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NUCLEON AXIAL COUPLING CONSTANTS AND QCD SUM RULES

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Abstract

QCD sum rules are used to obtain the isovector and "isoscalar" axial vector coupling constants, g_A and g_A^S. We find $g_A = 1.26 \pm 0.08$, $g_A^S = 0.13 \pm 0.08$ with sum rules for $g_A - 1$ and $g_A + g_A^S$. These sum rules also show that in the limit of chiral symmetry restoration, $g_A \rightarrow 1$ and $g_A^S \rightarrow -1$.

I. Introduction

Let me first say that it is a pleasure to be here to help celebrate this happy occasion of a colleague and friend. Although my talk is not directly in the field th t has been of primary concern to Abe Klein, I hope he will accept it as the tribute it is intended to convey.

One of the major questions for nuclear physics remains how to incorporate the basic hadronic theory QCD into its framework. To overcome the nonperturbative aspects of the theory, numerous methods have been proposed and used: lattice QCD, light-cone techniques, bag models, solitons, string and related flux-tube models, Nambu-Jona-Lasinio model, QCD sum rules, and many others. Each of them has been found useful in a limited realm, but none of them has so far been found to be widely applicable and successful without requiring added features. But many of the methods are still being developed, and their applicability continues to be tested and extended.

The method of QCD sum rules is, I believe, particularly useful for describing hadronic structures. It has been applied to both mesons and baryons and found to be remarkably successful. The method is to replace the non-perturbative long-range aspects by several (hopefully) universal vacuum expectation values (VEV's). The short-range aspects on the other hand, are calculated perturbatively. The method was developed by physicists in the USSR[1], who also extended it to computing hadronic properties in the presence of an external (e.g., electromagnetic) field.[2,3]

In this talk I want to discuss the calculation of $g_A^V \equiv g_A$ and g_A^S, the isovector and "isoscalar" axial vector coupling constants of the nucleon by means of QCD sum rules.[3,4] I will also show that $g_A \rightarrow 1$ and $g_A^S \rightarrow -1$ in the limit of chiral symmetry restoration at high nuclear densities or temperatures.

We are particularly interested in the response of nucleons to electroweak fields. These responses have been considered in the past.[3,4] One of the critical tests of a model of nucleons is its ability to generate the correct axial vector coupling constant, $g_A = 1.254 \pm .006$. Non-relativistic quark models predict 5/3 and relativistic ones predict $0.75 \lesssim g_A \lesssim 1.5$. QCD sum rules have been shown[3] to give $g_A = 1.25$, but this result has been obtained with non-standard values of the condensate. By including terms consistently up to dimension $d = 8$, we use a QCD sum rule for $g_A - 1$ to show that $g_A = 1.26$ with accepted values of these condensates. In addition we calculate g_A^S and compare it to recent EMC measurements.

II. Method

The method begins with a correlation operator $\pi(q)$ in an external axial field Z_μ

$$\pi(q) = i \int d^4x \, e^{iqx} < 0|T(\eta(x)\bar{\eta}(0))|0 > , \tag{1}$$

where the current η has the quantum number of the hadron in question. For a nucleon a useful form has been found to be[2-4]

$$\eta(x) = \epsilon^{abc} \left\{ u^a(x)^T \, C \, \gamma_\mu \, u^b(x) \right\} \gamma^\mu \gamma^5 d^c(x) , \tag{2}$$

where C is the charge conjugation operator and $a...c$ are color indices. If $v_N(p)$ is a nucleon spinor (normalized so that $\bar{v}v = 2M_N$, with M_N the nucleon mass) we can write

$$< 0|\eta(0)|N(p) >= \lambda_N \, v_N(p) , \tag{3}$$

where λ_N is a constant.

The correlator can be written in an operator product expansion

$$\pi = C_I \, I + \sum_n C_n(q^2) \mathcal{O}_n , \tag{4}$$

where I is the unit operator, the C_I, C_n are Wilson coefficients and $\mathcal{O}_n$ is an operator; the $\mathcal{O}_n$'s can be ordered by dimension and the Wilson coefficients fall off by corresponding powers of $|q^2|$. π is calculated in a region of $|q^2| \sim 1$ GeV^2, where the VEV's take into account nonperturbative QCD aspects, not included in the perturbative part.

The correlator π has various structure functions (labeled by j), each of which satisfies a dispersion relation

$$\pi^j(q^2) = \frac{1}{\pi} \int_0^\infty \frac{I_m \pi^j(s) ds}{s - q^2} + \text{subtraction constants} . \tag{5}$$

The subtraction terms are eliminated by assuring convergence via a Borel transform

$$B[F(q^2)] \equiv \lim_{\substack{n \to \infty \\ -q^2 \to \infty \\ q^2/n \to M_B^2 \text{ finite}}} \frac{1}{n!}(-q^2)^{n+1} \left(\frac{d}{dq^2}\right)^n F(q^2) \to \frac{1}{\pi} \int_0^\infty ds \, Im \, F(s) e^{-s/M_B^2} . \tag{6}$$

The correlator π in the presence of Z_μ is evaluated first for the quarks in the nucleon; it is equated to the nucleon-based dispersion calculation,

$$\pi(p) = -|\lambda N|^2 \frac{1}{\not p - M_N} g_A \not Z \gamma_5 \frac{1}{\not p - M_N} + \text{excited states and continuum} . \tag{7}$$

By comparing coefficients, several sum rules can be obtained for the coefficients of $p \cdot Z \not p \gamma^5$, $\not Z \gamma_5$, and $i\sigma^{\mu\nu} Z_\mu p_\nu \gamma^5$. Only the first of these is used to obtain g_A because (i) it contains g_A alone, and (ii) the excited and continuum state contributions are less important than for the other terms, because they would cancel for parity doublets.[3]

We begin with the quark propagator in configuration space in the presence of the external constant axial-vector field Z_μ. If we keep terms up to second order in an expansion of $<0|q(x)\bar{q}(0)|0>$, we obtain

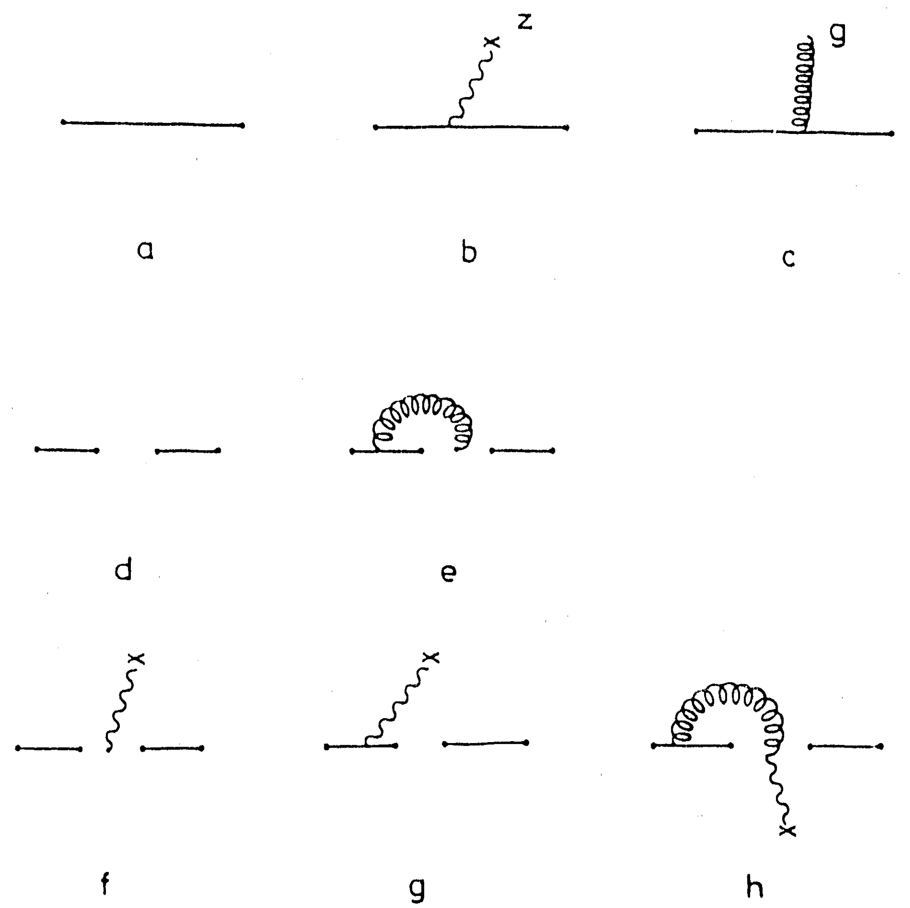
$$\begin{aligned} S^{ab} = & \frac{\delta^{ab}}{(2\pi)^4 x^4}(i\not x - g x \cdot Z \not x \gamma_5) + \frac{i}{32\pi^2 x^2} g_c \frac{\lambda^n_{ab}}{2} G^n_{\mu\nu}(\not x \sigma^{\mu\nu} + \sigma^{\mu\nu} \not x) \\ & + \delta^{ab} <\bar{q}q> \left\{ -\frac{1}{12}\left(1 + \frac{1}{16} x^2 m_0^2\right) + \frac{1}{12} g\chi \not Z \gamma_5 + \frac{1}{36} g x^\alpha Z^\beta \sigma_{\alpha\beta}\gamma_5 \right. \\ & \left. + \frac{1}{216} g\kappa \left(\frac{5}{2} x^2 \not Z - x \cdot Z \not x\right)\gamma_5 \right\} + \ldots . \end{aligned} \tag{8}$$

The various terms of this propagator are shown in Fig. 1. The constants g are the coupling of the quarks to the external field. In the standard model they are

$$g_u = -g_d = 1 \tag{9a}$$

for the up and down quarks; these are the coupling constants for the isovector g_A. The "isoscalar" coupling g_A^S is defined by

$$g_u = g_d = 1 . \tag{9b}$$



Quark propagator diagrams.

It should be noted that this is not truly the weak isoscalar axial coupling constant. The latter is given by the sum of g_A for the proton and neutron, and vanishes in the absence of strange quarks.[5]

The susceptibilities κ and χ and the mass m_0^2 in Eq. (8) are defined by

$$
\begin{aligned}
&< 0|\bar{q} g_c \sigma \cdot G q|0 > = -m_0^2 < \bar{q}q > , \\
&< 0|\bar{q} g_c \tilde{G}_{\mu\nu} \gamma^\nu q|0 > = g\kappa Z_\mu < \bar{q}q >, \\
&< 0|\bar{q} \gamma_\mu \gamma_5 q|0 > = g\chi Z_\mu < \bar{q}q > \ .
\end{aligned} \tag{10}
$$

The sum rule for g_A is then obtained from the coefficients of $p \cdot Z \not{p} \gamma^5$ in $\pi(p)$. After the application of a Borel transformation, we obtain

$$
\begin{aligned}
&\frac{M_B^6 E_2}{8L^{4/9}} + \frac{M_B^2}{32L^{4/9}} < g_c^2 G^2 > E_0 - \frac{M_B^2}{18L^{68/81}} \kappa a \, E_0 + \frac{5}{18} \, a^2 L^{4/9} \\
&\quad + \frac{1}{288L^{4/9}} \, \chi a < g_c^2 G^2 > \\
&\quad = \beta_N^2 (g_A + A M_B^2) \exp(-M_N^2/M_B^2) \, ,
\end{aligned} \tag{11}
$$

where $a = -(2\pi)^2 < \bar{q}q >$, the length L is $L = 0.621 \ln(10 M_B)$ corresponding to $\Lambda_{QCD} = 0.1$ GeV, with the Borel mass M_B given in GeV; the quantity $\beta_N^2 \equiv (2\pi)^4 \lambda_N^2/4$. In Eq. (11) we have kept anomalous dimensions. The functions E_0, E_1, and E_2 are given by

$$\begin{aligned} E_0 &= 1 - e^{-x} \\ E_1 &= 1 - (1+x)e^{-x} \\ E_2 &= 1 - (1 + x + \frac{1}{2}x^2)e^{-x} \ , \end{aligned} \tag{12}$$

with $x = W^2/M_B^2$. These functions account for contributions of excited states up to a mass W. We choose $W^2 \approx 2.3$ GeV^2. The constant A represents residual contributions of excited and continuum states. The terms which contribute to the left-hand side of Eq. (11) are shown in Fig. 2.

Similarly, for the "isoscalar" g_A^S we find

$$\begin{aligned} -\frac{M_B^6 E_2}{8L^{4/9}} + \frac{M_B^2}{32L^{4/9}} < g_c^2 G^2 > E_0 + \frac{1}{6L^{4/9}} \chi a M_B^4 E_1 - \frac{M_B^2}{18 L^{68/81}} \kappa a E_0 \\ - \frac{1}{18} a^2 L^{4/9} + \frac{1}{288 L^{4/9}} \chi a < g_c^2 G^2 > \\ = \beta_N^2 (g_A^S + A^S M_B^2) \exp(-M_N^2/M_B^2) \ . \end{aligned} \tag{13}$$

In order to obtain a sum rule for $g_A - 1$, we use the Belyaev-Ioffe[6] sum rule for the mass M_N

$$\begin{aligned} \frac{M_B^6}{8L^{4/9}} E_2 + \frac{M_B^2}{32L^{4/9}} < g_c^2 G^2 > E_0 + \frac{1}{6} a^2 L^{4/9} - \frac{1}{24 M_B^2} a^2 m_0^2 \\ = \beta_N^2 \exp(-M_N^2/M_B^2) \ . \end{aligned} \tag{14}$$

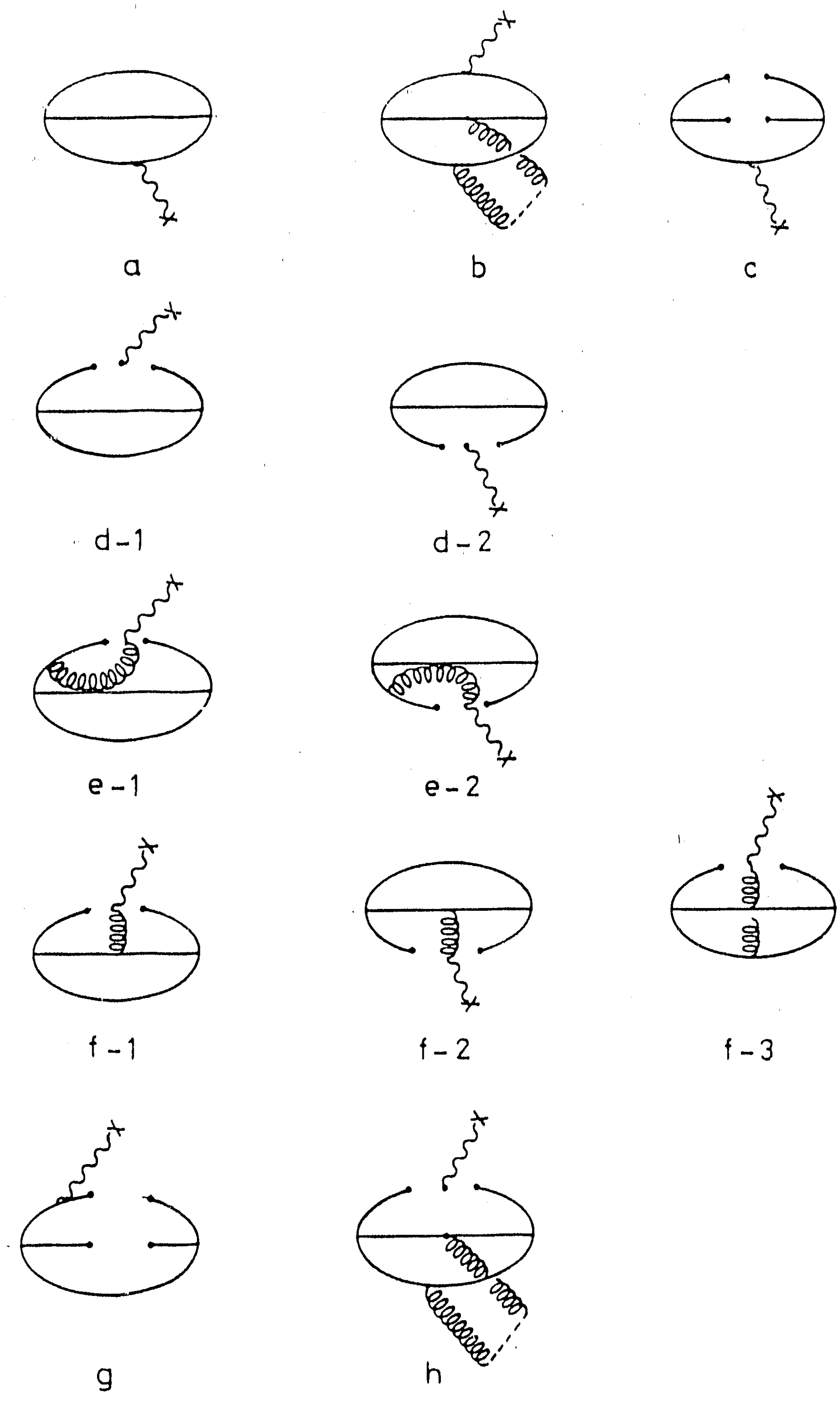
After subtraction of Eq. (14) from Eq. (11), we obtain

$$\begin{aligned} \frac{1}{9} a^2 L^{4/9} + \frac{1}{24} \frac{a^2 m_0^2}{M_B^2} - \frac{1}{18} \frac{\kappa a M_B^2}{L^{68/81}} E_0 + \frac{1}{288 L^{4/9}} \chi a < g_c^2 G^2 > \\ = \beta_N^2 \{(g_A - 1) + A M_B^2\} \exp(-M_N^2/M_B^2) \ . \end{aligned} \tag{15}$$

One advantage of the sum rule, Eq. (15) for $g_A - 1$ is that it has a smaller variation in powers of M_B^2 than that for g_A. This occurs because the first two terms of Eqs. (11) and (14) are equal to each other. From Eq. (15), we immediately see that if $a \to 0$, the left hand side $= 0$ and $g_A - 1 = 0$.

The constants required for the evaluation of the sum rules are obtained from other experiments. They are <u>not</u> adjusted. These constants are

$$\begin{aligned} a &\approx 0.55 \text{ GeV}^3 \ , \\ \kappa a &\approx 0.140 \text{ GeV}^4 \ , \\ \chi a &\approx 0.70 \text{ GeV}^2 \ . \\ < g_c^2 G^2 > &\approx 0.47 \text{ GeV}^4 \ , \\ m_0^2 &\approx 0.8 \text{ GeV}^2 \ . \end{aligned} \tag{16}$$



Diagrams included in the evaluation of the sum rules for g_A and g_A^S.

Of these values, κ is the least well known; we estimate the error in the sum rule by evaluating $g_A - 1$ for κ_a = -0.140 GeV^4 as well as for the value given in Eq. (16). The value of β_N^2 is obtained from the mass sum rule and the mass of the nucleon, $\beta_N^2 = 0.26$ GeV^6.

A reduction in the range of powers in M_B^2 can also be obtained for g_A^S through a sum rule for $g_A^S + g_A$,

$$\begin{aligned}\frac{M_B^2}{16L^{4/9}} < g_c^2 G^2 > \; E_0 + \frac{1}{6L^{4/9}}\,\chi a\, M_B^4 E_1 - \frac{M_B^2}{9L^{68/81}}\,\kappa a E_0 \\ + \frac{2}{9}\,a^2 L^{4/9} + \frac{1}{144L^{4/9}M_B^2}\,\chi a < g_c^2 G^2 > \\ = \beta_N^2 (g_A + g_A^S + A' M_B^2)\exp(-M_N^2/M_B^2)\,. \qquad (17)\end{aligned}$$

Again, we note that if $a \to 0$ and $< G^2 > \to 0$, the left hand side vanishes and $g_A^S = -g_A = -1$.

III. Results and Discussion

In the sum rule for $g_A - 1$, the quark condensate $< \bar{q}q >$ or a dominates the contribution on the left-hand side of Eq. (14) and the susceptibility terms proportional to κ and χ are relatively unimportant.

Solutions to the sum rules are found numerically; they are stable for $M_B \gtrsim 1.8$ GeV. Although this is a relatively large Borel mass, we find that the continuum contributions from A and A^S are very small. A smaller mass is desirable to emphasize the contribution of the nucleon and deemphasize excited states. We find

$$\begin{aligned} g_A &= 1.26 \pm 0.08 \\ g_A^S &= 0.13 \pm 0.08\,. \qquad (18)\end{aligned}$$

The central value of g_A is clearly consistent with experiment. The value of g_A^S differs from that found by Gupta, Murthy, and Pasupathy[4] who obtain $g_A^S = 0.35$ with a value of $< \bar{q}q >$ smaller by 20% than the "standard" one we use. This result shows the sensitivity of g_A^S to the susceptibilities and quark condensate.

From the EMC data and baryon decays, one obtains[7]

$$g_A^S \approx \Delta u + \Delta d \approx 0.31 \pm 0.08\,. \qquad (19)$$

Our value is not quite consistent with experiment if $\Delta\bar{u} - \Delta\bar{d} = 0$. If $\kappa = \chi = 0$, $g_A \to 1.3$ and $g_A^S = -0.56$. This again shows the sensitivity to g_A^S to the susceptibilities. If $a = 0$, we obtain $g_A = 1.00 \pm 0.02$ and $g_A^S = -0.68$; if $< GG >$ $= 0$ in addition, $g_A^S = -1.0$. The latter result is, indeed, a strange one. Together with $g_A = 1$, it implies $\Delta u = 0$, $\Delta d = -1$ for the proton when chiral symmetry is restored.

Can we use these results to predict what happens in nuclei? Although we expect a partial restoration of chiral symmetry in nuclei, we cannot simply interpolate between the free nucleon ($g_A = 1.26$) and chiral restoration ($g_A = 1$) limits. For instance, other condensates such as the vector $< q^\dagger q >$ can occur in nuclei. However, it is likely that $1 \lesssim g_A \lesssim 1.26$.

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* Talk given by EMH

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