+05 2.7920-03
3	7.10292-03	7.26549-03	-2.113	7.85062-01 -3.2803-03	3.70865+04 -2.5333-04	-1.33324+03 -2.2007-03	2.66422+05 -3.9257-03
4	6.60378-03	6.62415-03	-.265	8.98919-01 3.8466-04	3.96653+04 2.5076-03	-1.33324+03 1.2520-04	2.66422+05 -5.2331-04
5	6.26495-03	6.21609-03	.635	8.98919-01 -3.2603-04	3.45078+04 -2.3036-03	-1.33324+03 -2.1687-04	2.66422+05 -6.1044-04
6	6.57671-03	6.46878-03	1.403	8.98919-01 2.1841-03	3.70865+04 2.7305-04	-1.31589+03 2.5001-03	2.66422+05 1.0890-03
7	6.57269-03	6.65212-03	-1.032	8.98919-01 -2.1255-03	3.70865+04 -6.9022-05	-1.35059+03 -2.5918-03	2.66422+05 -2.2228-03
8	7.20098-03	7.49129-03	-3.773	8.98919-01 3.3882-03	3.70865+04 1.4558-04	-1.33324+03 1.6101-03	3.32381+05 4.0436-03
9	1.00776-02	9.75879-03	4.143	8.98919-01 -3.3296-03	3.70865+04 5.8451-05	-1.33324+03 -1.7018-03	2.00462+05 -5.1774-03
10	1.04846-02	1.06208-02	-1.771	9.55847-01 4.6186-03	3.83759+04 1.5898-03	-1.32456+03 3.2180-03	2.99401+05 4.2676-03
11	6.65097-03	6.69622-03	-.588	9.55847-01 1.2598-03	3.83759+04 1.5462-03	-1.32456+03 1.5621-03	2.33442+05 -3.4296-04
12	6.81009-03	6.73071-03	1.032	9.55847-01 2.4638-03	3.83759+04 1.4187-03	-1.34192+03 6.7209-04	2.99401+05 2.6116-03
13	6.04941-03	6.11796-03	-.891	9.55847-01 -8.9506-04	3.83759+04 1.3752-03	-1.34192+03 -9.8383-04	2.33442+05 -1.9989-03
14	9.88628-03	9.84685-03	.512	9.55847-01 4.2633-03	3.57972+04 -8.1581-04	-1.32456+03 3.0470-03	2.99401+05 4.2240-03
15	5.85249-03	6.00938-03	-2.039	9.55847-01 9.0441-04	3.57972+04 -8.5937-04	-1.32456+03 1.3911-03	2.33442+05 -3.8652-04
16	6.41552-03	6.29882-03	1.517	9.55847-01 2.1085-03	3.57972+04 -9.8685-04	-1.34192+03 5.0106-04	2.99401+05 2.5681-03
17	5.53401-03	5.77318-03	-3.109	9.55847-01 -1.2504-03	3.57972+04 -1.0304-03	-1.34192+03 -1.1549-03	2.33442+05 -2.0424-03
18	4.91517-03	4.69317-03	2.885	8.41990-01 1.3090-03	3.83759+04 1.2344-03	-1.32456+03 1.0632-03	2.99401+05 9.0869-04
19	7.35828-03	7.48630-03	-1.664	8.41990-01 -2.0498-03	3.83759+04 1.1909-03	-1.32456+03 -5.9273-04	2.33442+05 -3.7018-03
20	5.25827-03	5.11270-03	1.892	8.41990-01 -8.4579-04	3.83759+04 1.0634-03	-1.34192+03 -1.4827-03	2.99401+05 -7.4723-04
21	1.11611-02	1.12177-02	-.736	8.41990-01 -4.2047-03	3.83759+04 1.0198-03	-1.34192+03 -3.1386-03	2.33442+05 -5.3577-03

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22	4.68711-03	4.62989-03	.744	8.41990-01	3.57972+04	-1.32456+03	2.99401+05
23	7.41360-03	7.51015-03	-1.255	8.41990-01	3.57972+04	-1.32456+03	2.33442+05
24	5.41957-03	5.39149-03	.365	8.41990-01	3.57972+04	-1.34192+03	2.99401+05
25	1.17085-02	1.15836-02	1.624	8.41990-01	3.57972+04	-1.34192+03	2.33442+05
				-4.5600-03	-1.3858-03	-3.3097-03	-5.4013-03

AVERAGE ERROR IN FUNCTION = 1.539

EFFECT OF VARYING RAYK2 = 46.260 40.140
 EFFECT OF VARYING Z-28 = 33.653 29.249
 EFFECT OF VARYING S1-41 = 33.301 33.249
 EFFECT OF VARYING Z-40 = 41.415 78.802

MINIMUM OF EXPERIMENTS

1	4.01454-03	8.98919-01	3.70865+04	-1.33324+03	2.66422+05
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RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND Z-28

RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND S1-41

RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND Z-40

RELATIVE MINIMUM WITH RESPECT TO Z-28 AND S1-41

RELATIVE MINIMUM WITH RESPECT TO Z-28 AND Z-40

RELATIVE MINIMUM WITH RESPECT TO S1-41 AND Z-40

OPTIMUM OF FUNCTION

3.59153-03	7.91138-01	3.72374+04	-1.32674+03	3.11109+05
	-3.7835-10	-1.0459-10	-1.3242-09	2.9104-10

PREVIOUS VALUE	8.98919-01	3.70865+04	-1.33324+03	2.66422+05
PRESENT VALUE	7.91138-01	3.72374+04	-1.32674+03	3.11109+05

ABSOLUTE DIFFERENCE	1.07781-01	1.50912+02	6.50127+00	4.46874+04
EPSILON REQUESTED	5.00000-03	5.00000+02	2.00000+01	5.00000+03

ITERATION NO. 5

EXPERIMENT NUMBER	FUNCTION APPROX.	PERCENT ERROR	RAYK2	Z-28	S1-41	Z-40	
1	3.74433-03	3.74433-03	.000	7.91138-01 -7.4919-07	3.72374+04 4.2257-05	-1.32674+03 2.1981-05	3.11109+05 1.1694-05
2	4.85264-03	4.82209-03	1.270	8.58330-01 1.0785-03	3.72374+04 2.1343-04	-1.32674+03 8.6313-04	3.11109+05 8.8434-04
3	4.79298-03	4.82509-03	-1.334	7.23946-01 -1.0800-03	3.72374+04 -1.2892-04	-1.32674+03 -8.1916-04	3.11109+05 -8.6096-04
4	4.95856-03	4.98361-03	-1.041	7.91138-01 1.7043-04	3.90225+04 1.1970-03	-1.32674+03 1.2457-04	3.11109+05 3.8663-05
5	4.83807-03	4.81458-03	.976	7.91138-01 -1.7192-04	3.54524+04 -1.1125-03	-1.32674+03 -8.0603-05	3.11109+05 -1.5274-05
6	4.99218-03	4.97584-03	.679	7.91138-01 8.4040-04	3.72374+04 1.4484-04	-1.31506+03 1.2095-03	3.11109+05 5.3410-04
7	4.87001-03	4.88791-03	-.744	7.91138-01 -8.4189-04	3.72374+04 -6.0327-05	-1.33842+03 -1.1656-03	3.11109+05 -5.1071-04
8	4.69143-03	4.71349-03	-.917	7.91138-01 8.7190-04	3.72374+04 6.9226-05	-1.32674+03 5.4439-04	3.44089+05 9.5747-04
9	4.68722-03	4.666671-03	.852	7.91138-01 -8.7340-04	3.72374+04 1.5289-05	-1.32674+03 -5.0043-04	2.78129+05 -9.3408-04
10	6.15018-03	6.17981-03	-1.232	8.24734-01 1.4814-03	3.81299+04 7.7000-04	-1.32090+03 1.3488-03	3.27599+05 1.1956-03
11	4.73903-03	4.73440-03	.192	8.24734-01 6.0872-04	3.81299+04 7.4304-04	-1.32090+03 8.2642-04	2.94619+05 2.4982-04
12	4.68521-03	4.66972-03	.644	8.24734-01 6.4022-04	3.81299+04 6.6742-04	-1.33258+03 1.6128-04	3.27599+05 6.7319-04
13	4.27312-03	4.26912-03	.166	8.24734-01 -2.3243-04	3.81299+04 6.4045-04	-1.33258+03 -3.6113-04	2.94619+05 -2.7259-04
14	5.79340-03	5.79457-03	-.049	8.24734-01 1.3102-03	3.63449+04 -3.8476-04	-1.32090+03 1.2462-03	3.27599+05 1.1686-03
15	4.38237-03	4.40309-03	-.861	8.24734-01 4.3754-04	3.63449+04 -4.1173-04	-1.32090+03 7.2383-04	2.94619+05 2.2285-04
16	4.49646-03	4.48964-03	.283	8.24734-01 4.6905-04	3.63449+04 -4.8735-04	-1.33258+03 5.8691-05	3.27599+05 6.4622-04
17	4.10402-03	4.14298-03	-1.619	8.24734-01 -4.0360-04	3.63449+04 -5.1432-04	-1.33258+03 -4.6372-04	2.94619+05 -2.9956-04
18	4.33572-03	4.29634-03	1.637	7.57542-01 4.0210-04	3.81299+04 5.9883-04	-1.32090+03 5.0768-04	3.27599+05 3.2295-04
19	4.59055-03	4.59623-03	-.236	7.57542-01 -4.7055-04	3.81299+04 5.7186-04	-1.32090+03 -1.4729-05	2.94619+05 -6.2283-04
20	4.49040-03	4.46853-03	.909	7.57542-01 -4.3904-04	3.81299+04 4.9625-04	-1.33258+03 -6.7987-04	3.27599+05 -1.9946-04
21	5.81482-03	5.81323-03	.066	7.57542-01 -1.3117-03	3.81299+04 4.6928-04	-1.33258+03 -1.2023-03	2.94619+05 -1.1452-03

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22	4.25059-03	4.25345-03	-.119	7.57542-01	3.63449+04	-1.32090+03	3.27599+05
				2.3093-04	-5.5594-04	4.0509-04	2.9598-04
23	4.59219-03	4.60727-03	-.627	7.57542-01	3.63449+04	-1.32090+03	2.94619+05
				-6.4172-04	-5.8291-04	-1.1731-04	-6.4980-04
24	4.62660-03	4.63081-03	-.175	7.57542-01	3.63449+04	-1.33258+03	3.27599+05
				-6.1022-04	-6.5852-04	-7.8245-04	-2.2643-04
25	6.06022-03	6.02945-03	1.279	7.57542-01	3.63449+04	-1.33258+03	2.94619+05
				-1.4829-03	-6.8549-04	-1.3049-03	-1.1722-03

AVERAGE ERROR IN FUNCTION = .716

EFFECT OF VARYING RAYK2	= 46.067	43.588
EFFECT OF VARYING Z-28	= 50.470	45.461
EFFECT OF VARYING S1-41	= 51.867	46.789
EFFECT OF VARYING Z-40	= 39.366	39.191

MINIMUM OF EXPERIMENTS

1	3.74433-03	7.91138-01	3.72374+04	-1.32674+03	3.11109+05
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RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND Z-28

RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND S1-41

RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND Z-40

RELATIVE MINIMUM WITH RESPECT TO Z-28 AND S1-41

RELATIVE MINIMUM WITH RESPECT TO Z-28 AND Z-40

RELATIVE MINIMUM WITH RESPECT TO S1-41 AND Z-40

OPTIMUM OF FUNCTION

3.73857-03	8.03287-01	3.71431+04	-1.32772+03	3.06785+05
	7.0213-10	1.0300-10	1.0214-09	4.3292-10

PREVIOUS VALUE	7.91138-01	3.72374+04	-1.32674+03	3.11109+05
PRESENT VALUE	8.03287-01	3.71431+04	-1.32772+03	3.06785+05

ABSOLUTE DIFFERENCE	1.21497-02	9.43247+01	9.84863-01	4.32396+03
EPSILON REQUESTED	5.00000-03	5.00000+02	2.00000+01	5.00000+03

ITERATION NO. 6

EXPERIMENT NUMBER	FUNCTION APPROX.	PERCENT ERROR	RAYK2	Z-28	S1-41	Z-40	
1	3.73243-03	3.73243-03	.001	8.03287-01 5.4987-06	3.71431+04 1.1754-05	-1.32772+03 2.9283-06	3.06785+05 9.3924-06
2	4.04681-03	4.04198-03	.716	8.38528-01 3.0405-04	3.71431+04 5.6279-05	-1.32772+03 2.2234-04	3.06785+05 2.6911-04
3	4.01700-03	4.01998-03	-.442	7.68046-01 -2.9305-04	3.71431+04 -3.2772-05	-1.32772+03 -2.1648-04	3.06785+05 -2.5032-04
4	4.05090-03	4.05312-03	-.330	8.03287-01 5.0024-05	3.80482+04 3.0894-04	-1.32772+03 2.7973-05	3.06785+05 1.6252-05
5	4.01019-03	4.00611-03	.605	8.03287-01 -3.9027-05	3.62380+04 -2.8544-04	-1.32772+03 -2.2116-05	3.06785+05 2.5326-06
6	4.03513-03	4.03510-03	.004	8.03287-01 2.2491-04	3.71431+04 3.6798-05	-1.32189+03 2.9975-04	3.06785+05 1.5539-04
7	4.02521-03	4.02339-03	.270	8.03287-01 -2.1391-04	3.71431+04 -1.3291-05	-1.33356+03 -2.9389-04	3.06785+05 -1.3661-04
8	4.04719-03	4.05100-03	-.564	8.03287-01 2.6522-04	3.71431+04 1.8613-05	-1.32772+03 1.4893-04	3.25263+03 3.0918-04
9	4.01909-03	4.01343-03	.839	8.03287-01 -2.5422-04	3.71431+04 4.8938-06	-1.32772+03 -1.4307-04	2.88307+05 -2.9039-04
10	4.40737-03	4.41087-03	-.518	8.20908-01 4.1660-04	3.75957+04 1.9856-04	-1.32481+03 3.4657-04	3.16024+05 3.6557-04
11	3.98033-03	3.97951-03	.123	8.20908-01 1.5688-04	3.75957+04 1.9170-04	-1.32481+03 2.0057-04	2.97546+05 6.5787-05
12	4.01578-03	4.01455-03	.181	8.20908-01 1.9719-04	3.75957+04 1.7352-04	-1.33064+03 4.9747-05	3.16024+05 2.1957-04
13	3.87402-03	3.87519-03	-.173	8.20908-01 -6.2528-05	3.75957+04 1.6666-04	-1.33064+03 -9.6252-05	2.97546+05 -8.0211-05
14	4.31171-03	4.31093-03	.115	8.20908-01 3.7208-04	3.66906+04 -9.8627-05	-1.32481+03 3.2152-04	3.16024+05 3.5871-04
15	3.89031-03	3.89329-03	-.442	8.20908-01 1.1236-04	3.66906+04 -1.0549-04	-1.32481+03 1.7552-04	2.97546+05 5.8928-05
16	3.96517-03	3.96471-03	.068	8.20908-01 1.5266-04	3.66906+04 -1.2367-04	-1.33064+03 2.4702-05	3.16024+05 2.1271-04
17	3.83188-03	3.83906-03	-1.063	8.20908-01 -1.0705-04	3.66906+04 -1.3053-04	-1.33064+03 -1.2130-04	2.97546+05 -8.7071-05
18	3.88243-03	3.87621-03	.922	7.85667-01 1.1805-04	3.75957+04 1.5404-04	-1.32481+03 1.2715-04	3.16024+05 1.0586-04
19	3.96293-03	3.96429-03	-.202	7.85667-01 -1.4167-04	3.75957+04 1.4718-04	-1.32481+03 -1.8846-05	2.97546+05 -1.9393-04
20	3.92081-03	3.91873-03	.309	7.85667-01 -1.0136-04	3.75957+04 1.2899-04	-1.33064+03 -1.6967-04	3.16024+05 -4.0143-05
21	4.29706-03	4.29880-03	-.257	7.85667-01 -3.6108-04	3.75957+04 1.2213-04	-1.33064+03 -3.1566-04	2.97546+05 -3.3993-04

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22	3.86560-03	3.86533-03	.040	7.85667-01	3.66906+04	-1.32481+03	3.16024+05
23	3.96494-03	3.96712-03	-.323	7.85667-01	3.66906+04	-1.32481+03	2.97546+05
24	3.95614-03	3.95793-03	-.265	7.85667-01	3.66906+04	-1.33064+03	3.16024+05
25	4.35431-03	4.35172-03	.384	7.85667-01	3.66906+04	-1.33064+03	2.97546+05
				-4.0560-04	-1.7506-04	-3.4071-04	-3.4679-04

AVERAGE ERROR IN FUNCTION = .366

EFFECT OF VARYING RAYK2 = 46.579 42.162
 EFFECT OF VARYING Z-28 = 47.184 41.153
 EFFECT OF VARYING S1-41 = 44.848 43.379
 EFFECT OF VARYING Z-40 = 46.635 42.471

MINIMUM OF EXPERIMENTS

1	3.73243-03	8.03287-01	3.71431+04	-1.32772+03	3.06785+05
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RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND Z-28

RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND S1-41

RELATIVE MINIMUM WITH RESPECT TO RAYK2 AND Z-40

RELATIVE MINIMUM WITH RESPECT TO Z-28 AND S1-41

RELATIVE MINIMUM WITH RESPECT TO Z-28 AND Z-40

RELATIVE MINIMUM WITH RESPECT TO S1-41 AND Z-40

OPTIMUM OF FUNCTION

3.73135-03	8.06708-01	3.70984+04	-1.32789+03	3.04924+05
	9.7771-11	1.3128-10	8.8789-11	7.5715-11

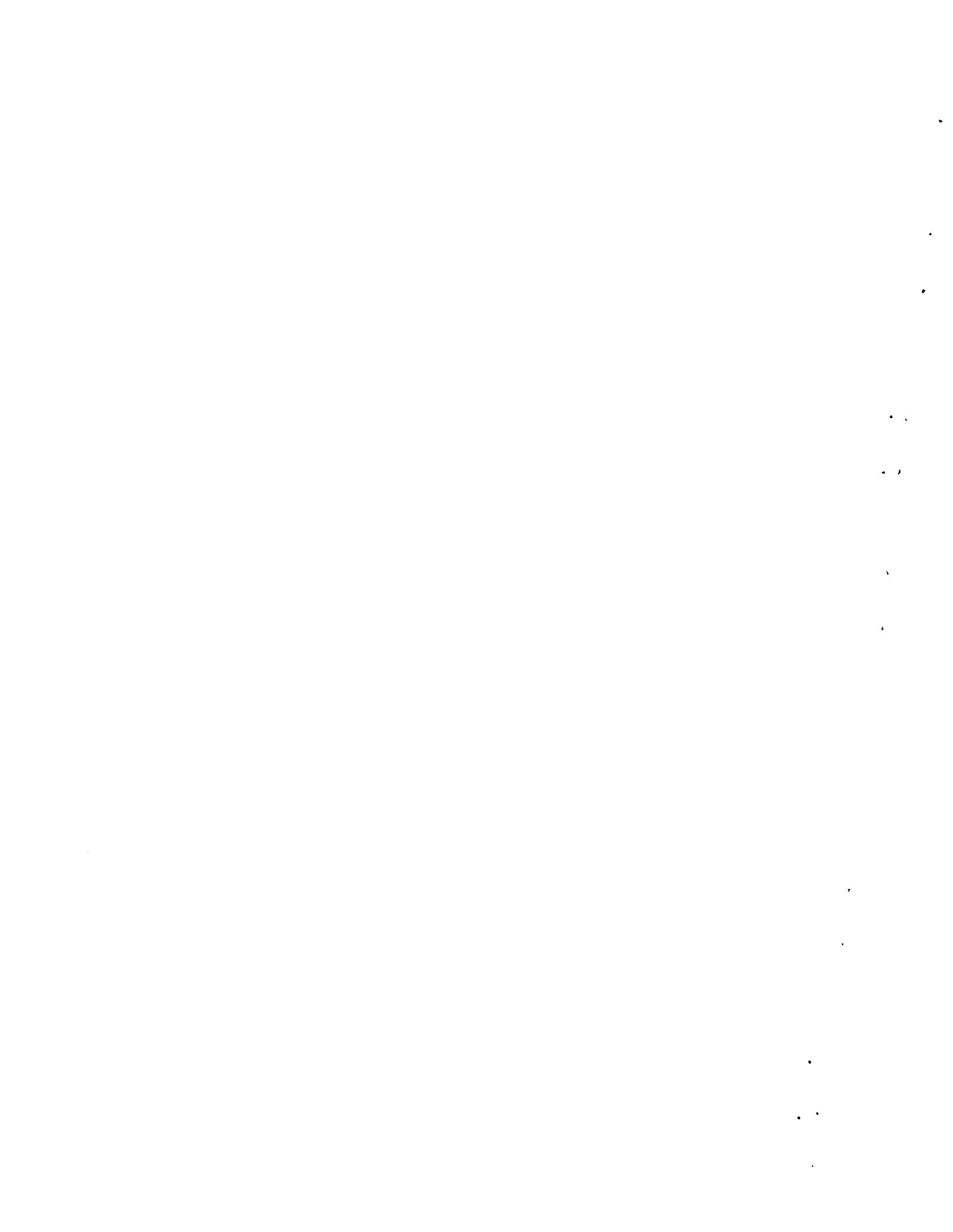
PREVIOUS VALUE	8.03287-01	3.71431+04	-1.32772+03	3.06785+05
PRESENT VALUE	8.06708-01	3.70984+04	-1.32789+03	3.04924+05

ABSOLUTE DIFFERENCE	3.42112-03	4.47251+01	1.63025-01	1.86077+03
EPSILON REQUESTED	5.00000-03	5.00000+02	2.00000+01	5.00000+03

THE ITERATION HAS CONVERGED TO THE FOLLOWING SOLUTION.

OPTIMUM OF FUNCTION

3.73135-03	8.06708-01	3.70984+04	-1.32789+03	3.04924+05
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