BEAM-BEAM INTERACTIONS FOR BUNCHED AND UNBUNCHED BEAMS

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ABSTRACT

The beam-beam interaction is analyzed in terms of Chirikov's stochasticity model. Stochastic blow-up occurs when the density of resonance regions in phase space becomes large, and Arnold diffusion is assumed to depend on the density parameter below the stochastic threshold. The relation between the density parameter and the tune shift ξ is affected by bunching of the beam and also by variations in the strengths of several interaction regions and by beam misalignment. It is seen that bunching can reduce the tolerable ξ by as much as an order of magnitude in proton storage rings.

We consider a model of the beam-beam interaction where the forces on particles in one beam due to interaction with the second beam are approximated by a localized nonlinear force field. We write the motion in terms of a Hamiltonian

$$H = \nu_1 J_1 + \nu_2 J_2 + V(J_1, J_2, \varphi_1, \varphi_2, \theta) \quad (1)$$

where V represents the beam-beam interaction (and possibly other nonlinear terms), and J and φ are action-angle variables.

The interaction term V can be written as a Fourier series

$$V = \epsilon_{krs} e^{i(r\varphi_1 + s\varphi_2 - k\theta)} \quad (2)$$

Let us consider the one-dimensional case, i.e., ignore J_2 and φ_2 . Then the ϵ_{00} term represents the variation of betatron tune with amplitude (octupole effect):

$$\alpha = \frac{d\omega}{dJ} = \frac{d^2}{dJ^2} \epsilon_{00}(J) \quad (3)$$

If V is small enough, each of the other terms of (1) produces a resonance at some amplitude J_0 where $\omega = \nu + \alpha J = k/r$, characterized by a set of islands in phase space whose width is (in J)

$$\Delta J = 4 \sqrt{2 \epsilon_{kr}(J)/\alpha} \quad (4)$$

frequency of oscillation within the center of the island

$$\omega_r = r \sqrt{2 \alpha \epsilon_{kr}} \quad (5)$$

and width in frequency of the unperturbed motion

$$\Delta\omega = \alpha \Delta J = 4\omega_r/r \quad (6)$$

The Chirikov criterion for stability¹ is: If the separation of different resonances is greater than their widths, then amplitudes are constant except for fluctuations of the order of the resonance width; if the resonance overlap, "chaotic" or "stochastic" motion will occur.

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To formulate the Chirikov condition quantitatively, we sum the width of all resonances in a unit frequency interval. This sum is

$$\frac{1}{\lambda} = \sum_r r \sqrt{a \varepsilon_{kr}} \quad (7)$$

and the Chirikov stability condition becomes

$$\lambda > 4 \quad (8)$$

For the beam-beam interaction occurring at one azimuth, we have (with a potential V which is Gaussian in shape)

$$\begin{aligned} \varepsilon_{k,2n} &= .25 J_1 I_n (J/2J_1) e^{-J/2J_1} \\ \varepsilon_{k,2n+1} &= 0 \\ \alpha &= \frac{\varepsilon}{2J_1} (2I_0 - (2+1/x)I_1) e^{-x} \end{aligned} \quad (9)$$

where J_1 corresponds to the rms width of the second beam, ξ is the nonlinear tune shift parameter, $x = J/2J_1$, and the I 's are modified Bessel functions of argument x . Note that only even order resonances are excited.

This gives λ as defined by (7) as a function of x . The largest value for $x \approx 1$, is approximately

$$1/\lambda = 2E \quad (10)$$

so that the threshold condition becomes

$$\varepsilon < 1/8. \quad (11)$$

It is assumed here that α , the tune dependence on amplitude, is due mainly to the beam-beam interaction (which is probably true in most actual storage rings). If α is due to the lattice, it is independent of ξ and therefore $1/\lambda$ is proportional to $\xi^{1/2}$.

In any particular case, however, the effective value of $1/\lambda$ is the density of islands near the actual working point, rather than the average over all conceivable tunes; therefore the effective value of λg may be less than the value of 0.5 given by (10).

In real systems the stability condition is more stringent than indicated above, for several reasons. The foremost reason is that the motion is in two dimensions. In two-dimensional systems the KAM theory no longer guarantees stability below the "threshold" for stochasticity. Rather, the narrow stochastic regions of phase space are now connected, and a particle can diffuse - slowly - to large amplitude (Arnold diffusion).

Precise estimates of the diffusion rate are difficult to obtain. However, Chirikov¹⁾ gives plausible, though not rigorous, arguments that the diffusion rate should scale with

$$1/T_d = \epsilon_c e^{-\pi\lambda} \quad (12)$$

where ϵ_c is a factor depending primarily on the strength of the coupling resonances, and λ is the same as above but with the widths of coupling resonances included. For tolerable lifetimes in proton or antiproton storage rings one desires lifetimes of the order of 10^{11} to 10^{12} orbit periods, and therefore one must have

$$\lambda \geq 10 \quad (13)$$

which is two or three times more stringent than in the one-dimensional case. Furthermore, there are more resonances, and the effective value of $1/\lambda$ is probably larger, for a given ξ , than in one dimension. If we guess that now $\lambda\xi = 1/4$, we arrive at a tolerable tune shift around 0.025.

Other factors tend to decrease the product $\lambda\xi$ further. The most important one is beam bunching². If the particle oscillates in an rf bucket, its energy is modulated at the synchrotron frequency ν_s . Because of the chromaticity of the machine lattice, this leads to a modulation of the free betatron oscillation frequency ν . Consequently each resonance is split into sidebands with separation ν_s/m for m -th order resonances; the resonance strength of the n -th sideband of an m -th order resonance is then

$$\epsilon_{\text{bunch}} = \epsilon_m J_n(m\nu_1/\nu_s) \quad (14)$$

where ν_1 is the range of tunes due to energy modulation; J_n is a Bessel function. The island width is the original width times J_n , and therefore the sum of the width, i.e., $1/\lambda$, is increased by the factor

$$\sum_n \sqrt{|J_n(y)|}, \quad y = m\nu_1/\nu_s.$$

This can be a large factor; for large y it is roughly $y^{3/4}$. In ISABELLE, $\nu_1 \sim 10^{-3}$ and $\nu_s \sim 10^{-5}$, and the factor is 30; on the other hand the CERN and FNAL $p\bar{p}$ colliders and HERA expect to have ν_s much larger, and the corresponding enhancement factor will be much smaller. Slow bunching is worse than fast because each resonance is traversed slowly. On the other hand, if ν_s very small as in ISABELLE, the modulation may simply move the islands back and forth adiabatically, and most particles move along with them; thus the foregoing analysis is surely too pessimistic for small ν_s . We may guess at an effective enhancement factor of 10 for ISABELLE, and 2 to 3 for the other projected proton rings.

Other factors affecting the relation between λ and ξ :

If there are several intersection regions instead of one, and they are equally spaced and identical, the period is simply $1/N$ of the circumference; the resonance widths and spacings both increase by the factor N , and λ is the same as with a single intersection region of the same strength. This is true if α , the detuning effect of amplitude, is due primarily to beam-beam interaction (which is probably the case in most machines). If on the other hand, α is due to other properties of the lattice, then the island widths only increase as $\sqrt{N}$, and the spacing increases with N ; and paradoxically we find $1/\lambda$ smaller with N

intersections.

But if the interaction regions are not equal, there is a further enhancement because the resonances are more closely spaced. For example, with $N = 6$ and strengths differing by 10% rms, one finds an enhancement factor in λ of 1.5; with five crossings active and one missing, the enhancement factor is 2.7.

Beam misalignment destroys the symmetry of the interaction geometry and introduces odd as well as even order resonances. This can increase $1/\lambda$ by another factor of 2.

Electron machines. Here synchrotron radiation damping will counteract the slow diffusion, and only the fast stochasticity need be considered; therefore the minimum λ is probably 4 rather than 10. Also here the synchrotron frequency ν_s tends to be large, therefore the enhancement factor from bunching is not very large, especially at high energy.

In summary: The severity of beam-beam interaction effects is determined by the parameter λ , which is defined as the ratio of average spacing to width of nonlinear resonances. Fast stochastic blow-up occurs for $\lambda = 4$; to ensure long-term stability against Arnold diffusion one requires $\lambda = 10$. The relation between λ and the tune-shift parameter depends on numerous factors. For a coasting beam and a single interaction region $\lambda\xi \sim 1/2$ unless the working point lies close to a low order resonances, in which case λ is effectively reduced. RF bunching of the beam reduces the effective value of $\lambda\xi$ by a factor which depends on the ratio of the amplitude ν_1 of the tune modulation to the frequency ν_s of synchrotron oscillations, and is estimated to be as large as 10 for large ν_1/ν_s . This consideration tends to favor designs with large ν_s for bunched beam colliders.

In the case of electron rings, ν_s tends to be larger for high energy machines because of the high rf voltage; for proton rings it is more favorable in the rings with high harmonic order (pp colliders and HERA) than those with low harmonics such as ISABELLE.

Several unequal intersection regions, and misalignments of the colliding beams, will also decrease the product $\lambda\xi$ and therefore the tolerable ξ . This factor will generally be no more than about 2 or 3. To test this factor, one might operate PEP with two of the normal three bunches; according to the current model this should result in a lower maximum ξ than operation with either one or three bunches.

As a result, the ranges of tolerable values of ξ are estimated to be (very roughly): Coasting beam (protons), equal intersections: $\lambda\xi = 0.5$, $\lambda > 10$, therefore $\xi < 0.05$.

Bunched beam: Enhancement factor 3-10 for protons, 2-4 for electrons; $\lambda > 4$ for electrons, therefore $\lambda < 0.005$ to 0.02 for protons, 0.03 to 0.06 for electrons.

Unequal intersection regions and beam misalignments cause another factor about 2: $\xi < 0.002$ to 0.001 for protons, 0.015 to 0.06 for electrons.

REFERENCES

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2. J. Tennyson, in "Nonlinear Dynamics and the Beam-Beam Interaction", AIP Conference Proceedings No. 57, 158-193 (1979).