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Signal Processing Algorithms on Parallel Architectures: A Performance Update

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Date Transmitted: February, 1991

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Signal Processing Algorithms on Parallel Architectures: A Performance Update^{*}

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The Burg algorithm is a widely applied and extensively studied signal processing procedure having a structure typical of a class of important batch signal processing algorithms. Its implementation and performance on four different parallel machines were reported in the 1990 *Journal of Parallel and Distributed Computing Special Issue on Massively Parallel Computation*. The machines were: the Intel iPSC/2, the Denelcor HEP, the NASA/Goodyear MPP, and the Cray X-MP/48. The objective of the work reported here was to extend that study to two new parallel machines: the nCUBE 2 and the MasPar MP-1. These computers are related to the distributed memory systems above (i.e., the iPSC and the MPP, respectively), but use newer technology. In addition to achieving significant performance gains on the new machines compared to machines in the same architectural class, we found that the original study underestimated the scalability of the algorithm. That is, the algorithm maps efficiently to small-scale as well as large-scale computers, including both SIMD and MIMD distributed memory systems. Improvements in the parallel algorithm are highlighted. Of special import is the use of appropriate performance metrics and performance visualization to characterize the parallelism of the algorithm and lend insight toward understanding and evaluating its performance.

1. INTRODUCTION

The Burg algorithm [1] is a linear signal processing procedure for fitting an autoregressive model to a time series data set. It is a popular, widely applied, extensively studied, and computationally efficient method; its structure is typical of a large class of important batch signal processing algorithms (for example, in power spectrum estimation) [7]. To investigate the mapping of signal processing algorithms onto general purpose parallel computers, Sammur and Hagan [14] implemented the Burg algorithm on four different parallel machines: the Intel iPSC/2, the Denelcor HEP, the NASA/Goodyear MPP, and the Cray X-MP/48. Results of their cross-architectural study indicate that the algorithm has a high degree of parallelism. The objective of the work reported here was to extend that study to two new parallel machines recently acquired by the Scalable Computing Laboratory (SCL) at the Ames Laboratory: the nCUBE 2, a distributed memory hypercube MIMD (Multiple Instruction Multiple Data) machine; and the MasPar MP-1, a distributed memory mesh SIMD (Single Instruction Multiple Data) machine. These computers are related to the distributed memory systems above (i.e., the iPSC and the MPP, respectively), but, most notably, use newer technology. The SCL currently has a 64-node nCUBE and a 8192-node MasPar; machines of varying sizes exist at sites outside of the SCL.

In addition to achieving significant performance gains on the new machines compared to their architectural counterparts, we found that the Sammur and Hagan study underestimated the scalability of the algorithm.

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That is, although the earlier study concluded that the parallelism of the Burg algorithm can only be exploited if a fine-grained, massively parallel machine such as the MPP is used, the algorithm *can* map efficiently to small-scale as well as large-scale computers, including both SIMD and MIMD distributed memory systems. It is this quality that we informally refer to as *scalable*. Scalability is one consideration in parallel system design (among programmability, flexibility, cost-effectiveness, speed, etc.). There have been several attempts at defining "scalability" in a formal manner, e.g. [11]. For purposes of this paper, we require a relative definition: an algorithm-machine combination is *more scalable* than another if and only if it has less parallel overhead for all possible numbers of processors greater than one. We believe this property is possessed by all extant definitions of scalability. "Parallel overhead" can arise from load imbalance, communication costs, redundant work, etc., and can be attributed to the machine or the algorithm design. The concept of *granularity* also commonly arises in this context, and formal definitions exist, e.g. [2, 6]. Informally, a computer is said to be "fine grained" if its communication is fast relative to its computation speed. An algorithm is "fine grained" if it *requires* a large amount of communication relative to the computation. This potentially gives rise to opposite design goals: whereas we wish to design computers as fine grained as possible to increase their flexibility and generality, we wish to create algorithms as coarse grained as possible to attain good performance on a broad range of architectures.

The *structure* of the Burg algorithm, being representative of a class of algorithms, has already been mentioned (as a reason it was selected for the Sammur and Hagan study). Indeed, the structure of a machine, an architecture, a program, or a problem is closely tied to its scalability and hence its breadth of effectiveness. In the context of the Burg algorithm, this paper brings to light the performance impact of using more scalable machines and algorithms. Of special import is the use of appropriate performance metrics and performance visualization to characterize the parallelism of the algorithm and lend insight toward understanding and evaluating its performance.

Section 2 summarizes the Sammur and Hagan study and the relevances to this study. Section 3 presents our parallel implementations, giving both SIMD and MIMD updates and the respective performance analyses. Section 4 concludes with a discussion of the results.

2. REVISITING THE EARLIER STUDY

This section summarizes the Sammur and Hagan study, in particular it presents the serial algorithm, performance results from the parallel implementations, and points of contention that will be addressed in the next section.

2.1. Serial Burg Algorithm

The objective of the Burg algorithm is to estimate reflection coefficients, and from these, autoregressive coefficients, so as to best fit a time series data sequence. Full derivation of the algorithm is detailed by Burg in [1]. Translation of this mathematical formulation into sequential code, as well as the sequential to parallel conversions for the four machines, is described by Sammur and Hagan in [14]. (See also [12] for the sequential program.) To maintain consistency with the Sammur and Hagan study, we obtained the original Burg programs from the researchers, including sequential versions in Fortran and Pascal and parallel versions in MPP Pascal and iPSC Fortran.

A standard sequential implementation of the Burg algorithm is shown in Fig. 1 [14]. The algorithm recursively estimates the parameters for an n -th order system from the known parameters of an $(n-1)$ -th order system. The parameters at each stage of the recursion are estimated so as to minimize the arithmetic mean of the

forward and backward prediction errors with respect to $a[n]$, the n -th coefficient of the autoregressive model. Specifically, it calculates a specified number of reflection coefficients (max) given a time series data set (x) having m data points. From the time series, a set of forward (e) and backward (b) prediction errors are determined. The e 's and b 's are recursively updated so the reflection coefficients (c) can be calculated. If desired, the autoregressive coefficients, a , can also be calculated. The main step in this algorithm, identified as *Loop 1*, is a time shift/inner product operation. It involves a time shift of the b array relative to the e array and incorporates three inner product operations. The step identified as *Loop 2* is the updating of the forward and backward prediction errors. It involves vector scale/add operations and statement-level data dependencies. As will be shown in the next section, these two steps, as well as the calculation of c , are amenable to parallelization. *Loop 1* incurs communication, and the calculation of c becomes a synchronization barrier between *Loop 1* and *Loop 2*.

```

FOR i=1 TO m DO
  e[i]=x[i]
  b[i]=x[i]
FOR n=1 TO max DO
  s1=0.0
  s2=0.0
  • Loop 1
  FOR i=n+1 TO m DO
    s1=s1+e[i]*b[i-n]
    s2=s2+e[i]^2+b[i-n]^2
  c[n]=-2.0*s1/s2
  IF n>1 THEN
    FOR i=1 TO n-1 DO
      a1[i]=a[i]+c[n]*a[n-i]
    FOR i=1 TO n-1 DO
      a[i]=a1[i]
    a[n]=c[n]
  • Loop 2
  FOR i=n+1 TO m DO
    temp=e[i]+c[n]*b[i-n]
    b[i-n]=b[i-n]+c[n]*e[i]
    e[i]=temp

```

FIG. 1. Sequential Burg algorithm

The operation count for this algorithm in terms of problem size parameters max and m , excluding the calculation of the autoregressive coefficients, is: $(10 \times m \times max) - (5 \times max^2)$. m is typically much larger than max . While exact values depend on the application, in practice it is common for max to take values between 10 and 50, and for m , between 1000 and 10000 [12]; larger m are practical for some applications [14]. Of these operations, *Loop 1* accounts for 1.5 times as many operations as *Loop 2*. Work, measured as the number of floating point operations performed, varies linearly with problem size; storage also varies linearly.

2.2. Summary of Performance Results

The Burg algorithm was implemented on four different parallel computers in the earlier study: the Intel iPSC/2, the Denelcor HEP, the NASA/Goodyear MPP, and the Cray X-MP/48. Table I compares execution

times observed for the parallel versions on the four computers (as reported in [14]) and, as a point of reference, for a serial version on a Sun 4 workstation. Also included in the list are the two parallel machines in this study: the MasPar MP-1 and the nCUBE 2. Each will be discussed in more detail in Section 3. The input problem parameters for this comparison were $max = 10$ and $m = 16384$. All machines used single precision (32-bit) floating point arithmetic, which was sufficient for this problem, except the Cray (always double precision).

TABLE I
Burg Execution Times

Machine	Execution Time (sec.)
Sun 4	2.85
HEP	1.68
iPSC/2	0.24
MPP	0.0552
nCUBE 2	0.0298
Cray X-MP/48	0.0169
MasPar MP-1	0.013

Of the four parallel computers in the earlier study, the fastest execution time was delivered by the Cray. Seeing such results was sufficient motivation for us to investigate the performance of the Burg algorithm on our new parallel computers. A result of our efforts is a MasPar execution time that surpasses the mark set by the Cray and that is more than four times faster than its earlier distributed memory counterpart, the MPP. The nCUBE execution time is also impressive; being more than eight times faster than its counterpart, the iPSC, it establishes itself as a competitive alternative. The reasons for such notable performance gains are twofold: technology and technique. Relevant changes in machine technology and algorithmic technique are noted in the discussion of each implementation in Section 3.

While revisiting the Sammur and Hagan study and reevaluating Burg algorithm performance, we identified the following assessments (in [14]) which no longer accurately characterize the performance of the algorithm on the distributed memory architectures: (1) The parallelism of the Burg algorithm can be fully exploited only if the number of processors is equal to the number of data points which is only feasible if a massively parallel machine like the MPP is used; (2) The calculation of the reflection coefficient at each iteration is a serial bottleneck that limits the algorithm performance (iPSC/2); and (3) The communication overhead incurred from the four-byte message between nodes limits linear speedup to small numbers of nodes and large numbers of data points (iPSC/2). These assessments hold in the context of the earlier study, but that is because they are a direct consequence of the machines and methods being used. For (1), the cross-architectural comparison yielded poor performance on the coarser grained parallel machines because these machines are less scalable than the MPP for executing the Burg program. For (2), the calculation of the reflection coefficient requires a global sum operation for which a costly serial algorithm was used (an efficient parallel algorithm is now well known for hypercubes). And for (3), single-word data transfers can occur in both linear shift and global sum operations, and the transfer time is dominated by the iPSC/2's large message startup overhead. Although such conclusions are indeed informative, care should be taken not to apply them more generally as an evaluation of the mapping of the algorithm onto an *architecture*. What is being evaluated is the implementation of the algorithm on a *machine*. Even though our ultimate objective is to correlate performance with *classes* of architectures and algorithms, it would be a mistake to exclusively let the performance of one machine in an architectural class stand as an indicator of performance for the whole class, thus possibly biasing the use of other machines. This is exemplified in the performance update presented in Section 3. The perfor-

mance update focuses not on the question, "How fast does the program run?", but on the more elucidating question, "What are the dynamics of program execution?"

3. PERFORMANCE UPDATE

The following sections describe our study, in particular the parallel Burg algorithm, its implementation on the MasPar MP-1 and nCUBE 2 computers, and the observed performance on these two machines.

3.1. General Parallel Implementation

A generalized parallel version of the Burg algorithm is shown in Fig. 2. The technique of domain decomposition or data parallelism (that is, distributing the data across the processor ensemble) is used. We incorporated the mappings specified in the Sammur and Hagan study. That is, a scattered decomposition [2], as was used for the MPP, is used for the MasPar implementation. And a block decomposition [2], as was used for the iPSC, is used for the nCUBE implementation. The concept of a processor "tile" is introduced in this study to accommodate mappings with multiple data points allocated to each processor. A tile is a set of distributed datums, one per processor. Access to a tile can be done in parallel. Scattered decomposition on MasPar-like machines typically results in large tiles and a small number of datums per processor, that is, a small number of tiles. Conversely, block decomposition on nCUBE-like machines typically results in smaller tiles and a larger number of datums per processor (i.e., number of tiles or block size).

A natural choice is to distribute the one-dimensional e and b arrays so as to parallelize *Loop 1* and *Loop 2* (see Fig. 1). To account for the time shift and have the calculations involving e and b local to each processor, communication is required to align the respective elements of e and b . This is done via a linear shift operation. Observe that the number of communication steps required for the linear shift of e in *Loop 1* depends on the decomposition (constant for block decomposition and varying linearly with the number of tiles for scattered decomposition). Each linear shift is accompanied by a zero assignment to b so as to maintain algorithm uniformity and then effectively nullify the superfluous calculations. A second communication phase is required during the calculation of c to accumulate the global sums from the partial sums among all processors. The cost of such an operation depends on the machine, and typically involves a logarithmic to linear number of steps with respect to the machine size; it is independent of the problem size.

3.2. MasPar Implementation: A SIMD Update

The MasPar MP-1 is a distributed memory massively parallel SIMD computer [9]. It has a high-speed two-dimensional toroidal mesh topology with eight-way nearest neighbor connections (the Xnet) and a router handling non-neighbor communications among subsets of processors. The Data Parallel Unit (DPU) consists of an Array Control Unit (ACU) and a Processing Element (PE) Array. The ACU operates at 12.5 MHz. Each PE is a custom load/store 4-bit processor with registers, floating-point hardware, and 16 KBytes of memory. The machine in the SCL currently has 8192 PEs arranged in a 128×64 array (which is the actual interconnection network supported by the Xnet). Two languages are supported: MPL, an extension of C, with explicit *plural* data constructs and interprocessor communication primitives; and MPFortran, an implementation of Fortran90.

The execution time reported in Table I is for an MPL version of the Burg algorithm using single-precision floating-point arithmetic. Phases of the parallel algorithm coincide with those in Fig. 2. A one-dimensional array of size m is distributed across the PEs, and a linear PE array is simulated (via a snakelike method as was done in the MPP study). The linear shift operation is implemented by a combination of two Xnet operations: a *send* to the north by Column 0's PEs, and a *send* to the west by all PEs. These are done for each tile in turn,

FIG. 2. Parallel Burg algorithm

with a final shift between tiles on the last PE only. Each global sum, enroute to calculating a coefficient, is obtained from an intrinsic *ReduceAdd* function applied to the partial sums. Fig. 3 is a timing profile illustrating the phases of execution (*a* - *h*) and processor activity (grayscale) over time for selected PEs (here, a slice of sixteen PEs in Row 0 and the last PE). The slice of time spans the iteration for the first reflection coefficient for a run with $m = 8192$ data points (i.e., one "tile" or one data point per processor). Idle time occurs when processors are disabled (removed from the "active set"). For example, during phase *b* only Column 0 PEs are active, as shown. Note also the longer time interval for phase *b* versus phase *c*, despite both being Xnet operations. This is because phase *b* incurs the overhead of disabling PEs. The Xnet is a resource to be utilized, especially in light of the fact that doing a single-precision multiply is two to three times more expensive than sending the operands to an adjacent PE and receiving the product [2]. Registers should also be utilized because small clusters of PEs are configured to access the same read/write memory unit; significantly better execution times are observed when plural data are explicitly declared as *register* types.

The scalability of the algorithm is illustrated in Fig. 4, which shows four timing profiles (*cf.* Fig. 3) for PE 0 corresponding to runs with $m = 8192$, 16384, 32768, and 262144 data points, respectively. That is, runs having 1, 2, 4, and 32 tiles, respectively. (Though data sets as large as 32 tiles are not typical in current applications using the Burg algorithm, they are interesting from a performance perspective.) The execution times of phases *a*, *e*, and, most notably, *g* are independent of problem size; times for the other phases increase linearly with the number of tiles. Phase *g* time depends on machine size, requiring a number of communication steps

FIG. 3. Execution profile for selected PEs of the MasPar (1 tile)

equal to $\log_2$ the number of PEs. Observe that PE utilization is high in each case, even though the global sum operations in phase *g* are relatively expensive for smaller problem sizes. The performance gain over the MPP has come largely from advanced technological and architectural features of the MasPar. Only minimal improvement can be attributed to algorithmic changes; e.g., tiling was introduced, and this required an optimal ordering of plural control constructs to reduce the overhead of disabling PEs.

FIG. 4. Execution profile for PE 0 of the MasPar for increasing problem sizes

The execution profiles, generated via program instrumentation and performance visualization [8], provide detailed information about the dynamics of the program executing on the machine. Complementing the graph-

ical representations of performance are a variety of statistics and other metrics that help to quantify the observations. Hockney and Jesshope [6] define two useful parameters that provide a quantitative means to individually characterize and collectively compare the performance of parallel (and other) computers for a given program: (1) the maximum or asymptotic performance r_∞ — the maximum rate of execution in floating-point operations performed per second (commonly stated as millions of FLOPS, or MFLOPS); and (2) the half-performance length $n_{1/2}$ — the number of datums required to achieve half of the maximum performance. Both are experimentally measured quantities obtained by varying the number of datums (problem size, n) and timing program execution on a machine of a specific ensemble size. r_∞ reflects both the peak speed of the machine and the goodness of fit between program and machine. It is calculable from the ratio of expressions: *operation count* (n) / *time*(n), as n approaches infinity. The expression in the numerator is determined via program analysis; and, in the denominator, via curve fitting the measured data. $n_{1/2}$ can be interpreted as a measure of the amount of parallelism in the system, specifically, in the program-machine mapping which produced r_∞ . It can be argued that a computer only solves efficiently those problems having n greater than $n_{1/2}$ [6]. Thus, there is a tradeoff: the higher the value of $n_{1/2}$, the greater the parallelism, but the more limited is the range of problems that can be solved efficiently.

We can apply these parameters to characterize the performance of the Burg algorithm on the MasPar MP-1 (and the nCUBE 2, to be discussed in the next section). Execution speed (r) versus time is shown in Fig. 5 for an 8192-PE system; the dashed line above the MasPar curve denotes maximum performance, $r_\infty \cong 325$ MFLOPS. Efficiency, defined here as the ratio of actual speed to peak speed (r/r_∞), is shown in Fig. 6; the dashed line indicates 50% efficiency, or half of the maximum performance, which corresponds to $n_{1/2} \cong 30600$ data points. Observe that there is good “acceleration” in approaching maximum performance; that $n_{1/2}$ indicates a high degree of parallelism; and that reasonable efficiencies can be obtained for a range of values of n used for the program. Time is used as the dependent variable (instead of n) in Figs. 5 and 6 to represent work done by each machine. This is because a fixed-time model of performance measurement has provided “simple” and consistent explanations of performance phenomena we have observed.

FIG. 5. Execution speed on the MasPar and nCUBE as a function of execution time

FIG. 6. Execution efficiency on the MasPar and nCUBE as a function of execution time

3.3. nCUBE Implementation: A MIMD Update

The nCUBE 2 is a distributed memory message-passing MIMD computer [10]. It has a hypercube interconnection topology with cut-through routing to non-neighboring processor nodes. Each node is a custom 32-bit CISC (Complex Instruction Set Computer) processor operating at 20 MHz. with floating-point hardware, DMA-based bidirectional interprocessor communication channels, and 1 MByte of memory. Standard operating system message-passing procedures incur a 60 μ sec. delay per message transmission due to software overhead. Communication channel bandwidth is 2.2 MBytes/sec. Single-word, near-neighbor messages typically require 70-80 μ sec. for total transmit time (with no contention). This is a factor of five to ten faster than earlier machines such as the nCUBE 1 and the iPSC/2. Indeed, the lower message latency is one reason that the nCUBE 2 gives a more scalable solution to the Burg algorithm than the iPSC/2 (at 2.8 MBytes/sec. bandwidth and $\approx$ 500 μ sec. overhead). Though not used yet in this study, new operating system message-passing procedures are now becoming available on the nCUBE that obviate some of the overhead in well-structured cases (for which the Burg algorithm would qualify). The machine in the SCL currently has 64 nodes (a six-dimensional hypercube). Two languages are supported, C and Fortran, each with similar features.

The execution time reported in Table I is for a C version of the Burg algorithm using single-precision floating-point arithmetic and assembler routines from a vector library [4]. Phases of the algorithm coincide with those in Fig. 2. The linear shift operation involves local and global operations: a local shift on the block of array elements "owned" by a processor; and one global linear shift to move elements from processor i to processor $i-1$ (from the start of a block to the end of the preceding block), where the i 's correspond to a one-dimensional ordering of the processors. These operations can be partially overlapped using the sequence: *write* to "left" neighbor, perform local shift, and *read* from "right" neighbor. This allows the computation to effectively absorb any excess communication delay. Each global sum operation is implemented with an efficient algorithm that utilizes the hypercube interconnect to its full potential. Pairs of nodes exchange partial sums across each hypercube dimension in turn, requiring a number of communication steps equal to the dimension of the hypercube ($\log_2$ of the number of processors). This logarithmic communication cost is in

stark contrast to the linear cost incurred by the uploading-downloading technique (between Node 0 and all other nodes) used for the iPSC/2 in the earlier study [14]. The global sum is parallel, not serial, in nature and is scalable. These algorithmic changes, primarily to minimize communication time, are the main reason for the greater scalability of the nCUBE solution to the Burg algorithm.

Performance visualization for the nCUBE was done using a combination of tools developed by researchers at Oak Ridge National Laboratory: PICL (Portable Instrumented Communication Library) [3] and ParaGraph [5]. Both are available through *netlib*. PICL produces an execution trace during an actual run of an instrumented program, and ParaGraph replays this trace using multiple views. Though the current version is limited to displaying a small number of processors, a new version of ParaGraph to be released will accommodate large numbers of processors; scalable views are being developed at the SCL for integration into tools like ParaGraph [13]. One view selected to illustrate the execution of the Burg algorithm is the *Feynman Diagram*. The Feynman Diagram is patterned after the diagrams that are used in physics to depict the interactions between particles (see Figs. 7 and 8). Processor activity is indicated by horizontal lines, one for each processor, while messages between processors are depicted by slanted lines between the sending and receiving processor activity lines, indicating the times at which each message was sent and received. A blank horizontal line indicates that a processor is idle waiting to receive a message. Fig. 7 shows a Feynman Diagram for the execution of the Burg algorithm on a 16-node subcube, spanning the iteration for the first reflection coefficient for a run with a block size of 16. Fig. 8 is similar, except with a block size of 256. In each view, a Gray-code ordering of the nodes was selected for the vertical axis. The first set of messages comprise the linear shift operation. The next four sets of messages constitute the pair-wise exchanges across consecutive dimensions of the hypercube. Observe the uniformity and synchronicity of the communication patterns; observe that no processors ever wait to receive a message. As seen for the MasPar, smaller problem sizes tend to be relatively expensive. These visual depictions of system activity reveal patterns and anomalies from which performance characteristics become perceptible to the user. Critical paths, for example, would suggest a need to redesign the algorithm (which we did).

FIG. 7. Execution profile for a 16-node subcube of the nCUBE (block size = 16)

FIG. 8. Execution profile for a 16-node subcube of the nCUBE (block size = 256)

As was done in the preceding section for the MasPar, we can apply the $(r_{\infty}, n_{1/2})$ parameters to characterize quantitatively the performance of the Burg algorithm on the nCUBE 2. Execution speed (r) versus time is shown in Fig. 5 for a 64-node system; the dashed line above the nCUBE curve denotes maximum performance, $r_{\infty} \cong 110$ MFLOPS. Efficiency (r/r_{∞}) is shown in Fig. 6 (also for 64 nodes); the dashed line indicates 50% efficiency, or half of the maximum performance, which corresponds to $n_{1/2} \cong 15900$ data points. By comparing the MasPar and nCUBE performance curves, we see that, although the MasPar has a higher maximum performance than the nCUBE, they have similar efficiencies. The factor of two difference in $n_{1/2}$ correctly reveals that the MasPar is a more highly parallel machine, but it also suggests that the nCUBE's performance characteristics for the Burg algorithm are not too unlike the MasPar's. Differences in maximum performance can be traced in part to the rated peak speed of the machines: 556 single-precision MFLOPS for the MasPar, and 171 for the nCUBE.

A final comparison between the two machines is shown in Fig. 9. A timing profile (*cf.* Fig. 3) for processor 0 on each machine is displayed for the iteration of the first reflection coefficient and for problem sizes that yield about equal execution time ($m = 4096$ data points, or 128 per each of 32 nodes, on the nCUBE; and $m = 8192$, or 1 per each of 8192 PEs, on the MasPar). Observe the analogous behavior on the two machines despite the contrasting machine granularities.

4. DISCUSSION

In the process of reevaluating Burg algorithm performance, we found that the algorithm maps efficiently to small-scale as well as large-scale computers, including both SIMD and MIMD distributed memory systems. Our study was done in the context of a massively-parallel SIMD computer, the 8192-PE MasPar, and a modestly-parallel MIMD computer, the 64-node nCUBE. Although the performance interplay between machine

FIG. 9. Comparison of execution profiles for processor 0 on the nCUBE and MasPar

ensemble size and problem size results in lower efficiencies for smaller problem sizes on both the MasPar and the nCUBE, the algorithm does exhibit a high degree of parallelism and scalability on both computers.

Regarding the assessments stated in Section 2, we make the following statements. (1) The parallelism of the Burg algorithm was exploited on both the MasPar and the nCUBE, and in each case, optimal performance was observed when the number of data points exceeded the number of processors. (2) The calculation of the reflection coefficient, in particular the global sum operation, is a parallel operation on both architectures, involving highly uniform and efficient communication. (3) Communication overhead on hypercube computers has been (and continues to be) reduced so that short messages are not as costly as they once were. For the algorithm on the nCUBE, communication is structured, both as a natural consequence of the algorithm and by explicit programming, so that little (if any) time is spent waiting for messages. As future machines impose shorter message transmission times, the result will be finer grained machines, making the algorithm-machine combination more scalable. Thus, efficiencies, especially for smaller problem sizes, will improve.

In conclusion, parallel supercomputers having a distributed memory architecture are suitable for signal processing algorithms similar to the Burg algorithm and are competitive alternatives to the more conventional vector supercomputers. Scalability remains a hard to define parallel processing concept, one that has and will result in changing perceptions of performance as it becomes better understood and applied. Finally, the utility of appropriate performance metrics and performance instrumentation and visualization to interpret the complex operation of large-scale systems cannot be overstated.

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