A Determination of the Asymptotic Ratio of the Deuteron

MASTER

D- and S-state Amplitudes

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ABSTRACT

We deduce the value 0.02649 ± 0.00043 for the deuteron asymptotic D- to S-state ratio from measurements of the $^{208}\text{Pb}(d,p)^{209}\text{Pb}$ tensor analyzing powers at sub-Coulomb bombarding energies $E_d = 7, 8$ and 9 MeV. The quoted uncertainty includes statistical errors, beam polarization measurement errors and systematic errors which result from a choice of optical model parameters used to describe the reaction. The result is compared to several theoretical deuteron wavefunctions.

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In spite of years of experimental effort, many properties of the deuteron are still poorly determined. A continued study of the deuteron is required if stringent tests of n-p interaction theories are to be possible. Although the measurability of the deuteron D-state probability has recently been called into question¹, the asymptotic ratio of the D- and S-state amplitudes (η) is accepted as a well defined measurable quantity. Two methods have been proposed in the literature to measure η . It has been suggested² that the tensor analyzing powers in p-d elastic scattering are sensitive to η . The sensitivity is a result of a pole in the scattering amplitude at an unphysical angle θ_p , where $\cos\theta_p < -1$. The extraction of η requires an extrapolation of tensor analyzing power and cross section angular distributions to θ_p . Such extrapolations have been performed³⁻⁴. Of course, in any experiment designed to measure a fundamental quantity to high accuracy, an accurate error analysis is most important. The difficulties in performing a rigorous error analyses of the extrapolation method have been discussed by Colby et al.⁵. The other method to measure η was proposed in a recent Letter by Knutson et al.⁶. The method is based on the observation that distorted-wave Born approximation (DWBA) calculations predict that the tensor analyzing powers of (d,p) stripping reactions have a strong (essentially linear) dependence on η . Moreover, at

bombarding energies below the Coulomb barrier, the fundamental assumptions in DWBA are particularly well satisfied and the dependence on parameters in the theory other than η is drastically decreased. Specifically, the dependence on the nuclear optical model parameters which describe the incoming deuteron and outgoing proton scattering waves decreases because the scattered particles do not penetrate into the nuclear interior. As a result, the scattering waves are perturbed Coulomb wavefunctions. By adjusting the value of η to fit measured tensor analyzing power angular distributions of $^{208}\text{Pb}(d,p)^{209}\text{Pb}$ at $E_d = 9$ MeV, Knutson et al. determined that⁷ $\eta = 0.0234 \pm 0.0017$. This value of η is smaller than that of many published deuteron wave functions. For example, the wavefunctions of Ref. 8-10 predict $\eta = 0.0262, 0.0260$ and 0.0260 , respectively. Unfortunately, because of the large error bar, the measurement is unable to conclusively rule out these wavefunctions and a measurement with a smaller error is desired. The sources of error in Ref. 6 were primarily from uncertainties in the beam polarization measurement and uncertainties in the nuclear optical model parameters. The former error can be reduced by careful polarimeter calibration. The latter uncertainty can be reduced by taking measurements at lower energies, at the expense of a more difficult experiment because of the reduced (d,p) cross section. As will be discussed

in this Letter, very little sensitivity to the optical model uncertainties remains, provided the bombarding energy is no larger than approximately 8 MeV.

In this Letter, we report new measurements of the $^{208}\text{Pb}(d,p)^{209}\text{Pb}$ tensor analyzing powers at $E_d = 7, 8$ and 9 MeV. The measurements were made with a highly accurate polarimeter with the result that uncertainties in the beam polarization measurement contribute less than ± 0.0002 to the uncertainty in η . Comparison of our $E_d = 9$ MeV data with those of Ref. 6 indicates that the T_{20} measurements of Ref. 6 were in error by 8% due to polarimeter calibration inaccuracies. When corrected for this error, the value of η from Ref. 6 becomes 0.0253. Transitions to the $9/2^+$, $5/2^+$ and $1/2^+$ states in ^{209}Pb ($E_x = 0.0, 1.57$ and 2.03 MeV) were used in the determination of η because of their relatively large tensor analyzing powers. Data taken at $E_d = 8$ MeV are shown in Fig. 1. The displayed errors are the result of counting statistics only. Data taken at $E_d = 7$ and 9 MeV are similar to that shown in Fig. 1.

For a given set of optical model parameters, η is the only free parameter in DWBA which affects the tensor analyzing powers and, as a result,

each measured point in the angular distributions represents an independent determination of η . Weighted averages of the η values have been calculated according to bombarding energy and are listed in Table 1 for three different sets of optical potentials. The internal agreement of the η values obtained at a given energy is indicated by χ^2 , which is also listed in Table 1. The sensitivity of the η determination to the optical parameters is indicated by the difference in η values obtained for the different potential sets.

Two (p,p) parameter sets and two (d,d) parameter sets are used, representing parameters obtained from fits to elastic scattering data both above and below the Coulomb barrier (approximately 12 MeV for deuterons or protons on Pb). As expected, the sensitivity to the optical parameters is largest at $E_d = 9$ MeV, where the scattering waves penetrate farthest into the nuclear interior. The values of χ^2 at $E_d = 9$ MeV are also larger than at 7 or 8 MeV, indicating poorer internal agreement. As a result, the measurements at $E_d = 9$ MeV are not considered useful in determining η , but serve to establish an upper limit on the bombarding energies which are useful in obtaining η .

In contrast, the determination of η is not sensitive to the optical parameters at $E_d = 7$ or 8 MeV. The largest variation in η obtained by changing the potential parameters is less than 1%. Because of ambiguities in optical

model fits, the potential parameter sets listed in Table 1 are not the only ones which reproduce the elastic scattering data. As is shown in the following, however, the parameter sets listed do give a reasonable measure of the uncertainties in η due to the choice of optical potentials. Even very crude changes in the potential parameters do not produce substantially larger variations in η than those shown in Table 1. The value of η was calculated using optical parameter sets in which the real and imaginary radius and diffuseness were each changed by 5 and 10% and in which the potential strengths were each changed by 5, 10, 20% and set to zero. The value of η was also calculated with all nuclear potential strengths set to zero so that the scattering waves were Coulomb wavefunctions. In no case were the changes in the $E_d = 7$ and 8 MeV values of η larger than 0.00036. Since these optical potentials obviously no longer fit the elastic scattering data, the effect on η is unreasonably large and this number is to be regarded as an upper limit.

The values of η which are midway between the values given by the different optical potentials in Table 1 are, respectively 0.02633 and 0.02655 at $E_d = 7$ and 8 MeV. The errors contributed by the uncertainties

in the optical parameters are calculated from the differences in η in Table 1 and are, at $E_d = 7$ and 8 MeV, the same value ± 0.00011 . The average of the $E_d = 7$ and 8 MeV values of η , weighted by the quadrature sum of the statistical and optical parameter errors, is

$$\eta = 0.02649 \pm 0.00043$$

The quoted error includes a polarimeter calibration error of ± 0.00016 added in quadrature. The DWBA calculations using this value of η are shown in Fig. 1. The χ^2 using this value of η and the $E_d = 7$ and 8 MeV measurements is 1.1.

The experimental value of η may have a dependence on three effects which have been neglected. In our DWBA calculations, we make use of the local-energy approximation¹¹ (LEA) because, at present, a suitable finite range code is not available. The accuracy of the LEA has not been precisely determined at sub-Coulomb energies but it has been estimated¹² on the basis of semi-classical arguments that the accuracy is better than 5%. Tensor terms in the $d + {}^{208}\text{Pb}$ optical potential have been neglected. The effect of these terms on the ${}^{208}\text{Pb}(d,p){}^{209}\text{Pb}$ analyzing powers has not been determined although it is expected that, in view of the insensitivity

of η to the central terms at $E_d = 7$ and 8 MeV, the tensor terms will also have a small effect on the $^{208}\text{Pb}(d,p)^{209}\text{Pb}$ tensor analyzing powers at $E_d \leq 8$ MeV. Finally, we have neglected the distortion of the deuteron in the Coulomb field of ^{208}Pb . Initial calculations¹³ show that the effect is of the order of 2-3%. It is hoped that the present experiment will stimulate further interest in calculating these effects.

The experimental result is compared with several theoretical values of η in Table 2. The first entry is an example of the separable potentials, which are used primarily in multiparticle calculations. As pointed out in Ref. 14, the multiparticle calculations made with a particular potential are strongly influenced by the deuteron properties of that potential. Thus, potentials such as the first entry in Table 2 which give the incorrect value of η will probably give incorrect results in multiparticle calculations. The other potential models all have a long range one- π -exchange potential tail and obtain the short range potential phenomenologically. The Paris (1975) potential is obsolete but is included to illustrate the changes in η that have occurred with theoretical progress. The most recent of these potentials is the Paris (1980) potential, in which

the intermediate and long range potential are calculated assuming one- π -exchange, two- π -exchange, and ω -exchange. The agreement between this potential and the present experimental result is indeed gratifying since the theoretical and experimental results were obtained independently.

References and Footnotes

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Table 1

Values of η obtained by comparing tensor analyzing power data at $E_d = 7, 8$ and 9 MeV with DWBA calculations using the indicated optical potential parameters.

Optical Potentials		η $\pm \delta\eta$ (χ^2)		
(d,d)	(p,p)	7 MeV	8 MeV	9 MeV
(a)	(b)	0.02644 ± 0.00064 (1.3)	0.02665 ± 0.0005 (0.9)	0.02649 ± 0.00032 (1.6)
(a)	(c)	0.02638 ± 0.00064 (1.3)	0.02644 ± 0.0005 (0.9)	0.02562 ± 0.00032 (1.7)
(d)	(b)	0.02622 ± 0.00064 (1.3)	0.02665 ± 0.0005 (0.9)	0.02151 ± 0.00027 (3.3)

- (a) from Ref. 16, in which the parameters were obtained by fitting the $^{208}\text{Pb}(d,d)^{208}\text{Pb}$ cross section and vector analyzing power at $E_d = 12.3$ MeV.
- (b) from Ref. 17, in which the parameters were obtained by fitting the $^{209}\text{Bi}(p,p)^{209}\text{Bi}$ cross section and analyzing power at $E_p = 10.76$ MeV.
- (c) from Ref. 18, in which the parameters were obtained by fitting cross section and analyzing power (p,p) data taken above the Coulomb barrier on targets with $A > 40$.
- (d) The $E_d = 9$ MeV parameters are from Ref. 19, where the parameters were obtained by fitting $^{208}\text{Pb}(d,d)^{208}\text{Pb}$ cross section and vector analyzing power at $E_d = 9$ MeV. The $E_d = 7$ and 8 MeV parameters were obtained by fitting $^{208}\text{Pb}(d,d)^{208}\text{Pb}$ cross section and vector analyzing power data taken at $E_d = 7$ and 8 MeV in conjunction with the present work. The parameters are: 8 MeV, $V_o = 122$, $r_o = 1.06$, $a_o = 0.72$, $w_I = 1.88$,

Table 1 (cont'd)

$r_I = 1.65$, $a_I = 0.88$; 7 MeV, $V_O = 126.7$, $r_O = 1.06$, $a_O = 0.72$,

$W_I = 1.58$, $r_I = 1.65$, $a_I = 0.88$. The $E_d = 7$ and 8 MeV potentials

had, in addition, a Coulomb distortion term $V(r) = -2400 r^{-4} (\text{MeV-fm}^4)$

as proposed in Ref. 20.

TABLE II. Values of P_D , Q and η given by
various deuteron wavefunctions

Author	Ref.	P_D (%)	Q (fm ²)	η
Mongan IV	14	1.4	0.279	0.0406
Reid	8	6.5	0.280	0.0262
dTS	9	5.5	0.279	0.0255
dTSR	9	5.8	0.279	0.0260
Paris(1975)	15	6.8	0.290	0.0293
Paris(1980)	10	5.4	0.278	0.0260
Experiment				0.02649 ± 0.00043

Figure Captions

Fig. 1) Angular distributions of the tensor analyzing powers T_{20} , T_{21} , and T_{22} for the reaction $^{208}\text{Pb}(d,p)^{209}\text{Pb}$ at $E_d = 8$ MeV. The displayed errors are statistical only. The curves are the result of DWBA calculations using $\eta = 0.02649$ and the optical potential parameters of Refs. 16 and 17.

C.M. ANGLE