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"J-INTEGRAL ELASTIC PLASTIC FRACTURE MECHANICS
EVALUATION OF THE STABILITY OF CRACKS IN
NUCLEAR REACTOR PRESSURE VESSELS"

FINAL REPORT

May 31, 1979 - September 30, 1979

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WASHINGTON UNIVERSITY TECHNOLOGY ASSOCIATES

Prepared for

Sandia Laboratories
Albuquerque, New Mexico
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PREFACE

The work presented in this report was conducted by Washington University Technology Associates (WUTA) for Sandia Laboratories, Albuquerque, with support from DOE/NPD, in fulfillment of one of the goals of Task Action Plan A-11 "Reactor Vessel Material Toughness," Subtask B of the NRC Unresolved Safety Issues Program. A preliminary version of this report was submitted to a group of reviewers selected by Sandia National Laboratories. Their comments and suggestions dealt mainly with either the technical approach or with editorial aspects. Except for a pair of omissions which have been corrected the technical approach was favorably received. Therefore, the present version is basically the same with the addition of some data which was not available at the time the preliminary report was released. From an editorial point of view most of the suggestions made by the reviewers have been taken into account in the preparation of this final version.

Dr. Mario P. Gomez, Professor of Mechanical Engineering, Washington University in Saint Louis, Missouri was the principal investigator on this contract. Dr. David Parks, Assistant Professor of Mechanical Engineering, Massachusetts Institute of Technology and Dr. Robert McMeeking, Assistant Professor of Theoretical and Applied Mechanics, University of Illinois, Urbana, were consultants to the project. They participated decisively, however, in the analysis of the problem, the development of the methodology and in the preparation of this report.

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I. INTRODUCTION

Present ASME-Code fracture methodology is conservatively based on linear elastic fracture mechanics (LEFM). The strict requirement for applicability of LEFM, namely that crack tip plastic zones be suitably small in comparison to crack dimensions, remaining ligament, etc., is generally met in the analysis of cracks in ferritic pressure vessels at temperatures at or below the transition region in curves representing the dependence of material toughness on temperature. At higher temperatures, the significant increase in material toughness often termed "upper shelf" behavior can result in crack tip plastic zones which are of the same order as pertinent specimen or structural dimensions. In such cases, the rationale for LEFM characterization of crack behavior tends to break down.

In order to reduce the uncertainties inherent in the use of LEFM methodology near this perceived limit of its applicability, it is evident that the effects of non-negligible spatial extents of plastic deformation should be accounted for in:

- a) the development of material fracture properties from laboratory tests;
- b) the analysis of cracks in structures such as reactor pressure vessels (RPV); and
- c) the overall methodology whereby material characterization and structural analysis are combined to provide a quantitative assessment of the margin of safety against unstable crack extension.

In developing such an extension of current LEFM-based methodology, it seems appropriate to consider the following experimental observations. Tests on tough RPV steel at upper shelf temperatures indicate that stable plane strain crack growth under increasing imposed deformation can take place following crack initiation.

The value of imposed deformation at which the stability of this ductile tearing mode of crack growth is lost can depend on the compliance of the loading system.

Finally, it would seem highly desirable that any new methodology developed to account for the presence of high toughness and large plastic zones should, in the limit of relatively lower toughness, compatibly interface with presently accepted LEMF-based approaches.

It is felt that an appropriate basis to account for the experimental observations in a manner which could conveniently and compatibly merge with LEFM can be found in elastic plastic fracture mechanics (EPFM) methods based on the J-integral approach (JEPFM). Further arguments for the use of JEPFM methodology are given in the discussion of Section IV.

The proposed approach, as presented in this report, requires for its engineering application the concurrent development of:

- a) experimental J_R curves providing J_{MAT} and dJ_{MAT}/da , measures of upper shelf material resistance to ductile tearing, and their dependencies on amount of crack extension, Δa , and environmental conditions such as level of exposure to neutron irradiation;
- b) analytical procedures to compute the loading parameters J_{APP} and dJ_{APP}/da and their dependencies on crack and structure geometry and material stress-strain relations.

The work herein reported deals mainly with the development of analytical techniques to determine the loading parameters and practical procedures for assessing crack stability by comparison of loading and material parameters.

A general description of the proposed methodology is presented in Section II. Section III presents a detailed discussion of two approaches used to compute the loading parameters. The second one, based on the "line-spring" model originally proposed by Rice, was developed in this investigation to a point where, with

little additional effort, it could be applied in the engineering analysis of cracked RPV's. It is expected that the line-spring model can yield results within 20% of the accuracy of 3-D finite element solutions at up to more than two orders in magnitude reduction in computer costs. The results of this investigation are discussed in Section IV, where some trends concerning the influence of mode of crack growth, as characterized by rate of change of crack shape, on the tendency toward crack instability are also presented. Sections V and VI present the Conclusion and Recommendations respectively. Appendix A discusses the possibility of having, in practice, conditions that tend to favor instability in RPV. Finally, Appendix B analyzes in some detail the formulation of instability in terms of local or global conditions and its practical consequences.

II. NEW METHODOLOGY

The present ASME Code, Sections III and XI, (1)(2) methodology may be too conservative in certain circumstances in that it may underestimate the actual remaining strength, or life, of a cracked pressure vessel under operational or accident conditions since the code does not account for possible increased cracking resistance beyond initiation. By using the present code, it is possible, however, to calculate practical lower bounds of the strength, or life, and upper bounds of the probability of failure. Present methodology is based on LEFM and, therefore, it is a one parameter approach. As long as the current value of $K_I = f(\sigma, \sigma_y, a, \text{geometry}) < K_{IC}$ the structure will be safe. If $K_I \geq K_{IC}$, the structure will fail. In principle the actual dependence of K_I on membrane stress σ , crack length a , and geometry is not relevant to the achievement of a critical, unstable situation. All that matters is the present value of K_I .

Experience, on the other hand, has shown that at temperatures above the upper shelf minimum temperature a crack may grow beyond initiation, in a stable manner, even under plane strain conditions, under increasing values of imposed deformation. This often happens in laboratory-sized specimens with large scale plastic deformation which vitiates the premises of LEFM.

If the problem is, simply, that conditions of plane stress prevail during crack growth, the stability of the structure could be determined by using the R-curve analysis (3)(4)(5). If a thin sheet of ductile material with a crack is loaded, and $K = K(a_{eff})$, where a_{eff} is a plasticity-corrected crack length, is calculated for the increasing loads and crack extensions Δa , a plot of K_R vs Δa can be made as shown in Figure 1. The K_R vs Δa plots are called R-curves and measure the resistance to crack extension as a function of actual, or effective, crack extension. As indicated in Figure 1, it is assumed that the

actual shape of the R-curve does not depend on the initial crack length a and that K_C is the value associated with the point of tangency between the R-curve and the K vs. " a " curve. This analysis makes implicit use of the conditions for stability:

$$\frac{1}{E'} \frac{dK^2}{da} = \frac{dG}{da} \leq \frac{dG_R}{da} \quad (1)$$

where $G = K^2/E'$, $E' = E/(1-\mu^2)$, μ = Poisson ratio, and E = Young's modulus.

On the other hand, if plane strain-like constraint is prevalent, but the in-plane scale of plastic deformation is such that LEFM is no longer applicable, one should turn to the methods of elastic plastic fracture mechanics (EPFM). For reasons to be given in the Discussion, we have chosen a methodology based on the J-integral (JEPFM).

In 1968 J. Rice (6) introduced an integral J , defined by

$$J = \int_{\Gamma} W dy - \vec{T} \cdot \left(\frac{\partial \vec{u}}{\partial x} \right) ds \quad (2)$$

where: Γ = any contour surrounding (counterclockwise) the crack tip, Figure 2.

W = the strain energy density = $\int_0^{\epsilon_{ij}} \sigma_{ij} d\epsilon_{ij}$

$\vec{T}$ = the traction vector defined according to the outward normal n along Γ , $T_i = n_j \sigma_{ji}$

$\vec{u}$ = displacement vector

ds = increment of arc-length along

For nonlinear elastic materials, J is path-independent and when multiplied by crack length increment is equal to the potential energy difference between two identical loaded planar bodies having crack sized differing by da :

$$J = - \frac{1}{B} \frac{\partial U}{\partial a} \quad (3)$$

where U is the potential energy and B is the plate thickness. The concept of J was developed using deformation theory of plasticity (material behavior is assumed non-linear elastic) and $J = J(P, a) = J(\Delta, a)$ where P is the load and Δ is the displacement. The utility of the method for incrementally plastic materials depends on there being predominantly proportional stressing in the vicinity of the crack tip. Hence when there are great deviations from proportional loading, significant unloading, or large amounts of crack growth, the method breaks down.

In small scale yielding

$$J = G = \frac{K^2}{E'} \quad (4)$$

and the critical value of J at the initiation of crack growth is

$$J_{IC} = G_{IC} = \frac{K_{IC}^2}{E'} \quad (5)$$

It has also been shown by Hutchinson (7) and Rice and Rosengren (8) that the intensity of the plastic stress-strain fields near the crack tip are characterized by J .

The potential energy expression for J , Equation (3) provides the analytical basis for an operational determination of J from load displacement records. From the expression:

$$dU = - JBda \quad (6)$$

it is obvious that J can be calculated by integrating the shaded area in Figure 3. Begley and Landes (9) used this approach for experimental J -determination, and showed that J_{IC} values thus obtained from specimens exhibiting large scale plasticity were the same as those G_{IC} values obtained from much larger specimens, fulfilling ASTM E-399 size requirements, which are formulated to insure small scale yielding. It was thus demonstrated that J could be used under certain circumstances to characterize crack growth initiation.

If the early stages of crack growth are analyzed in terms of increasing J vs. Δa it is often possible to distinguish an initial linear region that can be said to correspond to blunting of the original crack tip, Figure 4 (10), up to $J = J_{IC}$. Further increases in J are accompanied by stable crack extension up to a point where, depending on crack and specimen or structural geometry, and on loading conditions, the crack becomes unstable. In analogy to the plane stress R-curves one can experimentally develop J_R curves which are assumed to be characteristic of each material. Figure 5 shows an actual J_R curve obtained with compact tension specimens of ASTM 533-B vessel steel at 93°F (11). The stiff testing conditions caused J to reach values many times J_{IC} without instability. In actual structures, the maximum attainable J value prior to instability will depend on the rate dJ/da of variation of J with a , or Δa , and on whether the slope of the J_R curve is smaller or larger than the corresponding slope of the applied J , J_{APP} .

Schematically, as shown in Figure 6, if a structure has a crack characterized by its depth $a = a_0$ and is subjected to increasing loads $P_1, P_2 \dots$ etc. the crack will grow in a stable manner to $a_1 = a_0 + \Delta a_1$; $a_2 = a_0 + \Delta a_2$ etc. If for every value of a , $a_1, a_2 \dots$ we compute the J associated with a and P , a curve can be drawn, R_I , which represents all possible equilibria between the $J(a)$ applied, J_{APP} , and the $J(\Delta a)$ resistance, J_R (or J material, J_{MAT}). If the experiment is carried out at constant load conditions the $J_{APP} = J_{APP}(P, a)$ will usually exhibit a monotonically increasing dependence on " a " (linear at low loads and short cracks), represented by the lines labeled $P_1, P_2 \dots P_6$. At lower values of $P = P_1, P_2, P_3$ & P_4 the J_{APP} and the J_R curves intersect at the equilibrium values $J_{APP} = J_R$, and in all these cases $dJ_{APP}/da < dJ_R/da$. If the load is increased further, a value of $P = P_5$, $(J_{APP})_5 = (J_R)_5$ will be reached for which $dJ_{APP}/da = dJ_R/da$. This situation is unstable and any further increased in load and/or " a " will lead to

spontaneous crack propagation. The stable equilibrium value of J achievable under these conditions is J_5 . In practice, however, experiments to determine J_R curves are run at constant cross head velocity which, at any given instant, will result in loading conditions more like imposed displacement. The lines labeled $\Delta_1, \Delta_2, \dots, \Delta_7$ represent the $J(\Delta, a)$ dependence of J_{APP} . For illustration of the problem, the lines are shown with a negative slope, although the actual dependence may not be monotonic, as shown for some of the Δ 's. Real structural cases will exhibit somewhat intermediate behavior and the J_{APP} vs. " a " curves may, or may not, have several minima and maxima depending on the geometry.

At the point of incipient unstable crack growth, a critical condition can be defined by an equilibrium condition:

$$J_{APP} = J_{MAT} = J_R \quad (7)$$

and an instability condition:

$$\frac{dJ_{APP}}{da} \geq \frac{dJ_{MAT}}{da} \quad (8)$$

while stability would be assured if:

$$\frac{dJ_{APP}}{da} < \frac{dJ_{MAT}}{da} \quad (9)$$

The second members of these relations are obtained from experimentally determined J_R curves. J and dJ/da can be nondimensionalized as $J_0 - JE/\sigma_y t$ and $T = (dJ/da)E/\sigma_y^2$ where t is a linear dimension associated with the problem and σ_y is the yield strength of the material. The techniques used have been extensively reported and reviewed in the literature (12)(13)(14)(15). Recently, attempts have been made (16)(17) to rationalize the tearing modulus,

T_{MAT} concept by considering near-tip deformations for growing cracks. A tentative conclusion can be drawn that for small enough amounts of crack growth involving sufficiently large J_R slopes, use of the J_R curve can be justified. On the other hand, JEPFM has limitations imposed by the loss of J-dominance* after a certain amount of crack extension, at least when conventional flow theory plasticity analysis is used. This loss of dominance may happen (11) in a CT specimen after about $\Delta a/a_0 = 0.06$ due to the lack of predominantly proportional loading, but, on the other hand, even this small fraction is enough to result in J_{MAT} of the order of 6000 psi. in. $\approx 5 J_{IC}$ in A533B steel. From a practical standpoint loss of J-dominance due to crack growth in testing seems to lead towards apparently lower values of dJ_{MAT}/da , hence resulting in more conservative estimates of the critical conditions for crack propagation. One measure of the degree of J-dominance in a given situation may be judged by the parameter (19):

$$\omega = \frac{b}{J_{App}} \frac{dJ_{MAT}}{da} \quad (10)$$

where b is the remaining ligament size. J-dominance is obtained from conditions such that $\omega \gg 1$. How much larger than one ω should be is still the subject of research. It appears that the higher the degree of constraint at the crack tip the lower the admissible value of ω , i.e. $\omega > 5$ for bend bars and $\omega > 50$ for center cracked panels.

In order to assure J-dominance it is also necessary that certain minimum size conditions, dependent on geometry, be met (18)(20).

*Note: "J-dominance" is used here in the context of Ref. 18, where it was implicitly defined as the degree of J-based characterization of the stress and deformation fields at the crack tip region.

From an engineering point of view it is necessary to establish criteria for generating acceptably conservative J_R curves for the analysis of structural integrity. First, they should be representative of material behavior in the real structure under consideration or, if that were not possible, they should give the lower bound of J_{MAT} and dJ_{MAT}/da . It appears these lower bounds can be obtained by testing CT specimens with face grooves (21). This question remains open, however, because the ductile fracture model developed by Rice et al (16) suggests that the value of dJ_{MAT}/da at initiation ($J_{APP} = J_{MAT} = J_{IC}$) could be somewhat affected by the spatial extent of plastic deformation. In particular, the initial slope of the J_R curve in small scale yielding is suggested by their model to be somewhat smaller than that which would be obtained just after initiation in a smaller, fully plastic bend or compact tension specimen. By making certain assumptions as to the size of the region of dominance of a near tip field in fully plastic specimens, they estimate that the two slopes could differ by roughly $20\sigma_o^2/E$ for a very small fully plastic bend specimen with remaining ligament size $b = 25 J_{IC}/\sigma_o$. The difference is expected to be logarithmically dependent on b , so a somewhat larger remaining ligament of $b = 50 J_{IC}/\sigma_o$ should give slopes closer to that obtained at initiation in small scale yielding. There is also the possibility of other types of differences between small scale yielding and fully plastic J_R curves (16). More work, both experimental and theoretical, needs to be done in order to assure that conservative material J_R curves are to be incorporated into an overall JEPFM methodology. It is encouraging to note, however, that since fully plastic 4TCT specimens of unirradiated A533B give initially rather large slopes of $dJ_{MAT}/da \approx 200\sigma_o^2/E$, the absolute difference noted above represents only a 10 per cent lower slope in the case of small scale yielding. Until further investigation of the problem generates reliable and realistic J_R curves for structural application it seems advisable to use lower bound values. Size effects on J_R

curves are being investigated by Wessel et al (22) under the sponsorship of the EPRI. Second, the effect of irradiation at upper shelf temperatures on the J_R curves, schematically illustrated in Figure 6, should be thoroughly investigated. The limited amount of data available indicates decreases in J_{IC} and in dJ_{MAT}/da , but actual variations as a function of fluences are known only for specimens irradiated for 630 hours in an experimental reactor at a fluence of $1.5 \times 10^{19} \text{ n/cm}^2 > 1 \text{ Mev}$. which may be more severe irradiation than the one to which the surveillance specimens are subjected. These results are discussed further in the next two sections.

II.1 Calculation of J_{APP}

To enable us to implement JEPFM, we must calculate the dependence of J on a and P , or Δ , for the particular geometry, loading condition, and material under consideration.* The present state of the art is such that no realistic and accurate 3D solution for large scale yielding near a surface crack in a plate or shell is available although, in principle, certain of the required information could be obtained from 3D elastic plastic finite element solutions. At this stage, however, we choose to construct idealized models which are basically 2-dimensional, and to make assumptions that we hope will lead to conservative estimates of J_{APP} and dJ_{APP}/da . One such model was formulated

*At this point it should be emphasized that a $J_{APP} = J(a, P)$ relationship should not be construed as if a crack of length a_0 , subjected to a generalized constant load P , may grow indefinitely to any size a_f for which J_{APP} is given by $J(a_f, P)$. This could only be true for limited amounts of growth with J -dominance. The correct interpretation of the expression above is that if the crack has grown initially to a size a_0 by fatigue at lower loads, such that there is only a small plastic zone at the crack tip, and then the load is raised to a value P , with small crack growth Δa , the corresponding value of J_{APP} is $J(a_0 + \Delta a, P)$. If the structure is unloaded again to low values of the load, where LEFM applies, and the crack grows again by fatigue, or stress corrosion to a new value a_f , and the load is once more increased to P , the value of J_{APP} is $J(a_f + \Delta a, P)$.

by Zahoor et al. (23) who modeled a part-through thickness (P.T.T.) crack of depth a , in a plate or shell of thickness t , by a through thickness (T.T.) crack subjected to closure stresses on the free surfaces of the crack equal to $\sigma_o(1 - \frac{a}{t})$. The crack opening displacement (C.O.D.), δ_T , of the P.T.T. crack is considered equal to the crack opening displacement δ_1 at the center of the T.T. crack. J_{APP} is then taken as $J_{APP} = m\sigma_o\delta_T$ where m is a factor of order one.

Another possibility is to attempt to find an upper bound solution (with respect to surface cracks of finite length and same maximum depth) by finite element modeling of the cylinder, using computer codes with elastic plastic capabilities, and calculating J_{APP} for a longitudinal edge crack of constant depth a . The effects of constraints like variable uncracked length and shell bottom, can be conveniently simulated. This method has been successfully used by B. Szabo (24) to calculate J_{APP} and dJ_{APP}/da for nozzle cracks.

In our study we used two other models to obtain results for surface cracks subject to large scale yielding. One of them was developed by Erdogan, Irwin and Ratwani (EIR)(25) who solved the problem of a meridional rectangular crack, (PTT, TT or embedded) in a shallow cylindrical vessel wall, with the ligaments fully yielded and Dugdale plastic strips beside the crack ends. Their results are expressed in terms of crack tip opening displacements, which can also be converted to J_{APP} 's. These values should tend to be conservative as has been often found the case when ideal plastic constitutive laws are used. The contrary seems to be true for dJ_{APP}/da which ideal plastic behavior tends to underestimate. This latter point is illustrated by the results (24) shown in Figure 7. The EIR model was later refined by Krenk (26) who used a higher order shell theory.

The other model that was used is the so called line-spring model developed by Rice and Levy (27), who simulated a P.T.T. crack, in a plate or shell by a line spring of variable spring

constant. Although Rice and Levy used only a spring of linear elastic behavior, Rice (28) indicated how the method could be extended to an elastic analysis with thermal or residual stresses, and to an elastic-plastic line spring for large scale yielding. More details of these methods and sample calculations will be presented in the next section.

III. CALCULATIONS OF J_{APP} FOR PRESSURE VESSELS

As was indicated in the previous section, the analysis of the integrity of pressure vessels from a crack propagation standpoint, can be reduced to the analysis of the stability of cracks within the range of operational and conceivable accident conditions, taking into consideration stress gradients, geometrical factors, such as crack shape and location, vessel diameter and thickness, and material behavior and related properties.

The geometry of the vessel at different locations will be an important factor in determining local stress distribution and intensities. Finite element methods are being successfully used for accurate stress analysis even when the geometry of the vessel is rather complex and the material response is non-linear. However, it is unlikely that sufficient 3D finite element analyses could be performed, due to cost, to provide the required data on J_{APP} for surface cracks. Certainly these results are not available currently and we could not generate them. We thus sought results available from methods of analysis less accurate than 3D finite element calculations.

There were two methods reported in the literature in the early 70's which seem to have promise in regard to the analysis of surface cracks in plates and shells. These methods, due to Erdogan et al. (25,29) and to Rice (27,28), which will be summarily described in the next two sections, were used to estimate J_{APP} and T_{APP} for a wide range of crack sizes, under gross hoop stresses up to 90% σ_y , and different modes of crack growth. The results from both methods show good agreement.

A simpler method, proposed by Zahoor et al. (23) models the surface flaw by a through the thickness crack in a plate subject to both the applied (opening) stress and to tractions on the crack surface, tending to close the crack, of magnitude.

$$\sigma' = \sigma_0(1 - a/t) \quad (11)$$

where σ_o is taken as a flow stress average between the yield stress, σ_y , and the ultimate stress, σ_u , a is the crack depth and t is the vessel thickness. The length of the model through crack is $2c$, the length of the surface crack at the surface. The crack opening δ_T for the surface crack is calculated by assuming it is equal to δ_1 , the opening displacement at the center of the model through crack. The J_{APP} is then calculated with the formula:

$$J \approx \sigma_o \delta_T = \frac{4c}{E} \left[\sigma_o \sigma - \sigma_o \left(1 - \frac{a}{t}\right) \right] \quad (12)$$

and
$$T_{APP} = M \left(\frac{4c}{t} \right) \left[1 + \frac{t}{\sigma_o} \left(\frac{d\sigma}{da} \right) \right] \quad (13)$$

where M is a factor of order unity which accounts for shell effects (30). Zahoor et al. (23) had some success in correlating the crack behavior in two of the HSST intermediate test vessels (V-7 and V-1). Although the assumptions made in their analysis were intended to be conservative, a comparison of results obtained using their method with results generated from other models suggests that the Zahoor et al. model may not always be conservative, as is discussed in the next section.

Another analysis method for which results are currently available is a 2D finite element calculation for a part-through crack in a cylinder performed by Szabo et al. (24). Their results were obtained for deep edge cracks and are not directly relevant to our studies because the thickness to inside radius ratio was two, whereas our results are for vessels with this ratio about equal to ten. Furthermore, edge cracks may be considered as a limiting case of very long surface cracks which may yield upper bound values for J_{APP} but not necessarily for T_{APP} , since the latter is very sensitive to how the shape of the crack changes, as will be discussed below. On the other hand, Szabo's results were very useful for showing the importance

of material hardening in the analysis, especially for T_{APP} calculation. The assumption of perfect plasticity in a strip yield model tends to underestimate T_{APP} , as compared to strain hardening finite element solutions, as shown in Fig. 7, even though the non-hardening strip yield model gave generally good results for $J_{APP}(p,a)$.

III.1 Calculations for Surface Cracks in Plates and Shells Using the EIR Model

Erodgan, Irwin and Ratwani (25,29) analyzed the problem of a rectangular surface crack in a pressure vessel or a plate made from elastic, perfectly plastic material. The flaws they studied were part-through the thickness internal or external, cracks, through the wall thickness cracks, and cracks totally embedded in the vessel wall. The flaw dimension, the yield stress of the material and the loading are assumed to be such that in the neighborhood of the flaw the cylinder wall undergoes large-scale yielding. The cylinders are assumed to be thin-walled and of radius R . The crack is assumed to be in a meridional plane. The problem was solved by modelling the plastic zones as strips so that the net ligament of length $2c$ and of thickness $t-a$ carried only a membrane stress of magnitude σ_0 . There were also Barenblatt-Dugdale type yield zones of length l beside the cracks. Eighth-order elastic thin shallow shell theory was used to model the vessel. The flow stress σ_0 was taken as

$$\sigma_0 = (1 + n)\sigma_y \quad (14)$$

where σ_y is the tensile yield stress and n depends on the strain hardening of the material. Since bending effects are small compared to membrane stresses, the yield condition for the strip yield zone could be expressed by a linearized approximation to

the parabola represented by its tangent at the point $M = 0$, $N = \sigma_0 t$:

$$\frac{N}{t\sigma_0} + \frac{2|M|}{t^2\sigma_0} = 1 \quad (15)$$

where M and N are the moment and membrane stress resultants, respectively.

The shell problem is solved by adding the homogeneous solution given by $N = N_0 = Rp_0$ (with all other stress resultants equal to zero) to the perturbation solution obtained from the yield zone surface tractions. At the Dugdale zones the solution must meet the yield condition and the condition that stress resultants be finite at their tips. This leads to a singular equation which Erdogan et al. (25,29) solved by a collocation procedure.

Once the problem is solved the crack tip opening displacement δ_T may be obtained from

$$\delta(x, 0) = v(x, y=0^+) - v(x, y=0^-) \quad (16)$$

$$\theta_2(x) = 2 \frac{\partial}{\partial y} w(x, 0) \quad (|x| < c+l) \quad (17)$$

and, with $z \equiv a - t/2$

$$\delta_T(x, a - \frac{t}{2}) = \delta_T(x, z) = \delta(x, 0)(1 + \frac{z}{R}) + z\theta_2(x), \quad (18)$$

where x is the position along the crack front, y is measured normal to the crack surface plane, and z is, locally, the through the thickness direction. The center of the crack and shell-wall/plate is at $y = 0$, $z = 0$. The components of shell/plate mid-surface displacement are u , v , and w in the x , y , and z directions, respectively.

The data resulting from the calculations are given as δ_0/d_1 , δ_c/d_1 and θ_2/d_2 as functions of $N/t\sigma_0$ for different relative

crack sizes a/t and positions in the wall. δ_o is the COD at the wall center $\delta_o = \delta_T(0,0)$. σ_c is the COD for a through crack at the tip of the actual crack, $\delta_T(c,0)$. $\theta_2(x)$ is the relative crack surface rotation. The normalizing factors d_1 and d_2 are:

$$\begin{aligned} d_1 &= \frac{4c \sigma_o}{E} \\ d_2 &= \frac{4c \sigma_o}{tE} \end{aligned} \quad (19)$$

The data are given for each configuration as functions of the shell parameter

$$\lambda = \left[12(1-\mu^2) \right]^{1/4} \frac{c}{\sqrt{Rt}} \quad (20)$$

which for a flat plate ($R \rightarrow \infty$) is equal to zero.

More recently Krenk (20) repeated Erdogan's calculations using tenth order shell theory to account for the effects of the transverse shear deformations. His results are not very significantly different from Erdogan's results, especially in the context of our analysis, and will not be discussed any further in this report.

III.1.1 Results of Calculation of J_{APP} and T_{APP} Using the EIR Model

The model proposed by Erdogan and Irwin permits the estimation of J_{APP} and T_{APP} over wide ranges of crack sizes and loadings. On the other hand, it is only possible to compute results for loads sufficient to yield the ligament fully and the results end abruptly at a lower value of $\sigma_{NET} = \sigma / (1 - \frac{a}{f}) = \sigma_o$. Sources of inaccuracy in the way we have used the results are the conversion of δ_T into $J_{APP} = m\delta_T\sigma_o$ where m is a factor that depends on the extent and nature of the yield zone on which it is based. The m value is 1 for the contained Dugdale model, 1.15 for fully

plastic ligaments at edge cracks in tension and, say, 1.67 for small scale diffuse yielding, all in plane strain. There is also the possibility that the T_{APP} values obtained may not be conservative as is often found to be the case when perfectly plastic strip yield models are used.

For the calculations based on the E.I.R analysis, which are presented in this report, m was assumed equal to one, the radius of the vessel equal to 90 in., the thickness equal to 9 in., and $\sigma_y = \sigma_o (n=0)$.

Figure 8 shows the dependence of J_{APP} on membrane stress σ for cracks in a flat plate from $a = 2.7$ in. ($a/t = 0.3$) to $a = 7.2$ in. ($a/t = 0.8$) but with a constant length $2c = 54$ in. Two commonly used values of $K_{IR} = 170$, and $220 \text{ ksi} \sqrt{\text{in.}}$ are also indicated as well as the operating membrane stress in a reactor pressure vessel given by $\sigma_{op} = (R/t)p_{op} = 10 p_{op}$ with p_{op} , the operating pressure as 2250 psi. From Figure 8 it is seen that at σ_{op} cracks in a plate, up to 7.2 in. deep, are safe (for $K_{IR} = 220 \text{ ksi} \sqrt{\text{in.}}$) if for the time being it is assumed that T_{APP} is less than T_{MAT} . At twice the operating membrane stress, cracks up to 3.6 in. deep by 54 in. long are similarly found to be safe.

Since it may be objected that it is not realistic to analyze all cracks with the same assumed length at the surface, another set of calculations was made using homologous shape cracks with $2c = 6a$, similar to the ASME Code suggested shape. Figure 9 shows the results where shallower cracks in a flat plate differ more significantly from deep ones compared to the relationship shown in Figure 8. In the example in Figure 9 the safe cracks at the operating stress are practically of the same depth as in the previous example. However at twice the operating membrane stress, such cracks as deep as 4.5 in. could be safely tolerated. In plates at membrane stress as large as 60 ksi, an $a = 2.7$ in. deep crack would similarly appear to be safe. Figure 9 also shows curves for internal and external cracks 3.6 in. deep in a cylindrical shell of radius $R = 90$ in., thickness $t = 9$ in.

and $\lambda = 0.69$. Two of the curves were estimated using the EIR model (25) and the third curve, which is the same for external and internal cracks, was calculated using the Zahoor et al. model.

The effects of shell curvature are also shown in Figure 10. Calculations were made for two internal axial cracks 4.5 in deep ($a/t = 0.5$) but of different length 27 and 54 in. respectively, in a cylindrical shell with $R = 90$ in. The results are compared with the flat plate calculations shown in Figures 8 and 9. The importance of shell curvature is very significant at intermediate stresses and extremely important at higher ones $\sigma > .8\sigma_0$. However, despite the increased values of J_{App} , (even at $p = 3500$ psi, $\sigma = 35$ ksi) a 4.5 in. deep crack in the shell would be safe (if $2c < 54$ in.) even with the very restrictive criterion $K_{IC} = 220 \text{ ksi}\sqrt{\text{in.}}$. The effects on other crack depths and shapes is similar. Results from the model used by Zahoor et al (23) are also plotted for two cracks 4.5 in. deep ($a/t = 0.5$) and 27 and 54 in. long, respectively, in both a plate and a cylindrical shell of radius 90 in. In all cases the thickness was assumed to be 9 in. It would appear that the rather more detailed EIR model tends to give J_{App} estimates somewhat greater than those of the Zahoor et al. model.

In order to estimate T_{App} from the EIR model, the data was replotted as J_{App} vs. a for membrane stress held constant at values between 0.4 and 0.9 σ_0 . The yield stress σ_0 was taken to be 70 ksi, which is fairly representative of reactor pressure vessel steels. Figure 11 shows J_{App} vs. a , in plates for homologous cracks ($2c = 6a$). For comparison, Figure 11 also includes the J_{App} vs. a for a crack of constant length $2c = 54$ in. at $\sigma = 63$ ksi. As far as J_{App} is concerned this illustrates very well the significant effect of crack aspect ratio. Figure 11 also includes two curves calculated using the EIR model for internal and external cracks, respectively, in a shell of radius $R = 90$ in. and thickness $t = 9$ in., for a membrane stress $\sigma =$

49 ksi. Also for the same stress level a curve is shown which was computed using the Zahoor et al. model (23). In this latter case a single curve corresponds to external and internal cracks.

From the curves in Figure 11 the values of $(\partial J / \partial a)_\sigma$ were measured and plotted in Figure 12. The curves shown are for homologous cracks and one, at $\sigma = 63$ ksi, for a $2c = 54$ in. = constant family of cracks. The latter crack family exhibits, as expected, a linear "a-dependence" and therefore a constant T until large depths are reached. It is seen that, at least for flat panels, under most conditions $T_{APP} < 10$ except at the higher loads ($\sigma > 49$ ksi ($0.7\sigma_0$)) or larger cracks, $a > 6.3$ in. ($a/t > 0.7$). The shell and crack location effects are also illustrated in Figure 12 by two curves obtained using the EIR model (25) for a membrane stress $\sigma = 49$ ksi. A curve using the Zahoor et al. model is also shown. In this case, because $\sigma = \text{constant}$ equation (13) becomes

$$T_{APP} = M \frac{4c}{t} = M \frac{12a}{t} \quad (21)$$

and a single curve represents, using Zahoor's model, the $T = T_{(a)}$ relation, regardless of stress level or whether the crack is internal or external.

The results obtained from the analysis of flat plates for a homologous family of cracks has been plotted as T_{APP} vs. J_{APP} in Figure 13. For illustration the plot includes the boundaries of the stable region defined by the slope of the blunting line, J_{IC} , T_{MAT} , and $(J_{MAT})_{max}$ established from a 4TCT specimen of unirradiated A553B tested by G.E. at 93°C , with 25% face grooves (11). The value of $(J_{MAT})_{max}$ is the maximum value of J estimated from the experimental data. Figure 13 also shows how information concerning the limits of applicability of the J-integral methodology can be conveyed. In this case the size requirement $b > 25 J/\sigma_y$ and the $\omega = (b/J)(dJ/da)$ parameter requirement, $\omega \gg 1$, are illustrated.

In Figures 14 and 15, similar to Figure 13, data on (J_{MAT} , T_{MAT}) for irradiated specimens (31) have been included to define the stable region. In Figure 14, T_{APP} vs. J_{APP} data for the homologous cracks ($2c = 6a$, $\Delta 2c = 6\Delta a$) with $a = 5.4$ in. have been plotted for four membrane stresses. The effect of rate of length growth at the surface, $\Delta 2c$, is illustrated by points corresponding to $\Delta 2c = 12\Delta a$ and $\Delta 2c = 0$. The observed trend towards higher T 's, due to increase rate of loss of constraint associated with higher $\Delta 2c/\Delta a$ ratios, has important implications in structures like RPV where stress and property gradients tend to favor higher ratios (APPENDIX A). Figure 14 also shows a curve for an external crack in a shell, for $a = 5.4$ in., which can be compared with the flat plate data. Figure 15 shows the curves for T_{APP} vs. J_{APP} for internal and external 3.6 in. deep cracks in a cylindrical shell of radius $R = 90.0$ in. and thickness $t = 9.0$ in. obtained using the EIR and the Zahoor et al. models. We can point out two trends in the data shown in Figures 13, 14 and 15. First, it is worth noting that all the data from the family of homologous cracks in flat plates falls within a rather narrow band (even some data that were not included for clarity) extending in Figure 13, from $T_{APP} = 2$, $J_{APP} = 45$ psi in. to $T_{APP} = 20$, $J_{APP} = 8000$ psi in., and including data from crack depths $a = 2.7$ in. to 7.2 in. and membrane stresses from 28 to 63 ksi. The same trend is observed in the data for external cracks in shells in Figure 15. The second trend is illustrated by the data for external cracks which shows that for the same stress level and crack depth, T_{APP} increases with shell curvature. Data on internal cracks is still insufficient to draw conclusions beyond the fact that, for given a and σ , the (T_{APP} , J_{APP}) points fall above those of the flat plate data.

III.2 Line-Spring Model Calculations

The present calculations are based on the "line-spring" model of Rice and Levy (27,28,32). The essential idea of the line spring is that the presence of a part-through surface crack

in a thin plate or shell introduces an increased compliance of the body. Physically, this increased compliance manifests itself as an additional "cracked" extension δ and rotation θ of the shell/plate middle surface. Although this additional deformation is typically accommodated over a distance normal to the crack plane of a few plate thicknesses, in the line-spring model this additional deformation is lumped onto the line discontinuity of the shell surface.

It is required that the local values of $\delta(x)$ and $\theta(x)$, where x is a spatial coordinate along the discontinuity, be suitably related to the local axial force, N , and bending moment, M , per unit distance along the crack. In the linear elastic regime, these relations can be expressed as

$$\begin{bmatrix} M(x) \\ N(x) \end{bmatrix} = \begin{bmatrix} E_{11}(x) & E_{12}(x) \\ E_{21}(x) & E_{22}(x) \end{bmatrix} \begin{bmatrix} \theta(x) \\ \delta(x) \end{bmatrix} \quad (22)$$

where the stiffness matrix E depends on plate thickness, elastic constants, and the relative crack depth at location x (27,28,32).

Although to date, line-spring calculations have been performed mainly for a surface crack in a large elastic plate, the agreement between its predictions for K_I and those of recent 3D finite element (33) and boundary integral equation (34) analyses is generally good, especially considering the one or two orders of magnitude of computing costs by which they differ. Figure 16 shows a comparison of Raju and Newman's results (33) with calculations performed by Parks (35) using the line spring model. The agreement is quite good, especially considering the one or two orders of magnitude of computing costs by which they differ. The values of K_I agree to within 2 to 5% all along the crack front for crack depths to thickness ratios between 0.2 and 0.8, and $a/c = 0.2$.

We have incorporated Rice's (28) suggestions for developing an elastic-plastic line spring, based upon a nonhardening material model. Following Rice (28), at each coordinate x along the crack projection, a slice is made perpendicular to the plate middle surface. In cross-section the crack is as the single edge crack, with width t and crack length a . A yield surface $\phi(N, M, a, t) = 0$ for the generalized stresses is constructed from slip line analysis of the single edge crack geometry, and the incremental form of eq. (22) is used:

$$\begin{bmatrix} \dot{M} \\ \dot{N} \end{bmatrix} = [E^{el-pl}] \begin{bmatrix} \dot{\phi} \\ \dot{\delta} \end{bmatrix} \quad (23)$$

$E_{ij}^{el-pl} = E_{ij}$ for elastic response and

$$E_{ij}^{el-pl} = E_{ij} - \frac{\phi_{,m} E_{mi} E_{jk} \phi_{,k}}{\phi_{,p} E_{pq} \phi_{,q}} \quad (24)$$

for plastic loading. The components of the normal to the yield surface are $\phi_{,1} = \partial\phi/\partial M$ and $\phi_{,2} = \partial\phi/\partial N$.

Calculations were performed for a part-through crack in an (otherwise) elastic plate subjected to a farfield pure membrane stress. For this configuration the problem can be reduced to the solution of a pair of coupled integral equations along the crack (35).

These equations govern a specific load increment and the entire loading history is accomplished by solving a series of load increments, updating the generalized stress and deformation resultant after each load increment.

It should be noted that the elastic-plastic line spring model could be incorporated into a finite element shell/plate program for analysis of rather general surface crack configurations. For the present, however, attention was focussed on

the large plate which is elastic except for near crack front yielding because of the simplicity with which it could be investigated using singular integral equations, and the time and manpower limitations of the program leading to this report.

We have obtained estimates of midpoint crack tip opening displacement and J integral for various crack geometries. In every case, the plate thickness t was taken as 9 in. and the tensile yield stress σ_o was 70 ksi. A Mises criterion gives the shear yield stress τ_o used in constructing the slip-line yield surface as $\tau_o = \sigma_o / \sqrt{3}$.

In the linear elastic regime, the line spring results are close to those given previously by Rice and Levy (27). In the elastic-plastic regime, a J value at the midpoint of the crack was inferred in the following indirect manner. First, J was decomposed into the sum of an "elastic" and a "plastic" part. J_{elastic} was taken as K_I^2 / E' , where $K_I = \sigma f(a, t, c)$ is the linear elastic line-spring calibration for stress intensity factor. Thus, J_{elastic} , so defined, increases quadratically with loading. The plastic part of J was taken as a pure number, m , times the product of σ_o and the plastic crack tip opening displacement δ_t^P ;

$$J_{\text{plastic}} = m \sigma_o \delta_t^P \quad (25)$$

$$J = J_{\text{elastic}} + J_{\text{plastic}} \quad (26)$$

$$J = \frac{\sigma_o^2 f^2}{E'} + m \sigma_o \delta_t^P \quad (27)$$

The plastic part of the crack tip opening displacement increment was determined from the plastic parts of the increments of θ and δ using the kinematics of the slip line fields from which the yield surface was constructed. From experience in two dimensional problems, we expect that the appropriate value of the scalar m should depend on the geometry of the deformation

field. In the figures here, we took $m = 2/\sqrt{3}$ because our numerical results, to be discussed below, suggest that the generalized stress state along the remaining ligament tends toward the membrane state of mid-ligament loading of the plane strain edge crack geometry. Of course, this drift may well mean that the crack tip characterizing property of J as strength of an HRR singularity is breaking down (18)(20). Nonetheless, in the lower constraint configurations it is experimentally found that notional J resistance curves show considerably steeper slopes than in higher constraint, bend or compact geometries (36). In view of the simplicity of the present model, however, we have calculated what we believe to be conservative estimates of J . We may note, for example, that the idea of adding an "elastic" J which continues to increase quadratically with increasing load parameter may be appropriate in 2D configurations where load continues to rise after first yield due to strain hardening. In the present case, however, the increase in load after first yield is principally due to a transfer of load distribution to the larger ligaments toward the crack ends, as the yielded zone spreads from the center plane. Load does not dramatically increase at the center plastic ligament once it has yielded. In fact, if we take the local N, M values which evolve as plasticity spreads, and turn them into a local " K_I^* " by formally using the edge cracked stress intensity factor calibrations, it turns out that this local "elastic" K_I^* , or J , actually decreases, since M becomes much more negative while N increases very slightly. Consequently, we feel that our J results will generally be conservative estimates.

A portion of the normalized yield surface for relative crack depth $a/t = 0.5$ has been drawn in Figure 17, along with some of the mid-ligament data for the semielliptical surface crack of length $2c = 4t = 36$ " and maximum depth $a_{\max} = \frac{1}{2} t = 4.5$ ". The linear elastic solution comes to yield at the point $(-.25, .88)$,

and a membrane stress of $\sigma = 43.0$ ksi. As can be seen, the subsequent trajectory along the yield surface is heading toward the vertex at the point $(-.5, 1.)$, which, as Rice has noted, corresponds to mid-ligament loading of the edge crack. The change in the components of the outward normal, which are proportional to $\dot{\theta}_p$ and $\dot{\delta}_p$, is quite substantial.

The results of these calculations span the linear elastic analysis of surface flawed plates through the fully plastic strip yield results of Erdogan and Irwin. Indeed, the agreement with the crack opening displacements in their membrane model in the flat plate limit and our results is quite good. In fact, our more general model tends, in the fully plastic regime, to the membrane stress state which they assumed from the outset. This favorable comparison gives additional confidence in the J values for shells which are inferred.

Before closing this section, some observations should be made about the model used. First, the line spring itself should be most appropriate for large aspect ratio surface cracks, that is $2c \gg a$. Experience in the linear elastic range suggests that, for predominantly tensile loading, good results can be obtained for moderate aspect ratios of $2c/a$ of order 5 or so. In the plastic regime, the propriety of using the model for lower aspect ratios is undetermined. We emphasize, with Rice (28), that a shortcoming of the model as used is the neglect of contained plastic yielding prior to reaching the slipline yield surface, although for the problems investigated here, use of such features as a plasticity-adjusted crack length such as $a + \frac{1}{2\pi} (K_I/\sigma_o)^2$ for contained yielding could probably be accommodated.

Another important shortcoming of the model is the neglect of yielding at the points where the crack front intersects the free surface. Again, Dugdale zones as "uncracked" line-springs could be incorporated here as in the Erdogan and Irwin analysis.

The drift towards loss of triaxiality indicated here is an important topic which should be investigated further. It is likely that an isotropic hardening version of the yield surface, presently under investigation, would tend to suppress this drift. Also, it would seem likely that small applied positive bending loads would be very effective in this regard as well. Finally in view of the important consequences for crack stability analysis which may accompany such a loss of triaxiality in these structural applications, (36), it would seem highly desirable to conduct detailed 3D elastic-plastic analysis of some configurations to compare J and δ_T estimates and to look for evidence of triaxiality loss.

IV. DISCUSSION

The basic question addressed by this project was how to extend the present LEFM-based ASME Code methodology for RPV design and integrity assessment incorporating the research advances of the last ten years.

We believe that, despite its limitations, a viable method could be found in the use of Elastic-Plastic Fracture Mechanics (EPFM) analyses based on J-integral concepts (JEPFM). Another method which may be considered is the COD approach (37), but we believe that currently no other methods are sufficiently well documented theoretically, computationally, or experimentally, to justify the development of an associated design and analysis procedure.

The J-integral has a firm theoretical basis in view of the characterizing role of J in crack tip stress and deformation fields. Indeed, a similar characterizing role for K_I in linear elastic situations is the sound theoretical basis upon which linear elastic fracture mechanics has been included in current Code methodology. We note that much of the J method can be readily converted into a COD method which may extend the range of applicability of the approach.

The applicability of the J method is restricted to cases where the amount of crack growth is sufficiently small that not too much non-proportional stressing occurs in the near crack tip region. In addition, there may be not yet fully resolved questions concerning the application of a method developed and tested for through cracks in planar specimens to problems of part-through surface cracks.

It should be emphasized, however, that limitations to applicability are not unique to the J-integral method. They may be more conspicuous in this case because they have been studied to some extent, especially in the last few years.

We believe that significant advances can be made in the use of JEPFM concepts and methods to make realistic assessments of reactor pressure vessel integrity in circumstances beyond the limits of applicability of LEFM. LEFM can, and for the time being should, be used to deal with design problems where the temperature of the material is at or below the lower upper shelf temperature, since the intervention of a brittle cleavage-type fracture cannot then be ruled out.

At upper shelf temperatures where the material used in reactor pressure vessels exhibits very high toughness levels the LEFM methodology may be overly conservative and may lead to the unjustified decommissioning of very sound vessels. Alternatively, LEFM methodology may convey a sense of marginal safety when in reality the margin, as assessed by a more realistic methodology, may be even higher than the one estimated using LEFM calculations when the reactor started its operation. These problems may become acute in the near future if larger cracks are discovered by increasingly better nondestructive (N.D.) inspection methods, or if toughness, as measured by CVN, or K_{IC} , may fall, due to irradiation, below recommended levels.

IV.1 Calculations of J_{APP} and T_{APP}

The calculations of J_{APP} and T_{APP} performed for this project are not yet acceptable for immediate engineering use though they are representative of the most advanced state of the art. We believe that the levels of J_{APP} calculated are reasonably accurate for the limited number of configurations analyzed. However, more realistic calculations should be performed so that the effects of material hardening, and shell and crack geometry can be assessed. The calculations of T_{APP} on the other hand, may have yielded values too low compared to those that might be found in practice and/or computed with material hardening models. Again, material hardening, shell geometry, crack geometry and its rate of change, should be thoroughly investigated since T_{APP} seems to be much more sensitive than J_{APP} to changes in any of these parameters.

IV.1.1 Present Results

The results presented in this report cover ranges of membrane stress from 14 ksi to 63 ksi and crack depths a from 2.7 in. to 7.2 in. for a wall thickness t of 9 in. and an assumed flow stress $\sigma_0 = 70$ ksi. Calculations have been made mostly for flat plates. However, values for three shell parameters $\lambda = 0.69, 0.86$ and 1.71 with $a/t = 0.4$ and 0.5 have been computed. Two families of cracks have been analyzed: one of them exhibited a constant $2c = 54$ in. and varying depth a , and the other consisted of homologous cracks of $2c = 6a$.

The methods proposed by Erdogan and Irwin (25) and by Rice (28) which were applied and/or extended in this project gave good and very consistent results. The second in particular is very promising because it can provide a continuous transition from reasonable LEFM results (35) to the EPFM domain.

Figure 18 shows the results of calculations made using the line-spring with perfect plastic behavior (solid lines) and the calculation for same crack lengths and stresses using the EIR model for cracks with $2c = 54$ in. The agreement is remarkable. With some simple modification, it would seem that the line spring model could account for plasticity-corrected effective crack length and strain hardening in the fully plastic range (35). The curves in this case should show a smoother transition between both regimes. Figure 19 shows calculations using the line spring model for cracks with $2c = 36$ in. The only comparable curve using the EIR model for $2c = 32.4$ in. is also plotted. The agreement is very good.

The results obtained can also be used to illustrate the effects of crack shape and aspect ratio on T_{App} . From Figures 11 and 12 one can see that when $a = 5.4$ in. deep ($a/t = 0.6$) at 63 ksi, cracks of $2c = 54$ in. and 27 in. have $J_{App} = 13300$ and 8200 psi. in. while the $T_{App} = 14.5$ and 20, respectively. On the other hand, one can compare two cracks of equal depth $a = 5.4$ in. and base $2c = 32.4$ in., but belonging to two different

families: one family consists of homologous shapes $2c = 6a$, $\Delta 2c = 6\Delta a$, while a second consists of those of constant base $2c = 32.4$ in., $\Delta 2c = 0$, but varying depth a . At $\sigma = 63$ ksi, the J_{APP} are the same but the T_{APP} for the homologous family is 20 while for the constant base family it is only 9. Furthermore, if the same crack is considered as belonging to a family where the rate of growth at the free surface is twice the rate of the homologous family, $\Delta 2c = 12\Delta a$, $T_{APP} = 30$. It is conceivable that such surface growth situation could occur at belt line regions due to the stress gradients and irradiation, APPENDIX A. The two points for this last family and stress level and two more for $\sigma = 35000$ psi are shown in Figure 14.

In practice, this point deserves careful attention lest misleading conclusions be drawn. For a given shape the rate of change of the shape as a measure of the rate of loss of constraint should be carefully accounted for in the analysis of stability. It is also mandatory to determine whether local or global conditions of instability control the overall stability, as discussed in Appendix B.

IV.2 Application of the New Methodology

From the analyses and results presented and discussed in this report, one may speculate as to how to extend the present ASME methodology. Since current ASME methodology can be considered as a special case of the J-integral method, the advantages of a J-integral approach are obvious in this regard.

It appears that one possible first step would be to relax, after careful evaluation of all available data, the strict limitations on K_{IR} values allowable for computation of critical crack sizes above upper shelf temperatures since in this range of temperatures K_{IR} (or K_{IC}) is not a very meaningful parameter and cannot be very precisely determined.

A second step towards extension of current methods would be to assess the integrity of a RPV, in the spirit of ASME Code Section XI, using realistic J_R curves including irradiation effects, and calculating J_{APP} and T_{APP} for a wide range of crack sizes, shapes, and rate of change of shape, using realistic solutions as developed in this report but including hardening and actual cracked shell geometries.

Figures 13, 14, and 15 show how the new methodology could be applied for parametric analyses of crack problems. Figure 13, for instance, shows data points from unirradiated A-533B steel obtained from 25% face grooved compact specimens at 93°C. Assuming that these data are representative of actual structural behavior, the region enclosed by them defines all possible stable combinations of T_{APP} and J_{APP} . To illustrate this point we have also included a series of curves (T_{APP} vs. J_{APP}) for flat plates. Furthermore, the possible effects of irradiation on crack stability are shown in Figures 14 and 15, where data from A-533B steel specimens, irradiated to 1.5×10^{19} n/cm² > 1 Mev. in an experimental reactor for 630 hours at 288°C. According to F. Loss (31) the J_R -curves from which these data were obtained exhibited power law behavior, ($J_R = c\Delta a^n$), hence the regions of stability will be bounded by lines through the data points, from the top left side to the bottom right of Figures 14 and 15. Two bands of data points exhibiting such trend are shown for irradiated (I), irradiated-annealedirradiated (AR), and irradiated-annealed-irradiated-annealed-irradiated (ARAR) specimens. Each additional irradiation after annealing consisted of a fluence of 0.7×10^{19} n/cm² > 1 Mev. In Figure 14 we have included also T_{APP} vs. J_{APP} data for one crack depth, 5.4 in. ($a/t = 0.6$) in a flat plate, at four membrane stress levels, 28, 35, 49 and 63 ksi. Also shown are data for cracks also of depth 5.4 in. but different rate of surface growth: $\Delta 2c = 0$, $\Delta 2c = 6\Delta a$ and $\Delta 2c = 12\Delta a$. Figure 14 also includes a curve for a 5.4 in. deep

external crack in a shell of radius $R = 90$ in. and thickness $t = 9$ in.. Figure 15 shows results for 3.6 in. deep internal and external cracks in a shell of the same dimensions. A fourth curve is shown which represents T_{App} vs J_{App} points, for internal and external cracks, obtained using Zahoor et al. method (23). It is obvious from the data that for a given crack depth and stress level, larger shell curvature and surface rates of growth raise the T_{App} vs. J_{App} curves closer to the limits of stability.

The HSST pressure vessel tests (38) provide a wealth of data on the behavior of surface cracks in vessels. The methods of analysis, both structural and those concerning failure criteria, should be checked by comparison with HSST data. Since part-through yielding of the uncracked portion of the vessel occurred in most of these tests, this feature must be admitted by the analytical technique used for the structure. Our calculations were based on a linear elastic shallow shell theory description of the vessel (plate) which does not admit part-through yielding. However, an elastic-plastic shell/plate finite element program equipped with a line-spring element for modeling the surface crack would seem to be capable of analyzing these tests.

V. CONCLUSIONS

Contributions were made toward developing a new methodology to assess the stability of cracks in pressure vessels made from materials that exhibit a significant increase in toughness during the early increments of crack growth. It has a wide range of validity from linear elastic to fully plastic behavior.

The feasibility and desirability of using the "line-spring" method of analysis proposed by Rice (27) for surface cracks in plates and shells was shown. We also conclude that the development of the method into a workable, reliable and economic engineering tool is required. The line-spring model can be incorporated into sophisticated 3-D elasto-plastic shell analysis computer codes or it can be used in conjunction with simpler elastic thin shallow shell analyses. It is estimated that in the first case computer times can be reduced by more than one order of magnitude with respect to the more accurate three dimensional elastic plastic solution, while the accuracy could be within 15% of the latter. Shallow shell-line-spring analyses should reduce costs by another order of magnitude with anticipated accuracies within 30% of the 3-D solutions, but without the capability for analyzing thick shells, or shells loaded to levels so that they are partially yielded even in the absence of cracks. This would not necessarily be a severe limitation in the great majority of nuclear and nonnuclear applications. Comparison of the calculations with 3-D finite element results for some situations is necessary to provide documentation of the method.

The possibility of obtaining reasonably accurate results at low cost concerning the criticality of surface cracks in shells, would permit conducting extensive parametric JEPFM studies in the spirit of the ASME Boiler and Pressure Vessel Code, Section XI. This could eventually lead, as the worthiness of the method is fully tested, to its incorporation as standard procedure.

The HSST pressure vessel tests should be analyzed to validate the overall JEPFM methodology. This would contribute towards fulfilling two important objectives: validation of the model by testing it against unique experimental results and establishment of a more direct analytical link between the HSST model tests and full scale RPV behavior. Finally, consideration should be given to the formulation of precise stability criteria for surface cracks whose shape and/or rate of change of shape need not be restricted to within a single scalar value.

VI. RECOMMENDATIONS

In view of the good results already obtained with the line spring model in its elastic and elastic-perfectly plastic versions, and the very significant savings in time and efforts that can be achieved through its use, it is recommended that further research be supported to:

1. Include in the model material hardening, plastic zone crack depth correction, crack end plastic zones and stress gradient effects.
2. Incorporate the model in a 3-D elastic plastic finite element program to perform J_{APP} and T_{APP} calculations for surface cracks in thick shells, or where partial through the thickness yielding occurs. This capability would permit to test the validity of the model against HSST model tests and would provide a convenient analytical link between the latter and RPV.
3. Incorporate the model into a shallow shell computer code and compare the results with the predictions of the 3-D F.E. program. The resulting code could be used for extensive engineering parametric analyses of the stability of cracks in RPV in connection with design and integrity studies.
4. Perform 3-D elastic-plastic finite element analyses for part-through the thickness cracks in plates under tension and bending. The 3-D models should be used to investigate the values of J_{APP} and T_{APP} along the crack edge. The results should be used to estimate the accuracy of the line-spring model for a few loading and crack geometry combinations of interest.

5. Analyze in depth the stability criteria in its three dimensional formulation, in the spirit of Appendix B. The use of global condions could result in wider ranges of crack stability than those obtained using local conditions.

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APPENDIX A
CRACK SHAPE EFFECTS ON TEARING INSTABILITY

From the analyses of the variation of T_{APP} for different crack families in plates we observed a heretofore unreported sensitivity of T_{APP} to the mode of crack extension. Although we only analyzed three families with length changes $\Delta 2c = 0$, $\Delta 2c = 6\Delta a$ and $\Delta 2c = 12\Delta a$ the T varied by as much as a factor of 3. In all cases the T_{APP} was computed for the tip of the semiminor axis of the semiellipse. This observation has raised two questions: first how realistic is it to judge the stability of the whole crack on the basis of only a very local condition or in terms of a one-parameter representation of crack shape. Second how realistic is it to assume that the crack can advance at the surface faster than at the homologous ratio $\Delta 2c = 6\Delta a$? Without having good J_{APP} solutions, for at least the tips of the semiminor and major axes of the semiellipse it is impossible to provide quantitative answers. On the other hand considering the strong dependence of J on σ (of the order of $\sigma^{2.6}$ for the semiminor axis tip), the fact that the stresses decrease from the inside surface towards the outside surface, and the gradient in properties due to irradiation, it seems plausible to believe that under certain circumstances the crack may grow lengthwise sufficiently more rapidly than in the thickness direction to result in very high values of T_{APP} at the semiminor axis tip.

APPENDIX B
TEARING INSTABILITY IN THREE DIMENSIONS

The fact that rather different T_{APP} values were inferred from Erdogan and Irwin's calculations, depending on the assumed change in crack configuration (homologous crack shapes, or change in depth without a change in surface length, etc.) gives some cause for concern. In our implementation of recent research developments in the ductile tearing mode of crack extension, the final stability (or not) of the cracked, loaded structure depends critically on the value of a parameter T_{APP} which does not yet seem to be adequately defined for locally nonuniform conditions and resulting crack growth along three-dimensional crack fronts. We emphasize, however, that the methodology is essentially complete in cases such as the 25% face grooved CT specimen where rather uniform plane strain conditions (and resulting crack growth) along the crack front are enforced by the geometry.

In this appendix we present some first attempts to extend tearing stability analysis to cases of nonuniform crack front conditions in a manner which is consistent with the standard approach in the special case of uniform crack front conditions. In doing so, we shall rely heavily on the nonlinear elastic (deformation theory plasticity) basis of J. After deriving the salient results, we will critically discuss important assumptions and possible future areas of investigation.

Consider a 3D body with a given initial crack "shape" which can be parametrized in some convenient coordinate system by $a_0(s)$ where s is an arclength measuring coordinate along the crack front. Using the nonlinear elastic material model, we can, following Griffith, express the total potential

energy E_t of the body, apart from an arbitrary datum, associated with zero load and initial crack "shape" $a_0(s)$, as the sum of the usual mechanical potential energy, E_m , arising from the solution of a boundary value problem in continuum nonlinear elasticity, plus a "surface energy" term E_s given by

$$E_s = \int_{\text{crack front}} ds \Gamma(s) \quad (\text{B.1})$$

where

$$\Gamma(s) = \int_0^{\Delta a(s)=a(s)-a_0(s)} J_{\text{MAT}}(a',s) da'(s)$$

Here it is presumed that, locally, a material crack resistance force, J_{MAT} , can be defined which may depend explicitly on location s (through irradiation level, etc.) and on the local (small) crack advance $\Delta a(s)$ normal to the crack front in the plane of the crack. The total energy of the system can then be expressed by

$$E_t = E_m + E_s \quad (\text{B.2})$$

An equilibrium crack configuration under load, $a(s)$, can be obtained by rendering the first variation of E_t with respect to $a(s)$ equal to zero:

$$0 = \delta_a E_t = \delta_a E_m + \delta_a E_s. \quad (\text{B.3})$$

Now we use the fact that the J integral can be related to $\delta_a E_m$ by

$$\delta_a E_m = - \int_{\text{crack front}} J_{\text{APP}}(s; a(s')) \delta a(s) ds \quad (\text{B.4})$$

Here we wish to indicate that the crack driving force J_{APP} at point s depends parametrically on the entire shape of the crack front, denoted collectively by $a(s')$. When this result is combined with (B.1) and (B.2) and inserted into (B.3), there obtains

$$0 = \int_{\text{crack front}} ds \delta a(s) [J_{MAT}(s; \Delta a(s)) - J_{APP}(s; a(s'))] \quad (B.5)$$

If this is to hold for arbitrary $\delta a(s)$, we obtain the "equilibrium" crack configuration (implicitly) by solving

$$J_{MAT}(s; a(s) - a_0(s)) = J_{APP}(s; a(s'))$$

where, simultaneously, the notation emphasizes that J_{APP} must be determined from the solution of a boundary value problem in the body with crack configuration $a(s')$. We note that, for uniform (with respect to s) J_{APP} , J_{MAT} , and $\Delta a = a - a_0$, this result is simply $J_{MAT} = J_{APP}$ as has been already noted for crack fronts (and crack front advances) which are characterized by a single scalar parameter. Of course, for sufficiently small $\Delta a(s)$, $a(s) \cong a_0(s)$ and the boundary value problem can as well be solved in its original crack configuration.

We can examine the "stability" of this "equilibrium" by looking at the sign of a second variation. Let Q be defined by

$$Q \equiv \int_{\text{crack front}} ds \left[\frac{dJ_{MAT}}{da}(s; \Delta a(s)) \delta^* a(s) - \delta_a^* J_{APP}(s; a(s)) \right] \quad (B.6)$$

Here it is understood that $\delta_a^* J_{APP}(s; a(s'))$ is the variation in J_{APP} at crack front location s which is associated with a perturbation in the crack front shape from the "equilibrium" value given collectively by $a(s') = a_0(s') + \Delta a(s')$ to a perturbed configuration $a(s') + \delta a^*(s')$. The crack front perturbation function is just $\delta a^*(s)$. As the notation indicates, it is assumed that the variation in J_{MAT} at point s with respect to configuration is simply $\delta_a^* J_{MAT} = (dJ_{MAT}/da)\delta a^*$.

With these definitions and assumptions, it would seem that if $Q > 0$ for all $\delta a^*(s)$, then stability of the equilibrium crack configuration $a(s')$ is assured. Such a result may not be as useful as it might seem at first glance, since it would appear to require a rather extensive investigation of the functional forms $\delta a^*(s')$, especially as it manifests itself in the term $\delta_a^* J_{APP}$.

We may observe that the local variations of the two terms, J_{MAT} and J_{APP} , are fundamentally different. A change in material J at point s must be accompanied by nonzero crack advance δa^* at s . On the other hand, it would seem that, if crack advance occurred elsewhere, but not at s , it would still be possible to have a nonzero change $\delta_a^* J_{APP}$ at point s , due to load transfer.

If we restrict attention to functions $\delta a^*(s)$ which are everywhere nonzero, then we can multiply (B.6) by E/σ_o^2 , and take $(E/\sigma_o^2)\delta_a^* J_{APP} \equiv T_{APP}(s; a(s'))\delta a^*(s)$. Then the condition for stability becomes

$$\frac{EQ}{\sigma_o^2} = \int_{\text{crack front}} ds \delta a^*(s) [T_{MAT}(s; \Delta a(s)) - T_{APP}(s, a(s'))] > 0 \quad (B.7)$$

At this point we may note that we do not expect to lose stability due to crack "retreat", so we may presume that $\delta a^*(s) > 0$. Now suppose that the condition were uniform with respect to s . In this case, we again recover the stability criterion $T_{MAT} > T_{APP}$.

We can consider some important limiting cases. Namely, if in the integral in (B.7) $T_{MAT}(s, a(s))$ is replaced by its minimum value along s , $T_{MAT}(\min)$ and T_{APP} is replaced by its maximum value along s , $T_{APP}(\max)$, with $T_{MAT}(\min) > T_{APP}(\max)$, then stability is still assured for $\delta a^*(s)$ everywhere nonzero. We have emphasized that minimum J-R curves could be used to experimentally determine conservative values of $T_{MAT}(\min)$. On the other hand, it is not so clear how to effectively bound from above the $T_{APP}(s, a(s'))$, and, especially $\delta_a^* J_{APP}$ as we have defined it. This area would seem to be a worthwhile object of future study.

It will likely not suffice to locate a position $s_{\max}$ along the crack front of maximum J_{APP} and then choose a highly localized crack advance function $\delta a^*(s)$ which is very like the Dirac delta function centered around $s_{\max}$. In fact, such sharp local perturbations often serve to decrease the local J_{APP} value in the vicinity of its prior maximum. This was numerically verified in this investigation with the linear (elastic!) response of the line spring, when the relative crack depth at the central collocation point was increased by 1%, while relative crack depths at all other collocation points were maintained at their earlier values. A physical analogue of this phenomenon is the shape stability of fatigue cracks grown from surface defects. If a local "spike" in the shape of the crack front were to produce a local increase in the K_I (or J_{APP}), as compared to neighboring points along the crack front, then this spike would presumably

magnify itself due to increased fatigue crack growth rates and this is not observed in practice.

Although the rough basis for a three-dimensional stability criterion outlined above raises several fundamental questions, it does seem to be general enough to explore at least computationally. Again, it is emphasized that a nonlinear elastic Griffith-type formulation is at the heart of this analysis. Therefore, at least all of the two-dimensional requirements which have already been found necessary to obtain conditions of J-controlled crack growth will have to be satisfied here as well. In addition, it is likely that there may need to be other limitations, perhaps restricting the magnitude of $|\partial J_{APP}/\partial s|$, or of $|\partial \Delta a(s)/\partial s|$ for J controlled growth to be obtained along three-dimensional crack fronts as well. This area, too, should be explored further.

In closing, we emphasize that in true elastic-plastic materials, the formal energetics of the derivations presented here are not obtained (B-1). However, it would seem that formulation of three-dimensional conditions for J-controlled growth, in a similar spirit to the work of Hutchinson and Paris (B-2) for uniform two-dimensional situations, could provide a sound theoretical rationale for using the framework put forward here in assessing the stability against ductile tearing of flawed structures operating on the upper shelf, such as surface-cracked RPV's.

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Figure 1. Basic Principle of R-curves for Use in Determining K_c under Different Conditions of Initial Crack Length, a_c (Ref. 5).

Figure 2. Crack-tip Coordinate System and Arbitrary Line Integral Contour Γ (Ref. 5).

Figure 3. Interpretation of J-integral (Ref. 5).

Figure 4. The J-integral R-curves (Ref. 10).

Figure 5. J-Resistance Curves for A533B Steel Tested at 93°C. ΔT Side-Grooved Compact Specimens (Ref. 11).

Figure 6. Illustration of Equilibrium and Instability Conditions in Terms of J_{APP} , J_R , dJ_{APP}/da and dJ_{MAT}/da .

Figure 7. Variation of T_{APP} with J_{APP} for Two Tangent Moduli for $a/t_n = 0.95$ and $\sigma_o = 70$ ksi (24).

Figure 8. Dependence of J_{APP} on Membrane Stress σ for Various Crack Depths a and of Constant Length $2c = 54.0 \text{ in.}$ in a Flat Plate of Thickness $t = 9.0 \text{ in.}$ and Yield Strength $\sigma_0 = 70 \text{ ksi.}$ E.I.R. Model [25] [29].

Figure 9. Dependence of J_{APP} on Membrane Stress σ for Various Crack Depths a and Length $2c = 6a$ in a Flat Plate of Thickness $t = 9.0$ in. and $\sigma_o = 70.0$ ksi; E.I.R. Model [25][29]. Also Shown Results for 3.6 in. Deep Internal and External Crack in a Cylindrical Shell of Radius $R = 90.0$ in. and Thickness $t = 9.0$ in.; E.I.R. [25][29] and Zahoor et al. [23] Models.

Figure 10. Dependence of J_{APP} on Membrane Stress σ for Internal Crack of Depth $a = 4.5$ in. ($a/t = 0.5$) and Crack Length $2c = 54.0$ and 27.0 in. in a Flat Plate of Thickness $t = 9.0$ in. and a Cylindrical Shell of Radius $R = 90.0$ and Thickness $t = 9.0$ in. using E.I.R. Model [25][29]. Results using Zahoor et al. Model [23] also Shown as Dashed Lines.

Figure 11. Dependence of J_{APP} on Crack Depth a for Various Membrane Stresses σ and Crack Length $2c = 6a$ in a Flat Plate of Thickness $t = 9.0$ in. One Curve for $2c = 54.0$ in. also Shown; E.I.R. Model [25][29]. Results for Internal and External Cracks in a Cylindridal Shell of Radius $R = 90.0$ in. and Thickness $t = 9.0$ in. using E.I.R. [25][29] and Zahoor et al. [23] Models also Included.

Figure 12. Dependence of T_{APP} on Crack Depth a for Various Membrane Stresses σ and Crack Length $2c = 6a$ in a Flat Plate of Thickness $t = 9.0$ in.; E.I.R. Model [25][29]. Results for Internal and External Cracks in a Cylindrical Shell of Radius $R = 90.0$ in. and Thickness $t = 9.0$ in. using E.I.R. [25] [29] and Zahoor et al. [23] Models also Included.

Figure 13. Dependence of T_{APP} on J_{APP} and Limits of Stable T-J Region. Size and w Requirements also Shown. Data Plotted for Homologous Family of Cracks (2c-6a), in 9.0 in. Thick Flat Plates using E.I.R. Model [25] [29].

Figure 15. Dependence of T_{APP} on J_{APP} for 3.6 in. Deep Internal and External Cracks in a Cylindrical Shell of Radius $R = 90.0$ in. and Thickness $t = 9.0$ using E.I.R. [25][29] and Zahoor et al. [23] Models.

Figure 16. Comparison of Normalized Stress Intensity Factor Distributions Obtained from Line-Spring Model and Three-Dimensional Finite Element Solutions. Where Q is Equal to Square of the Elliptic Integral $E(k)$, Where $k = [1 - (a/c)^2]^{1/2}$. Inset: Schematic Representation of Semi-elliptical Surface Crack in a Plate.

Figure 17. A Portion of the Normalized Yield Surface for a Relative Crack Depth $a/t = 0.5$ Used in the Linear Spring Model Calculations.

Figure 18. Variation of J_{APP} with Membrane Stress σ for Various Crack Depths a Using the Line-Spring Model. E.I.R. Model Results Included for Comparison. Crack Length $2c = 54.0$ in. Flat Plate Thickness $t = 9.0$ in.

Figure 19. Variation of J_{APP} with Membrane Stress σ for Various Crack Depths a Using the Line-Spring Model. E.I.R. Model Results Included for Comparison for a Curve with $2c = 32.4$ in.

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