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ESTIMATION OF MOLTEN CORE SIZE
IN WASTE STORAGE VESSELS
AND FROZEN WALL WASTE MELTERS

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ESTIMATION OF MOLTEN CORE SIZE
IN WASTE STORAGE VESSELS
AND FROZEN WALL WASTE MELTERS

By

J. D. Kaser

Chemical Technology Department
Chemistry and Metallurgy Division

April 1969

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ESTIMATION OF MOLTEN CORE SIZE
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ABSTRACT

This report presents equations for calculating the size of molten cores in cylindrical and spherical vessels containing highly radioactive heat-generating waste. These equations may be used for cases where (1) all heat is generated by radioactive decay, and (2) internal heaters supply heat in addition to the radioactive decay heat.

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ESTIMATION OF MOLTEN CORE SIZE
IN WASTE STORAGE VESSELS
AND FROZEN WALL WASTE MELTERS

J. D. Kaser

INTRODUCTION

This report presents equations for calculating the size of molten cores in cylindrical and spherical vessels containing highly radioactive heat-generating waste. These equations may be used for cases where (1) all heat is generated by radioactive decay, and (2) internal heaters supply heat in addition to the radioactive decay heat. These equations include the case of cylindrical containers without axial heating, which was calculated earlier by W. V. DeMier.*

The equations are derived from simple idealizations and are therefore intended for preliminary sizing and evaluation of molten core storage vessels and frozen wall melters.

The following simplifying assumptions were used in the derivations:

1. The radioactive decay heat generation is uniform.
2. The thermal conductivity is uniform and independent of temperature in the solid portion of the waste.
3. The outer surface temperature of the vessel is uniform.
4. All temperatures are at their steady-state values.
5. The cylindrical vessels have insulated ends.
6. Any internal heaters are centrally located.
7. The heat transfer coefficient between the molten and solid waste is uniform, which means there are no thin spots in the frozen wall.

* W. V. DeMier. *Product Characteristics and Storage Measurements, Techniques, and Equipment*, BNWL-507, pp. 23-30. Pacific Northwest Laboratory, Richland, Washington, August 1967.

SIZE OF MOLTEN CORE IN CYLINDRICAL VESSELS

The differential heat balance in the frozen waste is

$$\frac{d}{dr} (rq_{rw}) = Ar \quad . \quad (1)$$

The differential heat balance for the container wall is

$$\frac{d}{dr} (rq_{rw}) = 0 \quad . \quad (2)$$

When Equation (1) is integrated we obtain

$$q_{rw} = \frac{Ar}{2} + \frac{C_{1w}}{r} \quad , \quad (3)$$

and likewise the integration of Equation (2) yields

$$q_{rw} = \frac{C_{1w}}{r} \quad . \quad (4)$$

The constants of integration, C_{1w} and C_{1w} may be evaluated using the following boundary conditions:

$$\text{B.C.1:} \quad q_{rw} = \frac{r_i^2 A}{2} + \frac{B}{2\pi r_i L} \quad \text{at } r = r_i$$

$$\text{B.C.2:} \quad q_r(c) = \frac{r_i^2 A}{2r_o} + \frac{B}{2\pi r_o L} \quad \text{at } r = r_o \quad .$$

Equation (3) then becomes

$$q_{rw} = \frac{Ar}{2} + \frac{B}{2\pi r L} \quad , \quad (5)$$

and Equation (4) becomes

$$q_{rc} = \frac{r_i^2 A}{2r} + \frac{B}{2\pi r L} \quad . \quad (6)$$

Inserting Fourier's law, $q_r = \frac{kdt}{dr}$, into the equations above, we get

$$\frac{-k_w dT_w}{dr} = \frac{Ar}{2} + \frac{B}{2\pi r L} \quad (7)$$

and

$$\frac{-k_c dT_c}{dr} = \frac{r_i^2 A}{2r} + \frac{B}{2\pi r L} \quad . \quad (8)$$

ERRATA

On page 3, change Equation (9) to read

$$\begin{aligned}
 T_w - T_o = & \frac{Ar_i^2}{4k_w} \left[1 - \left(\frac{r}{r_i} \right)^2 \right] + \frac{B}{2\pi L k_w} \ln \frac{r_i}{r} \\
 & + \frac{1}{k_c} \left[\frac{Ar_i}{2} + \frac{B}{2\pi L} \right] \ln \frac{r_o}{r_i} \quad , \quad (9)
 \end{aligned}$$

Integrating again and using the boundary conditions

$$\text{B.C.3: } T_c = T_w \text{ at } r = r_i$$

$$\text{B.C.4: } T_c = T_o \text{ at } r = r_o$$

Equation (7) becomes:

$$T_w - T_o = \frac{Ar_i^2}{4k_w} \left[1 - \left(\frac{r}{r_i} \right)^2 \right]^2 + \frac{B}{2\pi L K_w} \ln \frac{r_i}{r} + \frac{1}{k_c} \left[\frac{Ar_i}{2} + \frac{B}{2\pi L} \right] \ln \frac{r_o}{r_i}, \quad (9)$$

and Equation (8) becomes

$$T_c - T_o = 1/k_c \left(\frac{r_i^{2A}}{2} + \frac{B}{2\pi L} \right) \ln \frac{r_o}{r}. \quad (10)$$

At the boundary between the solid waste and molten waste, $r = r_m$ (the radius of the molten zone), and $T_w = T_m$ (the melting point of the waste). Thus, from Equation (9) the relationship between r_m and T_m is, in dimensionless form,

$$\frac{4(T_m - T_o)k_w}{Ar_i^2} = \left[1 - \left(\frac{r_m}{r_i} \right)^2 \right] - \frac{2B}{\pi L r_i^{2A}} \ln \frac{r_m}{r_i} + \frac{2k_w}{k_c} \left[1 + \frac{B}{\pi L r_i^{2A}} \right] \ln \frac{r_o}{r_i}. \quad (11)$$

Equation (11) cannot be solved explicitly for (r_m/r_i) when both A and B have nonzero values. For the special case of storage vessels with no additional internal heating ($B = 0$), Equation (11) can be solved explicitly for (r_m/r_i) :

$$\frac{r_m}{r_i} = \sqrt{1 - \frac{4(T_m - T_o)k_w}{Ar_i^2} + \frac{2k_w}{k_c} \ln \frac{r_o}{r_i}}, \text{ when } B = 0. \quad (12)$$

DERIVATION OF EQUATION FOR SPHERE

The differential heat balance for heat-generating waste in a spherical container is

$$\frac{d}{dr} (r^2 q_{rw}) = Ar^2 \quad . \quad (13)$$

The differential heat balance for the container wall is

$$\frac{d}{dr} (r^2 q_{rc}) = 0 \quad . \quad (14)$$

Integrating these two equations with the following boundary conditions,

$$\text{B.C.1:} \quad q_{rw} = \frac{Ar_i}{3} + \frac{B}{4\pi r_i^2} \quad \text{at } r = r_i$$

$$\text{B.C.2:} \quad q_{rc} = \frac{Ar_i^3}{3r_o^2} + \frac{B}{4\pi r_o^2} \quad \text{at } r = r_o \quad ,$$

we obtain from Equation (13)

$$q_{rw} = \frac{Ar}{3} + \frac{B}{4\pi r^2} \quad , \quad (15)$$

and from Equation (14)

$$q_{rc} = \frac{1}{r^2} \left[\frac{Ar_i^3}{3} + \frac{B}{4\pi} \right] \quad . \quad (16)$$

Again, after substitution of Fourier's law, $q_r = - \frac{kdT}{dr}$, Equation (15) for the solid waste becomes

$$\frac{-k_w dT_w}{dr} = \frac{Ar}{3} + \frac{B}{4\pi r^2} \quad , \quad (17)$$

and Equation (16) for the container wall becomes

$$\frac{-k_c dT_c}{dr} = \frac{1}{r^2} \left[\frac{Ar_i^3}{3} + \frac{B}{4\pi} \right] \quad . \quad (18)$$

With the boundary conditions

$$\text{B.C.3: } T_C = T_w \text{ at } r = r_i$$

$$\text{B.C.4: } T_C = T_O \text{ at } r = r_O,$$

solutions for (17) and (18) are respectively:

$$k_w (T_C - T_O) = \frac{A}{6} (r_i^2 - r^2) + \frac{B}{4\pi} \left(\frac{1}{r} - \frac{1}{r_i} \right) + \frac{k_w}{k_C} \left(\frac{1}{r_i} - \frac{1}{r_O} \right) \left(\frac{Ar_i^3}{3} + \frac{B}{4\pi} \right), \quad (19)$$

and

$$k_C (T_C - T_O) = \left(\frac{1}{r} - \frac{1}{r_O} \right) \left(\frac{Ar_i^3}{3} + \frac{B}{4\pi} \right). \quad (20)$$

Inserting the relationship $r = r_m$ when $T = T_m$ yields:

$$\left(\frac{r_m}{r_i} \right)^3 + \left[\frac{6k_w (T_m - T_O)}{r_i^2 A} - 1 + \frac{3B}{2\pi r_i^3 A} \right] \left(\frac{r_O - r_i}{r_O} \right) \left(2 + \frac{3B}{2\pi Ar_i^3} \right) \left(\frac{r_m}{r_i} \right) - \frac{3B}{2\pi Ar_i^3} = 0. \quad (21)$$

Equation (21) can be solved for (r_m/r_i) using any standard method for cubic equations. However, this equation can be simplified further for two interesting limiting cases.

For a nonheat generating material, Equation (21) simplifies to

$$\frac{r_m}{r_i} = \frac{1}{\frac{4\pi k_w r_i (T_m - T_O)}{B} + 1 - \frac{k_w}{k_C} \left(\frac{r_O - r_i}{r_O} \right)}. \quad (22)$$

For a heat-generating material without additional internal heating, Equation (21) can be simplified to

$$\left(\frac{r_m}{r_i}\right) = \sqrt{1 + \frac{2k_w}{k_c} \left(\frac{r_o - r_i}{r_o}\right) - \frac{6k_w(T_m - T_o)}{r_i^2 A}}. \quad (23)$$

Equation (23) for a sphere is analogous to Equation (12) for a cylinder.

EXAMPLES OF APPLICATIONS

SIZING A FROZEN WALL MELTER

In this example, we will estimate the size of a roughly spherical frozen wall melter for processing waste with a specific heat generation rate of 15,000 Btu/hr-ft³. The melter wall is to be 1/2 in. thick and the outer surface is to be kept at 600 °F. The thermal conductivity of the wall material is 5 Btu/hr-ft-°F, and the thermal conductivity of the frozen waste is 0.5 Btu/hr-ft-°F. The melting point of the waste is 1380 °F.

To size the melter, we first determine the maximum diameter that the sphere can have and still maintain a frozen wall 1 in. thick. Using Equation (23), we solve for r_i while expressing r_m as $r_i - a$ and r_o as $r_i + b$, where a and b are the thickness of frozen wall and the thickness of melter wall, respectively. Equation (23) rearranges into the following quadratic form:

$$\left(2a + \frac{2k_w b}{k_c}\right) r_i^2 + \left[2ab - a^2 - \frac{6k_w(T_m - T_o)}{A}\right] r_i - a^2 b - \frac{6k_w b(T_m - T_o)}{A_o} = 0.$$

Inserting numerical values, we find that $r_i = 0.933$ ft is the maximum value for the radius of the frozen wall melter.

Because of probable variation in the radioactivity and thermal properties of the waste, the radius of the actual melter should be less than the maximum diameter. It will be assumed that experience suggests reducing the radius 10%, which means about a 30% reduction in volume and radioactive decay heat load.

We will then estimate the practical melter diameter to be 20 in. Now the problem is to size the internal heaters needed to maintain a frozen wall thickness of 1 in. Therefore, we solve Equation (21) for B (the heater power) and find that the internal heater power must be at least 4000 Btu/hr in order to keep frozen wall 1 in. thick.

SIZE OF MOLTEN CORE IN CYLINDRICAL STORAGE CONTAINER

Here we will determine the molten core size when the heat generation is just great enough to keep the surface temperature of the container at 842 °F (450 °C), which is considered the safe limit for air storage.

Description of a container of waste:

Fill depth:	6 ft
Internal diameter:	0.667 ft
Thermal conductivity of waste:	0.5 Btu/hr-ft-°F
Thermal conductivity of container wall:	9 Btu/hr-ft-°F
Melting point of waste:	1380 °F
Wall thickness:	0.3 in.

From other calculations it has been estimated that the container will lose heat to the surroundings at a rate of 60,000 Btu/hr when the surface temperature of the container is 842 °F (450 °C).

This corresponds to a volumetric heat generation rate of $\frac{9 (60,000)}{6\pi} = 28,648 \text{ Btu/hr-ft}^3$.

From Equation (12) we find that $(r_m/r_i) = 0.818$. Thus, the radius of the molten core is 3.27 in. or 0.27 ft.

Nomenclature

- A = volumetric heat generation rate, Btu/hr-ft³
- a = thickness of frozen waste, ft
- B = power of axial or central heater, Btu/hr
- b = thickness of vessel wall, ft
- c = constant of integration
- k = thermal conductivity, Btu/hr-ft-°F
- L = length of filled portion of storage vessel, ft
- q_r = radial heat flux, Btu/hr-ft²
- r = radius of cylinder or sphere, ft
- T = temperature, °F
- π = 3.1415..., mathematical constant

Subscripts

- c = container wall material
- i = inner surface of vessel
- m = molten core boundary
- o = outer surface of vessel
- w = solid radioactive waste
- 1 = first integration
- 2 = second integration

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