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ERROR ANALYSIS OF THE FLASH THERMAL DIFFUSIVITY TECHNIQUE*

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ABSTRACT

An error analysis of the flash technique for thermal diffusivity measurements has been made by propagating errors in measured quantities through several data analysis techniques to determine the consequent errors in thermal diffusivity. The effects of extended pulse durations and of heat losses are considered. The results show that for most conditions actually encountered in experiments, relative measurement errors are reflected as relative errors of approximately the same size in the thermal diffusivity.

I. Introduction

Under ideal conditions, flash thermal diffusivity measurements⁽¹⁾ are made by causing a planar, instantaneous, uniform pulse of energy to be deposited on one face of a thin, flat sample and by analyzing the consequent temperature fluctuation which occurs at the opposite face (Fig. 1). Frequently, ideal conditions are not obtained and the effects of extended pulse durations or of significant heat losses must be considered.

Analysis of the temperature fluctuation to obtain the diffusivity and other parameters of the problem can be accomplished by a variety of techniques. Among the most widely employed are those which make use of the "half time", i.e., the time at which the fluctuation reaches half of its maximum value (Fig. 1, point (t_1, T_1)). Parker et al, (1) first suggested this technique for the ideal boundary and initial conditions listed above. Subsequently, a number of authors (2,3,4,5) provided modifications appropriate to the case of extended heat pulses. Cowen (6) and Heckman (5) have applied the technique to the case where heat losses are important.

In this paper, the effects of errors on the results of flash thermal diffusivity measurements are analyzed. This is done by propagating assumed errors of certain significant times or temperatures, including the half time, through data reduction methods appropriate to ideal, extended pulse or heat loss conditions.

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An alternative view of these calculations is that they constitute a sensitivity analysis of the flash technique, for the particular data analysis methods considered. Consequently, they provide a basis for comparing it on entirely theoretical grounds with other diffusivity measurement methods.

The word error, used in this discussion, has a particular meaning which should be emphasized. It is used to mean deviations of known magnitude and sign in the temperature-time points employed in data analysis, and to the consequent deviations in the deduced parameters of the analysis. No consideration has been given to the effects of random errors which would require statistical methods.

II. Heat Transfer Theory

Data analysis procedures require having at hand the results of heat transfer calculations for the particular set of boundary and initial conditions appropriate to the experiment. In addition, the propagation of errors requires these formulae for the calculation of certain partial derivatives. The formulae used in the present calculations are summarized below.

Ideal Case. When the ideal boundary and initial conditions mentioned earlier are met, the rear surface excess temperature, T , is given by (1):

$$\frac{T}{T_{\infty}} = 1 + \sum_{n=1}^{\infty} (-1)^n \exp(-n^2 \pi^2 \tau) \quad (1)$$

where $\tau = \alpha t / l^2$ and T_{∞} is the equilibrium excess temperature under adiabatic conditions. α is the thermal diffusivity, t is the time, and l is the specimen thickness. The behavior of this function is shown graphically in Fig. 1, curve a.

Extended Pulse Effects. When a rectangular heat pulse of duration t_p is incident on the specimen, the rear surface excess temperature is given by:

$$\frac{T}{T_{\infty}} = \frac{\tau}{\tau_p} - \frac{1}{6\tau_p} - \frac{2}{\tau} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 \pi^2} \exp(-n^2 \pi^2 \tau) \quad 0 \leq t \leq t_p \quad (2)$$

$$\frac{T}{T_{\infty}} = 1 + \frac{2}{\tau_p} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 \pi^2} \exp(-n^2 \pi^2 \tau) [\exp(n^2 \pi^2 \tau_p) - 1] \quad t \geq t_p \quad (3)$$

where $\tau_p = \alpha t_p / l^2$. Curve a, Fig. 1 is qualitatively descriptive of eqs. (2) and (3).

Heat Loss Effects. When heat losses are significant, but otherwise ideal conditions exist, the rear surface excess temperature is given by (5,6):

$$\frac{T}{T_{\infty}} = \sum_{n=1}^{\infty} Y_n(0) Y_n(\ell) \exp(-\beta_n^2 \tau) \quad (4)$$

where $Y_n(x) = \frac{2^{\frac{1}{2}} [\beta_n \cos(\beta_n x/\ell) + H \sin(\beta_n x/\ell)]}{[\beta_n^2 + 2H + H^2]^{\frac{1}{2}}}$ and $\beta_n (n = 1, 2, 3, \dots)$

are the positive roots of $(\beta^2 - H^2) \tan \beta = 2\beta H$. H is the Nusselt number describing the surface heat losses. The behavior of eq. (4) is shown in curve b, Fig. 1

III. Data Analysis.

Ideal Case. From eq. (1) for $T_1 = 0.5 T_{\infty}$, numerical evaluation shows that $\tau_1 = \tau(t_1) = .1388$. The simple data reduction scheme shown in Table I then follows (ref. 1).

Extended Pulse Effects. Table II shows the details of a data reduction method suggested in ref. (5). This method is similar but not identical to those suggested in refs. (2-4). Step 5 involves the use of tables or graphs of W vs X prepared by numerical evaluation of eqs. (2) or (3).

Heat Loss Effects: The analysis of a temperature fluctuation significantly influenced by heat losses proceeds differently, primarily because there are two parameters, α and H , whose values must be found from the temperature fluctuation. Cowen's procedure (6) for this analysis is shown in slightly modified form (5) in Table III. Steps 7 and 8 are accomplished using tables or graphs constructed by numerical evaluation of eq. (4) (see ref. (5)).

Table I. Ideal Case

<u>Data Reduction</u>	<u>Propagation of Errors</u>
1. Find $T_{\max}$	$\Delta T_{\max} = \delta T_{\max}$
2. $T_1 = 0.5 T_{\max}$	$\Delta T_1 = 0.5 \Delta T_{\max} + \delta T_1$
3. $t_1 = t(T_1)$	$\Delta t_1 = (t_T)_1 \Delta T_1 + \delta t_1$
4. $\alpha = .1388 \ell^2 / t_1$	$\Delta \alpha = -.1388 \ell^2 \delta t_1 / t_1^2$

Table II. Extended Pulse Effect

<u>Data Reduction</u>	<u>Propagation of Errors</u>
1. Find T_{∞}	$\Delta T_{\infty} = \delta T_{\infty}$
2. $T_1 = 0.5 T_{\infty}$	$\Delta T_1 = 0.5 \Delta T_{\infty} + \delta T_1$
3. $t_1 = t(T_1)$	$\Delta t_1 = (t_T)_1 \Delta T_1 + \delta t_1$
4. $X = t_p/t_1$	$\Delta X = (X_t)_1 \Delta t_1$
5. $W = W(X)$ ($W = \alpha/\alpha_0$)	$\Delta W = (W_X)_1 \Delta X$
6. $\alpha = W \cdot \alpha_0$	$\Delta \alpha = (\alpha_W)_1 \Delta W + (\alpha_t)_1 \Delta t_1$

Note: $\alpha_0 = .1388 \ell^2/t_1$, $(\alpha_W)_1 = \alpha_0$

Table III. Heat Loss Effects

<u>Cowen Data Analysis</u>	<u>Propagation of Errors</u>
1. Find $T_{\max}$	$\Delta T_{\max} = \delta T_{\max}$
2. $T_1 = 0.5 T_{\max}$	$\Delta T_1 = 0.5 \cdot \Delta T_{\max} + \delta T_1$
3. $t_1 = t(T_1)$	$\Delta t_1 = (t_T)_1 \Delta T_1 + \delta t_1$
4. $t_2 = 5.0 t_1$	$\Delta t_2 = 5.0 \Delta t_1 + \delta t_2$
5. $T_2 = T(t_2)$	$\Delta T_2 = (T_t)_2 \Delta t_2 + \delta T_2$
6. $V_2 = T_2/T_{\max}$	$\Delta V_2 = (V_T)_2 \Delta T_2 + (V_T)_{\max} \Delta T_{\max}$
7. $H = H(V_2)$	$\Delta H = (H_V)_2 \Delta V_2$
8. $\tau_1 = \tau(H)$	$\Delta \tau_1 = (\tau_H)_1 \Delta H$
9. $\alpha = \tau_1 \ell^2/t_1$	$\Delta \alpha = (\alpha_{\tau})_1 \Delta \tau_1 + (\alpha_t)_1 \Delta t_1$

IV. Error Analysis

The error analysis is carried out by propagating errors in the various measured quantities, i.e., δT_{∞} , $\delta T_{\max}$, δT_1 , δT_2 , δt_1 and δt_2 , through the data analysis schemes as shown in Tables I, II, and III. Notations such as $(t_T)_1$ represent partial derivatives, $(\frac{\partial t}{\partial T})_1$. $(t_T)_1$ and (T_t) are found by taking the derivatives, as appropriate, of eqs. (1), (2), (3), or (4), and

numerically evaluating the resulting function. (W_{X1}) , $(H_V)_2$ and $(\tau_H)_1$ are obtained from computer generated tables of their variables to which spline functions are fit. The derivatives are then evaluated from the spline coefficients. The remaining derivatives follow directly from the defining functions.

Those partial derivatives which require computer calculation are shown in Figs. 2 and 3. In order to make the calculations in a general form, the following substitutions have been made; $(t_T)_1 (T_1/t_1) = (\tau_T)_1 (T_1/\tau_1)$ and $(T_t)_2 (t_2/T_2) = (\tau_T)_2 (\tau_2/T_2)$.

Ideal Case. The results of the error analysis for the ideal case are:

$$\frac{\Delta\alpha}{\alpha} = - \frac{\delta t_1}{t_1}, \text{ for } \delta T_{\max} = 0, \delta t_1 \neq 0, \delta T_1 = 0.$$

$$\frac{\Delta\alpha}{\alpha} = -.769 \frac{\delta T_{\max}}{T_{\max}}, \text{ for } \delta T_{\max} \neq 0, \delta t_1 = 0, \delta T_1 = 0.$$

In both cases, relative measurement errors lead to approximately equal magnitude relative errors in the diffusivity.

Extended Pulse Effects. The results for the case of extended pulses are shown in Fig. 4 where the relative error in α is plotted against τ_p/t_1 . $\delta t_1/t_1 = 5\%$ or $\delta T_{\max}/T_{\max} = 5\%$ has been assumed. The results show that when pulse durations are negligible compared with diffusion times, errors do not depend on the pulse width. As the two times become comparable, the errors increase sharply. This conforms with the intuitive view that when the pulse duration is very long, the experiment will be rather insensitive to the diffusivity.

Heat Loss Effects. Figure 5 shows the results when heat losses are significant. Errors in both α and H are shown plotted against H . Relative errors in α display a generally weak dependence on H and are approximately of the same magnitude as the measurement relative errors. Relative errors in H on the other hand, are more strongly dependent on H and are substantially larger than those in α .

Sensitivities. It is convenient to define the sensitivity, S_i^α , as

$$S_i^\alpha = \left| \frac{\delta P_i}{\Delta\alpha} \right|$$

where P_i is one of the measured temperatures or times, i.e., $T_{\max}$, T_1 , T_2 , t_1 , or t_2 . According to this definition, S_i then provides a basis on

which to compare one diffusivity measurement technique with another, to compare one method with itself under a different set of conditions, or to compare one data analysis method with another.

The sensitivity of the flash experiment is, for all the measured quantities and for most values of H or t_p/t_1 , $S_1^\alpha \sim 1$. On the other hand, $S_1^H < .5$ for most values of H , suggesting that the flash method is a poorer technique for measurements of H than α . This is consistent with our laboratory experience where frequently in analyzing an experimental temperature-time curve, values of H outside the expected range are obtained while the same curve yields a reasonably accurate value of the diffusivity.

V. Concluding Remark

The foregoing results suggest that the flash method is reasonably sensitive for thermal diffusivity measurements. It should be noted, however, that the results depend critically on the data analyses procedures assumed. An error or sensitivity study based on different procedures would yield different results. This emphasizes that the method of data analysis is as important as the particular experimental technique adopted. If either is improperly selected, an insensitive overall experiment may result.

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Figure 1. Rear surface excess temperature fluctuation.
a. Ideal conditions. b. Heat losses.

Figure 2. Normalized partial derivatives vs. τ/t_1 using eqs. (2) and (3).

Figure 3. Normalized partial derivatives vs. H using eq. (4).

Figure 4. Relative error in α vs. τ/t_1 for +5% errors in $T_{\max}$ and t_1 .

Figure 5. Relative errors in α (curves 1, 2, 3) and H (4, 5, 6) vs. H for 5% errors in $T_{\max}$ (1, 4), t_1 (2, 5) and T_2 (3, 6).