**THE CONFORMAL MAPPING OF A CIRCLE
ONTO A RECTANGLE**

NOVEMBER 1968

**AEC RESEARCH &
DEVELOPMENT REPORT**

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THE CONFORMAL MAPPING OF A CIRCLE ONTO A RECTANGLE

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THE CONFORMAL MAPPING OF A CIRCLE ONTO A RECTANGLE

A. B. Schubert

INTRODUCTION

The problem of mapping the closed unit circle onto the closed rectangle was proposed to the Mathematical Analysis Section by the Theoretical Physics Section. The motivation for seeking this mapping was a desire to study the physics of the coupling of adjacent rectangular reactor cells. Such a study is analytically and numerically more feasible in polar coordinates than in rectangular coordinates.

The analysis of the problem was to be accompanied by a digital computer program which numerically performed the mapping within specified tolerance on a grid of interior points of the circle and the rectangle.

MAPPING

In order to determine the mapping of the circle onto the rectangle, we first seek the inverse mapping. Toward this end, consider the rectangle in the complex z -plane as shown in Figure 1.

FIGURE 1.

After translating the rectangle vertically by ib so that it lies entirely in the upper half z' -plane (Figure 2), where

$$z' = z + ib, \quad (1)$$

we appeal to the degenerate Schwarz-Christoffel transformation⁽¹⁾

$$z'' = k^{1/2} \operatorname{sn}(z'/\alpha) \quad (2)$$

where sn is the Jacobian elliptic function⁽²⁾ with parameter $0 \leq k^2 \leq 1$ and $\alpha = a/K$, where K is the real quarter-period of the sn function. Equation (2) maps the closed rectangle in the z' plane onto the upper half z'' plane, as indicated in Figure 3, with point D transforming into the point at infinity.

FIGURE 2.

1. H. Kober. *Dictionary of Conformal Representations*, Dover Publications, Inc., 1957. pp. 172-173.
2. M. Abramowitz and I. A. Stegun. *Handbook of Mathematical Functions*, National Bureau of Standards, AMS-55, June 1964. pp. 569 ff.

FIGURE 3.

Now the transformation⁽¹⁾

$$w = \frac{1 + iz''}{1 + z''} \quad (3)$$

maps the upper half z'' -plane onto the unit circle in the w -plane as indicated in Figure 4.

FIGURE 4.

1. R. V. Churchill. *Introduction to Complex Variables and Applications*, Second Edition, McGraw-Hill Book Co., N.Y., 1960, p. 287.

Using (2) and (1) in (3) we then have

$$w = \frac{1 + ik^{1/2} \operatorname{sn}[1/\alpha(z + ib)]}{i + k^{1/2} \operatorname{sn}[1/\alpha(z + ib)]} \quad (4)$$

for the transformation required to map the closed rectangle in the z -plane onto the closed unit circle in the w -plane.

Using values of special arguments of the sn function,⁽¹⁾ it can be shown that the vertices of the rectangle, given by $z = \pm a \pm ib$, map onto the points

$w = \frac{1}{1+k} [\pm 2k^{1/2} \pm i(1-k)]$, labeled by A, A', B, B' in Figure 4 and by $\pm \exp(\pm i\beta)$ in the graphical display on page D-2 of Appendix D, where

$$\beta = \tan^{-1} \frac{1-k}{2k^{1/2}}.$$

For the purpose of ease of evaluation of Transformation (4), we will rewrite it using the Jacobi theta functions,⁽²⁾ as these lend themselves readily to numerical evaluation via digital computer. The sn function may be written⁽³⁾ as

$$\operatorname{sn} u = \frac{2K}{\pi} \cdot \frac{\theta_1(\zeta)}{\theta_4(\zeta)} \cdot \frac{\theta_4(0)}{\theta_1'(0)}$$

where $\zeta = \pi u/2K$.

-
1. M. Abramowitz and I. A. Stegun. Handbook of Mathematical Functions, National Bureau of Standards, AMS-55, June 1964. p. 571.
 2. Whittaker and Watson. Modern Analysis, Cambridge University Press.
 3. M. Abramowitz and I. A. Stegun. Handbook of Mathematical Functions, National Bureau of Standards, AMS-55, June 1964. p. 578.

Further, using the identities⁽¹⁾

$$\theta'_1(0) = \theta_2(0) \theta_3(0) \theta_4(0) \quad ,$$

$$2K/\pi = \theta_3^2(0) \quad ,$$

$$\text{and } k^{1/2} = \theta_2(0)/\theta_3(0) \quad ,$$

we may write

$$\text{sn } u = k^{-1/2} \theta_1(\zeta)/\theta_4(\zeta) \quad . \quad (5)$$

$$\text{So, taking } u = \frac{1}{\alpha} (z + ib) = \frac{K}{a} (z + ib) \quad ,$$

$$\text{we have } \zeta = \pi/2a(z + ib) \quad , \quad (6)$$

and, if we use Equation (5), Equation (4) becomes

$$w = \frac{1 + i \frac{\theta_1(\zeta)}{\theta_4(\zeta)}}{i + \frac{\theta_1(\zeta)}{\theta_4(\zeta)}} \quad (7)$$

Now, if we can invert Transformation (7), we will have obtained the desired circle to rectangle transformation. Toward this end, rewrite Equation (7) as

$$(w - i) \theta_1(\zeta) = (1 - iw) \theta_4(\zeta)$$

or

$$f(\zeta) = (w - i) \theta_1(\zeta) + (iw - 1) \theta_4(\zeta) = 0 \quad . \quad (8)$$

The solution of Equation (8) for ζ and use of Equation (6) then yield the desired mapping $z(w)$.

1. M. Abramowitz and I. A. Stegun. Handbook of Mathematical Functions, National Bureau of Standards, AMS-55, June 1964. pp. 576 and 579.

METHOD OF EVALUATION

The solution of Equation (8) was obtained numerically using Newton's method. The algorithm is as follows:

- 1) Choose ζ_0
- 2) Generate the sequence $\{\zeta_n\}$ by

$$\zeta_{n+1} = \zeta_n - \frac{f(\zeta_n)}{f'(\zeta_n)}, \quad n = 0, 1, \dots \quad (9)$$

Then

$$\lim_{n \rightarrow \infty} \zeta_n = \zeta \text{ such that } f(\zeta) = 0.$$

Using Equation (8) write the iteration formula as

$$\zeta_{n+1} = \zeta_n - \frac{(w - i) \theta_1(\zeta_n) + (iw - 1) \theta_4(\zeta_n)}{(w - i) \theta'_1(\zeta_n) + (iw - 1) \theta'_4(\zeta_n)}, \quad n = 0, 1, 2, \dots$$

The generation of the sequence $\{\zeta_n\}$ required a method of evaluation of the functions $\theta_1(\zeta)$, $\theta_4(\zeta)$, and their derivatives. This was done by use of a subroutine (SUBROUTINE THETA Appendix A) which summed the rapidly convergent series expansions⁽¹⁾ for these functions in terms of powers of the nome $q = e^{i\pi\tau}$, where $\tau = 2ib/a$.

After having noted that the mapping is symmetric about each of the coordinate axes, it was decided that the numerical evaluation of the mapping would be restricted to the first quadrant of the w and z planes, so that, having obtained the magnitudes of the coordinates of the point $z(w)$, it was only necessary to give them the signs of the coordinates of w .

Because of the doubly-periodic nature of the theta functions, it was necessary to make a sufficiently good choice for ζ_0 before

1. Whittaker and Watson. Modern Analysis, Cambridge University Press.

iterating to insure that the solution obtained was the one which corresponded to the proper period cell of the theta functions. It was determined that if, for each value of w , ζ_0 was chosen to lie within the proper period cell and sufficiently close to the exact solution, then all succeeding iterates also lay within this same cell and convergence did obtain.

The initial value ζ_0 of the sequence of iterates was chosen by the following method:

- 1) Assume, for the sake of approximation, that the magnitude and argument of the point z are respectively proportional to the magnitude and argument of the point w on either side of the radial lines to the points z_c and w_c (see Figures 5 and 6).

FIGURE 5.

FIGURE 6.

- 2) So, if θ_e is an estimate of the argument of z , write

$$\frac{\theta_e}{\theta_0} = \frac{\phi}{\phi_0} \quad , \text{ if } \phi \leq \phi_0$$

$$\frac{\theta_e - \theta_0}{\pi/2 - \theta_0} = \frac{\phi - \phi_0}{\pi/2 - \phi_0} \quad , \text{ if } \phi > \phi_0 \quad (10)$$

where $\phi = \arg w$,

$$\begin{aligned}\phi_0 &= \arg w_c = \arg \frac{2k^{1/2} + i(1-k)}{1+k} \\ &= \tan^{-1} \frac{1-k}{2k^{1/2}},\end{aligned}$$

and $\theta_0 = \arg z_c = \arg (a + ib) = \tan^{-1} \left(\frac{b}{a}\right)$.

3) Then, if $|z_e|$ is an estimate of the magnitude of z , write

$$\frac{|z_e|}{|z_b|} = \frac{|w|}{|w_c|}, \quad (11)$$

where $|w_c| = 1$,

and $|z_b| = a/\cos \theta_e$, if $\theta_e \leq \theta_0$
 $= b/\sin \theta_e$, if $\theta_e > \theta_0$.

4) Upon computing θ_e and $|z_e|$ from Equation (10) and (11), we then have

$$z_e = |z_e|(\cos \theta_e + i \sin \theta_e).$$

5) Then, since we are iterating in the ζ -plane, compute from Equation (6)

$$\zeta_0 = \pi/2a (z_e + ib).$$

When convergence of the sequence $\{\zeta_n\}$ is obtained, i.e., when, for some $j > 0$,

$$\left| \frac{\zeta_j - \zeta_{j-1}}{\zeta_j} \right| < \varepsilon \quad (12)$$

for some specified relative error ϵ , take $\zeta = \zeta_j$, invert Equation (6) to get

$$z^* = \frac{2a}{\pi} \zeta - ib, \quad (13)$$

and restore signs of components to get

$$z(w) = \text{sign} [\text{Re}(z^*), \text{Re}(w)] + i \text{sign} [\text{Im}(z^*), \text{Im}(w)], \quad (14)$$

where the sign function is defined by

$$\text{sign} (A,B) = \frac{B}{|B|} |A|, \quad A,B \text{ real}, B \neq 0.$$

NUMERICAL PRECISION

Subroutines were written in single precision arithmetic with rounding for the Univac 1108 to calculate (1) theta functions with zero argument (FUNCTION THETA0, Appendix A);

(2) $\theta_1(z|\tau)$, $\theta_4(z|\tau)$, and their derivatives (FUNCTION THETA, Appendix A); (3) $w(z)$ (FUNCTION W, Appendix A); and (4) $z(w)$ (FUNCTION Z, Appendix A).

The theta functions were computed to 8 significant figures, so the calculation of $w(z)$ by Equation (7) was accurate to at least 7 significant figures (see Appendix B). In the calculation of $z(w)$ using the iteration defined by Equation (9) and with $\epsilon = 5 \times 10^{-7}$ in (12), accuracy was obtained to 7 significant figures except at points within a neighborhood of the vertex of the rectangle. Stability problems arose near the vertex because of the undue magnification of the correction term in iteration Equation (9) when $f'(\zeta)$ became small relative to the deviation of $f(\zeta)$ from zero.

The following table indicates the number n of significant figures obtained from the two-way mapping of points within a

square with side s at the vertex of a rectangle with dimensions 1 unit by 2 units:

<u>s</u>	<u>n</u>
1.	7
0.1	6
0.01	5
0.001	4
0.0002	3*

TIMING

An estimate of elapsed Univac 1108 time in the calculation of the mapping of a point from the circle to the rectangle was made (see Appendix B) using a rectangle of dimension 1 unit by 2 units** and a grid of points covering the entire rectangle with a spacing of 0.1 units in both directions. The average was taken for these points and was found to be approximately 60 msec per point.

Because of the singularity in the mapping at the vertex of the rectangle, hence a necessity for a greater number of iterations of Equation (9) to obtain convergence, a significant increase in mapping time was observed for points near the vertex.

* As may be evidenced from the printout on page B-7 Appendix B, all points on or within this region were, within computational error, mapped onto the point on the circle corresponding to the vertex of the rectangle, thus defining the resolution of the rectangle to circle mapping; hence, the inverse mappings of these points lay precisely on the vertex of the rectangle.

** Any size rectangle is a reasonable choice for a sample case, for the circle to rectangle mapping routine, FUNCTION Z (Appendix A), normalizes the input rectangle so that the smaller dimension is 2 units. Then, if necessary, the rectangle is rotated by 90° so that the greater sides parallel the y -axis of the $z(=x + iy)$ plane. This is desirable for the sake of maximizing the convergence rate of the series expansions for the theta functions, as the series are expressed in powers of $q = \exp(i\pi\tau)$, where $\tau = 2ib/a$, and $z = (a,b)$ is the location of the first quadrant vertex of the rectangle. (See Method of Evaluation.)

The following table indicates the average mapping time t for points on a square grid with spacing $0.1 s$ within a square with side s at the vertex of the rectangle:

<u>s</u>	<u>t(msec)</u>
0.1	90
0.01	125
0.001	230

ACKNOWLEDGMENTS

The analysis and program described above were requested by B. H. Duane of the Theoretical Physics Section. His helpful comments and suggestions for the analysis and his provision of the theta function and arithmetic rounding routines were significantly instrumental in the development of the program. Assistance in the analysis and computer programming was provided, respectively, by R. L. Richardson and Carol Lewis of the Mathematical Analysis Section. The graphical display in Appendix D was provided by Pauline Aldridge of the Theoretical Physics Section. The original manuscript was typewritten by Ethel Montgomery of the Mathematical Analysis Section. The contributing efforts of these people are gratefully acknowledged.

APPENDICES

APPENDIX A

SOURCE DECK LISTING FOR PROGRAM CIRREC

QIT FR5 CIRREC,CIRREC

C DRIVER PROGRAM FOR CIRCLE-RECTANGLE MAPPING ROUTINES
C PROGRAMMED BY A B SCHUBERT

```
      COMPLEX ZZ(400),WZ(400),ZW(400),C,WZD(400)
      COMPLEX W,Z
      DIMENSION NOIT(400)
      DATA MAXLIN/46/,PI/0202622077325/
99  FORMAT(8E10.0)
199 FORMAT(1H1 14X 59HMAPPING OF UNIT CIRCLE IN W-PLANE ONTO RECTANGLE
      * IN Z-PLANE//)
299 FORMAT(5X 37HCO-ORDINATES OF DIAGONAL OF RECTANGLE 2X 1H( F8.4,
      * 1H, F8.4, 5H) , ( F8.4, 1H, F8.4, 1H) )
399 FORMAT(1H0 3X 35HREGION OF RECTANGLE CONSIDERED X = F9.4, 3H TO
      * F9.4, 5H Y = F9.4, 3H TO F9.4/)
499 FORMAT(1H1 2X 59HMAPPING OF UNIT CIRCLE IN W-PLANE ONTO RECTANGLE
      * IN Z-PLANE//)
599 FORMAT(13X 37HCO-ORDINATES OF DIAGONAL OF RECTANGLE/11X 1H( F8.4,
      * 1H, F8.4,5H) , ( F8.4, 1H, F8.4,1H) )
699 FORMAT(1H0 8X 45HREGION OF CIRCLE CONSIDERED ( W=R*EXP(I*T) ) /
      * 7X 3HR = F7.4,3H TO F8.4,5H T = F8.4,3H TO F9.4,6H (DEG)/)
96  FORMAT(      14X 11HZ(I)--INPUT 16X 9HW( Z(I) ) 17X 9HZ( W(I) ) 7X
      * 4HITER /)
97  FORMAT(1X I3, 6F13.8, I5)
96  FORMAT(1H0 28X 2(4X 10HTOTAL TIME F8.3,1X 3HSEC) )
95  FORMAT(      3X 1HI 10X 11HW(I)--INPUT 16X 9HZ( W(I) ) 7X 4HITER /
      * 12X 1HR 10X 6HT(DEG) 9X 1HX 12X 1HY /)
94  FORMAT(1X I3,F13.8,F13.6,2F13.8,I5)
93  FORMAT(1H0 32X 10HTOTAL TIME F8.3,2X 3HSEC )
92  FORMAT(1H1)
      RAD=PI/180.
```

A-2

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```

10 READ(5,99) A,B,POLREC
   READ(5,99) X0,DX,XMAX,Y0,DY,YMAX
   C=CMPLX(A,B)
   AM=-A
   BM=-B
   NX=(XMAX-X0)/DX+1.005
   NY=(YMAX-Y0)/DY+1.005
   NZ=0
   IF(POLREC) 15,19,15
15 R0=X0
   TH0=RAD*Y0
   IF(X0) 17,16,17
16 WZ(1)=CMPLX(0.,0.)
   WZD(1)=WZ(1)
   NZ=1
   NX=NX-1
   R0=DX
17 RDTH=RAD*DY
   DO 18 J=1,NY
   THETA=TH0+FLOAT(J-1)*RDTH
   TH=Y0+FLOAT(J-1)*DY
   STH=SIN(THETA)
   CTH=COS(THETA)
   DO 18 I=1,NX
   R=R0+FLOAT(I-1)*DX
   NZ=NZ+1
   WZD(NZ)=CMPLX(R,TH)
18 WZ(NZ)=CMPLX(R*CTH,R*STH)
   GO TO 35
19 DO 20 J=1,NY
   Y=Y0+FLOAT(J-1)*DY
   DO 20 I=1,NX
   X=X0+FLOAT(I-1)*DX
   NZ=NZ+1
20 ZZ(NZ)=CMPLX(X,Y)
   CALL ETIME
   DO 30 I=1,NZ

```

A-3

BNWL-901

```

30 WZ(I)=W(ZZ(I),C,N)
   CALL ETIMEF(WZSEC)
35 CALL ETIME
   DO 40 I=1,NZ
40 ZW(I)=Z(WZ(I),C,NOIT(I))
   CALL ETIMEF(ZWSEC)
   IF(POLREC) 141,41,141
41 WRITE(6,199)
   WRITE(6,299) AM,BM,A,B
   WRITE(6,399) X0,XMAX,Y0,YMAX
   WRITE(6,98)
   LINE=9
   GO TO 241
141 WRITE(6,499)
   WRITE(6,599) AM,EM,A,B
   WRITE(6,699) X0,XMAX,Y0,YMAX
   WRITE(6,95)
   LINE=12
241 DO 60 I=1,NZ
   IF(LINE.LT.MAXLIN) GO TO 50
   WRITE(6,92)
   IF(POLREC) 42,43,42
42 WRITE(6,95)
   LINE=3
   GO TO 52
43 WRITE(6,98)
   LINE=2
50 IF(POLREC) 52,54,52
52 WRITE(6,94) I,WZD(I),ZW(I),NOIT(I)
   GO TO 60
54 WRITE(6,97) I,ZZ(I),WZ(I),ZW(I),NOIT(I)
60 LINE=LINE+1
   IF(POLREC) 62,63,62
62 WRITE(6,93) ZWSEC
   GO TO 10
63 WRITE(6,96) WZSEC,ZWSEC
   GO TO 10
   END

```

A-4

BNWL-901

```

@IT FR5 W,W
COMPLEX FUNCTION W(ZZ,C,N)

C THIS FUNCTION CONFORMALLY MAPS A POINT WITHIN OR ON A RECTANGLE
C IN THE Z-PLANE CENTERED AT THE ORIGIN AND WITH SIDES PARALLEL TO
C THE CO-ORDINATE AXES ONTO A POINT WITHIN OR ON A UNIT CIRCLE
C CENTERED AT THE ORIGIN OF THE W-PLANE.

C FUNCTION AND ARGUMENT DEFINITIONS

C ZZ = POINT IN RECTANGLE (INPUT)
C C = FIRST QUADRANT VERTEX OF RECTANGLE (INPUT)
C N = DUMMY ARGUMENT (OUTPUT)
C W = IMAGE OF ZZ UNDER RECTANGLE TO CIRCLE TRANSFORMATION (OUTPUT)

C PROGRAMMED BY C L LEWIS AND A B SCHUBERT

COMPLEX C,Z,TAU,ZP,TH(4),X,ZZ
COMPLEX CMR,CDR,CAR
DATA PIH/0201622077325/
TAU=CMPLX(0.,2.*FDR(AIMAG(C),REAL(C)))
Z=CMPLX(ABS(REAL(ZZ)),ABS(AIMAG(ZZ)))
ZP=CMR(CMPLX(PIH,0.),CDR(CAR(Z,CMPLX(0.,AIMAG(C))),CMPLX(REAL(C),
1 0.)))
CALL THETAS(ZP,TAU,TH)
IF(FAR(CABS(TH(1)), -CABS(TH(4))))1,1,2
1 X=CDR(TH(1),TH(4))
W=CDR(CAR(CMPLX(1.,0.),CMR(CMPLX(0.,1.),X)),CAR(CMPLX(0.,1.),X))
GO TO 4
2 X=CDR(TH(4),TH(1))
W=CDR(CAR(CMPLX(0.,1.),X),CAR(CMPLX(1.,0.),CMR(CMPLX(0.,1.),X)))
4 W=CMPLX(SIGN(REAL(W),REAL(ZZ)),SIGN(AIMAG(W),AIMAG(ZZ)))
END

```

A-5

BNWL-901

BIT FR5 Z,Z

COMPLEX FUNCTION Z(W,CC,NOIT)

C THIS FUNCTION CONFORMALLY MAPS A POINT WITHIN OR ON A UNIT CIRCLE
C CENTERED AT THE ORIGIN OF THE W-PLANE ONTO A POINT WITHIN OR ON A
C RECTANGLE IN THE Z-PLANE CENTERED AT THE ORIGIN AND WITH SIDES
C PARALLEL TO THE CO-ORDINATE AXES.

C FUNCTION AND ARGUMENT DEFINITIONS

C W = POINT IN UNIT CIRCLE (INPUT)
C CC = FIRST QUADRANT VERTEX OF RECTANGLE (INPUT)
C NOIT = NO. OF NEWTON ITERATIONS TAKEN TO OBTAIN SOLUTION (OUTPUT)
C Z = IMAGE OF W UNDER CIRCLE TO RECTANGLE TRANSFORMATION(OUTPUT)

C PROGRAMMED BY A B SCHUBERT

REAL K,KR
COMPLEX TH0(5),CP,ZE,WP,WC,CC
COMPLEX W,C,TAU,THATA(4),W1,W2,F,FP,Z1,Z2
COMPLEX CAR,CMR,CDR
DATA PI,MAXIT/0202622077325,30/
DATA EPS,EMAX,EP/5.E-7,1.E-5,5.E-8/
NOIT=0
EREL=EPS
WP=CMPLX(ABS(REAL(W)),ABS(AIMAG(W)))
IF(FAR(REAL(CC),-REAL(CP)))25,24,25
24 IF(FAR(AIMAG(CC),-AIMAG(CP)))25,26,25

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```

25 PI2=PI/2.
   C=CMPLX(1.,FDR(AMAX1(REAL(CC),AIMAG(CC)),AMIN1(REAL(CC),AIMAG(CC))
   *))
   TAU=CMPLX(0.,2.*FDR(AIMAG(C),REAL(C)))
   CALL THETA0(TAU,TH0)
   KR=CDR(TH0(3),TH0(4))
   K=FMR(KR,KR)
   PHI0=ATAN2(FAR(1.,-K),2.*KR)
   WC=CMPLX(FDR(2.*KR,FAR(1.,K)),FDR(FAR(1.,-K),FAR(1.,K)))
   T0=ATAN2(AIMAG(C),REAL(C))
   PIA2=FDR(PI2,REAL(C))
   CP=CC
26 IF (FAR(AIMAG(WP),-EP))27,27,29
27 IF (FAR(REAL(WP),-EP))28,28,29
28 Z=CMPLX(0.,0.)
   GO TO 6
29 IF (FAR(REAL(CC),-AIMAG(CC)))229,229,129
129 WP=CMPLX(AIMAG(WP),REAL(WP))
229 IF (FAR(ABS(FAR(REAL(WP),-REAL(WC))),-EP))329,329,529
329 IF (FAR(ABS(FAR(AIMAG(WP),-AIMAG(WC))),-EP))429,429,529
429 Z=C
   GO TO 105
529 IF (FAR(AIMAG(WP),-REAL(WP)))30,30,31
30 PHI=ATAN2(AIMAG(WP),REAL(WP))
   GO TO 32
31 PHI=FAR(PI2,-ATAN2(REAL(WP),AIMAG(WP)))
32 IF (FAR(PHI,-PHI0))33,33,34
33 TE=FMR(FDR(T0,PHI0),PHI)
   GO TO 35
34 TE=FAR(FMR(FDR(FAR(PI2,-T0),FAR(PI2,-PHI0)),FAR(PHI,-PHI0)),T0)
35 IF (FAR(TE,-T0))36,36,37
36 ZBAB=FDR(REAL(C),COS(TE))
   GO TO 38
37 ZBAB=FDR(AIMAG(C),SIN(TE))
38 ZAB=FMR(CABS(WP),ZBAB)
   ZE=CMR(CMPLX(ZAB,0.),CMPLX(COS(TE),SIN(TE)))

```

```

39  Z2=CMR(CMPLX(PIA2,0.),CAR(ZE,CMPLX(0.,AIMAG(C))))
    W1=CAR(WP,CMPLX(0.,-1.))
    W2=CAR(CMR(CMPLX(0.,1.),WP),CMPLX(-1.,0.))
139 DO 3 J=1,MAXIT
    NOIT=NOIT+1
    Z1=Z2
    CALL THETA(Z1,TAU,THATA)
    F=CAR(CMR(W1,THATA(1)),CMR(W2,THATA(3)))
    FP=CAR(CMR(W1,THATA(2)),CMR(W2,THATA(4)))
    Z2=CAR(Z1,CDR(-F,FP))
    IF (FAR(ABS(REAL(Z2)),-EPS))202,202,102
102 IF (FAR(ABS(FAR(FDR(REAL(Z2),REAL(Z1)),-1.)),-EREL))202,202,3
202 IF (FAR(ABS(AIMAG(Z2)),-EPS))5,5,302
302 IF (FAR(ABS(FAR(FDR(AIMAG(Z2),AIMAG(Z1)),-1.)),-EREL))5,5,3
    3 CONTINUE
    EREL=FMR(10.,EREL)
    IF (EREL-EMAX)139,139,5
    5 Z=CAR(CDR(Z2,CMPLX(PIA2,0.)),CMPLX(0.,-AIMAG(C)))
105 Z=CMR(Z,CMPLX(AMIN1(REAL(CC),AIMAG(CC)),0.))
    IF (FAR(REAL(CC),-AIMAG(CC)))305,305,205
205 Z=CMPLX(AIMAG(Z),REAL(Z))
305 Z=CMPLX(SIGN(REAL(Z),REAL(W)),SIGN(AIMAG(Z),AIMAG(W)))
    6 RETURN
    END

```

BIT FR5 THETA,THETA

C THIS ROUTINE EVALUATES THETA1(Z), D/DZ THETA1(Z), THETA4(Z),
C D/DZ THETA4(Z), WITH PERIOD RATIO TAU. VALUES ARE RETURNED
C IN THETAZ(I), I=1,4.
C REFERENCE - WHITTAKER AND WATSON, MODERN ANALYSIS

C PROGRAMMED BY A B SCHUBERT AS A MODIFICATION OF SUBROUTINE THETAS
C (CONTAINED IN THIS DOCUMENT) BY B H DUANE

```
SUBROUTINE THETA(Z,TAU,THETAZ)
COMPLEX Z,TAU,THETAZ(4),PIT,TERM,PHASE,SINCOS,SUM
COMPLEX CAR,CMR,CCOS,CEXP,CSIN
DIMENSION ERR(4)
DATA PI/0202622077325/
PIT=CMR(CMPLX(0.,PI),TAU)
DO 10 N=1,4
  THETAZ(N)=(0.,0.)
  IF (N.EQ.3) THETAZ(N)=(1.,0.)
10 ERR(N)=1.
  SIGN=-1.
  EN=0.
20 SIGN=-SIGN
  DO 100 M=1,3,2
    EN=FAR(EN,1./2.)
    NM=M+1
    DO 30 N=M,NM
      IF (ERR(N)) 40,30,40
30 CONTINUE
  GO TO 100
40 TERM=2.*CEXP(CMR(CMPLX(FMR(EN,EN),0.),PIT))
  PHASE=CMR(CMPLX(2.*EN,0.),Z)
  DO 90 N=M,NM
    IF (ERR(N)) 50,90,50
```

```

50 IF (N.EQ.1) SINCOS=SIGN*CSIN(PHASE)
   IF (N.EQ.2) SINCOS=SIGN*CMR(CMPLX(2.*EN,0.),CCOS(PHASE))
   IF (N.EQ.3) SINCOS=-SIGN*CCOS(PHASE)
   IF (N.EQ.4) SINCOS=SIGN*CMR(CMPLX(2.*EN,0.),CSIN(PHASE))
   SUM=-THETAZ(N)
   THETAZ(N)=CAR(THETAZ(N),CMR(TERM,SINCOS))
   ERR(N)=CABS(CAR(THETAZ(N),SUM))
90 CONTINUE
100 CONTINUE
   DO 110 N=1,4
   IF (ERR(N)) 20,110,20
110 CONTINUE
   RETURN
   END

```

```

GIT FKS THETA0,THETA0
C JACOBI THETA FUNCTION NORMS (WHITTAKER-WATSON--MODERN ANALYSIS)
SUBROUTINE THETA0(TAU,THETA)
COMPLEX TAU,THETA(5), PIT,TERM,SUM
COMPLEX CAR,CMR,CXP
DIMENSION ERR(5)
DATA PI/0202622077325/,ONE/02007777777776/
C COMPLEX INPUT PERIOD RATIO TAU
C COMPLEX OUTPUT THETA=(THETA1* THETA1** THETA2 THETA3 THETA4)
DO 100 N=1,5
THETA(N)=(0.,0.)
IF(N.GT.3) THETA(N)=(1.,0.)
100 ERR(N)=1.
600 PIT=CMR(CMPLX(0.,PI),TAU)
EN=0.
SIGN=1.
C ENTER JACOBI THETA SERIES LOOP
700 SIGN=-SIGN
DO 1500 M=1,4,3
EN=FAR(EN,1./2.)
NM=M/2+3
DO 800 N=M,NM
IF(ERR(N)) 900,800,900
800 CONTINUE
GO TO 1500
900 TERM=CXP(CMF(CMPLX(FMR(EN,EN),0.),PIT))
DO 1400 N=M,NM
IF(ERR(N)) 1000,1400,1000
1000 DOT=2.
IF(N=2) 1200,1100,1300
1100 DOT=FMR(-8.*EN,EN)
1200 DOT=FMR(-2.*EN,DOT)
1300 IF((N.LT.3).OR.(N.GT.4)) DOT=SIGN*DOT
SUM=-THETA(N)
THETA(N)=CAR(THETA(N),CMR(CMPLX(DOT,0.),TERM))
ERR(N)=CABS(CAR(THETA(N),SUM))
1400 CONTINUE

```

THETA0

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```

1500 CONTINUE
      DO 1600 N=1,5
        IF(ERR(N)) 700,1600,700
C      SUM JACOBI THETA SERIES TO SOLID CONVERGENCE
1600 CONTINUE
1700 RETURN
C      COMPLEX OUTPUT TAU,(THETA(N),N=1,5) HAS NO LIVE ZEROS
C      MAXIMUM ERROR ESTIMATE OF 2 BITS IF NO SPILL OR DIVISION BY ZERO
C      TIMING ESTIMATE OF 10 MILLISECONDS ON UNIVAC-1108      BH DUANE
      END

```

```

@IT FRB THETAS,THETAS
C JACOBI THETA FUNCTIONS (WHITTAKER-WATSON--MODERN ANALYSIS) THETAS
  SUBROUTINE THETAS(Z,TAU,THETAZ)
  COMPLEX Z,TAU,THETAZ(4), PIT,TERM,PHASE,SINCOS,SUM
  COMPLEX CMR,CMR, CCOS,CEXP,CSIN UNIVAC
  DIMENSION ERR(4)
  DATA PI/0202622077325/
C COMPLEX INPUT ARGUMENT Z PERIOD RATIO TAU AIMAG(TAU).GT.0.
C COMPLEX OUTPUT (THETAZ(J),J=1,4)=(THETA1 THETA2 THETA3 THETA4)
  PIT=CMR(CMPLX(0.,PI),TAU)
  DO 100 N=1,4
    THETAZ(N)=(0.,0.)
    IF(N.GT.2) THETAZ(N)=(1.,0.)
  100 ERR(N)=1.
    SIGN=1.
    EN=0.
C ENTER JACOBI THETA FUNCTION SERIES LOOP
  200 SIGN=-SIGN
    DO 1000 M=1,3,2
      EN=FMR(EN,1./2.)
      NM=M+1
      DO 300 N=M,NM
        IF(ERR(N)) 400,300,400
      300 CONTINUE
        GO TO 1000
      400 TERM=2.*CEXP(CMR(CMPLX(FMR(EN,EN),0.),PIT))
        PHASE=CMR(CMPLX(2.*EN,0.),2)
        DO 900 N=M,NM
          IF(N.EQ.3) GO TO 600
          IF(ERR(N)) 500,900,500
        500 IF(N.EQ.1) SINCOS=-SIGN*CSIN(PHASE)
          IF(N.NE.2) GO TO 700
        600 SINCOS=CCOS(PHASE)
        700 IF(ERR(N)) 800,900,800

```



```

      800 IF (N.EQ.4) SIN COS=SIGN*SIN COS
          SUM=-THETA Z(N)
          THETA Z(N)=CAR(THETA Z(N),CMR(TERM,SIN COS))
          ERR(N)=CABS(CAR(THETA Z(N),SUM))
      900 CONTINUE
     1000 CONTINUE
          DO 1100 N=1,4
          IF(ERR(N)) 200,1100,200
C      SUM JACOBI THETA SERIES TO SOLID CONVERGENCE
     1100 CONTINUE
          RETURN
C      COMPLEX OUTPUT (THETA Z(N),N=1,4) HAS NO LIVE ZEROS
C      MAXIMUM ERROR ESTIMATE OF 2 BITS IF NO SPILL OR DIVISION BY ZERO
C      TIMING ESTIMATE OF 25 MILLISECONDS ON UNIVAC-1108      BH DUANE
          END

```

C COMPLEX ZERO AND ARITHMETIC ROUNDING ROUTINES
C PROGRAMMED BY B H DUANE

@IT FR5 CZE,CZE
C COMPLEX FUNCTION CZE(A) RETURNS DEAD ZEROS FOR LIVE ZEROS
COMPLEX FUNCTION CZE(A)
COMPLEX A
CZE=CMPLX(ZER(REAL(A)),ZER(AIMAG(A)))
RETURN
END

@IT FR5 CAR,CAR
C COMPLEX ADD-AND-ROUND FUNCTION CAR(A,B) RETURNS A+B ROUNDED
COMPLEX FUNCTION CAR(A,B)
COMPLEX A,B,A0,B0,CZE
A0=CZE(A)
B0=CZE(B)
CAR=CMPLX(FAR(REAL(A0),REAL(B0)),FAR(AIMAG(A0),AIMAG(B0)))
RETURN
C INPUT MAY HAVE LIVE ZEROS OUTPUT HAS DEAD ZEROS
END

@IT FR5 CMR,CMR
C COMPLEX MULTIPLY-AND-ROUND FUNCTION CMR(A,B) RETURNS A*B ROUNDED
COMPLEX FUNCTION CMR(A,B)
COMPLEX A,B,A0,B0,CZE
A0=CZE(A)
B0=CZE(B)
CMR=CMPLX(FAR(FMR(REAL(A0),REAL(B0)),-FMR(AIMAG(A0),AIMAG(B0))),FA
1R(FMR(REAL(A0),AIMAG(B0)),FMR(REAL(B0),AIMAG(A0))))
RETURN
C INPUT MAY HAVE LIVE ZEROS OUTPUT HAS DEAD ZEROS
END

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```

@IT FR5 CDR,CDR
C COMPLEX DIVIDE-AND-ROUND FUNCTION CDR(A,B) RETURNS A/B ROUNDED
  COMPLEX FUNCTION CDR(A,B)
  COMPLEX A,B,B0,CZE,CMR
  B0=CZE(B)
  AB=ZER(CABS(B0))
  CDR=CMR(A,CMPLX(FDR(FDR(REAL(B0),AB),AB),-FDR(FDR(AIMAG(B0),AB),AB
1)))
  RETURN
C INPUT MAY HAVE LIVE ZEROS OUTPUT HAS DEAD ZEROS
END

```

C REAL ZERO AND ARITHMETIC ROUNDING ROUTINES
C PROGRAMMED BY B H DUANE

BIT ASM FAMDR,FAMDR

•	FUNCTION ZER(A) RETURNS (DEAD ZERO IF A IS LIVE ZERO, ELSE A)	FAMDR*00
•	FUNCTION FAR(A,B) RETURNS A+B ROUNDED (WITH GOOD ZEROS)	FAMDR 01
•	FUNCTION FMR(A,B) RETURNS A*B ROUNDED (WITH GOOD ZEROS)	FAMDR 02
•	FUNCTION FDR(A,B) RETURNS A/B ROUNDED (WITH GOOD ZEROS)	FAMDR 03
•	METHOD ADD TWICE LOW ORDER PART TO HIGH ORDER PART. BH DUANE BNW	FAMDR 04
A0	EQU 12	• LET A0=12 (14 OCTAL) FAMDR 05
A1	EQU 13	• LET A1=13 (15 OCTAL) FAMDR 06
B11	EQU 11	• LET B11=11 (13 OCTAL) FAMDR 07
\$(2)	LIT	• ASSIGN LITERALS TO BANK 2 FAMDR 08
\$(1)		• ASSIGN PROGRAM TO BANK 1 FAMDR 09
FAR*	LA A0,*0,B11	• FLOATING ADD AND ROUND FAMDR 10
	FA A0,*1,B11	• ADD A TO B FAMDR 11
	J FRN	• ROUND AND RETURN FAMDR 12
FMR*	LA A0,*0,B11	• FLOATING MULTIPLY AND ROUND FAMDR 13
	FM A0,*1,B11	• MULTIPLY A BY B FAMDR 14
FRN	FA A1,A1	• DOUBLE LOW-ORDER PART FAMDR 15
RND	FA A0,A1	• ROUND FAMDR 16
TOP	TOP A0,(0400400000000)	• IS ANSWER LIVE ZERO FAMDR 17
	SZ A0	• DEAD ZEROS ARE GOOD ZEROS FAMDR 18
	J 3,B11	• RETURN A0=ANSWER IF NO SPILL OR D/O FAMDR 19
FDR*	LA A0,*0,B11	• FLOATING DIVIDE-OF-PROCEED AND ROUND FAMDR 20
	FD A0,*1,B11	• DIVIDE A BY B FAMDR 21
	FA A1,A1	• NORMALIZE (AND DOUBLE) REMAINDER FAMDR 22
	FD A1,*1,B11	• DIVIDE REMAINDER BY B FAMDR 23
	J RND	• ROUND AND RETURN FAMDR 24
ZER*	LA A0,*0,B11	• MAP LIVE ZERO ON DEAD ZERO FAMDR 25
	JGD B11, TOP	• DECREMENT RETURN INDEX REGISTER FAMDR 26
	END	• UNIVAC-1108 FORTRAN-IV-CSC 19SEP67 FAMDR*27

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APPENDIX B

NUMERICAL OUTPUT FROM PROGRAM CIRREC FOR SAMPLE CASE 1:

MAPPING OF INPUT RECTANGLE WITH DIMENSIONS 1 UNIT
BY 2 UNITS ONTO UNIT CIRCLE ONTO ORIGINAL RECTANGLE

MAPPING OF UNIT CIRCLE IN W-PLANE ONTO RECTANGLE IN Z-PLANE

CO-ORDINATES OF DIAGONAL OF RECTANGLE (-.5000, -1.0000) , (.5000, 1.0000)

REGION OF RECTANGLE CONSIDERED X = .0000 TO .5000 Y = .0000 TO 1.0000

	Z(I)--INPUT		W(Z(I))		Z(W(I))		ITER
1	-.00000000	-.00000000	-.00000000	-.00000001	.00000000	.00000000	0
2	.10000000	-.00000000	.15951153	-.00000001	.10000000	-.00000004	4
3	.20000000	-.00000000	.32688550	-.00000001	.20000000	-.00000004	4
4	.30000000	-.00000000	.51176330	-.00000000	.30000000	-.00000004	4
5	.40000000	-.00000000	.72821840	-.00000001	.40000001	-.00000004	4
6	.50000000	-.00000000	.99999999	-.00000001	.50000001	-.00000004	1
7	-.00000000	.10000000	-.00000000	.15702451	-.00000000	.10000001	4
8	.10000000	.10000000	.15571804	.16068947	.10000000	.10000002	4
9	.20000000	.10000000	.31847267	.17245409	.20000001	.10000001	4
10	.30000000	.10000000	.49669853	.19496242	.30000001	.10000001	4
11	.40000000	.10000000	.70216479	.23400987	.40000001	.10000002	4
12	.50000000	.10000000	.95332468	.30194709	.50000001	.10000002	4
13	-.00000000	.20000000	-.00000000	.30695934	-.00000000	.20000003	4
14	.10000000	.20000000	.14510746	.31356909	.10000001	.20000003	4
15	.20000000	.20000000	.29510908	.33464080	.20000001	.20000003	4
16	.30000000	.20000000	.45542786	.37434550	.30000001	.20000005	4
17	.40000000	.20000000	.63244613	.44138100	.40000001	.20000005	3
18	.50000000	.20000000	.83335757	.55273426	.50000001	.20000003	4
19	-.00000000	.30000000	-.00000000	.44406643	-.00000000	.30000004	5
20	.10000000	.30000000	.12969229	.45244594	.10000000	.30000004	5
21	.20000000	.30000000	.26159858	.47889548	.20000002	.30000006	4
22	.30000000	.30000000	.39764488	.52766397	.30000002	.30000004	4
23	.40000000	.30000000	.53871983	.60696281	.40000002	.30000004	4
24	.50000000	.30000000	.68254425	.73084428	.50000002	.30000004	4
25	-.00000000	.40000000	-.00000000	.56483476	-.00000000	.40000002	5
26	.10000000	.40000000	.11202605	.57372341	.10000001	.40000004	5
27	.20000000	.40000000	.22380780	.60146674	.20000001	.40000004	5
28	.30000000	.40000000	.33439182	.65140146	.30000000	.40000004	4
29	.40000000	.40000000	.44097701	.72936699	.40000003	.40000004	4
30	.50000000	.40000000	.53676075	.84373453	.50000004	.40000004	4
31	-.00000000	.50000000	-.00000000	.66817863	-.00000000	.50000003	5
32	.10000000	.50000000	.09449698	.67651787	.10000001	.50000007	6
33	.20000000	.50000000	.18693371	.70226621	.20000002	.50000004	5
34	.30000000	.50000000	.27452065	.74755961	.30000001	.50000004	5
35	.40000000	.50000000	.35278308	.81567219	.40000001	.50000003	4
36	.50000000	.50000000	.41421359	.91017974	.50000004	.50000004	4
37	-.00000000	.59999999	-.00000000	.75508991	-.00000000	.60000004	5

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	Z(I)--INPUT		W(Z(I))		Z(W(I))		ITER
38	.10000000	.59999999	.07891308	.76214203	.10000001	.60000005	5
39	.20000000	.59999999	.15464056	.78372402	.20000003	.60000005	6
40	.30000000	.59999999	.22347964	.82098336	.30000002	.60000004	5
41	.40000000	.59999999	.28063609	.87534049	.40000004	.60000004	5
42	.50000000	.59999999	.31964502	.94753738	.50000002	.60000004	4
43	-.00000000	.70000000	-.00000000	.82804547	-.00000000	.70000009	5
44	.10000000	.70000000	.06643063	.83336493	.10000001	.70000009	5
45	.20000000	.70000000	.12905538	.84956337	.20000003	.70000009	5
46	.30000000	.70000000	.18383623	.87721784	.30000004	.70000008	5
47	.40000000	.70000000	.22628733	.91679800	.40000007	.70000008	4
48	.50000000	.70000000	.25137255	.96789042	.50000004	.70000006	4
49	-.00000000	.80000000	-.00000000	.89044985	-.00000000	.80000004	4
50	.10000000	.80000000	.05771533	.89377324	.10000001	.80000009	5
51	.20000000	.80000000	.11123710	.90393812	.20000003	.80000010	5
52	.30000000	.80000000	.15642180	.92138893	.30000006	.80000010	5
53	.40000000	.80000000	.18923482	.94641980	.40000012	.80000010	5
54	.50000000	.80000000	.20588148	.97857696	.50000007	.80000003	4
55	-.00000000	.89999999	-.00000000	.94628039	-.00000000	.90000011	4
56	.10000000	.89999999	.05320196	.94738913	.10000001	.90000010	5
57	.20000000	.89999999	.10177249	.95100766	.20000002	.90000010	5
58	.30000000	.89999999	.14141346	.95785776	.30000004	.90000010	5
59	.40000000	.89999999	.16843451	.96867575	.40000010	.90000011	6
60	.50000000	.89999999	.17997320	.98367156	.50000028	.90000005	5
61	-.00000000	.99999999	-.00000000	1.00000000	-.00000000	1.00000006	1
62	.10000000	.99999999	.05337756	.99857441	.10000000	1.00000009	4
63	.20000000	.99999999	.10134070	.99485181	.20000000	1.00000015	5
64	.30000000	.99999999	.13916252	.99026958	.30000002	1.00000015	6
65	.40000000	.99999999	.16329134	.98657794	.40000001	1.00000036	6
66	.50000000	.99999999	.17157289	.98517146	.50000000	1.00000000	0
	TOTAL TIME		.870 SEC	TOTAL TIME		4.465 SEC	

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MAPPING OF UNIT CIRCLE IN W-PLANE ONTO RECTANGLE IN Z-PLANE							
CO-ORDINATES OF DIAGONAL OF RECTANGLE (-.5000, -1.0000) , (.5000, 1.0000)							
REGION OF RECTANGLE CONSIDERED X = .4000 TO .5000 Y = .9000 TO 1.0000							
	Z(I)--INPUT		W(Z(I))		Z(W(I))		ITER
1	.40000000	.90000000	.16843451	.96867575	.40000010	.90000011	6
2	.42000000	.90000000	.17205542	.97134931	.42000016	.90000017	6
3	.44000000	.90000000	.17503536	.97419091	.44000027	.90000017	5
4	.46000000	.90000000	.17735774	.97719622	.46000008	.90000004	5
5	.48000000	.90000000	.17900793	.98035918	.48000016	.90000004	5
6	.50000000	.90000000	.17997320	.98367156	.50000028	.90000005	5
7	.40000000	.91999999	.16615713	.97252902	.40000010	.92000020	6
8	.42000000	.91999999	.16961739	.97457137	.42000014	.92000015	6
9	.44000000	.91999999	.17243908	.97676519	.44000008	.92000005	6
10	.46000000	.91999999	.17460748	.97910815	.46000037	.92000021	6
11	.48000000	.91999999	.17610982	.98159590	.48000006	.92000000	5
12	.50000000	.91999999	.17693543	.98422248	.50000018	.92000006	5
13	.40000000	.94000000	.16450203	.97622989	.40000011	.94000013	6
14	.42000000	.94000000	.16781825	.97764171	.42000023	.94000030	6
15	.44000000	.94000000	.17049642	.97919013	.44000021	.94000021	6
16	.46000000	.94000000	.17252359	.98087483	.46000035	.94000027	6
17	.48000000	.94000000	.17388882	.98269358	.48000030	.94000010	6
18	.50000000	.94000000	.17458334	.98464241	.50000014	.94000006	6
19	.40000000	.95999999	.16347031	.97979514	.40000013	.96000025	6
20	.42000000	.95999999	.16665746	.98057509	.42000012	.96000031	6
21	.44000000	.95999999	.16920511	.98147850	.44000010	.96000016	6
22	.46000000	.95999999	.17110198	.98250710	.46000014	.96000017	7
23	.48000000	.95999999	.17233893	.98366082	.48000040	.96000016	6
24	.50000000	.95999999	.17290897	.98493785	.50000053	.95999998	6
25	.40000000	.98000000	.16306503	.98323992	.40000007	.98000026	6
26	.42000000	.98000000	.16613651	.98338472	.42000007	.98000030	6
27	.44000000	.98000000	.16856494	.98364147	.44000010	.98000024	6
28	.46000000	.98000000	.17034066	.98401399	.46000003	.98000005	7
29	.48000000	.98000000	.17145629	.98450446	.48000018	.98000023	8
30	.50000000	.98000000	.17190662	.98511326	.50000037	.98000002	7
31	.40000000	.99999999	.16329134	.98657794	.40000001	1.00000036	6
32	.42000000	.99999999	.16625898	.98608215	.42000002	1.00000030	7
33	.44000000	.99999999	.16857783	.98568838	.44000002	1.00000045	7
34	.46000000	.99999999	.17023985	.98540268	.46000005	1.00000048	8
35	.48000000	.99999999	.17123935	.98522948	.48000008	1.00000092	8
36	.50000000	.99999999	.17157289	.98517146	.50000000	1.00000000	0
TOTAL TIME			.469 SEC	TOTAL TIME			3.146 SEC

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MAPPING OF UNIT CIRCLE IN W-PLANE ONTO RECTANGLE IN Z-PLANE

CO-ORDINATES OF DIAGONAL OF RECTANGLE (-.5000, -1.0000) , (.5000, 1.0000)

REGION OF RECTANGLE CONSIDERED X = .4900 TO .5000 Y = .9900 TO 1.0000

	Z(I)--INPUT		W(Z(I))		Z(W(I))	ITER
1	.49000000	.98999999	.17154381	.98500466	.49000102	.99000105 8
2	.49200000	.98999999	.17157965	.98503277	.49200056	.99000046 8
3	.49400000	.98999999	.17160882	.98506206	.49400113	.99000064 8
4	.49600000	.98999999	.17163132	.98509251	.49600051	.99000031 8
5	.49800000	.98999999	.17164715	.98512412	.49800063	.98999999 8
6	.50000000	.98999999	.17165630	.98515692	.50000122	.98999996 8
7	.49000000	.99199999	.17151961	.98504326	.49000131	.99200182 8
8	.49200000	.99199999	.17155429	.98506472	.49200226	.99200203 9
9	.49400000	.99199999	.17158229	.98508731	.49400137	.99200121 9
10	.49600000	.99199999	.17160362	.98511108	.49600092	.99200033 8
11	.49800000	.99199999	.17161828	.98513602	.49800089	.99199998 8
12	.50000000	.99199999	.17162627	.98516215	.50000161	.99199986 8
13	.49000000	.99399999	.17150208	.98508067	.49000089	.99400100 9
14	.49200000	.99399999	.17153559	.98509546	.49200118	.99400105 9
15	.49400000	.99399999	.17156243	.98511138	.49400051	.99400006 9
16	.49600000	.99399999	.17158259	.98512852	.49600220	.99400161 9
17	.49800000	.99399999	.17159609	.98514677	.49800086	.99400038 9
18	.50000000	.99399999	.17160291	.98516623	.50000348	.99400000 9
19	.49000000	.99599999	.17149121	.98511694	.49000061	.99600150 9
20	.49200000	.99599999	.17152356	.98512504	.49200053	.99600142 9
21	.49400000	.99599999	.17154923	.98513432	.49400093	.99600115 9
22	.49600000	.99599999	.17156824	.98514475	.49600109	.99600002 10
23	.49800000	.99599999	.17158057	.98515636	.49800317	.99600179 9
24	.50000000	.99599999	.17158623	.98516913	.50000262	.99600023 9
25	.49000000	.99799999	.17148700	.98515201	.49000014	.99800031 9
26	.49200000	.99799999	.17151819	.98515347	.49200043	.99800175 9
27	.49400000	.99799999	.17154270	.98515607	.49400013	.99800114 9
28	.49600000	.99799999	.17156055	.98515984	.49600076	.99800149 11
29	.49800000	.99799999	.17157173	.98516478	.49800543	.99800462 11
30	.50000000	.99799999	.17157622	.98517088	.50001021	.99800196 10
31	.49000000	.99999999	.17148948	.98518597	.48999989	1.00000170 9
32	.49200000	.99999999	.17151950	.98518074	.49200004	1.00000077 10
33	.49400000	.99999999	.17154286	.98517667	.49400055	1.00000057 10
34	.49600000	.99999999	.17155954	.98517378	.49600052	1.00000328 11
35	.49800000	.99999999	.17156955	.98517201	.49800189	1.00000128 13
36	.50000000	.99999999	.17157289	.98517146	.50000000	1.00000000 0

TOTAL TIME .504 SEC TOTAL TIME 4.597 SEC

MAPPING OF UNIT CIRCLE IN W-PLANE ONTO RECTANGLE IN Z-PLANE

CO-ORDINATES OF DIAGONAL OF RECTANGLE (-.5000, -1.0000) , (.5000, 1.0000)

REGION OF RECTANGLE CONSIDERED X = .4990 TO .5000 Y = .9990 TO 1.0000

	Z(I)--INPUT		W(Z(I))		Z(W(I))		ITER
1	.49900000	.99900000	.17157260	.98516980	.49901077	.99900898	31
2	.49920000	.99900000	.17157295	.98517007	.49920896	.99900560	12
3	.49940000	.99900000	.17157325	.98517037	.49941951	.99900988	12
4	.49960000	.99900000	.17157347	.98517063	.49959635	.99899583	15
5	.49980000	.99900000	.17157363	.98517097	.49981023	.99899565	14
6	.50000000	.99900000	.17157372	.98517130	.50001044	.99899741	13
7	.49900000	.99919999	.17157235	.98517015	.49900376	.99920118	12
8	.49920000	.99919999	.17157270	.98517039	.49921232	.99920909	31
9	.49940000	.99919999	.17157298	.98517062	.49941576	.99920750	11
10	.49960000	.99919999	.17157319	.98517083	.49960772	.99920125	13
11	.49980000	.99919999	.17157335	.98517109	.49981251	.99919666	14
12	.50000000	.99919999	.17157342	.98517136	.50001589	.99920057	16
13	.49900000	.99940000	.17157218	.98517054	.49900554	.99940701	13
14	.49920000	.99940000	.17157251	.98517068	.49920551	.99940078	28
15	.49940000	.99940000	.17157279	.98517086	.49942449	.99941708	31
16	.49960000	.99940000	.17157298	.98517101	.49960393	.99939568	31
17	.49980000	.99940000	.17157312	.98517121	.49982607	.99940187	15
18	.50000000	.99940000	.17157318	.98517141	.50002799	.99939741	12
19	.49900000	.99959999	.17157207	.98517089	.49900638	.99960673	12
20	.49920000	.99959999	.17157239	.98517099	.49920962	.99961089	31
21	.49940000	.99959999	.17157265	.98517106	.49940519	.99959891	16
22	.49960000	.99959999	.17157284	.98517120	.49962784	.99962522	17
23	.49980000	.99959999	.17157297	.98517130	.49983279	.99960488	32
24	.50000000	.99959999	.17157301	.98517142	.50002039	.99959582	13
25	.49900000	.99980000	.17157203	.98517126	.49900701	.99981062	12
26	.49920000	.99980000	.17157234	.98517127	.49920676	.99981292	13
27	.49940000	.99980000	.17157258	.98517128	.49940415	.99980898	31
28	.49960000	.99980000	.17157277	.98517135	.49962314	.99983624	17
29	.49980000	.99980000	.17157288	.98517141	.50000000	1.00000000	0
30	.50000000	.99980000	.17157292	.98517143	.50000000	1.00000000	0
31	.49900000	.99999999	.17157205	.98517159	.49900051	1.00001323	31
32	.49920000	.99999999	.17157235	.98517154	.49920218	1.00001293	14
33	.49940000	.99999999	.17157259	.98517149	.49940746	1.00001386	17
34	.49960000	.99999999	.17157275	.98517147	.49960054	1.00004113	24
35	.49980000	.99999999	.17157286	.98517147	.50000000	1.00000000	0
36	.50000000	.99999999	.17157289	.98517146	.50000000	1.00000000	0
TOTAL TIME			.467 SEC	TOTAL TIME			8.730 SFC

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MAPPING OF UNIT CIRCLE IN W-PLANE ONTO RECTANGLE IN Z-PLANE							
CO-ORDINATES OF DIAGONAL OF RECTANGLE (-.5000, -1.0000) , (.5000, 1.0000)							
REGION OF RECTANGLE CONSIDERED X = .4995 TO .5000 Y = .9995 TO 1.0000							
Z(I)--INPUT		W(Z(I))		Z(W(I))		ITER	
1	.49950000	.99950000	.17157281	.98517104	.49952440	.99951872	23
2	.49960000	.99950000	.17157289	.98517109	.49960727	.99950486	14
3	.49970000	.99950000	.17157298	.98517115	.49969825	.99949108	31
4	.49980000	.99950000	.17157304	.98517124	.49981107	.99949491	32
5	.49990000	.99950000	.17157307	.98517132	.49990346	.99949719	12
6	.50000000	.99950000	.17157310	.98517141	.50003214	.99949482	13
7	.49950000	.99959999	.17157276	.98517114	.49952190	.99961837	20
8	.49960000	.99959999	.17157284	.98517120	.49962784	.99962522	17
9	.49970000	.99959999	.17157291	.98517121	.49969406	.99958906	14
10	.49980000	.99959999	.17157297	.98517130	.49983279	.99960488	32
11	.49990000	.99959999	.17157300	.98517136	.49992502	.99960351	21
12	.50000000	.99959999	.17157301	.98517142	.50002039	.99959582	13
13	.49950000	.99969999	.17157271	.98517121	.49950608	.99970455	13
14	.49960000	.99969999	.17157279	.98517126	.49961487	.99971133	32
15	.49970000	.99969999	.17157286	.98517132	.49974116	.99973626	16
16	.49980000	.99969999	.17157291	.98517133	.49981733	.99969536	15
17	.49990000	.99969999	.17157294	.98517137	.49991132	.99969828	21
18	.50000000	.99969999	.17157295	.98517143	.50003570	.99971023	31
19	.49950000	.99980000	.17157268	.98517130	.49950848	.99980365	13
20	.49960000	.99980000	.17157277	.98517135	.49962314	.99983624	17
21	.49970000	.99980000	.17157283	.98517137	.49973899	.99986029	13
22	.49980000	.99980000	.17157288	.98517141	.50000000	1.00000000	0
23	.49990000	.99980000	.17157291	.98517140	.50000000	1.00000000	0
24	.50000000	.99980000	.17157292	.98517143	.50000000	1.00000000	0
25	.49950000	.99990000	.17157267	.98517139	.49950143	.99991105	23
26	.49960000	.99990000	.17157275	.98517141	.49961539	.99995299	20
27	.49970000	.99990000	.17157281	.98517141	.49971186	.99992630	12
28	.49980000	.99990000	.17157286	.98517144	.50000000	1.00000000	0
29	.49990000	.99990000	.17157289	.98517143	.50000000	1.00000000	0
30	.50000000	.99990000	.17157290	.98517146	.50000000	1.00000000	0
31	.49950000	.99999999	.17157267	.98517148	.49949886	1.00001344	32
32	.49960000	.99999999	.17157275	.98517147	.49960054	1.00004113	24
33	.49970000	.99999999	.17157281	.98517147	.49970805	1.00004718	16
34	.49980000	.99999999	.17157286	.98517147	.50000000	1.00000000	0
35	.49990000	.99999999	.17157288	.98517147	.50000000	1.00000000	0
36	.50000000	.99999999	.17157289	.98517146	.50000000	1.00000000	0
TOTAL TIME			.483 SEC	TOTAL TIME			7.669 SEC

APPENDIX C

NUMERICAL OUTPUT FROM PROGRAM CIRREC FOR SAMPLE CASE 2:

MAPPING OF INPUT UNIT CIRCLE ONTO RECTANGLE

WITH DIMENSIONS 8 UNITS BY 9 UNITS

MAPPING OF UNIT CIRCLE IN W-PLANE ONTO RECTANGLE IN Z-PLANE

CO-ORDINATES OF DIAGONAL OF RECTANGLE
(-4.0000, -4.5000) , (4.0000, 4.5000)

REGION OF CIRCLE CONSIDERED ($W=R*EXP(I*T)$)
R = .0000 TO 1.0000 T = .0000 TO 345.0000 (DEG)

I	W(I)--INPUT		Z(W(I))		ITER
	R	T(DEG)	X	Y	
1	.00000000	.000000	.00000000	.00000000	0
2	.20000000	-.000000	.90616883	.00000018	3
3	.40000000	-.000000	1.79075837	.00000018	4
4	.59999999	-.000000	2.62308080	.00000012	4
5	.79999999	-.000000	3.36743160	.00000018	4
6	.99999999	-.000000	4.00000010	.00000018	1
7	.20000000	15.000000	.87616619	.23307627	3
8	.40000000	15.000000	1.73829162	.45007569	4
9	.59999999	15.000000	2.56739940	.62622231	4
10	.79999999	15.000000	3.33346680	.73397881	4
11	.99999999	15.000000	4.00000010	.76607668	4
12	.20000000	30.000000	.78762311	.45154631	3
13	.40000000	30.000000	1.57798616	.88271123	3
14	.59999999	30.000000	2.38153410	1.26328480	3
15	.79999999	30.000000	3.20554700	1.53721184	3
16	.99999999	30.000000	4.00000010	1.63276708	4
17	.20000000	45.000000	.64526691	.64092046	4
18	.40000000	45.000000	1.30632386	1.27106422	4
19	.59999999	45.000000	2.01313260	1.88637578	4
20	.79999999	45.000000	2.64556890	2.48016230	4
21	.99999999	45.000000	4.00000010	2.85329970	3
22	.20000000	59.999999	.45769037	.78762823	4
23	.40000000	59.999999	.93219689	1.57862180	4
24	.59999999	59.999999	1.43467741	2.39310040	4
25	.79999999	59.999999	1.95785402	3.30539550	4
26	.99999999	59.999999	2.29710530	4.50000030	5
27	.20000000	75.000000	.23741330	.88051039	3
28	.40000000	75.000000	.48412196	1.77320814	3
29	.59999999	75.000000	.73187030	2.68691230	3
30	.79999999	75.000000	.93196076	3.61628030	4
31	.99999999	75.000000	1.00609533	4.50000030	4
32	.20000000	90.000000	.00000003	.91230470	3
33	.40000000	90.000000	.00000000	1.83917880	3
34	.59999999	90.000000	.00000009	2.77750760	3

I	W(I)--INPUT		Z(W(I))		ITER
	R	T(DEG)	X	Y	
35	.79999999	90.000000	.00000012	3.68737950	3
36	.99999999	90.000000	.00000012	4.50000030	1
37	.20000000	105.000000	-.23741325	.88051039	3
38	.40000000	105.000000	-.48412185	1.77320814	3
39	.59999999	105.000000	-.73187017	2.68691230	3
40	.79999999	105.000000	-.93196057	3.61628030	4
41	.99999999	105.000000	-1.00609507	4.50000030	4
42	.20000000	119.999999	-.45769034	.78762823	4
43	.40000000	119.999999	-.93219680	1.57862198	4
44	.59999999	119.999999	-1.43467729	2.39310050	4
45	.79999999	119.999999	-1.95785382	3.30539560	4
46	.99999999	119.999999	-2.29710500	4.50000030	5
47	.20000000	135.000000	-.64526686	.64092052	4
48	.40000000	135.000000	-1.30632378	1.27106422	4
49	.59999999	135.000000	-2.01313240	1.88637596	4
50	.79999999	135.000000	-2.84556880	2.48016260	4
51	.99999999	135.000000	-4.00000010	2.85330020	3
52	.20000000	150.000000	-.78762308	.45154643	3
53	.40000000	150.000000	-1.57798611	.88271135	3
54	.59999999	150.000000	-2.38153400	1.26328498	3
55	.79999999	150.000000	-3.20554680	1.53721195	3
56	.99999999	150.000000	-4.00000010	1.63276726	4
57	.20000000	165.000000	-.87616618	.23307627	3
58	.40000000	165.000000	-1.73829162	.45007581	4
59	.59999999	165.000000	-2.56739940	.62622255	4
60	.79999999	165.000000	-3.33346680	.73397899	4
61	.99999999	165.000000	-4.00000010	.76607686	4
62	.20000000	180.000000	-.90616881	.00000024	3
63	.40000000	180.000000	-1.79075839	.00000036	4
64	.59999999	180.000000	-2.62308080	.00000030	4
65	.79999999	180.000000	-3.36743160	.00000036	4
66	.99999999	180.000000	-4.00000010	.00000036	1
67	.20000000	195.000000	-.87616620	-.23307627	3
68	.40000000	195.000000	-1.73829165	-.45007557	4
69	.59999999	195.000000	-2.56739950	-.62622225	4
70	.79999999	195.000000	-3.33346690	-.73397875	4
71	.99999999	195.000000	-4.00000010	-.76607645	4
72	.20000000	210.000000	-.78762314	-.45154625	3
73	.40000000	210.000000	-1.57798620	-.88271111	3
74	.59999999	210.000000	-2.38153410	-1.26328462	3
75	.79999999	210.000000	-3.20554700	-1.53721154	3
76	.99999999	210.000000	-4.00000010	-1.63276678	4
77	.20000000	225.000000	-.64526694	-.64092046	4

I	W(I)--INPUT		Z(W(I))		ITER
	R	T(DEG)	X	Y	
78	.40000000	225.000000	-1.30632392	-1.27106416	4
79	.59999999	225.000000	-2.01313260	-1.88637561	4
80	.79999999	225.000000	-2.84556900	-2.48016220	4
81	.99999999	225.000000	-4.00000010	-2.85329940	3
82	.20000000	240.000000	-.45769041	-.78762823	4
83	.40000000	240.000000	-.93219697	-1.57862180	4
84	.59999999	240.000000	-1.43467754	-2.39310030	4
85	.79999999	240.000000	-1.95785424	-3.30539540	4
86	.99999999	240.000000	-2.29710570	-4.50000020	5
87	.20000000	255.000000	-.23741338	-.88051039	3
88	.40000000	255.000000	-.48412213	-1.77320808	3
89	.59999999	255.000000	-.73187060	-2.68691220	3
90	.79999999	255.000000	-.93196114	-3.61628020	4
91	.99999999	255.000000	-1.00609572	-4.50000030	4
92	.20000000	270.000000	-.00000010	-.91230470	3
93	.40000000	270.000000	-.00000020	-1.83917880	3
94	.59999999	270.000000	-.00000030	-2.77750760	3
95	.79999999	270.000000	-.00000039	-3.68737950	3
96	.99999999	270.000000	-.00000041	-4.50000030	1
97	.20000000	285.000000	.23741320	-.88051039	3
98	.40000000	285.000000	.48412173	-1.77320820	3
99	.59999999	285.000000	.73187000	-2.68691240	3
100	.79999999	285.000000	.93196034	-3.61628050	4
101	.99999999	285.000000	1.00609486	-4.50000030	4
102	.20000000	300.000000	.45769030	-.78762823	4
103	.40000000	300.000000	.93219673	-1.57862198	4
104	.59999999	300.000000	1.43467714	-2.39310050	4
105	.79999999	300.000000	1.95785359	-3.30539580	4
106	.99999999	300.000000	2.29710470	-4.50000030	5
107	.20000000	315.000000	.64526684	-.64092052	4
108	.40000000	315.000000	1.30632374	-1.27106434	4
109	.59999999	315.000000	2.01313230	-1.88637602	4
110	.79999999	315.000000	2.84556870	-2.48016280	4
111	.99999999	315.000000	4.00000000	-2.85330030	3
112	.20000000	330.000000	.78762304	-.45154649	3
113	.40000000	330.000000	1.57798603	-.88271153	3
114	.59999999	330.000000	2.38153390	-1.26328510	3
115	.79999999	330.000000	3.20554680	-1.53721225	3
116	.99999999	330.000000	4.00000010	-1.63276762	4
117	.20000000	345.000000	.87616614	-.23307639	3
118	.40000000	345.000000	1.73829156	-.45007586	4
119	.59999999	345.000000	2.56739940	-.62622267	4
120	.79999999	345.000000	3.33346670	-.73397923	4
121	.99999999	345.000000	4.00000010	-.76607704	4
TOTAL TIME			7.325	SEC	

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BNWL-901

APPENDIX D

GRAPHICAL DISPLAY OF NUMERICAL OUTPUT IN APPENDIX C

CONFORMAL TRANSFORMATION: RECTANGLE TO CIRCLE

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