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ISOMERIC CROSS-SECTION RATIOS FOR
SOME (n,γ) AND (λ,xn) REACTIONS

by

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ABSTRACT

Isomeric cross section ratios for several nuclear reactions in the medium mass region were measured. The cross section ratios of isomers produced by the thermal neutron irradiation of Se-80, Rh-103, Pd-108, Te-128 and Cs-133 were measured. Isomeric cross section ratios for the formation of the isomeric pair In-110 and In-110m, produced in the Ag-107(α ,n) and the Ag-109(α ,3n) reactions were also measured at several energies of the incoming projectile. The neutron irradiations were carried out in the Argonne Research Reactor CP-5. Helium ions were accelerated with the Argonne constant frequency 60" Cyclotron to a maximum energy of 43 Mev. The relative amounts of the isomers produced in the reactions were determined by scintillation counting. In the case of the In-110 isomers the efficiencies of the scintillation counter for detecting each isomer were measured, so that absolute cross sections as well as cross section ratios could be determined for these isomers.

The isomeric cross section ratios were also calculated theoretically using a statistical model, in which angular momentum effects were included. By comparing experimentally measured cross section ratios for the formation of isomeric pairs to the calculated ratios, information about the dependence of the nuclear level density on angular momentum was obtained. The results for both the (n, γ) and the (α ,xn) reactions gave an average nuclear spin density parameter σ , of 3 to 5. This parameter is defined in the equation, $\rho(J) = \rho(0) (2J+1) \exp [-(J+\frac{1}{2})^2 / 2\sigma^2]$, in which $\rho(J)$ is the density of levels with spin J, $\rho(0)$ is the density of levels with spin zero, and σ is the nuclear spin density parameter which characterizes the distribution in spin. A more detailed analysis of the isomeric cross section ratios obtained by the Ag-107(α ,n) and the Ag-109(α ,3n) reactions showed that a theoretically calculated energy dependent value of σ , using a moment of inertia less than the rigid value at low excitation energies, was in agreement with the experimental results.

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I. INTRODUCTION

One of the important factors to consider in a nuclear reaction is angular momentum. In this investigation angular momentum effects in a number of nuclear reactions have been studied by comparing experimentally measured cross section ratios for the formation of isomeric pairs, to those calculated on the basis of a compound nucleus mechanism. From this comparison, information about a nuclear spin density parameter has been obtained. In two of the reactions studied the energy of the incoming projectile was varied to determine additional information about this parameter.

The spin or total angular momentum of the nucleus may be thought of as a vector sum of the spin angular momentum and the orbital angular momentum of the individual protons and neutrons within the nucleus. If a nucleus is in its ground state or in a low excited state one can predict the spin (or angular momentum) of that state by use of the shell model¹ or the collective model of Bohr and Mottelson.² If the nucleus is excited to more than a few Mev one finds that there are very many levels available and one may apply a statistical model.³⁻⁵ In fact at high excitation energies one speaks of level densities (i.e. the number of levels per Mev), which increase exponentially with the excitation energy of the nucleus. Now instead of speaking of the angular momentum of each individual level, at high excitation energies, one speaks of the distribution in angular momentum of levels within a small energy interval. Realizing that

there are many protons and neutrons within the nucleus and, at high excitation energy, many possible excited states of these individual particles, one can apply the idea of random coupling to the angular momenta of the individual particles. If we consider the component M , of the angular momentum J , on an arbitrary axis, then the distribution of M becomes a Gaussian. Furthermore it can be shown ^{6,7} that at a given excitation energy E , the density of levels having an angular momentum J , is given by:

$$\rho(J) = \rho(0) (2J + 1) e^{-(J + \frac{1}{2})^2 / 2 \sigma^2}$$

where $\rho(0)$ is the density of levels having angular momentum 0, and σ is a nuclear spin density parameter characterizing the distribution. It is this parameter σ that we wish to determine experimentally. Since a highly excited nucleus is very short lived (generally $< 10^{-13}$ sec.) σ must be determined indirectly through nuclear reactions in which these highly excited levels occur as intermediates in the nuclear reaction.

In addition to energy, charge, radius and parity of the nuclei involved in a nuclear reaction, angular momentum plays an important role.⁸ In any nuclear reaction total angular momentum must be conserved. Likewise in a nuclear reaction there is a centrifugal barrier limiting the amount of angular momentum that can be imparted to the target nucleus by the incoming projectile and also limiting the amount of angular momentum that can be carried away by a particle leaving the nucleus. And finally in the deexcitation of the nucleus formed in the nuclear reaction, gamma transitions are im-

pared considerably if the angular momentum change is large. These factors together with the factor $\rho(J)$, are involved when one wishes to examine angular momentum effects in nuclear reactions.

If the product nucleus of a nuclear reaction has a long lived excited level of measurable life time, then such an excited level and its ground state, called isomers, will both be populated in a nuclear reaction. An isomeric pair differs considerably in spin, but only slightly in energy. The extent to which each isomer is populated in the nuclear reaction may be expressed by the isomeric cross section ratio. The isomeric cross section ratio is defined here as the ratio of the cross section for the formation of the high spin isomer (usually the metastable state) to the total cross section for the formation of both isomers. Since isomers differ in their spin it is possible, taking into consideration the angular momentum effects discussed above, to calculate this cross section ratio theoretically using a certain mechanism for the nuclear reaction. Such a calculation requires the use of the above spin density equation involving σ . Thus σ can be determined by comparing experimental cross section ratios with the theoretically calculated values.

To examine angular momentum effects in nuclear reactions and more specifically to determine the parameter σ and its variation with excitation energy, the isomeric cross section ratios of the products of a number of reactions were measured. The reactions studied were chosen on the basis of the ease with which the isomeric cross section ratio could be measured

experimentally, and on the basis of the knowledge of the spins of the initial and final nuclei involved in the reaction. The latter are necessary in the calculation of the isomeric cross section ratio. (n,γ) reactions using thermal neutrons enabled one to compare theory and experiment where only one or two initial compound nucleus spin states were involved. The isomeric cross section ratios of the isomers formed in the (n,γ) reactions on Se-80, Rh-103, Pd-108, Te-128 and Cs-133 were measured. Also determined was the isomeric cross section ratio for the formation of In-110 and In-110m by the reactions, Ag-107 (α,n) and Ag-109 $(\alpha,3n)$ at several alpha energies. In these reactions a number of initial spin states of the compound nucleus are produced since the alpha particles, of relatively high energy, can impart considerable angular momentum to the target nucleus. The study of these two reactions likewise enabled one to study the isomeric cross section ratio over a large energy range, 10 to 22 Mev for the Ag-107 (α,n) reaction and 26 to 39 Mev for the Ag-109 $(\alpha,3n)$ reaction.

The parameter σ has been determined for other reactions both by the method of measuring isomeric cross section ratios^{9,10} and by other methods.¹¹⁻¹⁵ The purpose of the present investigation was to determine σ by additional reactions and to get some idea of the excitation energy dependence of σ .

II. EXPERIMENTAL PROCEDURE

A. Targets

Neutron-gamma reactions on Se-80, Rh-103, Pd-108, Te-128 and Cs-133 were studied. Rh-103 and Cs-133 are the only stable isotopes of their respective elements, so that natural targets could be used for neutron irradiation in these two cases. In the case of Se-80, Pd-108 and Te-128 enriched isotopes* were used as targets. Enriched isotopes* were also used as targets in the Ag-107 (α, n) reaction and the Ag-109 ($\alpha, 3n$) reaction. The percentage enrichments of the isotopes used in these experiments are shown in Table 1.

Table 1

Target Descriptions

Target	Se-80	Rh-104	Pd-108	Te-128	Cs-133	Ag-107	Ag-109
% isotopic enrichment	94.4	100	94.2	96.5	100	99.6	99.8
Chem. and phys. form	Metal powder	Metal wire	Metal chips	Metal powder	CsNO ₃ crystals	Metal foil	Metal foil

The chemical and physical form of the targets used for irradiation are also shown in this table. The stable isotopes of silver were obtained in the form of powdered metal. This powder was melted down and rolled into very thin foils of uniform thickness. Silver foils were $3/4$ " in diameter and had a thickness of about 2 Mg/cm².

* Purchased from Isotopes Division of Oak Ridge National Laboratory, Oak Ridge, Tenn.

B. Irradiation Procedures

1. (n, γ) Reactions

Irradiations for producing (n, γ) reactions were carried out in the Argonne Research Reactor, CP-5.¹⁶ Except for the isomers of Te-129 and Cs-134, the half-lives of the isomers produced in the (n, γ) reactions were very short, such that it was necessary to determine the activity of these isomers as soon as possible after the irradiation. For such cases, a pneumatic tube going through the edge of the reactor core and equipped with a helium gas system, which carried a cartridge in and out of the reactor, known as the "rabbit facility", was used for quick irradiation of the sample. Counting equipment near the place of removal of the sample from the reactor enabled one to measure the activity of the isomers before considerable decay. It was possible to begin counting the sample 2 minutes after its removal from the reactor. The rabbit facility was also used for the irradiation of Cs-133. Although the half-lives of the isomers produced by the (n, γ) reaction on this target were long, short irradiations were desired and could be conveniently carried out with the rabbit facility. The neutron flux obtained with the reactor operating at 2200 kw* by use of the rabbit facility was about 7×10^{12} neutrons/cm²sec. The cadmium

*Most irradiations were carried out at a power level of 2200 kw, but later the power level was increased to 4600 kw and the flux at 4600 kw is about 2.5 times the flux at 2200 kw.

ratio" was about 32 for irradiations carried out with the Rabbit facility.

Irradiation of Te-126 was carried out in a tunnel, "isotope tunnel no. 1", passing through the graphite reflector beneath the reactor. The Te-126 samples were irradiated at the vertical centerline of the reactor where the neutron flux was 2.1×10^{12} neutrons/cm² sec. (at 2200 kw) and the cadmium ratio was 1100.

All samples for neutron irradiation were placed in a small glass or plastic tube which was then placed in a standard container for irradiation in the CP-5 reactor.

2. (α ,xn) Reactions.

Irradiations for producing the Ag-107 (α ,n) and the Ag-109(α ,3n) reactions were performed with the Argonne constant frequency 60" Cyclotron.¹³ A diagram of the irradiation set-up is shown in Figure 1. The cyclotron accelerates helium ions to an energy of about 42.5 Mev. The accelerated helium ions are removed from the cyclotron by a deflector system and the alpha beam is focused by use of a magnet system.

* The cadmium ratio is defined as¹⁷ "the ratio of the neutron intensity as recorded on some detector, to the intensity when the detector is surrounded by cadmium (usually about 0.03" thick). The bare detector responds to the resonance plus the thermal flux; when surrounded with cadmium the detector responds to the resonance flux only, because the cadmium cover stops all the thermal neutrons, but is transparent to those neutrons of energy greater than about 0.4 ev."

FIGURE 1.
HELIUM ION IRRADIATION SETUP.

Scale 1" $\approx$ 80"

The energy of the alpha particles* will vary slightly from day to day, so that it is necessary to determine this energy before each irradiation. Before irradiation of the target, the alpha beam is directed into an apparatus for determining the energy of the alpha particles.¹⁹ This energy is obtained by measuring the mean range of the alpha particles in aluminum. Experimentally the percentage beam transmission through aluminum foils is measured as a function of foil thickness. From this curve, shown in Figure 2, the Al foil thickness which reduces the percentage of alpha particles transmitted to 50% can be obtained. This thickness is defined as the mean range of alpha particles of aluminum. This range may be converted into a particle energy by use of a range vs. energy curve for alpha particles. Such a curve may be obtained from an experimental range energy relation for protons^{20,21} by assuming that the energy loss of alpha particles has the same velocity dependence as protons.

After the energy measurement is made the alpha beam is directed to the target by means of a switching magnet. The beam passes through a one mil Al foil, $7/8$ " of air and a stack of Al foils before striking the target. The purpose of the stack of aluminum foils is to reduce the energy of the alpha particles to the desired energy of irradiation. In passing through matter an alpha particle loses only a

*The term alpha particles will be used from now on. It is well to remember however, that our source of "alpha particles" was produced by the acceleration of helium ions.

FIGURE 2.

TRANSMISSION OF ALPHA
PARTICLES THROUGH ALUMINUM.

small fraction of its energy in a single collision with an electron²² (nuclear collisions are negligible compared to atomic collisions); in fact of the order of 10^5 collisions are necessary to reduce the energy of the alpha particle by a few Mev. Consequently when a large number of alpha particles pass through an absorber of uniform thickness, the alpha particles emerging from the other side of the absorber will all be reduced by the same amount of energy within narrow limits. The reduction in energy depends upon the number of collisions which in turn depends upon the thickness of the Al foil through which the alpha particles pass.

The thickness of the Al foils in front of the silver target was varied so that the (α, xn) reactions could be studied at several energies. In most cases two silver foils were irradiated at different alpha energies by placing these foils behind different thicknesses of aluminum. The target assembly consisted of an aluminum "Faraday Cup" which contained the silver target foils and the aluminum degrading foils. This "Faraday Cup" was used to measure the current of the alpha particles on the target so that absolute cross sections as well as cross section ratios could be obtained for the In-110 isomers produced in the Ag-107 (α, n) and the Ag-109 $(\alpha, 3n)$ reactions.

C. Scintillation Counting

1. General

The isomers produced in the (n,γ) reactions on Se-80, Rh-103, Pd-108, Te-128 and Cs-133, in the (α,n) reaction on Ag-107, and in the $(\alpha,3n)$ reaction on Ag-109 were all radioactive. The isomers studied, their half lives, and the reactions used to produce these isomers are shown in Table 2. Also shown in this table are the scintillation detectors used, as well as the radiation detected. The irradiated samples could be counted directly (i.e. no chemical separations were necessary). The samples were mounted with scotch tape on a 2 1/2" x 3 1/8" cardboard card for counting. Absorbers were used in some cases (cf Table 2) to absorb the beta radiation or to decrease the intensity of the low energy gamma radiation. Absorbers were placed directly over the samples, which were placed on shelves at a given distance from the face of the scintillation crystal.

A block diagram of the counting set-up used in the scintillation counting is shown in Figure 3. A more elaborate system was used for the counting of In-110 and In-110m, and will be discussed in the next section. The scintillation crystals were canned in hermetically sealed aluminum containers. One surface of the crystal is optically coupled to a photomultiplier tube, while the remaining surfaces of the crystal are coated with MgO, which serves as a light reflector. When radiation enters the scintillator, the light produced enters the photomultiplier tube. The output voltage pulse from this photomultiplier tube is

Table 2

Reactions Studied and Radiation Detected

Reaction	Isomers produced	$t_{1/2}$ of isomers*	Radiation detected*	Absorber	Scintillator
Se-80 (n, γ)	Se-81 Se-81m	18m } 57m }	1.38 Mev β^- of Se-81	None	1.5" x 1 4" Anthracene
Rh-103 (n, γ)	Rh-104 Rh-104m	42s } 4.4m }	556 kev γ of Rh-104	1.8 g/cm ² Pb	1.5" x 1" NaI (Tl)
Pd-108 (n, γ)	Pd-109	13.45h ^a	88 kev γ of Pd-109	.612 g/cm ² Al	1.5" x 1" NaI (Tl)
	Pd-109m	4.69m ^a	188 kev γ of Pd-109m		
Te-128 (n, γ)	Te-129 Te-129m	1.10h } 33d }	1120 kev γ of Te-129	3.05 g/cm ² Pb	3" x 3" NaI (Tl) (1)
Cs-133 (n, γ)	Cs-134 Cs-134m	2.1y ^b } 2.895h ^b }	706 and 801 kev γ 's of Cs-134	10.2 g/cm ² Pb	3" x 3" NaI (Tl) (2)
Ag-107 (α ,n) } Ag-109 (α ,3n) }	In-110 In-110m	70.2m ^c 5.2h ^c	656 kev γ of In-110 670 + 940 + 885 + 656 kev γ 's of In-110m	.697 g/cm ² Al	Two 4" x 4" NaI (Tl)

* Unless otherwise indicated all decay scheme information here and elsewhere in this report is taken from references 23 and 24.

- a) Cf reference 25
- b) Cf reference 26
- c) Present Author

FIGURE 3.
SCINTILLATION COUNTING SETUP.

amplified, and analyzed by a 256 channel analyzer. The use of scintillation crystals in gamma ray spectroscopy has been treated in detail by many authors.²⁷⁻³² Further discussion on the use of a single crystal for gamma ray detection will not be given here, but the reader is referred to the above references.

Samples were counted in shielded caves to cut down background radiation (cf. Table 3). Because of their short half lives, the isomers of Se-81, Rh-104 and Pd-109 were counted near the reactor, where only a small lead "pig" was available for shielding. The radioactivity produced in these samples was very high, such that the background even with this small amount of shielding was very small compared to the counting rates of the samples. The large caves shielding the 3" x 3" NaI(Tl) crystals used in detecting the activity of the Te-129 and the Cs-134 isomers, served also to cut down on backscattering since the samples were placed near the center of the cave.

2. Total Absorption Counter

The scintillation counter used in the detection of In-110 and In-110m was somewhat different in that it was a total absorption gamma ray spectrometer. This counter has been described by Hansen³³ and is similar to other counters described in the literature.^{34,35} It consists essentially of two NaI(Tl) crystals surrounded by a plastic anti-compton shield (cf. Figure 4). The NaI(Tl) crystals, 4" in diameter and 4" high, are mounted face to face $3/8$ " apart in a 6" diameter hole through the center of the plastic scintillator.

Table 3

Shielding and Analyzers used with Scintillation Crystals

Scintillator	Shielding (Type and approx. external dimensions)	(Wall thickness)	Analyzer used
1.5" NaI (Tl), 1.5" x 1/4" NaI (Tl) or anthracene	Lead "Pig" 14" high 9.5" in diameter	1.75"	Nuclear Data ND 101
3" x 3" NaI (Tl)(1)	Steel Cave 44" high 48" wide 48" long	6"	R.C.L. ^a 20609
3" x 3" NaI (Tl)(2)	Lead Cave 18" high 18" wide 18" long	4"	R.I.D.L. ^b 3302
Two 4" x 4" NaI (Tl)	Lead Cave 32" high 29" wide 29" long	2"	R.C.L. ^a 20609

a) Radiation Counter Laboratories

b) Radiation Instrument Development Laboratory

FIGURE 4.
TOTAL ABSORPTION GAMMA
RAY SPECTROMETER.

The plastic scintillator, 24" in diameter and 24" high, is NE-102 brand phosphor supplied by Nuclear Enterprises Ltd. of Winnipeg, Canada. The sample is introduced between the two NaI(Tl) crystals on a slide which is inserted through a slot in the plastic scintillator.

The outputs of the two NaI(Tl) preamplifiers are added and fed into an ANL model A-61 linear amplifier (cf. Figure 5 for diagram of electronics). The A-61 linear output is fed into the analog to digital converter circuit of a R.C.L. 256 channel analyzer which is operated in the delayed coincidence mode. The A-16 discriminator output is fed into a gate generator which generates a "yes" pulse, which in turn is fed into an anti-coincidence circuit. The four outputs of the tubes viewing the plastic scintillator are added and fed into a second A-61 linear amplifier. The discriminator output from this amplifier is fed into a second gate generator which generates a "no" pulse. This "no" pulse is fed into the same anti-coincidence circuit as was the "yes" pulse.

Unless a "yes" and a "no" pulse occur simultaneously in the anti-coincidence circuit, the "yes" pulse is fed into the delayed coincidence input of the R.C.L. 256 channel analyzer and causes the analyzer to process the pulse produced by the two NaI(Tl) crystals. When simultaneous pulses occur in both the NaI(Tl) crystals and in the plastic scintillator, the "no" pulse blocks the "yes" pulse in the anti-coincidence circuit, and consequently the pulse produced by the two NaI(Tl)

FIGURE 5.
TOTAL ABSORPTION COUNTER
ELECTRONICS.

crystals is not analyzed or recorded. Thus when a gamma ray is compton scattered in one of the two NaI(Tl) crystals and escapes into the plastic scintillator,* producing in the NaI(Tl) a pulse which is smaller than the pulses which fall in the photopeak, it is rejected, thereby reducing the compton pulse height distribution from the gamma ray pulse height spectrum. This anti-coincidence feature similarly reduces background due to cosmic radiation and natural radioactivity.

*The lower bias on the plastic scintillator is rather high, i.e. about 50 kev, because one does not wish to eliminate true counts by chance coincidence with electrical noise from the four photomultiplier tubes.

D. Spectra Analysis

1. General

The output of the 256 channel analyzer gives a gamma ray spectrum when a NaI(Tl) scintillator is used or a beta ray spectrum when anthracene is used. The actual output consists of the number of counts in each of 256 channels. Each channel corresponds to a certain range of pulse heights. Each pulse height in turn corresponds to the energy lost by the gamma ray or beta particle in the crystal. In order to determine which channel corresponds to which energy it is necessary to use standard radioactive sources having gamma rays (or a β^- in the case of a β^- spectrum) of known energies. The energy standards used in this work were: Cs-137, with a 662 kev conversion electron; Na-22, with 1276 and 511 kev gamma rays, the latter coming from positron annihilation; and Hg-203, with a 279 kev gamma ray and a 72 kev X-ray. The sum peaks of Na-22 gamma rays, 1022, 1787 and 2298 kev, were also used in the energy calibration of the total absorption counter. The spectra of these standards were measured and from the energy and the location of the peaks, the number of kev per channel as well as the energy at a certain channel number could be determined.

Knowing the number of kev per channel it is possible to examine a gamma ray spectrum and determine the energy of the peaks. By measuring the peak intensity as a function of time it is also possible to determine the half life of the nuclide responsible for the peak. Using these two criteria

with reference to known decay schemes, one can identify peaks corresponding to the nuclides whose activities one wishes to determine. The decay schemes of the isomers whose activities one wishes to determine. The decay schemes of the isomers whose activities were determined in this work are shown in Figures 6 through 11. Most of this decay scheme information is taken from the tables of Strominger et. al.²³ or the Nuclear Data Tables²⁴ (the notation used in these decay schemes is that of reference 24). The half lives given for In-110 and In-110m were determined by the present author. The 3.15 Mev level of Cd-110 was found in the decay of In-110m, in experiments carried out in the study of the Ag-107 (α ,n) reaction.

To obtain absolute disintegration rates from the spectra obtained by scintillation counting, it is necessary that the efficiency of the detecting system be known. The counting rates obtained from the spectra are proportional to the disintegration rates:

$$A = \epsilon \lambda N \quad (1)$$

where A is the counting rate or activity, λN the disintegration rate, and ϵ the efficiency of the scintillation counter for determining the disintegration rate. The efficiency ϵ is a product of several factors including geometry, crystal efficiency, peak to total ratio, etc. (cf. Appendix A). For the calculation of cross section ratios, it is not necessary to obtain absolute disintegration rates, but only a ratio of of the disintegration rates is required. For an isomeric pair

FIGURE 6.
DECAY SCHEME OF Se-81, Se-81m.

FIGURE 7.
DECAY SCHEME OF Rh-104, Rh-104m.

FIGURE 8.

DECAY SCHEME OF Pd-109, Pd-109m.

FIGURE 9.

DECAY SCHEME OF Te-129, Te-129m.

FIGURE 10.
DECAY SCHEME OF Cs-134, Cs-134m.

FIGURE II.
DECAY SCHEME OF In-110, In-110m.

the ratio of the disintegration rates at the end of the irradiation is given by:

$$\frac{\lambda_2 N_2}{\lambda_1 N_1} = \frac{\epsilon_1 A_2^0}{\epsilon_2 A_1^0} \quad (2)$$

where subscript 1 refers to the metastable state, subscript 2 refers to the ground state and superscript 0 refers to the activity at the end of the irradiation. In the case of the Se-81, Rh-104 and Cs-134 isomers, the same radiation was detected to determine the activity of both isomers so that ϵ_1/ϵ_2 was unity. In the case of the Pd-109 and the In-110 isomers a different gamma peak was used to determine the activity of each isomer, while in the case of the isomers of Te-129 a different geometry was used in determining the activity of each isomer. In measuring the absolute disintegration rates of these last three isomeric pairs then, ϵ_1/ϵ_2 was not unity and had to be determined experimentally.

2. Selenium - 81 Isomers

In the case of Se-81 and Se-81m, the activity of the isomers was followed by measuring the 1.38 Mev β^- of Se-81 (cf. Figure 6). Very little or no β^- radiation from other activities was present. Although Se-78 is present to the extent of 2.85% in the irradiated target, its (n, γ) product Se-79, which is a beta emitter, has a half life of 6.5×10^4 y, so that very little of this interfering activity is produced. The amount of beta radiation due to Se-81 was determined by adding all the counts, except for the first

five channels,* under the entire beta ray spectrum. The activity was followed for several hours.

3. Gamma Ray Spectra

In all other cases the amounts of the isomers produced were determined by measuring the intensity of a gamma ray (or gamma rays) in their decay. The total number of counts under a particular gamma ray peak is determined by adding up the counts in all the channels falling under the peak and subtracting out counts due to compton and natural background. The gamma ray peaks used in determining counting rates are shown in figures 12 through 18. Also indicated are number of kev per channel** and the compton background subtracted from the peak where this subtraction was necessary. The complete gamma ray spectra for the isomers of Pd-109 and In-110[†] are given. The two spectra given in these cases correspond to different times after the end of the irradiation. The gamma ray spectrum given for Pd-109m was recorded 10 min. after the irradiation; that for Pd-109 was taken using the same sample, but was recorded about one day after irradiation. Similarly the gamma ray spectrum given for In-110 was recorded about 1 hour after the irradiation, while that for

*The output data in the first few channels of a multi-channel analyzer, is usually inaccurate, because of the electronics of the system.

**Zero channel no. does not necessarily represent zero energy in every spectrum.

[†]The spectra of In-110 and In-110m were recorded with an electronic graphic recorder

FIGURE 12.
556 kev GAMMA RAY PEAK OF Rh-104.

FIGURE 13.
GAMMA RAY SPECTRUM OF Pd-109m.

FIGURE 14.
GAMMA RAY SPECTRUM OF Pd-109.
(Natural background subtracted)

FIGURE 15.
1120 kev GAMMA RAY PEAK OF Te-129.

FIGURE 16.
796 AND 801 kev GAMMA
RAY PEAK OF Cs-134.

FIGURE 17.
GAMMA RAY SPECTRUM OF In-110.

FIGURE 18.
GAMMA RAY SPECTRUM OF In-110m.

In-110m was recorded about 12 hours after irradiation. The intensity of the gamma ray peak is followed in most cases for several half lives of the longest lived isomer. Analysis of the decay curve, or the growth in the case of the Cs-134 isomers, to obtain the activities of each isomer produced, will be discussed in a later section.

4. Rhodium - 104 Isomers

Measuring the activity of the Rh-104 from the gamma ray spectrum, was simply a matter of measuring the gamma ray intensity under the 556 kev peak. Although the method of subtracting out the compton distribution from higher energy gamma rays, and counts from the tail end of lower energy peaks was somewhat arbitrary (cf. Figure 12), it made little difference as long as the same method of subtraction was used throughout the decay. This is true because Rh-104 and Rh-104m are practically the only activities produced by irradiation of Rh-103 with thermal neutrons, such that the activity that is subtracted out is decaying with practically the same half-life as the activity under the 556 kev gamma peak, which is due to the photoelectric process. This also applies to other cases in which the two isomers produced are practically the only activities produced in the reaction.

5. Palladium-109 Isomers

The activity of Pd-109 and Pd-109m were determined from the intensity of the gamma peaks at 88 and 188 kev respectively. The target material did contain some isotopes

of palladium other than Pd-108 which might interfere with the measurement of the activity of the Pd-109 isomers, because their (n, γ) products have gamma rays close to those of Pd-109 and Pd-109m (cf. Table 4.) Although Pd-107m may be produced

Table 4

Palladium Isotopes in Pd-108 Target

Nuclides in target	Per cent isotopic abundance	(n, γ) products	Cross section ³⁶ (mb)	$t_{1/2}$	γ 's in product (kev)
Pd-106	3.2	Pd-107m Pd-107		21s 7x10 ⁶ y	213 none
Pd-108	94.7	Pd-109m Pd-109	.26 10.4	4.7m 13.4h	188 88
Pd-110	1.1	Pd-111m Pd-111	<0.05 .21	5.5h 22m	170 65

to a considerable extent, its presence will be negligible after five minutes of decay (about 14 half-lives of Pd-107m), so that in following the decay of the 188 kev peak of Pd-109m, the first count was not taken until at least five minutes after the removal of the sample from the reactor. Pd-107 has no gamma rays and thus will not interfere. Although Pd-111m has a 170 kev gamma ray which may interfere with the measurement of the Pd-109m activity, the cross section for its formation is negligible. Finally the Pd-109 activity is measured a few hours after irradiation so that the Pd-111, having a 65 kev gamma ray, will be gone because of its short half life.

In view of the fact that the activity of the two isomers of Pd-109 were measured using two different gamma rays, the

efficiency ratio ϵ_1/ϵ_2 will differ from unity. There are a number of factors to consider in calculating the efficiency for determining disintegration rates. (cf. Appendix A).

However in the case of the measurement of the activities of Pd-109 and Pd-109m the same counting system was used, so that a number of these factors were practically the same. The factors that differed to any great extent were the photopeak efficiency, the internal conversion coefficients of the transitions measured, the X-ray escape peak factor and the absorption by the aluminum absorber. For the measurement of the Pd-109 activities then:

$$\epsilon = C \epsilon_P \epsilon_\gamma \epsilon_X \epsilon_A \quad (3)$$

where C contains those factors which are constant to both gamma rays measured,

ϵ_P is the fraction of gamma rays striking the crystal that will cause a pulse to fall under the full energy peak (photopeak efficiency).

ϵ_γ is the fraction of transitions giving a gamma ray as opposed to those giving a conversion electron.

ϵ_X is the fraction of pulses falling under the photopeak as opposed to those falling under the X-ray escape peak.

ϵ_A is the fraction of gamma rays not absorbed by the absorber covering the sample.

The photopeak efficiency is obtained directly from graphs of photopeak efficiency vs. gamma ray energy for a 1.5" diameter by 1" high NaI(Tl) crystal.³⁷ These curves are given

for various source to crystal face distances and in the case of the Pd-109 counting this distance was approximately 6cm. The ϵ_γ 's are determined from the internal conversion coefficients of Pd-109m and Ag-109m; Ag-109m is a delayed state in the decay of Pd-109 emitting the 88 kev gamma ray.

$$\epsilon_\gamma = 1/(1 + \alpha) \quad (4)$$

where α is the internal conversion coefficient. This conversion coefficient for Ag-109m can be calculated from existing experimental data.²³ The experimental K internal conversion coefficient for Pd-109m was available but the L and M coefficients were interpolated from the theoretical data of Rose.³⁸ The X-ray escape peak correction factors, ϵ_x , were interpolated from the theoretical data of Axel.³⁹

Table 5

Efficiencies* in the Detection of Pd-109 Isomers

E_γ (kev)	ϵ_P	ϵ_γ	ϵ_x	ϵ_A
88	0.97	0.0413	0.934	0.894
188	0.73	0.538	0.989	0.927

* Cf equation 3 for a definition of these four efficiencies.

Absorption corrections were made using the absorption coefficients for gamma radiation as given by White.⁴⁰

$$\epsilon_A = e^{-\mu(E_\gamma)t} \quad (5)$$

$\mu(E_\gamma)$ is the mass absorption coefficient for a gamma ray of energy E_γ , and t is the thickness of the absorber. Table 5 lists the efficiency factors for an 88 and a 188 kev gamma ray under the experimental conditions for which the Pd-109 isomers were measured.

6. Tellurium - 129 Isomers

Because of the long half life of Te-129m, the activity of this isomer was small in comparison to the activity of Te-129. To obtain better counting statistics in determining the activity of Te-129m, the sample was moved to within $\frac{1}{2}$ " (shelf 1) of the crystal face. The sample was at a distance of 6" (shelf 3) in determining the activity of the Te-129. Thus ϵ_1/ϵ_2 is different from unity. This ratio was determined experimentally by irradiating a sample of Te-128 for a very long period of time (7 Mg of the separated isotope were irradiated for 48 hours at a neutron flux of 2.1×10^{13} neutrons per cm^2 per sec.), allowing the short lived isomer to decay away, and measuring the intensity of the 1120 kev gamma ray peak with the sample on shelf 1 and with the sample on shelf 3. Thus since the same gamma ray was used in determining the activity of Te-129 and Te-129m on shelf 3 and shelf 1 respectively:

$$\epsilon_1/\epsilon_2 = A_1^*/A_3^* \quad (6)$$

where A_1^* is the activity of the 48 hour irradiated Te-128 on shelf 1 and A_3^* is the activity of the 48 hour irradiated Te-128 on shelf 3 at the same time. ϵ_1/ϵ_2 was found to be 21.2 for the counting setup used to measure the 1120 kev gamma ray intensity of Te-129.

7. Cesium-134 Isomers

In the case of the Cs-134 isomers the growth of the ground state isomer was measured. Because of the large difference in the half lives of these two isomers, the growth

amounted to only about 5%. Thus in measuring the intensity of the 796 and 801 kev gamma peak, one had to be very careful in integrating the counts under the peak. It was necessary to check the exact location of the peak, with respect to channel number, at each count because of slight drifts in the gamma ray spectrum due to the slight instability of the analyzer electronics. Activities were then determined by adding all those counts in a given number of channels on either side of the peak and dividing by the time interval of the count.

8. Indium - 110 Isomers

Gamma ray spectra from a total absorption scintillation counter were most suitable for determining the activities of In-110 and In-110m. The activity of In-110 was followed by measuring the intensity of the 656 kev gamma ray. From the decay scheme (Figure 11) it can be seen however that this gamma ray emission is preceded by positron emission in 72% of the cases. These positrons annihilate to produce two 511 kev gamma rays which are in coincidence with the 656 kev gamma ray. The use of a total absorption counter will then cause most of the 511 kev gamma rays to appear in sum peaks (511 + 511, 511 + 656, or 511 + 511 + 656 kev) so that the intensity of the 511 kev peak is very small and will interfere very little with the determination of the number of counts under the 656 kev peak (cf. Figure 17). Using a single 3" x 3" NaI(Tl) crystal, the 511 kev peak was greater in intensity than the 656 kev peak such that there was overlapping of these two peaks in the gamma ray spectrum, making

the analysis difficult. It can also be seen that the 656 kev gamma ray occurs in the decay of In-110m. If a single 3" x 3" NaI(Tl) crystal were used, there would be considerable interference from this gamma ray, but using a total absorption counter there is little chance that the gamma ray from In-110m appear under the 656 kev peak. It is much more likely to appear in a sum peak, since this gamma ray is in coincidence with 2 or 3 other gamma rays (cf. Figure 11). The use of a total absorption counter to measure the activity of In-110 has obvious advantages.

Since higher energy peaks occur in the gamma ray spectrum of In-110, it is necessary to subtract from the counts under the 656 kev peak, those counts which are due to compton distributions of higher energy peaks. Here again the use of a total absorption counter is advantageous because of its ability to reduce the compton distribution of the gamma ray spectrum. This compton distribution was subtracted from the 656 kev peak by extrapolation of the straight line portion of the spectrum on the right hand side of this peak, under this peak, and subtracting out those counts falling under this line. This method is somewhat arbitrary since it is doubtful that the compton distribution under the 656 kev peak is exactly a straight and horizontal line, but since this method was also used in determining the efficiency, the errors will tend to cancel one another.

The 3.15 Mev peak occurring in the gamma ray spectrum of In-110m (cf. Figure 18) as measured with the total absorp-

tion counter, is the peak due to the sum of 4 gamma rays (cf. Figure 11). The intensity of this peak is not too great, but because of the long half live of In-110m, a large number of counts can be accumulated by counting for a long period of time. Also because of the high energy of this peak, the background was very low (about 4 counts per minute under the 3.15 Mev peak compared to about 50 counts per minute under the 656 kev peak). Another advantage to determining the activity of In-110m by measuring the intensity of the 3.15 Mev peak, is the fact that at such a high energy there is little chance of contamination from other radioactivities which may be present in the sample.

E. Total Absorption Counter Efficiencies

1. General

In the case of the In-110 isomers the absolute values of the counting efficiencies ϵ_1 and ϵ_2 were determined. This enabled one to obtain absolute disintegration rates as well as the ratio of these disintegration rates. As mentioned previously the efficiencies depend upon a number of factors. To determine each of these factors individually for the total absorption counter would be quite difficult, and the errors involved in each factor would add up to give a large error in the overall efficiency. It would be much simpler to obtain samples of In-110 and In-110m whose absolute disintegration rates are known, and then count these samples directly with the total absorption counter to determine the overall efficiencies experimentally. This is what was done, and a different procedure was used in the case of each isomer.

2. Efficiency for Counting In-110

A sample containing In-110 and also a small amount of In-110m, prepared by alpha irradiation of Ag-107, is counted with a 3" x 3" NaI(Tl) crystal at a fixed geometry. The efficiency for counting 656 kev gamma rays at this geometry had been previously determined by use of a Cs-137 intensity standard. Cs-137 has a single gamma ray of 662 kev in its decay. The absolute disintegration rate of this standard had been determined by 4π counting. Knowing the efficiency of the 3" x 3" NaI(Tl) crystal and assuming that

every disintegration of an In-110 nucleus produces a 656 keV gamma ray (i.e. every In-110 nucleus decays via the 656 keV excited state of Cd-110, and conversion electrons in the transition of this state to the ground state are negligible), one can readily obtain the disintegration rate of In-110:

$$(\lambda N)_{\text{In-110}} = \frac{A_{\text{In-110 } 3" \times 3" \text{ NaI(Tl)}}}{\epsilon_{656} \epsilon_A \epsilon_S} \quad (7)$$

In this equation $(\lambda N)_{\text{In-110}}$ is the absolute disintegration rate of the In-110. $A_{\text{In-110 } 3" \times 3" \text{ NaI(Tl)}}$ is the counting rate of the In-110 656 keV peak on the 3" x 3" NaI(Tl) crystal; this counting rate has been corrected for any In-110m 656 keV gamma radiation that is present in the sample. ϵ_{656} is the efficiency for detecting a 656 keV gamma ray under the conditions which the Cs-137 standard was counted, ϵ_A is a factor due to absorption by Al absorbers covering the In-110 sample, and ϵ_S is a factor due to the possibility of losing a count under the 656 keV peak because of summing with one of the 511 keV gamma rays produced in the annihilation of the positron preceding the emission of the 656 keV gamma ray. The factor ϵ_{656} was determined experimentally to be 5.93×10^{-3} (this was at a distance of about 6" from the face of the NaI(Tl) crystal). The aluminum absorbers covering the In-110 sample had a thickness of $.697 \text{ g/cm}^2$. Using the total absorption coefficient obtained from the data of White⁴⁰ the factor ϵ_A was found to be .9492. The efficiency factor ϵ_S is very close to unity. Because of the large distance of the sample from the crystal, the probability of a sum peak

occurring was very small. ϵ_s was calculated to be 0.983.

Now that the absolute disintegration rate of an In-110 sample can be found, it is possible to determine the efficiency of the total absorption counter by counting this same sample with that counter:

$$\epsilon_{\text{In-110}} = \frac{A_{\text{In-110 TA}}}{(\lambda N)_{\text{In-110}}} \quad (8)$$

where $\epsilon_{\text{In-110}}$ is the efficiency for determining the disintegration rate of In-110 on the total absorption counter from the activity of the 656 kev gamma peak, $A_{\text{In-110 TA}}$ is the counting rate of In-110 under the 656 kev gamma peak on the total absorption counter. The silver-107 foil was irradiated with alpha particles at 12.4 Mev such that at this low energy, practically no interfering activities were produced. The activities as determined by the two counters at the end of the irradiation, after making the necessary corrections for the In-110m present, were found by analysis of their decay to be: $A_{\text{In-110 } 3"x3" \text{ NaI(Tl)}} = 3,490$ counts per minute and $A_{\text{In-110 TA}} = 59,660$ counts per minute. Using equations (7) and (8), $\epsilon_{\text{In-110}}$ was found to be 0.0952.

3. Efficiency for Counting In-110m

In order to determine the efficiency for counting the 3.15 Mev peak in the gamma ray spectrum of In-110m (Figure 18), it was necessary to prepare an almost radioactively pure sample of In-110m. This was done by irradiating Ag-107 with very low energy alpha particles (13.7 Mev) and then allowing the In-110 to decay away. At this low energy

the only reaction other than the (α, n) , occurring to any extent, might be the (α, p) reaction. Its threshold is only 3.4 Mev, but the product of the reaction, Cd-110, is stable and thus does not interfere with the counting. This sample is then counted in a 5"x5" NaI(Tl) well crystal in conjunction with a single channel analyzer. Such a counting system produces one count for every gamma ray absorbed by the crystal. This counter acts as a 4π counter in the detection of In-110m (i.e. every disintegration of In-110m will produce a count). The reason that every disintegration of In-110m produces a count is not directly evident. It can be seen from the decay scheme of In-110m that for practically every disintegration at least 3 gamma rays occur in cascade. Thus if the first gamma ray escapes detection, there is a chance that the second gamma ray will be detected. If both the first and second escape, there is still a chance that the third gamma ray will be detected. The probability of detection of a single gamma ray can be calculated since the absorption coefficients for gamma radiation in NaI(Tl) are known as a function of the gamma ray energy⁴¹ (cf. Appendix B). Using the efficiencies calculated in Appendix B, the probability of detecting at least one of the three gamma rays, ϵ_γ , in the decay of In-110m with a 5"x5" NaI(Tl) well crystal can be calculated:

$$\epsilon_\gamma = 1 - (1 - \epsilon_{656})(1 - \epsilon_{885})(1 - \epsilon_{940}) \approx .993 \quad (9)$$

This equation is based on probability considerations, where $(1 - \epsilon_{656})$, $(1 - \epsilon_{885})$ and $(1 - \epsilon_{940})$ actually represent the

probability of escape of a 656, an 885 and a 940 kev gamma ray respectively. From this quantity and on the basis that a small fraction, 0.006, of the In-110m might also decay to In-110 and possibly produce two counts per In-110m disintegration, it was assumed to a very good approximation that one In-110m disintegration occurred for every count recorded with the 5"x5" NaI(Tl) crystal.

In following the decay of In-110m with the 5"x5" NaI(Tl) crystal, it was found that a long lived component was present (cf. Figure 19). This was found to be In-111 ($t_{\frac{1}{2}} = 2.8d$) which was produced by the (α, γ) reaction on Ag-107. This activity was subtracted from the total activity to leave only the activity due to In-110m. The decay of the In-110m was followed on both the 5"x5" NaI(Tl) crystal and the total absorption counter. The activities were determined at a common time after irradiation and since the activity on the 5"x5" NaI(Tl) crystal corresponds to the absolute disintegration rate of In-110m, it is possible to determine the efficiency for counting the 3.15 Mev peak as recorded by the total absorption counter:

$$\epsilon_{\text{In-110m}} = \frac{A_{\text{In-110m TA}}}{(\lambda N)_{\text{In-110m 5"x5" NaI(Tl)}}} \quad (10)$$

$\epsilon_{\text{In-110m}}$ is the efficiency for determining the disintegration rate of In-110m on the total absorption counter from the activity of the 3.15 Mev gamma peak, $A_{\text{In-110m TA}}$ is the counting rate of In-110m under the 3.15 Mev gamma peak on the total

FIGURE 19.
In 110 m DECAY.

absorption counter, and $(\lambda N)_{\text{In-110m } 5"x5" \text{ NaI(Tl)}}$ is the disintegration rate of In-110m as measured by the 5"x5" NaI(Tl) crystal. For a given sample, $A_{\text{In-110m TA}}$ and $(\lambda N)_{\text{In-110m } 5"x5" \text{ NaI(Tl)}}$ were found to be 113.5 and 11,150 counts per minute respectively at a time 1400 minutes after the end of the irradiation. This gives an efficiency $\epsilon_{\text{In-110m}}$ of 0.01018.

III. EXPERIMENTAL RESULTS

A. Decay Analysis

1. Selenium-81, Rhodium-104 and Tellurium-129 Isomers

In determining the ratio of the disintegration rates of the isomers produced in the (n, γ) reactions on Se-80, Rh-103 and Te-128, only the activity of the ground state isomer was measured experimentally. It can be seen from the decay schemes of these three isomeric pairs (Figures 6,7,9) that they exhibit a "parent-daughter" relationship. Thus it is possible using the decay curve of the ground state isomer (the daughter), and the equation for the growth and decay of this daughter²²,

$$A_2 = \frac{\lambda_2}{\lambda_2 - \lambda_1} A_1^0 (e^{-\lambda_1 t} - e^{-\lambda_2 t}) + A_2^0 e^{-\lambda_2 t} \quad (11)$$

to determine the activities of both isomers at the end of the irradiation, A_1^0 and A_2^0 . The subscripts 1 and 2 in the above equation refer to the parent and daughter respectively. The λ 's are the decay constants, and A_2 is the activity of the daughter determined at a time t after the end of the irradiation. Equation (11) assumes that all of the metastable state decays directly to the ground state by an isomeric transition. This is true to within less than 0.5% for the cases of the Se-81, Rh-104 and Te-129 isomeric pairs.

The method of calculating A_2^0 and A_1^0 for these three cases may be demonstrated by using the Rh-104 isomers as an example. Figure 20 shows the decay of the daughter activity Rh-104, followed by measuring the 556 kev gamma ray

FIGURE 20.
DECAY OF Rh-104.
(556 kev γ peak)

intensity. The two components of the curve indicate the presence of the long and short lived isomers. In this case, where the parent is longer lived than the daughter ($\lambda_2 > \lambda_1$), the second term on the right hand side of equation (11) will become very small compared to the first term for times of the order of several half lives of the daughter. A_1^0 then can be calculated from equation (11) using values of A_2 obtained after several half lives of the daughter, and estimating A_2^0 graphically. A_2^0 is estimated graphically by extrapolating the straight line portions of the two component curve back to zero time, and taking the difference of these two activities to be approximately equal to the initial daughter activity. Sometimes the short lived component curve is extrapolated mathematically by use of the simple decay equation:

$$A = A^0 e^{-\lambda t} \quad (12)$$

The method of estimating A_2^0 as described above may be as great as 15% in error, but the error introduced in the calculation of the initial parent activity A_1^0 , is very small (<1%) since A_2^0 appears only as a small correction term in its calculation.

Using the value of A_1^0 as calculated above, is now possible to calculate a more accurate value of A_2^0 using equation (11) in conjunction with the data obtained early in the decay of the ground state isomer. $\frac{\lambda_2 N_2}{\lambda_1 N_1}$, necessary for the calculation of the cross section ratio, can now be calculated from equation (2), since ϵ_1/ϵ_2 is unity for the selenium and rhodium isomers and was determined experimentally for the counting of the Te-129 isomers.

2. Cesium - 134 Isomers

In the case of the Cs-134 isomers, the growth of the

ground state isomer from the metastable state isomer, was determined by measuring the intensity of the 801 and 796 keV gamma peak. Since the resolution of the 3" x 3" NaI(Tl) crystal used to detect these gamma rays was of the order of 10%, they appeared as one peak. The evaluation of A_2^0/A_1^0 for the Cs-134 isomers proceeds as follows. If equation (11) is divided through by $e^{-\lambda_2 t}$, an equation for a straight line of the form, $y = mx + b$ is obtained:

$$A_2 e^{\lambda_2 t} = \frac{\lambda_2}{\lambda_1 - \lambda_2} A_1^0 \left(1 - e^{-(\lambda_1 - \lambda_2)t}\right) + A_2^0 \quad (13)$$

where $y = A_2 e^{\lambda_2 t} \quad (14)$

$$x = \frac{\lambda_1}{\lambda_1 - \lambda_2} \left(1 - e^{-(\lambda_1 - \lambda_2)t}\right) \quad (15)$$

$$b = A_2^0 = y \text{ intercept} \quad (16)$$

$$m = \frac{\lambda_2}{\lambda_1} A_1^0 = \text{slope} \quad (17)$$

For the Cs-134 isomers $\lambda_1 = 0.2394 \text{ h}^{-1}$ and $\lambda_2 = 3.765 \times 10^{-5} \text{ h}^{-1}$, so that to a very good approximation $\lambda_1 - \lambda_2$ may be replaced by λ_1 and therefore:

$$x = (1 - e^{-\lambda_1 t}). \quad (15a)$$

Also since $\lambda_2 t$, for $t < 10\text{h}$, is very small, $e^{\lambda_2 t} \simeq 1$ to a very good approximation and

$$y = A_2. \quad (14a)$$

Thus by plotting A_2 vs. $1 - e^{-\lambda_1 t}$, a straight line is obtained. From the slope of this straight line and the y intercept, A_2^0/A_1^0 or $\lambda_2 N_2/\lambda_1 N_1$ can be obtained. Figure 21 shows a plot of A_2 vs. $(1 - e^{-\lambda_1 t})$ for a typical experiment. A straight line is fitted to the points by the method of least squares.

FIGURE 21.
GROWTH OF Cs-134.

3. Palladium - 109 Isomers

The activity of the Pd-109m was followed by measuring the decay of the 188 kev gamma ray peak. To obtain its activity at the end of the irradiation is a matter of applying the simple decay equation (eq. 12). The activity of the ground state, Pd-109, was measured after the Pd-109m had decayed away. To determine the initial activity of the ground state it was necessary to apply equation (11). For times long after the end of the irradiation this equation reduces to:

$$A_2 = \frac{\lambda_2}{\lambda_1 - \lambda_2} A_1^0 e^{-\lambda_2 t} + A_2^0 e^{-\lambda_2 t} \quad (18)$$

enabling one to calculate A_2^0 using the A_1^0 obtained above, and using values of A_2 for t much greater than the half life of the Pd-109m. As mentioned previously it is necessary to take into account the different counting efficiencies in calculating the ratio of the absolute disintegration rates.

4. Indium - 110 Isomers

From the decay scheme of the In-110 isomers (Figure 11), it can be seen that these isomers decay essentially independently of one another. The decay of In-110m is followed by measuring the intensity of the 3.15 Mev gamma peak. The decay of In-110 is followed by measuring the intensity of the 656 kev gamma peak. In the case of the latter it may be necessary to subtract out long lived components due to In-110m or In-109. In-109, which is produced by the Ag-107(α ,2n) or the Ag-109(α ,+n) reaction, has a 632 kev gamma ray, unresolvable from the In-110 656 kev gamma ray. The In-109 produced by these reactions will interfere considerably in the decay analysis when their reaction thresholds have been

exceeded by a few Mev. Figure 22 shows the decay of the 656 kev gamma peak activity. The long lived component is due to In-109 and In-110m with half-lives of 4.3h and 5.2h respectively. After the long lived component is subtracted out, the remaining activity is due to In-110 alone. A straight line is drawn through these points to fit the experimental data and to fit the previously measured half-life of In-110. To determine the initial activity of In-110, one usually chooses some point near the experimental points and calculates A^0 using equation (12) with $\lambda = 9.874 \times 10^{-3} \text{m}^{-1}$. This method is also used to calculate the initial activity of In-110m using $\lambda = 2.222 \times 10^{-3} \text{m}^{-1}$. The disintegration rates of these two isomers are readily found using the efficiencies calculated previously.

FIGURE 22.
In-110 DECAY.

B. Calculations and Results of Cross Sections and Isomeric Cross Section Ratios

1. Independently Decaying Isomers.

To calculate experimental cross section ratios of independently decaying isomers, it is necessary to solve the rate equation for the formation of a radioactive nuclide produced at a constant rate. For the irradiation of a very thin target (this applies to the silver targets used in the (α, n) and $(\alpha, 3n)$ reactions) the rate of change of the number of product atoms, $\frac{dN}{dt}$, is given by its rate of formation, R , minus its rate of decay, λN :

$$\frac{dN}{dt} = R - \lambda N \quad (19)$$

R is given by:

$$R = n f \sigma \quad (20)$$

where n is the number of target atoms per square centimeter producing the desired reaction, f is the number of particles per second striking the target, and σ is the cross section in centimeters squared. If R is to remain constant during the irradiation, the flux of alpha particles produced by the cyclotron must be kept constant. Actually there are slight fluctuations in the beam current during an irradiation ($\pm 20\%$) but as long as the length of the irradiation is short compared to the half life of the shortest lived isomer, this will not effect the results to any considerable degree.

The solution of equation (19) is:

$$\lambda N = n f \sigma (1 - e^{-\lambda t}) \quad (21)$$

where λN is the disintegration rate of the product after an

irradiation time t . For the Ag-107 (α, n) and the Ag-109 ($\alpha, 3n$) reactions, λN , n , f and t were determined from experimental data so that σ could be calculated directly from equation (21). To calculate the cross section ratio for the production of In-110 and In-110m by these reactions, it is not necessary to know n or f , provided $\lambda_1 N_1$ and $\lambda_2 N_2$ are evaluated from the activities of the isomers in the same irradiated target. If equation (21) is written for each isomer then it follows that:

$$\frac{\sigma_2}{\sigma_1} = \frac{\lambda_2 N_2}{\lambda_1 N_1} \frac{(1 - e^{-\lambda_1 t})}{(1 - e^{-\lambda_2 t})} \quad (22)$$

where the subscripts 1 and 2 again refer to the metastable and ground state isomers respectively. The isomeric cross section ratio, $\sigma_1/(\sigma_1 + \sigma_2)$ can then be calculated from σ_2/σ_1 .

2. Parent-Daughter Isomers

In the case of an isomeric pair in which the metastable state decays completely to the ground state, the rate equation for the production of the ground state is given by:

$$\frac{dN_2}{dt} = R_2 - \lambda_2 N_2 + \lambda_1 N_1 \quad (23)$$

where the terms have the same meaning as in equation (19). The solution of this equation (cf. Appendix C) is:

$$\lambda_2 N_2 = n f \left\{ (\sigma_1 + \sigma_2) (1 - e^{-\lambda_2 t}) + \frac{\lambda_2}{\lambda_1 - \lambda_2} \sigma_1 (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \right\} \quad (24)$$

For the metastable state, equation (21) applies:

$$\lambda_1 N_1 = n f \sigma_1 (1 - e^{-\lambda_1 t}) \quad (25)$$

The terms and subscripts in these last two equations have the same meaning as before. If the amounts of both isomers are

studied from one irradiation, then from equations (24) and (25):

$$\frac{\sigma_2}{\sigma_1} = \frac{1}{(1-e^{-\lambda_2 t})} \left\{ \frac{\lambda_2 N_2}{\lambda_1 N_1} (1-e^{-\lambda_1 t}) - \frac{\lambda_2}{\lambda_1 - \lambda_2} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \right\} - 1 \quad (26)$$

For short irradiation periods:

$$(1-e^{-\lambda t}) \simeq 1 - 1 + \lambda t = \frac{.693}{t_{\frac{1}{2}}} t \quad (27)$$

and equation (26) reduces to:

$$\frac{\sigma_2}{\sigma_1} = \frac{\lambda_2 N_2 t_{\frac{1}{2}}(2)}{\lambda_1 N_1 t_{\frac{1}{2}}(1)} \quad (28)$$

where $t_{\frac{1}{2}}(1)$ and $t_{\frac{1}{2}}(2)$ are the half lives of the metastable and ground state isomers respectively. Equation (26) or (28) was used in calculating cross section ratios for isomers produced in (n,γ) reactions.

3. Numerical Results

Isomeric cross section ratios of isomers produced in the (n,γ) reactions were determined with thermal and epi-cadmium neutrons. Thermal neutrons were obtained by direct irradiation with the "rabbit facility". Epi-cadmium neutrons were obtained by irradiation of the samples wrapped in cadmium foil (20 mil thick in most cases). The isomeric cross section ratios are shown in Table 6. Additional experimental data can be found in Appendix D. Isomeric cross section ratios, as well as absolute cross sections for the formation of In-110 and In-110m in the Ag-107 (α,n) and the Ag-109 $(\alpha,3n)$ reactions were determined at several energies, and are shown in Tables 7 and 8 (cf. Appendix D for additional experimental data). The alpha particle energy range given in these tables refers to the average energy of the alpha

Table 6
Thermal Neutron and Epi-Cadmium
Isomeric Cross Section Ratios

Isomers	Exp. no.*	Isomeric Cross Thermal Neutrons	Section Ratio Epi-Cadmium Neutrons	Error
Se-81, Se-81m	{ N20 N28	0.115	0.161 }	±10%
Rh-104, Rh-104m	{ N16 N22 N21	0.0685 0.0729	0.0688 }	±10%
Pd-109, Pd-109m	{ N13 N27	0.0216	0.0219 }	±20%
Te-129, Te-129m	{ N 7 N26	0.0736	0.070 }	±20%
Cs-134, Cs-134m	{ N 5 N32 N24 N29 N30 N31	0.0925 0.0792	0.0760 0.0723 0.0689 0.0747 }	±15%

*Experiment numbers are given so that reference to the experimental data given in Appendix D can readily be made. This also applies to Tables 7 and 8.

Table 7

Cross Sections and Isomeric Cross Section Ratios
for the Ag-107(α ,n) In-110, In-110m Reaction

Exp. no.	α energy* (Mev)	$\sigma_{\text{In-110m}}$ (mb)	$\sigma_{\text{In-110m}}$ (mb)	$\frac{\sigma_{\text{In-110m}}}{\sigma_{\text{In-110}} + \sigma_{\text{In-110m}}}$
A10	10.6-11.0	1.99	0.30	0.131
A 8(2)	11.9-12.3	33.2	6.63	0.167
A 9	13.4-13.7	78.1	21.5	0.216
A12(2)	14.85-15.15	159	69.0	0.303
A11(2)	16.8-17.1	187	123	0.397
A11(1)	18.5-18.8	157	188	0.546
A 8(1)	18.6-18.9		211	
A12(1)	20.1-20.35	59.7	178	0.749
A13(2)	20.3-20.55	74.9	214	0.740
A13(1)	21.8-22.1	30.6	128	0.806 ₅

*This is the alpha particle energy, E_α , in the laboratory system. The alpha particle energy (or E_α) referred to in other tables, and throughout the text likewise refers to the alpha particle energy in the laboratory system.

Table 8

Cross Sections and Isomeric Cross Section Ratios
for the Ag-109(α ,3n) In-110, In-110m Reaction

Exp. no.	α energy (Mev)	$\sigma_{\text{In-110}}$ (mb)	$\sigma_{\text{In-110m}}$ (mb)	$\frac{\sigma_{\text{In-110m}}}{\sigma_{\text{In-110}} + \sigma_{\text{In-110m}}}$
B 4(2)	25.9-26.2		6.2	
B 4(1)	27.5-27.8	33	68.5	0.675
B 2(2)	29.2-29.5	72.9	193	0.726
B 2(1)	31.0-31.3	103	363	0.778
B 1(2)	33.1-33.4	116.4	487	0.807
B 1(1)	35.1-35.3	135.6	672	0.832
B 3(2)	36.8-37.0	141.4	811	0.851 ₅
B 3(1)	38.6-38.75	115	799	0.874

particle upon entering, and upon leaving, the silver foil. The difference is due to the finite thickness of the silver foil. The alpha particle beam at full energy actually has an energy spread (i.e. the full width at half maximum beam intensity) of about 0.4 Mev. This spread will increase when the beam is degraded because of scattering effects in the aluminum foils; the beam spread is about 0.8 Mev at an alpha energy of 15 Mev.

4. Cross Section for the Ag-107 (α, γ) Reaction

As was mentioned previously the product of the Ag-107 (α, γ) reaction, In-111, was observed by means of the 5"x5" NaI(Tl) well crystal. The activity of this product was followed for about a week, so that it was possible to determine the activity of the In-111 at the end of the irradiation by extrapolation of its decay curve. The detection efficiency of the 5"x5" NaI(Tl) crystal for detecting In-111 can be assumed to be close to 100% since two low energy gamma rays (172 and 247 kev) occur in cascade.* Since the efficiency for detecting In-110m disintegrations is also ~100%, then from equation (21) it follows that:

$$\frac{\sigma_{\text{In-110m}}}{\sigma_{\text{In-111}}} = \frac{A_{\text{In-110m}}}{A_{\text{In-111}}} \frac{(1-e^{-\lambda_{\text{In-111}}t})}{(1-e^{-\lambda_{\text{In-110m}}t})} \quad (29)$$

where $A_{\text{In-110m}}$ and $A_{\text{In-111}}$ are the counting rates observed with the 5"x5" NaI(Tl) crystal. And since the irradiations were short compared to the half lives of In-110m and In-111:

$$\frac{\sigma_{\text{In-110m}}}{\sigma_{\text{In-111}}} = \frac{A_{\text{In-110m}}}{A_{\text{In-111}}} \frac{t_{\frac{1}{2}}(\text{In-110m})}{t_{\frac{1}{2}}(\text{In-111})} \quad (30)$$

*Actually there is a delay of ~0.1 μ s in emitting the second gamma ray, but the resolving time of the counter was greater than 5 μ sec, so that there is no chance that two counts are recorded by the single channel analyzer.

Using half lives of .217 days and 2.81 days for In-110m and In-111 respectively, the ratio $\sigma_{\text{In-110m}}/\sigma_{\text{In-111}}$ could be calculated. This ratio was calculated at two energies for which experimental data on the In-111 activity was available. The absolute cross section for the formation of In-110m had been previously calculated for these two energies, so that the absolute cross section ratio for the formation of In-111 could be obtained. The results are as follows: $\sigma_{\text{In-111}} = 0.46\text{mb}$ for an average alpha particle energy of 12.0 to 12.4 Mev and $\sigma_{\text{In-111}} = 0.56\text{mb}$ for an average alpha particle energy of 13.4 to 13.7 Mev. The error in these cross sections is about $\pm 30\%$.

5. Discussion of Errors

The principal errors in determining isomeric cross section ratios for the (n, γ) reactions arise because of uncertainties in the decay schemes and the decay constants of the isomers whose activities were measured. For the Pd-109 and the Te-129 isomers it was also necessary to determine efficiencies or an efficiency ratio in the case of the latter. This gives rise to additional errors. Poor counting statistics in measuring the Te-129m activity also produces error in the isomeric cross section ratio. Additional errors in determining the isomeric cross section ratio for the Cs-134 isomers arise because the method of determining the ratio involves the measurement of a very slight growth (only about 5%) over a period of several hours. Approximate errors in the isomeric cross section ratios of isomers produced by (n, γ) reactions are given in Table 6.

In determining the cross sections and isomeric cross section ratios for the In-110 isomers a number of errors are involved:

- (1) Determination of In-110 counting efficiency: these errors arise because of uncertainties in the decay schemes of In-110 and Cs-137, and because of the rather indirect method of determining this efficiency: $\pm 6\%$
- (2) Determination of In-110m counting efficiency: $\pm 4\%$
- (3) Decay constants of In-110 and In-110m: $\pm 2\%$ each
- (4) Gamma ray spectra analysis: This error is greatest at the lowest energies and at the highest energies of each reaction because at these energies activities from competing reactions make the analysis of the gamma ray spectrum more difficult: $\pm 3\%$ to $\pm 8\%$.
- (5) Miscellaneous small errors: loss of small bits of the sample, inability to reproduce exact geometry, etc.: $\pm 2\%$
- (6) Weight of silver target: $\pm 1.5\%$
- (7) Current on Target: $\pm 4\%$

The overall error in the determination of the individual cross sections may be as high as 20%. However we are interested principally in the isomeric cross section ratio. Since equation (22) was used in determining these cross section ratios, the last two errors above do not enter into the error of the isomeric cross section ratio. Also since these ratios are measured at a number of energies, it is meaningful to consider constant and random errors.⁴² Of the errors listed above, the first three will be constant for each measurement of the cross section ratio, while the fourth and fifth are random in nature. Table 9 gives an idea of the overall errors involved in the isomeric cross section ratios at various energies.

Table 9

Errors in Isomeric Cross Section Ratios for
Ag-107(α ,n) and Ag-109(α ,3n) Reactions

Exp. no.	α Energy (Mev)	$\frac{\sigma_{\text{In-110m}}}{\sigma_{\text{In-110}} + \sigma_{\text{In-110m}}}$	Constant* error	Random error
A 8(2)	11.9-12.3	0.167	± 0.021	± 0.007
A11(2)	16.8-17.1	0.397	± 0.036	± 0.015
A13(1)	21.8-22.1	0.807	± 0.023	± 0.015
B 4(1)	27.5-27.8	0.675	± 0.033	± 0.022
B 1(2)	33.1-33.4	0.807	± 0.023	± 0.013
B 3(1)	38.6-38.75	0.874	± 0.017	± 0.011

*The constant error in $\frac{\sigma_{\text{In-110m}}}{\sigma_{\text{In-110}}}$ will be the same percentagewise for every measurement of this quantity. The percentage error in $\frac{\sigma_{\text{In-110}}}{\sigma_{\text{In-110}} + \sigma_{\text{In-110m}}}$ due to the constant error in $\frac{\sigma_{\text{In-110m}}}{\sigma_{\text{In-110}}}$ however

becomes smaller as this quantity approaches zero or one.

An additional error of $\pm 10\%$ occurs in $\sigma_{\text{In-110}}$, and consequently in $\frac{\sigma_{\text{In-110m}}}{(\sigma_{\text{In-110}} + \sigma_{\text{In-110m}})}$, of experiment A12(1), because of a delay in measuring the In-110 activity. The error in measuring the alpha particle energy from range measurements is about $\pm 2\%$ at 38 Mev and could be as high as $\pm 10\%$ at 12 Mev.

IV. THEORETICAL CALCULATION METHODS

A. Reaction Mechanism and Energetics

In order to determine how to calculate the isomeric cross section ratio theoretically, it is necessary first to examine the mechanism of the reaction. It is assumed first of all, that the target nucleus absorbs the incoming projectile and a compound nucleus is formed. The excitation energy of this compound nucleus, E_c , is equal to the binding energy of the incoming particle, B , plus the kinetic energy of the incoming projectile in the center of mass coordinates, E_{cm} (some of the incoming energy is lost in the form of recoil energy of the compound nucleus because of the law of conservation of linear momentum):

$$E_c = B + E_{cm} \quad (31)$$

The binding energies for the reactions of interest are given in Table 10. For the (n,γ) reactions it can be assumed that a thermal neutron has negligible kinetic energy, so that the excitation energy of the compound nucleus is given by the binding energy of the neutron alone. As mentioned previously the alpha particles striking the silver targets have a spread of energies (the full width at half maximum beam intensity is about 0.4 Mev at an alpha energy of 42 Mev, and about .8 Mev at an alpha energy of 15 Mev). Consequently when an average alpha energy is used in computing E_c , one must realize that an average compound nucleus excitation energy is obtained.

De-excitation of the compound nucleus formed by slow neutron capture occurs principally by emission of gamma radiation. A number of gamma rays are emitted until finally de-excitation to

Table 10

Some Neutron and Alpha Particle Binding Energies*

Reaction	B (Mev)	Error ($\pm$ Mev)
$^{80}_{34}\text{Se} + n \rightarrow ^{81}_{34}\text{Se}$	6.71	.13
$^{103}_{45}\text{Rh} + n \rightarrow ^{104}_{45}\text{Rh}$	6.82	.27
$^{108}_{46}\text{Pd} + n \rightarrow ^{109}_{46}\text{Pd}$	6.10	.46
$^{128}_{52}\text{Te} + n \rightarrow ^{129}_{52}\text{Te}$	6.51	.52
$^{133}_{55}\text{Cs} + n \rightarrow ^{134}_{55}\text{Cs}$	6.75	.10
$^{107}_{47}\text{Ag} + {}^4_2\text{He} \rightarrow ^{111}_{49}\text{In}$	1.56	.50
$^{109}_{47}\text{Ag} + {}^4_2\text{He} \rightarrow ^{113}_{49}\text{In}$	2.68	.46
$^{113}_{49}\text{In} \rightarrow ^{112}_{49}\text{In} + n$	- 9.28	.40
$^{112}_{49}\text{In} \rightarrow ^{111}_{49}\text{In} + n$	- 8.18	.44
$^{111}_{49}\text{In} \rightarrow ^{110}_{49}\text{In} + n$	- 9.61	.50
$^{110}_{49}\text{In} \rightarrow ^{109}_{49}\text{In} + n$	- 7.63	.46

*These are taken from reference 43; a positive value indicates energy released in the nuclear reaction.

the ground state or the metastable state occurs. In the case of a compound nucleus formed by absorption of an energetic alpha particle, the nucleus has sufficient energy to de-excite by particle emission. For the same available energy of the emitted particle, neutron emission is much more probable than proton emission, which in turn is more probable than alpha emission. The reason for this is the emergent coulomb barrier in the case of a charged particle emission. In the case of neutron emission the average energy of the residual nucleus, E_r , is given by:

$$E_r = E_c - B_n - E_n \quad (32)$$

where E_c is the average compound nucleus energy, B_n is the neutron binding energy, and E_n is the average neutron kinetic energy.

Neutron binding energies necessary for calculating average residual nucleus energies for the Ag-107 (α, n) and the Ag-109 ($\alpha, 3n$) reactions are also listed in Table 10. The calculation of average neutron kinetic energies will be discussed later. They are of the order of a few Mev. When particle emission is no longer possible, de-excitation will occur by gamma ray emission as in the (n, γ) reactions, eventually leading to the metastable or ground state isomer.

To calculate the isomeric cross section ratio, a method described by Vandenbosch and Huizenga^{9,10} was used. In this method the important factors to consider are the following:⁹

- (1) The spins of the compound nuclear states. (This requires a knowledge of the target spin and the process by which the compound nucleus is formed).
- (2) The number and types of steps in the de-excitation of the compound state.

- (3) The angular momentum carried away at each step.
- (4) The probability of forming states of different spins during each step of the cascade.
- (5) The spins of the metastable and ground state finally populated.

On the basis of the nuclear reaction mechanism described above, the effect that these factors play in determining the isomeric cross section ratio will be discussed in the following sections.

B. Spin Distributions

1. Compound Nucleus

For thermal neutrons it is assumed that only s-wave ($\ell=0$) neutrons are captured. In other words no orbital angular momentum is imparted to the target nucleus, so that the spin of the capturing state in the compound nucleus is $J_c = (I + \frac{1}{2})$ or $J_c = (I - \frac{1}{2})$, where I is the spin of the target nucleus and $\frac{1}{2}$ is the intrinsic spin of the neutron. For the target nuclei Se-80, Pd-108 and Te-128, $I = 0$, so that the capturing state can only be $J_c = \frac{1}{2}$.

For the formation of a compound nucleus with a high energy alpha particle (>10 Mev) a distribution in spins of the compound nucleus is obtained, since the alpha particle can carry in orbital angular momentum of various amounts. This distribution can be calculated from a knowledge of the cross section for the formation of a compound nucleus of spin J_c , at a bombarding energy E^4 . This cross section is given by:

$$\sigma(J_c, E) = \pi \lambda^2 \sum_{S=|I-s|}^{I+s} \sum_{\ell=|J_c-S|}^{J_c+S} \frac{2J_c+1}{(2s+1)(2I+1)} T_\ell(E) \quad (33)$$

where λ = de Broglie wavelength of the incoming projectile

I = spin of the target nucleus

s = spin of the projectile

$T_\ell(E)$ = barrier transmission coefficient* of a particle

with orbital angular momentum ℓ and energy E .

For the targets Ag-107 and Ag-109, $I = \frac{1}{2}$, and for the projectile He-4, $s = 0$, such that in this case equation (33) reduces to:

*The transmission coefficient may be defined as⁸ the probability for the transmission of a single particle of energy E and angular momentum ℓ through the barrier.

$$\sigma(J_c, E) = \pi\lambda^2 (J_c + \frac{1}{2}) \left\{ T_{(J_c - \frac{1}{2})}(E) + T_{(J_c + \frac{1}{2})}(E) \right\} \quad (34)$$

The total cross section for the formation of a compound nucleus of any spin is given by the sum of the cross sections for all the individual spins, such that the probability, $P_{J_c=y}$, of producing a compound nucleus of spin $J_c = y$ is given by:

$$P_{J_c=y} = \frac{[2(J_c=y) + 1][T_{(J_c=y) - \frac{1}{2}}(E) + T_{(J_c=y) + \frac{1}{2}}(E)]}{\sum_{\text{All } J_c} (2J_c + 1)[T_{J_c - \frac{1}{2}}(E) + T_{J_c + \frac{1}{2}}(E)]} \quad (35)$$

This equation enables one to calculate the spin distribution of compound nuclei formed in the reactions Ag-107 + He-4 and Ag-109 + He-4, provided the transmission coefficients for the incoming alpha particles are known. The latter have been calculated by Huizenga and Igo⁴⁴ using an optical model potential(cf. Appendix E). The effects of the energy of the bombarding alpha particles on the spin distribution of the compound nuclei are shown in Figures 23 and 24. In these figures In-111* and In-113* refer to excited states of these nuclei.

2. Distribution of Spins After Neutron Emission

The compound nuclei formed by the absorption of an alpha particle of energy greater than 10 Mev will have sufficient energy to de-excite by neutron emission. One must now determine the distribution in spin of the nuclei after neutron emission. For a particular spin of the compound nucleus J_c , the distribution in final spin J_f , after neutron emission will depend upon two factors. The first factor is the density of available levels having a final spin J_f . The second is the angular

FIGURE 23.
DISTRIBUTION IN SPIN J_c OF
THE COMPOUND NUCLEUS In-III.

FIGURE 24.

DISTRIBUTION IN SPIN J_c OF THE
COMPOUND NUCLEUS In-113.

momentum carried away by the neutron. It is possible to de-excite to a final state of spin J_f , from an initial state of spin J_c , in a number of different ways (i.e. the neutron may carry away angular momentum $\ell = 0, 1, 2 \dots$). If the probability of de-excitation by a neutron of orbital angular momentum ℓ , is proportional to its barrier transmission coefficient $T_\ell(E)$, then the relative probability for a compound nucleus of spin J_c , for emitting a neutron leading to a final state of spin J_f , is given by:¹⁰

$$P(J_f)_{J_c} = \rho(J_f) \sum_{S=J_f-\frac{1}{2}}^{J_f+\frac{1}{2}} \sum_{\ell=J_c-S}^{J_c+S} T_\ell(E) \quad (36)$$

where the $\frac{1}{2}$ refers to the intrinsic spin of the emitted neutron. The density of levels $\rho(J_f)$, having a spin J_f , is predicted theoretically to be of the form:^{5,7}

$$\rho(J_f) = \rho(0)(2J_f+1) e^{-(J_f+\frac{1}{2})^2/2 \sigma^2} \quad (37)$$

where $\rho(0)$ is the density of levels with spin 0 and σ is a parameter characterizing the spin distribution.* The neutron transmission coefficient, $T_\ell(E)$, is a function of the energy of the emitted neutron and of the radius of the nucleus emitting the neutron. Feld et. al.⁴⁵ have calculated these transmission coefficients using a square well nuclear potential. These transmission coefficients can also be obtained from the optical model calculations of Campbell et. al.⁴⁶. The neutron transmission coefficients used in the calculations of the isomeric

* This parameter is not to be confused with the cross section which is also represented by the symbol σ . The context should make it clear as to whether one is referring to the spin density parameter of equation (37) or the cross section (the symbol for the cross section is usually accompanied by a subscript referring to a particular cross section).

cross section ratios are given in Appendix E.

If one normalizes the relative probabilities for each final state of a given spin and sums over all the compound nuclear states, then an expression for the absolute probability $P_{J_f=x}$ of populating a final state of spin $J_f = x$, by neutron emission, is obtained:

$$P_{J_f=x} = \sum_{J_c=a}^b P_{J_c=y} \frac{\rho(J_f=x) \sum_{S=|(J_f=x)-\frac{1}{2}|}^{(J_f=x)+\frac{1}{2}} \sum_{\ell=|(J_c=y)-S|}^{(J_c=y)+S} T_{\ell}(E)}{\sum_{J_f=a'}^{b'} \rho(J_f=z) \sum_{S=|(J_f=z)-\frac{1}{2}|}^{(J_f=z)+\frac{1}{2}} \sum_{\ell=|(J_c=y)-S|}^{(J_c=y)+S} T_{\ell}(E)} \quad (33)$$

The numerator of the second term on the right hand side of this equation represents the relative probability of populating a final spin, $J_f = x$, from a particular compound nucleus spin, $J_c = y$. When this numerator term is divided by the sum of all the relative probabilities of all possible final spins J_f , arising from the compound nucleus spin $J_c = y$, then an absolute probability for going from an initial state of spin $J_c = y$ to a final state of spin $J_f = x$ is obtained. The term $J_f = z$ in the denominator then indicates that the double sum on the right hand side of this denominator must be evaluated for all the final states J_f , arising from the compound nucleus state of spin $J_c = y$. To obtain the absolute probability of going to a final spin $J_f = x$, from any compound nucleus spin J_c , it is finally necessary to sum over all the compound nuclear states of spin J_c . The first summation on the right hand side of the above equation performs this operation. The $P_{J_c=y}$'s here have been

normalized (cf. equation 35).

For the compound nuclei formed in the Ag-107 + He-4 and the Ag-109 + He-4 reactions J_c will be half integer since both targets have a spin of $\frac{1}{2}$, and the alpha particle has a spin of zero and carries in integer values of angular momentum. In the above equation then, a will be $\frac{1}{2}$ and the upper limit b, is determined by when $P_{J=y}$ becomes negligible. Upon emission of a neutron ($s = \frac{1}{2}$, $l = 0, 1, 2, \dots$) the nuclei will have integer spins and the limits a' and b' will be integer. a' will be 0 and b' is determined by when $P_{J=x}$ becomes negligible. Equation (38) then enables one to calculate the spin distribution of the nuclei after the emission of one neutron.

For the Ag-109($\alpha, 3n$) reaction in which one wishes to obtain the spin distribution of nuclei after the emission of three neutrons, it is necessary to recalculate the distribution using equation (38) for each neutron emitted. To calculate the spin distribution after the emission of three neutrons using equation (38) is quite tedious. Even after programming this equation on the IBM 650 computer, the calculation takes about 50 minutes of computing time for a typical calculation.

3. Gamma Ray Emission and Final Population of the Isomers

It is now necessary to consider the change in the distribution of the spin by the emission of gamma radiation. If one is in a region of relatively high excitation energy (> 2 Mev) where there are very many nuclear levels both before and after gamma ray emission having many values of J, then de-

excitation by dipole ($\ell=1$) gamma ray emission is expected,* since the probability of gamma ray emission decreases rapidly as ℓ increases ($\ell = 0$ is forbidden and $\ell = 1$ is about 10^5 times more probable than $\ell = 2$ for a 1 Mev gamma ray). Thus at fairly high excitation energies the probability of going from a state of spin J to a state of spin $J + 1$, J or $J - 1$ is determined solely by the level density factor $(2J + 1) e^{-(J+\frac{1}{2})^2 / 2 \sigma^2}$ (cf. equation 37). The probability of going from a state of spin J to a state of spin $J+1$ is given by:

$$P_{J \rightarrow J+1} = \frac{\rho(J+1)}{\rho(J+1) + \rho(J) + \rho(J-1)} \quad (39)$$

and similarly for going to states of spin J or $J-1$:

$$P_{J \rightarrow J} = \frac{\rho(J)}{\rho(J+1) + \rho(J) + \rho(J-1)}, \quad (40)$$

$$P_{J \rightarrow J-1} = \frac{\rho(J-1)}{\rho(J+1) + \rho(J) + \rho(J-1)}. \quad (41)$$

In equation (39) $P_{J \rightarrow J+1}$ represents the probability of going from a state of spin J to another state of spin $J+1$. $P_{J \rightarrow J}$ and $P_{J \rightarrow J-1}$ are similarly defined. $\rho(J)$, $\rho(J+1)$ and $\rho(J-1)$ represent the relative density of levels having a spin J , $J+1$ or $J-1$ respectively. Thus each spin state population of the excited nucleus redistributes itself among three spin states upon the emission of one dipole gamma ray, and a new spin distribution results. A number of gamma rays will be emitted depending upon the excitation energy of the nucleus before gamma ray emission so that the above calculation must be repeated for each gamma ray emitted. Of course with the emission of

*This is not necessarily true for collective excitations where quadrupole transition lifetimes are greatly reduced over single particle estimates and hence, quadrupole transitions will compete with dipole transitions.

the last gamma ray only two states will be available for population, namely the ground and metastable states of the product isomers. Thus it is assumed that the last de-excitation chooses to populate either the high spin state or the low spin state, depending on which transition has the smaller spin change. If one considers a case in which four gamma rays are emitted in going to the metastable or ground state of In-110, the high and low spin states being $I = 7$ and $I = 2$, then it is assumed that states which after the third gamma ray de-excitation have spins 0,1,2,3 or 4 will populate the spin 2 isomer with the emission of the last gamma ray. States with spin 5 or greater will populate the spin 7 isomer with the emission of the last gamma ray. If P_{J_f}' represents the absolute probability that the nucleus will have a spin J_f before the emission of the last gamma ray, then

$$\sigma_{\text{In-110m}} / (\sigma_{\text{In-110m}} + \sigma_{\text{In-110}}) = \sum_{J=5}^{\infty} P_{J_f}' \quad . \quad (42)$$

This method may also be applied to the (n,γ) reactions using the proper spins of the isomers involved.

C. Parameters for the Calculation of Spin Distributions

1. Nuclear Temperature and Neutron Kinetic Energy.

For neutrons emerging from a highly excited compound nucleus, an energy distribution of the form⁴:

$$N(E_n) \propto E_n e^{-E_n/T} \quad (43)$$

is expected. $N(E_n)$ is the relative intensity of neutrons of kinetic energy E_n , and T is the nuclear temperature of the residual nucleus. T is defined by⁴:

$$\frac{1}{T} = \frac{d \ln \rho(A,E)}{dE} \quad (44)$$

where $\rho(A,E)$, the level density, is a function of the excitation energy of the nucleus and of the number of nucleons within the nucleus. This level density is roughly of the form⁴:

$$\rho(A,E) = C e^{2\sqrt{aE}} \quad (45)$$

where C and a are parameters containing the A dependence of $\rho(A,E)$, and the energy dependence of C has been neglected.

Using the level density as given by equation (45) and the definition of the nuclear temperature (equation 44), it follows by differentiation that:

$$T = \sqrt{E_R/a} \quad (46)$$

for the residual nucleus of energy E_R .

If one wishes to calculate the spin distribution after neutron emission using equation (38) it is necessary to use a set of neutron transmission coefficients for a neutron of a particular kinetic energy. It was found (cf. section V.C.1.) that the average neutron kinetic energy ($2T$ if the distribution is of the form given by equation (43)) is a very good approximation

as far as the calculation of the isomeric cross section ratio is concerned. Using $E_n = 2T$, then from equations (32) and (46) it is found that:

$$T = \frac{-1 + \sqrt{1 + \frac{a}{2} (E_c - B_n)}}{\frac{a}{2}} \quad (47)$$

To obtain the nuclear temperature and the average neutron kinetic energy, it is necessary that the parameter a be known. For nuclei in the region of In-111, a is about $10 \text{ Mev}^{-1.47,48}$

2. Number of Gamma Rays Emitted

After particle emission is no longer energetically possible, de-excitation of an excited nucleus occurs by emission of gamma radiation. For a nucleus of excitation energy E_0 , a distribution of gamma rays of energy E_γ , is obtained. If the level density of the residual nucleus is of the form given by equation (45) then this distribution is of the form⁴⁹:

$$N(E_\gamma) \propto E_\gamma^3 e^{-2\sqrt{\frac{a}{2}(E_0 - E_\gamma)}} \quad (48)$$

for dipole gamma radiation, where $N(E_\gamma)$ is the relative intensity of gamma rays having an energy E_γ . Assuming a distribution of this form, it is possible to obtain an approximate expression for the average number of gamma rays emitted, $\bar{N}$, in the de-excitation of a nucleus of energy E_0 ⁵⁰:

$$(2\ell + 2) \bar{N} \approx 2 \sqrt{\frac{a}{2}} E_0^{\frac{1}{2}} \quad (49)$$

where ℓ is the multipolarity of the gamma ray, and a is the level density parameter of equation (45). For dipole gamma radiation this equation reduces to:

$$\bar{N} \approx \frac{\sqrt{\frac{aE_0}{2}}}{2} \quad (50)$$

In the (n, γ) reactions, $\bar{N}$ was treated as a variable but in the

(α, xn) reactions with silver, equation (50) was used to obtain the approximate average number of gamma rays emitted. Unless otherwise indicated all calculations were performed assuming dipole gamma ray emission, however a few calculations were performed assuming quadrupole ($\ell=2$) gamma ray emission. In this case equation (49) is valid only if the transitions arise from single particle states.

3. σ used in Calculations

In many of the calculations the spin density parameter σ was treated as a constant. Calculations of the isomeric cross section ratio using $\sigma=3$ or $\sigma=5$ were compared with the experimental ratios. These two values were chosen since other experiments indicated that σ was in this range.^{9,10} It has been shown however that¹²:

$$\sigma^2 = \frac{\mathcal{I}}{\hbar^2} T \quad (51)$$

where $\mathcal{I}$ is the moment of inertia, T is the nuclear temperature, and $\hbar$ is Planck's constant divided by 2π . If T is given by $\sqrt{E/a}$, then from the above equation it follows that there is a slight variation of σ with energy. At high excitation energies of the nucleus $\mathcal{I}$ is believed to be given by the rigid moment of inertia³:

$$\mathcal{I}_{\text{RIGID}} = (2/5)m AR^2 \quad (52)$$

where m is the nucleon mass, A is the number of nucleons within the nucleus, and R is the nuclear radius. At low energies however, $\mathcal{I}$ is believed to be less than the rigid moment of inertia.^{2,51} Several calculations have been carried out using a σ calculated from equation (51) with $T = \sqrt{E/a}$, and a moment of inertia given by equation (52).

V. CALCULATIONS AND DISCUSSION

A. Isomeric Cross Section Ratios for (n,γ) Reactions

1. Calculations

Calculations of the isomeric cross section ratio for products of the (n,γ) reactions were performed for $\sigma=3$ and $\sigma=5$. The average number of gamma rays emitted from a nucleus formed by slow neutron capture is about four.⁵⁰ A sample spin distribution after 1,2,3, and 4 gamma rays emitted, for $\sigma=5$ and $J_0=\frac{1}{2}$, is shown in Table 11. The method of determining the

Table 11

Spin Distribution after Gamma Ray Emission
($\sigma=5$, $J_0=\frac{1}{2}$)

No. of γ rays emitted	$J \rightarrow$	$1/2$	$3/2$	$5/2$	$7/2$	$9/2$
0		1.000				
1		0.347*	0.653			
2		0.240	0.453	0.307		
3		0.166	0.392	0.319	0.123	
4		0.130	0.325	0.331	0.170	0.044

*These numbers are the fraction of states having a spin J after the emission of 1,2,3, or 4 gamma rays.

isomeric cross section ratio from spin distributions before the emission of the last gamma ray was discussed earlier (Section IV. B.3.). The spins of the isomers finally populated in the (n,γ) reactions are given in Table 12.

In the case of the Cs-134 isomers and the Rh-104 isomers, a level between the metastable and ground states is found. In considering the Rh-104 isomers then, it is assumed that states with spins 1,2, and 3 populate the ground state isomer, having $I=1$, or the $I=2$ level (which immediately decays to the $I=1$ level)

Table 12

Calculated Isomeric Cross Section Ratios for (n, γ) Reactions

Target	Target Spin	Competing levels (spins)	Isomeric Cross Section Ratio						
			Experimental ^a	Calculated					
				$\sigma=3$			$\sigma=5$		
				$N\gamma^b=3$	$N\gamma=4$	$N\gamma=5$	$N\gamma=3$	$N\gamma=4$	$N\gamma=5$
Capturing State $J_c=I+\frac{1}{2}$									
Se-80	0	7/2, 1/2	0.115±0.012	0.262	0.372	0.459	0.307	0.442	0.545
Pd-108	0	11/2, 5/2	0.022±0.004	0	0	0.023	0	0	0.044
Te-128	0	11/2, 3/2	0.074±0.015	0	0.043	0.085	0	0.062	0.129
Rh-103	1/2	5,1,(2) ^c	0.071±0.007	0	0.053	0.088	0	0.087	0.149
Cs-133	7/2	8,4,(5)	0.086±0.013	0	0.008	0.011	0	0.032	0.053
Capturing State $J_c=I-\frac{1}{2}$									
Rh-103	1/2	5,1,(2)	0.071±0.007	0	0	0.052	0	0	0.079
Cs-133	7/2	8,4,(5)	0.086±0.013	0	0	0.002	0	0	0.012

a) These ratios are for thermal neutron irradiation; the epi-cadmium neutron ratios were the same within experimental error except in the case of the selenium isomers, where this ratio was 0.161 $\pm$ 0.016.

b) $N\gamma$ = Number of gamma rays emitted.

c) A level in parentheses indicates a level between the ground and metastable state.

upon the emission of the last gamma ray; states with spin 4 or greater populate the metastable state ($I=5$). In considering the Cs-134 isomers it is assumed that states with spins 1 to 6 populate the ground state isomer, having $I=4$, or the $I=5$ level (which immediately decays to the $I=4$ level) upon the emission of the last gamma ray; states with spin 7 or greater populate the metastable state ($I=8$). Table 12 shows the results of the calculations of the isomeric cross section ratios for 3, 4, and 5 gamma rays emitted from the compound nucleus. The calculations for the Rh-104 and the Cs-134 isomers were carried out for the capturing states of $J_c = I + \frac{1}{2}$ and $J_c = I - \frac{1}{2}$.

2. Discussion

Having measured the isomeric cross section ratio experimentally and having made several calculations of this ratio (cf. Table 12), we are able to draw a few conclusions about the results of the (n, γ) reactions. First of all consider the three nuclides which can have a capturing state of $J_c = \frac{1}{2}$ only (assuming $\ell=0$), namely the (n, γ) reactions on Se-80, Pd-108 and Te-128. In the case of the isomers produced by the (n, γ) reactions on the latter two nuclides, the calculations of the isomeric cross section ratio using $\sigma=3$ or 5 and $N_\gamma=4$ or 5, agree fairly well with the experimental ratio. This is in agreement with other results.⁹

To get agreement between the experimental and calculated isomeric cross section ratio for the formation of the Se-81 isomers, it would be necessary to use a smaller σ and (or) a smaller number of gamma rays in the de-excitation process. For

$\sigma=3$ and $N_\gamma=2.5$ the isomeric cross section ratio is calculated to be 0.13, which is in good agreement with the experimental value of 0.115 ± 0.012 . A smaller σ is expected for a small A nuclide (from equations (46), (51) and (52), $\sigma^2 = \left[(2/5) \pi A R^2 \sqrt{E_{r/a}} \right] / \hbar^2$, and since $R \propto A^{1/3}$ and $a \propto A$, then $\sigma^2 \propto (A^{2/3}) / (A^{1/2})$ or $\sigma \propto A^{7/12}$. The small number of emitted gamma rays might be expected since experiments measuring the gamma ray spectra from a selenium target upon thermal neutron capture, have indicated a relatively large number of high energy gamma rays⁵² (this means that on the average, fewer gamma rays will be emitted in the de-excitation of the nucleus formed by neutron capture).

It was also observed in the case of the selenium-81 isomers that the isomeric cross section ratio increased when the sample was irradiated in cadmium foil. By wrapping the sample in cadmium most of the low energy neutrons are absorbed in this cadmium, because of its high thermal neutron cross section. Consequently a relatively much larger number of high energy neutrons compared to thermal neutrons, will penetrate the cadmium foil. These neutrons are capable of imparting additional angular momentum to the target nucleus, so that capture in a spin state greater than $J_c = \frac{1}{2}$ is possible. The increase in the isomeric cross section ratio using epi-cadmium neutrons over the ratio obtained using thermal neutrons, can then be explained by partial capture in a state of spin greater than $J_c = \frac{1}{2}$ (the formation of a higher spin state by neutron capture will increase the yield of the high spin isomer upon gamma ray emission).

Now consider the two nuclides which can have a capturing state of $J_c = I + \frac{1}{2}$ or $J_c = I - \frac{1}{2}$ (Rh-103 and Cs-133). If σ is to be between 3 and 5 in agreement with the above results, then it appears that the calculations assuming a capturing state of $J_c = I + \frac{1}{2}$ best fit the experimental data. The isomeric cross section ratio calculated with σ slightly greater than 5, would be a better fit to the experimental ratio obtained with the Cs-134 isomers. This higher σ is expected for a nuclide of higher A (the A dependence of σ was mentioned earlier in relation to the selenium isomers).

B. Total Cross Sections

In addition to measuring isomeric cross section ratios, absolute cross sections for the formation of In-110 and In-110m, by the Ag-107(α ,n) and the Ag-109(α ,3n) reactions were determined (cf. Tables 7 and 8). The excitation functions for the formation of each isomer in both reactions, are shown in Figures 25 and 26 (as mentioned previously the error in the individual cross sections is higher than the error in the cross section ratio, and is about 15-20%). The displacement of the excitation functions in the Ag-107(α ,n) reaction is a result of angular momentum effects, the relative yield of the high spin isomer increasing with increasing energy. The total cross section for the formation of both isomers can be obtained directly from the individual cross sections. If the reaction occurs by compound nucleus formation, then one would expect the total cross section for the Ag-107(α ,n) reaction (i.e. $\sigma_{\text{In-110}} + \sigma_{\text{In-110m}}$) to equal the compound nucleus cross section, when this is the only reaction occurring. Below an alpha particle energy of about 15.5 Mev, the Ag-107(α ,n) reaction is the only reaction occurring to any extent. The Ag-107(α , γ) reaction is negligible (cf. Section III.B.4.), while the threshold for the Ag-107(α ,2n) reaction is about 15.7 Mev. Although the threshold for the (α ,p) reaction is only 3.3 Mev, proton emission is negligible below an alpha particle energy of 15.5 Mev because of the coulomb barrier. Nuclear evaporation calculations using a method described by Chasman,⁵³ showed that proton emission from the compound nucleus In-111*,

FIGURE 25.

EXCITATION FUNCTIONS FOR
Ag-107(α , n) In-110, In-110m REACTION.

FIGURE 26.

EXCITATION FUNCTIONS FOR
Ag-109(α , 3n) In-110, In-110m REACTION.

does not occur below an alpha particle energy of 15.5 Mev.

Reaction thresholds for the (α, an) , (α, d) and (α, pn) reactions on Ag-107 are 9.4, 10.7 and 12.9 Mev respectively, but these reactions are also negligible below an alpha particle energy of 15 Mev because of the coulomb barrier.

The compound nucleus cross section can be calculated from the equation⁴⁴

$$\sigma_c = \pi \lambda^2 \sum_{\ell=0}^{\infty} (2\ell+1) T_{\ell}(E) \quad (53)$$

where λ is the de Broglie wavelength of the incoming projectile, ℓ is the angular momentum of the incident particle in units of $\hbar$, and $T_{\ell}(E)$ is the transmission coefficient of the incident particle of energy E . The compound nucleus cross sections

Table 13

Compound Nucleus Cross Sections

E_{α} (Mev)	σ_c (mb)	E_{α} (Mev)	σ_c (mb)
12	21.8	26	1342
14	175.4	29	1479
16	455.0	32	1585
18	713.9	35	1670
20	923.4	38	1739
22	1092		

calculated from the above equation, using the alpha particle transmission coefficients listed in Appendix E, are given in Table 13. Figure 27 shows the compound nucleus cross section and the Ag-107(α, n) cross section. The qualitative agreement (the error in the experimental cross section for the formation of both isomers may be as high as 40%), between these two curves below an alpha particle energy of 15.5 Mev indicates compound nucleus formation. Above an alpha particle energy of about 19

FIGURE 27.

TOTAL CROSS SECTION FOR Ag-107(α , n) REACTION
vs. COMPOUND NUCLEUS CROSS SECTION.

FIGURE 28.

TOTAL CROSS SECTION FOR Ag-109(α , 3n) REACTION
vs. COMPOUND NUCLEUS CROSS SECTION.

Mev, the Ag-107(α ,n) reaction cross section begins to fall because of considerable competition from the Ag-107(α ,2n) reaction.

Figure 28 shows the cross section for compound nucleus formation compared to the Ag-109(α ,3n) cross section. The Ag-109(α ,3n) excitation function falls below the cross section for the formation of the compound nucleus, since at these high alpha particle energies the (α ,p), (α ,n), (α ,d), (α ,pn), (α ,2n) and (α ,p2n) reactions on Ag-109, with thresholds of 3.3, 6.6, 10.6, 12.7, 14.8 and 19.6 Mev respectively, are also likely to occur to some extent.

C. Isomeric Cross Section Ratios for (α, xn) Reactions

1. Effect of Neutron Energy Spectrum on Calculations

It can be seen from equation (43) that a neutron emerging from a compound nucleus has a distribution of energies. To perform the calculation of the isomeric cross section ratio more exactly then, would require that the neutron energy spectrum be divided up into several bins. In this calculation it is necessary to calculate the isomeric cross section ratio for each bin, using the neutron transmission coefficients for the average neutron kinetic energy of that bin, and to weigh each bin according to the relative number of neutrons in that bin. A number of these calculations were performed for the $\text{Ag-107}(\alpha, n)$ reaction, using the compound nucleus spin distribution obtained with an alpha particle energy, E_α , of 16 Mev. The neutron kinetic energy evaporation spectrum as given by equation (43) for a nuclear temperature of 0.77 Mev, is shown in Figure 29. This is the nuclear temperature of the residual nucleus In-110 , calculated from equation (47) using $a = 10 \text{ Mev}^{-1}$, $B_n = 9.61 \text{ Mev}$, and $E_c = 16.98 \text{ Mev}$ ($E_\alpha = 16 \text{ Mev}$). Both optical model and square well neutron transmission coefficients^{45,46} were used. The transmission coefficients used in the calculations are listed in Appendix E. The calculations were performed using a constant σ of 3 or 5 for both the calculation of the spin distribution after neutron emission, and the calculation of the spin distribution after gamma ray emission.

For the calculations involving optical model neutron transmission coefficients, the neutron energy spectrum was divided

FIGURE 29.

NEUTRON KINETIC ENERGY EVAPORATION SPECTRUM.

$$N(E_n) \propto E_n e^{-E_n/T} \quad T = 0.77 \text{ Mev}$$

into five bins. From the average energy of the neutrons in each bin, the neutron transmission coefficients could be calculated using the data of Campbell et al.⁴⁶. They have tabulated the quantity $\left[\frac{\sigma_c(\ell)}{\pi R^2} \right]$, for a number of values of the parameters X' and X_0 . In this quantity $\sigma_c(\ell)$ is the cross section for the inverse process (n+In-110, in this case), ℓ being the orbital angular momentum of the neutron, and R is the optical model nuclear radius in fermi ($1 \text{ fermi} = 10^{-13} \text{ cm}$), given by:

$$R = (1.15A^{1/3} + 0.4) \text{ fermi.} \quad (54)$$

The parameters X' and X_0 are dimensionless and are defined by the equations:

$$X' = 0.22 \sqrt{\frac{A}{A+1}} E_n R \quad (55)$$

in which $A+1$ is the mass number of the nucleus emitting the neutron of energy E_n (E_n is expressed in Mev), and

$$X_0 = 0.22 \sqrt{\frac{A}{A+1}} V_0 R \quad (56)$$

in which the nuclear potential V_0 , is taken to be 52 Mev.

The transmission coefficient is given by the equation⁴⁶:

$$T_\ell = X'^2 \left[\frac{\sigma_c(\ell)}{\pi R^2} \right] \frac{1}{2\ell+1} \quad (57)$$

and can thus be calculated from the quantity $\left[\frac{\sigma_c(\ell)}{\pi R^2} \right]$.

The value of X_0 for In-110 was calculated⁴⁶ to be 9.35.

Values of $\left[\frac{\sigma_c(\ell)}{\pi R^2} \right]$ have been calculated and tabulated for

$X_0 = 9.2$ and $X_0 = 9.6$ by Campbell et al.⁴⁶. The neutron

transmission coefficients could then be calculated from equation

(57) using the values of $\left[\frac{\sigma_c(\ell)}{\pi R^2} \right]$ for these two values of X_0 ,

and using the value of X' calculated from the average neutron energy of each bin of the spectrum. Using these transmission coefficients the isomeric cross section ratios for each bin

were calculated. The results of the calculations for $E_\alpha=16$ Mev and $\sigma=3$ for each bin of the neutron energy spectrum, as well as the weighted isomeric cross section ratio, are shown in Table 14. The results show that the isomeric cross section ratio obtained by using the average neutron energy, $2T$, is very close to the ratio obtained when the neutron kinetic energy spectrum is divided into several bins, and weighed according to the relative number of neutrons in each bin. The results also show, by virtue of the calculations for $X_0=9.2$ and $X_0=9.6$ that the isomeric cross section ratio varies very little for a change in the nuclear radius.

Calculations have also been performed using square well model neutron transmission coefficients calculated by Feld et al.⁴⁵. Transmission coefficients for a square well radius of 8×10^{-13} cm were used (the actual square well radius of In-110 is 7.2×10^{-13} cm), since transmission coefficients were tabulated graphically as a function of the parameter X , at this value of the nuclear radius. The parameter X is dimensionless and is defined by:

$$X = 0.22 R \sqrt{E_n} \quad (58)$$

in which R is the square well nuclear radius in fermi (equal to $1.5A^{1/3}$). The results of these calculations (cf. Table 15) again show that the average neutron kinetic energy, $2T$, is a very good approximation to use in the calculation of the isomeric cross section ratio for reactions in which neutron emission occurs. In this table both the isomeric cross section ratio assuming only one gamma ray emitted, and the ratio assuming the

Table 14

Isomeric Cross Section Ratios for a Spectrum of
Neutron Energies Using Optical
Model Neutron Transmission Coefficients

Energy Range of bin (Mev)	Rel. wt. of bin ^a	Ave. n energy of bin (Mev)	X_0^b	No. of γ -rays emitted ^{c,d}	^e $\frac{\sigma_{\text{In-110m}}}{\sigma_{\text{In-110}} + \sigma_{\text{In-110m}}}$	
					$X_0=9.6$	$X_0=9.2$
0-0.6	0.184	0.38	0.8	4.5	0.344	0.341
0.6-1.2	0.278	0.86	1.2	4	0.346	0.345
1.2-1.9	0.245	<u>1.52^f</u>	1.6	4	<u>0.331</u>	0.329
1.9-2.9	0.184	2.37	2.0	3	0.342	0.342
2.9-7.4	0.109	3.42	2.4	3	0.320	0.328

Weighted average of all 5 bins = 0.338 for both $X_0=9.6$ and $X_0=9.2$

- a) Calculated by integration of equation (43).
- b) Calculated from equations (54) and (55).
- c) Approximate average number of gamma rays emitted, as calculated by equation (50), after a neutron of kinetic energy E_n , is emitted.
- d) In this table and in all others, dipole gamma ray emission is assumed unless otherwise specified.
- e) Calculations for the $\text{Ag-107}(\alpha, n)$ reaction with $E_\alpha = 16$ Mev, $\sigma=3$ and $a=10$ Mev⁻¹.
- f) This is the average neutron kinetic energy, $2T$, of the entire spectrum.

Table 15

Isomeric Cross Section Ratios for a Spectrum of
Neutron Energies Using Square Well Model
Neutron Transmission Coefficients

Energy Range of bin (Mev)	Rel. wt. of bin ^a	Ave. n energy of bin (Mev)	X ^b	No. of γ-rays emitted ^{c,d}	$\frac{\sigma_{\text{In-110m}}^e}{\sigma_{\text{In-110}} + \sigma_{\text{In-110m}}} -$ $\sim \bar{N} \gamma' s g \quad 1 \gamma^h$	
0-0.5	0.139	0.30	0.87	5	0.328	0.460
0.5-0.8	0.140	0.64	1.27	4	0.346	0.442
0.8-1.26	0.208	1.00	1.58	4	0.336	0.429
1.26-1.80	0.191	<u>1.54</u> ^f	1.96	4	<u>0.324</u>	0.413
1.80-2.46	0.151	2.10	2.30	3	0.340	0.399
2.46-3.30	0.099	2.84	2.66	3	0.328	0.385
3.30-7.40	0.072	4.10	3.20	2	0.337	0.365

Weighted average of all 7 bins = 0.334, when the number of gamma rays emitted is equal to the number listed in the preceding column.

a), c), d), e), f) As in Table 14.

b) Calculated from equation (58).

g) These are the ratios assuming the number of gamma rays emitted, to be equal to the number of gamma rays listed in the preceding column (cf. footnote c).

h) These are the ratios, if only one gamma ray were emitted after neutron emission.

number of gamma rays emitted to be equal approximately to $\sqrt{\frac{a E_0}{2}}$ (cf. equation 50), are listed. A comparison of these quantities shows how the number of gamma rays emitted, effects the calculation of the isomeric cross section ratio. It can also be seen that these results differ very little from the results using optical model neutron transmission coefficients. The calculations using a constant σ of 5 also showed that $2T$ for the average neutron kinetic energy, is a good approximation to use in calculating the isomeric cross section ratio.

2. Calculations Using a Constant σ

Several calculations of the isomeric cross section ratio for the Ag-107(α ,n) and the Ag-109 (α ,3n) reactions were performed using a constant value of σ in all the de-excitation steps. Figure 30 exemplifies the spin distribution of nuclei at various stages of the reaction for $\sigma=3$ at an alpha particle energy of 16 Mev. The isomeric cross section ratios using $\sigma=3$ and $\sigma=5$ at several alpha particle energies are given in Table 16. In these calculations the average neutron kinetic energy $2T$, was used except at the alpha particle energies of 20 and 22 Mev for the Ag-107(α ,n) reaction. At these two energies the (α ,n) cross section is considerably less than the total compound nucleus cross section (cf. Figure 27), principally because of competition from the (α ,2n) reaction. This means that only the high energy neutrons from the neutron kinetic energy evaporation spectrum of In-111 lead to the (α ,n) products, since after the emission of a high energy neutron, the nucleus will have insufficient energy to emit a second

FIGURE 30.

DISTRIBUTION IN SPIN J - COMPOUND NUCLEUS - AFTER
NEUTRON EMISSION - AFTER GAMMA EMISSION.

Table 16

Isomeric Cross Section Ratios for the (α ,xn)
Reactions Using $\sigma=3$ and $\sigma=5$

E_α (Mev)	E_n^a (Mev)	No. of γ -rays emitted ^b	$\sigma_{\text{In-110m}}$	
			$\sigma_{\text{In-110}} + \sigma_{\text{In-110m}}$	
			$\sigma = 3$	$\sigma = 5$

Ag-107(α ,n) Reaction				
12	1.00	2.5	0.165	0.263
14	1.29	3	0.225	0.363
16	1.53	4	0.324	0.518
18	1.74	4	0.447	0.643
20	3.60	4	0.503	0.721
22	5.53 ^c	4	0.537	0.769
20	1.92	5	0.510	0.723
22	2.10	5	0.590	0.779
Ag-109(α ,3n) Reaction				
32	2.92 2.10 ^d 0.59	1.5	0.728	0.885
38	3.27 2.53 1.42	3.5	0.676 ^e	0.898

- a) The value of the parameter a used in the calculation of the neutron energy, E_n , was 10 Mev⁻¹.
- b) Approximate average number of gamma rays emitted, as calculated from equation (50).
- c) $E_n \approx 2T$ in these two cases.
- d) These are the energies of the first, second and third neutrons respectively.
- e) For $a = 7 \text{ Mev}^{-1}$, which reduces the number of gamma rays emitted to about 2.5, this ratio is 0.686.

neutron. Thus since only the high energy neutrons lead to the (α, n) products, the use of $2T$ for the neutron energy is not reasonable at energies where the (α, n) cross section is low compared to the $(\alpha, 2n)$ cross section. Rather, at the alpha particle energies of 20 and 22 Mev, the neutron kinetic energy was chosen so that the residual nucleus energy after the emission of this neutron, was equal to the binding energy of the next neutron. This assumption is not too serious however, since the isomeric cross section ratio using these neutron energies, differs little from the ratios using even a neutron energy of $2T$ (these values are also given in Table 16 for comparison).

The results of the theoretical calculations of the isomeric cross section ratio for the $\text{Ag-107}(\alpha, n) \text{In-110, In-110m}$ reaction are compared to the experimental values in Figure 31. The data here indicate that the calculated ratios using a spin density parameter σ , of 3 to 5 are in agreement with the experimental ratios. From these curves however, there is also some indication that σ may increase with increasing energy. Additional calculations (cf. section V. C. 3.) were performed using an energy dependent σ .

The nuclear level density parameter, $\underline{a}$, used in these calculations (an $\underline{a}$ of 10 Mev^{-1} was used to calculate the nuclear temperature, $T = \sqrt{E_r/\underline{a}}$) is by no means definitely established.*

*According to Bramblett and Bonner⁴⁷, $\underline{a} = 0.094A \text{ Mev}^{-1}$, and in particular $\underline{a} = 10.0 \text{ Mev}^{-1}$ for Ag-109 , but $\underline{a}$ has also been given by, $\underline{a} = (1/8) A \text{ Mev}^{-1}$ (reference 48).

FIGURE 31.

EXPERIMENTAL ISOMERIC CROSS SECTION RATIOS
vs. CALCULATED RATIOS WITH $\sigma = 3$ AND $\sigma = 5$.

Ag-107 (α, n) In-110, In-110m

Several calculations at an alpha particle energy of 16 Mev for the $\text{Ag-107}(\alpha, n)$ reaction using $\sigma=3$ however, showed that the results of the calculation of the isomeric cross section ratio are very insensitive to this parameter. The results are shown in Table 17. It should also be noted however, that a change in the parameter $\underline{a}$, will also change the number of gamma rays emitted (cf. equation 50). To make a more accurate comparison

Table 17

Isomeric Cross Section Ratios for Various
Values of the Parameter $\underline{a}$ with $\sigma=3$

$$E_{\alpha} = 16 \text{ Mev} \quad \sigma = 3 \quad E_n = 2T$$

$\underline{a}$ (Mev^{-1})	No. of γ -rays emitted	$\frac{\sigma_{\text{In-110m}}}{\sigma_{\text{In-110}} + \sigma_{\text{In-110m}}}$
4.8	1	0.400
4.8	4	0.315
7.44	1	0.407
7.44	3*	0.347
7.44	4	0.320
10.0	1	0.412
10.0	3	0.350
10.0	4*	0.324
17.1	1	0.421
17.1	4	0.330

*Approximate average number of gamma rays emitted, as calculated by equation (50).

then, one could compare the isomeric cross section ratio obtained with $\underline{a}=10 \text{ Mev}^{-1}$ and 4 gamma rays emitted, to the ratio obtained with $\underline{a}=7.44 \text{ Mev}^{-1}$ and 3 gamma rays emitted, to see the real change that a different value of the parameter $\underline{a}$ makes in the calculation (cf. Table 17 for a comparison of these two ratios). A calculation at an alpha particle energy of 38

Mev for the $\text{Ag-109}(\alpha, 3n)$ reaction using $\underline{a}=7 \text{ Mev}^{-1}$, also showed the insensitivity of the isomeric cross section ratio to the parameter $\underline{a}$ (cf. footnote e, Table 16).

In calculating the isomeric cross section ratio, it was necessary that the calculations be performed assuming that a certain number of gamma rays are emitted after neutron emission is no longer possible. To decide how many gamma rays are emitted, an approximate equation (eq. 50) was used. It is therefore of some value to compare the isomeric cross section ratios obtained assuming various numbers of gamma rays emitted from the excited nucleus after neutron emission, to see how this ratio varies with the number of gamma rays emitted.

Calculations of the isomeric cross section ratio at an alpha particle energy of 16 Mev for the $\text{Ag-107}(\alpha, n) \text{ In-110}$, In-110m reaction, and at an alpha particle energy of 38 Mev for the $\text{Ag-109}(\alpha, 3n) \text{ In-110}$, In-110m reaction, using both a σ of 3 and a σ of 5 in all the steps of the calculation (i.e. for both neutron and gamma ray emission), are shown in Table 18 for various numbers of gamma rays emitted. Whether the isomeric cross section ratio will increase or decrease upon the emission of an additional gamma ray, depends upon the distribution of spins of the excited nuclei after neutron emission, and the distribution of spins to which these nuclei can decay. The latter distribution depends upon the value of the σ used in the spin density equation (eq. 37). The distribution of spins for the two values of σ used in the above calculations are shown in Figure 32. One observes that for a σ of 3, the most probable

Table 18

Effect of the Number of Gamma Rays Emitted
on the Isomeric Cross Section Ratio^a

No. of γ -rays emitted	$\sigma = 3$		$\sigma = 5$	
	$\ell=1^b$	$\ell=2^c$	$\ell=1$	$\ell=2$
	Ag-107(α, n) In-110, In-110m		$E_\alpha = 16$ Mev	
1	0.412	0.412	0.511	0.511
2	0.380	0.332	0.513	0.520
3	0.350	0.277	0.516	0.531
4	0.324	0.239	0.518	0.541
5	0.301		0.521	
	Ag-109($\alpha, 3n$) In-110, In-110m		$E_\alpha = 38$ Mev	
1	0.762	0.762	0.908	0.908
2	0.730	0.670	0.904	0.897
3	0.695	0.563	0.900	0.884
4	0.658	0.458	0.896	0.868

a) Calculations for $\underline{a} = 10 \text{ Mev}^{-1}$ and $E_n = 2T$.

b) Calculations for dipole gamma rays.

c) Calculations for quadrupole gamma rays.

FIGURE 32.

RELATIVE DENSITY $\rho(J)$ OF STATES
OF SPIN J FOR $\sigma = 3$ AND $\sigma = 5$.

spin ($\sigma = \frac{1}{2}$) in the states available for population after gamma ray emission differs from the most probable spin of the nuclei which are decaying (from Figure 30, which shows the distribution of spins of the nuclei after the emission of one neutron, the most probable spin is about 4). Consequently when a gamma ray is emitted the distribution of spins will be shifted to lower spin values.

The change in the spin distribution after the emission of three gamma rays is shown in Figure 30 for $\sigma=3$ at an alpha particle energy of 16 Mev. With the emission of an additional gamma ray then, the population of the high spin states decreases relative to the population of the low spin states. Consequently the greater the number of gamma rays emitted, the lower will be the value of $\sigma_{\text{In-110m}} / (\sigma_{\text{In-110}} + \sigma_{\text{In-110m}})$ (recall that the spin of In-110m is 7, while that of In-110 is 2). For $\sigma=5$ however the distribution of spins of the available states (i.e. the states to which the nucleus can decay upon gamma ray emission), is slightly greater than the spin distribution of the states which are decaying, and consequently the ratio $\sigma_{\text{In-110m}} / (\sigma_{\text{In-110}} + \sigma_{\text{In-110m}})$ increases slightly upon the emission of gamma radiation.

Some calculations were also performed assuming quadrupole gamma ray emission. These results are also shown in Table 18. The effect of increasing or decreasing the isomeric cross section ratio upon the emission of gamma radiation, is enhanced over the effect with dipole gamma radiation, since quadrupole gamma radiation is capable of carrying away more angular momentum.

This is evident when one compares the results for the two types of gamma radiation (cf. Table 18).

3. Calculations Using an Energy Dependent σ

It has already been observed that the calculated isomeric cross section ratios using a σ of 3 to 5, were in fair agreement with the experimental ratios. It was noted however that there seemed to be a slight energy dependence of the parameter σ . If σ is energy dependent, then this means that there is a different value of σ at each step of the nuclear de-excitation process, since the nucleus decays to a lower excitation energy upon the emission of each neutron or gamma ray. Since only the amounts of the isomers finally populated after de-excitation were measured experimentally, it would be impossible to predict σ as a function of energy solely from the experimental data. One might, for example, be able to get agreement with the experimental results with a function that predicts a σ varying from 2 at 1 Mev excitation energy, to 9 at 20 Mev excitation energy; but another function in which σ might be 0.6 at 1 Mev excitation energy and 15 at 20 Mev excitation energy, might fit the experimental data just as well. What one can do however, is to test the agreement of the calculated isomeric cross section ratios, using a theoretically predicted value of σ , with the experimental ratios.

The parameter σ has been predicted theoretically to be energy dependent and is given by³:

$$\sigma^2 = \frac{5}{h^2} \sqrt{\frac{E_r}{a}} \quad (59)$$

according to equations (46) and (51). Several calculations of the isomeric cross section ratios for the Ag-107(α ,n) and the Ag-109(α ,3n) reactions, were performed using a value of σ predicted by equation (59), in which a rigid moment of inertia given by equation (52) was used. The results of the calculations are shown in Table 19. The calculation of σ_{RIGID} (i.e. σ calculated from equation (59) using a moment of inertia given by the rigid value of equation (52)) for the nuclear states after neutron emission, was performed using an average neutron energy $2T$, except in three cases. A neutron energy greater than $2T$ ($E_n = 3.6$ Mev at $E_\alpha = 20$ Mev and $E_n = 5.53$ Mev at $E_\alpha = 22$ Mev, compared to the average neutron energies of $2T = 1.92$ Mev and $2T = 2.10$ Mev respectively), was used at these two alpha particle energies for reasons described previously. Neutron energies less than $2T$ were used at $E_\alpha = 29$ Mev for the Ag-109(α ,3n) reaction, since at this alpha particle energy the emission of high energy neutrons results in the (α ,2n) product rather than the (α ,3n) product. An average value of σ_{RIGID} was used in the gamma ray cascade. A comparison of these calculated isomeric cross section ratios with the experimental ratios for the Ag-107(α ,n) and the Ag-109(α ,3n) reactions, is shown in Figures 33 and 34. The results do not differ too greatly from the results using a constant σ of 5 in all of the steps of the de-excitation process.

A few calculations were also performed for the Ag-109(α ,3n)

Table 19

Isomeric Cross Section Ratios Using a σ Obtained
from a Rigid Moment of Inertia

E_α (Mev)	E_n^a (Mev)	σ_{RIGID}^a for neutron emission	No. of γ -rays emitted ^b	Ave. σ_{RIGID} for γ -ray emission	Isomeric Cross Section Ratio
Ag-107(α, n) In-110, In-110m					
12	1.00	4.19	2.5	3.4	0.218
14	1.29	4.78	3	3.7	0.324
16	1.53	5.16	4	4.0	0.494
18	1.74	5.53	4	4.3	0.633
20	3.60	5.53	4	4.3	0.717
22	5.53	5.53	4	4.3	0.767
Ag-109($\alpha, 3n$) In-110, In-110m					
29	1.7 1.2 0.4 } ^c	7.10 6.00 2.50 } ^d	1		0.858
32	2.92 2.10 0.59	7.27 6.11 3.24	1.5	2.5	0.879
35	3.10 2.33 1.07	7.49 6.44 4.34	2.5	3.4	0.898
33	3.27 2.53 1.42	7.69 6.72 5.00	3.5	3.9	0.910

a) An a of 10 Mev^{-1} was used in calculating these quantities.

b) Approximate average number of gamma rays emitted, as calculated from equation (50).

c) These are the energies of the first, second and third neutrons respectively.

d) These are the values of σ_{RIGID} calculated from the residual nucleus energy after the emission of one, two and three neutrons respectively.

FIGURE 33.

EXPERIMENTAL ISOMERIC CROSS SECTION
RATIOS vs. CALCULATED RATIOS WITH

$$\sigma = \sigma_{\text{RIGID}} \cdot$$

Ag-107 (α , n) In-110, In-110m

FIGURE 34.

EXPERIMENTAL ISOMERIC CROSS SECTION
RATIOS vs. CALCULATED RATIOS WITH

$$\sigma = \sigma_{\text{RIGID}} \cdot$$

Ag-109 (α , 3n) In-110, In-110m

reaction, at an alpha particle energy of 38 Mev, using different values of the parameter a . A different value of a will change the value of the neutron kinetic energy, the value of σ_{RIGID} , and also the number of gamma rays emitted. The results are given in Table 20. Again (the effect has already been determined using a constant value of σ in calculating the isomeric cross section ratio for the $\text{Ag-107}(\alpha, n)$ reaction, at an alpha particle energy of 16 Mev) it is found that a large change in the parameter a , has little effect in the calculation of the isomeric cross section ratio. This is not suprising since a is a nuclear level density parameter related to the absolute level density (cf. equation 45), and in the calculation of the isomeric cross section ratio, one is concerned only with relative level densities in relation to spin.

If one compares the experimental data of Figure 33 with the calculations, it can be seen that at alpha particle energies of 18 Mev and below, the calculated ratios are somewhat higher than the experimental ratios. These data then could indicate that σ is less than the value calculated from the rigid moment of inertia at low excitation energies of the nucleus. A value of σ less than the rigid value is expected at low excitation energies of the nucleus, since at these low energies the moment of inertia is expected to be less than the rigid moment.^{2,51} The reduced moment of inertia is believed to occur in the nucleus at low excitation energies, because of pairing effects of nucleons at these low energies.^{54,55} The extent to

Table 20

Effect of the Parameter a on the calculation of
the Isomeric Cross Section Ratio with
 $\sigma = \sigma_{\text{RIGID}}$ at $E_{\alpha} = 38$ Mev

$\frac{a}{(\text{Mev}^{-1})}$	E_n (Mev)	σ_{RIGID} for neutron emission	No. of γ -rays emitted ^a	Ave. σ_{RIGID} for γ -ray emission	Isomeric Cross Section Ratio
7	$\left. \begin{array}{l} 3.87 \\ 2.93 \\ 1.50 \end{array} \right\}^b$	$\left. \begin{array}{l} 8.36 \\ 7.23 \\ 5.13 \end{array} \right\}^c$	2.5	3.8	0.920
10	$\left. \begin{array}{l} 3.27 \\ 2.53 \\ 1.42 \end{array} \right\}$	$\left. \begin{array}{l} 7.69 \\ 6.72 \\ 4.99 \end{array} \right\}$	3.5	3.9	0.911
13	$\left. \begin{array}{l} 2.89 \\ 2.27 \\ 1.33 \end{array} \right\}$	$\left. \begin{array}{l} 7.22 \\ 6.35 \\ 4.83 \end{array} \right\}$	4.5	4.0	0.903

- a) Approximate average number of gamma rays emitted, as calculated from equation (50).
- b) These are the energies of the first, second and third neutrons respectively.
- c) These are the values of σ_{RIGID} calculated from the residual nucleus energy, after the emission of one, two and three neutrons respectively.

which the moment of inertia is reduced below the rigid value as a function of the excitation energy of the nucleus is not definitely established, although some theories have been advanced.⁵⁴⁻⁵⁷ It was observed however, that a calculated isomeric cross section ratio for the Ag-107(α ,n) reaction at 14 Mev, using a σ of 2.6 (compared to a σ_{RIGID} of 3.7) for the gamma ray cascade, gave good agreement with the experimental ratio (cf. Figure 33). This calculated ratio,

$\sigma_{\text{In-110m}}/(\sigma_{\text{In-110}} + \sigma_{\text{In-110m}})$, was 0.252. Values of σ lower than σ_{RIGID} at alpha particle energies of 12, 16 and 18 Mev could also be chosen to agree with the experimental data. The results for alpha particle energies below 18 Mev then are consistent with a nuclear model which predicts reduced moments of inertia, and consequently reduced values of the parameter σ , at low excitation energies of the nucleus.

Still observing the results of the Ag-107(α ,n) In-110, In-110m reaction, it is seen that beyond alpha particle energies of 20 Mev, the calculated ratios using rigid σ 's (i.e. σ_{RIGID}) begin to fall below the experimental values (cf. Figure 33). The calculated ratio at $E_{\alpha}=22$ Mev, using σ_{RIGID} in all of the steps of the de-excitation process, agrees with the experimental value considering the errors. But since a σ less than σ_{RIGID} for the gamma ray cascade at low excitation energies, fits the experimental data at an alpha particle energy of 14 Mev, at first thought one might also expect a σ less than σ_{RIGID} for the gamma ray cascade, to fit the experimental data at an alpha particle energy of 22 Mev.

If however, at an alpha particle energy of 22 Mev σ for the gamma ray cascade is reduced below σ_{RIGID} by approximately the same percentage that it was reduced at 14 Mev, then the ratio obtained does not agree with the experimental value (cf. Figure 33, which shows the calculated isomeric cross section ratio for $\sigma = \sigma_{\text{RIGID}}$ for neutron emission, and $\sigma = 3.0(\sigma_{\text{RIGID}}=4.3)$ for gamma ray emission). The fact that some change in the variation of the isomeric cross section ratio with alpha particle energy is occurring beyond an alpha particle energy of about 17 Mev, is also evidenced by observing the experimental data alone. From Figure 35 it can be seen that the slope of the $\log (\sigma_{\text{In-110m}}/\sigma_{\text{In-110}})$ versus alpha particle energy curve changes at about 17 Mev.

To interpret the above results it is necessary to consider the effect of interference from the $\text{Ag-107}(\alpha, 2n)$ reaction. Recall that at an alpha particle energy of 22 Mev, the $\text{Ag-107}(\alpha, n)$ reaction cross section is small compared to the compound nucleus cross section (cf. Figure 27), since most of the reaction is occurring by way of the $\text{Ag-107}(\alpha, 2n)$ reaction. It has already been stated that in the energy spectrum of the neutrons evaporated from the compound nucleus In-111^* , it is the high energy neutrons that lead to the (α, n) product, since the emission of a low energy neutron leaves the nucleus with sufficient energy to emit a second neutron. However if σ is energy dependent, then it is necessary to consider the distribution in spin of the residual nuclei after the emission of one neutron, as well as the distribution in excitation energies.

FIGURE 35.
EXPERIMENTAL ISOMER RATIOS
($\sigma_{\text{In-110m}}/\sigma_{\text{In-110}}$) FOR THE
Ag-107(α , n) REACTION.

For those energy states just above the binding energy of the next neutron, gamma ray emission will compete with the emission of another neutron⁵⁸. Now consider the spin distribution of the states of In-110* just above the binding energy of the next neutron. If a second neutron is to be emitted, it cannot carry away a large amount of angular momentum because it will be of fairly low energy, consequently most of the angular momentum must be left in the nucleus. But if σ is relatively small at low excitation energies, there will be few high spin states to which it is possible to decay, and consequently the decay of the high spin states by neutron emission will be hindered. The high spin states will rather tend to decay by gamma ray emission, since a gamma ray can take away little energy, and populate higher energy states where σ is relatively large and hence a large number of high spin states are available.

If there is this preferential tendency for the high spin states of In-110* to decay to the In-110 isomers by gamma ray emission just beyond the Ag-109(α ,2n) threshold, then at these alpha particle energies, the experimental isomeric cross section ratio is expected to be higher than the calculated value (recall that In-110m has a spin of 7, and In-110 has a spin of 2), since in the calculation of this ratio the effect of interfering reactions was not considered. In other words, when more than one reaction is possible, and when one is just beyond the threshold of one of these reactions, it is necessary to consider angular momentum differences as well as energy differences in interpreting the results.

If the calculated isomeric cross section ratios using values of σ_{RIGID} in all the steps of the de-excitation process, are compared with the experimental ratios for the $\text{Ag-109}(\alpha, 3n)$ reaction, it can be seen that the experimental values fall below the calculated values (cf. Figure 34). Using a σ less than the rigid value at low excitation energies would bring the calculated ratios closer to the experimental ratios, but in addition there are two other factors to consider. These are, 1) the effect of competition with the $\text{Ag-109}(\alpha, 2n)$ reaction just beyond the $\text{Ag-109}(\alpha, 3n)$ threshold of 24.4 Mev and, 2) the possibility of quadrupole gamma ray emission at the higher alpha particle energies, where there are a large number of high spin states in the nucleus after the emission of three neutrons. The presence of quadrupole gamma rays in the de-excitation of a compound nucleus containing a significant number of high spin states, has been detected in recent experiments by Mollenauer.⁵⁹

First consider the alpha particle energy region just beyond the $\text{Ag-109}(\alpha, 3n)$ threshold. From the experimental data alone, a change in the slope of the isomeric cross section ratio versus alpha particle energy curve is observed when this threshold is approached (cf. Figure 34). This increased tendency to populate the low spin isomer of In-110 , just beyond the $\text{Ag-109}(\alpha, 3n)$ threshold (when one approaches the threshold from the high alpha particle energy side), may again be explained by an energy dependent σ , and the effect of competition from the $\text{Ag-109}(\alpha, 2n)$ reaction. Consider the residual nucleus

after the emission of two neutrons from In-113*, namely In-111*. If a neutron is to be emitted, it must populate a low excited state of In-110, and if σ is relatively low at this low excitation energy, there will be few high spin states available for population. For gamma ray emission from In-111* however, de-excitation occurs to a high excitation energy of In-111*, where σ is relatively high, and a larger number of high spin states are available. Consequently the high spin states will tend to decay by way of gamma ray emission (provided σ is relatively low at low excitation energies, and relatively high at high excitation energies). This means that a smaller number of high spin states than expected, assuming no interference from the $(\alpha, 2n)$ reaction just beyond the $(\alpha, 3n)$ threshold, will decay to the In-110 isomers. This in turn means that the isomeric cross section ratio, $\sigma_{\text{In-110m}} / (\sigma_{\text{In-110}} + \sigma_{\text{In-110m}})$, will be lower than expected assuming no interference from the $(\alpha, 2n)$ reaction (recall again that In-110m is the high spin isomer).

At alpha particle energies of 32 and 35 Mev, it is possible to fit the experimental ratio by using a value of σ less than the rigid value for the gamma ray cascade, and by assuming partial de-excitation by quadrupole rather than dipole gamma ray emission. Both of these factors tend to decrease the calculated isomeric cross section ratio, giving better agreement with the experimental ratio. For example it is possible to calculate a ratio in good agreement with the experimental ratio at an alpha particle energy of 38 Mev, if the following

assumptions are made: 1) $\sigma=3$ for the gamma ray cascade ($\sigma_{\text{RIGID}}=3.9$), 2) 3 gamma rays are emitted in the cascade and, 3) 75% of the gamma rays emitted (excluding the last gamma ray) are quadrupole rather than dipole. The ratio

$\sigma_{\text{In-110m}} / (\sigma_{\text{In-110}} + \sigma_{\text{In-110m}})$, making these assumptions is 0.861, and is in good agreement with the experimental ratio (cf. Figure 34). Summarizing then in regard to the $\text{Ag-109}(\alpha, 3n)$ reaction, no quantitative conclusions about the energy dependence of σ , or the de-excitation of a nucleus of high spin by quadrupole gamma ray emission can be made, but only qualitative agreement in regard to these concepts is observed.

D. Summary

A great deal of work has been done in determining isomeric cross section ratios in recent years^{9,10,60-66} but considerable care must be taken in interpreting the results. In the investigation of the $\text{Ag-107}(\alpha, n)$ and the $\text{Ag-109}(\alpha, 3n)$ reactions, it has been shown that in order to interpret the isomeric cross section ratios the effect of interfering reactions must be considered. It was also found that the calculation of isomeric cross section ratio using the method of Vandenbosch and Huizenga,^{9,10} is somewhat sensitive to the number and multipolarity of the gamma rays emitted, depending upon the initial spin distribution of the compound nuclei, and the spin distribution of the states to which these nuclei can decay. The ratios however are very insensitive to the nuclear level density parameter a , and the kinetic energy of the neutron emitted by the decaying nucleus.

Even after considering the above mentioned factors, a few conclusions could be made from the present results. It can be concluded that the average value of σ for medium mass nuclides is in the range of three to five, for nuclear reactions in which the compound nucleus has excitation energy less than about 25 Mev. For the $\text{Ag-109}(\alpha, 3n)$ reaction involving compound nuclei of very high angular momentum the same conclusions cannot be made, because of the uncertainty in the nature of the gamma ray de-excitation process. A more detailed investigation of the isomeric cross section ratios for the $\text{Ag-107}(\alpha, n)$ reaction producing the In-110 isomers showed however, that an energy dependent σ in which this parameter varies from about 2.5 at low excitation energies, to about 7 at high excitation

energies, was in agreement with the calculations. These results were in agreement with a model predicting σ as a function of energy, in which at low excitation energies of the nucleus the moment of inertia is less than the rigid value.

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Appendix A
Efficiency Factors in Scintillation Counting
of Gamma Radiation

In determining the disintegration rate of a sample by measuring the intensity of a peak in a gamma ray spectrum, it is necessary to consider a number of factors. It is assumed that the peak intensity has been corrected for background, Compton distribution of higher energy gamma rays in the spectrum, and impurities in the sample (if any), such that the peak intensity represents the counting rate of the photoelectric events in the scintillation crystal, produced by the gamma ray being measured. The factors involved will not be discussed in detail since this has been done by others,^{27,28,32} but they are listed here for reference. These factors are:

- 1) Geometry
- 2) Peak to total ratio
- 3) Crystal efficiency
- 4) Absorption by
 - a) sample
 - b) absorber covering sample
 - c) air between sample and crystal
 - d) crystal housing
- 5) Backscattering
- 6) Spread source effect
- 7) Coincidence summing
- 8) X-Ray escape peak
- 9) Decay Scheme
 - a) % decay by emission of gamma ray measured
 - b) internal conversion

The first eight factors enable one to determine the absolute gamma ray intensity of the sample from the peak intensity. The last two factors (9a and 9b) take into consideration the fact that not every disintegration produces the gamma ray whose intensity is being measured. The factors 4a, 4c, 4d, 5 and 6

can generally be neglected for gamma radiation when high accuracy is not desired. Coincidence summing can be neglected for large sample to crystal distances, and the escape peak factor may also be neglected for high energy gamma rays (>200 kev). The peak to total ratio (the ratio of the counts under the photopeak to the total counts in the gamma ray spectrum of a pure gamma ray emitter), and the crystal efficiency (the fraction of gamma radiation incident on the crystal face which interacts to produce a measurable scintillation), are sometimes combined and called the "photopeak efficiency".

Appendix B

Determination of Efficiency of a 5" x 5"

NaI(Tl) Crystal for Detecting Gamma

Rays Occuring in the Decay of In-110m

Efficiencies for detecting gamma rays in a 5" x 5" NaI(Tl) well crystal were determined by calculations, in which the following assumptions were made:

- 1) The sample being counted was a point source at the center of the crystal.
- 2) Absorption by the nickel well and the glass tube in which the sample was placed was negligible.
- 3) The crystal was a solid 5" diameter by 5" high cylinder of NaI(Tl) (i.e. the loss of NaI(Tl) absorber due to the presence of the well was negligible).

In the experiment to determine the disintegration rate of In-110m, the sample, a folded Ag-107 foil, was placed in the bottom of a pyrex glass tube, 1/32" thick. This tube was inserted in the well 5/16" in diameter and 2 5/8" deep. The nickel covering the well had a thickness of 1 mil. In view of these facts, the above assumptions are very good.

If one considers a gamma ray source at the center of a 5" x 5" NaI(Tl) crystal, then from the symmetry of the cylindrical crystal it can be seen that the probability of absorption will be equivalent through any plane passing through the axis of the cylinder. Likewise the probability of absorption through any half quadrant of this plane will be equivalent (cf. Figure B-1). Thus the probability of absorp-

tion of a gamma ray in a cylinder of equal height and diameter with the source at the center of the cylinder is given by:

$$\epsilon(E_\gamma) = \int_0^{\pi/4} (1 - e^{-\mu(E_\gamma)r}) d\theta / \int_0^{\pi/4} (1 - e^{-\mu(E_\gamma)\infty}) d\theta \quad (\text{B-1})$$

$$= \frac{4}{\pi} \int_0^{\pi/4} (1 - e^{-\mu(E_\gamma)x \sec \theta}) d\theta \quad (\text{B-2})$$

$\epsilon(E_\gamma)$ is the efficiency of the crystal for detecting a gamma ray of energy E_γ , $\mu(E_\gamma)$ is the total linear absorption coefficient for a gamma ray of energy E_γ , in NaI(Tl), and x is the radius of the cylinder (2.5" or in this case). This integral (equation B-2) may be evaluated if the exponential term is expanded:

$$\begin{aligned} \epsilon(E_\gamma) = & \frac{4}{\pi} \left\{ [\mu(E_\gamma)x] \int_0^{\pi/4} \sec \theta d\theta \right. \\ & - \frac{[\mu(E_\gamma)x]^2}{2!} \int_0^{\pi/4} \sec^2 \theta d\theta \\ & + \frac{[\mu(E_\gamma)x]^3}{3!} \int_0^{\pi/4} \sec^3 \theta d\theta \\ & - \frac{[\mu(E_\gamma)x]^4}{4!} \int_0^{\pi/4} \sec^4 \theta d\theta \\ & \left. + \dots \right\} \quad (\text{B-3}) \end{aligned}$$

$\epsilon(E_\gamma)$ for $E_\gamma = 656$ keV was calculated from equation (B-3) using an absorption coefficient extrapolated from the tables of Grodstein.⁴¹ Efficiencies for $E_\gamma = 610, 885$, and 940 keV were calculated using an approximation to equation (B-3), namely:

$$\epsilon(E_\gamma) = (1 - e^{-\mu(E_\gamma)r'}) \quad (\text{B-4})$$

where r' is the mean distance travelled by the gamma ray in leaving the crystal. r' was determined from equation (B-4)

FIGURE B-1
QUADRANTS OF A 5" X 5" SQUARE

using the $\epsilon(E_\gamma)$ obtained by equation (B-3) for $E_\gamma = 656$ kev. The efficiencies obtained, as well as the absorption coefficients used in the calculations, are shown in Table B-1.

Table B-1
Detection Efficiencies for Some Gamma
Rays in a 5" x 5" NaI(Tl) Well Crystal

E_γ (kev)	$\mu(E_\gamma)$ (cm^{-1})	$\epsilon(E_\gamma)^*$
656	0.272	0.852
670	0.269	0.849
885	0.227	0.797
940	0.219	0.785

*Cf. equation (B-1) for definition of terms.

Appendix C

Cross Section Ratio for a Parent-Daughter Isomeric Pair

For the case of an isomeric pair in which the metastable state decays completely to the ground state, the rate equation for the production of the ground state is given by:

$$\frac{dN_2}{dt} = R_2 - \lambda_2 N_2 + \lambda_1 N_1 \quad (C-1)$$

where $\frac{dN_2}{dt}$ = the rate of change of the number of ground state nuclei.

R_2 = the rate of formation of ground state nuclei in the particle flux.

$\lambda_2 N_2$ = the rate of decay of ground state nuclei.

$\lambda_1 N_1$ = the rate of growth of ground state nuclei from the metastable state.

But

$$\lambda_1 N_1 = R_1 (1 - e^{-\lambda_1 t}) \quad (C-2)$$

This equation is obtained by solving

$$\frac{dN_1}{dt} = R_1 - \lambda_1 N_1 \quad (C-3)$$

by the method of separation of variables. $\frac{dN_1}{dt}$ and R_1 are defined as were $\frac{dN_2}{dt}$ and R_2 , the subscript 1 in this case referring to the metastable state. From equations (C-1) and (C-2)

$$\frac{dN_2}{dt} + \lambda_2 N_2 = R_2 + R_1 - R_1 e^{-\lambda_1 t} \quad (C-4)$$

To solve this equation it is necessary to obtain the general solution of the reduced equation (eq. C-5) and add it to a particular solution of the given equation (eq. C-4). The

general solution of the reduced equation follows thus:

$$\frac{dN_2}{dt} + \lambda_2 N_2 = 0 \quad (C-5)$$

$$\frac{dN_2}{N_2} = -\lambda_2 dt \quad (C-6)$$

$$\ln N_2 = -\lambda_2 t + \ln C \quad (C-7)$$

$$N_2 = C e^{-\lambda_2 t} \quad (C-8)$$

Now assume a particular solution of the given equation using the method of undetermined coefficients:

$$N_2 = a e^{-\lambda_1 t} + b \quad (C-9)$$

where a and b are the coefficients to be determined. From equation (C-9):

$$\frac{dN_2}{dt} = -a\lambda_1 e^{-\lambda_1 t} \quad (C-10)$$

and from equations (C-4), (C-9) and (C-10):

$$-a\lambda_1 e^{-\lambda_1 t} + \lambda_2 a e^{-\lambda_1 t} + \lambda_2 b = R_2 + R_1 - R_1 e^{-\lambda_1 t} \quad (C-11)$$

By equating coefficients:

$$-a\lambda_1 + a\lambda_2 = -R_1 \quad (C-12)$$

$$a = R_1 / (\lambda_1 - \lambda_2) \quad (C-13)$$

$$\lambda_2 b = R_2 + R_1 \quad (C-14)$$

$$b = (R_2 + R_1) / \lambda_2 \quad (C-15)$$

so that equation (C-9) becomes:

$$N_2 = \left[R_1 / (\lambda_1 - \lambda_2) \right] e^{-\lambda_1 t} + (R_2 + R_1) / \lambda_2 \quad (C-16)$$

Thus in view of equations (C-8) and (C-16) the solution of equation (C-4) is:

$$N_2 = C e^{-\lambda_2 t} + \left[R_1 / (\lambda_1 - \lambda_2) \right] e^{-\lambda_1 t} + \frac{(R_2 + R_1)}{\lambda_2} \quad (C-17)$$

The constant C may be evaluated by realizing that the number of ground state nuclei at zero time is zero, so that from the previous equation, (C-17):

$$0 = C e^0 + \left[\frac{R_1}{\lambda_1 - \lambda_2} \right] e^0 + \frac{R_2 + R_1}{\lambda_2} \quad (C-18)$$

and

$$C = \frac{-R_1}{\lambda_1 - \lambda_2} - \frac{(R_2 + R_1)}{\lambda_2} \quad (C-19)$$

so that equation (C-17) becomes

$$\begin{aligned} \lambda_2 N_2 &= (R_1 + R_2) (1 - e^{-\lambda_2 t}) \\ &+ \frac{\lambda_2 R_1}{\lambda_1 - \lambda_2} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \end{aligned} \quad (C-20)$$

In the case where both activities are determined from one irradiation:

$$R_1 = n f \sigma_1 \quad (C-21)$$

$$R_2 = n f \sigma_2 \quad (C-22)$$

where n = number of target nuclei

f = particle flux

σ_1 = cross section for formation of
metastable state

σ_2 = cross section for formation of ground
state

And from the decay rates of the ground and metastable state isomers, equations (C-20) and (C-2):

$$\begin{aligned} \frac{\lambda_2 N_2}{\lambda_1 N_1} (1 - e^{-\lambda_1 t}) &= \frac{1}{\sigma_1} \left\{ (\sigma_1 + \sigma_2) (1 - e^{-\lambda_2 t}) \right. \\ &+ \left. \frac{\lambda_2 \sigma_1}{\lambda_1 - \lambda_2} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \right\} \end{aligned} \quad (C-23)$$

Upon rearranging equation (C-23) one obtains an expression for the cross section ratio:

$$\sigma_2/\sigma_1 = \frac{1}{(1-e^{-\lambda_2 t})} \left\{ \frac{\lambda_2 N_2}{\lambda_1 N_1} (1-e^{-\lambda_1 t}) - \frac{\lambda_2}{\lambda_1 - \lambda_2} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \right\} - 1 \quad (\text{C-24})$$

Appendix D
Experimental Data

Table D-1

Experimental Data - (n, γ) Reactions

Experi- ment no.	Target	t ^a	Approx. neutron flux per CM ² per sec.	Wt. of target (Mg)	Cd foil around sample	Activity ^b of high spin isomer (c.p.m.)	Activity ^b of low spin isomer (c.p.m.)
N20	Se-80	15s	7×10^{12}	6.3	none	195,900	475,500
N28	Se-80	50s	1.7×10^{13}	9.3	20 mil	18,470	30,450
N16	Rh-103	2s	7×10^{12}	2.4	none	6,072	519,800
N22	Rh-103	2s	7×10^{12}	2.1	none	8,637	690,400
N21	Rh-103	15s	7×10^{12}	29	20 mil	8,395	646,500
N13	Pd-108	15s	7×10^{12}	1.0	none	122,600	32,340
N27	Pd-108	60s	1.7×10^{13}	3.1	20 mil	284,700	79,580
N 7	Te-128	4h	2.1×10^{12}	3.8	none	1.873	6,170
N26	Te-128	10h	5.3×10^{12}	7.3	10 mil	7.95	12,000

a) Length of irradiation

b) These are the activities in counts per minute at the end of the irradiation as determined by the counting set-ups listed in Tables 2 and 3 of the text.

c) The activities of both the high and low spin isomers of Pd-109 and Te-129 given here are corrected for any difference in counting efficiency.

Table D-1 (continued)

Experimental Data - (n, γ) Reactions

Experiment no.	Target	t^a	Approx. neutron flux per cm^2 per sec.	Wt. of target (Mg)	Cd foil around sample	Activity ^b of high spin isomer (c.p.m.)	Activity ^b of low spin isomer (c.p.m.)
N 5	Cs-133	10m	7×10^{12}	44	none	$4,252 \frac{\lambda_1}{\lambda_2}^d$	41,690
N32	Cs-133	2m	1.7×10^{13}	112	none	$2,038 \frac{\lambda_1}{\lambda_2}$	23,700
N24	Cs-133	5m	7×10^{12}	440	20 mil	$626 \frac{\lambda_1}{\lambda_2}$	7,611
N29	Cs-133	3m	1.7×10^{13}	540	20 mil	$1,950 \frac{\lambda_1}{\lambda_2}$	25,040
N30	Cs-133	3m	1.7×10^{13}	446	20 mil	$1,595 \frac{\lambda_1}{\lambda_2}$	21,550
N31	Cs-133	4m	1.7×10^{13}	327	20 mil	$1,543 \frac{\lambda_1}{\lambda_2}$	19,125

d) These are the decay constants for the Cs-134 isomers; the subscript 1 refers to the metastable state and the subscript 2 refers to the ground state.

Table D-2

Experimental Data - Ag-107 (α n) Reaction

Experi- ment no.	t* (min.)	Total charge on target (μ amp sec.)	Wt. of Ag foil (Mg)	Foil thickness in front of Ag foil (Mg/cm ²)	Range of α 's striking first Al foil (Mg/cm ²)	In-110 activity† (d.p.m.)	In-110m activity† (d.p.m.)
A10	18.2	437	5.16	159.9 Al	179.4	249,300	9,038
A 8(2)	14	320.2	5.52	154.56 Al + 1.68 Ag	178.9	3,316,900	157,200
A 9	10	174.9	4.92	150.26 Al	178.0	3,876,700	249,200
A12(2)	10	59.75	4.83	145.04 Al + 1.73 Ag	178.8	2,641,500	268,470
A11(2)	3	17.48	5.22	138.08 Al + 1.84 Ag	179.6	1,020,000	152,600
A11(1)	3	17.48	5.23	132.72 Al	179.6	855,500	234,100
A 8(1)	14	320.2	4.80	131.79 Al	178.9		4,356,000
A12(1)	10	59.75	4.93	125.41 Al	178.8	1,014,000	705,500
A13(2)	4	10.74	4.57	121.37 Al + 1.92 Ag	177.0	218,500	142,300
A13(1)	4	10.74	5.48	115.68 Al	177.0	107,200	102,200

*Length of Irradiation

†These are the absolute disintegration rates per minute at the end of the irradiation.

Table D-3

Experimental Data - Ag-109 ($\alpha, 3n$) Reaction

Experi- ment no.	t* (min)	Total charge on target (μ amp sec.)	Wt. of Ag foil (Mg)	Foil thickness in front of Ag foil (Mg/cm ²)	Range of α 's striking first Al foil (Mg/cm ²)	In-110 activity† (d.p.m.)	In-110m activity† (d p.m.)
B4(2)	2	12.63	6.55	92.09 Al + 2.10 Ag	175.6		6,876
B4(1)	2	12.63	6.00	84.72 Al	175.6	147,800	69,480
B2(2)	3	8.837	6.25	73.40 Al + 2.35 Ag	175.5	236,400	142,700
B2(1)	3	8.837	6.70	64.26 Al	175.5	359,000	286,900
B1(2)	2	6.500	8.21	52.19 Al + 1.87 Ag	178.0	366,700	347,700
B1(1)	2	6.500	5.34	40.85 Al	178.0	277,800	311,900
B3(2)	1	5.469	6.06	27.18 Al + 2.34 Ag	177.0	278,100	360,300
B3(1)	1	5.469	6.66	15.73 Al	177.0	248,800	390,200

*Length of irradiation

†These are the absolute disintegration rates per minute at the end of the irradiation.

Appendix E

Alpha Particle and Neutron Transmission Coefficients

Alpha particle transmission coefficients were calculated by Huizenga and Igo⁴⁴ using an optical model potential. Calculations were made at several alpha energies for the Ag-108 + He-4 reaction (practically the same as the Ag-107 + He-4 or the Ag-109 + He-4 reaction as far as the calculation of transmission coefficients is concerned). In all calculations the following Woods-Saxon Potential Parameters were used: $V = -50$ Mev, $W = -14.9$ Mev, $r_0 = (1.17 A^{1/3} + 1.77)$ fermi, $d = 0.576$ fermi and $n = 7.75$ (cf. reference 44 for a discussion of these parameters and their use in calculating charged particle transmission coefficients).

In a few calculations (Section IV.D.2.) optical model neutron transmission coefficients from Cambell et al.⁴⁶ were used; these are listed in Table E-2. In most calculations neutron transmission coefficients from Feld et al.⁴⁵ were used in calculating spin distributions after neutron emission; these are listed in Table E-3 for several neutron energies.

Table E-1

Alpha Particle Transmission Coefficients

E_α (Mev)* ℓ	12	14	16	18	20	22
0	1.011	5.931	9.101	9.791	9.941	9.981
1	8.832	5.611	8.991	9.771	9.941	9.981
2	6.692	5.001	8.771	9.721	9.921	9.981
3	4.392	3.981	8.331	9.621	9.901	9.971
4	2.492	2.801	7.581	9.431	9.851	9.961
5	1.222	1.671	6.341	9.071	9.761	9.931
6	5.283	8.432	4.581	8.351	9.581	9.881
7	2.043	3.632	2.691	7.031	9.181	9.771
8	7.214	1.382	1.261	4.961	8.341	9.531
9	2.354	4.823	4.952	2.711	6.651	8.951
10	7.205	1.583	1.732	1.151	4.201	7.651
11	2.062	4.984	5.753	4.112	1.951	5.331
12		1.494	1.853	1.382	7.302	2.691
13		4.255	5.814	4.573	2.512	1.051
14		1.132	1.754	1.493	8.483	3.682
15			4.995	4.754	2.893	1.282
16				1.444	9.654	4.553
17				4.102	3.094	1.593
18					9.295	5.354
19						1.694
20						4.995

Note--The underlined number is the negative exponent of 10 by which the three-digit number is to be multiplied, e.g. 1.011 = 1.01×10^{-1} (This also applies to the data given in Tables E-2 and E-3).

*The alpha particle energies are given in the laboratory system.

Table E-1 (continued)

Alpha Particle Transmission Coefficients					
E_{α} (Mev)	26	29	32	35	38
0	10.001	10.001	10.001	10.001	10.001
1	10.001	10.001	10.001	10.001	10.001
2	10.001	10.001	10.001	10.001	10.001
3	10.001	10.001	10.001	10.001	10.001
4	9.991	10.001	10.001	10.001	10.001
5	9.991	10.001	10.001	10.001	10.001
6	9.991	10.001	10.001	10.001	10.001
7	9.981	10.001	10.001	10.001	10.001
8	9.951	9.991	10.001	10.001	10.001
9	9.901	9.981	9.991	10.001	10.001
10	9.761	9.961	9.991	10.001	10.001
11	9.411	9.881	9.971	9.991	10.001
12	8.491	9.711	9.931	9.981	9.991
13	6.381	9.161	9.831	9.961	9.991
14	3.531	7.681	9.461	9.891	9.981
15	1.441	5.001	8.421	9.601	9.911
16	5.242	2.281	6.081	8.841	9.691
17	1.962	8.732	3.051	6.751	9.071
18	7.523	3.382	1.231	3.641	7.091
19	2.863	1.362	4.922	1.541	4.021
20	1.043	5.523	2.072	6.322	1.761
21	3.574	2.173	8.873	2.772	7.402
22	1.144	8.034	3.713	1.252	3.372
23	3.385	2.784	1.473	5.503	1.582
24		8.945	5.454	2.313	7.323
25		2.275	1.884	9.084	3.233
26			6.025	3.334	1.343
27			1.795	1.134	5.174
28				3.585	1.854
29					6.175
30					1.915

Table E-2
Neutron Transmission Coefficients
(Campbell et al. ⁴⁶)

x^a	T_0	T_1	T_2	T_3	T_4	T_5	T_6
$x_o^b = 9.2$							
0.8	2.441	5.061	1.192	2.803			
1.2	3.421	6.571	6.812	2.922	1.704		
1.6	4.271	6.881	1.901	1.041	2.153	8.105	
2.0	5.001	6.761	3.521	2.011	1.572	6.904	
2.4	5.631	6.521	5.081	2.841	8.292	3.643	9.204
$x_o^b = 9.6$							
0.8	3.431	2.151	1.632	8.204			
1.2	4.701	3.541	1.021	9.803	2.804		
1.6	5.741	4.381	3.061	4.392	3.903	5.405	
2.0	6.551	4.851	5.641	1.111	3.312	5.004	1.854
2.4	7.161	5.131	7.581	1.971	2.061	2.963	1.503

$$\begin{aligned}
 \text{a) } X' &= 0.22 \left\{ \sqrt{\frac{A}{A+1} E_n} R \right\} \\
 \text{b) } X_o &= 0.22 \left\{ \sqrt{\frac{A}{A+1} V_o} R \right\}
 \end{aligned}
 \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{ (cf. Sec. V.C.1. for an explanation of these terms) }$$

Table E-3

Neutron Transmission Coefficients

(Feld et al.⁴⁵)

X^*	T_0	T_1	T_2	T_3	T_4	T_5	T_6	T_7
0.50	2.201	5.002	3.003					
0.71	2.981	1.121	8.003	5.004				
0.87	3.501	1.681	2.102	1.103				
1.00	3.881	2.171	3.902	2.203	6.004			
1.22	4.481	3.001	8.502	5.603	1.103			
1.27	4.631	3.171	9.802	7.303	1.303			
1.58	5.421	4.301	1.961	2.822	3.703			
1.63	5.551	4.441	2.101	3.402	4.103	5.004		
1.75	5.781	4.751	2.531	5.202	6.003	7.004		
1.80	5.871	4.921	2.681	6.052	7.003	7.004		
1.89	6.071	5.151	3.031	7.702	9.203	9.004		
1.94	6.181	5.261	3.221	8.802	1.102	1.003		
1.96	6.221	5.301	3.271	9.302	1.162	1.103		
2.09	6.451	5.631	3.731	1.261	1.632	1.703		
2.19	6.631	5.871	4.101	1.541	2.132	2.303		
2.30	6.821	6.081	4.431	1.861	2.942	3.203		
2.43	6.981	6.361	4.771	2.261	4.352	4.403	8.004	
2.54	7.181	6.561	5.131	2.621	6.052	6.203	1.103	
2.66	7.281	6.781	5.431	3.021	8.102	8.703	1.403	
2.73	7.401	6.881	5.621	3.261	9.502	1.082	1.603	
2.81	7.481	7.021	5.821	3.481	1.131	1.382	2.003	
2.89	7.551	7.181	5.981	3.761	1.331	1.722	2.603	
3.00	7.641	7.301	6.221	4.101	1.621	2.402	3.403	
3.14	7.801	7.501	6.551	4.551	2.031	3.502	4.803	8.004
3.20	7.821	7.581	6.621	4.751	2.231	4.102	5.503	1.003
3.75	8.301	8.101	7.551	6.281	3.971	1.431	2.302	3.403

$$*X = 0.22 R \sqrt{E_n} \quad (\text{cf. equation 58})$$