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RESULTS OF A STUDY TO INVESTIGATE
THE RELIABILITY OF PREVENTIVE MAINTENANCE
STANDARD TIMES

J. M. Walsh

PLANT ENGINEERING DIVISION

SEPTEMBER 15, 1975

MASTER

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T A B L E O F C O N T E N T S

	<u>Page</u>
1.0 INTRODUCTION.....	1-1
2.0 RESULTS.....	2-1
2.1 DECISION CRITERIA.....	2-1
2.2 ACCURACY IMPACT.....	2-2
3.0 RECOMMENDATIONS.....	3-1
4.0 COVERAGE.....	4-1
5.0 DATA BASE ADJUSTMENT.....	5-1
5.1 PROPOSED MODEL.....	5-1
5.2 APTNESS (PROPOSED).....	5-1
5.3 REVISED MODEL.....	5-3
5.4 APTNESS (REVISED).....	5-6
5.5 PRECISION.....	5-6
6.0 STUDY COSTS.....	6-1
7.0 CRITIQUE.....	7-1
7.1 REPETITIONS OF THE OBSERVATIONS.....	7-1
7.2 ESTIMATION OF THE SAMPLE SIZE.....	7-1
7.3 REPRESENTATIVENESS OF THE DATA.....	7-2
7.4 INITIAL DESIGN OF THE STUDY.....	7-2
7.5 PREDETERMINED ACCURACY OF THE STANDARDS.....	7-3
7.6 ACCURACY IMPACT ON WORKLOAD.....	7-3
APPENDICES	
APPENDIX A. Test of Normality.....	A-1
Test of Representativeness.....	A-2
APPENDIX B. Proposed Model Output.....	B-1

	<u>Page</u>
APPENDIX C. Test of Linearity.....	C-1
APPENDIX D. Plots of Transformations.....	D-1
APPENDIX E. Revised Model Output.....	E-1
APPENDIX F. Aptness of Revised Model.....	F-1

1.0 INTRODUCTION

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In January of 1975, a report entitled "The Design of a Study to Investigate the Reliability of Preventive Maintenance Standard Times" was submitted to Plant Engineering's division management for review and approval. The report outlined (1) the proposed approach to establish reliable PM standard times, (2) the investment required to employ the specified techniques, and (3) the expected benefits to be gained from the project. Now that the proposed study has been approved, implemented, and completed, it is appropriate to present results of the effort and offer recommendations to management regarding follow-up activities.

To avoid redundancy, the reader will be referred to the Study Design for details concerning background, purpose, scope, and techniques during the course of this discussion. However, the following fast facts provide a synopsis of the approach:

1. In order to implement the PM program on a timely basis, estimates by qualified maintenance foremen were used to establish the standards data base.
2. Now that the implementation phase of the program has been completed, the accuracy of estimated times must be measured and improved as necessary.
3. To measure the accuracy of existing standards, time studies were conducted on a number of randomly selected PM jobs. Estimated times were compared with the actual or true times as provided by the time studies.
4. With the information developed from the sample data, the acceptability of measured accuracy could be determined and, if necessary, adjustments to the data base could be performed to correct serious standards deficiencies.

2.0 RESULTS

2.0 RESULTS

The statistic selected to measure accuracy, designated as $\bar{d}$, is the expected average difference between estimated and actual times of individual job standards assigned to a particular craft. It is important to recognize that since the data is derived from a sampling plan approach, it would be misleading to present results as single, exact values rather than a range of values^a which with some degree of confidence would include the true average difference of actual versus estimated job times.

2.1 DECISION CRITERIA

By comparing this range of values with limits approved by management regarding acceptable^b accuracy, a decision to adjust the existing times is facilitated. For the purpose of this particular study, the established limits will be implemented according to the following criteria as specified in the Study Design.

If $\bar{d} \pm z_{\alpha/2} (\hat{\sigma}/\sqrt{n})(\sqrt{1-f})$ falls between +0.20 hours and -0.20 hours, no adjustment to the existing standards data base will be made.

To clarify, if the true value of the average difference is less than 0.20 hours, either overestimated or underestimated, the accuracy of the present job times will be considered acceptable. Of course, since the computed $\bar{d}$ is only an estimate of the true difference, a confidence interval about $\bar{d}$ is used in the decision rule rather than the single value.

^aAlso known as a confidence interval.

^bFor a more complete discussion on the logic of accepting or rejecting existing standards accuracy, see Section 7.0 of this report.

A summary of results is graphically portrayed in Fig. 1. Each curve represents a probability distribution of possible values of μ for each craft^c. The best point estimate of μ , which is $\bar{d}$, is illustrated by the line located at the center of each curve. For instance, of the Area Mechanics' jobs studied, the average difference between the estimated and the actual time of a job standard was identified at +0.01 hours^d. However, there exists a relatively small probability that $\bar{d}$ is exactly equal to +0.01 hours. Therefore, the interval between -0.11 hours and +0.13 hours is used since there is a 90% probability that the true difference lies in that region. The solid vertical lines represent the prescribed limits that will be employed to accept or to modify the existing standards data base.

2.2 ACCURACY IMPACT

The primary interest of PM standards application at Pantex is in the areas of (1) weekly schedules, (2) quarterly performance reports, and (3) yearly manpower projections. It is appropriate, therefore, to identify the total impact of standards discrepancies associated with these three areas of concern. Once again, the following results are provided in the form of confidence intervals as computed according to the criteria presented in the Study Design.

To place the significance of the confidence intervals in perspective, the information is given in terms of percentages (graphs) as well as in hours (tables).

^c μ is defined as the true population average of the differences, actual versus estimated.

^dA + sign means overestimated and, conversely, a - sign means underestimated.

PROBABILITY DISTRIBUTION OF MEANS (HOURS)

Fig. 1

For example, the first curve in Fig. 2, representing the Area Mechanics' shop in Building 12-05, illustrates the following facts. Assuming $\bar{d}$ is the true population mean, for any given weekly schedule there is a 90% probability that the difference between the total time estimated and the total time actually required will be between +6 and -6 hours. In terms of percentages, the expected total difference is within approximately -11% and +13% of the total time needed^e.

Through statistical properties of the normal distribution, the confidence interval decreases in proportion as the number of jobs increases. As a result, for any given quarterly performance report, the expected difference is between -17 and +27 hours or -3% and +4%. Finally, at the present level of work, the difference between actual and estimated manhours requested to support PM from this shop in a year would be within -25 and +62 hours or -1% and +2%.

To this point in the discussion, two underlying concepts have been presented without proof that must be of significant concern to the statistician. First, the observed data has been shown in the form of a normal probability distribution. Second, the reader must assume that the sample data is representative of all jobs within the scope of the study. Appendix A provides a discussion of the tests employed to validate the assumption of normality and representativeness.

^e Rounding effects explain the ostensible inconsistency of hours and percentages.

AREA MECHANICS

90% Confidence Intervals

Hours			
Item	A0	A1	A2
Weekly Schedules	$-6 \leq \mu_3 \leq +6$	$-4 \leq \mu_3 \leq +4$	$-5 \leq \mu_3 \leq +6$
Quarterly Performance Reports	$-17 \leq \mu_2 \leq +27$	$-12 \leq \mu_2 \leq +16$	$-16 \leq \mu_2 \leq +22$
Yearly Manpower Projections	$-25 \leq \mu_1 \leq +62$	$-20 \leq \mu_1 \leq +35$	$-24 \leq \mu_1 \leq +52$

Fig. 2

ELECTRICIANS

90% Confidence Intervals

Hours			
Item	E0	E1	E2
Weekly Schedules	$-2 \leq \mu_3 \leq +6$	$-2 \leq \mu_3 \leq +2$	$-2 \leq \mu_3 \leq +3$
Quarterly Performance Reports	$+8 \leq \mu_2 \leq +38$	$-2 \leq \mu_2 \leq +11$	$0 \leq \mu_2 \leq +19$
Yearly Manpower Projections	$+61 \leq \mu_1 \leq +122$	$+5 \leq \mu_1 \leq +32$	$+18 \leq \mu_1 \leq +56$

Fig. 3

ELECTRONIC TECHNICIANS

90% Confidence Intervals

Hours

Item	R0	R2
Weekly Schedules	$-3 \leq \mu_3 \leq +6$	$-3 \leq \mu_3 \leq +4$
Quarterly Performance Reports	$+4 \leq \mu_2 \leq +38$	$-2 \leq \mu_2 \leq +20$
Yearly Manpower Projections	$+49 \leq \mu_1 \leq +117$	$+13 \leq \mu_2 \leq +57$

Fig. 4

INSTRUMENT MECHANICS

90% Confidence Interval

Hours	
Item	I0
Weekly Schedules	$+6 \leq \mu_3 \leq +18$
Quarterly Performance Reports	$+134 \leq \mu_2 \leq +179$
Yearly Manpower Projections	$+582 \leq \mu_1 \leq +671$

Fig. 5

MACHINE TOOL GROUP

90% Confidence Interval

Hours

Item	A3
Weekly Schedules	$-3 \leq \mu_3 \leq +5$
Quarterly Performance Reports	$+1 \leq \mu_2 \leq +31$
Yearly Manpower Projections	$+33 \leq \mu_1 \leq +92$

Fig. 6

PIPEFITTERS

90% Confidence Intervals

Fig. 7a

PIPEFITTERS (Continued)

90% Confidence Intervals

Item	Hours		
	F0	F1	F2
Weekly Schedules	$-10 \leq \mu_3 \leq +6$	$-7 \leq \mu_3 \leq +5$	$-9 \leq \mu_3 \leq +6$
Quarterly Performance Reports	$-31 \leq \mu_2 \leq +8$	$-13 \leq \mu_2 \leq +7$	$-18 \leq \mu_2 \leq +8$
Yearly Manpower Projections	$-84 \leq \mu_1 \leq -7$	$-32 \leq \mu_1 \leq +8$	$-46 \leq \mu_1 \leq +6$

Fig. 7b

3.0 RECOMMENDATIONS

3.0 R E C O M M E N D A T I O N S

In view of the observed results and previously established decision criteria, the following course of action will be initiated with approval of division management.

1. The standard times presently established for the Area Mechanics, Electricians, Electronic Technicians, and Machine Tool Group are considered reasonably accurate and should not be adjusted.

In order to effectively improve standards reliability for these crafts, all jobs would have to be studied individually using M-T-M, time studies, or published standard data. Due to the average size of a PM job, one to two hours, the use of any of these methods could not be economically justified.

2. The analysis of results strongly suggests that the estimated times established for the Instrument Mechanics are significantly overestimated and therefore should be adjusted.

A complete discussion of standards adjustment for this craft is presented in Section 5.0 of this report.

3. The confidence intervals shown for the Pipefitters indicate that even though the point estimate of the true average difference between actual and estimated is only -0.12 hours, there is a sufficient probability that the true value could be significantly worse. Therefore, some course of action should be taken to correct the deficiency.

However, in view of the width of the confidence intervals, the use of the corrective method discussed in the Study Design would be somewhat ineffective at this time. Since an in-depth review of the statistics associated with this problem would be quite lengthy, a few brief comments will be offered to, hopefully, provide the reader with a general understanding of the logic of the proposed course of action.

The confidence interval about the estimated mean, $\bar{d}$, as shown in Fig. 1 could be reduced by simply taking more time studies^f. From a statistical viewpoint, the width of the interval is dictated by the standard deviation, $\hat{\sigma}$. In accordance with the mathematics of computing a confidence interval about the mean, the standard deviation decreases as n increases and will approach zero as n approaches N . In essence, the more time studies taken, the smaller the interval becomes, so that a sufficient degree of confidence can be placed in the selection of the true average difference.

The confidence intervals associated with the total expected difference of a given number of jobs as presented by Figs. 2 through 7b will not decrease significantly as more time studies are added to the scope of the study^g. The standard deviation applicable to the distribution of possible total differences associated with a select number of jobs as opposed to the population mean, is not proportional to the number of jobs sampled. Therefore, more time studies would provide a better estimate of μ but would not appreciably reduce the spread of the interval concerning total impact.

^fIn view of the observed standard deviation, approximately 200 additional time studies would be required to reduce the confidence interval to a reasonable width (± 0.10 hours).

^gThe width of the interval can be significantly increased or decreased by specifying a higher or lower level of confidence respectively.

The use of linear modeling as proposed in the Study Design will adjust individual standard times so that $\bar{d}$, which is estimated to be -0.12 hours, will become closer to zero. However, the confidence intervals as illustrated in Figs. 7a and 7b will remain unchanged. Thus, for any one week, quarter, or year, the true impact of standard discrepancies may be significant and somewhat unacceptable.

To overcome this particular problem, a review of the time studies was conducted. During the review, it became apparent that the standard times associated with steam traps have predominantly caused the large standard deviation and therefore the wide spread of the confidence interval. Specifically, the craftsman is required to clean the trap "as necessary." When cleaning was required the standard was significantly underestimated and conversely when cleaning was not necessary the existing standard was inordinately high. Estimates for servicing traps will be revised to include only the inspection time. Cleaning activities will be handled as a repair and scheduled as shop dispatch work rather than PM. This course of action should suffice to provide sufficient accuracy of the data base. The available alternative is to study each job individually using M-T-M, time studies, or published standard data. As discussed in No. 1 of this section, the economics of this latter approach would be questionable.

4.0 C O V E R A G E

4.0 C O V E R A G E

Due to the random selection of jobs, the scope of the study has been inherently restricted to the range of time estimates included in the sample. For example, if all jobs evaluated ranged from 0.25 hours to 4.00 hours, then results could not be extended for jobs that are estimated at, say, 80.00 hours. This is particularly true when applying the linear modeling techniques as discussed in Section 5.0. For an overview of the coverage of the study in terms of the number of jobs and quantity of hours, the following table is provided.

% Coverage

<u>Craft</u>	<u>Jobs</u>	<u>Hours</u>
Area Mechanics	93	55
Electricians	95	57
Electronic Technicians	91	57
Instrument Mechanics	97	66
Machine Tool Group	94	40
Pipefitters	<u>94</u>	<u>50</u>
Average	94	54

As shown, there is excellent coverage in terms of the number of jobs performed in a year but somewhat less than ideal coverage in terms of hours. At this point in the discussion, it is important to reiterate the advantages associated with the general approach of the study. The significance of the coverage can then be placed in perspective.

The technique selected to establish a reliable standard times data base must be viewed in relation to the investment required to implement the technique. In addition, the use of standards must be evaluated before a specific method is adopted. M-T-M, time studies, and published data provide excellent reliability but each requires a substantial investment.

These techniques can be justified for large jobs, but for small jobs, similar to the majority of those found in a PM program, the point of diminishing returns is reached. The real concern of small PM jobs is not the accuracy of individual standards. The primary interest is the net accuracy associated with scheduling a given number of jobs in a week, measuring craft performance during a quarter, and predicting manpower requirements in a year. This particular study design provides a technique to measure these significant areas of concern.

For large jobs, individual accuracy does become important. The PM Group will continue to evaluate the few large jobs that are not included in the scope of this study. Specifically, time study methods, et. al. will be employed to improve the reliability of existing estimates.

For example, the Machine tool Group shows 40% study coverage of the total number of hours scheduled in a year. By reviewing one job, the coolant change route which is conducted weekly, the hourly coverage would improve to 99%.

5.0 DATA BASE ADJUSTMENT

5.0 DATA BASE ADJUSTMENT

5.1 PROPOSED MODEL

A review of the results obtained from the time studies strongly indicates that revisions to the estimated standards pertaining to Instrument Mechanics are necessary. As stipulated in the Study Design, regression analysis techniques will be employed to (1) identify the functional relationship between estimated and actual job times, and (2) based on this relationship, recompute the existing standards so they will more closely represent the true maintenance times required.

The proposed regression model was given as:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 X_i^3 + \epsilon_i$$

Where Y_i = mean response of the true standard
 X_i = existing standard
 β_j = regression coefficients ($j = 0, 1, 2, 3$)
 ϵ_i = random error term

The LAN Regression Model, developed by Dr. Frank Landram of West Texas State University, was employed to identify the regression coefficients. As a result, the relationship between actual and estimated times was in the form,

$$\hat{Y}_i = 0.455 + 0.078 X_i + 0.209 X_i^2 - 0.017 X_i^3$$

5.2 APTNESS (PROPOSED)

One feature of the LAN Model is the test to determine if each term in the regression equation is appropriate. According to the computerized

results, the cubic and quadratic terms were found to be insignificant and the model was reduced to the form^h,

$$\hat{Y}_i = 0.060 + 0.699 X_i$$

Initially, four basic assumptions are made concerning the aptness of a simple regression model.

1. Linearity of the regression function; since by design the regression function is linear in the parameters, the statistician is concerned with whether or not a linear independent variable is sufficient for the data being analyzed.
2. Normality of the error terms, ϵ_i .
3. Constant variance of the error terms.
4. Statistical independence of the error terms.

Now that the coefficients of the regression function have been derived, it is important to further analyze the model to insure that it indeed reflects the proper relationship between estimated and true standard times.

No discussion was given in the Study Design concerning linearity due to the fact that the initial proposed model contained curvilinear terms. However, since the model has been reduced to a linear function, a test for linearity is now deemed necessary. The need for such a test may appear contradictory in view of the fact that the cubic and quadratic terms were determined to be insignificant. However, there are other curvilinear terms, e.g. logarithms, reciprocals, etc. which might be applicable.

^hThe model output is provided in Appendix B with general comments concerning analysis.

Informally, a good indication of the aptness of the model with respect to constant variance as well as linearity is provided by a residual plot. The data points shown on the plot should be uniformly dispersed both in the e and X axial direction if the model is appropriate. However, a visual inspection of the illustration in Fig. 8 suggests the following:

1. The data points can be represented by a curved line suggesting the need for some curvilinear influence in the model.
2. The variation of the spread of the data points is proportional to the value of X suggesting that the error variance is not constant.

In order to verify the observation concerning the linearity discrepancy, a formal test was conducted using the Analysis of Variance (ANOVA) approach. The results are depicted by the table in Appendix D. To summarize, the hypothesis that the proposed model sufficiently describes the relationship between actual and estimated times cannot be accepted and, therefore, further revisions to the proposed model are requiredⁱ.

5.3 REVISED MODEL

In the book, Applied Linear Statistical Models, Neter and Wasserman provide several techniques that serve as remedial measures to overcome the problem of non-linearity. Since, for this particular model, quadratic and cubic terms are not appropriate, transformations of the data will be investigated. The data may be transformed by using reciprocals or logarithms. The different variations encountered in transforming the data include:

- (1) $X' = 1/X$, $Y = Y$
- (2) $X = X$, $Y' = 1/Y$
- (3) $X' = 1/X$, $Y' = 1/Y$

ⁱAnalysis concerning statistical independence and normality are not necessary, since the model must be revised to correct error variance and linearity problems.

Fig. 8. Residual Plot

- (4) $X' = \log X, Y = Y$
- (5) $X = X, Y' = \log Y$
- (6) $X' = \log X, Y' = \log Y$

The choice of the particular transformation to adopt is facilitated by plotting each of the above and selecting the one whose data can best be represented by a straight line.

In analyzing the graphs provided in Appendix D, it is apparent that technique number 6 would be the most appropriate transformation for the data in question.

After employing the LAN Regression Model, the revised regression function appears in the form^j,

$$\hat{Y}'_i = -0.137 + 0.846 X'_i$$

where $X'_i = \log X_i$
 $\hat{Y}' = \log \hat{Y}$

For example, if one were interested in the mean response of the true standard when the existing standard is 2 hours, the model would be employed as follows.

Let $X'_i = \log X_i$
 $= \log 2.000$
 $= 0.3010$

therefore, $\hat{Y}'_i = -0.137 + 0.846 (0.3010)$
 $= 0.118$

^jRefer to Appendix E for model output with general comments concerning analysis.

and $\hat{Y}_i = \text{antilog } 0.118$
 $= 1.312 \text{ hours}$

5.4 APTNESS (REVISED)

Once again it is necessary to test the revised model to insure the validity of the assumptions of (1) linearity, (2) uniform variance, (3) statistical independence, and (4) normality.

The residual plot, Fig. 9, reveals the data points are uniformly dispersed both in the e and $\hat{Y}'$ axial directions indicating the model is appropriate with respect to uniform variance and linearity. In order to verify the observation concerning linearity, a formal test was conducted using the Analysis of Variance (ANOVA) approach. The results, as depicted by the table in Appendix F, reveal the revised model sufficiently describes the relationship between actual and estimated times with respect to linearity.

In accordance with Study Design, a plot of e_i versus time reflects no significant trend thus upholding the assumption of statistical independence. In addition, 70% of the standardized residuals, $e_i/\sqrt{MSE}$, fall between the values of +1 and -1 thereby supporting the assumption of normality. Applicable charts as discussed are also provided in Appendix F.

5.5 PRECISION

A graph of the regression function with the associated 90% confidence band for the entire regression line is provided in Fig. 10. As a matter of information, the LAN model provides confidence intervals for specified levels of X which are also shown on the graph.

5-7

Fig. 9. Residual Plot

Fig. 10. Revised Model (Instrument Mechanics)

6.0 STUDY COSTS

6.0 STUDY COSTS

For an update of the costs previously identified in the Study Design, the following table is provided:

PERSONNEL

<u>Group</u>	<u>Activity</u>	<u>Man-Hours Required</u>	<u>Cost¹</u>
PM	Planning	90	\$ 600
PM	Conducting Time Studies	1070	7,000
PM	Tabulating Data	80	600
PM	Evaluating Model	40	300
PM	Developing Bench-Mark Jobs ²	125	900
PM	Upgrading Maintenance Instructions ²	80	600
Systems	Programming Data Base Changes ²	35	300
PM	Documenting Design and Results ³	320	<u>2,300</u>
	Sub-Total		\$12,600

EQUIPMENT

<u>Item</u>	<u>Use</u>	<u>Machine-Hours Required</u>	<u>Cost</u>
IBM 360/65 Computer	Process Model	1	\$ 300
	Change Data Base ²		
Stop Watches	Time Studies	N/A	<u>200</u>
			\$ 500

SUMMARY

Actual To-date	\$11,000
Projected	<u>2,100</u>
Total Study Cost	\$13,100

¹Rounded to the nearest \$100

²Projected

³Includes clerical support

7.0 C R I T I Q U E

7.0 CRITIQUE

In this report and in the Study Design, various benefits of the overall approach have been highlighted. To place these various benefits in perspective, it is appropriate to (1) point out limiting factors and assumptions, (2) provide clarification of various design points, and (3) submit improvements to the approach now that the study has been completed. It is through reasonable assumptions and statistical risk-taking that management is able to minimize costs associated with studies of this type.

7.1 REPETITIONS OF THE OBSERVATIONS

In the classic approach of identifying a standard time for a job using time studies, several observations are made and the average is used as the standard. In the particular study in question, only one observation was made for each job selected in the sample. However, this presents no real problem in view of the intended use of the data. In essence, the classical time study approach is used for the purpose of establishing a reliable standard for a specific job. Here, the interest is the average difference, actual versus estimated, of jobs included in the scope of the study and not the accuracy of any one particular job. Therefore multiple observations were not deemed necessary.

7.2 ESTIMATION OF THE SAMPLE SIZE

In the study design, a "general case" equation was presented that enables the planner to avoid making a prediction about the value of the standard deviation in determining the required sample size. However, the planner must be willing to make a guess concerning the value of the ratio of the standard deviation versus the population mean. Unfortunately, any advantage gained in estimating the ratio instead of the standard deviation itself deteriorates as μ becomes small^k.

^kAs discussed in the Study Design, a limit must be established for the relative accuracy as μ becomes small. When this limit is reached, the planner is once again forced to make some estimate of the standard deviation.

As depicted in the results of the study, μ was small in all cases. Therefore, without further discussion, in studies of this type it is recommended that a more realistic approach in determining a sufficient sample size would be to conduct a preliminary study to estimate the value of σ . The importance of predetermining the sample size is the ability to estimate study costs in advance.

7.3 REPRESENTATIVENESS OF THE DATA

Before conclusions can be made from a sampling plan approach, the analyst must be confident that the observed data is representative of the population under study. The best way to ensure representativeness is to use the stratified random sampling approach. This technique is considered more efficient than simple random sampling in that fewer observations are necessary to provide a specified accuracy.

7.4 INITIAL DESIGN OF THE STUDY

In the Study Design, the sampling plan was designed to make inferences concerning each craft. However, in this report the impact of standards discrepancies are extended for each craft-shop. This apparent inconsistency between design and evaluation does not violate statistical principles if two basic assumptions are valid. First, as stated in the Study Design, "This procedure is made possible under the assumption that a craft is, in general, assigned the same type of work whether the location is Zone 11, Area C, or elsewhere." Second, assuming the data comes from a normal population², impact of standards discrepancies for each craft shop can be determined in the same manner as computing statistical tolerances.

²This assumption has been verified. See tests in Appendix A.

7.5 PREDETERMINED ACCURACY OF THE STANDARDS

The accuracy limits established in the Study Design for the purpose of deciding whether to make changes to the existing data base should be considered as guidelines only. Certainly, the accuracy required should take into account the economics of balancing expected savings associated with making standard times more reliable and costs necessary to make adjustments to presently used job times. However, management must keep in mind that an hour spent on revising the estimated standards is a tangible cost, whereas an hour adjustment made to the data base may or may not constitute a tangible savings. Some judgment is necessary to determine how many hours added to or deleted from the existing standard times would precipitate real savings.

Assuming $\bar{d}$ is the true value of μ , the recommendations presented in this report are consistent with accuracy limits dictated by economics alone. Of course, since $\bar{d}$ is only an estimate of μ , this assumption is subject to the law of probabilities associated with statistical estimation.

7.6 ACCURACY IMPACT ON WORKLOAD

The exact locations of the confidence intervals with respect to the Y axis in Figs. 2 through 7b are contingent upon the value of μ , the true population mean of the average difference for each craft. Since the value of μ is estimated by $\bar{d}$, the location of the confidence intervals represent an estimated position based on the value of $\bar{d}$.

The problem of uncertainty could be minimized by increasing the sample size which means additional costs. However, in view of the computed accuracy of $\bar{d}$ for each craft, the true locations of the confidence intervals should not deviate significantly from the positions as illustrated.

A P P E N D I C E S

A P P E N D I X A

TEST OF NORMALITY

In order to compute the confidence intervals as presented in this report, the assumption was made that the sample data was normally distributed. Following the completion of the study, a Chi-Square Goodness of Fit Test^a was employed for each craft. The tests revealed that the data associated with the Electronic Technicians, and the Instrument Mechanics could be considered normally distributed with little reservation. However, for the Pipefitters, Electricians, Area Mechanicis, and Machine Tool Group, the formal test did not verify the assumption of normality at the same level of confidence. A departure from normality is not considered prohibitively significant in view of the following:

1. According to Duncan's Quality Control and Industrial Statistics, "if the distribution of $X[d]$ is unimodal^b, the probability that $X[d]$ should deviate from its mean by more than K times its standard deviation is equal to or less than $1/2.25 K^2$." This fact is better known as the Camp-Meidel extension of Tchebycheff's Inequality.

Therefore, even if the distribution is not precisely normal, when $K = 1.645^c$ there is a greater than an 84% probability that the confidence intervals presented in this study will include the parameter in question, which is only 6% less reliability proposed when the distribution is, in fact, normal.

^aThe test was conducted at the $\alpha = 0.01$ level.

^bHistograms of the data for each craft reflect that the probability density curves are unimodal and symmetric about the mean thus supporting the assumption of normality.

^cThe value of $z_{\alpha/2}$ at the 90% confidence interval.

2. A plot of the data points on normal probability paper showed no significant deviation from normality.
3. The shape of the probability density curves for the Area Mechanics, Electricians, and Machine Tool Group is described as leptokurtic, that is, each curve exhibits a high peak and close grouping of the data about the mean^d. For this reason, it is believed that the number of standard deviations provided by the normal distribution tables, when applied, will result in a wider confidence interval than the true distribution would require.

TEST OF REPRESENTATIVENESS

To insure that the sample data reflected the applicable time study data base, the distributions of each were compared and tested for significant difference. Using the Kolmogorov-Smirnov Test^d, it was determined that for each craft studied, the sample data was indeed representative of the population. Therefore, sample results can realistically be applied to the entire data base within limits of probabilities as discussed.

NOTE: Copies of tests discussed in this report are available on request.

^dThe test was conducted at the $\alpha = 0.01$ level.

A P P E N D I X B

PROPOSED MODEL OUTPUT

The LAN Regression Model is a computerized statistical package that calculates the regression coefficients and provides various statistical information appropriate for determining aptness and precision. The following brief discussion serves to describe the output used to determine the relationship between estimated and actual times for the Instrument Mechanics and to identify the significant factors used in evaluating results.

OUTPUT DESCRIPTION

Section B-1 of the model output provides a listing of the input data. There are 66 rows reflecting the number of time studies made for the Instrument Mechanics. Each row contains a value for X_0 , X , X^2 , X^3 and Y in columns 1 through 5, respectively^a.

Section B-2 provides the coefficient of determination matrix, various tableaus of the Gauss-Jordan algorithm used to compute the regression coefficients, and statistical measurements of the regression function as each variable is included in the model. The coefficient of determination reveals the degree of correlation between the dependent and independent variables and between the independent variables themselves. The Gauss-Jordan tableaus are simply the mathematical iterations that are used to compute statistical measurements required to identify and evaluate the regression equation each time a variable is added to the model.

Section B-3 of the output lists the residuals which can be plotted in various ways to examine the aptness of the model as previously discussed in Section 5.0.

^aThe general regression equation is $\hat{Y} = b_0X_0 + b_1X + b_2X^2 + \dots$ where X_0 is assigned the value of 1.000.

Section B-4 identifies the 90% confidence intervals that are associated with the mean of each Y distribution and with individual values of Y for all levels of X included in the scope of the study (see Fig. 10).

MODEL ANALYSIS

The statistical measurements for the four variable equation in Section B-3 of the output lists the regression coefficients in the first column. As shown, the proposed model in the form, $\hat{Y}_i = 0.4552 + 0.0778 X_i + 0.2091 X_i^2 - 0.0165 X_i^3$.

The partial R^2 values provided in the third column are used in a standard F-test for significance to determine the applicability of each independent variable. As a result, the cubic and quadratic terms are dropped from the proposed model.

Since the model is reduced to a relationship between X and Y, the analysis is based on the statistical measurements for the two variable equation. The proposed model now takes the form, $\hat{Y}_i = 0.0597 + 0.6991 X_i$.

An in-depth discussion of all the terms in the statistical measurement section of the output is beyond the scope of this report. However, it is important to note the values assigned to R^2 , TV, EV, and UV. R^2 , which is the coefficient of determination, identifies the portion of the variance in Y explained by X. For the reduced model, R^2 is equal to 0.69842. TV, EV, and UV, which are the abbreviations for total, explained, and unexplained variance respectively, are used in the Analysis of Variance Tables in tests for linearity.

WALSH - INSTRUMENT MECHANICS

SECTION B-1

VARIABLE FORMAT USED WAS (BFI0.)

NVAL = 56 NCOPY = 0 SELECT = 0.00

NCOL = 5 KUTPUT = -1 N2 = 1 NDEGRE = 0 NEST = 9 NORDER = 0

ROW 1	1.000	2.500	6.250	15.625	0.700
ROW 2	1.000	2.500	0.250	0.125	0.500
ROW 3	1.000	1.000	1.000	1.000	1.100
ROW 4	1.000	2.000	4.000	8.000	0.700
ROW 5	1.000	2.000	4.000	8.000	1.050
ROW 6	1.000	3.000	9.000	27.000	1.500
ROW 7	1.000	1.000	1.000	1.000	0.650
ROW 8	1.000	2.000	4.000	8.000	0.900
ROW 9	1.000	0.500	0.250	0.125	0.450
ROW 10	1.000	1.000	1.000	1.000	0.600
ROW 11	1.000	2.500	0.250	0.125	0.300
ROW 12	1.000	2.000	4.000	8.000	0.400
ROW 13	1.000	2.000	4.000	8.000	1.300
ROW 14	1.000	4.000	16.000	64.000	2.050
ROW 15	1.000	1.000	1.000	1.000	1.300
ROW 16	1.000	3.000	9.000	27.000	2.000
ROW 17	1.000	3.000	9.000	27.000	2.400
ROW 18	1.000	4.000	16.000	64.000	2.100
ROW 19	1.000	1.000	1.000	1.000	1.800
ROW 20	1.000	1.000	1.000	1.000	0.550
ROW 21	1.000	2.000	4.000	8.000	2.200
ROW 22	1.000	3.500	12.250	42.875	4.250
ROW 23	1.000	3.500	12.250	42.875	4.450
ROW 24	1.000	1.000	1.000	1.000	0.750
ROW 25	1.000	1.000	1.000	1.000	0.550
ROW 26	1.000	0.500	0.250	0.125	1.650
ROW 27	1.000	1.000	1.000	1.000	1.150
ROW 28	1.000	0.500	0.250	0.125	0.600
ROW 29	1.000	1.000	1.000	1.000	0.950
ROW 30	1.000	2.000	4.000	8.000	0.800
ROW 31	1.000	2.500	6.250	15.625	0.900
ROW 32	1.000	2.500	6.250	15.625	1.300
ROW 33	1.000	1.000	1.000	1.000	0.500
ROW 34	1.000	1.000	1.000	1.000	0.800
ROW 35	1.000	8.000	64.000	512.000	6.000
ROW 36	1.000	1.000	1.000	1.000	1.000
ROW 37	1.000	1.000	1.000	1.000	0.750
ROW 38	1.000	2.500	6.250	15.625	1.050
ROW 39	1.000	0.500	0.250	0.125	1.300
ROW 40	1.000	0.500	0.250	0.125	0.300
ROW 41	1.000	2.500	0.250	0.125	0.350
ROW 42	1.000	3.000	9.000	27.000	3.400
ROW 43	1.000	0.500	0.250	0.125	0.250
ROW 44	1.000	0.500	0.250	0.125	0.400
ROW 45	1.000	2.000	4.000	8.000	1.250
ROW 46	1.000	2.500	0.250	0.125	0.400
ROW 47	1.000	1.000	1.000	1.000	0.800
ROW 48	1.000	1.000	1.000	1.000	0.450
ROW 49	1.000	1.000	1.000	1.000	0.350
ROW 50	1.000	0.500	0.250	0.125	0.300
ROW 51	1.000	1.000	1.000	1.000	0.800
ROW 52	1.000	1.000	1.000	1.000	0.600
ROW 53	1.000	0.500	0.250	0.125	0.250
ROW 54	1.000	1.000	1.000	1.000	0.500

B-3

SECTION B-1 (Cont'd)

ROW 55	1.000	2.000	4.000	8.000	1.300
ROW 56	1.000	1.000	1.000	1.000	0.500
ROW 57	1.000	3.000	9.000	27.000	1.750
ROW 58	1.000	2.000	4.000	8.000	1.450
ROW 59	1.000	1.000	1.000	1.000	1.500
ROW 60	1.000	0.500	0.250	0.125	0.400
ROW 61	1.000	0.500	0.250	0.125	0.350
ROW 62	1.000	0.500	0.250	0.125	0.300
ROW 63	1.000	0.500	0.250	0.125	0.350
ROW 64	1.000	2.000	4.000	8.000	1.800
ROW 65	1.000	1.500	2.250	3.375	1.750
ROW 66	1.000	2.000	4.000	8.000	1.300

SECTION B-2

MULTIPLE REGRESSION MODEL

NORMAL EQUATIONS

ROW 1	66.000	104.000	267.000	1046.750	76.650
ROW 2	104.000	267.000	1046.750	5689.500	192.875
ROW 3	267.000	1046.750	5689.500	37886.188	765.958
ROW 4	1046.750	5689.500	37886.188	279435.500	4218.930
ROW 5	76.650	192.875	765.958	4218.930	161.182

COEFFICIENT OF DETERMINATION MATRIX
(TOP TRIANGLE OF SYMMETRIC MATRIX)

	(2)	(3)	(4)
(Y)	0.5984	0.6248	0.4755
(2)	1.0000	0.8245	0.6022
(3)	*****	1.0000	0.9347
(4)	*****	*****	1.0000

TABLEAU 2

ROW 1	1.576	4.045	15.860	1.161
ROW 2	103.121	626.023	4040.076	72.093
ROW 3	626.023	4609.363	33651.609	455.876
ROW 4	4040.076	33651.609	262834.188	3003.279
ROW 5	72.093	455.876	3003.278	72.165

B 5

STATISTICAL MEASUREMENTS FOR 1 VARIABLE EQUATION

TV =	72.165	UV =	72.165	S2 =	1.110
EV =	0.0	R2 =	0.0	F =	0.0

REGRESSION
COEFFICIENTBETA
COEFFICIENTPARTIAL
R2-VALUEPARTIAL
VARIATIONEXTRA SUM-
OF-SQUARES

(1)	1.1614	1.1614	0.5523	89.018
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TABLEAU 3

ROW 1	-5.521	-45.875	0.060
ROW 2	6.071	39.178	0.699
ROW 3	808.938	9125.344	18.215
ROW 4	9125.340	104552.500	178.807
ROW 5	18.215	178.808	21.763

SECTION B-2 (Cont'd)

STATISTICAL MEASUREMENTS FOR 2 VARIABLE EQUATION

TV = 72.165 UV = 21.763 S2 = 0.340
 EV = 50.401 R2 = 0.69842 F = 49.406

	REGRESSION COEFFICIENT	BETA COEFFICIENT	PARTIAL R2-VALUE	PARTIAL VARIATION	EXTRA SUM- OF-SQUARES
(1)	0.0597	0.0597	0.0042	0.091	
(2)	0.6991	0.8357	0.6984	50.401	50.401

TABLEAU 4

RCW 1	16.401	0.184
RCW 2	-29.304	0.562
RCW 3	11.281	0.023
RCW 4	1612.813	-26.670
RCW 5	-26.673	21.353

B-6

STATISTICAL MEASUREMENTS FOR 3 VARIABLE EQUATION

TV = 72.165 UV = 21.353 S2 = 0.339
 EV = 50.812 R2 = 0.70411 F = 49.972

	REGRESSION COEFFICIENT	BETA COEFFICIENT	PARTIAL R2-VALUE	PARTIAL VARIATION	EXTRA SUM- OF-SQUARES
(1)	0.1840	0.1840	0.0202	0.440	
(2)	0.5624	0.6723	0.2114	5.725	50.401
(3)	0.0225	0.1800	0.0188	0.410	0.410

SECTION B-2 (Cont'd)

X'X-INVERSE MATRIX

ROW	1	0.244	-0.355	0.122	-0.010	0.455
ROW	2	-0.355	0.588	-0.212	0.018	0.078
ROW	3	0.122	-0.212	0.080	-0.007	0.209
ROW	4	-0.010	0.018	-0.007	0.001	-0.017
ROW	5	-0.455	-0.078	-0.209	0.017	20.912

STATISTICAL MEASUREMENTS FOR 4 VARIABLE EQUATION

TV =	72.165	UV =	20.912	S2 =	0.337
EV =	51.253	R2 =	0.71022	F =	50.652

	REGRESSION COEFFICIENT	BETA COEFFICIENT	PARTIAL R2-VALUE	PARTIAL VARIATION	EXTRA SUM- OF-SQUARES
--	---------------------------	---------------------	---------------------	----------------------	--------------------------

(1)	0.4552	0.4552	0.0391	0.850	
(2)	0.0778	0.0930	0.0005	0.010	50.401
(3)	0.2091	1.6708	0.0254	0.545	0.410
(4)	-0.0165	-0.9980	0.0207	0.441	0.441

SECTION B-3

RESIDUAL ANALYSIS

	ACTUAL Y-VALUES	ESTIMATED Y-VALUES	DIFFERENCE
ROW 1	0.700	1.698	-0.998
ROW 2	0.500	0.544	-0.044
ROW 3	1.100	0.726	0.374
ROW 4	0.700	1.315	-0.615
ROW 5	1.050	1.315	-0.265
ROW 6	1.500	2.124	-0.624
ROW 7	0.650	0.726	-0.076
ROW 8	0.900	1.315	-0.415
ROW 9	0.450	0.544	-0.094
ROW 10	0.600	0.726	-0.126
ROW 11	0.300	0.544	-0.244
ROW 12	0.400	1.315	-0.915
ROW 13	1.300	1.315	-0.015
ROW 14	2.050	3.053	-1.003
ROW 15	1.300	0.726	0.574
ROW 16	2.000	2.124	-0.124
ROW 17	2.400	2.124	0.276
ROW 18	2.100	3.053	-0.953
ROW 19	1.800	0.726	1.074
ROW 20	0.550	0.726	-0.176
ROW 21	2.200	1.315	0.885
ROW 22	4.250	2.580	1.670
ROW 23	4.450	2.580	1.870
ROW 24	0.750	0.726	0.024
ROW 25	0.550	0.726	-0.176
ROW 26	1.650	0.544	1.106
ROW 27	1.150	0.726	0.424
ROW 28	0.600	0.544	0.056
ROW 29	0.950	0.726	0.224
ROW 30	0.800	1.315	-0.515
ROW 31	0.900	1.698	-0.798
ROW 32	1.800	1.698	0.102
ROW 33	0.500	0.726	-0.226
ROW 34	0.800	0.726	0.074
ROW 35	6.000	5.991	0.009
ROW 36	1.000	0.726	0.274
ROW 37	0.750	0.726	0.024
ROW 38	1.050	1.698	-0.648
ROW 39	1.000	0.544	0.456
ROW 40	0.300	0.544	-0.244
ROW 41	0.350	0.544	-0.194
ROW 42	3.400	2.124	1.276
ROW 43	0.250	0.544	-0.294
ROW 44	0.400	0.544	-0.144
ROW 45	1.250	1.315	-0.065
ROW 46	0.400	0.544	-0.144
ROW 47	0.800	0.726	0.074
ROW 48	0.450	0.726	-0.276
ROW 49	0.350	0.726	-0.376
ROW 50	0.300	0.544	-0.244
ROW 51	0.800	0.726	0.074

B-8

SECTION B-3 (Cont'd)

RCW 52	0.600	0.726	-0.126
RCW 53	0.250	0.544	-0.294
RCW 54	0.500	0.726	-0.226
RCW 55	1.300	1.315	-0.015
RCW 56	0.500	0.726	-0.226
RCW 57	1.750	2.124	-0.374
RCW 58	1.450	1.315	0.135
RCW 59	1.500	0.726	0.774
RCW 60	0.400	0.544	-0.144
RCW 61	0.350	0.544	-0.194
RCW 62	0.300	0.544	-0.244
RCW 63	0.350	0.544	-0.194
RCW 64	1.800	1.315	0.485
RCW 65	1.750	0.987	0.763
RCW 66	1.300	1.315	-0.015

SECTION B-4

STATISTICAL ESTIMATION

X-VALUES FOR ESTIMATE 1

1.00 0.50 0.25 0.13

T-VALUE 1.670
 VARIANCE FOR Y-AVERAGE 0.02
 STANDARD ERROR OF REGRESSION 0.13
 INTERVAL ESTIMATE FOR Y-AVERAGE
 0.33 TO 0.76

POINT ESTIMATE FOR Y 0.54
 VARIANCE FOR Y-INDIVIDUAL 0.35
 STANDARD ERROR OF FORECAST 0.59
 INTERVAL ESTIMATE FOR Y-INDIVIDUAL
 -0.45 TO 1.54

X-VALUES FOR ESTIMATE 2

1.00 1.00 1.00 1.00

T-VALUE 1.670
 VARIANCE FOR Y-AVERAGE 0.01
 STANDARD ERROR OF REGRESSION 0.09
 INTERVAL ESTIMATE FOR Y-AVERAGE
 0.58 TO 0.87

POINT ESTIMATE FOR Y 0.73
 VARIANCE FOR Y-INDIVIDUAL 0.35
 STANDARD ERROR OF FORECAST 0.59
 INTERVAL ESTIMATE FOR Y-INDIVIDUAL
 -0.26 TO 1.71

B-10

X-VALUES FOR ESTIMATE 3

1.00 1.50 2.25 3.38

T-VALUE 1.670
 VARIANCE FOR Y-AVERAGE 0.01
 STANDARD ERROR OF REGRESSION 0.11
 INTERVAL ESTIMATE FOR Y-AVERAGE
 0.80 TO 1.17

POINT ESTIMATE FOR Y 0.99
 VARIANCE FOR Y-INDIVIDUAL 0.35
 STANDARD ERROR OF FORECAST 0.59
 INTERVAL ESTIMATE FOR Y-INDIVIDUAL
 -0.00 TO 1.97

X-VALUES FOR ESTIMATE 4

1.00 2.00 4.00 8.00

T-VALUE 1.670
 VARIANCE FOR Y-AVERAGE 0.01

POINT ESTIMATE FOR Y 1.31
 VARIANCE FOR Y-INDIVIDUAL 0.35

SECTION B-4 (Cont'd)

STANDARD ERROR OF REGRESSION 0.12
 INTERVAL ESTIMATE FOR Y-AVERAGE
 1.12 TO 1.51

STANDARD ERROR OF FORECAST 0.59
 INTERVAL ESTIMATE FOR Y-INDIVIDUAL
 0.33 TO 2.30

 X-VALUES FOR ESTIMATE 5

1.00 2.50 6.25 15.63

T-VALUE 1.670
 VARIANCE FOR Y-AVERAGE 0.01
 STANDARD ERROR OF REGRESSION 0.12
 INTERVAL ESTIMATE FOR Y-AVERAGE
 1.50 TO 1.89

POINT ESTIMATE FOR Y 1.70
 VARIANCE FOR Y-INDIVIDUAL 0.35
 STANDARD ERROR OF FORECAST 0.59
 INTERVAL ESTIMATE FOR Y-INDIVIDUAL
 0.71 TO 2.69

 X-VALUES FOR ESTIMATE 6

1.00 3.00 9.00 27.00

T-VALUE 1.670
 VARIANCE FOR Y-AVERAGE 0.02
 STANDARD ERROR OF REGRESSION 0.14
 INTERVAL ESTIMATE FOR Y-AVERAGE
 1.89 TO 2.36

POINT ESTIMATE FOR Y 2.12
 VARIANCE FOR Y-INDIVIDUAL 0.36
 STANDARD ERROR OF FORECAST 0.60
 INTERVAL ESTIMATE FOR Y-INDIVIDUAL
 1.13 TO 3.12

 X-VALUES FOR ESTIMATE 7

1.00 3.50 12.25 42.88

T-VALUE 1.670
 VARIANCE FOR Y-AVERAGE 0.04
 STANDARD ERROR OF REGRESSION 0.20
 INTERVAL ESTIMATE FOR Y-AVERAGE
 2.25 TO 2.91

POINT ESTIMATE FOR Y 2.58
 VARIANCE FOR Y-INDIVIDUAL 0.38
 STANDARD ERROR OF FORECAST 0.61
 INTERVAL ESTIMATE FOR Y-INDIVIDUAL
 1.55 TO 3.61

 X-VALUES FOR ESTIMATE 8

1.00 4.00 16.00 64.00

SECTION B-4 (Cont'd)

T-VALUE 1.670
 VARIANCE FOR Y-AVERAGE 0.03
 STANDARD ERROR OF REGRESSION 0.28
 INTERVAL ESTIMATE FOR Y-AVERAGE
 2.58 TO 3.52

POINT ESTIMATE FOR Y 3.05
 VARIANCE FOR Y-INDIVIDUAL 0.42
 STANDARD ERROR OF FORECAST 0.65
 INTERVAL ESTIMATE FOR Y-INDIVIDUAL
 1.98 TO 4.13

X-VALUES FOR ESTIMATE 9

1.00 8.00 64.00 512.00

T-VALUE 1.670
 VARIANCE FOR Y-AVERAGE 0.34
 STANDARD ERROR OF REGRESSION 0.58
 INTERVAL ESTIMATE FOR Y-AVERAGE
 5.02 TO 6.96

POINT ESTIMATE FOR Y 5.99
 VARIANCE FOR Y-INDIVIDUAL 0.67
 STANDARD ERROR OF FORECAST 0.82
 INTERVAL ESTIMATE FOR Y-INDIVIDUAL
 4.62 TO 7.36

B-12

A P P E N D I X C

TEST OF LINEARITY

The following analysis of variance (ANOVA) table constitutes a formal test for determining whether or not the proposed regression model, $Y_i = 0.060 + 0.699 X_i$, is appropriate with respect to linearity.

ANOVA Table				
Source of Variation	SS	df	MS	F* - Ratio
Regression (EV)	50.401	1	50.401	
Error (UV)	21.763	64	0.340	
Lack of Fit	11.429	7	1.633	9.022
Pure Error	10.334	57	0.181	
Total	72.164	65		

TEST OF HYPOTHESIS FOR LINEARITY:

H_0 : $E(Y) = \beta_0 + \beta_1 X$; the linear model is appropriate

H_1 : $E(Y) \neq \beta_0 + \beta_1 X$; a curvilinear term is needed

$$\alpha = 0.10$$

$$F^* = 9.022 \text{ (from ANOVA table)}$$

$$F(0.90; 7, 57) = 1.83$$

DECISION RULE

If $F^* \leq 1.83$ conclude H_0

If $F^* > 1.83$ conclude H_1

CONCLUSION

Since F^* (9.022) is greater than 1.83, the conclusion that the model should include a curvilinear influence must be made.

A P P E N D I X D

PLOTS OF TRANSFORMATIONS

The selection of the technique that will transform the data to overcome the problem of linearity associated with the proposed model is facilitated by an analysis of the various plots are presented in Figs. D1 through D6. It is apparent that the transformation, $X' = \log X$, and $Y' = \log Y$, provides the best adjustment to account for the curvilinear effect required in the model.

Fig. D1

$$Y' = 1/Y$$

Fig. D2

D-4

Fig. D3

Fig. D4

Fig. D5

Fig. D6

A P P E N D I X E

REVISED MODEL OUTPUT

For general comments concerning description and analysis of the LAN Regression Model, the reader is referred to Appendix B.

As shown in the statistical measurement section for the 2 variable equation, the regression function takes the form, $\hat{Y}_i = -0.1373 + 0.8456 X_i$.

In adopting the revised model (Sections E-1 through E-4), there is some decrease in the coefficient of determination, R^2 , from the proposed model. However, the revised model does meet all the tests for aptness and is considered more appropriate to describe the relationship between actual and estimated times.

Section E-4 of the output provides the 90% confidence band for the mean response of the true standard times as presented in Fig. 10 of this report.

J.M. WALSH INSTRUMENT MECHANICS TRANSFORMED VARIABLES

VARIABLE FORMAT USED WAS (BF10.)

NVAL = 66 NCOPY = 0 SELECT = 0.04
 NCCL = 3 KINPUT = -1 N2 = 1 NDEGRF = 0 NEST = 9 NORDER = 0

ROW 1	1.000	0.398	-0.155
ROW 2	1.000	-0.301	-0.301
ROW 3	1.000	0.0	0.041
ROW 4	1.000	0.301	-0.155
ROW 5	1.000	0.301	0.021
ROW 6	1.000	0.477	0.176
ROW 7	1.000	0.0	-0.187
ROW 8	1.000	0.301	-0.046
ROW 9	1.000	-0.301	-0.347
ROW 10	1.000	0.0	-0.222
ROW 11	1.000	-0.301	-0.523
ROW 12	1.000	0.301	-0.398
ROW 13	1.000	0.301	0.114
ROW 14	1.000	0.602	0.312
ROW 15	1.000	0.0	0.114
ROW 16	1.000	0.477	0.301
ROW 17	1.000	0.477	0.380
ROW 18	1.000	0.602	0.322
ROW 19	1.000	0.0	0.255
ROW 20	1.000	0.0	-0.260
ROW 21	1.000	0.301	0.342
ROW 22	1.000	0.544	0.628
ROW 23	1.000	0.544	0.648
ROW 24	1.000	0.0	-0.125
ROW 25	1.000	0.0	-0.260
ROW 26	1.000	-0.301	0.217
ROW 27	1.000	0.0	0.061
ROW 28	1.000	-0.301	-0.222
ROW 29	1.000	0.0	-0.022
ROW 30	1.000	0.301	-0.097
ROW 31	1.000	0.398	-0.046
ROW 32	1.000	0.398	0.255
ROW 33	1.000	0.0	-0.301
ROW 34	1.000	0.0	-0.097
ROW 35	1.000	0.903	0.778
ROW 36	1.000	0.0	0.0
ROW 37	1.000	0.0	-0.125
ROW 38	1.000	0.398	0.021
ROW 39	1.000	-0.301	0.0
ROW 40	1.000	-0.301	-0.523
ROW 41	1.000	-0.301	-0.456
ROW 42	1.000	0.477	0.531
ROW 43	1.000	-0.301	-0.602
ROW 44	1.000	-0.301	-0.398
ROW 45	1.000	0.301	0.097
ROW 46	1.000	-0.301	-0.398
ROW 47	1.000	0.0	-0.097
ROW 48	1.000	0.0	-0.347
ROW 49	1.000	0.0	-0.456
ROW 50	1.000	-0.301	-0.523
ROW 51	1.000	0.0	-0.097
ROW 52	1.000	0.0	-0.222
ROW 53	1.000	-0.301	-0.602
ROW 54	1.000	0.0	-0.301

E-2

SECTION E-1 (Cont'd)

ROW 55	1.000	0.301	0.114
ROW 56	1.000	0.0	-0.301
ROW 57	1.000	0.477	0.243
ROW 58	1.000	0.301	0.161
ROW 59	1.000	0.0	0.176
ROW 60	1.000	-0.301	-0.398
ROW 61	1.000	-0.301	-0.456
ROW 62	1.000	-0.301	-0.523
ROW 63	1.000	-0.301	-0.456
ROW 64	1.000	0.301	0.255
ROW 65	1.000	0.176	0.243
ROW 66	1.000	0.301	0.114

SECTION E-2

MULTIPLE REGRESSION MODEL

NORMAL EQUATIONS

ROW 1	66.000	5.844	-4.117
ROW 2	5.844	6.563	4.748
ROW 3	-4.117	4.748	7.099

 COEFFICIENT OF DETERMINATION MATRIX
 (TOP TRIANGLE OF SYMMETRIC MATRIX)

(Y)	0.6319
(2)	1.0000

TABLFAU 2

ROW 1	0.089	-0.062
ROW 2	6.045	5.112
ROW 3	5.112	6.842

STATISTICAL MEASUREMENTS FOR 1 VARIABLE EQUATION

TV =	6.842	UV =	6.842	S2 =	0.105
EV =	0.0	R2 =	0.0	F =	0.0

REGRESSION
COEFFICIENTBETA
COEFFICIENTPARTIAL
R2-VALUEPARTIAL
VARIATIONEXTRA SUM-
OF-SQUARES

(1)	-0.0624	-0.0624	0.0362	0.257
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SECTION E-2 (Cont'd)

X'X-INVERSE MATRIX

ROW 1	0.016	-0.015	-0.137
ROW 2	-0.015	0.165	0.846
ROW 3	0.137	-0.846	2.519

STATISTICAL MEASUREMENTS FOR 2 VARIABLE EQUATION

TV =	6.342	UV =	2.519	S2 =	0.039
FV =	4.323	R2 =	0.63186	F =	109.848

	REGRESSION COEFFICIENT	PETA COEFFICIENT	PARTIAL R2-VALUE	PARTIAL VARIATION	EXTRA SUM- OF-SQUARES
(1)	-0.1373	-0.1373	0.3126	1.145	
(2)	0.8456	0.7949	0.6319	4.323	4.323

RESIDUAL ANALYSIS

	ACTUAL Y-VALUES	ESTIMATED Y-VALUES	DIFFERENCE
ROW 1	-0.155	0.199	-0.354
ROW 2	-0.301	-0.392	0.091
ROW 3	0.041	-0.137	0.179
ROW 4	-0.155	0.117	-0.272
ROW 5	0.021	0.117	-0.096
ROW 6	0.176	0.266	-0.090
ROW 7	-0.187	-0.137	-0.050
ROW 8	-0.046	0.117	-0.163
ROW 9	-0.347	-0.392	0.045
ROW 10	-0.222	-0.137	-0.085
ROW 11	-0.523	-0.392	-0.131
ROW 12	-0.398	0.117	-0.515
ROW 13	0.114	0.117	-0.003
ROW 14	0.312	0.372	-0.060
ROW 15	0.114	-0.137	0.251
ROW 16	0.301	0.266	0.035
ROW 17	0.380	0.266	0.114
ROW 18	0.322	0.372	-0.050
ROW 19	0.255	-0.137	0.393
ROW 20	-0.260	-0.137	-0.122
ROW 21	0.342	0.117	0.225
ROW 22	0.628	0.323	0.306
ROW 23	0.648	0.323	0.326
ROW 24	-0.125	-0.137	0.012
ROW 25	-0.260	-0.137	-0.122
ROW 26	0.217	-0.392	0.609
ROW 27	0.061	-0.137	0.198
ROW 28	-0.222	-0.392	0.170
ROW 29	-0.022	-0.137	0.115
ROW 30	-0.097	0.117	-0.214
ROW 31	-0.046	0.199	-0.245
ROW 32	0.255	0.199	0.056
ROW 33	-0.301	-0.137	-0.164
ROW 34	-0.097	-0.137	0.040
ROW 35	0.778	0.626	0.152
ROW 36	0.0	-0.137	0.137
ROW 37	-0.125	-0.137	0.012
ROW 38	0.021	0.199	-0.178
ROW 39	0.0	-0.392	0.392
ROW 40	-0.523	-0.392	-0.131
ROW 41	-0.456	-0.392	-0.064
ROW 42	0.531	0.266	0.265
ROW 43	-0.602	-0.392	-0.210
ROW 44	-0.398	-0.392	-0.006
ROW 45	0.097	0.117	-0.020
ROW 46	-0.398	-0.392	-0.006
ROW 47	-0.097	-0.137	0.040
ROW 48	-0.347	-0.137	-0.210
ROW 49	-0.456	-0.137	-0.319
ROW 50	-0.523	-0.392	-0.131
ROW 51	-0.097	-0.137	0.040

SECTION E-3 (Cont'd)

רנר 52	-0.222	-0.137	-0.085
רנר 53	-0.602	-0.392	-0.210
רנר 54	-0.301	-0.137	-0.164
רנר 55	0.114	0.117	-0.003
רנר 56	-0.301	-0.137	-0.164
רנר 57	0.243	0.266	-0.023
רנר 58	0.161	0.117	0.044
רנר 59	0.176	-0.137	0.313
רנר 60	-0.398	-0.392	-0.006
רנר 61	-0.456	-0.392	-0.064
רנר 62	-0.523	-0.392	-0.131
רנר 63	-0.456	-0.392	-0.064
רנר 64	0.255	0.117	0.138
רנר 65	0.243	0.012	0.231
רנר 66	0.114	0.117	-0.003

STATISTICAL ESTIMATION

X-VALUES FOR ESTIMATE 1

1.00 -0.30

T-VALUE 1.670
 VARIANCE FOR Y-AVERAGE 0.00
 STANDARD ERROR OF REGRESSION 0.04
 INTERVAL ESTIMATE FOR Y-AVERAGE
 -0.46 TO -0.33

POINT ESTIMATE FOR Y -0.39
 VARIANCE FOR Y-INDIVIDUAL 0.04
 STANDARD ERROR OF FORECAST 0.20
 INTERVAL ESTIMATE FOR Y-INDIVIDUAL
 -0.73 TO -0.05

X-VALUES FOR ESTIMATE 2

1.00 0.0

T-VALUE 1.670
 VARIANCE FOR Y-AVERAGE 0.00
 STANDARD ERROR OF REGRESSION 0.03
 INTERVAL ESTIMATE FOR Y-AVERAGE
 -0.18 TO -0.09

POINT ESTIMATE FOR Y -0.14
 VARIANCE FOR Y-INDIVIDUAL 0.04
 STANDARD ERROR OF FORECAST 0.20
 INTERVAL ESTIMATE FOR Y-INDIVIDUAL
 -0.47 TO 0.20

X-VALUES FOR ESTIMATE 3

1.00 0.18

T-VALUE 1.670
 VARIANCE FOR Y-AVERAGE 0.00
 STANDARD ERROR OF REGRESSION 0.03
 INTERVAL ESTIMATE FOR Y-AVERAGE
 -0.03 TO 0.05

POINT ESTIMATE FOR Y 0.01
 VARIANCE FOR Y-INDIVIDUAL 0.04
 STANDARD ERROR OF FORECAST 0.20
 INTERVAL ESTIMATE FOR Y-INDIVIDUAL
 -0.32 TO 0.35

X-VALUES FOR ESTIMATE 4

1.00 0.30

T-VALUE 1.670
 VARIANCE FOR Y-AVERAGE 0.00

POINT ESTIMATE FOR Y 0.12
 VARIANCE FOR Y-INDIVIDUAL 0.04

SECTION E-4 (Cont'd)

STANDARD ERROR OF REGRESSION 0.03
 INTERVAL ESTIMATE FOR Y-AVERAGE
 0.07 TO 0.17

STANDARD ERROR OF FORECAST 0.20
 INTERVAL ESTIMATE FOR Y-INDIVIDUAL
 -0.22 TO 0.45

 X-VALUES FOR ESTIMATE 5

1.00 0.40

T-VALUE 1.670
 VARIANCE FOR Y-AVERAGE 0.00
 STANDARD ERROR OF REGRESSION 0.03
 INTERVAL ESTIMATE FOR Y-AVERAGE
 0.14 TO 0.26

POINT ESTIMATE FOR Y 0.20
 VARIANCE FOR Y-INDIVIDUAL 0.04
 STANDARD ERROR OF FORECAST 0.20
 INTERVAL ESTIMATE FOR Y-INDIVIDUAL
 -0.14 TO 0.54

 X-VALUES FOR ESTIMATE 6

1.00 0.48

T-VALUE 1.670
 VARIANCE FOR Y-AVERAGE 0.00
 STANDARD ERROR OF REGRESSION 0.04
 INTERVAL ESTIMATE FOR Y-AVERAGE
 0.20 TO 0.33

POINT ESTIMATE FOR Y 0.27
 VARIANCE FOR Y-INDIVIDUAL 0.04
 STANDARD ERROR OF FORECAST 0.20
 INTERVAL ESTIMATE FOR Y-INDIVIDUAL
 -0.07 TO 0.60

 X-VALUES FOR ESTIMATE 7

1.00 0.54

T-VALUE 1.670
 VARIANCE FOR Y-AVERAGE 0.00
 STANDARD ERROR OF REGRESSION 0.04
 INTERVAL ESTIMATE FOR Y-AVERAGE
 0.25 TO 0.40

POINT ESTIMATE FOR Y 0.32
 VARIANCE FOR Y-INDIVIDUAL 0.04
 STANDARD ERROR OF FORECAST 0.20
 INTERVAL ESTIMATE FOR Y-INDIVIDUAL
 -0.02 TO 0.66

 X-VALUES FOR ESTIMATE 8

1.00 0.60

SECTION E-4 (Cont'd)

T-VALUE 1.670
 VARIANCE FOR Y-AVERAGE 0.00
 STANDARD ERROR OF REGRESSION 0.05
 INTERVAL ESTIMATE FOR Y-AVERAGE
 0.29 TO 0.45

POINT ESTIMATE FOR Y 0.37
 VARIANCE FOR Y-INDIVIDUAL 0.04
 STANDARD ERROR OF FORECAST 0.20
 INTERVAL ESTIMATE FOR Y-INDIVIDUAL
 0.03 TO 0.71

X-VALUES FOR ESTIMATE 9

1.00 0.90

T-VALUE 1.670
 VARIANCE FOR Y-AVERAGE 0.00
 STANDARD ERROR OF REGRESSION 0.07
 INTERVAL ESTIMATE FOR Y-AVERAGE
 0.51 TO 0.74

POINT ESTIMATE FOR Y 0.63
 VARIANCE FOR Y-INDIVIDUAL 0.04
 STANDARD ERROR OF FORECAST 0.21
 INTERVAL ESTIMATE FOR Y-INDIVIDUAL
 0.28 TO 0.98

A P P E N D I X F

APTNESS OF REVISED MODEL

Before adopting the revised model for usage, the assumptions of linearity, normality, uniform variance, and statistical independence must be verified. The residual plot in Fig. 9 of the report shows the revised model is consistent with the uniform variance assumption and does indicate that the curvilinear effect is appropriate. The assumption concerning linearity is verified by the ANOVA table presented on the following page.

The data is considered to be statistically independent from an analysis of the plot in Fig. F1. Deviations from statistical independence would be reflected by a discernable trend as time progresses.

Since (1) approximately 68% of the data points are located between -1.0 and +1.0 on the $e_i/\sqrt{MSE}$ scale, and (2) the pattern closes represents a straight line on the chart in Fig. F2, the assumption of normality is upheld.

TEST OF LINEARITY

ANOVA Table

Source of Variation	SS	df	MS	F* - Ratio
Regression (EV)	4.277	1	4.277	
Error (UV)	2.504	64	0.039	
Lack of Fit	0.453	7	0.065	1.81
Pure Error	2.051	57	0.036	
Total	6.781			

TEST OF HYPOTHESIS FOR LINEARITY

H_0 : $E(Y') = \beta_0 + \beta_1 X'$; the curvilinear effect adopted is appropriate

H_1 : $E(Y') \neq \beta_0 + \beta_1 X'$; the curvilinear effect adopted is not appropriate

$$\alpha = 0.10$$

$$F = 1.81$$

$$F(0.90; 7, 57) = 1.83$$

DECISION RULE

If $F^* \leq 1.83$, conclude H_0

If $F^* > 1.83$, conclude H_1

CONCLUSION

Since $F^* (1.81)$ is less than 1.83, it is appropriate to conclude that the adopted transformation sufficiently accounts for the curvilinear effect of the data.

Fig. F1. Test of Statistical Independence

Fig. F2. Test of Normality