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UNIFICATION

By

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Color in Internal Supersymmetry Unification

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In [1], a scheme was introduced for determining the charge of the quarks and leptons in successive generations: A family of representations of the superalgebra $sl(n/1)$ was constructed, depending on a complex parameter, b . For each b , the underlying space of the representation is Λ^n , the exterior algebra of the standard n dimensional vector space. Under the even part of $sl(n/1)$ the space Λ^n decomposes into the direct sum of the i -th exterior powers, Λ^i , with $0 \leq i \leq n$. Let us write a diagonal matrix, U , in this even part as

$$U = \text{diag} (c_1, \dots, c_n / T)$$

where the c_i and T are the diagonal entries of U and the condition that U be supertraceless requires that

$$(1) \quad T = c_1 + \dots + c_n$$

Then the eigenvalues of the operator corresponding to U on Λ^i are

$$(2) \quad c_{j_1} + \dots + c_{j_i} + (b-i)T, \quad j_1 < \dots < j_i$$

The third component of the weak Isospin was introduced in the form

$$(3) \quad I_3 = \text{diag} (\frac{1}{2}, -\frac{1}{2}, 0, \dots, 0/0)$$

We choose the weak hypercharge U to commute with I_3 , with the electric charge given by the standard formula $Q = I_3 + \frac{1}{2}U$. To reproduce the observed values of electric charge, the eigenvalues of U must be restricted to $2, 4/3, 1, 2/3, 1/3, 0, -1/3, -2/3, -1, -4/3$ and -2 . In 1 it was shown how to choose U with these

properties and the eigenvalue 2 in Λ^0 and the eigenvalue -2 in Λ^n , under the hypothesis of three colors, i.e. $r = 3$. We show here that the number of colors is fixed by the observed charges and the hypothesis that U is 2 in Λ^0 and -2 in Λ^n . Indeed, it follows from (1) and the construction in 1, that the eigenvalues of U on Λ^0 and Λ^n are bT and $(b - (n-1)T)$ respectively. If 2 is to occur in Λ^0 and -2 in Λ^n this requires that $(n-1)T = 4$ and $bT = 2$ so

$$(4) \quad T = \frac{4}{n-1}$$

and

$$(5) \quad b = \frac{1}{2}(n-1)$$

The leptonic eigenvalues of U , 1 and -1 must reccur in the I doublets in ΛV , to accommodate the e^+ and e^- and ν_L , $\bar{\nu}_L$ of $1 = \frac{1}{2} + \frac{1}{2}$, $0 = \frac{1}{2} - \frac{1}{2}$, $0 = -\frac{1}{2} + \frac{1}{2}$, $-1 = -\frac{1}{2} - \frac{1}{2}$. The eigenvalues $4/3$ and $2/3$ must occur as I_3 singlets, to accomodate the quark charges of $2/3$ and $1/3$. Similarly, 0 must occur as an I_3 singlet.

Let us now examine the eigenvalues on Λ^1 , reflecting by (2) the linear action of (3). An I_3 doublet appears. It must correspond to $U = 1$ eigenvalues, since the value 1 must occur as an I_3 doublet somewhere, and is not allowed to occur for an isosinglet. Using eq.(2) systematically, one finds that there is no choice of the eigenvalues for Λ^1 isosinglets that would produce $U = 1$ for an isodoublet anywhere else in ΛV , if we select in Λ^1 any one of the alternative allowed isodoublet assignments ($1/3$, $-1/3$ or -1). Thus the I_3 doublet eigenvalues of U in Λ^1 are 1.

The eigenvalue $1/3$ is not allowed to occur as a singlet in Λ^1 since there is no corresponding physical system. The eigenvalue $4/3$ must occur, reasoning as in the case of $U = 1$. No eigenvalues smaller than $1/3$ can occur as eigenvalues for Λ^1 because this would imply wrong eigenvalues somewhere else in ΛV .

The eigenvalues of U are thus:

- (1,1) occuring once
- (2) occuring say s times, $s \geq 1$
- (4/3) occuring say k times, $k \geq 1$
- (2/3) occuring say l times, $l \geq 0$

yielding for the trace of U on Λ^1 the value

$$\text{tr}(U)_{\Lambda^1} = 2 + 2s + 4/3 k + 2/3 l = (n(b-1)+1)T$$

but $2 + s + k + l = n$, and $(n(b-1)+1)T = 2n - 4$ by equations (4) and (5).

As a result:

$$(6) \quad \frac{2}{3} k + \frac{4}{3} l = 2$$

The only solutions of eq.(6) for non-negative integer k and l are

$$(7) \quad k = l = 1$$

or

$$(8) \quad k = 3, \quad l = 0$$

If we interpret k as the number of colors, then solution (7) corresponds to only one color degree of freedom (i.e. color is reduced to quark number, at most).

This solution can occur for example in $sl(4/1)$: Taking $b = 3/2$ and

$$U = \text{diag} (1/3, 1/3, 2/3, 0/4/3) .$$

The eigenvalues of U are then

Λ_0	Λ_1				Λ_2						Λ_3				Λ_4
2	1	1	4/3	2/3	0	1/3	1/3	-1/3	-1/3	0	-1	-1	-2/3	-4/3	-2

Thus $s = 0$ and $r = 1$ and all the right eigenvalues occur.

The case (8) corresponds to 3 colors, i.e. we interpret U as commuting with an $su(3)$ which sits as a single block-diagonal in $sl(n/1)$. There are then altogether only three distinct eigenvalues for U acting on V , with multiplicities 2, s and r respectively. Furthermore, s gives rise to 2^s generations.

The charge-symmetric subset of the representations we constructed in [1] thus allows "one" or three colors only. The number three can of course be traced to the denominator of the quark charges i.e. to the original classification $su(3)$. Ofer Gabber has pointed out to us a more fundamental reason why we should expect (5) to hold: the structure of the anticommutation relations for the creation and annihilation operators for the particles, together with the requirement that conjugate particles occur in Λ^i and Λ^{n-i} , requires that there exist a bilinear pairing between Λ^i and Λ^{n-i} , invariant under the even part of $sl(n/1)$. In

particular, eigenvalues of $\text{diag}(1, 1, \dots, 1/n)$ on these two spaces must be the negatives of one another. It follows from (2) that $b = \frac{1}{2}(n-1)$. We should remark, that under this condition, there exists a bilinear form invariant under the whole superalgebra. Indeed, as discussed in [1], the space Λ^i is really $\Lambda^i \otimes F^{b-i}$ where F^{b-i} denotes the one dimensional space of homogeneous functions of degree $b-i$. Exterior multiplication on the Λ factor and ordinary multiplication on the F factor gives a bilinear map of $(\Lambda^i \otimes F^{b-i}) \times (\Lambda^{n-i} \otimes F^{b-(n-i)})$ into $\Lambda^n \otimes F^{-1}$ if (5) holds. But, under the action of the even part of $sl(n/1)$, this space has a canonical identification with the complex numbers. We thus get an invariant bilinear pairing on the representation space $\Lambda \otimes F$. It is straightforward to check that this is invariant (in the superalgebra sense) under the entire superalgebra $sl(n/1)$.

- [1] Y. Ne'eman and S. Sternberg, "Internal supersymmetry and unification", to appear in the Proc. of the Nat. Acad. of Sci.