MONTE CARLO COMPUTATIONS OF THE HADRONIC MASS SPECTRUM

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ABSTRACT

This paper summarizes two talks presented at the Orbis Scientiae Meeting, 1982. Monte Carlo results on the mass gap (or glueball mass) and on the masses of the lightest quark-model hadrons are illustrated.

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MASTER

During the last few years Monte Carlo simulations have emerged as a very powerful technique to obtain numerical information on quantum gauge field theories¹. They provide the necessary bridge between the perturbative results and the strong coupling results which the lattice formulation allows to derive. Monte Carlo computations have produced very substantial evidence for the validity of the lattice regularization, and for the correctness of quantum chromodynamics (QCD) as the theory of strong interactions. Moreover it has been possible to estimate the value of several quantities of great experimental and theoretical interest.

I wish to present here recent Monte Carlo results on the lightest masses in the spectrum of a quantized SU(2) or SU(3) gauge theory. I shall consider the states occurring in a pure gauge system as well as those present in a theory of coupled gauge and fermionic fields. In QCD the former states (frequently called glueballs) correspond to quantized colorless excitations of the gluonic field. The latter states would of course constitute the full hadronic spectrum in a complete theory. In a first approximation, however, they are the states of the quark model, i.e. states of a quark and an antiquark or of three quarks, bound together by the gluonic field. As we shall see, the computation has not proceeded yet beyond this approximation, but the results are already very satisfactory.

The common denominator to both analyses is the procedure by which the masses are evaluated from the rate of decay of suitable Green's functions. Let $O(\vec{x}, t)$ be an operator with non-vanishing matrix elements between the vacuum and the state under consideration. O is defined in the lattice-regularized theory and $\vec{x}, t$ label lattice sites. By Monte Carlo simulations one calculates the quantum expectation value of the product of two such operators and forms the connected Green's function

$$G(\vec{x}, t) = \langle O(\vec{x}, t) O(0) \rangle - \langle O \rangle^2. \quad (1)$$

Expressing $G(\vec{x}, t)$ as a sum over a complete set of intermediate states one obtains

$$G(\vec{x}, t) = \sum_{n \neq 0} \left| \langle n | O(0) | 0 \rangle \right|^2 e^{-E_n t + i \vec{p}_n \cdot \vec{x}}, \quad (2)$$

where $|0\rangle$ stands for the vacuum state and $E_n, \vec{p}_n$ are energy and momentum of state $|n\rangle$ (recall that a Wick rotation to Euclidean space-time has been performed). It is convenient to project onto intermediate states of zero momentum by summing over all space lattice positions $\vec{x}$. In this way one arrives at

$$G_0(t) \equiv \sum_{\vec{x}} G(\vec{x}, t) = \sum_{\substack{n \neq 0 \\ \vec{p}_n = 0}} \left| \langle n | O(0) | 0 \rangle \right|^2 e^{-m_n t}, \quad (3)$$

where now m_n is the mass of the state. One can then estimate the mass of the lightest state from the rate of decay of $G_0(t)$ with t sufficiently large.

The problems met in the numerical determination of the Green's functions are however different in the pure gauge theory and in the theory with fermions and I shall proceed with a separate discussion of the two cases.

The simplest operator one can use to excite a pure gauge state out of the vacuum is the character of the parallel transporter around a plaquette, $U_{\square}$, in the fundamental representation. It is convenient to sum over all orientations of the plaquette in order to project onto the states of lowest lattice angular momentum. Consequently, we define

$$O = \sum_{\text{orientations}} \frac{1}{2} \text{Tr}(U_{\square}). \quad (4)$$

This operator reduces to the Lagrangian density $1/4 \text{Tr}(F_{\mu\nu} F^{\mu\nu})$ in the continuum limit.

The difficulty of the computation resides in the fact that the plaquette-plaquette correlation, i.e. $\langle O(\vec{x}, t) O(0) \rangle_{\text{connected}}$, remains of very short range throughout the domain of couplings, where the simulation can produce meaningful results. This is a consequence of the rather large value of the mass-gap, when expressed in terms of the natural lattice scale Λ . When the separation between the plaquettes exceeds a few lattice units, the correlation falls below the level of statistical fluctuations inherent in the Monte Carlo procedure and cannot be reliably determined. In practice one is limited to a time separation $t \leq 3$ in Eq. (3).

In a $SU(2)$ gauge theory the Monte Carlo simulation produces the results displayed in Fig. 1.² The graph shows the values of the effective mass-gap

$$m(t) = -\frac{1}{t} \ln \frac{G_0(t)}{G_0(0)} \quad (5)$$

as function of the coupling parameter $\beta = 4/g^2$, g being the unrenormalized coupling constant. The true mass-gap is given by

$$m(\beta) = \lim_{t \rightarrow \infty} m(t, \beta). \quad (6)$$

The lines in Fig. 1 illustrate the leading term in the strong coupling expansion for $m(\beta)$ (dashed line, all $m(t)$ are identical to leading order), the expected scaling behavior

$$m(\beta) = c \left[\frac{6\pi^2}{11} \beta \right]^{\frac{51}{121}} \exp \left[-\frac{3\pi^2}{11} \beta \right], \quad (7)$$

with three values of c representing a fit to the Monte Carlo results and estimated error (solid lines), and the constant asymptotic limits for $m(1)$, $m(2)$ and $m(3)$ (short dashed lines to the right), as derived from a perturbative weak-coupling expansion.

One notices that the Monte Carlo data agree well with the strong coupling expansion up to $\beta \approx 2$. After that, for a limited range of β , the values for $m(t, \beta)$ are consistent with a scaling behavior of the envelope $m(\beta) \equiv m(\infty, \beta)$. The interval over which the data appear to scale is rather small, because the effective masses $m(t, \beta)$ must eventually tend to the constant, weak-coupling limits. The overall pattern is reminiscent of the behavior of the effective string tension K_{eff} , as determined from finite size Wilson loops^{3,4}. The range over which the effective string tension appears to scale is however more extended, also because all K_{eff} tend to zero rather than to constant values in the weak coupling limit.

The computation of the mass gap can be ameliorated by selecting a more appropriate operator O to excite the glueball state from the vacuum. Indeed, if one could choose an operator such that $\langle n|O|0\rangle$ is different from zero only for a specific state $|n\rangle$, then of course all $m(t)$ in Eq. (5) would be identical and equal to m_n . Knowing the exact form of the operator with the above property would be tantamount to mastering the wave function of the lowest excited state, an unattainable feat for the moment. But in analogy with the variational technique used to find the lowest eigenstate of a Hamiltonian, one can allow for some free parameters in the definition of O and determine these so as to obtain the slowest rate of decay. This provides better upper bounds for the mass gap.

The variational procedure has been followed in Refs. (2), (5) and (6) for the SU(2) theory, and in Ref. (7) for the SU(3) theory. Linear combinations of characters of parallel transporters around various kinds of closed loops were used to excite the glueball state from the vacuum. Fig. 2 illustrates the results obtained in Ref. (2). The higher statistical errors deriving from the inclusion of several operators restrict the range of time separations to $t \leq 2$. Yet one sees that the minimization procedure allows to obtain, with $t \leq 2$, an estimate comparable to the one obtained with $t \leq 3$ and a single plaquette operator. Using the earlier determination of the string tension K , the value² found for the constant in Eq. (7) corresponds to

$$m_g = (2.4 \pm 0.6) \sqrt{K}$$

for the mass-gap, i.e.

$$m_g \approx 1.2 \text{ GeV}$$

assuming that the passage from SU(2) to SU(3) and the inclusion of fermions do not alter too much $m_g/\sqrt{K}$.

In Refs. (5) and (6) (where estimates for the masses of higher spin glueballs are also presented) the values $m_g = (2.9 \pm 0.4)\sqrt{K}$ and $(2.5 \pm 0.3)\sqrt{K}$ have been found. The authors of Ref. (7) give instead $m_g = (2.6 \pm 0.3)\sqrt{K}$ for a pure SU(3) gauge theory.

Earlier Monte Carlo results on the glueball mass were derived from direct consideration of the correlation function, with no sum over space positions^{4,8}. The estimates were higher, not surprisingly since states of all momenta contribute then to the sum. Other estimates have relied on strong coupling expansions⁹, which can give results compatible with the ones just quoted, but unfortunately quite sensitive to the extrapolation being used. Monte Carlo determinations of m_g from studies of the bulk properties of the system have also been presented in the literature¹⁰. The values found for m_g tend to be slightly lower than those mentioned above; the difficulty in these procedures is that one has little control over the degeneracy of intermediate states.

Let me now turn to the quark-model states in a theory with fermions. To excite mesonic states from the vacuum one considers operators

$$O(\vec{x}, t) = \bar{\psi}(\vec{x}, t) \Gamma \psi(\vec{x}, t), \quad (8)$$

where $\psi, \bar{\psi}$ represent the fermionic fields and Γ stands for a suitable matrix acting over spin and flavor degrees of freedom. In the study of baryonic states one would instead consider expressions trilinear in the fermionic fields. The relevant Green's functions are given by

$$\begin{aligned} \langle O(\vec{x}, t) O(0) \rangle &= Z^{-1} \int DA \int D\psi D\bar{\psi} O(\vec{x}, t) O(0) \times \\ &\times \exp\{-S_G(A) - \bar{\psi}(\not{D}(A) + m)\psi\}. \end{aligned} \quad (9)$$

In this formula one integrates over all values of the gauge dynamical variables A , associated with the links of the lattice, and over the values of the anticommuting (Grassmann) variables ψ and $\bar{\psi}$, defined at the lattice sites. $S_G(A)$ stands for the pure gauge part of the action, whereas the fermionic action is given by the expression $\bar{\psi}(\not{D}(A) + m)\psi$, quadratic in the Fermi fields. I have used the continuum notation $\not{D}(A) + m$ to represent the lattice Dirac operator as well. I shall not discuss here the various ways to formulate the Dirac equation on the lattice, nor the difficulties one encounters. Let me only state that in the Monte Carlo applications both the method proposed by Wilson¹¹ and an extension of the method proposed by Susskind and collaborators¹² have been followed. Finally, Z in Eq. (9) stands for the vacuum to vacuum permanence amplitude, given by an integral analogous to the one shown in the formula, but without the O operators.

In the path integral formulation of theories with fermions one introduces Grassmann variables to represent sums over occupation numbers which can be 0 or 1. In a lattice of finite extent, the number of fermionic states which can be occupied or empty is finite. Thus one might think that the integration over fermionic degrees of freedom, representing a finite sum, should also be amenable to the Monte Carlo procedure. Unfortunately, if the quantum averages are recast as sums over occupation numbers the crucial positive-definiteness property of the measure is lost.

Attempts to perform Monte Carlo simulations for systems with fermions have followed a different route. The Gaussian integral over fermionic variables can be done explicitly and one arrives at

$$\begin{aligned}
\langle O(\vec{x}, t) O(0) \rangle &= \\
&= Z^{-1} \int DA \langle O(\vec{x}, t) O(0) \rangle_A e^{-S_G(A)} \text{Det} (\not{D}(A) + m), \quad (10)
\end{aligned}$$

where $\langle O(\vec{x}, t) O(0) \rangle_A$ stands now for the Green's function of the operator O in the presence of a fixed background gauge field A . Assuming that $\text{Det} (\not{D} + m)$ is positive (which in QCD is guaranteed by the existence of a γ_5 transformation: $\gamma_5 (\not{D} + m) \gamma_5 = -\not{D} + m = (\not{D} + m)^\dagger$), then one can evaluate the quantum averages as averages over gauge field configurations only with the new measure $\exp\{-S_{\text{eff}}(A)\}$, where

$$\exp\{-S_{\text{eff}}(A)\} \equiv \exp\{-S_G(A) + \ln \text{Det} (\not{D}(A) + m)\}. \quad (11)$$

This does not solve all problems, because the new, effective action $S_{\text{eff}}(A)$ is non-local and the computation of the variation ΔS_{eff} induced by a change ΔA becomes much more difficult. The evaluation of ΔS_{eff} is at the core of the Monte Carlo procedure and indeed the major efforts in the application of the method to fermions have gone precisely into finding efficient ways to calculate ΔS_{eff} .

Techniques to include the fermionic determinant in the measure are now available and have been tested on a variety of simplified models^{14,15}. One can however conjecture that the major dynamical effects of the interaction between the gauge field and the fermions would already be manifest if the fixed-background Green's functions $\langle O(\vec{x}, t) O(0) \rangle_A$ were averaged with the pure gauge measure $\exp\{-S_G(A)\}$. In diagrammatic terms, leaving out the $\text{Det} (\not{D} + m)$ factor in Eq. (10) corresponds to considering only Feynman graphs without internal closed fermion loops (although of course non-perturbative effects are also included in a Monte Carlo computation). This goes beyond the so-called planar approximation which has often been advocated as a viable first order approximation for the theory of strong interactions.

Monte Carlo results for the massive Schwinger model, obtained with or without the inclusion of the $\text{Det} (\not{D} + m)$ factor, have been compared in Ref. 15. Borrowing from the theory of spin glasses, the term "quenched approximation" has been introduced to denote the neglect of the fermionic determinant in the measure. A crucial parameter is the expectation value $\langle \bar{\psi} \psi \rangle$, which does not vanish as the mass m of the fermion goes to zero because of spontaneous chiral symmetry breaking. In the continuum limit $\langle \bar{\psi} \psi \rangle_{m=0}$ is known from the exact solution of the theory. The Monte Carlo numerical analysis gave¹⁵

$$\langle \bar{\psi} \psi \rangle_{m=0} \approx 0.84 \langle \bar{\psi} \psi \rangle_{m=0, \text{ cont.}},$$

when the effects of $\text{Det} (\not{D} + m)$ were included, and

$$\langle \bar{\psi} \psi \rangle_{m=0} \approx 1.38 \langle \bar{\psi} \psi \rangle_{m=0, \text{ cont.}}$$

in the quenched approximation. Monte Carlo simulations making use of the quenched approximation have been performed for an $SU(2)$ model of quarks and gluons in Refs. 16, 17 and 18; for the $SU(3)$ theory in Ref. 19.

In Refs. 16 and 17 the fermionic action was defined adapting to the Euclidean lattice a formulation proposed by Susskind and collaborators in a Hamiltonian context¹³. The Lagrangian maintains then even in the lattice version a chiral invariance broken explicitly only by the mass term m_q for the quarks. In the limit $m_q = 0$ the action is chirally invariant (the conserved chiral charge is not a singlet with respect to flavor transformations, thus avoiding a conflict with theorems on the anomaly), but one expects the symmetry to be dynamically broken. Indeed, the Monte Carlo simulation gives evidence for a non-vanishing expectation value of $\bar{\psi}\psi$ (see Fig. 3), which moreover appears to scale with β in agreement with the renormalization group prediction (solid line). In Ref. 16 correlation functions of operators exciting pseudoscalar, vector and scalar mesons from the vacuum were evaluated. (Relaxation techniques have been used in all investigations¹⁶⁻¹⁹ to compute the Green's functions in a fixed background field.) Fig. 4 illustrates the typical time dependence of a definite correlation (namely, the correlation of the pseudoscalar operator on a $8^3 \times 16$ lattice, with $m_q = 0.2$ in units of inverse lattice spacing and $\beta = 2.2$). The numerical computation shows that the mass squared of the lightest pseudoscalar object goes to zero together with the quark mass parameter. One can then use the experimental value of m_π and the independently determined value of the string tension^{3,4} to fix m_q , which turns out approximately equal to 7 MeV (this must be interpreted as an average mass for the u and d quarks which are not differentiated in the analysis). The masses of the vector and scalar excitations, m_ρ and m_0 , exhibit much less dependence on the quark mass and tend to non-vanishing limits as $m_q = 0$. One finds $m_\rho = 800 \pm 80$ MeV and $m_0 = 950 \pm 100$ MeV. Finally, the value measured for $\langle \bar{\psi}\psi \rangle$ allows one to determine $f_\pi = 150 \pm 10$ MeV for the pion decay constant.

In Ref. 17 the parameters of the system (β and m_q) were modified so as to adapt the analysis to the states of charmonium. Using $m_{J/\psi} = 3.10$ GeV as input, values

$$m_{\eta_c} = (2.95 \pm 0.03)\text{GeV (exp 2.979 GeV)},$$

$$m_{X_0} = (3.40 \pm 0.10)\text{GeV (exp 3.414 GeV)},$$

$$m_{\rho_c} = (3.60 \pm 0.15)\text{GeV (exp 3.507 GeV)},$$

$$m_{\eta_c'} = (3.65 \pm 0.15)\text{GeV, and}$$

$$m_\rho = (3.75 \pm 0.15)\text{GeV have been derived.}$$

In Ref. 18 the lightest mesonic states have been studied in a SU(2) model and with Wilson's action for the lattice fermions. The results are in agreement with those of Ref. 16.

In Ref. 19 the analysis was carried out with an SU(3) model, which is certainly more realistic and also allows considering baryonic states. Wilson's formulation of the lattice fermionic action was followed. The results: $m_\rho = 800 \pm 100$ MeV,

$$m_\delta = 1000 \pm 100 \text{ MeV},$$

$$m_{A_1} = 1200 \pm 100 \text{ MeV},$$

$$f_\pi = 95 \pm 10 \text{ MeV},$$

for the mesonic part of the spectrum, and

$$m_p = 950 \pm 100 \text{ MeV},$$

$$m_\Delta = 1300 \pm 100 \text{ MeV},$$

for the baryonic sector, are in excellent agreement with the experimental data.

Altogether, the results on hadronic spectroscopy obtained with Monte Carlo simulations are highly satisfactory. Much work remains to be done. In particular, one needs to go beyond the quench approximation: research along this line is in progress. Moreover, the most rewarding, analytical understanding of the properties of the theory is still evading us. But what has been achieved up to now is very gratifying and reinforces our conviction of the correctness of QCD as the theory of strong interactions.

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FIGURE CAPTIONS

- 1) Estimate of the SU(2) mass-gap from the plaquette-plaquette correlation function.
- 2) Estimate of the mass gap using a variational technique.
- 3) Monte Carlo measurement of $\langle \psi \bar{\psi} \rangle_{m_q=0}$ in an SU(2) model with fermions.
- 4) Monte Carlo measurement of the correlation function for the pseudoscalar state.

FIGURE 1

FIGURE 2

FIGURE 3

FIGURE 4