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**Magnetoelastic Instabilities and Vibrations
of Superconducting-Magnet Systems**

Francis C. Moon

March 1982

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**Final Summary Report to the
U.S. Department of Energy
September 1975 - May 1979**

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Abstract

This report describes the research accomplished under Department of Energy/NSF grants associated with the structural design of superconducting magnets for magnetic fusion reactors. The main results pertain to magneto-mechanical instabilities in toroidal and poloidal field magnets for proposed fusion reactors. One major accomplishment was the building and testing of a $1/75^{\text{th}}$ scale superconducting structural model of a 16 coil Tokamak reactor. Using this model the buckling of toroidal and poloidal field coils under different constraints was observed.

A series of dynamic tests were performed, including the effect of currents on natural frequencies, poloidal-toroidal coil interaction, and buckling induced superconducting-normal quench of the coils. The stability of poloidal coils in a toroidal magnetic field were investigated with the 16 coil torus. A superconducting poloidal coil was observed to become statically unstable or buckle as the current approached a certain value.

Magnetoelastic buckling of other magnet systems such as a yin-yang pair of magnets, Ioffe coils, and discrete coil solenoids were also studied. It is concluded that in high field superconducting structures where mass is to be minimized, magnetoelastic buckling is always possible. Structural designs for superconducting magnet systems should be checked for such instabilities since current structural engineering design codes do not account for magnetoelastic effects.

Introduction

The research program conducted under these grants has sought to explore a range of problems where electromagnetic fields create nontrivial stress, deformations and dynamics in superconducting structures. As operating fields have increased, the limiting constraint becomes the structures ability to withstand the magnetic forces.

The problems studied under these grants have focused on those in which the magnetic forces and structural deformations are coupled. It has been shown that for many different systems of superconducting magnets an elastic buckling instability can occur under magnetic forces. This class of phenomena is called magnetoelastic instability and has been shown to be possible in arrays of toroidal field coils, poloidal field coils, mirror machine magnets and in discrete coil solenoids. The conditions under which these structural instabilities can occur, and the design procedures to be taken to avoid such instabilities, has been the main focus of this research.

One of the ironies that have come out of this research is that the magnets that must stabilize the plasma, may themselves become unstable under magnetic forces. As a result of this research, many design groups are now aware of this phenomenon and will choose structural designs which will avoid such problems. At one time it was proposed to develop a short design handbook on magnetoelastic instabilities with more practical and simplified procedures for magnet designs. However, this proposal was not funded.

A brief description of our major findings are summarized in the following sections with references given to one or more of our many research papers and reports.

Chronology

The research summarized in this report on the Mechanics of Superconducting Magnets covers the period September 1974 - May 1979. The program was initially supported by a grant from the National Science Foundation to Princeton University beginning in 1971 on the subject of magnetoelastic instabilities. In 1974 the Atomic Energy Commission provided a grant to complement the NSF effort. In that study the importance of magnetoelastic instabilities in magnetic fusion magnets was investigated theoretically. After F.C. Moon's move to Cornell University the ERDA/DOE provided three grants to the principal investigator supplemented by NSF funds to study experimentally magnetoelastic effects on superconducting magnets for fusion. This effort was phased out by DOE in August 1978 through a no-cost extension to Cornell ending in May 1979.

Continuing work on magneto-mechanical problems relating to fusion was supported by the National Science Foundation through August 1980.

The results of this research has been reported to AEC/ERDA/DOE officials in ten letter, progress and annual reports covering work through June 1978. In addition fourteen papers have been published in the applied mechanics, applied physics and fusion related journals. During this period, the principal investigator reported his results to magnet designers in the fusion community through personal visits to six U.S. DOE National Laboratories as well as to several laboratories in Japan.

Summary of Research Accomplishments

I. Elastic Buckling of Toroidal Field Coils.

1. The basic phenomenon of magnetoelastic buckling of superconducting structures was discovered by the Cornell group. A theory was developed based on magnetic forces and structural mechanics which led to the development of a computer code to predict the possibility of this instability in full scale reactors. (Reports 1,2,3, Publ. 1,2).

2. Magnetoelastic buckling of TF coils was demonstrated experimentally using three, five, eight, and sixteen coil models. (Reports 3, 6, 8, Publ. 3, 5, 6, 7, 10). In the most extensive experimental program, a 16 coil, 1/75th superconducting model of an Argonne National Laboratory design was tested. (Figure 1), (Publ. 10). A film was made showing actual buckling of the coils in liquid helium, and shown to DOE in 1977.

3. The experiments showed that out of plane buckling was the most critical mode of failure and that the coils would buckle in pairs in an alternating buckling pattern around the torus. (Figure 2), (Publ. 6, 10). Lateral buckling stiffness was found to be critical to assure that the buckling current was below the design current in the coils.

4. The critical buckling current was determined as a function of the number of lateral constraints around the TF coils. (Table I), (Report 8, Publ. 14). It was discovered through tests that the internal winding or pancake structure would add little to the lateral stiffness (Figure 3) and that the coil case design itself would be a critical parameter in keeping the buckling current above the design current. (Report 8, Pub. 14).

5. It was demonstrated experimentally that ferromagnetic material outside the torus could act as a "sky hook" and structurally stabilize the TF coils with respect to lateral deflections without direct contact with the TF coils. (Report 6).

II. Toroidal Coil Misalignment Forces and Perturbed Magnetic Fields.

1. It was discovered that small out of plane imperfections in the construction of the coils or misalignments around the torus could lead to pairwise lateral bending of the TF coils. (Figure 2), (Report 8).

2. Calculations of the perturbed toroidal field in the plasma region due to laterally bent TF coils were made. (Figure 4, 5), (Report 8).

3. It was observed that small misalignments in the TF coils could lower the critical buckling current under certain coil constraint conditions. (Figure 6).

III. Elastic Buckling of Poloidal Field Coils.

1. A theory was developed for the buckling of poloidal field coils under magnetic forces between PF and TF coils. (Report 6, 8, Publ. 6,9). One theory considers an elastic coil on rigid pin supports. The buckling mode consists of a helical deformed shape around the circumference. (Figure 7). Another theoretical analysis considered a rigid coil on elastic supports. The buckling mode in a toroidal field was shown to consist of a planar displacement and a rotation about an axis along a diameter of the coil (Figure 8).

2. Experiments were conducted using a PF coil inside of the 16 coil torus which demonstrated the rigid coil instability on elastic supports. (Figure 9), (Report 8, Publ. 9). Fair agreement with theory was found. (Publ. 9). Photographs of the instability were taken in liquid helium.

3. The number of structural constraints necessary to avoid helical buckling in a PF coil was determined as a function of the current, toroidal field, and the coil case bending and torsional stiffnesses. (Report 6).

IV. Vibrations and Transient Dynamics of Magnets in a Superconducting Torus.

1. It was discovered that the mutual magnetic interactions between toroidal field coils would couple the vibrations of all the TF coils. (Report 2, 3, Publ. 2, 12). The natural out of plane bending vibration frequencies were shown to decrease with the current in the coils. (Figure 10), (Publ. 3, 6, 10). In-plane vibrations were found to increase with current. The lowest lateral natural frequency was shown to approach zero near the critical buckling current. (Figure 11), (Report 3, 6, Publ. 3, 6, 10).

2. The change in natural frequency with increases in current in a toroidal arrangement of coils was observed in experiments on the 3, 5, 8 and 16 coil models. The decrease in natural frequency with current was found to provide an experimental method to determine the critical buckling current. (Table II), Publ. 3, 5, 6, 10).

3. Dynamic interaction between a pulsed poloidal field coil and a toroidal field coil were measured on the 5 coil partial torus. (Report 6, Publ. 5). It was found that dynamic strains could exhibit 100% overshoot over the strains produced by quasi-steady magnetic forces, depending on the rise time of the poloidal field and the natural frequencies of the TF coil. (Figure 12), (Report 6, Publ. 6).

4. It was demonstrated experimentally that elastic bending in the TF coils could induce quenching of the coils. Also during sudden quench, the stored elastic energy in the coils could lead to vibration, bending stress reversal and dynamic overshoot. (Report 8, Publ. 12).

5. The periodic array of coils in a torus was shown to exhibit wave-like behavior in numerical simulation of transient dynamics. (Report, 2, 3, 8, Publ. 2, 12), (Figure 13). This phenomenon was observed experimentally in the 16 coil torus. The group velocity of the waves was shown to depend on the coil current. (Publ. 12). This phenomenon may be important in the response of the magnets to earthquakes or one coil quench or other non-symmetric transient events.

V. Behavior of Superconducting Coils near Ferromagnetic Structures.

1. The Cornell group developed a ferromagnetic toroidal simulator using one coil between two steel plates. For fields below saturation of the hot rolled steel, the magnetic behavior of the coil between the plates was similar to that of the 16 coil torus. (Figure 14). Elastic buckling currents for the single coil toroidal simulator and the 16 coil torus were found to be in good agreement with theoretical predictions. (Table III, Figure 15). The possibility of using this scheme for the Large Coil Program tests was proposed informally.

2. It was shown experimentally that the presence of ferromagnetic material near superconducting coils could provide positive or negative magnetic stiffness for the lateral bending of the coils. (Report 6). For example, a circular pancake coil between two ferroplates was found to have decreased lateral stiffness leading to elastic buckling of the coils. On the other hand, ferromagnetic steel placed outside the TF coils was found to stiffen the coils laterally, (Report 6).

VI. Magnetoelastic Buckling of Other Magnet Systems.

1. Theoretical studies by the principal investigator based on an extension of Earnshaw's theorem to magnet systems, has shown that all sets of

magnets operating under constant current control will have at least one unstable rigid body or elastic deformation mode. (Publ. 14). Magnet systems which have been shown to exhibit this behavior include yin-yang mirror magnets, discrete solenoid coils for tandem mirror reactors, and an Ioffe coil in a solenoidal field.

2. A rotation instability in a yin-yang pair of magnets has been demonstrated experimentally on a small superconducting model. (Figure 16, 17), (Publ. 13). Each magnet can carry 30×10^3 Amp-turns. Rotational motion of one magnet with respect to the other was observed to increase as the current approached a critical value. The discovery of this phenomenon had some influence on the design of the connecting structure between the yin-yang magnets designed, built and tested at Lawrence Livermore Laboratory.

3. Experiments with two circular superconducting coils between ferromagnetic steel plates have demonstrated lateral buckling of the coils. The steel plates provide additional magnetic fields which simulate image magnets in an infinite set of coils in a discrete coil solenoid. Recent theoretical studies by the principal investigator for the Mirror Fusion Test Facility program at Lawrence Livermore Laboratory have predicted this instability. This phenomenon puts constraints on the type of intercoil connections between the solenoid magnets for tandem mirror devices.

4. Experiments were conducted which demonstrated a rigid body rotational instability of an Ioffe magnet in the field of a pair of split solenoid magnets.

VII. Eddy Current Forces in Magnetic Fusion Reactors.

One of the objectives of the last funded year's proposal was to investigate the effect of eddy current related forces on tokamak structures

due to pulsed magnetic fields using an infrared scanning technique to detect eddy currents in plate-like structures developed under a grant supported by the Department of Transportation. This technique was used in this study to examine eddy currents in cylindrical thin conductors and plates with cracks. (Figure 18, 19). This part of the project was not completed under this grant but was completed under NSF and funding from the Office of Naval Research. Under this continuing grant, numerical techniques, including finite element and boundary integral methods have been used to develop codes to predict eddy current patterns, magnetic forces, and elastic transient deformation in plate-like conductors due to pulsed magnetic fields. (Publ. 15, 16, 17). These codes have been verified using the infrared scanning technique supported in part by this DOE grant. Also, the principal investigator has participated in a design review at Argonne National Laboratory for an eddy current test facility to be built there.

Published Research Papers
Supported in Part by DOE/NSF Funds

1. F.C. Moon and S. Chattopadhyay, "Elastic Stability and Vibration of a Superconducting Coil for a Fusion Reactor," Proc. 5th Sym. on Engineering problems of Fusion Research, Princeton, N.J., November 1973. IEEE Pub. No. 73-CHO 843-3-NPS.
2. F.C. Moon and C. Swanson, "Vibration and Stability of a Set of Superconducting Toroidal Magnets," J. Applied Physics, Vol. 47, No. 3, March 1976, p. 914-919.
3. F. C. Moon, "Buckling of a Superconducting Coil Nested in a Three Coil Toroidal Segment," J. Applied Physics, Vol. 47, No. 3, March 1976, p. 920-921.
4. F. C. Moon: "Buckling and Coupled Lateral Vibration Problems of Superconducting Magnets," Proc. Workshop on Structural Analysis Needs for Magnetic Fusion Energy Superconducting Magnets, p. 377-392, Brookhaven National Laboratory Report CONF-760984, Sept. 1976.
5. F.C. Moon, C. Swanson and S.King, "An Eight Coil Superconducting Torus for Elastic Buckling Studies," Cryogenics June 1977, pp. 341-344.
6. C. Swanson and F.C. Moon, "Buckling and Vibrations in a Five Coil Superconducting Partial Torus," J. of Applied Physics, July 1977, pp. 3110-15.
7. F.C. Moon, C. Swanson, "Experiments on Buckling and Vibration of Superconducting Coils," J. Applied Mechanics December 1977, July 44, pp. 701-13.
8. F.C. Moon, "Problems in Magnetic-Solid Mechanics," review chapter in Mechanics Today, Vol. 4, 1978, S. Nemat Nassar (Editor), Pergamon Press.
9. F.C. Moon, "Buckling of a Superconducting Ring in a Toroidal Magnetic Field," J. Applied Mechanics, 46, No. 1, pp. 151-155, March 1979.
10. F. C. Moon, "Experiments on Magnetoelastic Buckling in a Superconducting Torus," J. Applied Mechanics, 46, No. 1, pp. 145-150, March 1979, presented at 8th U.S. National Congress of Applied Mechanics.
11. F. C. Moon and B.S. Kim, "Buckling of a Superconducting Energy Storage Shell," Proc. 3rd Engineering Mechanics Conf., Am. Soc. Civil Engr., Austin Texas, Sept. 1979, pp. 69-73.
12. F. C. Moon, "Elastic Vibrations and Waves in a Superconducting Torus," IEEE Trans. Magnetics, Vol. MAG-16, No. 5, Sept. 1980.
13. F. C. Moon and K. Hara, "Elastic Buckling of Superconducting Yin-Yang Magnets for Fusion," Applied Superconductivity Conference, Sept. 1980. IEEE Trans. on Magnetics, Vol. MAG-17, No. 1, Jan. 1981, p. 207-210.

14. F. C. Moon (Editor), Mechanics of Superconducting Structures, ASME Symposium Winter Annual Meeting, November 1980: "Introduction to Superconducting Structures," p. 1-12; "Magnetoelastic Instabilities in Superconducting Structures and Earnshaw's Theorem," p. 77-90, published by ASME, N.Y., AMD, Vol. 41.
15. Yuan, K-Y, F. C. Moon, and J. F. Abel, "Magnetic forces in plates using finite elements," Proc. 3rd Engineering Mechanics Conf., Am. Soc. Civil Engineers, Austin, Texas, Sept. 1979, pp. 730-733.
16. M. A. Morjaria, S. Mukherjee, F.C. Moon, "A Boundary Integral Method for Eddy Current Flow Around Cracks in Thin Plates," Proc. of COMPUMAG, September 1981, IEEE Trans. on Magnetics, Vol. MAG-18, No. 2, March 1982.
17. M. Morjaria, F. C. Moon, S. Mukherjee, "Eddy Currents Around Cracks in Thin Plates Due to a Current Filament," Electric Machines and Electromechanics, Vol. 7, pp. 57-71, 1982.

Reports to AEC/ERDA/DOE 1974-1978

1. F.C. Moon and C. Swanson, "Vibration and Stability of a Set of Superconducting Toroidal Magnets," Progress Report to AEC, Contract No. AT(11-1)-2493.
2. F.C. Moon and C. Swanson, "Vibration and Stability of a Set of Superconducting Toroidal Magnets," Topical Report to AEC, June 1975, Contract No. AT(11-1)-2493.
3. F.C. Moon and C. Swanson, "Elastic Stability and Vibration of Toroidal Magnets for Fusion Reactors," Final Report to Energy Research and Development Administration, September 1975, Report No. C00-2493-3.
4. F.C. Moon, "Lateral Force Problems in Superconducting Magnets," Letter Report to ERDA, February 1976.
5. F.C. Moon, "Magnetoelastic Stability and Vibrations of Superconducting Magnets, " Progress Report to ERDA, Contract No. E(11-1)-2780, May 1976.
6. F.C. Moon and C. Swanson, "Elastic Stability and Vibrations of Superconducting Magnets for Magnetic Fusion Reactors," ERDA Report No. 000-2780-2, October 1976.
7. F.C. Moon, "Lateral Buckling and Vibrations of Superconducting Magnets for Magnetic Fusion Reactors," Progress Report to ERDA, May 1977, Contract No. (EY-76-S-02-2780).
8. F.C. Moon and B.S. Kim, "Elastic Stability and Vibrations of Superconducting Magnets for Magnetic Fusion Reactors," Annual Report to Department of Energy, December 1977.
9. F.C. Moon, "Magnetoelastic Research in Superconducting Magnets for Magnetic Fusion Reactors," Progress Report to Department of Energy, June 1978, Contract No. EY-76-S-02-2780.

Research Lectures

1975 *Energy Research and Development Administration (ERDA), Washington, D.C.
"Magnetoelastic Instabilities in Superconducting Magnets for Fusion Reactors," April 1975

1976 *Argonne National Laboratory, Chicago, Ill.
"Elastic Instabilities in Superconducting Magnets," May 1975.

Brookhaven National Laboratory Conference on Structural Problems in Magnetic Fusion Reactors

"Experiment in Magnetoelastic Buckling of Superconducting Magnets", September 1976

1977 National Science Foundation, Washington, D.C.

"Research in Magneto-Solid Mechanics", May 1977

*ERDA, Washington, D.C.

"Experiments on Buckling of Superconducting Magnets in a 16 Coil Torus", May 1977

*Oak Ridge National Laboratory

"Experiments on Buckling of Superconducting Magnets in a 16 Coil Torus", May 1977

*Laboratory for Plasma Studies, Cornell University

"Magnetoelastic Instabilities in Superconducting Magnets for Magnetic Fusion Reactors", October 1977

Society of Engineering Science Meeting, Lehigh University

"Magnetoelastic Instabilities in Magnetic Fusion Reactors", November 1977

American Society of Mechanical and Engineering Annual Meeting, Atlanta, GA

"Experiments on Buckling and Vibration of Superconducting Coils", November 1977

1978 Society of Natural Philosophy Meeting, Williamsburg, VA

"Magnetoelastic Catastrophes", April 1978

* Invited Lectures.

- 1978 United States National Congress of Theoretical and Applied Mechanics,
Los Angeles
- "Experiments on Magnetoelastic Buckling in a Superconducting
Torus, June 1978
- *Welding Society of Japan, Tokyo, Japan
- "Instabilities in Superconducting Magnets for Fusion",
November 1978
- *University of Japan, Department of Nuclear Engineering, Tokyo,
Japan
- "Magneto-Solid Mechanics Research", November 1978
- 1979 IEEE 1979 INTERMAG Meeting, New York, New York
- "Infrared Techniques for Eddy Current Distributions",
July 1979
- American Society of Civil Engineers, Engineering Mechanics Division,
Austin, Texas
- "Magnetoelastic Buckling of a Superconducting Solenoid for
Energy Storage", September 1979
- 1980 *Massachusetts Institute of Technology, National Magnet Lab
- "Instabilities in Superconducting Magnets for Fusion Reactors",
April 1980
- IEEE 1980 INTERMAG Meeting, Boston, Massachusetts
- "Vibrations and Waves in a Superconducting Torus", April 1980
- Applied Superconductivity Meeting, Arizona
- "Elastic Instabilities in Superconducting Ying-Yang Magnets
for Fusion", September 1980
- American Society of Mechanical Engineers, Chicago, Illinois
Symposium on Mechanics of Superconducting Structures
(organized by F. Moon)
- "Elastic Instabilities in Superconducting Structures and
Earnshaw's Theorem", November 1980

* Invited Lectures.

Figure 1

Figure 1. Photograph of 16 coil superconducting torus for buckling studies; 130 KILOAMP-TURNS MAXIMUM TOTAL CURRENT.

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

Figure 7. Helical Buckling Shape of a Poloidal Coil in a Toroidal Magnetic Field B_T .

The top diagram illustrates the magnetic field configuration in a tokamak. It shows a cross-section of a torus with a central column. The toroidal field magnet is labeled, and the poloidal field magnet is also labeled. The toroidal magnetic field is denoted by B_T , the poloidal current by I_p , and the poloidal magnetic field by B_p . The bottom diagram shows a cross-section of a torus with a central column, labeled with I_p and ϕ .

Figure 8. COMBINED TRANSLATION-PITCH INSTABILITY OF A POLOIDAL COIL IN A TOROIDAL MAGNETIC FIELD

Figure 9

VIBRATION MODES IN A 16 COIL SUPER CONDUCTING TORUS

Figure 10

Figure 11

Figure 12

Figure 13

Figure 14

Figure 15

ELASTIC BUCKLING OF SUPERCONDUCTING YIN-YANG MAGNETS FOR FUSION

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Summary

A small superconducting model of yin-yang coils for the Mirror Fusion Test Facility has exhibited a magnetoelastic buckling instability. The 1:22 model, wound with 150 turns of niobium titanium wire and potted in epoxy, can carry up to $30 \cdot 10^3$ Amp-turns. Rotational motion of the magnets is observed to increase as the current approaches a critical value, while the torsional natural frequency is observed to decrease with current. A prediction of the buckling current based on a theoretical model is found to be higher than the measured values. The phenomenon is similar to that observed for tokamak toroidal field magnets. Structural design implications for Yin-Yang magnets are discussed.

Yin-Yang Magnets for a Mirror Fusion Reactor

Figure 17

FIGURE 4

Comparison of Static, Dynamic , and Calculated Buckling Currents

STATIC	DYNAMIC	CALCULATED
64-74	68-73	109 AMPS/TURN

Figure 6. Eddy current stream lines by boundary integral method for a square plate with a crack ($a/L = 0.5$) in a uniform magnetic field.

Figure 19

Figure 19. Eddy current stream lines in a plate with a crack induced by a parallel current filament.

Table I

Table I

CURRENT LIMITATIONS ON EPR SCALE TOROIDAL FIELDS DUE TO BUCKLING

16 Coil Torus
Geometry

Design Current

$I = 7 \cdot 10^6$

Amps/Coil

Cross Section Stainless Steel Case

<u>Number</u>	<u>Modal Bending</u>	<u>Buckling Current</u>	<u>Buckling Current</u>
<u>of Pin</u>	<u>Stiffness K_0</u>	<u>Clamped at Base</u>	<u>Pinned at Base</u>
<u>Supports</u>	<u>Clamped Base</u>	<u>10^6 amp-turns/coil</u>	<u>10^6 amp-turns/coil</u>
	<u>10^6 N/m</u>		
0	33.3	4.56	**
1	147.7	8.75	4.94
2	780.3	16.4	10.8

TABLE II

Comparison of Theoretical and Experimental Buckling Currents

No. of Coils	Mode	Coil No.	Experimental Values Amps/Turn		Theoretical Values Amps/Turn	
			Static	Dynamic	$\frac{m_o \omega^2}{4L_1}$ (Eqn (7))	With Tension Correction (Eqn 9)
8	1	2	104	109	87.6	-
		3	114		88.5	-
16	1	3	41.8	44.8	46.5	-
		4	42.4	44.5	47.2	-
	2	3	-	117	96.9	107
		4	102-149	117	99.8	111
		7	213	-	-	-
		8	-	153	98.0	110
		9	125-161	-	-	-
	3	7	214	-	-	-
		8	214	-	186	239
		9	291	328	198	266
		10	248		189	246

Table III

Comparison of Buckling Currents from a Single Coil
Ferromagnetic Toroidal Simulator and 16 Coil Super-
conducting Torus.

Number of Pin Supports	Buckling Current (AMPS/TURN)				Theory Table 2
	Single Coil - "Ferro Torus"		16 Coil Torus		
	Static	Dynamic	Static	Dynamic	
0	41.8	45.5	42.2	44.5	47.2
1	145	-	102-149	117-153	111
2	-	265-327	214-248	328	243