## TORSION AHD GEONETROSTASIS IN CONARIANT SUPERSTRINCS

## Conmas Zachos

HEP Divialion, Argonne Mational Lshoratory, Argonne, IL 60439
The covariant action for freely propagating heterotio superstrings consists of a metric and a torsion term with a special rolative strength. It is hown that the otrength for which torsion flattens the underlying 10-dimensional superspace geometry is procisely that which yislds fres oscillators on the light cone. This is in complete analogy with the gaometrostasis of two-dimensional o-models with Weor-Zumino intemetions.

I am reporting on some observations made in collaboration with $T$. Curtright and L. Mezincescul concerning the geonetrical structure of covariant free auperstringe. To thia end, ithall exploit the otriking analogy of thit formulation to the geometrostasis ${ }^{2}$ of cwo-dimensional omodels with torsion ${ }^{3}$. i.e. a Wess-Zumino interaction ${ }^{4}$.

In o-models, when the atrength of the torsion tern in the action ia such that the underlying group geometry is parallelized, the renormalization of this geometry ceases and the system reduces to a free theory ${ }^{2}$. The covariantly formulated heterotic atring is a omodel whose fiber (target) coset is lo-diaensional superspace. it appears like an interacting theory, but it reduces to the light-cone oscllators comprising the free superstring when the strength of the torsion term relative to the metric tern is such that the underlying superspace is flattench--and only then. This specific connection to the $\sigma$ model should be contradiscingulahed from treatments which asaciate effective descriptions of strings propagating in non-trivial backgrounds with qeneralized ormodels whose cosete are not, in any case, superspace ${ }^{5}$. Recently, however, witten has formulated the problem super-covariantiy in curved superspace ${ }^{6}$.

I briefly review some relevant background on plain bosonic chiral o-models with corsion ${ }^{2}$. The base space is flat iwo-dimensional spacetime, whlle the group fiber is the coset $G_{L} \mathcal{K}_{\mathrm{i}} / \mathcal{G}_{V}$. The projective coordinates of this group mantfold are the "plons" ${ }^{\text {a }}$, i.e. the Nambucoldstone bosons which shift under the nonlinear axial transfornations Exponentiating $1 \oint^{A} T^{a}$ ylelds the gtandard group elements $U$, and hence $U^{-1} d U$ are elements in the algehra. All group manifold tensor functions

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of $\boldsymbol{p}^{\text {a }}$ are expressed in terme of flat tangent space indices (i, $, k, \ldots$, converted from curved kroup indices ( $a, b, c, \ldots$ ) throuph the vielbein one-form and the counterpart currents defined in the fibration,

$$
\begin{equation*}
v^{\prime} \equiv \frac{1}{2} \operatorname{Tr} T_{U}^{d} d u \equiv J_{u}^{1} d x^{\mu}, \quad J_{\mu}^{j} \equiv \frac{1}{2} \operatorname{Tr}^{1} T_{U}^{-1} \partial_{U} U \tag{1}
\end{equation*}
$$

The conventional o-model is then simply the flat contraction of spacetime and sromp indices in the current bilinear, amounting to a metric term, the "Sugajara nction":

$$
\begin{equation*}
I_{1}=\frac{1}{2 \lambda^{2}} \int d^{2} \times n^{\mu v_{6}} k_{k} J_{j} j_{v}^{k} \tag{2}
\end{equation*}
$$

where $\lambda$ is the dimensionless coupling ( $-1 / f_{\mathrm{F}}$ ).
The properties of this nodel are modified dramatically if a corgion (Hesa-Zumino) interaction term $I_{2}$ ia introduced. The group manifold considered is typified by a fundamental invariant three-form constructed from the structure constants of the algebra:
$\Omega_{3} \equiv \eta f_{1 j k} v^{1} u^{j} v^{k}=-\frac{\eta}{2} \operatorname{Tr} U^{-1} d U U^{-1} d U U^{-1} d U \quad-S_{a b c} d \phi^{\theta} d \phi_{d}^{b} \phi^{c}$.

Where $n$ is an arhitrary numerical atrength. Thie form is antaciated to Cartan's torsion two-form, and the corresponding rank-three antisymsetric tensor $s^{\text {a }}$ bc is the coraion completitiz the Christoffel sumhols $\Gamma^{3}$ be insife covariant derivatives on the group manifold. $n_{3}$ is closed, $d \Omega_{3}=0$, but not exact. l.e. it may only be derived from a potential two-form $\Omega_{2}$ locally on the group manffold: $\Omega_{1}=d \Omega_{2}$.
llsing this, the coraton interaction term

$$
\begin{equation*}
I_{2}=\frac{1}{2 \lambda^{2}} \frac{2 \eta}{3} \int d^{3} \times c^{\lambda \nu v_{f}}{ }_{i j k} J_{\lambda}^{1} J_{H}^{J} J_{v}^{k} \tag{4}
\end{equation*}
$$

may he converted to a two-digenstonal interaction throunh integration hy parts. However, the coeffictent $n$ is constralned by the cohomolony of the manifold ${ }^{4}$ to equal $N \lambda^{2} / c$, where $N$ is an arbitrary integer and $c$ Is a normalization chararieristic of the group; e.f. $c=2 \pi$ for the conventional pyperspherlcal model based on $\operatorname{SU}(2)_{1} \mathrm{KSU}^{\mathrm{S}}(2)_{R} / \mathrm{SU}(2)_{V}$.

At $n= \pm 1$. it may he seen by use of the Maurer-Cartan equation

f.e. the one dictating the vanishing of the genecaltsed curvature twofom $\mathcal{G R}^{\text {i]. ( }}$. For $n=-1$ this is obvious, since the epin connection vandshes, while the curvature io definable as its covarlant curl.) When the seumetry which drives the renomalization of these models is flattencd, remorralization cones to atandstill, as the infrared geometroscatic fixed point is reached ${ }^{2}, 3,4$. Thit fact was checked to wo loops in ref. $\left\{2 \mid\right.$ and and ahould not he unexpected ${ }^{4}$; it was finsily proved to all orders in perturbation theory in ref. [71. The only bosonic geometrlee with this property are actually the group manifolds discussed above. At the geometrostatic point, $n= \pm l$, these theories msy also be shown co he free ${ }^{4}$.

This result persists upon $N=1$ supersymmetrization, i.e. when Maforana apinor superpartiners of the are introduced, which are not proiective coordinates themselves, however ${ }^{3}$.


Figure. The renomalization of the bosonic hypersphere omodel with torsion: At high energies the coupling $\lambda$ dsereases indefinitely, i.e. the radius of the group hypersphere $r=f_{T}=1 / \lambda$ increase with anergy. At low energies howsvar, in lisu of infrared slavery, $\lambda$ increases to a critical value $1 / r_{c}=$ $\sqrt{2 \pi / N}$ and stops evolving with decreasing anergy, i.e. achieves confomal invariance.

Formulating free superstrings in a covariant language, so that chelr spacetime symetries and supersymetry are manifent, resultsin lageandans which appear at first to describe nonlinear interactions. Cuvariant superstrings 8 may be regarded as boeonic two-dimensional $\sigma^{-m o d e l}{ }^{9}$ In curved spacetime (che world-sheet), with conscralnts which arlse from the extra variations of the world-sheet metric, and from the appropilate boundary conditions. I will diseuss the lo-dimensional heterotic superstring. ${ }^{10}$ The fither coset of chis gering is In-dinensional $N=1$ superspace. fre snncrast to ordinary spacetime which may be flatcened withoul corston, flat superspace needs torsion,
allil hence, unlike the case of bosonic otringe, otorston term is necessary in formulating gerinks covariancly.

The projective coordinatea correaponding to the nonlinearly realIfed 10-translation and the 32 (16)-component M.jorana-Heyl superfymmetry are $x^{\nu}$ and $\theta^{m}$ reapecitively, whe, $u$ and are that langent space vector and (Matorana-deyl) spinor indices, respectively. In addition, che fermionic representacion of the hecerotic atring also utilizes $\mathbf{3 2}$ ancillary Maforana-Heyl world-sheet spinorn $\psi$. 10 uhich are wealars under the action of the above aperapace conet. They are thus co be contrasted to ine above projective coordinates ( $X^{\nu}, \theta^{m}$ ) which are vorld-sheel scalars hut, of course, crensform nontrivially in 10 -dimensional superspace.

The supervielbefn one-form and the counterpart currents are

$$
\begin{equation*}
\omega^{M} \equiv\left(\omega^{\mu}, \omega^{m}\right)=\left(d x^{\mu}-1 \bar{\theta} \Gamma^{\mu} d \theta, d \theta^{m}\right) \equiv \omega_{a}^{M} d \xi^{a}, \tag{S}
\end{equation*}
$$

Where $\xi^{a}$ denote the two worlid-sheet coordinatea. The metric term of the action (which alons contains the bosonic string as $\theta+0$ ) is

$$
\begin{equation*}
J_{1}=\frac{1}{2} \int d^{2} \xi \sqrt{-g} g^{a \beta} \eta_{\mu \nu} \omega_{a} \mu_{\omega_{B}}^{\nu} \tag{6}
\end{equation*}
$$

Tris is manifestly world-shect reparameterization tovariant, and globally toretit and supersymetry invariant in cen dimenstons.

The fundamental invariant three-form constructed from the supersymmery structure constant 16

$$
\begin{equation*}
\Omega_{3}=-\ln \left(C r_{\nu}\right)_{m n} w^{\mu} \omega^{n \pi} \omega^{n}=-1 n d x^{\mu}\left(d \bar{\theta} r_{\nu} d \theta\right) . \tag{7}
\end{equation*}
$$

Hecause of incomplete antisymmetry of the ahove structure ronstant, $w^{\|}$ appears not thrice hut once in the above expression, which ulll result In an extra factor of 3 in the nomalization of the action analog of FA. (4).
clearly, $\Omega_{3}$ is not only closed, hut also exact: $\Omega_{3}=$ d $\Omega_{2}$,


The refulting torston term for the heterotic superstring is thus

which contalns the strink tension factor lust like the metric term.

This term has no analof for the purely bosonic string in flat space. Note that because of the lack of a nontrivisi topology in the superapace considered, there are no cohomological reasons to constrain the coefficient $n$ to he a priori quantized. ${ }^{11}$

The parallelizing torsion in superapace is (dw, 0 ): for ihat value the spin connection and hence the curvature vanishes. The fundamental three-fort corresponds to thit value for $\eta-1$, binee $\Omega_{3}=$ $\eta \omega_{u} d \omega^{\mu}$. For both omodels and superatringe, the torgion and its nomalization way be identified from equationg of motion o: $\mathbf{I}_{\mathbf{1}}+\mathfrak{I}_{2}$. The heterotic supersiting is not chirally symetric, and thus the value $\eta=-1$ amounts to reversing the $s i g n$ of the $r_{\mu}$ matrices and reversing the chiralities of the cwo sectora of the string discussed helow to yield and equivalent mitror-image theory-actually this theory conforms to the sense chosen in ref. [10].

The geometrical correspondences between oraodels and the covariant euperstring covered so far are collected below:

CHIRAL G MODEL WITH TORSION
Base space: 2- +1 m. spacelime
Coser: $G_{q} \times G_{R} / G_{V}$
Projective coordinates: $\phi^{i} \equiv V_{a}^{i} \phi^{a}$
Vielbein form: $v_{a}^{1} d \phi_{=1}^{a}{ }_{4}^{1} d x^{\mu}$
raurer-Cartan Eq, $\quad \Delta V^{i}=-f_{j k}^{i} V^{j} V^{k}$


Thision terin:
$I_{2}=\frac{1}{2 \lambda^{2}} \frac{2 n}{3}!j^{3}{ }_{k} \varepsilon^{\lambda \mu v_{f}}{ }_{i j k} J_{\lambda}^{i}, j_{j}{ }_{v}^{k}$

## COVARIANT N=1 HETEROTIC SUPERSTRING

2-dim. curved world-sheet

N=1 Susy \& Poincare / SO(9,1)
( $\mathrm{x}^{\mathrm{L}}, \mathrm{\theta}^{\mathrm{m}}$ )

$d \omega^{\Gamma}=\left(-1\left(r^{\nu}\right)_{m n} \omega^{m} \omega^{n}, 0\right)$

$n_{3}=-i n\left(C r_{u}\right)_{m n} \omega^{\nu^{m} \omega^{m}} \omega^{n}$
$d R_{3}=n$
$\Omega_{3}-\mathrm{d} \Omega_{2}$
${ }^{1}{ }_{2}-\frac{-21 n}{2 \pi} \int d^{3} \xi c^{\alpha \beta \gamma} \gamma_{\alpha} x^{\nu} \partial_{\beta} \bar{\theta} r_{u}{ }^{\partial}{ }_{\gamma}{ }^{\theta}$
tiqs. of molion:

$$
\partial_{u} J^{\mu I}-n f_{j k}^{1} J_{u}^{j} J_{v}^{k} \varepsilon^{\mu \nu}=0
$$

At $r= \pm 1, \Omega_{j}=W^{j} d x^{i}$ and $x^{i j}=0$ free theory

$$
a^{a} \omega_{a}{ }_{a}+1 \bar{n}_{a} r{ }^{\mu} \omega_{B} \varepsilon^{a B_{0}}=0
$$

At $n=1, \Omega_{3}=\omega_{\mu} d \omega^{\nu}$ and $g R^{H N}=0$ free string

The central observation in this discussion is that it is only for the special atrength $n=1$ selected by the geometric criterion of perallelizability that the etring action $I_{1}+I_{2}$ possesses a local fermionic invariance: this invariance ia essential for its reduction to free oscillators on the light-cone, i.e. the free superstring, to be detalled below. Naturally, one experts that thia is not mere colncidence, but no complete causal connection le avallable so far.

The following full covariant formilation of the heterotic superstring, in addition to $I_{1}$ and $I_{2}$ with $n=1$, alec contalns ine erparameterization invarian, kinetic term for the anclliary, non-supersymmetrle fermions:

Where $r_{G}=c_{a}{ }^{0} r_{A}$ are world-shect it race matrices defires from tampent space ones throuph the world-sheet $z$ weibein $e_{a}^{a}, \gamma_{p}$ is the nsendoscalar matrix, and $P_{ \pm}^{a \beta} \equiv\left(R^{\alpha \beta} \pm \frac{e^{a \beta}}{\sqrt{-g}}\right) / 2$ is a projection operator which
enforces the weyl chicalicy on enforces the Weyl chlealicy on w:

$$
\begin{equation*}
P_{+}^{a B} Y_{\alpha} a_{B}=\gamma_{a}^{a_{a}} \frac{\left(1+Y_{p}\right)}{2} \tag{10}
\end{equation*}
$$

As a restilt, the 'r's describe only it degreas of freearm. I will conclude ly sketching hou this action merely describes a free stering,

As already remarked, the action (9) possesses the following local fermionic symmetryg, characterized by a lo-dinenstonal spinor, worldshect vector parameter ${ }^{(W n}$, with $P_{-}=k$ :

$$
\begin{align*}
& \delta \theta=1 \Gamma_{\mu} \omega_{a}^{\nu} \alpha^{\alpha} \\
& \delta X^{u}=1 \bar{\theta} r^{\mu} j \\
& \delta_{a}^{\alpha}=-4 P_{-}^{\alpha \beta \partial_{\beta}} \bar{\theta}_{a \gamma} P_{-}^{\gamma \delta}{ }_{\delta}  \tag{11}\\
& 8 \Psi^{j}=0
\end{align*}
$$

This，along with the other symutries of the string，may be used to prune out the superfluous degrees of frepdamgin the equafions of motion． The light cone is defincd by $x^{ \pm} \equiv \frac{x^{\prime} \pm x^{9}}{\sqrt{2}}$ and $r^{ \pm}=\frac{r^{2} r^{9}}{\sqrt{2}}$ ． Recalling that $\Gamma^{+} \Gamma^{+}=0$ ，the invariance（ 11 ）permits rotating half of the components of $\theta$ away（ $a$ eotal of 8 ，corresponding to $r^{-} x$ ），so as co gauge fix on the light cone frame：$r^{+} \theta=0$ ．Reparameterization invarlance is then uged to go to the conformal gauge，$\sqrt{-g} g^{\alpha \beta}=\eta^{\alpha \beta}$ ； and conformal invariance to fix to the evolution time，$X^{+}=p^{+} r$ ． Lastly，consistently to the above，a two－dimensional tangent bpace rotation fixes che zweibeln to $\left(e_{T}^{0}-e_{0}^{0}\right)+\left(e_{0}^{1}-e_{\tau}^{l}\right)=2$.

The equations of motion of（12）then collapse to free equations derlvable from the light cone action

$$
\begin{equation*}
1=\frac{1}{2 \pi} \int d^{2} \xi\left|-\partial_{a} x^{k} \partial \alpha^{k}+\frac{1}{\sqrt{2}} \bar{\theta} r^{-}\left(\partial_{\sigma}-\partial_{\tau}\right) \theta+\frac{1}{\sqrt{2}} \bar{F}^{1} \gamma^{+}\left(\partial_{\sigma}+\partial_{\gamma}\right) r^{1}\right| \tag{12}
\end{equation*}
$$

where the transverse coordisates $X^{k}$ take 8 values．This yields the bybrld spectrum of the heteracte string．The left－moving sectar contalns $S$ bosonlc components and $R$ fermionlc onis（ $\theta$ ）and is super－ symmetric；the right－moving sector contains 8 bosonic components and 16 fermionic ones（ $\boldsymbol{y}$ ）without supersymetry ${ }^{12}$－upon bosonization of the latter，they all add up to 24 ，equal to the transverse dimensions of the busontc string．To recapltulate，for the gpecial value $\eta=1 a$ local supersymmetry emerges so as to refuce the generalized superspace o－model to the free heterotic string．
a teasonable question ie ask at this point is whether properly defined quantur cheorles with $n \neq 1$ are consistent，and if so，whecher they are related to interacting strings．Furthermore，in that case， would the renormalization group push them to the $n=1$ ifmit in the Infrared，analopously to o－models？

Extension of this seometric connection to the Green－Schwarz super－ string is aot gtralghtforward．This is becauge the fundamental closed form $\Omega_{3}$ breaks the $0(2)$ symmetry which connects the two fermionir coorilnates with each other．in the 10 －dimenstonal $\mathrm{N}=2$ guperspare on

Which they are defined．On the other hand，the parallelizing corgion does nnt ${ }^{1}$ ．Aspecta of this feature may he understood in terms of the recent curved superapsce formulato．．f the IIf superstring． 13

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[^0]:    Invited talk presented ac che npf meuting of the Amerlcan Physical Sociecy in Eurene, Orekon, August 13, 1985.

