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ON THE MEASUREMENT OF THE DYNAMIC PROPERTIES
OF THE STEAM VOID FRACTION IN
BOILING WATER CHANNELS

by

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NOMENCLATURE

A	area
A_f	water area
A_g	steam area
A_R	blade aspect ratio
a	parameter in expression for the steam void fraction
$a(x,t)$	modified steam void fraction
$a_2(t)$	modified exit steam void fraction
$b(t)$	term in expression for the gamma-ray signal
b	term in expression for the average gamma-ray signal
C	capacitance
C	mean chord of a blade element
c	chord of a blade element
$f(t)$	deviation from the estimated mean of $i(t)$
f	frequency
f_b	break frequency
$G(f)$	transfer function
$g(t)$	system impulse response
I	inertia constant
$i(t)$	gamma-ray signal
$i_e(t)$	gamma-ray signal for an empty channel
$i_f(t)$	gamma-ray signal for a full channel
i_T	estimate of the mean of $i(t)$
k	attenuation constant
k	number of degrees of freedom
k_v	velocity correction factor
k_T	time constant correction factor
L	length
L_b	boiling length

L_{nb}	nonboiling length
L'	force per unit blade length perpendicular to the effective velocity
M	maximum lag number in autocorrelation computation
n	exponent in expression for the steam void fraction
n	number of sources
$P_{yy}(f)$	power density spectrum of $y(t)$
$P_{xy}(f)$	cross power density spectrum of $x(t)$ and $y(t)$
p	number of records
p	percentage
$Q(f)$	spectral window
$q(x,t)$	small steam source
q	amplitude of sinusoidal variation of a small steam source
R	resistance
R	mean radius of a blade
$R_{yy}(f)$	estimate of $P_{yy}(f)$
r	radius
r_1	radius at the foot of a blade
r_2	radius at the tip of a blade
s	complex frequency
T	torque
T	recording time
T_n	time corresponding to n sampled points
T_m	maximum time lag used in autocorrelation computation
t	time
u	axial velocity of the fluid
v	steam perturbation velocity
v	effective velocity of fluid
v_f	water velocity
v_g	steam velocity

W	mass rate of flow
w	angular frequency
x	coordinate
x_0	inlet coordinate
x_1	boiling boundary coordinate
x_2	outlet coordinate
$x(t)$	random time function
y	coordinate
$y(t)$	random time function
z	variable

Greek Symbols

α	blade angle with pipe axis
α_0	effective angle of attack
$\alpha(x,t)$	steam void fraction
γ	slip ratio
δ	angle between the axial and effective velocities of fluid
ϵ_a	amplitude error in the simplified model
ϵ_ϕ	phase error in the simplified model
η	coordinate
η	blade "airfoil" efficiency
θ	angular displacement
ξ	coordinate
ρ	density
ρ_f	density of water
ρ_g	density of steam
σ	standard deviation
τ	time lag
τ_a	time lag beyond which autocorrelation function is zero

τ_c	largest time lag for which autocorrelation function is computed
τ_m	transit time of steam perturbation
τ_r	rotor time constant
$\phi_{xx}(\tau)$	autocorrelation function of $x(t)$
$\varphi_{xx}(\tau)$	normalized autocorrelation function of $x(t)$
$\phi_{xy}(\tau)$	crosscorrelation function of $x(t)$ and $y(t)$
$\varphi_{xy}(\tau)$	normalized crosscorrelation function of $x(t)$ and $y(t)$
$T\phi(\tau)$	estimate of the autocorrelation function

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ABSTRACT

The problem of determining the dynamic properties of the steam void fraction undergoing random variations at a particular location in a boiling channel was studied. Emphasis was placed on a gamma-ray attenuation method and also on a method employing sensitive flowmeters at suitable locations of the channel. The dynamic properties of interest were the autocorrelation function and the power density spectrum of the variations.

Equations were derived for computing the desired quantities on the basis of gamma-ray records obtained during actual boiling experiments and from runs with empty and full (nonboiling) channels. The equations take into account the statistical variations of the gamma-ray source and detector. A procedure was also outlined for estimating the measurement effort to obtain a predetermined accuracy.

Two models were developed to correlate the variations of flow with fluctuations of the steam void. Both models were based on the mass continuity equation for a two-phase fluid. The simpler model leads to a first-order, linear differential equation with randomly varying coefficients. The desired quantities can be computed from the equation. The second model leads to a linear integral equation of the third kind. This equation relates the power density spectrum of the void fraction with the spectra of the water velocities.

A frequency range from zero to five cycles per second was investigated. A comparison of the results obtained with the gamma-ray and velocity methods on a 42-atm heat transfer loop indicated that neither model satisfactorily relates the velocity variations with the steam void variations over the entire range of frequencies studied. However, if the steam transit time is about 0.1 sec and the frequency range of interest less than one cycle per second, both models are useful and have approximately the same accuracy. At frequencies greater than one cycle per second, the deviation from the gamma-ray results is more than three decibels for a 0.1-sec steam transit time. The second model gives slightly better results than the first. The accuracy of the models increases with decreasing steam transit time.

CHAPTER I INTRODUCTION

The purpose of the work reported here was to study the usefulness of certain methods for the determination of the dynamic properties of the steam void fraction in boiling water channels. These methods have proved to be reasonably reliable in obtaining the steady-state void fraction, but much less attention has been paid to their use in the dynamic situation, or even to the problem as to how the often inevitable statistical fluctuations of the measured quantities affect the results obtained for the average values. The scope of the study is limited to the gamma-ray attenuation method and to the method employing sensitive, suitably located flowmeters. The gamma-ray method was chosen as a representative of methods which use the variation of attenuation or cross section of some type of beam as a function of the void content, the source being external of the steam water channel. Here the method is developed to include the dynamic measurements of the void. In the evaluation of the use of the flowmeters an assumption is made that the gamma-ray method gives reliable results with which the results obtained from the flowmeter measurements may be compared.

By the knowledge of the dynamic properties of a quantity is meant the knowledge of either its space-time dependence or of certain statistical properties, like the autocorrelation function or the power density spectrum. As appears later, these statistical properties are very descriptive of the quantitative behavior and quite useful for the determination of the system dynamics. In the latter case, even more descriptive than the autocorrelation function is the cross-correlation function.

The significance of the statistical methods is hinted in the above. Their position among the methods of study of system dynamics is further clarified by the following. Consider a linear system or a system which can reasonably well be approximated by linear differential equations in the space-time domain. Rather than study its dynamics in the time domain, it has become quite customary to transform the equations into the space-frequency domain. In the frequency domain the relations between the system variables are given by the corresponding transfer functions. In many systems, like the boiling water system, this theoretical approach is hampered by the fact that the space-time dependences of some system parameters are not known. So to obtain the desired transfer functions one sometimes resorts to the phenomenological approach, which is to describe the system by means of empirical constants to match the theory to the data. The third, purely empirical approach is to measure the transfer functions, using either the actual system or suitable simulators and some convenient excitation for the purpose. The simplest excitation or perturbation is the step input. The method is called ringing. A rather often used perturbation is the sinusoidal signal.

But there are some limitations in the use of the perturbation signals of a particular form. First, it is sometimes very hard to vary the considered input in a desired manner. Secondly, the noise level of the output may be so high that for a reliable detection of the response so large an input signal should be used that the linearity assumption is no longer valid. Here the statistical methods bring considerable help. For instance, in the case of the sinusoidal input, one can improve the signal-to-noise ratio of the measurements by the autocorrelation and even more by the cross-correlation. Also, it is not necessary to use an externally induced input signal. Often the "steady" operation of the system involves a random noise superimposed on the steady-state values of the quantities. If the noise of one quantity can be considered as an input signal and the noise of another quantity as the response to this, one obtains the transfer function between the quantities in question by using the corresponding noises for the statistical analysis. For the above reasons, throughout the present study emphasis is placed on the statistical approach.

The dynamics of the boiling water systems, particularly of the boiling water reactors, where the behavior of the steam void plays an important role, has been the subject of study of several investigators. Without going to any extensive literature review, a few references dealing especially with the boiling water reactor stability are mentioned. Theoretical studies emphasizing the transfer function approach are those of Beckjord,^{(1)*} Thie,⁽²⁾ and Akcasu,⁽³⁾ whereas Fleck⁽⁴⁾ obtains the time-domain solution. The phenomenological approach is used by DeShong, Jr. and Lipinski⁽⁵⁾ to obtain the EBWR feedback transfer function on the basis of experimental data. To the purely experimental class belongs the work of DeShong, Jr.,⁽⁶⁾ in which the EBWR transfer function was determined using a sinusoidal reactivity input, and the work of Zivi,⁽⁷⁾ in which the power-void transfer function was determined for the SPERT IA reactor simulator with the aid of a sinusoidal heat input. A paper by Anderson and Lottes⁽⁸⁾ deals with the boiling stability in a general manner and gives an extensive bibliography on the topic.

The generalized harmonic analysis, which is a more descriptive name for the statistical methods employed in the present study, got its mathematical foundation largely through the work of N. Wiener.⁽⁹⁾ Two of the later contributors in the field are Rice, whose paper on mathematical analysis of random noise is published, for instance, in the good collection of related papers edited by Wax,⁽¹⁰⁾ and Y. W. Lee, who, in his book,⁽¹¹⁾ gives a lucid presentation of the methods and brings up the practical applications which are the result of his and his collaborators' efforts. Some of the papers, which emphasize the use of the statistical methods for the determination of the system characteristics under normal operating conditions, are those of Goodman and Reswick,⁽¹²⁾ Moore,⁽¹³⁾ Thie,⁽¹⁴⁾ and

*For all numbered references, see bibliography.

DeShong.⁽¹⁵⁾ The last-mentioned author used direct Fourier analysis instead of correlation techniques in his work. A comprehensive account of the autocorrelation method in the practical power spectrum measurements is given by Blackman and Tukey.⁽¹⁶⁾ Akcasu has also studied both the direct Fourier analysis⁽¹⁷⁾ and correlation techniques⁽¹⁸⁾ in connection with the noise analysis.

In several of the above-listed papers the feedback diagram of a boiling system is given. It appears that the steam void fraction is an important node in the network, both in the sense that it is the termination of several branches and that it affects directly the reactivity. Therefore, the reliable measurements of the dynamic properties of the void play an important role in the experimental determination of the boiling system stability and heat transfer properties.

CHAPTER II

REVIEW OF THE STATISTICAL POWER SPECTRAL ANALYSES

Fourier Analysis

Blackman and Tukey in their book about the measurement of power spectra⁽¹⁶⁾ outline briefly the direct Fourier analysis in determination of the power spectrum of a noise. Akcasu⁽¹⁷⁾ develops the theory with finite sampled data in more detail, obtaining the formulae for designing an experiment which would meet specified resolution and accuracy requirements. In comparing the method with the autocorrelation analysis, he comes to the conclusion that it can be used equally well, the choice of one being a matter of personal preference.

In the following formulation, it is assumed that the noise being considered is an ergodic, stationary random process with zero mean value. The power density spectrum of the noise $y(t)$ by direct Fourier analysis is defined as

$$P_{yy}(f) = \lim_{T \rightarrow \infty} \frac{1}{T} \left| \int_{-T/2}^{T/2} y(t) \exp(-i\omega t) dt \right|^2. \quad (1)$$

Having available sampled data from a finite record, one can only compute an estimate of $P_{yy}(f)$. Let it be $R_{yy}(f)$. In both of the mentioned references, it is shown that the relation between $P_{yy}(f)$ and the ensemble average of $R_{yy}(f)$ is

$$\text{Av}[R_{yy}(f)] = \int_{-\infty}^{\infty} P_{yy}(f') Q(f-f') df' \quad , \quad (2)$$

where the function $Q(f)$ is called a "spectral window." Equation (2) represents smoothing of $P_{yy}(f)$ over a frequency range near f . Assuming that the noise is Gaussian and that the distribution law of $R_{yy}(f)$ is a chi-square distribution, Akcasu shows on the basis of equation (2) that the number of degrees of freedom, k , which characterizes the distribution is two regardless of the length of the observation time, but that the resolution improves with the length. To obtain more stable results than those corresponding to $k=2$, one has to use higher resolution than eventually needed and then to average estimates of $R_{yy}(f)$ over the frequency band of desired resolution.

Correlation Analyses

The autocorrelation function of the type of noise described in the previous paragraph is defined by

$$\varphi_{yy}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} y(t) y(t+\tau) dt \quad . \quad (3)$$

The well-known Wiener's theorem states that the power density spectrum of a function is the Fourier transform of its autocorrelation function, i.e.,

$$P_{yy}(f) = \int_{-\infty}^{\infty} \varphi_{yy}(\tau) \exp(-i\omega\tau) d\tau \quad , \quad (4)$$

and conversely also

$$\varphi_{yy}(\tau) = \int_{-\infty}^{\infty} P_{yy}(f) \exp(i\omega\tau) df \quad . \quad (5)$$

Blackman and Tukey⁽¹⁶⁾ give an extensive treatment of the planning of measurement to obtain predetermined resolution and accuracy of the power spectral density estimates. The best one can get with a record of a finite length available is an estimate $R_{yy}(f)$ which represents an average of the true power density spectrum $P_{yy}(f)$ over a frequency range near f . The ensemble average of this estimate is related to $P_{yy}(f)$ by a similar expression as in the case of direct Fourier analysis, but the spectral window enters into the picture in a different manner. Here again it is assumed that the estimates $R_{yy}(f)$ have chi-square distribution. The authors then show that the number of degrees of freedom, if $P_{yy}(f)$ does not have any sharp peaks, is approximated by

$$k \approx \frac{2T_n}{T_m} - \frac{2p}{3} \quad , \quad (6)$$

where T_n is the length of the record, T_m the maximum time lag used for computing the autocorrelation functions, and p the number of records. In the case of a single sharp peak of $P_{yy}(f)$ $k=2$. Different from the direct Fourier analysis, k is not independent of the length of the record and also depends on the time lag T_m . The resolution depends on the width of the spectral window or windows used to calculate the estimates $R_{yy}(f)$. A commonly used window is called "hamming," in which case the estimates separated by $1/T_m$ can be considered resolved.

The cross correlation function of two noises $x(t)$ and $y(t)$ is defined by

$$\varphi_{xy}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) y(t+\tau) dt \quad (7)$$

and the cross power spectral density by

$$P_{xy}(f) = \int_{-\infty}^{\infty} \varphi_{xy}(\tau) \exp(-i\omega\tau) d\tau \quad . \quad (8)$$

The cross power spectral density contains two components, cospectral density and quadrature spectral density, which determine the magnitude and phase angle of $P_{xy}(f)$, and which separately can be estimated following the above outlined techniques.

Transfer Functions from the Power Spectra

The power spectral densities from the Fourier or correlation analyses can be used to obtain the transfer function between variables. The relation between the magnitude of the transfer function and the power density spectra of the considered system input and the response to it is given by

$$\left| G(f) \right|^2 = \frac{P_{yy}(f)}{P_{xx}(f)} \quad . \quad (9)$$

Since $P_{yy}(f)$ and $P_{xx}(f)$ do not contain any information about the phase angle of the transfer function, only the magnitude is obtained.

In the time domain, the system input and output are related by the following superposition integral involving the system impulse response $g(t)$:

$$y(t) = \int_{-\infty}^{\infty} g(t') x(t - t') dt' \quad . \quad (10)$$

Substituting equation (10) into equation (7), one obtains

$$\varphi_{xy}(\tau) = \int_{-\infty}^{\infty} g(t') \varphi_{xx}(\tau - t') dt' \quad , \quad (11)$$

which shows that the autocorrelation function $\varphi_{xx}(\tau)$ and the cross-correlation function $\varphi_{xy}(\tau)$ are also related by the similar superposition integral as $x(t)$ and $y(t)$ are. By the theory of convolution, one gets from equation (11)

$$P_{xy}(f) = G(f)P_{xx}(f) \quad . \quad (12)$$

Thus, the transfer function is the cross power spectrum divided by the input power spectrum. As $P_{xy}(f)$ contains both the amplitude and phase information, $G(f)$ obtained by this method also contains them both.

CHAPTER III

DESCRIPTION OF EQUIPMENT

Scope of the Experimental Program

The main purpose of the experimental study was to get information about the relative merits of the different methods for measuring the dynamic properties of the steam void fraction. The test apparatus is shown in Fig. 1 and schematically in Fig. 2. It consists of an AC electrically heated vertical test section, riser, steam separation section, and downcomer. In all experiments a tubular, stainless steel test section was used, whose inner diameter was 1.58 cm and length 91.5 cm. The riser section had the same diameter and was 108 cm long. A sensitive flowmeter was placed both at the inlet of the heated section and at the point where the steam void volume was to be measured, in this study the outlet of the test section. At the latter location were also placed the gamma-ray source and detector. The other variables measured during the runs included the power to the heated section, the system pressure, the inlet temperature, the temperature of the saturated steam, and the water level. The runs were made at pressures of about 10.5 atm and 42 atm, and at several power levels. The power levels were in all cases kept constant and such that marked random fluctuations of the gamma-ray and flow traces were observed, but that no single frequency oscillation was noticeable. At very high power levels, the system started to oscillate with a single frequency. The flow conditions were such that water entering the heated test section was near saturation temperature.

An approximate signal flow diagram of the system is given in Fig. 3 which shows the most important variables and interlinkages between them. Each branch in the diagram represents a transfer function between the corresponding variables in a linear representation. It is assumed that no power perturbations occur, for which reason the effect of power on other variables is shown as dotted lines. In this case, the recorded random variations could be, for instance, due to the statistical nature of the steam generation or the feedwater flow. Directly or indirectly, both cause variations of the steam void volume, which is also affected by the water velocity through the feedback loop. This feedback loop or the other loop involving the flow friction may be a factor leading to single-frequency oscillations under suitable conditions. The cause of the oscillation phenomenon is not well understood.

Instrumentation for Gamma-ray Attenuation Detection

A successful use of the gamma-ray attenuation method for the steady state or average value measurements of the steam void volume has been reported by several investigators.⁽¹⁹⁻²¹⁾ A detailed description of the apparatus used, the theory on which it is based, and sources of error are given in any of these reports, the most comprehensive being that of Hooker and Popper.⁽²⁰⁾

Fig. 1. Overall View of the Test Apparatus

Fig. 2
Schematic Diagram
of the Test Loop

Fig. 3
Signal-Flow Diagram of
a Boiling Water System

The apparatus is reviewed here briefly. A block diagram of it appears in Fig. 4. The radioactive source is a thulium-170 pellet (diameter 0.48 cm) enclosed in an aluminum container and irradiated to produce about 9 r/hr at a distance of 15 cm of air. Thulium-170 has a half-life of about 129 days, and its energy spectrum shows two peaks: 0.053 Mev and 0.084 Mev.

Fig. 4. Block Diagram of the Gamma-ray Void Measuring Equipment

It has been established that for the best results in the dynamic measurements of the steam void fraction, no additional absorber has to be used to eliminate part of the spectrum. The gamma rays are directed through the mixture in the flow channel and the attenuated radiation detected by a scintillation crystal attached to a photomultiplier tube. The crystal is thallium-activated sodium iodide, 32 mm in diameter, and the photomultiplier tube DuMont-6296 or RCA-5819. The output signal of the tube is amplified by a linear DC amplifier, whose output voltage is proportional to the input current, and is then fed through a voltage amplifier to a Visicorder recorder. The Honeywell Model 1012 Visicorder, which was used for the recording, has light beam galvanometers as sensing elements. These can be chosen to have a fast enough response. The filter is needed to eliminate the high-frequency variations of the current amplifier output, which are mainly due to the statistics of the source intensity and the scintillation crystal. The Krohn-Hite Model 330-C filter used has a good cutoff characteristic, which is shown in Fig. 5; here is given the measured gain near the upper cutoff frequency, here selected at 20 cps. Thus, the cutoff frequency can be set quite close to the highest frequency of interest in the measurements (≈ 5 cps) to provide a clear trace at a reasonable recording speed. The voltage amplifier (Electro Instruments Model A 12) preceding the filter is for eliminating the effect of a slight zero drift noticed in the filter behavior. A Brown recorder is also connected at the outlets of the current amplifier to obtain the DC value of the signal.

Fig. 5
Characteristic of Krohn-Hite
Model 330-C Band-Pass Filter

To check the statistical characteristics of the gamma-ray source and the transfer characteristics of the accompanying equipment, gamma rays were beamed through a cold empty channel and the power spectrum of the recorded signal computed. A sample of the record is seen in Fig. 6. The result of computation, shown in Fig. 7, indicates that the source is a white noise source and that the instrumentation does not cause any attenuation of the signal in the frequency range of interest.

Fig. 6. Gamma-ray Signal Beamed Through Cold Empty Channel

Fig. 7. Power Density Spectrum of the
Gamma-ray Signal Beamed
Through Cold Empty Channel

Instrumentation for the Velocity Measurements

For the detection of the velocities, turbine-type flowmeters designed by Potter Aeronautical Company were used. The design of the Pottermeter sensing element is such that the rotor operates at a position completely independent of the housing and support. Therefore the friction is very small. The rotational speed of the turbine is proportional to the velocity of the fluid being measured. There is a permanent magnet attached to the rotor and a fixed coil outside of the meter housing. As the rotor revolves, an AC current is induced in the pick-up coil. The frequency of this current is proportional to the fluid velocity. The signal from the pick-up coil is led to a frequency converter whose output is a direct indication of the fluid velocity.

In all measurements, both the inlet and the outlet Pottermeters were of Model 3/4 - 102 B with a 1.67-cm inner diameter housing. The meters were enclosed in triple-layer magnetic shields to prevent the strong 60-cycle field near the bus bars from inducing an error signal. The frequency converters were Potter Model 3 C-1. They were coupled to the Visicorder through Model A 12 amplifiers designed by Electro Instruments.

The question naturally arises how good this type of flowmeter is for transient measurements. Grey⁽²³⁾ studied the problem in the case of a single-phase fluid assuming a step change of the flow rate. His results show that the time constant τ_T is smallest (~ 1 ms) for a 1.27-cm sensing element, increasing when the meter size either decreases or increases. It also increases with a decreasing volumetric flow rate and with an increasing blade angle. It appears from the results that in the most usual cases the time constant is below 15 ms, which is satisfactory when the frequency range of interest is only up to 5 cps. In Appendix A, the time constant is derived for the case of a two-phase flow following a similar procedure as given by Grey. In one case it is assumed that the slip ratio remains constant during the step change of flow, and in the other case that the change occurs in one phase only. It is seen that the time constant increases by a factor

$$k_T = \frac{1}{(1 - \alpha_2) + \alpha_2 (\rho_g/\rho_f) \gamma} \quad (13)$$

due to the presence of steam. It appears also from the derivation that, if one is interested in obtaining the water velocity, the following correction factor has to be applied to the meter readings:

$$k_V = \frac{(1 - \alpha_2) + \alpha (\rho_g/\rho_f) \gamma}{(1 - \alpha_2) + \alpha (\rho_g/\rho_f) \gamma^2} \quad (14)$$

As an example, if the system pressure is 42 atm, the steam void fraction 0.5, and the slip ratio 2, one obtains $k_T = 1.90$ and $k_V = 0.95$.

The second consideration is the transient response of the instrumentation connected to the flowmeter outputs. Here mainly the transfer function of the frequency converter comes into question. This was both measured and calculated analytically. The agreement of the results is good, the break frequency in both cases being about 9 cps. The analytical expression for the transfer function of the filter stage of the frequency converter is given in Appendix B. The transfer function is practically independent of the range setting of the converter, but the output resistance plays a significant role in determining the transfer characteristics. A fast-slow switch connects an extra condenser in the circuit in the slow position, making the break frequency about one-third of the value at the high position.

CHAPTER IV GAMMA-RAY ATTENUATION METHOD

Theory

According to the previously mentioned references, (19-21) the relation between the output signal $i(t)$ of the current amplifier and the steam void fraction $\alpha(t)$ can be expressed as

$$i(t) = i_f(t) \exp[k\alpha(t)] \quad , \quad (15)$$

where $i_f(t)$ is the output signal in the case of the full-flow channel, i.e., $\alpha = 0$, and contains the statistical intensity variations of the source and the statistical variations of the system transfer characteristics. The combined attenuation constant k is given by

$$k = \ln(\bar{i}_e / \bar{i}_f) \quad . \quad (16)$$

Here i_e denotes the output signal for the empty channel, i.e., $\alpha = 1$, and the overbars denote the mean values.

Equation (15), however, is useless for the purpose of solving the true instantaneous values of $\alpha(t)$, for it is practically impossible to obtain simultaneous records of $i(t)$ and $i_f(t)$. But starting from the assumption that i 's are stationary random processes, one can obtain the statistical properties of α , in which one really is interested, directly from the statistical properties of i 's.

Both $\alpha(t)$ and $i_f(t)$ can be taken as statistically independent, for the steam generation and the gamma-ray source strength are in no way correlated. Hence,

$$\langle i(t) \rangle = \langle i_f(t) \rangle \langle \exp[k\alpha(t)] \rangle \quad , \quad (17)$$

$$\langle i^2(t) \rangle = \langle i_f^2(t) \rangle \langle \exp[2k\alpha(t)] \rangle \quad , \quad (18)$$

and

$$\langle i(t)i(t+\tau) \rangle = \langle i_f(t)i_f(t+\tau) \rangle \langle \exp k[\alpha(t) + \alpha(t+\tau)] \rangle \quad , \quad (19)$$

where $\langle x \rangle = \bar{x}$ = the mean value or the mathematical expectation of the random variable. Use the following notations:

$$A = \frac{\langle i(t) \rangle}{\langle i_f(t) \rangle}$$

$$B^2 = \frac{\langle i^2(t) \rangle}{\langle i_f^2(t) \rangle} = \frac{\sigma_i^2 + \bar{i}^2}{\sigma_{i_f}^2 + \bar{i}_f^2}$$

$$C^2(\tau) = \frac{\langle i(t) i(t+\tau) \rangle}{\langle i_f(t) i_f(t+\tau) \rangle} = \frac{\sigma_i^2 \varphi_{ii}(\tau) + \bar{i}^2}{\sigma_f^2 \varphi_{ff}(\tau) + \bar{i}_f^2} ,$$

where

$$\sigma_x^2 = \langle (x - \bar{x})^2 \rangle = \overline{x^2} - \bar{x}^2 = \text{standard deviation}$$

$\varphi_{xx}(\tau)$ = correlation coefficient defined by

$$\sigma_x^2 \varphi_{xx}(\tau) = \langle [x(t) - \bar{x}][x(t+\tau) - \bar{x}] \rangle .$$

It is known that the expectation value of an exponential is given by

$$\langle \exp z \rangle = \exp \left(\bar{z} + \frac{1}{2} \sigma_z^2 \right) \quad (20)$$

when z is Gaussian. From (20) it is readily obtained that

$$\langle \exp[k\alpha(t)] \rangle = \exp \left(k\bar{\alpha} + \frac{1}{2} k^2 \sigma_\alpha^2 \right) , \quad (21)$$

$$\langle \exp[2k\alpha(t)] \rangle = \exp(2k\bar{\alpha} + 2k^2 \sigma_\alpha^2) , \quad (22)$$

and

$$\langle \exp k[\alpha(t) + \alpha(t+\tau)] \rangle = \exp[2k\bar{\alpha} + k^2 \sigma_\alpha^2 + k^2 \sigma_\alpha \varphi_{\alpha\alpha}(\tau)] . \quad (23)$$

Substituting (21), (22), and (23) into (17), (18), and (19), and solving for $\bar{\alpha}$, σ_α^2 , and $\varphi_{\alpha\alpha}(\tau)$, one obtains

$$\bar{\alpha} = \frac{1}{k} \ln A - \frac{1}{2} k \sigma_\alpha^2 = \frac{1}{k} \ln \frac{A^2}{B} , \quad (24)$$

$$\sigma_\alpha^2 = \frac{1}{k^2} \ln \frac{B^2}{A^2} , \quad (25)$$

and

$$\varphi_{\alpha\alpha}(\tau) = \frac{1}{k^2 \sigma_\alpha^2} \ln \left[\frac{C^2(\tau)}{A^2} \right] = \ln \left[\frac{C^2(\tau)}{A^2} \right] / \ln \left[\frac{C^2(0)}{A^2} \right] . \quad (26)$$

The quantities A , B^2 , and $C^2(\tau)$ are completely determined by the statistical properties of $i(t)$ and $i_f(t)$, and thus the properties of α can be computed. Knowing $\varphi_{\alpha\alpha}(\tau)$, the spectrum of α can be readily computed, too.

If there were no random fluctuations in the steam void as well as in the source strength, $i(t)$ and $i_f(t)$ would have constant values and α would be given by

$$\alpha = \frac{1}{k} \ln \frac{i}{i_f} . \quad (27)$$

One may intuitively tend to think that the average values of $i(t)$ and $i_f(t)$ could simply be used in the above steady-state equation to calculate the average value of α , and also in the actual case where both $i(t)$ and $i_f(t)$ contain random fluctuations. But the presented analysis leads to an equation containing an additional term $\frac{1}{2} k \sigma_\alpha^2$, which is due to the exponential dependence of $\alpha(t)$ and $i(t)$. Although this additional term appears to be small in practical situations, it will be retained in the applications of the given formulas for the sake of rigor.

Error Analysis

Hooker and Popper⁽²⁰⁾ give a comprehensive analysis of errors arising from the following factors:

- 1) errors arising from the electronics of the system;
- 2) sources of error in the measurement technique; and
- 3) errors due to the limitations of the system when preferential distribution patterns are encountered.

The authors concluded that, with a uniform bubble distribution, the maximum possible error in void fraction is approximately 0.03 over the full void range for the test facility used.

In the measurements of the statistical properties of the steam void fluctuations, one meets still another source of error, namely, the uncertainty of the results obtained from finite time records. The errors in the mean value and the autocorrelation function are measured by their standard deviations about the true values. The formal calculation of these standard deviations is outlined below.

Use the following notations:

$$i_T = \frac{1}{T} \int_0^T i(t) dt = \text{estimate of the average of } i(t).$$

$$f(t) = i(t) - i_T = \text{deviations from the estimated mean.}$$

$$T_{\phi_{ff}}(\tau) = \frac{1}{T-\tau} \int_0^{T-\tau} f(t) f(t+\tau) dt = \text{estimate of the autocorrelation function of } f(t).$$

It is seen that the true mean value of i_T is

$$\bar{i}_T = \frac{1}{T} \int_0^T \langle i(t) \rangle dt = \bar{i} \quad . \quad (28)$$

The standard deviation of the i_T 's about $\bar{i}$ is obtained from

$$\begin{aligned}\sigma_{iT}^2 &= \frac{1}{T^2} \int_0^T \int_0^T \langle i(t) i(t') \rangle dt dt' - \bar{i}^2 \\ &= \frac{2}{T} \int_0^T \left(1 - \frac{x}{T}\right) \phi_{ii}(x) dx, \quad (29)\end{aligned}$$

where $\phi_{ii}(\tau) = \sigma_i^2 \varphi_{ii}(\tau)$ is the true autocorrelation function of the process $i(t)$.

The mean value of the autocorrelation function $T\phi_{ff}(\tau)$ is

$$\begin{aligned}\langle T\phi_{ff}(\tau) \rangle &= \frac{1}{T-\tau} \int_0^{T-\tau} \langle f(t) f(t+\tau) \rangle dt \\ &= \phi_{ii}(\tau) - \sigma_{iT}^2. \quad (30)\end{aligned}$$

Thus the average of the estimates never gives the right autocorrelation function, but there is a correction whose magnitude is σ_{iT}^2 . The standard deviation of the estimates $T\phi_{ff}(\tau)$ is

$$\begin{aligned}\sigma_{\phi_f}^2(\tau) &= \frac{1}{(T-\tau)^2} \int_0^{T-\tau} \int_0^{T-\tau} \langle f(t) f(t+\tau) f(t') f(t'+\tau) \rangle dt dt' - \phi_{ff}^2(\tau) \\ &= \frac{2}{T-\tau} \int_0^{T-\tau} \left(1 - \frac{x}{T-\tau}\right) [\phi_{ff}^2(x) + \phi_{ff}(x-\tau)\phi_{ff}(x+\tau)] dx. \quad (31)\end{aligned}$$

For the details of this derivation see, for instance, Bendat.⁽²⁴⁾

In order to plan an experiment in such a manner that a predetermined accuracy will be obtained, one ought to know the correlation function $\phi_{ii}(\tau)$ in the equations (29) and (31). However, $\phi_{ii}(\tau)$ is one of the quantities to be measured and thus unknown. On the other hand, one often has a rough idea of the nature of the correlation function on the basis of either previous experiments or an approximate physical analysis of the system. Let us say, for instance, that we know $\phi(\tau)$ to be essentially zero after $\tau = \tau_a$. Then we get the upper bound of the error by taking $\phi(\tau) = \phi(0) = \text{constant}$ up to $\tau = \tau_a$ and zero thereafter. In this case the above-mentioned standard deviations are obtained from equations (29) and (31):

$$\begin{aligned}\sigma_{i_T}^2 &= \frac{2\phi(0)}{T} \left(\tau_a - \frac{1}{2} \frac{\tau_a^2}{T} \right) \\ &\cong 2\phi(0) \frac{\tau_a}{T}\end{aligned}\quad (32)$$

$$\begin{aligned}\sigma_{\phi_f}^2(\tau) &\leq \frac{2\phi^2(0)}{T-\tau} \left\{ \left(\tau_a - \frac{1}{2} \frac{\tau_a^2}{T-\tau} \right) + \left[\frac{T(\tau_a - \tau)}{(T-\tau)} - \frac{(\tau_a^2 - \tau^2)}{2(T-\tau)} \right] \right\} \\ &\cong 2\phi^2(0) \left(\frac{\tau_a}{T} + \frac{\tau_a - \tau}{T} \right).\end{aligned}\quad (33)$$

The approximations are obtained under the assumption that $\tau_a \ll T$ and $\tau \ll T$. The standard deviation of the standard deviation is (33) with $\tau = 0$. As the measured values are assumed to be normally distributed, then, for instance, for 95% certainty that a measured value lies within p per cent of the true mean, the following holds:

$$0.01 \bar{p}i = 2\sigma_{i_T} \quad . \quad (34)$$

In the equation (26), $\varphi_{\alpha\alpha}(\tau)$ is a function of the mean lagged products contained in $C(\tau)$ and the average values contained in A . The errors of both the lagged products and the averages can be obtained by means of the above-described techniques, after which the error of $\varphi_{\alpha\alpha}(\tau)$ can be calculated in a normal manner.

The error of the power spectrum stems from two reasons:

- 1) $\varphi_{\alpha\alpha}(\tau)$ is known only for a finite length of time.
- 2) $\varphi_{\alpha\alpha}(\tau)$ contains an error $\Delta\varphi_{\alpha\alpha}(\tau)$.

The upper limit of the error due to the first source is

$$\left| \Delta P_1(f) \right| \leq 2 \int_{\tau_c}^{\infty} \left| \varphi_{\alpha\alpha}(\tau) \right| d\tau \quad . \quad (35)$$

Though $\varphi_{\alpha\alpha}(\tau)$ is not known for $\tau > \tau_c$, we can get an estimate of the error by extrapolation of the known portion of the correlation function for time lags greater than τ_c . The error due to the second source is bounded by

$$\left| \Delta P_2(f) \right| \leq 2 \int_0^{\tau_c} \left| \Delta\varphi_{\alpha\alpha}(\tau) \right| d\tau \leq 2\tau_c \left| \Delta\varphi_{\alpha\alpha}(\tau) \right|_{\max} \quad . \quad (36)$$

The upper bound of the total error is the sum of equations (35) and (36).

The above-described method, though giving the correct procedure for determining the stability of the estimates, is quite cumbersome and, because of the mentioned uncertainties, may easily lead to false conclusions. For a quick evaluation of the measurement effort, the following short cut may sometimes be useful. The steam void fraction solved from equation (15) is

$$\alpha(t) = \frac{\ln \frac{i(t)}{i_f(t)}}{\ln \frac{\bar{i}_e}{\bar{i}_f}} . \quad (37)$$

Writing $i(t) = i_f(t) + b(t)$ and $\bar{i}_e = \bar{i}_f + b$, one obtains

$$\alpha(t) = \frac{\ln \left[1 + \frac{b(t)}{i_f(t)} \right]}{\ln \left[1 + \frac{b}{\bar{i}_f} \right]} . \quad (38)$$

If $\frac{b(t)}{i_f(t)} \ll 1$, $\frac{b}{\bar{i}_f} \ll 1$, and $\frac{i_f}{i_f(t)} \approx 1$, one gets the approximation

$$\alpha(t) \approx \frac{i(t) - \bar{i}_f}{\bar{i}_e - \bar{i}_f} . \quad (39)$$

Thus the variations of the steam void fraction are approximately proportional to the variations of the output signal $i(t)$, and one can use the ideas presented in Chapter II for the measurement of the power spectrum of a single time function. If the estimates turn out to be too unstable, one has already gained valuable information about the magnitude of the measurement effort actually needed

CHAPTER V VELOCITY METHOD

General

The determination of the steam void fraction by the velocity method is based on the mass continuity equation of the two-phase fluid and, as mentioned earlier, employs sensitive flowmeters at the inlet of the channel under study and at the point where the void is to be measured. Depending on the assumptions made about the variables involved, one can develop models of different degrees of accuracy with respect to the relation between the steam void and the flowmeter indications. In the following, two models are developed. The less accurate one leads to a first-order, linear differential equation with randomly varying coefficients. The second model leads to an integral equation relating the power density spectra of the steam void fraction and the inlet and outlet water velocities.

If the flow is considered to be one dimensional and, assuming that the superposition of the contributions of different phases is allowed, the continuity equation in the integral form states

$$W_0 - W_2 = \frac{d}{dt} \int_{x_0}^{x_2} (\rho_f A_f + \rho_g A_g) dx \quad , \quad (40)$$

i.e., the difference between the inlet and outlet mass rate of flow is equal to the mass change rate in the volume in question. The inlet flow consists purely of water phase. Thus the quantities in equation (40) are

$$W_0 = \rho_{f0} A_{T0} v_{f0} \quad (41)$$

$$W_2 = \rho_{f2} (1 - \alpha_2) A_{T2} v_{f2} + \rho_{g2} \alpha_2 A_{T2} v_{g2} \quad (42)$$

$$A_f = (1 - \alpha) A_T \quad (43)$$

$$A_g = \alpha A_T \quad . \quad (44)$$

Equations (41) and (42) are time dependent only, but equations (43) and (44) are both space and time dependent, even though, for the sake of simplification, this is not indicated.

In the present study the total channel cross section is constant. Taking this into account and writing $v_{g2} = \gamma v_{f2}$, where γ is the slip ratio, equation (40) becomes, after rearrangement of terms,

$$\rho_{f0} v_{f0} - \rho_{f2} v_{f2} + (\rho_f - \gamma \rho_{g2}) \alpha_2 v_{f2} = \frac{d}{dt} \int_{x_0}^{x_2} [\rho_f - (\rho_f - \rho_g) \alpha] dx \quad . \quad (45)$$

Allowing the assumptions made so far, equation (45) is a general one. The interest is now to obtain from it either the time variations of the outlet steam void fraction as a function of variations of inlet and outlet velocities or, directly, the statistical properties of the void fraction in terms of the statistical properties of the velocities. Both approaches are presented in the following.

Simplified Model

Make the assumptions

- 1) ρ_f is constant in time and space;
- 2) ρ_g is constant in time and space;
- 3) time and space variations of α are separable;
- 4) α varies as some positive (integral or fractional) power of x , i.e.,

$$\alpha = 0, \text{ when } x_0 \leq x \leq x_1$$

$$\alpha = \frac{\alpha_2(t)}{(x_2 - x_1)^n} (x - x_1)^n, \text{ when } x_1 \leq x \leq x_2 \quad . \quad (46)$$

The first two assumptions are quite well justified, but the last two have their limitations. They imply, firstly, that the perturbation in steam production is to the same direction throughout the boiling length and, secondly, that the perturbation velocity is high enough to make the transit time small compared to the period of highest perturbation frequency. Supposing that the first implication holds, the accuracy of the results depends on the perturbation transit time and also on the particular frequency of variations. It is shown in Appendix C that, in this case, with a constant axial weighting of the heat flux and a constant perturbation velocity v , the amplitude of a certain frequency component is obtained by

$$A_f = A \frac{\sin \pi f \tau_m}{\pi f \tau_m} \quad , \quad (47)$$

where A is the amplitude in the case of zero perturbation transit time and $\tau_m = L_b/v$. Thus the error is given by

$$\epsilon_f = \frac{\sin \pi f \tau_m}{\pi f \tau_m} - 1 \quad . \quad (48)$$

Thus, if τ_m is large, one cannot expect accurate results except at very low frequencies.

Under the given assumptions, equation (45) simplifies to the following linear differential equation:

$$\frac{d\alpha_2}{dt} + (n+1) \frac{(\rho_f - \gamma \rho_g) v_{f2}}{(\rho_f - \rho_g) L_b} \alpha_2 + \alpha_2 \frac{1}{L_b} \frac{dL_b}{dt} = (n+1) \frac{\rho_f}{(\rho_f - \rho_g)} \frac{1}{L_b} (v_{f2} - v_{f0}) \quad , \quad (49)$$

which gives the exit steam void fraction as a function of the variations of the velocities and the boiling length. Under the conditions in which the heat flux into the channel is constant and the inlet velocity variations can be neglected, the term containing dL_b/dt drops out.

If, instead of equation (46), one assumes an exponential relation for the variation of α with the channel length, i.e.,

$$\alpha = 0 \quad , \text{ when } x_0 \leq x \leq x_1$$

$$\alpha = \frac{\alpha_2(t)}{1 - \exp(-aL_b)} \left\{ 1 - \exp[a(x_1 - x)] \right\}, \text{ when } x_1 \leq x \leq x_2 \quad , \quad (50)$$

a differential equation similar to equation (49) is obtained, but with somewhat different constants. Equations (46) and (50) give a multitude of choices to fit with the expected dependence of the steam void fraction on the channel length in a particular case under study.

At low pressures, $\rho_g \ll \rho_f$ and also as the slip ratio in a wide variety of test conditions is less than three, (19,21,22) one can neglect the terms ρ_g and $\gamma \rho_g$ as compared with ρ_f . Thus equation (49) simplifies further to

$$\frac{d\alpha_2}{dt} + (n+1) \frac{v_{f2}}{L_b} \alpha_2 + \frac{1}{L_b} \frac{dL_b}{dt} \alpha_2 = (n+1) \frac{1}{L_b} (v_{f2} - v_{f0}) \quad , \quad (51)$$

or if the boiling boundary variations are negligible,

$$\frac{d\alpha_2}{dt} + (n+1) \frac{1}{L_b} v_{f2} \alpha_2 = (n+1) \frac{1}{L_b} (v_{f2} - v_{f0}) \quad . \quad (52)$$

In the steady state, time derivatives are zero and one obtains from equation (49)

$$\alpha_2 = \frac{\rho_f}{\rho_f - \gamma \rho_g} \frac{v_{f2} - v_{f0}}{v_{f2}} \quad , \quad (53)$$

and from equation (52)

$$\alpha_2 = \frac{v_{f2} - v_{f0}}{v_{f2}} \quad . \quad (54)$$

When equation (49) is used to solve for the time variations of the exit steam void fraction, one has to express the boiling length L_b in terms of measurable quantities like the inlet velocity and the power input. Approximate expressions for these relationships have been derived, for instance, by Beckjord⁽¹⁾ and Akcasu.⁽³⁾ When test conditions are such that equation (49) with $dL_b/dt = 0$ or equation (52) can be used, one has to record only the inlet and outlet velocities to obtain a solution for α_2 . Because the velocities are randomly varying quantities the solution can best be obtained by digital or analog computers. The solution is also a random time function from which the desired statistical properties have to be extracted by the methods described in Chapter II. For a predetermined accuracy and resolution, the length of the record and the interval Δt at which the record is digitized are determined also by the rules given in Chapter II for a single time function.

Theory has been developed by Tikhonov⁽²⁵⁾ to obtain the statistical properties of α_2 directly from the properties of the variable coefficients of the differential equation (52) without actually solving it. The equations, however, appear quite complicated and their programming does not seem justified when other methods are available.

Model Two

Another approach of obtaining the statistical properties of the exit steam void fraction is to assume that the variations of α about the average value are caused by an infinite number of small sources along the boiling section of the heated channel. The following assumptions are made about the sources:

- 1) They are statistically independent.
- 2) The weighting function along the boiling section is constant, i.e., all sources vary with the same standard deviation and autocorrelation function about the zero mean.

Consider equation (45) with constant densities and a constant boiling boundary. It can then be written as

$$v_0(t) - v_2(t) [1 - a_2(t)] = - \frac{d}{dt} \int_{x_1}^{x_2} a(x, t) dx \quad , \quad (55)$$

where v 's are water velocities and

$$\left. \begin{aligned} a_2(t) &= \left(1 - \frac{\gamma \rho_g}{\rho_f}\right) \alpha_2(t) = c_2 \alpha_2(t) \\ a(x, t) &= \left(1 - \frac{\rho_g}{\rho_f}\right) \alpha(x, t) = c \alpha(x, t) \end{aligned} \right\} . \quad (56)$$

Assume further that the perturbation in steam void fraction caused at some location of the channel will propagate upwards with a constant velocity v .

The following notations are introduced:

$$I(t) = \int_{x_1}^{x_2} a(x,t) dx = \int_0^L a(x,t) dx$$

and

$$q(x,t) = \left(1 - \frac{\rho_g}{\rho_f}\right) x \text{ (source strength)} .$$

Then the variations of a at any point x are given by

$$a(x,t) = \int_0^x q\left(\xi, t - \frac{x-\xi}{v}\right) d\xi \quad (57)$$

and

$$I(t) = \int_0^L \int_0^x q\left(\xi, t - \frac{x-\xi}{v}\right) dx d\xi . \quad (58)$$

Mean Value of a_2 . To obtain the mean value of a_2 , the mean of both sides of equation (55) is taken. For the derivative

$$\begin{aligned} \left\langle \frac{dI(t)}{dt} \right\rangle &= \lim_{\tau \rightarrow 0} \left\langle \frac{I(t+\tau) - I(t)}{\tau} \right\rangle \\ &= \lim_{\tau \rightarrow 0} \frac{\langle I(t+\tau) \rangle - \langle I(t) \rangle}{\tau} = 0 , \end{aligned} \quad (59)$$

for

$$\langle I(t) \rangle = \int_0^L \int_0^x \left\langle q\left(\xi, t - \frac{x-\xi}{v}\right) \right\rangle d\xi dx = 0 .$$

Thus one obtains

$$\bar{v}_0 - \bar{v}_2 (1 - \bar{a}_2) + \phi_{v_2} a_2(0) = 0 , \quad (60)$$

from which

$$\bar{\alpha}_2 = \frac{\bar{v}_2 - \bar{v}_0 - \phi_{v_2 a_2}(0)}{c_2 \bar{v}_2} . \quad (61)$$

This shows that when random fluctuations are present, the average value of the steam void fraction is not merely equation (53) with average values of the velocities substituted for the steady velocities, but contains an extra term $-\phi_{v_2 a_2}(0)/c_2 \bar{v}_2$, where $\phi_{v_2 a_2}(0)$ is the value of the cross correlation function of v_2 and a_2 at zero correlation time.

Autocorrelation Function of α_2 . This is obtained by equating the autocorrelation functions of both sides of equation (55). The autocorrelation function of a derivative is

$$\left\langle \frac{dI(t)}{dt} \frac{dI(t+\tau)}{dt} \right\rangle = -\phi_{II}''(\tau) . \quad (62)$$

This result has been derived, for instance, by Rice.⁽¹⁰⁾ The expression for $\phi_{II}''(\tau)$ is obtained in Appendix D and is as follows:

$$\phi_{II}''(\tau) = -2v^2 \left\{ \phi_{a_2 a_2}(\tau) - \frac{1}{2L} \int_0^L \left[\phi_{a_2 a_2}\left(\tau + \frac{x}{v}\right) + \phi_{a_2 a_2}\left(\tau - \frac{x}{v}\right) \right] dx \right\} . \quad (63)$$

The autocorrelation function of the left-hand side of equation (55) is

$$\left\langle \left\{ v_0(t) - v_2(t)[1 - a_2(t)] \right\} \left\{ v_0(t+\tau) - v_2(t+\tau)[1 - a_2(t+\tau)] \right\} \right\rangle . \quad (64)$$

This is expanded in Appendix D. The final result is

$$D(\tau) + E(\tau)\phi_{a_2 a_2}(\tau) + F(\tau) + G(\tau) , \quad (65)$$

where

$$D(\tau) = \phi_{v_0 v_0}(\tau) - (1 - \bar{\alpha}_2) \left[\phi_{v_0 v_2}(\tau) + \phi_{v_0 v_2}(-\tau) \right] + (1 - \bar{\alpha}_2)^2 \phi_{v_2 v_2}(\tau) \quad (66)$$

$$E(\tau) = \phi_{v_2 v_2}(\tau) + \bar{v}_2^2 \quad (67)$$

$$F(\tau) = \bar{v}_2 \left[\phi_{v_0 a_2}(\tau) + \phi_{v_0 a_2}(-\tau) \right] \quad (68)$$

and

$$G(\tau) = -\bar{v}_2(1 - \bar{\alpha}_2) \left[\phi_{v_2 a_2}(\tau) + \phi_{v_2 a_2}(-\tau) \right] + \phi_{v_2 a_2}(\tau) \phi_{v_2 a_2}(-\tau) . \quad (69)$$

Equating (63) and (65) with proper signs, one obtains an equation which relates the autocorrelation function of a_2 with other correlation functions, some of which can be obtained directly from the measurements. The equation reads

$$D(\tau) + E(\tau) \phi_{a_2 a_2}(\tau) + F(\tau) + G(\tau) = 2v^2 \left\{ \phi_{a_2 a_2}(\tau) - \frac{1}{2L} \int_0^L \left[\phi_{a_2 a_2} \left(\tau + \frac{x}{v} \right) + \phi_{a_2 a_2} \left(\tau - \frac{x}{v} \right) \right] dx \right\} \quad (70)$$

Standard Deviation of α_2 . Setting $\tau = 0$ in equation (70) and solving for $\sigma_{a_2^2}$, one gets

$$\sigma_{a_2^2} = \frac{D(0) + F(0) + G(0)}{2v^2 \left[1 - \frac{1}{L} \int_0^L \phi_{a_2 a_2} \left(\frac{x}{v} \right) dx \right] - E(0)} \quad (71)$$

The quantities in $D(0)$ and $E(0)$ can easily be obtained from the velocity measurements, but the terms $\phi_{v_0 a_2}(0)$ and $\phi_{v_2 a_2}(0)$ in $F(0)$ and $G(0)$ contain crosscorrelation of velocities with the unknown quantity a_2 and are thus unknown. Equation (71) contains also $\phi_{a_2 a_2}(\tau)$ which is the autocorrelation function of a_2 . However, in order to get an estimate of σ_{a_2} one can make the following approximations. It is quite conceivable that $\phi_{v_0 a_2}(\tau)$ and $\phi_{v_2 a_2}(\tau)$ have relatively small values at $\tau \leq 0$, for the steam void fraction does not follow instantly the variations of the water velocity and is thus practically uncorrelated with the velocity at small or negative values of τ . This being the case, $F(0)$ and $G(0)$ can be neglected as compared with $D(0)$ in equation (71). If, also, the autocorrelation function $\phi_{a_2 a_2}(\tau)$ dies out rapidly, the integral term in equation (71) will be small compared to one and one gets as an appropriate expression for σ_{a_2}

$$\sigma_{a_2^2} \cong \frac{D(0)}{2v^2 - E(0)} \quad (72)$$

In the case that σ_{a_2} and $\phi_{a_2 a_2}(\tau)$ are known from other measurements, like gamma-ray measurements, equation (71) can be used to obtain an estimate of the steam perturbation velocity in the channel.

Power Density Spectrum of α_2 . Taking the Fourier transform on both sides of equation (70), the following is obtained:

$$P_{DD}(f) + \int_{-\infty}^{\infty} P_{EE}(f-f') P_{a_2a_2}(f') df' + P_{FF}(f) + P_{GG}(f) = 2v^2 \left(1 - \frac{v}{wL} \sin \frac{wL}{v}\right) P_{a_2a_2}(f) \quad , \quad (73)$$

or, taking equation (67) into account and rearranging,

$$H(f) P_{a_2a_2}(f) = P_{DD}(f) + P_{FF}(f) + P_{GG}(f) + \int_{-\infty}^{\infty} P_{v_2v_2}(f-f') P_{a_2a_2}(f') df' \quad , \quad (74)$$

where

$$H(f) = 2v^2 \left(1 - \frac{v}{wL} \sin \frac{wL}{v}\right) - \bar{v}_2^2 \quad . \quad (75)$$

Equation (74) is a linear integral equation of the third kind. As in practice the power spectral densities vanish after a certain frequency, the limits of integration are finite. The kernel $P_{v_2v_2}(f)$ does not have any singularities and all the other terms are finite. Thus, according, for instance, to Schmeidler,⁽²⁶⁾ the solution exists.

Here again $P_{FF}(f)$ and $P_{GG}(f)$ contain cross power spectral densities of the velocities and the unknown α_2 . These can be approximately obtained by computing the time function of α_2 by the Simplified Model and correlating it with the measured velocities. The solution for $P_{a_2a_2}(f)$ from equation (74) in this manner should yield a close approximation at low frequencies, i.e., below 5 cps. An order of magnitude of the error can be calculated by means of equation (48) combined with the techniques of estimating the error of different power spectral density estimates as outlined in Chapter II.

Another approximation is obtained as follows. Assume that at low frequencies the cross power spectral densities are related to $P_{a_2a_2}(f)$ as

$$\left. \begin{aligned} P_{v_0a_2}(f) &= \frac{\sigma_{v_0}}{\sigma_{a_2}} P_{a_2a_2}(f) \\ P_{v_2a_2}(f) &= \frac{\sigma_{v_2}}{\sigma_{a_2}} P_{a_2a_2}(f) \end{aligned} \right\} \quad , \quad (76)$$

which presupposes that the spectral density of the steam void fraction is proportional to the spectral densities of the water velocities and that the cross spectral densities $P_{v_0a_2}(f)$ and $P_{v_2a_2}(f)$ are real at low frequencies.

Further, if one makes a similar assumption about $\phi_{v_2 a_2}(\tau)$ for $\tau \leq 0$, as in connection with getting an estimate of the standard deviation of a_2 , the convolution term in $P_{GG}(f)$ is dropped out. Thus equation (74) is put in a simple form, from which the approximate solution of $P_{a_2 a_2}(f)$ is readily obtained.

Comparison of Simplified Model and Model Two

Considering the initial assumptions, one sees that in terms of Model Two the simplified model means that

- 1) all the sources are completely correlated, and
- 2) the steam perturbation velocity is infinite.

It is realized that the latter model is far more general than the former and involves assumptions all of which can be considered moderate. However, as it was pointed out, it involves cross-correlation terms which cannot be obtained directly from measurement, but the form of which either has to be assumed on the basis of physical consideration or previous knowledge, or calculated using the simplified model. In this respect, the simplified model complements the more accurate one in a useful manner.

CHAPTER VI EXPERIMENTAL RESULTS

Data Manipulation

Both the gamma-ray and the inlet and outlet velocity traces were recorded simultaneously on a 30-cm-wide photographic chart whose speed was chosen as 5 cm/sec to allow the digitation of the traces at 0.05-sec intervals. According to the theories outlined in Chapter II, this reading density will yield the power spectral densities up to $f_{\max} = 1/(2\Delta t) = 10$ cps. Digitizing the traces constituted a rather big effort, even though a Benson-Lehner chart reader was used, which could be connected with a keypunch to punch the data on IBM cards simultaneously while being read. To punch 1000 data points in this manner required several hours of effort. The accuracy of the chart reader is very good. Thus the reading error is practically entirely due to other reasons, like the thickness of the trace, and can be considered negligible. In addition, when the correlation methods are used, the reading error just generates a very low-amplitude white noise whose contribution is practically zero. The reading effort can be eliminated almost completely by using analogue to digital data recording systems, which consequently make it easy to use very long records.

To compute the power spectral densities of the steam void fraction, equations (24), (25), and (26) were programmed for IBM 704 computer. The Argonne code is RE 245. Direct autocorrelation analyses were performed by means of an existing code.⁽²⁷⁾ The cross-correlation code (RE 243) was initiated but was not made available within the time limit of the appointment.

Test Conditions of Analyzed Runs

Following is a list of the test condition of the runs which were chosen as typical representatives and of which at least one of the traces was analyzed.

General Features (Armadilla test loop):

Test channel	Stainless Steel Tube		
Test channel length	91.5 cm		
Test channel inner diameter	1.58 cm		
Riser length	104 cm		
Riser inner diameter	1.58 cm		
Run	71	ST5	ST9
Power density (kw/liter)	70	112	168
System pressure (atm)	11.5	42.5	42.5.
Subcooling (°C)	1.8	18.4	23
Water level from test channel outlet (cm)	~145	~145	~150
Average inlet water velocity (cm/sec)	63	95	95
Average outlet water velocity (cm/sec)	150	161	205
Average outlet steam void fraction	0.58	0.41	0.54

Also, a gamma-ray trace was recorded, using the so-called small-scale loop. In this particular run the conditions were:

Test channel	Rectangular Tube
Test channel length	127 cm
Test channel cross section	4.45 x 1.11 cm
Power density	48 kw/liter
System pressure	42 atm
Subcooling	12.0°C
Average inlet water velocity	77 cm/sec
Average outlet water velocity	133 cm/sec
Average outlet steam void fraction	0.42

A sample of the recorded traces from Run ST5 is given in Fig. 8.

Fig. 8

Typical Recording of (A) Coolant Velocity and (B) Gamma-ray Signals (Run ST5)

Gamma-ray Attenuation Method

The code RE 245 was used to compute the power density spectrum of α_2 for the Run ST5. One thousand points were sampled at 0.05-sec intervals. The largest time lag $M\Delta t$ for computing the autocorrelation function was 5 sec. The result, shown in Fig. 9, has a very large scatter, the estimates having errors of several hundred per cent. This is partly due to a short recording time and partly due to a very low signal-to-noise ratio in the measurement (~ 0.25). The low signal-to-noise ratio was caused by the particular geometry, i.e., steam and water constituted only a small fraction of the attenuation of the gamma signal, whereas the tube walls and air constituted the major part. Thus the variations of the steam void fraction practically vanished in the statistical variations of the source.

Fig. 9. Power Density Spectrum of the Exit Steam Void Fraction (Run ST5)

In spite of the large scatter, the results in Fig. 9 seem to indicate that the power spectral density is a white noise. To check this, a gamma-ray trace was recorded with the Small Scale Heat Transfer Loop which had a rectangular channel and in which the steam-water path constituted the major attenuation of gamma rays. Here the signal-to-noise ratio was about four. Two thousand points were used for the analysis, with $M = 100$. The result is given in Fig. 10 and indicates clearly that the power density spectrum of the variations of the steam void fraction is a white noise. In this case, the bandwidth of about 1.9 db contains 60% of the spectral density estimates. If the approxi-

mation of equation (39) is considered valid, one would calculate that the corresponding bandwidth is 1.6 db. The slight increase in the actual case is due to the presence of the source and scintillation crystal noise. The two examples serve to demonstrate the roles of the recording time and the signal-to-noise ratio in using the gamma-ray method.

Fig. 10. Power Density Spectrum of the Exit Steam Void Fraction (Small Scale Loop Run)

Velocity Methods

In applying the velocity methods for obtaining the power density spectrum of the exit steam void variations, the computed spectra of the velocities have to be corrected for the effects of the flowmeter time constant and the transfer characteristics of the frequency converter. Likewise, the correction given in equation (14) has to be taken into account.

In the previously mentioned runs, the flow corresponding to the outlet velocity was around 25 liters/min. According to the curves given by Grey,⁽²³⁾ the Potter-meter sensing element which was used in the measurements has a time constant of about 23 ms at this particular flow rate. Taking into account the increase due to the presence of steam as given in equation (13) for α around 0.5, a time constant of 45 ms resulted. It corresponds to a break frequency of 3.5 cps. The frequency converter has break frequencies at 9 cps and 19 cps, according to Appendix B. The combined correction curve is plotted in Fig. 11.

Fig. 11. Correction Factor for Power Density Spectra of Velocities

The code⁽²⁷⁾ which was used to analyze the velocity traces not only computes the autocorrelation function and power spectrum but also the distribution of data around the mean to provide a check as to how close to the Gaussian distribution they are. Such a histogram is given in Fig. 12 for the outlet velocity data obtained from Run 71. The distribution can be considered nearly Gaussian. The autocorrelation function of the same data appears in Fig. 13.

Fig. 12. Histogram of the Outlet Velocity Data (Run 71)

Fig. 13. Autocorrelation Function of the Outlet Velocity (Run 71)

The power density spectra of outlet velocities from Runs 71, ST5, and ST9 are plotted in Fig. 14 in a normalized form. In all of these computations, 1000 sampled points were used with $M = 100$. The sampling interval was 0.05 sec, except for the Run 71, for which it was 0.1 sec. The inlet velocity variations were so small in all runs that they were not analyzed. In other words, v_0 was taken as zero. The power density spectra of the outlet velocities with the correction of Fig. 11 taken into account are plotted in Fig. 15.

To compute the power density spectrum of α_2 by the simplified model, equation (52) was solved with the aid of a PACE-Analog computer for Run ST5. Here the boiling length L_b was calculated to be 32 cm and n was chosen as 0.5 to approximate the dependence of the steam void fraction with the channel length in equation (46). This value of n corresponds to having $a = 2$ in equation (50), and the multiplier of terms containing the velocities equal to 1.5 in the differential equation. A sample of the solution is shown in Fig. 16 together with the velocities. The power density spectrum of the solution with the correction of Fig. 11 is plotted in Fig. 17.

Fig. 14. Measured Power Density Spectra of Outlet Velocities

Fig. 15. Corrected Power Density Spectra of Outlet Velocities

Fig. 16. Analog Computer Solution of the Exit Steam Void Fraction by Simplified Model (Run ST5)

Fig. 17. Power Density Spectra of the Exit Steam Void Fraction by Velocity Methods (Run ST5)

Because the cross-correlation code was not available, the approximations presented in equation (76) were used. As the variations of inlet velocity are negligible, equation (74) simplifies to

$$\left[2v^2 \left(1 - \frac{v}{wL} \sin \frac{wL}{v} \right) + 2\bar{v}_2 (1 - \bar{\alpha}_2) \frac{\sigma_{v_2}}{\sigma_{\alpha_2}} - \bar{v}_2^2 \right] P_{a_2 a_2}(f) =$$

$$(1 - \bar{\alpha}_2)^2 P_{v_2 v_2}(f) + \int_{-f_m}^{f_m} P_{v_2 v_2}(f - f') P_{a_2 a_2}(f') df' \quad (77)$$

For Run ST5, the values of the quantities are:

$$\begin{array}{ll} \bar{v}_0 = 95 \text{ cm/sec} & v = 322 \text{ cm/sec} \\ \bar{v}_2 = 161 \text{ cm/sec} & L = 32 \text{ cm} \\ \sigma_{v_0} = 0 & \bar{\alpha}_2 = 0.41 \\ \sigma_{v_2} = 12.6 \text{ cm/sec} & \sigma_{\alpha_2} = 0.036 \end{array}$$

Substituting these in equation (77), it is noticed that the integral term is negligible compared to the other terms, which further simplifies the equation. Figure 17 shows the power density spectrum of the exit steam void fraction calculated from it.

The velocity methods as presented here seem to yield power spectral densities which differ from that obtained by the gamma-ray method. In the case of the simplified model, the deviation is, of course, partly due to the initial assumptions; but in the case of the more accurate model, the assumptions are moderate and the deviation was not anticipated to be as big as shown in Fig. 17. It is possible that the time constants calculated by Grey for the Potter sensing element are actually larger than given, which would explain the differences from the gamma-ray results.

Because the power density spectrum of α_2 is white in the frequency range for which the analysis was made, the power spectra of outlet velocities as given in Fig. 14 indicate the Potter-meter output-steam void transfer functions for the given test conditions. If this curve is determined beforehand for a particular flowmeter and accompanying equipment, then the results obtained from the actual measurements can be corrected for its effect. In this case, one does not have to use the above presented models for computing the power density spectra of the steam void fraction.

CHAPTER VII CONCLUSIONS

In the present study, the use of the gamma-ray attenuation method, which previously has been successfully applied for the determination of the steady-state value and periodic variations of the steam void fraction in boiling channels, was extended to measure the dynamic properties of this quantity from the recordings of the so-called boiling noise. The equations were derived to compute the standard deviation, the autocorrelation function, and the power density spectrum of the steam void fraction. They take into account the statistical noise of the gamma-ray source, the scintillation crystal, and the photomultiplier tube, the standard deviation of this noise appearing in the equations in an essential manner. To reduce the measuring effort, one should minimize the level of this noise by choosing a strong source, a low-noise photomultiplier tube, and, if possible, such a geometry that the steam-water mixture constitutes the major part of the attenuation of the gamma-ray beam. A procedure was also outlined to calculate the error of measurements and to predetermine the recording length in the general case; an approximation to this was given which is closely valid in a low-noise case.

The second part of the work was devoted to the study of using sensitive flowmeters for the determination of the dynamic properties of the steam void. Two models, both based on the mass continuity equation of the two-phase fluid, were developed to correlate the variations of the steam void with the variations of the water velocities detected at the channel inlet and at the location where the properties of the void are to be measured. The simpler model assumes that the perturbations of the void fraction are completely correlated and that the velocity by which perturbations are propagated is infinite. It leads to a simple first-order linear differential equation with randomly variable coefficients. The more sophisticated model starts from the assumption that the steam void perturbations are caused by a row of statistically independent bubble sources along the boiling channel. It leads to a linear integral equation of the third kind, which relates the power density spectrum of the steam void with the power density spectra of the velocities.

A comparison of the results obtained with the gamma-ray and velocity methods from the same experiment indicates that neither model satisfactorily relates the velocity variations with the steam void variations over the entire range of frequencies studied. However, if the steam transit time is about 0.1 sec and the frequency range of interest less than one cycle per second, both models are useful and have approximately the same accuracy. At frequencies greater than one cycle per second, the deviation from the gamma-ray results is more than three decibels for a 0.1-sec steam transit time. The second model gives slightly better results than the first. The accuracy of the models increases with decreasing steam transit time.

If the flowmeters are preferred or the only feasible way to obtain information about the steam void variations, it is recommended whenever possible to predetermine the power density spectra of the velocity variations for the particular flowmeter size together with the gamma-ray void spectrum from a simulated experiment, such as was used in this study. The correction factor thus obtained can then be applied to the results from the actual experiments.

APPENDIX A

DERIVATION OF THE TIME CONSTANT OF POTTER SENSING
ELEMENT IN A TWO-PHASE FLOW

For the purpose of analysis assume a step change of the fluid velocity the slip ratio remaining constant. During the subsequent speeding up period of the rotor the hydrodynamic forces due to one phase are shown in the diagram of Fig. 18. Assume further that the fluid is incompressible and nonviscous, that the rotor bearing friction is negligible, and that the superposition holds for the contributions of each phase. Then the equation of the rotor can be written as

$$T = I \frac{d^2\theta}{dt^2} \quad . \quad (77)$$

Fig. 18

Hydrodynamic Forces on a
Blade Element During a
Transient in a Single Phase
Flow

During the transient, the torques exerted by each phase on a blade element of chord c and length dr are (referring to Fig. 18)

$$\left. \begin{aligned} dT_1 &= L'_1 \cos \delta_1 (1 - \alpha_2) r dr \\ dT_2 &= L'_2 \cos \delta_2 \alpha_2 r dr \end{aligned} \right\} , \quad (78)$$

where L' is the force per unit blade angle perpendicular to the effective velocity v and α_2 is the steam void fraction. The force L' is related to the lift coefficient C_L by the expression

$$L' = C_{L'} \left(\frac{1}{2} \rho v^2 c \right) , \quad (79)$$

and $C_{L'}$ is given by

$$C_{L'} = \frac{2\pi\eta \sin \alpha_0}{1 + (2\eta/A_R)} , \quad (80)$$

which introduces the effect of the induced drag. The following equations also hold:

$$v = u / \cos \delta \quad (81)$$

$$\delta = \tan^{-1} \frac{r(d\theta/dt)}{u} \quad (82)$$

$$u_2 = \gamma u_1 \quad . \quad (83)$$

Thus the torque becomes

$$\begin{aligned} dT &= dT_1 + dT_2 \\ &= \frac{2\pi\eta \sin(\alpha - \delta_1)}{1 + (2\eta/A_R)} \left[\frac{1}{2} \rho_1 \left(\frac{u_1}{\cos \delta_1} \right)^2 c \right] \left[\cos \delta_1 (1 - \alpha_2) \right] r dr \\ &= \frac{2\pi\eta \sin(\alpha - \delta_2)}{1 + (2\eta/A_R)} \left[\frac{1}{2} \rho_2 \left(\frac{u_2}{\cos \delta_2} \right)^2 c \right] \left[\cos \delta_2 \alpha_2 \right] r dr \\ &= \frac{\pi\eta \rho_1 u_1^2 c}{1 + (2\eta/A_R)} \left[(1 - \alpha_2) \frac{\sin(\alpha - \delta_1)}{\cos \delta_2} + \alpha_2 \left(\frac{\rho_2 \gamma^2}{\rho_1} \right) \frac{\sin(\alpha - \delta_2)}{\cos \delta_2} \right] r dr \quad .(84) \end{aligned}$$

Due to the general geometry of the blades, only a small error will be introduced by taking a mean radius R and a mean chord C . Thus one obtains

$$\begin{aligned} \frac{d^2\theta}{dt^2} &= \frac{1}{I} \int_{r_1}^{r_2} dT \\ &= \frac{N\pi\eta \rho_1 u_1^2 CR (r_2 - r_1)}{I[1 + (2\eta/A_R)]} \cos \alpha \left[(1 - \alpha_2) \left(\tan \alpha - \frac{R}{u_1} \frac{d\theta}{dt} \right) \right. \\ &\quad \left. + \alpha_2 \left(\frac{\rho_2 \gamma^2}{\rho_1} \right) \left(\tan \alpha - \frac{R}{\gamma u_1} \frac{d\theta}{dt} \right) \right] \quad . \quad (85) \end{aligned}$$

Let

$$K = \frac{N\pi\eta \rho_1 u_1^2 CR^2 (r_2 - r_1)}{I[1 + (2\eta/A_R)]}$$

$$L = 1 - \alpha_2 + \alpha_2 \frac{\rho_2 \gamma^2}{\rho_1}$$

$$M = 1 - \alpha_2 + \alpha_2 \frac{\rho_2 \gamma}{\rho_1}$$

$$z = \frac{R}{u_1} \frac{d\theta}{dt} \quad . \quad (86)$$

Then equation (85) simplifies to

$$\frac{dz}{dt} = K \cos \alpha (L \tan \alpha - Mz) \quad (87)$$

The rotor time constant τ_r is determined by

$$\int_0^{\tau_r} dt = \int_{z_0}^{z_T} \frac{dz}{A \cos \alpha (L \tan \alpha - Mz)} \quad . \quad (88)$$

Here z_0 corresponds to the initial velocity u_0 and z_T to the velocity $u_0 + [1 - (1/e)](u_1 - u_0)$ where u_1 is the final velocity after the step.

$$z_0 = \frac{u_0}{u_1} \frac{L}{M} \tan \alpha$$

$$z_T = \left[1 - \frac{1}{e} \left(1 - \frac{u_0}{u_1} \right) \right] \frac{L}{M} \tan \alpha \quad . \quad (89)$$

Carrying out the integration one finally gets

$$\tau_r = \frac{1}{KM \cos \alpha} \quad . \quad (90)$$

It is seen that τ_r is increased by a factor of $k_T = 1/M$ because of the presence of the second phase.⁽²³⁾ This factor is given below as a function of the exit steam void fraction for the case that the pressure is 42 atm and the slip ratio γ is 2.

If one assumes that the step increase of velocity occurs only in one phase, it can be shown by a similar procedure as above that the time constant is given by equation (90) when the final slip ratio is substituted in M .

Setting equation (85) equal zero, one obtains the steady-state velocity of the rotor:

$$\left(\frac{d\theta}{dt} \right)_0 = \frac{u_0}{R} \frac{L}{M} \tan \alpha \quad . \quad (91)$$

Thus if one is interested in measuring water velocities in the presence of steam a correction, $k_v = M/L$ has to be taken into account. It is given below for the same conditions as k_T .

<u>α_2</u>	<u>k_T</u>	<u>k_v</u>
0.1	1.11	0.995
0.2	1.23	0.988
0.3	1.40	0.979
0.4	1.61	0.968
0.5	1.90	0.953
0.6	2.32	0.933
0.7	2.98	0.902
0.8	4.14	0.854
0.9	6.81	0.759
1.0	19.2	0.500

APPENDIX B

TRANSFER FUNCTION OF THE FILTER STAGE OF
POTTER FREQUENCY CONVERTER MODEL 3C

The circuit diagram of the filter stage of Potter frequency converter Model 3C is given in Fig. 19.

Fig. 19

Diagram of the Filter Stage of
Potter Frequency Converter
Model 3C

$$R_e = 945 \text{ ohms to } 1080 \text{ ohms}$$

$$R_1 = 330 \text{ ohms}$$

$$R_o = 60 \text{ ohms}$$

$$C_1 = 50 \mu f$$

$$C_2 = 200 \mu f$$

$$C_3 = 500 \mu f$$

Its transfer function is derived readily to be

$$G_f = \frac{1}{A_1 s^2 + A_2 s + A_3}, \quad (92)$$

where

$$A_1 = R_e R_1 C_1 C_2$$

$$A_2 = R_e \left(\frac{R_1 + R_o}{R_o} C_1 + C_2 \right) + R_1 C_2$$

$$A_3 = \frac{R_e + R_1}{R_o} + 1.$$

The break frequencies, for given values of the quantities and different positions of the switch, are as follows:

R_e , ohms	Switch Position	$f_{b_1'}$ cps	$f_{b_2'}$ cps
945	Fast	9.4	19.4
945	Slow	3.75	13.7
1080	Fast	9.0	19.3
1080	Slow	3.25	13.8

It may be noted that the break frequencies decrease with increasing R_o .

APPENDIX C

ERROR CAUSED BY THE FINITE PERTURBATION
VELOCITY IN THE SIMPLIFIED MODEL

Assume that there is a row of n bubble sources, separated by equal distances L_b/n , along the heated channel, each of which varies sinusoidally with the same magnitude and frequency and without phase lag. In other words, the variations of the sources are to the same direction at any time. Assume also a constant steam perturbation velocity. Then the total steam void fraction at the outlet of the channel is the sum

$$\alpha_2 = \sum_{i=1}^n q \sin w \left[t - \frac{iL_b}{(n+1)v} \right] \quad . \quad (93)$$

Expanding, one obtains

$$\alpha_2 = q \frac{\sin \left[\frac{n}{(n+1)} \frac{wL_b}{2v} \right]}{\sin \left[\frac{1}{(n+1)} \frac{wL_b}{2v} \right]} \sin \left(wt - \frac{wL_b}{2v} \right) \quad . \quad (94)$$

Because n is large, this simplifies to

$$\alpha_2 \cong nq \frac{2v}{wL_b} \sin \left(\frac{wL_b}{2v} \right) \sin \left(wt - \frac{wL_b}{2v} \right) \quad . \quad (95)$$

If the steam perturbation velocity is infinite, one gets

$$\alpha_2 = nq \sin wt \quad . \quad (96)$$

Thus the amplitude and phase errors due to the finite velocity are

$$\epsilon_a = \frac{2v}{wL_b} \sin \left(\frac{wL_b}{2v} \right) - 1 \quad (97)$$

$$\epsilon_\varphi = wL_b/2v \quad . \quad (98)$$

For a typical value $\tau_m = L_b/v = 0.25$ sec, there are found $\epsilon_a = -10$ per cent and $\epsilon_\varphi = 45^\circ$ when $f = 1$ cps, and $\epsilon_a = -26$ per cent and $\epsilon_\varphi = 90^\circ$ when $f = 2$ cps.

APPENDIX D

DERIVATION OF EQUATIONS OF MODEL TWO

To obtain $\phi_{II}''(\tau)$, equation (58) is used to derive first the expression for $\phi_{II}(\tau)$. One has

$$\begin{aligned}\phi_{II}(\tau) &= \langle I(t) I(t + \tau) \rangle \\ &= \int_0^L dx \int_0^L dy \int_0^x d\xi \int_0^y d\eta \left\langle q\left(\xi, t - \frac{x-\xi}{v}\right) q\left(\eta, t + \tau - \frac{y-\eta}{v}\right) \right\rangle.\end{aligned}\quad (99)$$

The independency condition for the sources can be expressed as

$$\langle q(x_1, t_1) q(x_2, t_2) \rangle = \delta(x_1 - x_2) \phi_{qq}(t_1 - t_2). \quad (100)$$

Thus

$$\phi_{II}(\tau) = \int_0^L dx \int_0^L dy \int_0^x d\xi \int_0^y d\eta \delta(\xi - \eta) \phi_{qq} \left[\tau + \frac{(x-y) - (\xi - \eta)}{v} \right].$$

When performing the double integration over x and y , one has to note that the integrand has a different value for $x < y$ and $x > y$, i.e.,

$$\begin{aligned}\int_0^x d\xi \int_0^y d\eta \delta(\xi - \eta) \phi_{qq} \left[\tau + \frac{(x-y) - (\xi - \eta)}{v} \right] &= x \phi_{qq} \left(\tau + \frac{x-y}{v} \right), \text{ when } x < y \\ &= y \phi_{qq} \left(\tau + \frac{x-y}{v} \right), \text{ when } y < x.\end{aligned}$$

Thus one gets

$$\begin{aligned}\phi_{II}(\tau) &= \int_0^L dx \int_0^x y \phi_{qq} \left(\tau + \frac{x-y}{v} \right) dy + \int_0^L dy \int_0^y x \phi_{qq} \left(\tau + \frac{x-y}{v} \right) dx \\ &= \int_0^L dx \int_0^x y \left[\phi_{qq} \left(\tau + \frac{x-y}{v} \right) + \phi_{qq} \left(\tau - \frac{x-y}{v} \right) \right] dy.\end{aligned}\quad (102)$$

From this

$$\phi_{II}''(\tau) = \int_0^L dx \int_0^x y \left[\phi_{qq}'' \left(\tau + \frac{x-y}{v} \right) + \phi_{qq}'' \left(\tau - \frac{x-y}{v} \right) \right] dy. \quad (103)$$

The integration can be performed by noting that, for instance,

$$\phi''_{qq} \left(\tau + \frac{x-y}{v} \right) = -v \frac{d}{dy} \phi'_{qq} \left(\tau + \frac{x-y}{v} \right) .$$

The result is

$$\phi''_{II}(\tau) = -2v^2 L \phi_{qq}(\tau) + v^2 \int_0^L \left[\phi_{qq} \left(\tau + \frac{x}{v} \right) + \phi_{qq} \left(\tau - \frac{x}{v} \right) \right] dx \quad . \quad (104)$$

Now, ϕ_{qq} is related to the autocorrelation function $\phi_{a_2 a_2}(\tau)$. This relation is obtained in the following manner with the aid of equations (57) and (100):

$$\begin{aligned} \phi_{a_2 a_2}(\tau) &= \int_0^L \int_0^L \left\langle q \left(\xi, t - \frac{L-\xi}{v} \right) q \left(\eta, t + \tau - \frac{L-\eta}{v} \right) \right\rangle d\xi d\eta \\ &= \int_0^L d\xi \int_0^L d\eta \delta(\xi - \eta) \phi_{qq} \left(\tau + \frac{\xi - \eta}{v} \right) = L \phi_{qq}(\tau) \quad . \quad (105) \end{aligned}$$

The autocorrelation function of the left-hand side of equation (55) becomes

$$\begin{aligned} &\left\langle \left\{ v_0(t) - v_2(t) [1 - a_2(t)] \right\} \left\{ v_0(t + \tau) - v_2(t + \tau) [1 - a_2(t + \tau)] \right\} \right\rangle \\ &= \left\langle v_0(t) v_0(t + \tau) \right\rangle - \left\langle v_0(t) v_2(t + \tau) [1 - a_2(t + \tau)] \right\rangle \\ &\quad - \left\langle v_0(t + \tau) v_2(t) [1 - a_2(t)] \right\rangle + v_2(t) v_2(t + \tau) \left\langle [1 - a_2(t)] [1 - a_2(t + \tau)] \right\rangle \quad . \quad (106) \end{aligned}$$

Here

$$\left\langle v_0(t) v_0(t + \tau) \right\rangle = \phi_{v_0 v_0}(\tau) + \bar{v}_0^2 \quad . \quad (107)$$

Assuming that the quantities have normal distributions, the second term in equation (106) is obtained from the relation⁽²⁴⁾

$$\left\langle [v_0(t) - \bar{v}_0] [v_2(t + \tau) - \bar{v}_2] [1 - a_2(t + \tau) - (1 - \bar{a}_2)] \right\rangle = 0 \quad . \quad (108)$$

Expanding this, one gets the relation

$$\begin{aligned} \left\langle v_0(t) v_2(t + \tau) [1 - a_2(t + \tau)] \right\rangle &= (1 - \bar{a}_2) \phi_{v_0 v_2}(\tau) - \bar{v}_0 \phi_{v_2 a_2}(0) - \bar{v}_2 \phi_{v_0 a_2}(\tau) \\ &\quad + \bar{v}_0 \bar{v}_2 (1 - \bar{a}_2) \quad . \quad (109) \end{aligned}$$

The third and fourth terms in equation (106) can be derived in a similar manner. Substituting the final results into the equation, one gets, taking equation (60) into account also,

$$\begin{aligned}
 & \left\langle \left\{ v_0(t) - v_2(t) [1 - a_2(t)] \right\} \left\{ v_0(t+\tau) - v_2(t+\tau) [1 - a_2(t+\tau)] \right\} \right\rangle \\
 &= \phi_{v_0 v_0}(\tau) - (1 - \bar{a}_2) \left[\phi_{v_0 v_2}(\tau) + \phi_{v_0 v_2}(-\tau) \right] + (1 - \bar{a}_2)^2 \phi_{v_2 v_2}(\tau) \\
 &+ \left[\phi_{v_2 v_2}(\tau) + \bar{v}_2^2 \right] \phi_{a_2 a_2}(\tau) + \bar{v}_2 \left[\phi_{v_0 a_2}(\tau) - \phi_{v_0 a_2}(-\tau) \right] \\
 &- \bar{v}_2 (1 - \bar{a}_2) \left[\phi_{v_2 a_2}(\tau) + \phi_{v_2 a_2}(-\tau) \right] + \phi_{v_2 a_2}(\tau) \phi_{v_2 a_2}(-\tau) \quad . \quad (110)
 \end{aligned}$$

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