UCC-ND

**NUCLEAR
DIVISION**

**UNION
CARBIDE**

MASTER

**NUREG/CR-1616
ORNL/NUREG/CSD/TM-16**

The Effect of Reflector Location on Array Criticality

J. T. Thomas

**Prepared for the U.S. Nuclear Regulatory Commission
Office of Nuclear Regulatory Research
Under Interagency Agreement DOE 40-550-75**

**DO NOT MICROFILM
COVER**

**OPERATED BY
UNION CARBIDE CORPORATION
FOR THE UNITED STATES
DEPARTMENT OF ENERGY**

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

The following pages are an exact
representation of what is in the original
document folder.

Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road, Springfield, Virginia 22161

Available from
GPO Sales Program
Division of Technical Information and Document Control
U.S. Nuclear Regulatory Commission
Washington, D.C. 20555

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DO NOT MICROFILM
COVER

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

NUREG/CR--1616

TI86 002781

THE EFFECT OF REFLECTOR LOCATION ON ARRAY CRITICALITY

J. T. Thomas

PREPARED FOR THE U.S. NUCLEAR REGULATORY COMMISSION
OFFICE OF NUCLEAR REGULATORY RESEARCH
UNDER INTERAGENCY AGREEMENT DOE 40-550-75
NRC FIN No. B0419

Techmonitor: Donald E. Solberg

Manuscript Completed: July 17, 1980

Date Published : November 1980

MASTER

NOTICE This document contains information of a preliminary nature.
It is subject to revision or correction and therefore does not represent a
final report.

COMPUTER SCIENCES DIVISION
at
Oak Ridge National Laboratory
Post Office Box X
Oak Ridge, Tennessee 37830

Union Carbide Corporation, Nuclear Division
operating the
Oak Ridge Gaseous Diffusion Plant • Paducah Gaseous Diffusion Plant
Oak Ridge Y-12 Plant • Oak Ridge National Laboratory
under Contract No. W-7405-eng-26
for the
Department of Energy

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

peg JW

**THIS PAGE
WAS INTENTIONALLY
LEFT BLANK**

TABLE OF CONTENTS

<u>Title</u>	<u>Page</u>
LIST OF FIGURES.	v
LIST OF TABLES	vii
ACKNOWLEDGMENTS.	ix
ABSTRACT	xi
INTRODUCTION	1
CRITICALITY OF UNREFLECTED AND REFLECTED CUBIC ARRAYS.	3
<u>Reflectors and Cuboidal Array Geometries.</u>	7
<u>Reflector Location and Array Shape.</u>	16
<u>Array Reflector Thickness</u>	24
<u>Discussion.</u>	25
<u>Conclusions</u>	33
REFERENCES	35
APPENDIX	37
<u>Array Reactivity.</u>	37

LIST OF FIGURES

<u>Figure</u>	<u>Page</u>
1. Parametric representation of calculated criticality of cubic arrays of U(93.2) metal as spheres and cubes under three reflector conditions.	4
2. Total reactivity worth of thick concrete and of thick water as reflectors closely fitting cubic arrays of air-spaced U(93.2) metal units	8
3. The increasing mass of units in an array is displayed as a function of the distance a thick water reflector is located from the array surfaces	11
4. The increasing mass of units in an array is displayed as a function of the distance a thick concrete reflector is located from the array surfaces	12
5. Representation of array reactivity loss as a thick reflector material is moved a distance from the array. The distance is expressed in multiples of the array dimension	13
6. Parametric representation of criticality displaying the mass required to maintain criticality as three sides of a reflector recede from an array initially reflected on six faces by thick concrete.	15
7. Representation of array reactivity loss as a thick water reflector recedes from five surfaces of a 32 x 16 x 1 planar array of U(93.2) metal units of several shapes.	21
8. Calculated neutron multiplication factors of a single cubic having a centered 22.5 kg U(93.2) metal cylinder as a function of reflector location and reflector conditions. The 32 x 16 x 1 array data from Table 7a are shown for comparison.	23
9. Generic representation of reactivity controlled by the water reflector thickness on arrays of arbitrary shape.	26
10. Equivalence between the location of a thick reflector from the surfaces of an array and the thickness of a closely fitting water reflector on arrays	27

LIST OF TABLES

<u>Table</u>	<u>Page</u>
1. Calculated Neutron Multiplication Factor of Water Reflected Cubic Arrays of U(93.2) Metal Spheres as the Reflector Recedes Uniformly From Six Faces of the Array	9
2. Calculated Neutron Multiplication Factor of Concrete Reflected Cubic Arrays of U(93.2) Metal Spheres as the Reflector Recedes Uniformly From Six Faces of the Array	10
3. Calculated Neutron Multiplication Factor of Concrete Reflected Arrays of U(93.2) Metal Spheres as the Reflector Recedes From Three Faces of an Array of 64 Units Each Centered in a 60.96 cm Cubic Cell.	14
4. Comparison of Estimated and Monte Carlo Calculated K-eff as Number of Units is Increased Inside a Reflecting Cavity of Constant Dimension.	17
5. Monte Carlo Calculated K-eff for an Infinite Planar Array of 14.94 kg U(93.2) Metal Spheres Centered in 58.39 cm Cubic Cells Four Tiers High as a Thick Water Reflector Recedes From the Cell Boundaries	18
6. Calculated Neutron Multiplication Factor of a Planar 32 x 16 x 1 Water-Reflected Array of U(93.2) Metal Spheres as the Reflector Recedes from Five Faces of the Array. Units are 20.2 kgU Centered in Cubic Cells with Dimension of 32.908 cm	19
7. Calculated Neutron Multiplication Factor of a 32 x 16 x 1 Planar Array of U(93.2) Metal Cylinders for Varying Water Reflector Conditions. Units are Centered in 32.908 cm Cubic Cells	20
8. Calculated Neutron Multiplication Factor of a Cubic Water-Reflected Array of U(93.2) Metal Cylinders as the Reflector Recedes From Five Faces of a 64-Unit Array with Units Centered in 83.2 cm Cubic Cells	24
9. Summary of Analyses for Operational Area Evaluations.	31

ACKNOWLEDGMENTS

The author is indebted to Mrs. P. B. Fox for the many calculations performed with the KENO IV code, checking the data and preparing it in tabular form.

ABSTRACT

The motion of a thick reflector away from a critical array of subcritical units of fissile material results in an array reactivity loss. The fraction of the total reactivity worth of a reflector is related to the distance a reflector is located from the surfaces of an array. The magnitude of reactivity associated with the reflector location may be used in the assessment of nuclear criticality safety of operations with fissile materials to establish margins of subcriticality. An equivalence is established between the reactivity associated with reflector location and the reactivity associated with the thickness of a water reflector closely fitting an array.

INTRODUCTION

One of the types of basic information used in establishing nuclear criticality safety limits is that of criticality data for fissile materials having a thick closely fitting water reflector. The subcritical safety limit corresponds to some accepted k_{eff} value less than unity under the condition of a closely fitting water reflector. An operation in a process evaluated for subcriticality through application of a safety limit should have included a demonstration that materials and other operations surrounding the system in question do not return neutrons more effectively to the system than would the reflector condition of the safety limit.¹ A validated computational method² may be employed to determine the k_{eff} of the operation and the resultant k_{eff} compared to that of the safety limit. Generally, the value is found to be less than that of the limit, and the reactivity difference may be included in or be an addition to an acceptable margin of subcriticality for the operation. On a case-by-case basis and for a single vessel or operation, that is not a complicated evaluation and inclusion of the reactivity loss in the margin of subcriticality can be considered acceptable practice. However, when a number of operations are conducted in an area and there is a high potential for their being neutronically coupled, the demonstration can become complex, especially when the safety limit may have a parametric dependence.

The criticality studies presented in the following suggest a method for estimating the effect of reflector location on the criticality of arrays of subcritical units of fissile materials. The study was conducted by a validated method³ and employed calculations performed

with the KENO-IV Monte Carlo Criticality Code⁴ in conjunction with the Hansen-Roach sixteen-group neutron cross section sets⁵ as was used in the validation. The response of an array k_{eff} to motion of a closely fitting reflector away from the array, ultimately to the unreflected array condition (vacuum boundary condition), is expressed in terms of the distance of the reflector from the array. A previous study⁶ on the evaluation of reflector materials closely fitting an array resulted in an expression of the reactivity worth of a reflector in terms of the thickness of a water reflector surrounding an array. Combining the results of these two studies permits expression of reactivity equivalence between the reflector location and the thickness of a closely fitting water reflector on the array. The reactivity associated with a reflector condition intermediate to thick, closely fitting water and unreflected arrays is shown to depend on the mass of fissile material in a unit, its shape and the array shape. A condition in which no allowance in k_{eff} for reflector location should be permitted is planar arrays, or continuous slabs of fissile materials, unlimited in two dimensions.

CRITICALITY OF UNREFLECTED AND REFLECTED CUBIC ARRAYS

Experimental and Monte Carlo calculated criticality data for air-spaced subcritical units of fissile material arranged in cuboidal arrays, reflected by water and concrete, have been represented³ analytically by the expression

$$\sigma(m) = \frac{nm}{(2a_n)^2} (1 - c/\sqrt{N})^2 \quad (1a)$$

and
$$\sigma(m) = c_2 (m_0 - m). \quad (1b)$$

In these expressions of criticality, N is the total number of units in the array, $n^3 = N$, m is the mass of fissile material centered in a cubic cell of dimension $2a_n$, m_0 is the unreflected critical mass in the geometry of the unit, c_2 is a characteristic slope for the data base resulting from a linear regression of the data, and $c = 0.55 \pm 0.18$. Presented in Figure 1 are data of arrays of U(93.2) metal spheres in cubic array for three array reflector conditions, unreflected, 30-cm-thickness of water, and 41-cm-thick concrete. Each data point, $[m, \sigma(m)]$, shown represents several calculations of arrays with constant m but different N and a_n ; i.e., a point represents any N unit array for $N \geq 27$ by Equation (1a). The reflector location, when present is adjacent to the outer planar surfaces of the array defined by the outer boundaries of the peripheral cells.

Figure 1. Parametric representation of calculated criticality of cubic arrays of U(93.2) metal as spheres and cubes under three reflector conditions.

Consider an N unit cubic array with cells of dimension $2a_n$ cm. Let the geometry of the array remain unchanged and suppose for this unreflected condition a mass m is required for criticality. Adding a 30-cm-thick water reflector about the array will necessitate reducing the mass from m to some value m' in order to maintain criticality. Again, if 41-cm-thick concrete replaces the water reflector, the mass must be further reduced from m' to some m'' to compensate for the excess reactivity and restore k_{eff} to unity. These changes are represented analytically by Equations (1) as

$$\frac{\sigma(m)}{m} = \frac{\sigma(m')}{m'} = \frac{\sigma(m'')}{m''} = \frac{n}{(2a_n)^2} (1 - c/\sqrt{N})^2, \quad (2)$$

and are the equations of the lines passing through the origin in the figure. The relation expresses equivalent masses under the constraint of criticality when the reflector condition is changed. It also may express mass equivalence when the array and reflector remain unchanged but the fissile material is changed or is of a different form and/or shape. A slope c_2 is generic and its value is determined when all parameters of the array are unchanged but the mass of the unit is varied.

It is possible to interpret the reactivity change caused by adding a reflector to or changing a reflector condition of an array in terms of the unit mass change or change in unit reactivity. This is similar to

the concept of reflector savings in single unit criticality analyses. The relation between array reactivity and unit reactivity is expressed* in the following statement.

In order that an array k_{eff} be changed in reactivity by an amount ρ , it is sufficient that the reactivity of each unit in the array be changed by the same ρ .

For critical arrays with a median energy of fission greater than about 0.1 eV a good approximation of the resultant array k_{eff} due to a change in the mass of the units is

$$\frac{k'_{eff}}{k_{eff}} \approx \frac{r'}{r} . \quad (3)$$

For arrays having an appreciable fraction of fissions in the thermal neutron energy region, there would be needed as input to Equation (2) the additional information of the k_{eff} response to reductions in radius for the single unreflected unit. Under the constraint of criticality, i.e., array $k_{eff} = 1$, for both the reflected and unreflected arrays, the total Δk associated with the reflector is $(1 - r/r')$, where r' is the radius of units in the critical unreflected array. This Δk is also the reactivity associated with removing the reflector from the reflected array, i.e., $\rho = (k' - k)/k'$. Applying this relation to the data of Figure 1 gives the total reactivity worth of the reflector (or total Δk change on

*See discussion in Appendix.

reflector removal) as a function of the fraction of the unreflected critical mass of the U(93.2) metal units in an array. The relation is shown in Figure 2 for reflectors of water and concrete.

Reflectors and Cuboidal Array Geometries

The condition intermediate to the unreflected and closely reflected arrays represented in Figure 1 and the associated total k by Figure 2 is exhibited by a series of Monte Carlo calculations in which the reflector of constant thickness recedes from the array boundary. The location of the inner surfaces of the reflector is defined in terms of the dimension of the cubic array, i.e., the length of an edge of the array. Letting the six faces of the reflector recede from the stationary array, the cubic array is considered to be centered in a cubic cavity of dimension R in the reflector material. The ratio of the array dimension, $A = n_2 a_n$, to an inside edge dimension of the reflector cavity, R , is a measure of the distance between the surface of the array and the inner surface of the reflector. Table 1 presents Monte Carlo calculations of critical cubic arrays as a 30-cm-thick water reflector recedes from the array. Table 2 reports similar calculations for concrete reflected arrays. These data are shown in Figures 3 and 4, respectively. It is apparent that for a given reflector location, i.e., A/R value, the data of different arrays can be easily correlated in the mass range of the unreflected arrays characterized by linearity. The correlation is extended over the entire mass range $(0, m_0)$ by expressing the reactivity change associated with the reflector location as a fraction of the total reactivity worth $(1 - r/r')$ of the reflector. This has been done for the data of Tables 1 and 2 and the results are shown in Figure 5.

Figure 2. Total reactivity worth of thick concrete and of thick water as reflectors closely fitting cubic arrays of air-spaced U(93.2) metal units.

TABLE II. Calculated Neutron Multiplication Factor of Water Reflected Cubic Arrays of U(93.2) Metal Spheres as the Reflector Recedes Uniformly From Six Faces of the Array

Dimension of Array:Refl.	N = 64, $2a_0 = 60.36$ cm			N = 216, $2a_0 = 50.8$ cm			N = 512, $2a_0 = 45.72$ cm			N = 729, $2a_0 = 30.8$ cm			N = 1000, $2a_0 = 25.4$ cm		
	Radius, (cm)	k_{eff}	Calculated ^a Rad. for Critic'y (cm)	Radius, (cm)	k_{eff}	Calculated ^a Rad. for Critic'y (cm)	Radius, (cm)	k_{eff}	Calculated ^a Rad. for Critic'y (cm)	Radius, (cm)	k_{eff}	Calculated ^a Rad. for Critic'y (cm)	Radius, (cm)	k_{eff}	Calculated ^a Rad. for Critic'y (cm)
1:1	7.568	0.999	7.568	6.682	1.018	6.682	6.001	1.015	6.001	4.727	1.005	4.727	4.118	1.015	4.118
9:10		0.990	7.652		0.995	6.790		0.991	6.129		0.978	4.874		0.965	4.267
3:4		0.957	7.342		0.945	7.115		0.938	6.451		0.910	5.114		0.906	4.515
6:10		0.940	8.317		0.914	7.300		0.894	6.709		0.847	5.428		0.831	4.822
1:2		0.936	8.164		0.891	7.533		0.864	6.926		0.808	5.660		0.782	5.011
1:4		0.908	8.359		0.855	7.870		0.810	7.294		0.733	6.129		0.694	5.355
1:8		0.913	8.431		0.845	7.917		0.802	7.481		0.705	6.205		0.666	5.522
1:∞			8.438			8.017			7.481			6.202			5.518

^aStandard deviation for 30,000 neutron histories was less than ± 0.005 .

TABLE 2. Calculated Neutron Multiplication Factor of Concrete Reflected Cubic Arrays of U(93.2) Metal Spheres as the Reflector Recedes Uniformly From Six Faces of the Array

Dimension of Array: Refl.	N = 64, 2a _s = 60.96 cm			N = 216, 2a _s = 50.8 cm			N = 512, 2a _s = 45.72 cm			N = 729, 2a _s = 30.48 cm			N = 1000, 2a _s = 25.4 cm		
	Radius, (cm)	k _{eff}	Calculated ^a Rad. for Critic'y (cm)	Radius, (cm)	k _{eff}	Calculated ^a Rad. for Critic'y (cm)	Radius, (cm)	k _{eff}	Calculated ^a Rad. for Critic'y (cm)	Radius, (cm)	k _{eff}	Calculated ^a Rad. for Critic'y (cm)	Radius, (cm)	k _{eff}	Calculated ^a Rad. for Critic'y (cm)
1:1	7.090	0.996	7.090	6.056	1.00	6.056	5.338	1.009	5.335	4.107	1.005	4.105	3.552	1.007	3.550
9:10			7.276		0.95E	6.310		0.945	5.571		0.954	4.283		0.951	3.717
3:4		0.933	7.593		0.89E	6.647		0.887	6.038		0.860	4.783		0.845	4.039
6:10		0.902	7.890		0.84E	7.139		0.822	6.381		0.781	5.046		0.757	4.462
1:2		0.893	7.992		0.81E	7.280		0.782	6.621		0.710	5.391		0.694	4.700
1:4		0.863	8.357		0.782	7.785		0.728	7.166		0.619	6.065		0.580	5.317
1:8		0.849	8.386		0.769	7.917		0.706	7.385		0.586	6.174		0.546	5.520
1:∞			8.438			8.017			7.481			6.202			5.518

^aStandard deviation for 30,000 neutron histories was less than ± 0.005 .

ORNL-DWG 79-16586

Figure 3. The increasing mass of units in an array is displayed as a function of the distance a thick water reflector is located from the array surfaces.

ORNL-DWG 79-16584

Figure 4. The increasing mass of units in an array is displayed as a function of the distance a thick concrete reflector is located from the array surfaces.

Figure 5. Representation of array reactivity loss as a thick reflector material is moved a distance from the array. The distance is expressed in multiples of the array dimension.

The relationship presented in Figure 5 may be considered as generic for finite cuboidal arrays and applied to fissile materials other than U(93.2) metal by application of the mass equivalence relation, Equation (2). The estimated k_{eff} for such systems can be expected to be within a few percent of a Monte Carlo calculation of the configuration.

Experimental and calculated data^{3,7,8} for the criticality of arrays when three sides of the array are reflected and the remaining three sides recede from the array also are characterized by Figure 5. Typical data are given in Table 3 and displayed in Figure 6. Also shown are two experiments with U(93.2) metal cylinders. The arrays contained 8 and 27 cylindrical units and were reflected on three surfaces by 15 cm of paraffin.

Table 3. Calculated Neutron Multiplication Factor of Concrete Reflected Arrays of U(93.2) Metal Spheres as the Reflector Recedes from Three Faces of an Array of 64 Units Each Centered in a 60.96 cm Cubic Cell

Dimension Ratio array:reflector	Sphere Radius (cm)	Calculated ^a k_{eff}
1:1	7.090	0.996
9:10	7.172	0.997
3:4	7.345	0.998
3:5	7.522	1.000
1:2	7.668	1.009
1:4	7.837	0.997
1:8	7.939	0.998
1:0	8.019	---

^aStandard deviation for 30,000 neutron histories was less than ± 0.006 .

A test of Equations (1) and Figures 2 and 5 is the following hypothetical experiment. Suppose an array is assembled in a large reflecting cavity (or vault) and it is desired to eliminate the k_{eff} as the the assembly progresses until criticality is achieved with the volume filled to capacity. Beginning with a 5^3 ($5 \times 5 \times 5$) array of 8.5 kg U(93.2)

ORNL-DWG 79-16582

Figure 6. Parametric representation of criticality displaying the mass required to maintain criticality as three sides of a reflector recede from an array initially reflected on six faces by thick concrete.

spheres centered in 38.1-cm cubic cells, assume the array is centered in a 495.3 cm cube surrounded by a thick water reflector. The volume will be filled by the 13^3 array resulting in criticality and the k_{eff} is to be estimated as succeeding arrays of 7^3 , 9^3 and 11^3 are completed. Applying Equations (1) to the 5^3 array with a reflector closely fitting yields a critical radius of 6.150 cm; Figure 2 gives the total reflector worth for 8.5 kg U ($r = 4.764$) as 0.238 Δk . The 5^3 array in the cavity is represented by an A/R value of 5/13 giving an $F(\rho)$ of 0.80 and a ρ of 0.19 ($0.80 \times .238$) which must be added to the 5^3 array to maintain criticality. The radius for criticality is then $6.15/(1 - 0.18) = 7.60$ cm and the estimated k_{eff} is $4.764/7.60$ or 0.627 for the 8.5 kg U loading. This procedure was repeated for each configuration and a summary appears in Table 4. The last column of Table 4 reports the Monte Carlo calculation for each array condition and the estimate is in reasonable agreement.

Reflector Location and Array Shape

The results of Figure 5 obviously do not apply to infinite planar arrays, since it is known that the k_{eff} of an infinite slab of fissile material reflected on its two surfaces is not sensitive to motion of the reflector at finite distances between the surfaces of the material and reflector. As an illustration of this point, the k_{eff} of a water-reflected infinite planar array of spherical units was calculated as the reflector location was varied from contact with the array at the peripheral cell boundaries to an infinite distance represented by the unreflected condition for the array. These data are given in Table 5. The entry for A:R = 1:8 represents the reflector at about 8 m from the array, and its effectiveness cannot be said to differ significantly from

Table 4. Comparison of Estimated and Monte Carlo Calculated K_{eff} as Number of Units is Increased Inside a Reflecting Cavity of Constant Dimension

Array	Critical radius ^a , cm	Criticality in Cavity				Est. k_{eff}	Calc. ^b k_{eff}
		A/R	R(ρ)	ρ	r, cm		
5 ³	6.150	5/13	0.80	0.190	7.60	0.627	0.649
7 ³	5.635	7/13	0.63	0.150	6.63	0.718	0.707
9 ³	5.296	9/13	0.42	0.100	5.88	0.810	0.801
11 ³	4.978	11/13	0.21	0.050	5.24	0.909	0.897
13 ³	4.764	13/13	0	0	4.764	1.000	1.000

^aReflector closely fitting the array.

^bStandard deviation for 30×10^3 neutron histories was ± 0.005 .

that of the reflector at the array cell boundaries. The addition of a reflector to an infinite slab of fissile material results in a reactivity addition characterized by a delta-function, i.e., the boundary condition is either a vacuum or total reflection.

Finite planar arrays should, therefore, exhibit a reactivity change with reflector relocation having a behavior intermediate to that of cuboidal arrays and the " δ -effect" of infinite planar arrays. This is illustrated by calculations of a $32 \times 16 \times 1$ planar array of spheres and of cylinders with h to d ratios of 0.5 and 4.0. The calculated results for the spherical units appear in Table 6 and those for the cylinder in Table 7. The results are presented in Figure 7 as the fraction of the total reactivity associated with the reflector location as a function distance from the array expressed as $1 - A/R$. Comparison with Figure 5 confirms the displacement of the curve toward the infinite planar result

Table 5. Monte Carlo Calculated K_{eff} for an Infinite Planar Array of 14.94 kg U(93.2) Metal Spheres Centered in 58.39 cm Cubic Cells Four Tiers High as a Thick Water Reflector Recedes from the Cell Boundaries

Dimension Ratio	Reflector Location	Calculated ^a k_{eff}
A:R	$\pm Z$	
1:1	116.78	1.006
9:10	129.75	1.005
3:4	155.70	1.005
3:5	194.63	1.007
1:2	233.55	1.006
1:4	467.12	0.999
1:8	934.24	1.007
1: ∞	∞	0.774

^aStandard deviation less than ± 0.006 .

of a δ -effect at $A/R = 0$. It is to be noted that these calculations represent the motion of five reflecting surfaces away from the arrays. Also shown in the tables are cases for a reflector surface in contact with the units, i.e., violating the array boundary defined by the cells to represent units supported by the reflector. This is not a significant effect in the case of spherical units but it is significant for the cylindrical units. As expressed by Equation (3), the array reactivity change will not be greater than that experienced by the units in an array, a result confirmed by the data shown in Figure 8. These data are calculations of a single cell of the cylinders described in Table 7a. The k_{eff} of the single cell was calculated as the bottom

Table 6. Calculated Neutron Multiplication Factor of a Planar
 $32 \times 16 \times 1$ Water-Reflected Array of U(93.2) Metal Spheres
 as the Reflector Recedes from Five Faces of the Array.
 Units are 20.2 kg U Centered in Cubic Cells with
 Dimension of 32.908 cm.

Reflector location, (cm)				Calculated ^a k_{eff}
+Z	-Z	$\pm X$	$\pm Y$	
16.454	10.096 ^b	526.5	263.3	1.002
	10.732			0.998
	11.686			1.004
	13.275			0.996
	14.884			0.999
	15.659			0.996
	16.454 ^c			1.000
18.282	16.454	585.0	292.5	0.989
21.939		702.0	351.0	0.976
32.908		1053.0	526.5	0.959
65.816		2106.1	1053.0	0.933
131.632		4212.2	2106.1	0.903
263.264		8424.4	4212.2	0.877
∞		∞	∞	0.825

^aStandard deviation for 30,000 neutron histories was less than ± 0.005 .

^bBottom reflector in contact with spherical units.

^cBottom reflector at cell boundary.

Table 7. Calculated Neutron Multiplication Factor of a $32 \times 16 \times 1$ Planar Array of U(93.2) Metal Cylinders for Varying Water Reflector Conditions. Units are Centered in 32.908 cm Cubic Cells.

22.5 kg U(93.2), h:d = 1:2						
Dimension Ratio ^a A:R	Reflector				Monte Carlo	
	Location				Calculated	
	-Z	±X	±Y	+Z	k _{eff}	±σ
1:1	16.454 ^b	526.5	263.3	16.45	0.998	0.005
---	3.628 ^b	526.5	263.3	16.45	1.046	0.006
9:10	16.454	585.0	292.5	18.28	0.987	0.005
3:4		702.0	351.0	21.94	0.977	0.005
3:5		877.5	438.8	27.43	0.965	0.005
1:2		1053.0	526.5	32.91	0.964	0.005
1:4		2106.1	1053.0	65.82	0.925	0.005
1:8		4212.2	2106.1	131.63	0.881	0.004
1:16		8424.4	4212.2	263.26	0.838	0.004
1:∞		∞	∞	∞	0.797	0.004
1:∞	3.628 ^b	∞	∞	∞	0.933	0.005
1:∞	∞	∞	∞	∞	0.756	0.004

Cylinders with h:d = 4:1 and variable mass

A:R	Radius, cm	Mass, Kg U	k _{eff} ^c
1:1	3.675	23.4	0.997
9:10	3.760	25.1	1.000
3:4	3.794	25.7	0.999
3:5	3.917	28.3	1.004
1:2	3.986	29.9	0.996
1:4	4.183	34.5	1.003
1:8	4.491	42.7	1.000
1:∞	4.960	57.5	1.001

^aRatio of any array dimension to the corresponding inside dimension of "reflector cavity."

^bBottom reflector in contact with base of cylinders.

^cStandard deviation for 30,000 neutron histories was less than ±0.006.

ORNL-DWG 79-16581

Figure 7. Representation of array reactivity loss as a thick water reflector recedes from five surfaces of a $32 \times 16 \times 1$ planar array of U(93.2) metal units of several shapes.

reflector moved from the cell boundary to contact with the base of the cylinder. Two reflector conditions for the remaining five faces of the cell were considered and were unchanged as the reflector location was varied. These were unreflected and reflected by 30 cm of water. The similar reflector conditions for the $32 \times 16 \times 1$ planar array are included for comparison. The Δk_{eff} (reactivity change) associated with the bottom reflector displacement to contact is 0.16 ($\rho \sim 0.22$) and 0.13 ($\rho \sim 16$) for the unreflected and reflected cell, respectively, while it is 0.12 ($\rho \sim 0.15$) and 0.05 ($\rho \sim 0.05$) for the arrays with similar reflector conditions. The data suggest that the reflected single-cell reactivity change simulating moving a unit from the center of a cell onto a reflecting surface is comparable in magnitude to the effect obtained for an array when the reflector is moved from the cell boundaries to contact with all the units. The magnitude of the reflected single-cell reactivity change would be conservative if used to define a required margin of subcriticality to compensate for units against one reflector face of a planar array or cubic array. Further, the reactivity change associated with a cell having five faces unreflected would have additional conservatism in application. These are conservative in the sense they would represent an overestimate of the amount to reduce the total reflector reactivity worth yielding a higher estimate of k_{eff} for a reflector location than would occur.

It has not been implied here that the k_{eff} worth of a reflector be that of total reflection, i.e., the six faces of the array. Figure 5 is applicable to cuboidal arrays where the total Δk_{eff} represents the conditions of interest as, for example, the reflector worth of three or five faces on an array. The data of Table 8 is an additional illustration and evidences applicability of Figure 5 to units of extreme shape.

Figure 8. Calculated neutron multiplication factors of a single cubic cell having a centered 22.5 kg U(93.2) metal cylinder as a function of reflector location and reflector conditions. The $32 \times 16 \times 1$ array data from Table 7a are shown for comparison.

Table 8. Calculated Neutron Multiplication Factor of a Cubic Water-Reflected Array of U(93.2) Metal Cylinders as the Reflector Recedes from Five Faces of a 64-Unit Array with Units Centered in 83.2 cm Cubic Cells.

Dimension Ratio ^a array:reflector	Cylinder h:d = 0.1 Radius (cm)	Calculated ^b k_{eff}
1:1	27.868	1.004
9:10	28.177	0.999
3:4	28.877	0.999
6:10	29.672	0.986
1:2	30.063	0.998
1:4	31.423	1.003
1:8	31.778	1.002
1:∞	32.143	0.996

^aRatio of an array dimension to the corresponding inside dimension of "reflector cavity."

^bStandard deviation less than ± 0.006 .

Array Reflector Thickness

Reducing the water reflector thickness on an array will result in an array reactivity loss to the array which can be compensated by increasing the mass of the units in the array to maintain criticality. A study⁶ of this effect on array criticality resulted in a characterization similar to that for the effect of reflector location summarized in Figure 5. The relation between array reactivity response and reflector thickness is summarized in Figure 9. Comparison is made in the figure to a similar treatment of data for single units of different neutron spectra. The application of Figure 9 to arrays is generic to a finite number of units in arrays of arbitrary shape.

For equal fractions of total reflector worth the results of Figure 5 on reflector location may be empirically related to the results of Figure 9 on reflector thickness. The relationship is displayed in Figure 10 which establishes the equivalence between reflector location and thickness of a water reflector on an array. The results presented in Figure 10 are significant in that they provide data for water reflected arrays, which are more reliably calculated than large complex operational areas, to be used as a basis to estimate the reflector worth of an environment encountered in actual operations with fissile materials. For example, the c_2 value for three-sided array reflection⁷ by thick concrete is $2.54 \times 10^{-3} \text{ cm}^{-2}$ as shown in Figure 6. This corresponds to about an A/R value of 0.5 when interpreted in Figure 4. Figure 10 provides the equivalent water thickness on six sides of the array as about 4 cm.

Discussion

Nuclear criticality safety limits presented in standards and guides are necessarily based upon specific reflector conditions. Application of a safety limit to an operation being conducted with fissile materials should be investigated to establish that the environment of the operation does not constitute a reflector effectiveness that is more efficient than the conditions stated for the safety limit. If the reflector results in a k_{eff} for an operation less than that corresponding to the safety limit, the difference in reactivity between the safety limit value and the result of the assessment for the operation may be used in two ways. It may be considered as an additional margin of subcriticality to an established subcritical k_{eff} for an area or it may be considered

Figure 9. Generic representation of reactivity controlled by the water reflector thickness on arrays of arbitrary shape.

ORNL-DWG 79-16579

Figure 10. Equivalence between the location of a thick reflector from the surfaces of an array and the thickness of a closely fitting water reflector on arrays.

in the establishment of an acceptable margin of subcriticality for operations. As an illustration of the results of this study, consider the following hypothetical problem.

Assume the location of a process line for fabrication of a fissile material product is to be selected from among several possibilities. The engineering operations have been delineated, i.e., the flow chart established, optimization of engineering features such as unit sizes, work stations, units in transit, temporary storage locations in the operations area, etc. The proposed unit sizes are to be such that they would be subcritical if submerged. It is estimated that about 400 units will be in the area and that a 25% margin is desired for contingencies. The minimum volume associated with a unit is 2.84 ℓ corresponding to a cubic cell with dimension $2a_u = 30.5$ cm.

A request has been made to the nuclear criticality safety specialist to provide an estimate of the maximum unit size resulting in a k_{eff} not exceeding 0.9 for the operation if the process were to be conducted in each of three locations having dimensions of $42 \times 21 \times 5.8$ m, $21 \times 21 \times 4$ m and $10 \times 7 \times 3$ m. The one-story structures are of typical 8" concrete block having concrete floors with tar and gravel roofs. It is conservative to use concrete having a thickness of 41 cm at the boundaries of the areas. The comparative analyses begins with a specified array selected from the American National Standard N16.5 (1975) Guide to Nuclear Criticality Safety in the storage of fissile materials. Table 5.1 of the guide presents units of U(93.2) metal spheres as a function of cubic cell size and number of units when assembled in cubic arrays reflected by water having a k_{eff} not exceeding 0.95. The entry for $N = 512$ units in

cubic cells of dimension $2a_u = 30.5$ cm cells for a mass limit of 7.1 by U(93.2). The evaluation for each site proceeds as follows:

- a) The mass equivalent to the 7.1 kg by U(93.2) when the reflector is 41-cm of concrete is given by Equations (1a) and (2) as

$$m_1 = \frac{m_o}{1 + \frac{c_2}{c'_2} \left(\frac{m_o}{m} - 1 \right)}$$

- b) The highest k_{eff} of the cubic array in an area will occur when the array is in a corner. The difference between m_1 and the m_2 determined for the array reflected on three sides gives the total reactivity worth of three reflector surfaces on the array. The mass m_2 is also given by the relation in (a) above where c'_2 and m'_o are values for three-sided reflection by concrete. ($m'_o = 55.4$ kg is a constant satisfying data characterized by Equation (1a).)
- c) The total ρ of b) above is reduced for reflector location by the $F(\rho)$ from Figure 5, using the average of the array/reflector dimension ratios. This defines an m_3 as $m_2 / (1 - F(\rho)\rho)^3$.
- d) Depending on the arrangement of the cells over the area, the m_3 may be increased because of increased neutron leakage from the less favorable array geometry. An array shape factor $R = \sqrt[3]{N/3} \left(\frac{1}{n_x} + \frac{1}{n_y} + \frac{1}{n_z} \right)$ is used to define c'_2 in the relation

$$\frac{n_z c_2}{nc'_2} = \frac{1}{4} \left(SR^{-0.672} - 1 \right)$$

and gives m_4 by the equivalent mass relation for array shape as

$$m_4 = \frac{m_o}{1 + \frac{n_z c_2}{n c'_2} \left(\frac{m_o}{m_3} - 1 \right)} .$$

- e) The estimated mass of the unit to produce a $k_{eff} = 0.9$ in the operational areas is determined by applying Equation (3),

$$m_{est} = \left(\frac{k'}{k} \right)^3 m_4 = \left(\frac{0.9}{0.95} \right)^3 m_4 .$$

A summary of the calculations for the areas appears in Table 9.

It should be noted that the mass for each of the effects evaluated in Table 9 maintains the k_{eff} of the reference array, i.e., $k_{eff} = 0.95$ of the array from N16.5. It would be normal to request a Monte Carlo calculation to verify that indeed the masses distributed as assumed here would produce the k_{eff} indicated. This is not necessary for we can convert the array in the largest area to the conditions described in Table 6 for 20.2 kg units in a $32 \times 16 \times 1$ array reflected by water. Thus

$$m'_4 = \frac{52.1}{1 + \frac{1.085}{1.762} \left(\frac{52.1}{20.4} - 1 \right)} = 26.62 \text{ kg} .$$

TABLE 9. Summary of Analyses for Operational Area Evaluations.

Operations Area:	42 x 21 x 5.8 m	21 x 21 x 4 m	10 x 7 x 3 m
a) concrete reflection:			
$m_1 = m_o / \left(1 + \frac{c_2}{c_1} (m_o/m - 1) \right)$	$m_1 = 52.1 / \left[1 + \frac{1.762}{1.085} \left(\frac{52.1}{7.1} - 1 \right) \right]$ $m_1 = 4.61 \text{ kg}$	$m_1 = 4.61 \text{ kg}$	$m_1 = 4.61 \text{ kg}$
b) m_2 and total reactivity worth of 3 rd reflector faces:			
$\rho_r = 1 - \left(\frac{m_1}{m_2} \right)^{1/3}$	$m_2 = 55.4 / \left[1 + \frac{1.762}{2.54} \left(\frac{52.1}{4.61} - 1 \right) \right]$ $m_2 = 6.80 \text{ kg}$ $\rho_r = 1 - \frac{4.61}{6.8}^{1/3} = 0.122$	$m_2 = 6.80 \text{ kg}$ $\rho_r = 0.122$	$m_2 = 6.08 \text{ kg}$ $\rho_r = 0.122$
c) Reflector location: $A = n2a_n = 8(30.5) = 2.44 \text{ m}$			
A/R_1	2.44/42 = 0.06	2.44/21 = 0.12	2.44/10 = 0.24
A/R_2	2.44/21 = 0.12	= 0.12	2.44/7 = 0.35
A/R_3	2.44/5.8 = 0.42	2.44/4 = 0.61	2.44/3 = 0.81
A/R	$A/R = 0.2 \Rightarrow F(e) = 0.44$	$A/R = 0.28 \Rightarrow F(e) = .89$	$A/R = 0.47 \Rightarrow F(e) = 0.725$
$m_3 = \frac{m_2}{[1 - F(e)e_r]^3}$	$m_3 = 6.8 / (1 - .115)^3$ $m_3 = 9.8 \text{ kg}$	$m_3 = 6.8 / (1 - 0.109)^3$ $m_3 = 9.61 \text{ kg}$	$m_3 = 6.8 / (1 - .088)^3 \text{ c}$ $m_3 = 8.98 \text{ kg}$
d) Array shape:			
n_x, n_y, n_z	32, 16, 1	16, 16, 2	14, 12, 3
$R = \left(\frac{1}{n_x} + \frac{1}{n_y} + \frac{1}{n_z} \right)^{113/3}$	$R = 2.917$	$R = 1.667$	$R = 1.295$
$\frac{n_z}{n} \frac{c_2}{c_1} = \frac{1}{4} (sp^{-0.672} = 1)$	$\frac{n_z}{n} \frac{c_2}{c_1} = 0.359$	$\frac{n_z}{n} \frac{c_2}{c_1} = 0.637$	$\frac{n_z}{n} \frac{c_2}{c_1} = 0.80$
$m_4 = \frac{m_o}{1 + \frac{n_2}{n} \frac{c_2}{c_1} \left(\frac{m_o}{m_2} - 1 \right)}$	$m_4 = \frac{52.1}{1 + .359 \left(\frac{52.1}{9.8} - 1 \right)}$ $m_4 = 20.4 \text{ kg}$	$m_4 = \frac{52.1}{1 + .637 \left(\frac{52.1}{9.61} - 1 \right)}$ $m_4 = 13.65 \text{ kg}$	$m_4 = \frac{52.1}{1 + 0.8 \left(\frac{52.1}{8.98} - 1 \right)}$ $m_4 = 10.76 \text{ kg}$
e) Estimated mass:			
$M_{est} = \left(\frac{k_1}{k} \right)^3 m_4 = \left(\frac{.9}{.95} \right)^3 m_4 = 0.85 m_4$	17.4 kg	11.6 kg	9.2 kg

The adjustment in mass to compensate for a change in spacing from 30.5 cm to 32.908 cm of the table is given by

$$\frac{m}{m'_4} = \left(\frac{32.908}{30.3} \right)^2 \left(1 - \frac{26.62}{52.1} \right) + \frac{26.62}{52.1}$$

$$m = (1.08)(26.62) = 28.76 \text{ kg}$$

$$\text{so that } k'_{\text{eff}} = \left(\frac{20.2}{28.76} \right)^{113} (0.95) = 0.844$$

Since the $\pm$ reflector location would be about 5.6 m from the array, the k_{eff} of 0.844 would be expected to be between the last two entries of Table 6, i.e., between 0.825 and 0.877. Thus, the results of Table 9 are not simply an exercise with numbers and formulae but are what may be expected from a Monte Carlo calculation of the configurations.

The basic analyses given above was conducted with U(93.2) metal; however, the results are applicable to other materials for which c'_2 and m'_0 have been determined. In particular, Table 4.1 of the Nuclear Safety Guide, TID-7016 Revision 2 (NUREG/CR-0095), is a table of equivalent masses for different fissionable materials. The above analyses are easily expressed in terms of any of the more than 40 different forms of fissile materials tabulated for which the characteristic constant have been established.

This example has illustrated the use of the American National Standard N16.5 to evaluate reflector conditions, and, in particular, that

the reactivity associated with reflector location for finite planar arrays is adequately approximated by the effects of a three-sided reflector on cubic arrays and array shape conversions.

A complete safety assessment would include consideration for the effects of unit shape, i.e., other than spherical, for units set on the floor, for fire protection systems, etc. Each of these would result in a reactivity change to the array. One could compensate for the positive reactivity effects by reducing the mass of the units. The negative reactivity effects could be considered in establishing the margin of subcriticality for the operation or they could be left as an arbitrary source of an additional bit of safety.

Conclusions

- Three-sided reflection of an array by thick concrete is equivalent to a water reflector thickness of about 4 cm surrounding the array.

- In establishing the subcriticality of general operations with fissile materials, it is reasonable to consider relaxation of nuclear criticality safety limits (data) by some portion of the reactivity associated with reflector location. It would be prudent to allow no more than some fraction of the reactivity controlled by three surfaces of the reflector.

- Nuclear criticality safety limits established by reference to slabs or arrays of fissile materials unlimited in two dimensions should not be relaxed because of reflector location.

REFERENCES

1. American National Standard N16.1-1975, "Nuclear Criticality Safety in Operations with Fissionable Materials Outside Reactors."
2. American National Standard N16.9-1975, "Validation of Calculational Methods for Nuclear Criticality Safety."
3. J. T. Thomas, Nucl. Sci. Eng., 62, 424-437 (1977).
-----, "Generic Array Criticality," Nuclear Criticality Safety, p. 66, TID-26286, U.S. Atomic Energy Commission (1974).
4. L. M. Petrie and N. F. Cross, "KENO IV - An Improved Monte Carlo Criticality Program," ORNL-4938, Oak Ridge National Laboratory (1975)
5. Gordon E. Hansen and William H. Roach, "Six and Sixteen Group Cross Sections for Fast and Intermediate Critical Assemblies," LAMS-2543, Los Alamos Scientific Laboratory (1961).
6. J. T. Thomas, "Comparative Method for the Evaluation of Array Reflector Materials," ORNL/CSD-42, Oak Ridge National Laboratory (1979).
7. J. T. Thomas, "Evaluation of Criticality Criteria for Fissile Class II Packages in Transportation," Proceedings on Symposium on Transport Packaging for Radioactive Materials, IAEA-SR-10/10, International Atomic Energy Agency, Vienna (1976).
8. J. T. Thomas, Nucl. Sci. Eng., 52, 350-359 (1973).
9. R. Avery, "Theory of Coupled Reactors," Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva (1958).

APPENDIX

Array Reactivity

Suppose we are given N units of fissile material, each subcritical, arranged into an array of units that results in a subcritical system. Let the quantity $1 - k_i$ ($i = 1, 2, \dots, N$) be a measure of the subcriticality of unit i where k_i is the k_{eff} of the unit without contributions from the remaining $N-1$ units. Let ν represent the number of neutrons per fission occurring in the system and suppose the $\nu_c > \nu$ neutrons per fission are needed for criticality. For the noncritical system, the fission source in one generation is reproducing itself in the next generation with a magnitude which differs by a factor k , where $k = \nu/\nu_c$, the k_{eff} of the system. The reactivity of the system may be expressed as

$$\rho = \frac{k-1}{k} = \frac{\nu-\nu_c}{\nu} = -\delta\nu/\nu \quad (1A)$$

Assume that in each unit, i , ν_i is the number of neutrons per fission, and suppose ν_i is changed by an amount $\delta\nu_i$ to the value $\nu_i + \delta\nu_i$. It has been shown⁹ that the fraction of the system reactivity in a unit α_i is the ratio of the overall reactivity to $\delta\nu_i/\nu_i$ and is given by

$$\alpha_i = \frac{1/1 - k_i}{\sum_{j=1}^N 1/(1-k_j)} \quad (2A)$$

The reactivity of the systems of units is thus expressible in terms of the individual reactivity of the unit by

$$\rho \approx \sum_{i=1}^N \alpha_i \frac{\delta v_i}{v_i} \quad (3A)$$

In application to a uniform array of units, i.e., identical units, k_i is the k_{eff} of the unit (k_u) and is constant for all i . If an array has a $k_{\text{eff}} = k_a$ initially and suffers a change resulting in a $k'_a < k_a$, then from Equation (1A) the array reactivity is

$$\rho = \frac{k_a - k'_a}{k'_a} = 1 - k_a/k'_a$$

If the reactivity has resulted from each unit in the array having its value of k_u changed to k'_u , then from Equation (3A)

$$\rho = \sum_{i=1}^N \alpha_i \left(\frac{k'_u - k_u}{k'_u} \right)_i$$

Now α_i from Equation (2A) is

$$\alpha_i = \frac{1/(1-k_u)_i}{\sum_j 1/(1-k_u)_j} = \frac{1}{N} \quad \text{for all } i ;$$

therefore,

$$\rho = \sum_{i=1}^N \frac{1}{N} = \frac{k'_u - k_u}{k'_u} = 1 - \frac{k_u}{k'_u}$$

This, with the above result for ρ , implies

$$\frac{k_a}{k'_a} = \frac{k_u}{k'_u} \quad (4A)$$

which is the basis for the statement:

In order that an array k_{eff} be changed in reactivity by an amount ρ , it is sufficient that the reactivity of each unit in the array be changed by the same ρ .

Applications of these results to storage arrays provides some insight to the consequences of maloperations in handling fissile materials, e.g., double-batching. If one or several, say n , units in an N unit array are double-batched such that each of the n units suffers an increase in reactivity, ρ_u , then the array reactivity is

$$\rho = \frac{n}{N} \rho_u ,$$

that is, $\rho < \rho_u$. Thus, if the margin of subcriticality for an array $(1-\rho)$ is sufficient to allow for double-batching, and double-batching a unit does not result in the unit being critical, the array cannot achieve criticality. It is also apparent from this that the consequences of a single unit achieving criticality accidentally in an array will not necessarily lead to the array being critical. The consequences may not differ significantly from a typical single unit incident for the fissile material.

Let us return to the application made in the text of these concepts to the reactivity worth of reflectors. Let k_1 be the k_{eff} of a unit in a critical reflected array and k_2 , the k_{eff} of a unit in the same array, unreflected but critical. The reactivity lost in removing the reflector can be expressed as

$$\rho = \frac{k_2 - k_1}{k_2} = 1 - k_1/k_2$$

while that gained in surrounding an unreflected array with a reflector is

$$\rho = \frac{k_1 - k_2}{k_1} = 1 - k_2/k_1 \quad .$$

These are not equal reactivities but are equal Δk 's. For a reflector location intermediate to the above conditions, there exists a unit with a corresponding k_x which will produce criticality. The reactivity with respect to the reflected condition, k_1 , is

$$\rho = 1 - \frac{k_1}{k_x}$$

and with respect to the unreflected array, $\rho = 1 - k_2/k_x$. Again these are not equal; however, in terms of the fraction of total reactivity worth of the reflector, they are complementary with respect to unity. That is,

$$\frac{k_1/k_x - 1}{k_1/k_2 - 1} + \frac{1 - k_2/k_x}{1 - k_2/k_1} = 1$$

so that Figure 5 of the text may be used to also estimate the reactivity due to motion of a reflector to a location x on an unreflected array. This, of course, requires the ordinate value $(1-A/R)$ be subtracted from unity. It is clear, also from Equation (4A), that the ρ may be evaluated either from the array k_{eff} or the unit k_{eff} .

NUREG/CR-1616
ORNL/NUREG/CSD/TM-16
Distribution Category RC

INTERNAL DISTRIBUTION

1. J. W. Arendt	13. L. M. Petrie
2. G. H. Burger	14. J. S. Tang
3. H. P. Carter/A. A. Brooks/
CSD X-10 Library	15. R. G. Taylor
4- 5. H. R. Dyer	16-30. J. T. Thomas
6. C. M. Hopper	31-36. G. E. Whitesides
7. N. F. Landers	37. Central Research Library
8. J. H. Marable	38. Y-12 Document Reference
Section
9. P. J. Maudlin	39-40. Laboratory Records
10. J. D. McLendon	41. Laboratory Records - RC
11. F. R. Mynatt	42. Radiation Shielding
Information Center
12. C. E. Newlon	43. ORNL Patent Section

EXTERNAL DISTRIBUTION

44. J. N. Rogers, Division 8324, Sandia Laboratories, Livermore, California 94550
- 45- 46. D. E. Solberg, Office of Nuclear Regulatory Research, U. S. Nuclear Regulatory Commission, Washington, DC 20555
47. Chief, Mathematics and Geoscience Branch, Department of Energy, Washington, DC 20545
48. Office of Assistant Manager for Energy Research and Development, Department of Energy, Oak Ridge Operations Office, Oak Ridge, Tennessee 37830
- 49- 50. Technical Information Center, Department of Energy, Oak Ridge Operations, Oak Ridge, Tennessee 37830
- 51-315. Distribution Category RC (10 copies to NTIS)