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**LANDAU DAMPING IN THE IMPROVED SLC LINAC:
A. THE SENSITIVITY TO INJECTION JITTER***

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INTRODUCTION

Injection jitter and machine errors can greatly degrade the emittance of a high current beam in a linac through the interaction of the beam's orbit and the transverse wakefields, in a process sometimes known as single bunch beam break-up. By now it is well known that a longitudinal energy variation can stabilize a high current beam in a linac against the beam break-up instability, by a phenomenon that is similar to Landau damping.¹⁻⁶

The question of current dependent emittance growth in the old SLC linac due to jitter and static machine errors – including the effects of Landau damping – was investigated in Ref. 2. Since the time of publication of Ref. 2 a large portion of the beginning of the linac – the whole of Sectors 2-4 – has been greatly modified. In particular, many quadrupoles have been added to these sectors in order to strengthen the beam focusing where it will do the most good: at the low energy end of the linac. Since the time of publication of Ref. 2 the nominal bunch length has also been changed from $\sigma_z = 1.0$ mm to $\sigma_z = 1.5$ mm. And it is now hoped that eventually a bunch population of $N = 7 \times 10^{10}$ will be realized. In this report we study the effects of injection jitter on emittance growth, and hence on luminosity, in the improved linac with stronger focusing, and for the new bunch parameters. We consider both cases when Landau damping has been invoked, and when it has not been. The effects of machine errors will be discussed in a sequel report.

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Inducing an Energy Variation Along the Beam

Normally a high current beam rides in front of the rf crest at the phase ϕ_0 that minimizes its energy spread. As in Ref. 2 we propose inducing an energy variation in the beam by setting it behind the rf crest at phase ϕ_s for the first n_s klystrons of the linac – we go behind the crest because an energy spread is most effective against beam break-up when tail particles are at a lower energy than head particles. Then, in order that the energy spread at the final focus is still minimised, we need to set the beam at the proper phase ϕ_f in front of ϕ_0 in the remainder of the linac. The energy variation induced in the front of the linac greatly lessens the emittance growth due to a given level of injection jitter. The price that is paid for this phase juggling, however, is a reduction in the bunch's overall energy gain.

If the accelerating sections were all of the same length, then the following two equations would need to be obeyed:

$$\begin{aligned} n_s \sin \phi_s + (n - n_s) \sin \phi_f &= n \sin \phi_0 \\ n_s \cos \phi_s + (n - n_s) \cos \phi_f &= n \cos \phi_0 - \Delta E / (E/n) \end{aligned} \quad (1)$$

with n the total number of klystrons, ΔE the reduction in energy gain due to phase switching, and E/n the energy gain per klystron ($\phi = 0$ is at the rf crest). The first equation leaves the final energy spread minimized, the second gives the energy balance. If we know the optimal average phase ϕ_0 and the amount of overall energy gain we are willing to sacrifice ΔE , then given ϕ_s we can solve for n_s . In the SLC all the acceleration sections are not of the same length, but this change involves only a minor modification in the above equations.

To get some insight into the properties of Eqs. (1) let us, for the moment, let ϕ_0 be 0. Then a geometric solution of Eqs. (1) can easily be constructed.

Fig. 1. Geometric representation of Eq. (1) when $\phi_0 = 0$.

Consider an ellipse with foci at points P and Q separated by the distance $n - \Delta E/(E/n)$ (see Fig. 1). An arbitrary point on the ellipse is designated as O . We further choose the properties of the ellipse such that the length $|P\vec{O}| + |O\vec{Q}|$ equals n . If we associate the length $|P\vec{O}|$ with the value n_a , the length $|O\vec{Q}|$ with the value $n_b = n - n_a$, then the angle $\angle QPO = -\phi_a$, the angle $\angle OQP = \phi_b$.

The maximum energy variation induced along the bunch (ignoring for the moment the contribution of the longitudinal wake) equals the length of $|P\vec{O}|$ times the negative of the sine of $\angle QPO$, that is, $-n_a \sin \phi_a$. From the figure we clearly see that this quantity is maximum when $\phi_a = -\phi_b$. On the other hand, the energy variation is induced into the beam most quickly when $\phi_a = -90^\circ$. For effective Landau damping in a linac we want that a *sufficient amount* of energy variation be induced in the beam at a *sufficiently early* point in the linac, where the beam is very sensitive to beam break-up problems. Thus we expect that for a given ΔE there is an optimum phase ϕ_a somewhere between the extremes of $\phi_a = -90^\circ$ and $\phi_a = -\phi_b$. We see this feature in the simulation results even when ϕ_0 is not 0 and the longitudinal wakefield is included in the calculations.

The Effective Emittance Growth

In the SLC the beam is ejected from its damping ring by the extraction kicker; it then follows the RTL transfer line before entering the linac. Injection jitter in the linac is caused by the pulse-to-pulse variation of the strength of the extraction kicker. It is normally confined to be in the horizontal - x - direction. Let us suppose that the static errors of the linac are well corrected, and, on a certain pulse, the kicker strength varies by a certain amount, which in turn changes the beam's initial conditions in the linac. This change in initial conditions, when amplified in the linac by the interaction of the beam's orbit with the transverse wakefields, will result in the degradation of beam quality, and hence to a luminosity reduction. The amount the beam quality is degraded depends on the oscillation amplitude, divided by the beam size, at the beginning of the linac.

In this paper we simulate the effects of injection jitter by tracking a beam, with initial offset x_0 , in the error-free linac lattice using the computer program LTRACK.² This program also computes an effective emittance growth factor that we use as our measure of beam quality. We expect that our calculations for the error-free machine agree reasonably well with the more complicated procedure of first correcting the orbit of the beam in a machine with alignment errors, and then finding the incremental emittance growth due to an added offset x_0 . Simulation results presented in Ref. 2 give us cause to believe that this expectation is valid.

The program LTRACK cuts the bunch transversely into a finite number of slices. It then follows the beam ellipses of the slices through the lattice elements of a linac to first order, by matrix multiplication, as is done, for example, by the

computer program TRANSPORT.⁷ In addition, when the beam traverses an acceleration section, the program also calculates the effects of the longitudinal and dipole wakefields. The wakefields used are those of the so-called "average SLAC cell".⁸ As beam quality criterion we use the effective normalized emittance growth δ_{ez} due to a certain initial beam offset.

The total effective x -emittance of a bunch is defined as in Ref 9:

$$c_x^2 = \sigma_{xx}\sigma_{x'x'} - \sigma_{xx'}^2, \quad (2)$$

where

$$\sigma_{xx} = \frac{1}{N} \sum_i^M N_i (\sigma_{xxi} + (x_i - \bar{x})^2), \quad (3)$$

N is the total bunch population, M is the number of slices in the bunch; N_i is the population, $\sigma_{xxi} = \sigma_{xi}^2$ is the square of the standard deviation and x_i is the center of slice i ; and the centroid of the bunch $\bar{x}$ is given by

$$\bar{x} = \frac{1}{N} \sum_i^M N_i x_i. \quad (4)$$

The other total moments $\sigma_{x'x'}$ and $\sigma_{xx'}$ are defined in an analogous fashion. In essence we have replaced all the beam slices by one representative slice - a slice with the same covariance matrix that all the slices taken together would have - and taken its area as the effective emittance. Then the normalized emittance growth factor δ_{ez} is defined as

$$\delta_{ez} = \frac{(\bar{\gamma}\epsilon_x) - (\bar{\gamma}\epsilon_x)_0}{(\bar{\gamma}\epsilon_x)_0}, \quad (5)$$

with $\bar{\gamma}$ the average energy of the bunch, and the subscript 0 denoting the conditions at the beginning of the linac. If we begin with an unperturbed beam, offset

in x by x_0 , then δ_{ex} at the end of the linac is a function of x_0/σ_{x0} , with σ_{x0} the initial beam size.

The effective emittance growth is a measure of the luminosity reduction, as long as δ_{ex} is not too large. It includes both dispersive and chromatic effects; that is, effects due to, respectively, the misalignments of the slice centroids, and the relative distortions of the slices due to their energy differences. In the specific case of injection jitter – which we assume to be confined to the horizontal plane – we expect the luminosity to be reduced, roughly, by the factor

$$r' = \frac{1}{\sqrt{1 + \delta_{ex}}} \quad , \quad (6)$$

if two identical beams are injected with about the same jitter amplitude. Note, however, that when δ_{ex} becomes large compared to 1 due to beam break-up it is not a good measure of luminosity reduction; it then tends to give too much importance to tail slices at large phase space amplitudes, and to greatly overestimate the luminosity degradation.

The Luminosity Reduction Factor

When δ_{ex} is large how does one estimate the luminosity reduction? Let us construct a luminosity reduction factor. Consider the situation of two beams that follow two linacs to position s and then collide head on. At one particular pulse beam A, with population N_A , has an initial jitter offset x_0^A , whereas beam B, with population N_B , has an initial offset x_0^B . Neither beam is offset in y . Let us represent the longitudinal distribution of each beam by M slices, as before. Then the luminosity of the two beams colliding is given by

$$\mathcal{L}/f_{rep} = \int \int \int \int dx dy dz d(ct) \lambda_A(x, y, z) \lambda_B(x, y, z - ct) \quad , \quad (7)$$

with f_{rep} the repetition rate. The particle distribution of beam A is represented by

$$\lambda_A(x, y, z) = \sum_i^M \lambda_i^A(x, y) \delta(z - z_i) \quad , \quad (8)$$

and similarly for λ_B . We assume the particle distribution of every slice is uncoupled in the $x - y$ plane and varies as a gaussian in both x and y . Therefore the transverse particle distribution of any slice i of either beam can be written as

$$\lambda_i(x, y) = \frac{N_i}{2\pi\sigma_{xi}\sigma_{yi}} \exp\left(-\frac{(x - x_i)^2}{2\sigma_{xi}^2} - \frac{y^2}{2\sigma_{yi}^2}\right) \quad . \quad (9)$$

Let us define the luminosity reduction factor r as the ratio of the luminosity of the perturbed beams in collision to the luminosity of two unperturbed beams, with beam sizes σ_{xr} and σ_{yr} . Performing the integration of Eq. (7) yields:

$$r(x_0^A, x_0^B) = \frac{2\sigma_{xr}\sigma_{yr}}{N_A N_B} \sum_{i,j}^M \frac{N_i^A N_j^B}{\sigma_{xij}\sigma_{yij}} \exp\left(-\frac{(x_i^A - x_j^B)^2}{2\sigma_{xij}^2}\right) \quad , \quad (10)$$

with

$$\sigma_{xij}^2 = \sigma_{xxi}^2 + \sigma_{xxj}^2 \quad , \quad (11)$$

and similarly for σ_{yij} . The dependance of r on the initial beam offsets x_0^A, x_0^B , is to be found in the slice centroid values x_i^A, x_j^B . The factor r is about 1 for unperturbed identical beams with small offsets; it becomes smaller as the effective emittances of the two beams increase. Unlike an emittance it oscillates as a function of s ; at locations where the tails of the beams are optimally aligned with their heads r will be at a maximum, at locations where they are most badly aligned r will be at a minimum.

If the injection jitter of each beam is randomly distributed according to the function $f(x_0)$, then the expectation value of the luminosity reduction over many pulses is

$$\langle r \rangle = \int \int dx_0^A dx_0^B f(x_0^A) f(x_0^B) r(x_0^A, x_0^B) \quad (12)$$

Let us now limit our consideration to two beams that are initially identical and that follow identical linacs before meeting in collision. We assume that the injection jitter of the two beams is normally distributed with rms values of $\sigma_{x_0^A}$ and $\sigma_{x_0^B}$ for beams A and B, respectively. Since the slice offsets x_i^A and x_j^B , at any point s along the linac, vary linearly with the initial offsets x_0^A and x_0^B we need only perform a single simulation to find the dependence of x_i^A on x_0^A , and of x_j^B on x_0^B , at any point in the linac. Then, in terms of the results of this simulation, Eq. (12) becomes

$$\langle r \rangle = \frac{2\sigma_{x_i^A}\sigma_{x_j^B}}{N^2} \sum_{i,j} \frac{N_i N_j}{\sigma_{x_{ij}^A} \sigma_{x_{ij}^B}} \left[1 + \left(\frac{\sigma_{x_0^A} x_i^A}{\sigma_{x_{ij}^A} x_0^A} \right)^2 + \left(\frac{\sigma_{x_0^B} x_j^B}{\sigma_{x_{ij}^B} x_0^B} \right)^2 \right]^{-1/2} \quad (13)$$

The initial beam offset used in the simulation is x_0 ; the number of particles is N . Note that for small jitter the luminosity reduction varies quadratically with the rms jitter amplitudes.

Finally we should point out that there is more to the question of luminosity reduction in the SLC due to the beam break-up phenomenon than is suggested here. For one thing, to get a better estimate of the real luminosity reduction, the perturbed beams that we find at the end of the linac should subsequently be tracked through the arcs and through the final focus. In this paper we assume that these systems are ideal and do not reduce the luminosity. Another question

that affects the luminosity is the impact of beam tails on the machine performance. When the tails become large they need to be scraped to avoid damage to accelerator components and to avoid an unbearably large background count in the detector. This extra complication has been ignored in the present analysis.

The Improved SLC Linac Lattice

Hardware modifications performed two years ago allow for stronger focusing in Sectors 2-4 of the linac. Instead of the 4 cells per sector that are found in the rest of the linac, there are 16 cells in Sector 2, and there are 8 cells in each of Sectors 3 and 4. A normal sector is 102 m long, containing 8 quads and 8 acceleration sections, each of which is fed by a klystron. The total number of klystrons is 232. For the simulations to be presented the phase advance per cell is 90° in Sectors 2-4, 76° in Sector 5, 90° in Sectors 6-19. (With the 76° transition region the beam was found to be less susceptible to chromatic emittance growth, according to simulations, than if 90° per cell were the phase advance in all of Sectors 2-19.) In the rest of the linac the quad strengths are set approximately to their maximum of 110 kG. The first quad the bunch encounters focuses it in the x plane. A plot of the matched β -function is shown in Fig. 2.

In all the simulations to be presented in this paper the beam enters the middle of the first quad upright in each plane, with normalized emittance 3×10^{-5} rm, beam size $\sigma_{x0} = 250 \mu\text{m}$ and energy $E_0 = 1.21$ GeV. The average accelerating gradient is 18 MeV/m, sufficient for the nominal bunch ($N = 5 \times 10^{10}$, $\sigma_z = 1.5$ mm) to attain 50 GeV in the linac, when phase switching is not done. The bunches have a longitudinal charge distribution that is gaussian; in the simulations the longitudinal distribution is represented by 16 slices, ranging from $3\sigma_z$ in front of bunch center to $2\sigma_z$ behind it. We study injection jitter by

Fig. 2. The matched β -function in Sectors 2-30.

tracking a beam with initial offset x_0 through an error-free machine. In the SLC the current relative kicker jitter rating of 10^{-3} would translate into about a value of $x_0 = 0.1\sigma_{x0} = 25 \mu\text{m}$.

SIMULATION RESULTS

The Effect of Landau Damping on the Injection Jitter Tolerance

As our measure of beam sensitivity to jitter we define the jitter tolerance t as the ratio x_0/σ_{x0} that causes δ_{tx} to grow by 25%. In the old machine with weaker focusing we found that $t = 0.007$ when the bunch population N is 5×10^{10} and the bunch length σ_z is 1.0 mm, when Landau damping is not used (see Ref. 2). The beam phase $\phi_0 = 12^\circ$, the residual energy spread $\sigma_E/E = 0.4\%$. With the same bunch properties, but in the current, stronger focusing lattice, we find $t = 0.016$. Thus according to this comparison the new lattice is twice as strong as the old one.

Landau damping results for a bunch with $N = 5 \times 10^{10}$ and $\sigma_z = 1.0 \text{ mm}$

in the current SLC lattice are shown in Fig. 3. The graph gives values of t vs ϕ_a , with contours in ΔE . The computed results, given by the diamonds, at $\phi_a = -45^\circ, -30^\circ, -15^\circ, 0^\circ$, have been connected with smooth curves to guide the eye. We observe that there is an interior peak in effectiveness of Landau damping – at somewhere near $\phi_a = -15^\circ$ – as was anticipated in the Introduction. We note that an energy sacrifice of $\Delta E = 1.0$ GeV is needed to keep the emittance growth to 0.25 when there is an injection jitter amplitude of 10% of the initial beam size σ_{x0} . In the old, weaker lattice, for the same bunch properties, we found that we need to give up 1.5 GeV to achieve this same level of stability to jitter.

Fig. 3. The jitter tolerance factor t when $N = 5 \times 10^{10}$ and $\sigma_z = 1.0$ mm, for several values of ΔE , in the current lattice. The computed results are given by the diamonds.

The nominal bunch properties of the current design are $N = 5 \times 10^{10}$ and $\sigma_z = 1.5$ mm. In this case the average phase position to minimize the final energy spread ϕ_0 is 5.9° , resulting in a final energy spread of $\sigma_E/E = 0.4\%$. (The energy

spreads quoted here include all bunch particles in the calculation. The design value of $\sigma_E/E = 0.2\%$ is obtained when a small fraction of the beam is not included in the standard deviation calculation. The phase ϕ_0 needed to achieve the design value, however, remains at 5.9° .) Since the bunch is longer the dipole wakefield is stronger than in the earlier example with $\sigma_z = 1.0$ mm. In this case we find that $t = 0.009$. Thus, by strengthening the focusing and at the same time increasing the bunch length, the sensitivity to injection jitter has remained approximately unchanged, when phase shifting is not used.

Simulation results for the nominal bunch in the present lattice, when phase shifting is employed, are summarized in Fig. 4. The top frame gives t for various values of ϕ_s and ΔE . Comparing this plot to Fig. 3 we notice that the beam is actually *more* stable to jitter, for $\Delta E = 0.5, 1.0$ GeV, than the shorter, 1.0 mm bunch was. Since the energy spread induced in the beam by the rf wave depends on the bunch length times the rf slope at the bunch, more energy spread is induced in a longer bunch; and apparently, in these cases, the resulting increased damping more than compensates for the stronger dipole wake experienced by the longer bunch. We need to give up 0.6 GeV of energy to keep the emittance growth to 0.25 due to a jitter of $0.1\sigma_{z0}$. Finally, we also note that the effectiveness of Landau damping seems to saturate at $\Delta E = 1.5$ GeV over most of the range in ϕ_s : increasing the energy loss to 2.0 GeV does not lessen the jitter sensitivity.

In the bottom frame of Fig. 4 we give the emittance growth of a nominal bunch, offset by $x_0 = 0.1\sigma_{x0}$, for various values of ϕ_s and ΔE . The saturation at $\Delta E = 1.5$ GeV is here again observed. The dotted curve gives δ_{rx} for a beam launched on axis with $\Delta E = 2.0$ GeV. This emittance growth is purely due to chromatic effects, and has nothing to do with the dipole wakefield. So we see that

Fig. 4. The jitter tolerance t (top) and the emittance growth when $x_0 = 0.1\sigma_{x0} = 25\mu\text{m}$ (bottom), if $N = 5 \times 10^{10}$ and $\sigma_x = 1.5$ mm, and for several values of ΔE , in the current lattice. The diamonds display the computed results. The dotted curve in the bottom frame gives the emittance growth of an on-axis beam if $\Delta E = 2.0$ GeV.

at a jitter level of $0.1\sigma_{x0}$ with $\Delta E = 2.0$ GeV the emittance growth due to the dipole wake is negligible compared to that due to chromatic effects – chromatic effects that were caused by the energy variation that we induced along the beam in order to invoke Landau damping.

We have chosen a case with $\phi_s = -20^\circ$ and $\Delta E = 1.0$ GeV to present in more detail. This combination of parameters results in $n_s = 41$ and $\phi_i = 11.4^\circ$. The development of the energy variation and the emittance growth along the linac, when $x_0 = 0.16\sigma_{x0}$, are given in Fig. 5. The top frame gives the energy spread in the beam, which peaks at a value of 3.6% at $s = 500$ m, after the 41st klystron. The bottom frame shows the emittance growth in the linac. We note that the emittance grows in the first 500 m, is flat for the next 1500 m, and rises again in the last 1000 m, where the energy spread is again small and the focusing becomes weak.

Fig. 6 shows the jitter tolerance for various values of ϕ_s and ΔE when $N = 3.0 \times 10^{10}$ and $\sigma_x = 1.5$ mm. The optimal average phase ϕ_0 is 3.5° , the final energy spread is again 0.4%. The jitter tolerance is 0.4 when $\Delta E = 1.0$ GeV. In contrast, when phase shifting is not employed we find that $t = 0.04$.

According to the Blue Book the number of particles per bunch will eventually be upgraded to $N = 7 \times 10^{10}$. Fig. 7 displays the jitter tolerance for this bunch population, again for $\sigma_x = 1.5$ mm. In this case $\phi_0 = 8.2^\circ$, the final value of $\sigma_E/E = 0.5\%$. The jitter tolerance is 0.07 when $\Delta E = 1.0$ GeV. With no Landau damping we find $t = 0.003$. Finally, we summarize some of the results of the previous figures in Table 1.

Fig. 5. The development of the energy spread (top) and of the emittance growth when $x_0 = 0.16\sigma_{x0}$ (bottom) in the error-free linac, if $N = 5 \times 10^{10}$, $\sigma_x = 1.5$ mm, $\phi_0 = -20^\circ$ and $\Delta E = 1.0$ GeV.

Fig. 6. The jitter tolerance factor t when $N = 3 \times 10^{10}$ and $\sigma_x = 1.5$ mm. Curves are given for several values of ΔE . The diamonds indicate the computed results.

Fig. 7. The jitter tolerance t when $N = 7 \times 10^{10}$ and $\sigma_x = 1.5$ mm. Curves are given for several values of ΔE . The diamonds indicate the computed results.

$N/10^{10}$	$\Delta E / \text{GeV}$			
	0.0	0.5	1.0	1.5
3	.038	.256	.428	.604
5	.009	.072	.172	.232
7	.003	.016	.068	.076

Table 1. The jitter tolerance t maximized with respect to ϕ_s , when $\sigma_z = 1.5$ mm, in the current SLC linac lattice. The initial beam size $\sigma_{z0} = 250$ μm .

The Luminosity Reduction for Jitter in the Absence of Landau Damping

What if we operate the linac with no Landau damping and the injection jitter is large, yielding a large emittance growth? What is the effective luminosity reduction? As pointed out in the Introduction a large value of δ_{cz} tends to be an overly pessimistic measure of luminosity degradation. A better measure would be the luminosity reduction factor r , presented in the Introduction.

In Fig. 8 we show the development of the luminosity reduction factor r (see Eq. (10)) along the SLC linac, for two identical beams following the linac to position s before colliding head on. The two beams have the same initial offset $x_0 = 0.1\sigma_{x0}$; the parameters of each bunch are $N = 5 \times 10^{10}$ and $\sigma_z = 1.5$ mm; Landau damping is not invoked. In this case the emittance growth $\delta_{cz} = 20$ at the end of the linac. We have plotted the values of r at the quads.

We note the oscillatory character of r . A local maximum of r represents a

Fig. 8. The luminosity reduction factor r at the quads for identical beams in the current SLC lattice; each beam has an initial offset $x_0 = 0.1\sigma_{x0} = 25\mu\text{m}$. The bunch properties are $N = 5 \times 10^{10}$, $\sigma_z = 1.5\text{ mm}$. No Landau damping is used. The dashes give the factor $r' = (1 + \delta_{iz})^{-1/3}$.

point in the linac where the tail of each beam lines up in an optimal way with the head. The frequency of oscillation is approximately twice the betatron frequency, since the tail oscillates approximately at the betatron frequency. The factor r' calculated from the effective emittance as

$$r' = \frac{1}{\sqrt{1 + \delta_{iz}}} \quad , \quad (14)$$

is shown by the dashed curve. The factor r' agrees well with the average value of r in the very beginning of the linac, until about the 500 m point. At the end of the machine, however, we clearly see that r' is much more pessimistic than r : in this region of the linac r has an average value of 0.5, with a spread of ± 0.1 , whereas $r' = 0.2$,

If we know the distribution function of injection jitter for the two beams we can compute the expectation value of luminosity reduction over many pulses $\langle r \rangle$. If we assume that the two beams have the same bunch population and the same bunch length, and that the jitter of the two kickers are normally distributed and uncorrelated, then the factor $\langle r \rangle$ can be written in terms of the results of a single LTRACK simulation, and is given by Eq. (13). Fig. 9 displays the development of the factor $\langle r \rangle$ as a function of s in the SLC linac, for two beams with $N = 5 \times 10^{10}$ and $\sigma_z = 1.5$ mm. The standard deviation of the injection jitter amplitude $\tilde{\sigma}_{x_0}$ is set to $0.1\sigma_{x0}$ for each beam. (The tilde is used in the symbol designating the rms jitter amplitude in order that it is less easily confused with the symbol for the initial beam size.)

Fig. 9. The expectation value of the luminosity factor r over many pulses. The kickers each have normally distributed jitter; the rms amplitude of injection jitter $\tilde{\sigma}_{x_0} = 0.1\sigma_{x0} = 25 \mu\text{m}$. Both bunches have $N = 5 \times 10^{10}$ and $\sigma_z = 1.5$ mm. No Landau damping is used.

The plot of Fig. 9 resembles that of Fig. 8: the function $\langle r \rangle$ also oscillates in s , with the maxima and minima of $\langle r \rangle$ occurring at the same linac locations as for the function r in the earlier plot. This is to be expected since the locations – in phase space – of all the centroids of the slices that represent the beam scale linearly with the initial beam offset; therefore the tail of the beam will optimally line up with the head at the same linac positions on every pulse, irrespective of the pulse-to-pulse variation in the jitter amplitude. In the example shown in Fig. 9 we find that $\langle r \rangle = 0.6 \pm 0.1$ at the end of the linac.

Let us continue to consider the case of initially identical beams, with identical rms injection jitter amplitudes $\tilde{\sigma}_{x_0}$. Fig. 10 displays the averaged value of $\langle r \rangle$ – averaged in s over an oscillation cycle – at the end of the SLC linac as a function of $\tilde{\sigma}_{x_0}$ for bunch populations per beam of $N = 3, 5, 7 \times 10^{10}$. Again the bunch length is $\sigma_x = 1.5$ mm, and phase shifting has not been done. When $\tilde{\sigma}_{x_0} = 0.1\sigma_{x0}$ we find that $\langle r \rangle = 0.75 \pm 0.05, 0.50 \pm 0.10, 0.40 \pm 0.15$ at the end of the linac, for bunch populations of $N = 3, 5, 7 \times 10^{10}$, respectively. If we take the averaged values of $\langle r \rangle$ as indicative of the luminosity reduction, then for jitter amplitudes of $\tilde{\sigma}_{x_0} = 0.1\sigma_{x0}$ we gain only a factor of 3 in luminosity by changing the bunch population from $N = 3 \times 10^{10}$ to $N = 7 \times 10^{10}$, instead of the factor of 5.5 we would expect if we ignored wakefields.

In the three examples with different bunch populations we find the maxima and minima of $\langle r \rangle$ to be in approximately the same locations in the linac – but not in exactly the same locations. For two low current beams we would expect the maxima of $\langle r \rangle$ to be at integral multiples of half the betatron wavelength from the kicker, since at these points both beams would be on axis. From the simulations, however, we find that for $N = 3, 5, 7 \times 10^{10}$ the last maximum of

Fig. 10. The averaged value of $\langle r \rangle$ at the end of the linac, as function of the rms injection jitter amplitude $\tilde{\sigma}_{x_0}$, when phase shifting is not employed (the solid lines). The values of $\tilde{\sigma}_{x_0}$ are the same for both beams; the two beams have the same bunch properties. Curves are given for $N = 3, 5, 7 \times 10^{10}$, with $\sigma_x = 1.5$ mm. A case with phaseshifting, with $N = 5 \times 10^{10}$, $\sigma_x = 1.5$ mm $\Delta E = 1.0$ GeV, $\phi_s = -20^\circ$, is also displayed (dashes).

$\langle r \rangle$ is positioned respectively at about $20^\circ, 40^\circ, 60^\circ$, – in terms of the betatron wavelength at the end of the linac, which is about 200 m – past the point where the maximum for a low current beam would occur. This behavior is caused by the relative phase lag of tail particles with respect to head particles which develops in a beam undergoing single bunch break-up in a linac, when Landau damping is not used. (For a discussion of this subject see, for example, Ref. 10.)

From the foregoing discussion we see that if the SLC is to be run at high currents, with no Landau damping, the effect of injection jitter on luminosity can be minimized by allowing the machine tune – the total phase advance between the kicker and the final focus, divided by 2π , for a low current beam – to be

variable. If the fractional part of the tune is variable over the range .0 to .2, or the range .5 to .7, then the effect of injection jitter can be minimized – $\langle r \rangle$ can be maximized – for any bunch population up to 7×10^{10} by a tune adjustment. At $N = 5 \times 10^{10}$ such an adjustment can at best make a difference of 50% in the luminosity achieved, at $N = 7 \times 10^{10}$ the improvement can be a factor of 2. Of course, this analysis again ignores any indirect effects on the luminosity that the beam tails may have, such as through their effect on the detector background, etc.

For comparison, Fig. 10 also displays – by the dashed curve – an example when phase shifting has been employed. The parameters for this example are $N = 5 \times 10^{10}$, $\sigma_x = 1.5$ mm, $\Delta E = 1.0$ GeV and $\phi_s = -20^\circ$. A jitter of $\delta_{\phi_s} = 0.1\sigma_{\phi_s}$ results in the luminosity reduction $\langle r \rangle = 0.92 \pm 0.02$. Note that this value is well approximated by the factor $(1 + \delta_{\phi_s})^{-1/2}$, since $\delta_{\phi_s} = 0.16$ for this example.

Finally, in Figs. 11 and 12 we display phase space plots and beam profiles at two quadrupoles near the end of the linac. We consider two beams in these figures, both with bunch population $N = 5 \times 10^{10}$ and bunch length $\sigma_x = 1.5$ mm; both were launched into the error-free linac lattice with initial offset $x_0 = 0.1\sigma_{x0}$. In one case Landau damping was not used; in the other it was, with $\Delta E = 1.0$ GeV and $\phi_s = -20^\circ$. To make these plots we first ran simulations in which 61 slices were used to represent the gaussian charge distribution, out to $\pm 3\sigma_x$ from the center of the bunch; then 2000 random points were generated to produce scatter plots of the beams at the two linac positions.

Fig. 11 shows the beams at a point in the linac where r – for each beam colliding with its mirror image – is at a local minimum; i.e. at a position where

the beam quality appears especially bad in the $x - z$ plane. Fig. 12 displays the same beams at positions that are approximately 90° further downstream, where r is at a maximum. In both figures the case in which phase shifting has not been employed is given by the top two frames, the case in which it has been is shown in the bottom two frames. The left two frames of each figure display the beams in phase space; the right two frames show them projected in the $x - z$ plane (the head is to the right). In Fig. 12, note that particles more than $1.5\sigma_z$ behind the bunch center, for the case in which phase shifting was not used, are outside the frame of the plots.

We clearly see that the beam that has been Landau damped is a better quality beam. In the Landau damped case we can discern a small tail in the $x - z$ plot of Fig. 11 that is absent in Fig. 12. In the non Landau damped case the beam has a large tail at both linac positions. However, in Fig. 12 the tail does not begin in earnest until about $z \approx -\sigma_z$, whereas in Fig. 11 it becomes already noticeable much closer to $z = 0$. If we suppose the beams collide with their mirror images, then the values of the luminosity reduction factor r for the first plot are 0.37 and 0.39, respectively, in the case Landau damping has not been invoked and in the case that it has been; for the second plot the values are 0.62 and 0.99.

Fig. 11. Scatter plot of two beams at a point near the end of the SLC linac where r is at a minimum; one beam has not been Landau damped (the top two frames), the other has been (the bottom two frames). In both cases $N = 5 \times 10^{10}$ and $\sigma_z = 1.5$ mm; both beams were launched with the offset $x_0 = 0.1\sigma_{x0}$. In the Landau damped case $\Delta E = 1.0$ GeV, $\phi_s = -20^\circ$. The two left frames display the beams in phase space; the right two frames show them projected in the $x - z$ plane (the head is to the right).

Fig. 12. Scatter plot of the same two beams shown in Fig. 11, but at a point approximately 90° further down the linac, where r is at a maximum; one beam has not been Landau damped (the top two frames), the other has been (the bottom two frames). In both cases $N = 5 \times 10^{10}$ and $\sigma_z = 1.5$ mm; both beams were launched with the offset $x_0 = 0.1\sigma_{x0}$. In the Landau damped case $\Delta E = 1.0$ GeV, $\phi_a = -20^\circ$. The two left frames display the beams in phase space; the right two frames show them projected in the $x - z$ plane (the head is to the right).

CONCLUSIONS

The modifications done to the front end of the SLC linac – in Sectors 2-4 – in order to strengthen the focusing have increased the jitter tolerance ϵ by a factor of 2, according to our simulations, when no Landau damping is used. Here by jitter tolerance we mean the ratio of the jitter amplitude x_0 and the beam size σ_{x0} at the entrance of the linac that increases the effective beam emittance at the end of the linac by 25%. To invoke Landau damping we induce a coherent energy variation in the beam near the beginning of the linac by shifting the beam's phase behind the rf crest – at phase ϕ_a – for the first n_a klystrons. Then, in order that the final energy spread at the final focus is still minimized the beam is shifted to the proper phase ϕ_b in front of the rf crest for the rest of the linac. The phase shifting makes the beam less sensitive to injection jitter, at the expense of some energy. We have presented plots of the dependence of the jitter tolerance ϵ on the phase ϕ_a and on the energy sacrifice ΔE , for bunch populations $N = 3, 5$, and 7×10^{10} and bunch length $\sigma_z = 1.5$ mm. The jitter tolerance is only 0.9% for the nominal bunch of the SLC – $N = 5 \times 10^{10}$ and $\sigma_z = 1.5$ mm – when no phase shifting is done. If, however, we do use phase shifting, and are willing to give up 1 GeV in final energy, we found that the jitter tolerance can be increased to 17%. For this example the optimal initial phase ϕ_a is somewhere between -25° and -15° ; the maximum induced energy spread – 3.5% – occurs at about the 500 m point in the linac.

When the effective emittance – which is our measure of beam quality – becomes large due to the beam break-up phenomenon, it tends to give too much importance to tail particles with large phase space amplitudes and therefore to overestimate the luminosity reduction. So we have defined a luminosity reduction

factor r that – at any position s in the linac – represents the relative luminosity of two beams colliding head on. And if we can assume the injection jitter is random, and is normally distributed, we can calculate the expectation value of the luminosity reduction factor over many pulses $\langle r \rangle$. This function allows us to compare the effect on luminosity of large kicker jitter when Landau damping is used with when it is not. Unlike an emittance, the factor $\langle r \rangle$ oscillates as the beam moves down the linac; a maximum is at a location where the tail is optimally aligned with the head of the beam. Suppose we have two nominal bunches, and also suppose we have kicker jitter that, for each beam, results in a position jitter, with an rms value of $0.1\sigma_{x0}$, at the front of the linac. Then, in this case, we found that $\langle r \rangle = 0.5 \pm 0.1$ at the end of the linac when Landau was not used, whereas $\langle r \rangle = 0.92 \pm 0.02$ when it was, if $\Delta E = 1.0$ GeV.

The positions in the linac where the minima and maxima of the function $\langle r \rangle$ occur do not vary with the jitter amplitude. The positions of the extrema do change slightly, however, when the beam current is changed: as the current is increased the extremum positions move further down the linac, when phase shifting is not used. If the SLC is to be run at high currents with no Landau damping, we suggest that the tune – from kicker to final focus – be adjustable. Then, whatever the running current may be, the effect of jitter on the luminosity can always be minimized by changing the tune. For the nominal bunch properties adjusting the tune can improve the luminosity by 50% when Landau damping is not used; for bunches of 7×10^{10} particles the improvement can be a factor of 2. Note that our analysis has ignored any indirect effects the tails might have on the luminosity, for example, by increasing the background count in the detector.

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