Aerojet Nuclear Company

550 SECOND STREET
IDAHO FALLS, IDAHO 83401

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Mr. R. E. Swanson, Director
Reactor Safety Operations Division
Idaho Operations Office - ERDA
Idaho Falls, Idaho 83401

MASTER

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LOFT Technical Report 141-16, "Determination of Fluid-Induced Loads from Strain Gages Theory, Equations and Error Analysis," is forwarded herewith for your information.

This LTR presents a discussion of the strain gage installation configurations on the LOFT piping to determine dynamic subcooled blowdown loads during loss-of-coolant experiments.

L. J. Ybarrondo
L. J. Ybarrondo, Manager
LOFT Program

GH:jlk

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LOFT TECHNICAL REPORT
LOFT PROGRAM DIVISION

AEROJET NUCLEAR COMPANY

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ABSTRACT

Strain gages are to be used on the LOFT piping to determine load resultants during the subcooled portion of blowdown in Loss-of-Coolant Experiments. This report discusses the configuration of strain gages to be installed. The selected configuration is shown to:

- (a) be sufficient to determine the desired load resultants
- (b) provide for measurement redundancy
- (c) provide an evaluation of the applicability of the analytical assumptions made in obtaining load resultants from the measured strains.

Data reduction equations are derived and an error analysis is performed. Data obtained from an analytical model of a Loss-of-Coolant Experiment is analyzed, showing that the error bounds on the load resultants are less than 12% of the load, provided the analytical assumptions are not violated.

SUMMARY

This report presents a discussion of the strain gage configurations to be installed on the LOFT piping in order to determine dynamic subcooled blowdown loads. Two configurations are discussed, the redundant 10-gage configuration discussed in Section 2.3 and the 12-gage configuration discussed in Section 2.4. Both of these configurations are shown to provide measurement redundancy. The data from the 12-gage configuration will permit an evaluation of the applicability of the underlying analytical assumptions which are made in deriving equations for obtaining the load resultants from the measured strains. In order to standardize the data reduction process and to minimize the computer programming effort, two sets of data reduction equations are derived and presented in Sections 2.2 and 2.3. With these two sets of equations, the data can be reduced in a routine fashion even though any one strain gage might fail.

Secondly, the equations and procedures for statistically testing the underlying analytical assumptions are derived and presented in Section 2.4. To use these, it is necessary to record data from all twelve strain gages at each location. Such an evaluation need be done only once for each location.

Finally, an error analysis is performed encompassing all the errors which impinge on the calculation of the load resultants at each location. It is found that a reasonable estimate of the upper bound to the error is $\pm 12\%$ of the computed value provided the pipe thickness at each location is measured by ultrasonic methods prior to installation of the gages.

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1.0 INTRODUCTION

One of the most serious accidents that could occur to a typical commercial pressurized water reactor would be to have a rupture in one or more of the main coolant pipes. Immediately following such a rupture, the coolant would depressurize from its subcooled, liquid state to a state of a state of saturated steam and water, and the ability of the coolant to remove heat from the reactor core would be greatly diminished. Inasmuch as a loss of coolant accident (LOCA) has never occurred in a commercial pressurized water reactor, no data exists on the behavior of a large-scale reactor during such an accident.

To obtain such data, the U. S. Energy Research and Development Administration and Nuclear Regulatory Commission are sponsoring the construction of an experimental reactor called the Loss of Fluid Test (LOFT) facility. This reactor is a scaled version of a typical commercial pressurized water reactor, and will incorporate quick-opening valves in a blowdown loop to allow for a planned rapid depressurization.

During such a depressurization, pressure waves propagate throughout the coolant piping, giving rise to hydraulically-induced forces on the piping, its supporting structure, and within the reactor vessel itself.

One of the objectives of the tests planned for the LOFT facility is to measure the dynamic response of the piping and support structure to these hydraulically-induced forces and to compare the response to that predicted analytically by the methods currently being utilized in reactor safety analyses. Measurements of both the mechanical motion and of the internal forces and moments in the pipes are to be made. The accelerations will be measured

with commercially available transducers, and are routinely-made types of mechanical measurements. The internal load resultants (the forces and moments in the pipes) cannot, however, be measured directly. Instead, it is necessary to measure strains on the outside surface of the pipes, and from these strain values, infer the load resultants which act inside the pipes.

This report describes the strains which must be measured, the number and configuration of the strain gages which can be utilized in measuring the required strains, and derives the equations which are used to relate the strain values to the load resultants. Besides permitting the calculation of the desired load resultants, the configuration of strain gages to be described will also:

- i) Provide for measurement redundancy, so that if any one strain gage at a location should fail to function, computation of the load resultants will still be possible.
- ii) Permit a qualitative evaluation of the applicability of the analytical assumptions of technical beam theory which are implicitly made in the computation of the load resultants.

In general, there are six unknown load resultants at each cross-section of the pipe, three moment components and three force components as shown in Figure 1. The force and moment components are related to the stresses acting at the cross-section by

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1. Force and Moment Resultants at a Cross-Section.

$$M_x = \iint_A y \sigma_{zz} dA$$

$$M_y = \iint_A x \sigma_{zz} dA$$

$$M_z = \iint_A r \sigma_{z\theta} dA$$

$$F_x = \iint_A \sigma_{zx} dA$$

$$F_y = \iint_A \sigma_{zy} dA$$

$$F_z = \iint_A \sigma_{zz} dA$$

where A is the area of the pipe. The objective here is to make a number of strain measurements on the outside of the pipe from which the values of the resultant moments and forces can be determined. This can only be done if the distribution of stresses due to the applied loads is known. For a long, prismatic body, the stress distributions due to applied moments M_x , M_y , and M_z and due to an axial force F_z are known with a high degree of accuracy. So, provided that the cross-section at which the resultants are to be determined is sufficiently remote from any discontinuities, the load resultants F_z and M_x , M_y , and M_z can be confidently determined from strain measurements. By contrast, the distribution of stresses due to shear loads depends both on the shape of the cross-section and on the manner in which the shear loads are applied, and hence is not known precisely. Thus the magnitudes of the shear force resultants F_x and F_y cannot usually be determined with a high degree of confidence.

Hence, in general, one seeks to find the three moment components and the axial force component at each cross-section of interest. When a pressurized pipe is being considered, these four unknowns are augmented by the pressure, and then there are five quantities to be determined. It will be shown that these five quantities can be determined if six strains are measured on the surface of the pipe, and that no fewer than six will suffice. This configuration is discussed in Section 2.3. However, should any one of these six gages malfunction, not all of the five unknown load resultants could be determined.

To provide for measurement redundancy, four additional strain gages can be added to each section of piping. With these ten strain gages, the failure of any one gage does not preclude the determination of all five load resultants at that section. This configuration of ten gages is discussed in Section 2.4.

Finally, if it is desired to obtain a quantitative check on the analytical assumptions utilized in converting measured strains to load resultants, then, as will be shown, a total of twelve strain gages are required at each location. The assumptions in question are those upon which "technical beam theory" is based, these being:

1. The pipe is assumed to remain circular and plane sections are assumed to remain plane during deformation. These conditions will be violated if the cross-section in question is too close to a geometric discontinuity, such as a junction with the reactor vessel or an abrupt change in pipe diameter.

2. The stresses in the pipe remain below the yield stress limit.
3. Shear strains due to the torque M_z do not vary around the circumference of the pipe.

A check on the first assumption can be obtained by evaluating the hoop strain and axial strain at four separate locations around the pipe location. The hoop stress due to the internal pressure should not vary around the circumference, while the "strain surface" associated with the axial strains should be planar. The second assumption is checked by comparing the computed stresses with the appropriate yield stress. The third assumption is checked by comparing the values of the shear strain (measured at different locations on the circumference of the pipe) after removing any shear strain due to the transverse shear forces.

2.0 DATA REDUCTION EQUATIONS

2.1 Strain Distribution at Each Section

Let (x, y, z) be a set of coordinates with the origin at the centroid of the cross section, Figure 2. The z axis is the prismatic axis of the pipe, and (x, y) are coordinates in the plane of the cross section. Introduce the angle θ in the x, y plane, measured in the positive sense from the x axis. Let I_{xx} , I_{yy} and $J = I_{xx} + I_{yy}$ denote the second area moments of inertia of the cross section. For a pipe, $I_{xx} = I_{yy} = \frac{\pi}{4}(b^4 - a^4)$, where b, a are the outside and inside radii of the pipe, respectively.

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2. Local Coordinates at a Cross-Section.

The moments which act about the three axes are M_x , M_y and M_z .

The force resultant in the axial (z) direction is denoted F_z .

The internal pressure which acts at that axial location is denoted as P. According to beam theory, the stresses on the pipe surface due to the moment components are

$$\sigma_{zz} = \frac{M_x}{I_{xx}} b \sin \theta - \frac{M_y}{I_{yy}} b \cos \theta$$

$$\sigma_{\theta\theta} = 0$$

(1)

$$\sigma_{z\theta} = \frac{M_z b}{J}$$

The only stress resulting from the axial load F_z is $\sigma_{zz} = F_z/A$ where A is the area of the pipe cross section. The axial stress due to pressure will be included in F_z . The hoop stress at the outside surface of a thick-walled cylinder due to internal pressure is given by

$$\sigma_{\theta\theta} = \frac{a^2(a^2 + b^2)P}{b^2(b^2 - a^2)} \quad (2)$$

As stated earlier, the distribution of stresses corresponding to the transverse shear force is not known precisely. From symmetry conditions, however, it is known that the shear stress distribution is symmetric with respect to the line of action of the shear force. For example, consider Figure 3 where the shear resultant F_y acts. The stress distribution which is equivalent to F_y must be symmetric with respect to the y axis. Furthermore, at the points

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3. Stress Distribution Due to Transverse Shear Force.

$y = \pm b$, $x = 0$, the shear stress $\sigma_{z\theta}$ must vanish. Thus due to the action of the transverse shear forces, the form of the shear stress distribution is taken as

$$\begin{aligned} \sigma_{z\theta} = & F_y [A_1 \cos \theta + B_1 \cos 3\theta] \\ & - F_x [A_1 \sin \theta + B_1 \sin 3\theta] \end{aligned} \quad (3)$$

It is shown in the theory of elasticity (Reference 1) that this form is exact for certain loading conditions. In any case, this form does reflect the symmetry conditions which must always hold. For the purpose at hand, the constants A and B need not be explicitly identified. By contrast, the shear stresses $\sigma_{\theta z}$ due to the twisting moment M_z are constant around the circumference of the pipe, as shown in Figure 4. This provides the means of eliminating the effects of the transverse shear forces. Since the shear strain $\epsilon_{z\theta}$ is directly proportional to the shear stress $\sigma_{z\theta}$, we can measure the shear strain at two points diametrically opposite one another, and sum them. By symmetry, the shear strain due to transverse shear forces will cancel, and the result will equal twice the shear strain due to the twisting moment M_z .

Combining the stress distributions due to the individual load resultants given by the equations above, the state of stress on the outside surface of the pipe is given by

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4. Stress Distribution Due to Twisting Moment M_z .

$$\left\{ \begin{array}{l} \sigma_{zz} = \frac{M_x b}{I_{xx}} \sin \theta - \frac{M_y b}{I_{yy}} \cos \theta + \frac{F_z}{A} \\ \sigma_{\theta\theta} = \frac{a^2(b^2 + a^2)p}{b^2(b^2 - a^2)} \\ \sigma_{z\theta} = \frac{M_z b}{J} + F_y [A_1 \cos \theta + B_1 \cos 3\theta] \\ \quad - F_x [A_1 \sin \theta + B_1 \sin 3\theta] \end{array} \right. \quad (4)$$

All other stress components are zero since the surface of the pipe is assumed to be traction free.

Since $\sigma_{rr} = 0$ on the outside surface, the strains are related to the stresses via Hooke's Law in the form

$$\begin{aligned} \epsilon_{zz} &= \frac{1}{E} [\sigma_{zz} - \nu \sigma_{\theta\theta}] \\ \epsilon_{\theta\theta} &= \frac{1}{E} [\sigma_{\theta\theta} - \nu \sigma_{zz}] \\ \epsilon_{z\theta} &= \frac{1+\nu}{E} \sigma_{z\theta} \end{aligned} \quad (5)$$

which apply at each point on the outside surface. These can be inverted to give

$$\begin{aligned} \sigma_{zz} &= \frac{E}{1-\nu^2} [\epsilon_{zz} + \nu \epsilon_{\theta\theta}] \\ \sigma_{\theta\theta} &= \frac{E}{1-\nu^2} [\epsilon_{\theta\theta} + \nu \epsilon_{zz}] \\ \sigma_{z\theta} &= \frac{E}{1+\nu} \epsilon_{z\theta} \end{aligned} \quad (6)$$

Note that thermal strains are not considered. It is assumed that the strain gages will be "zeroed" just prior to the onset of the dynamic loads, so that thermal strains will not be recorded.

A typical 45° strain gage rosette is shown in Figure 5. At any point on which a rosette is located, the pressure P and M_z can be found immediately by eliminating $\sigma_{\theta\theta}$ from (4) and (6) to get

$$\frac{P}{E} \left[\frac{a^2(b^2+a^2)}{b^2(b^2-a^2)} \right] = \frac{1}{1-\nu^2} [\epsilon_{\theta\theta} + \nu\epsilon_{zz}] \quad (7)$$

and thus if $\epsilon_{\theta\theta}$ and ϵ_{zz} are measured at a point, P can be computed.

As mentioned previously, to relate M_z to the measured strains, it will be necessary to measure two values of $\epsilon_{z\theta}$ on opposite sides of the pipe. From the last of Equations (4) and (6) we can write

$$\begin{aligned} \frac{E}{1+\nu} \epsilon_{z\theta} &= \frac{M_z b}{J} + F_y [A_1 \cos \theta + B_1 \cos 3\theta] \\ &\quad - F_x [A_1 \sin \theta + B_1 \sin 3\theta] \end{aligned}$$

Without loss of generality, we will suppose that there are two rosettes on the pipe located at $\theta = 0^\circ$ and $\theta = 180^\circ$ as shown in Figure 6. [Note that the relative orientation of the two ϵ_{45} strain gages must be as shown in this figure.] Let the strains measured at $\theta=0^\circ$ be denoted with a superscript 1, while those measured at $\theta = 180^\circ$ have the superscript 3. Then from the above equation, one can write

$$\frac{E}{1+\nu} \epsilon_{z\theta}^{(1)} = \frac{M_z b}{J} + F_y [A_1 + B_1]$$

$$\frac{E}{1+\nu} \epsilon_{z\theta}^{(3)} = \frac{M_z b}{J} - F_y [A_1 + B_1]$$

5. Typical 45° Strain Gage Rosette.

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6. Diametrically Opposed Pair of Rosettes

and hence by addition one has

$$\frac{M_z b}{J} = \frac{E}{2(1+\nu)} [\epsilon_{z\theta}^{(1)} + \epsilon_{z\theta}^{(3)}] \quad (8)$$

Now one cannot measure $\epsilon_{z\theta}$ directly (since it is the relative angular change between two lines at right angles before deformation and a strain gage can only measure a change in length), but it can be computed if ϵ_{45} is measured. Using the equations for transformation of strain, it can be shown that

$$\epsilon_{45} = \epsilon_{z\theta} + \frac{1}{2}[\epsilon_{zz} + \epsilon_{\theta\theta}]$$

and hence (8) becomes

$$\frac{M_z b}{J G} = \left\{ \epsilon_{45}^{(1)} - \frac{1}{2}[\epsilon_{zz}^{(1)} + \epsilon_{\theta\theta}^{(1)}] + \epsilon_{45}^{(3)} - \frac{1}{2}[\epsilon_{zz}^{(3)} + \epsilon_{\theta\theta}^{(3)}] \right\} \quad (9)$$

where the shear modulus $G \triangleq \frac{E}{2(1+\nu)}$ has been introduced. Thus it is seen that by having two rosettes on opposite sides of the pipe, the twisting moment M_z can be computed.

It remains to determine the resultants M_x , M_y and F_z . These can be determined by measuring the axial strains at different points around the pipe. By eliminating σ_{zz} from the first of Equations (4) and (6) there results

$$\frac{M_x b}{I_{xx}} \sin\theta - \frac{M_y b}{I_{yy}} \cos\theta + \frac{F_z}{A} = \frac{E}{1-\nu^2} [\epsilon_{zz} + \nu\epsilon_{\theta\theta}] \quad (10)$$

Thus at least three axial strains will have to be measured to determine M_x , M_y , and F_z . The equations which apply to these axial strain gages are obtained as follows. The expressions for σ_{zz} and $\sigma_{\theta\theta}$ from Equation (4) are substituted in the first of Equation (5), giving

$$\epsilon_{zz} = \frac{M_x b}{EI_{xx}} \sin\theta - \frac{M_y b}{EI_{yy}} \cos\theta + \frac{F_z}{EA} - \nu \frac{P}{E} \left[\frac{a^2 (b^2 + a^2)}{b^2 (b^2 - a^2)} \right] \quad (11)$$

Then by measuring three independent values of ϵ_{zz} , we get three independent algebraic equations which may be solved for the remaining load resultants since P is known from Equation (7).

2.2 The Four-Rosette Piping Configuration

Consider four 45° rosettes installed at 90° intervals around the pipe as shown in Figure 7. Note that the superscripts denote the rosette location, and that the rosettes are numbered sequentially starting at $\theta = 0^\circ$. In the following, equations for determining the load resultants are presented for three situations:

- (a) Minimum Six-Gage Hookup, in which data from only six of the twelve strain gages are recorded.
- (b) Redundant Ten-Gage Hookup, in which data from ten of the twelve strain gages are recorded.
- (c) Complete Twelve-Gage Hookup, in which data from all twelve gages is recorded, and a check on the validity of the underlying analytical assumptions is obtained.

7. Four Rosette Configuration

2.3 Minimum Six -Gage Hookup

Consider six strain gages in the configuration shown in Figure

8. This configuration will be denoted Case A. With no loss in generality, the rosette is taken at location 1, with $\theta = 0^\circ$.

From Equation (7), one obtains

$$\frac{P}{E} \left[\frac{a^2(b^2+a^2)}{b^2(b^2-a^2)} \right] = \frac{1}{1-\nu} [\epsilon_{\theta\theta}^{(1)} + \nu \epsilon_{zz}^{(1)}]$$

Applying equation (11) to each axial gage yields the set of equations below:

$$\begin{bmatrix} \epsilon_{zz}^{(4)} \\ \epsilon_{zz}^{(2)} \\ \epsilon_{zz}^{(1)} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 & -\nu \\ 1 & 0 & 1 & -\nu \\ 0 & -1 & 1 & -\nu \end{bmatrix} \begin{bmatrix} \frac{M_x b}{EI_{xx}} \\ \frac{M_y b}{EI_{yy}} \\ \frac{F_z}{EA} \\ \frac{P}{E} \left[\frac{a^2(b^2+a^2)}{b^2(b^2-a^2)} \right] \end{bmatrix}$$

In these equations, P is already known from equation (7). From the first two equations above,

$$\frac{M_x b}{EI_{xx}} = \frac{1}{2} [\epsilon_{zz}^{(2)} - \epsilon_{zz}^{(4)}].$$

From the first and third equations,

$$\epsilon_{zz}^{(4)} - \epsilon_{zz}^{(1)} = \frac{M_x b}{EI_{xx}} + \frac{M_y b}{EI_{yy}}$$

or

$$\frac{M_y b}{EI_{yy}} = \frac{1}{2} [\epsilon_{zz}^{(2)} + \epsilon_{zz}^{(4)}] - \epsilon_{zz}^{(1)}.$$

Finally, adding the first two equations and using the expression for the pressure already obtained gives

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8. Minimum Six-Gage Configuration (Case A)

$$\frac{F_z}{EA} = \frac{1}{2} [\epsilon_{zz}^{(4)} + \epsilon_{zz}^{(2)}] + \frac{\nu}{1-\nu} [\epsilon_{\theta\theta}^{(1)} + \nu \epsilon_{zz}^{(1)}].$$

It remains to determine the twisting moment M_z by the use of Equation (9). Now in Equation (9), it was assumed that there was a complete rosette on both sides of the pipe. In this configuration, however, there is a rosette on one side, but only an ϵ_{45} gage on the opposite side. However, expressions can be obtained for the missing $\epsilon_{zz}^{(3)}$ and $\epsilon_{\theta\theta}^{(3)}$ values in terms of the other five measured strain values. In fact, using Equation (4) it is seen that

$$\frac{1}{2} [\epsilon_{zz}^{(1)} + \epsilon_{\theta\theta}^{(1)} + \epsilon_{zz}^{(3)} + \epsilon_{\theta\theta}^{(3)}] = (1-\nu) \frac{F_z}{AE} + \frac{a^2(b^2 + a^2)P}{b^2(b^2 - a^2)}$$

Using the equations for F_z and P in terms of the measured strains in the right hand side of the above yields

$$\begin{aligned} \frac{1}{2} [\epsilon_{zz}^{(1)} + \epsilon_{\theta\theta}^{(1)} + \epsilon_{zz}^{(3)} + \epsilon_{\theta\theta}^{(3)}] &= [\epsilon_{\theta\theta}^{(1)} + \nu \epsilon_{zz}^{(1)}] \\ &+ \frac{1-\nu}{2} [\epsilon_{zz}^{(2)} + \epsilon_{zz}^{(4)}] \end{aligned}$$

and when this expression is substituted in Equation (9) we finally obtain

$$\frac{M_z b}{JG} = [\epsilon_{45}^{(1)} + \epsilon_{45}^{(3)}] - [\epsilon_{\theta\theta}^{(1)} + \nu \epsilon_{zz}^{(1)}] - \left(\frac{1-\nu}{2}\right) [\epsilon_{zz}^{(2)} + \epsilon_{zz}^{(4)}]$$

Thus all five desired load resultants can be determined from the six strain gages shown in Figure 8. These equations are summarized on Table I.

TABLE I

SUMMARY OF SIX-GAGE, 90° CONFIGURATION (CASE A)

$$\frac{M_x b}{EI_{xx}} = \frac{1}{2} [\epsilon_{zz}^{(2)} - \epsilon_{zz}^{(4)}]$$

$$\frac{M_y b}{EI_{yy}} = \frac{1}{2} [\epsilon_{zz}^{(2)} + \epsilon_{zz}^{(4)}] - \epsilon_{zz}^{(1)}$$

$$\frac{P}{E} \left[\frac{a^2(b^2+a^2)}{b^2(b^2-a^2)} \right] = \frac{1}{1-\nu} [\epsilon_{\theta\theta}^{(1)} + \nu \epsilon_{zz}^{(1)}]$$

$$\frac{F_z}{EA} = \frac{1}{2} [\epsilon_{zz}^{(4)} + \epsilon_{zz}^{(2)}] + \frac{\nu}{1-\nu} [\epsilon_{\theta\theta}^{(1)} + \nu \epsilon_{zz}^{(1)}]$$

$$\frac{M_z b}{JG} = [\epsilon_{45}^{(1)} + \epsilon_{45}^{(3)}] - [\epsilon_{\theta\theta}^{(1)} + \nu \epsilon_{zz}^{(1)}] - \left(\frac{1-\nu}{2} \right) [\epsilon_{zz}^{(2)} + \epsilon_{zz}^{(4)}]$$

where

a = Inside radius

b = Outside radius

A = Area of pipe cross section

ν = Poisson's ratio

E = Young's Modulus

G = Shear Modulus = $\frac{E}{2(1+\nu)}$

2.4 Redundant Ten-Gage Hookup

In this case, data from ten of the twelve strain gages is recorded in the configuration shown in Figure 9a. If one of these ten gages fails, one can always select two gages (out of those which remain) which constitute either Case A or Case B as shown in Figure 9b.

Case A has been discussed in the previous section, and the load resultants can be found from the equations given there. The derivation of the data reduction equations for Case B follows that presented in the previous section for Case A, and the details are omitted. The results are summarized in Table II.

It should be noted that the six gages in Case A or Case B are not the only sets of six gages from which all the load resultants can be determined. For any set of six gages, it is only necessary that

1. Two of the six gages be ϵ_{45} gages on diametrically opposite sides of the pipe.
2. The four linear algebraic equations relating the measured strains and the load resultants which can be written for the remaining four strain gages must be linearly independent.

As shown in the previous section, the twisting moment M_z is found from the two ϵ_{45} strain gages via

$$\frac{M_z b}{JG} = \epsilon_{45}(\theta) + \epsilon_{45}(\theta+\pi) + (1-\nu) \left[\frac{F_z}{AE} + \frac{a^2(b^2 + a^2)P}{b^2(b^2 - a^2)E} \right] \quad (12)$$

9a Redundant Ten Gage Configuration

9b Case A and Case B

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TABLE II

SUMMARY OF SIX-GAGE, 90° CONFIGURATION (CASE B)

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$$\frac{M_x b}{EI_{xx}} = \epsilon_{zz}^{(2)} - \frac{1}{2} [\epsilon_{zz}^{(1)} + \epsilon_{zz}^{(3)}]$$

$$\frac{M_y b}{EI_{yy}} = \frac{1}{2} [\epsilon_{zz}^{(3)} - \epsilon_{zz}^{(1)}]$$

$$\frac{M_z b}{JG} = 2\epsilon_{45}^{(1)} - \epsilon_{zz}^{(1)} - \epsilon_{\theta\theta}^{(1)}$$

$$\frac{F_z}{EA} = \frac{1}{2} [\epsilon_{zz}^{(1)} + \epsilon_{zz}^{(3)}] + \frac{\nu}{1-\nu} [\epsilon_{\theta\theta}^{(1)} + \nu \epsilon_{zz}^{(1)}]$$

$$\frac{P}{E} \left[\frac{a^2(b^2+a^2)}{b^2(b^2-a^2)} \right] = \frac{1}{1-\nu} [\epsilon_{\theta\theta}^{(1)} + \nu \epsilon_{zz}^{(1)}]$$

$$\frac{M_z b}{JG} = \epsilon_{45}^{(2)} + \epsilon_{45}^{(4)} - [\epsilon_{\theta\theta}^{(1)} + \nu \epsilon_{zz}^{(1)}] - \frac{(1-\nu)}{2} [\epsilon_{zz}^{(1)} + \epsilon_{zz}^{(3)}]$$

where

a = Inside radius

b = Outside radius

A = Area of pipe cross section

 ν = Poisson's ratio

E = Young's Modulus

G = Shear Modulus = $\frac{E}{2(1+\nu)}$

and thus M_z can be found once F_z and P are determined. For each of the remaining four gages, one of the following pair of equations will apply:

$$\epsilon_{\theta\theta} = \frac{P}{E} \left[\frac{a^2(b^2+a^2)}{b^2(b^2-a^2)} \right] - \nu \frac{M_x b}{EI_{xx}} \sin\theta + \frac{M_y b}{EI_{yy}} \cos\theta - \nu \frac{F_z}{EA} \quad (13)$$

$$\epsilon_{zz} = \frac{M_x b}{EI_{xx}} \sin\theta - \frac{M_y b}{EI_{yy}} \cos\theta + \frac{F_z}{EA} - \frac{\nu P}{E} \left[\frac{a^2(b^2+a^2)}{b^2(b^2-a^2)} \right] \quad (14)$$

Thus one forms the four appropriate equations from the above pair, and then determines if they are linearly independent by verifying that the determinant of coefficients does not vanish. If this is the case, then the four equations may be solved for M_x , M_y , P , and F_z by well-known methods. For purposes of routine data evaluation, however, the equations presented for Case A and Case B will be sufficient to handle all cases which involve the failure of one of the eight strain gages.

2.5 Complete Twelve-Gage Hookup

The main purpose of recording data from all twelve gages is to evaluate the applicability of the analytical assumptions made in deriving expressions for the load resultants. The load resultants themselves can be determined from the equations derived for Case A or Case B in the previous sections. Somewhat simpler expressions can be written if the data from more than

six gages are utilized. However, since it is anticipated that the data from all twelve gages will be recorded only once at each cross section, it is not worth the additional effort to program a separate set of data reduction equations for the twelve-gage configuration.

A qualitative check on the assumptions of "technical beam theory" can be obtained as follows. Consider the assumption that the shear strain $\epsilon_{z\theta}$ due to M_z should be the same at all points of the circumference of the pipe. The shear strain $\epsilon_{z\theta}$ can be independently evaluated at each pair of opposing rosettes using Equation (9).

Now the two values of $\epsilon_{z\theta}^{(k)}$ will not, in general, be identical. They will certainly not be the same if local discontinuities are present. However, they will also differ due to the fact that the strain gages are not perfect, and each strain gage reading has an inherent error in it which is random and beyond our control. Thus when the two values of $\epsilon_{z\theta}^{(k)}$ are compared, it is necessary to be able to determine how much of the discrepancy is due to errors in the strain gages, and how much (if any) is due to local perturbations in the stress field.

A measure of the errors inherent in the strain gages is obtained from the variance of each strain gage reading, and this variance can be determined from data supplied by their manufacturer. The errors in the strain gages give rise to an uncertainty in the

determination of each $\epsilon_{z\theta}^{(k)}$. However, if the variances of the strain gage readings are known, the variance in each $\epsilon_{z\theta}^{(k)}$ can be computed as shown below.

The variance in a quantity which is a function of several independent variables whose variances are known can be computed using the so-called "Equation of Linear Error Propagation", as shown in Reference 2. Thus, if y is a function of n variables p_i ,

$$y = f(p_1, p_2, \dots, p_n) \quad (15)$$

and each p_i has an associated variance σ_i^2 , then the total variance of y is given by

$$\sigma_y^2 = \sum_{i=1}^n \left(\frac{\partial f}{\partial p_i} \right)^2 \sigma_i^2 \quad (16)$$

assuming that the p_i are uncorrelated. Since the shear strain due to M_z is one half the sum of two measurements of $\epsilon_{z\theta}$ on opposite sides of the pipe, one has from Equations (8) and (9),

$$2 \epsilon_{z\theta} = \epsilon_{45}^{(1)} + \epsilon_{45}^{(3)} - \frac{1}{2} \left[\epsilon_{zz}^{(1)} + \epsilon_{zz}^{(3)} + \epsilon_{\theta\theta}^{(1)} + \epsilon_{\theta\theta}^{(3)} \right]$$

and hence the variance is

$$\sigma_{\epsilon_{z\theta}}^2 = 1/4 \left[\sigma_{\epsilon_{45}}^2 + \sigma_{\epsilon_{45}}^2 \right] + \sigma_{\epsilon_{zz}}^2 + \sigma_{\epsilon_{zz}}^2 + \sigma_{\epsilon_{\theta\theta}}^2 + \sigma_{\epsilon_{\theta\theta}}^2 \quad (17)$$

This gives the variance in the value of $\epsilon_{z\theta}$ obtained from rosettes 1 and 3. By an obvious interchange of indices one also obtains the variance in $\epsilon_{z\theta}$ obtained from rosettes 2 and 4.

A detailed discussion of the uncertainty in the output of each strain gage is presented in Section 3.0. There it is shown that the variance in each measured strain value depends both on the magnitude of the indicated strain and on the accuracy of the recording system, and is given by

$$\sigma_{\epsilon}^2 = \left[\frac{0.079}{2} \epsilon \right]^2 + [3.0 \times 10^{-6}]^2 \quad (18)$$

where ϵ is the indicated strain at each strain gage. This expression applies to the output of each gage. Using this, the variance $\sigma_{\epsilon_{z\theta}}^2$ can be computed from Equation (17). A different variance will be calculated at each pair of rosettes, and these will be denoted by $\sigma_{\epsilon_{z\theta}}^{(k)}$, $k = 1, 2$.

The question of whether or not the strain $\epsilon_{z\theta}$ is constant around the perimeter can now be considered. If $\epsilon_{z\theta}$ is assumed to be constant, then its most probable value is the average of the two measured values, as given by

$$\bar{\epsilon}_{z\theta} = \frac{1}{2} \sum_{k=1}^2 \epsilon_{z\theta}^{(k)}$$

Now the two measured values $\epsilon_{z\theta}^{(k)}$ will, in general differ from the average, $\bar{\epsilon}_{z\theta}$, and the question to be answered is whether these differences are due only to the intrinsic error in the strain gages, or whether the differences are large enough to indicate that $\epsilon_{z\theta}$ is not the same around the circumference of the pipe.

This question can be answered in a statistical sense by the use of the "F-Ratio" test as described in Reference 2. The F-Ratio is the ratio of the total variance of the measurements with respect to the assumed model to the intrinsic variance of the individual measurements. In this case, the total variance is

$$\sigma_{\text{total}}^2 = \sum_{k=1}^2 (\epsilon_{z\theta}^{(k)} - \bar{\epsilon}_{z\theta})^2$$

and the intrinsic variance of the measurements can be taken as the average of the $\sigma_{\epsilon_{z\theta}}^2(k)$, i.e.,

$$\bar{\sigma}_{\epsilon_{z\theta}}^2 \triangleq \frac{1}{2} \sum_{k=1}^2 \sigma_{\epsilon_{z\theta}}^2(k)$$

The total variance is said to have one degree of freedom, since it represents two independent measurements of a single quantity, and hence one is redundant. The intrinsic variance is determined from the variance in the strain gage values, which is based on the manufacturer's published data. It is assumed that this data is based on a large number of tests, and hence the variance would have a large number of degrees of freedom. Thus, the intrinsic variance of the measurement (the denominator of the F-Ratio) is taken to have an infinite number of degrees of freedom. The F-Ratio appropriate to this case is then

$$F_{1,\infty} = \frac{\sum_{k=1}^4 [\epsilon_{z\theta}^{(k)} - \bar{\epsilon}_{z\theta}]^2}{\frac{1}{4} \sum_{k=1}^4 \sigma_{\epsilon_{z\theta}}^2(k)}$$

If the total variance (the numerator) is due only to the variance in the individual strain measurements, then the F-Ratio should be nearly unity. If the strain $\epsilon_{z\theta}$ is not constant around the perimeter of the pipe, however, then the total variance must be larger than the intrinsic variance, and this is reflected by the F-Ratio being greater than one.

Values of the F-Ratio (for different degrees of freedom) are tabulated in Reference 2, from which one finds that any value of $F_{1,\infty}$ less than 3.84 would not be considered a rare event (with 95% confidence). Thus for values of $F_{1,\infty}$ less than 3.84, the data does not reject the hypothesis that the shear strain $\epsilon_{z\theta}$ is constant around the perimeter of the pipe.

In order to verify the second assumption that plane sections remain plane, it is necessary to evaluate the planarity of the axial strain surface, i.e., the surface

$$\epsilon_{zz} = \epsilon_{zz}(x,y)$$

where (x,y) are coordinates in the plane of the cross section.

As seen from equation (12), ϵ_{zz} depends not only on M_x , M_y , and F_z , but also on the internal pressure. The hoop stress due to internal pressure will be constant provided the cross section

is sufficiently remote from any bends or discontinuities. This can be checked by evaluating equation (7) at each of the four rosettes. The right hand side of (7) involves both the material property ν and the measured strain values $\epsilon_{\theta\theta}$ and ϵ_{zz} . At any given cross section, however, the value of ν should be nearly constant, and since we are interested in the uniformity of the hoop stress (due to the pressure) around the circumference and not its absolute value, the variance of ν will be neglected. If $\sigma_{P^*}^2$ denotes the variance of the quantity P^* defined by

$$\frac{P(1-\nu^2)}{E} \left[\frac{a^2(b^2+a^2)}{b^2(b^2-a^2)} \right] = P^* \quad (22)$$

then $\sigma_{P^*}^2$ is computed from equation (7) using equation (16) as

$$\sigma_{P^*}^2 = \sigma_{\epsilon_{\theta\theta}}^2 + \nu^2 \sigma_{\epsilon_{zz}}^2 \quad (23)$$

where the variance in each strain gage reading is obtained from equation (18) as before. A value of $\sigma_{P^*}^2$ is obtained at each rosette. Independent values of P^* are also computed at each rosette, which will be denoted as $P^*(i)$ for $i = 1, 4$. Then the most probable constant value of P^* is the average of the four values, $\bar{P}^*$, as computed from

$$\bar{P}^* = \frac{1}{4} \sum_{i=1}^4 P^*(i) \quad (24)$$

As was done for the shear strains, the F-Ratio Test can be used to evaluate the assumption that P^* does not vary around the circumference. In this case, the F-ratio is given by

$$F_{3,\infty} = \frac{\sum_{i=1}^4 [P^*(i) - \bar{P}^*]^2}{\frac{1}{4} \sum_{i=1}^4 \sigma_{P^*}^2(i)} \quad (25)$$

where, as before, the average of the variances computed at each rosette location is used in the denominator. The numerator has three degrees of freedom since there are four independent measurements of P^* , and hence three are redundant. The critical value of $F_{3,\infty}$ is 2.60. Thus, it is concluded that for values of $F_{3,\infty}$ less than 2.60, the data does not reject the hypothesis that the hoop stress due to internal pressure is constant around the circumference of the pipe.

If P^* is sensibly constant as determined above, then one can proceed to evaluate the planarity of the axial strain surface, knowing now that any nonplanarity is not due to local disturbances in the hoop stress. The assumption that plane sections remain plane implies that the axial strain ϵ_{zz} due to M_z , M_y , and F_z is a linear function of x and y . If we denote the right hand side of equation (10) as $F(x,y)$,

$$F(x,y) = \frac{1-\nu^2}{E} \left[\frac{M_x b}{I_{xx}} \sin\theta - \frac{M_y b}{I_{yy}} \cos\theta + \frac{F_z}{A} \right] \quad (26)$$

then $F(x,y)$ is proportional to the axial strain due to M_x , M_y , and F_z , with any component due to pressure removed. A value of F is computed at each rosette, and these four values should

be in a common plane. The "best-fitting" plane is determined by the Least Squares Technique, as described below. We seek the best fitting equation for $F(x,y)$ in the form

$$F(x,y) = B_0 + B_1x + B_2y \quad (27)$$

The coefficients B_0 , B_1 , and B_2 will be evaluated from the values of F obtained at each of the four rosettes, using equation (10) which becomes

$$F^{(k)} = \epsilon_{zz}^{(k)} + v\epsilon_{\theta\theta}^{(k)} \quad k = 1, 4 \quad (28)$$

Again it is assumed that v does not vary around the circumference. Associated with each $F^{(k)}$ is a variance $\sigma_F^2(k)$, computed using equation (16) as

$$\sigma_F^2(k) = \sigma_{\epsilon_{zz}}^2(k) + v^2 \sigma_{\epsilon_{\theta\theta}}^2(k) \quad k = 1, 4.$$

As before, the variances in the individual strain values are computed using equation (18). The values of B_0 , B_1 , B_2 are obtained by solving a set of linear simultaneous equations given in Reference 2 as

$$X^T V^{-1} X B = X^T V^{-1} Y \quad (30)$$

where

$$X = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \\ 1 & x_4 & y_4 \end{bmatrix}$$

$$V = \begin{bmatrix} \sigma_F^2(1) & 0 & 0 & 0 \\ 0 & \sigma_F^2(2) & 0 & 0 \\ 0 & 0 & \sigma_F^2(3) & 0 \\ 0 & 0 & 0 & \sigma_F^2(4) \end{bmatrix}$$

$$B = \begin{bmatrix} B_0 \\ B_1 \\ B_2 \end{bmatrix}$$

$$Y = \begin{bmatrix} F(1) \\ F(2) \\ F(3) \\ F(4) \end{bmatrix}$$

and $[]^T$ denotes the transpose and $[]^{-1}$ denotes the inverse. If the four values of variance $\sigma_F^2(k)$ are not appreciably different, one can replace V^{-1} by the identity matrix. A closed-form solution for equations (3) could be written using Cramer's Rule, but no appreciable labor is saved over using a standard numerical inversion subroutine on a digital computer.

Once equations (30) have been solved for the "best-fitting" values of B_0 , B_1 , B_2 , an analysis of variance can be performed to determine how well the four values $F^{(k)}$ fit the plane. For this purpose, one may again use the "F-Ratio" test as described in Reference 2. Here, the numerator is the sum of squares of the deviations of the measured data from the "best-fit" plane, i.e.,

$$\sum_{k=1}^4 [F^{(k)} - F(x_k, y_k)]^2$$

and this is an estimate of the total variance of the data. Since there were three constants in $F(x,y)$ determined by the Least Squares procedure and four independent measurements, this estimate of variance has only one degree of freedom. The total variance includes the variance due to lack of fit of the data with the "best-fit" plane and also the intrinsic variability of the measurement, that is, the $\sigma_F^2(k)$ computed at each rosette. The denominator of the F-Ratio is (as before) the intrinsic variance of the measurement, which is assumed to have an infinite number of degrees of freedom.

If the "best fitting" plane (as determined by Least Squares) closely matches the data, then the F-ratio should be nearly unity, since the total variance should be nearly the same as the intrinsic variance. For the denominator, it is appropriate to use the average of the $\sigma_F^2(k)$, i.e.,

$$\bar{\sigma}_F^2 = \frac{1}{4} \sum_{k=1}^4 \sigma_F^2(k) \quad (27)$$

Thus, the F-ratio is given by

$$F_{1,\infty} = \frac{\sum_{k=1}^4 [F^{(k)} - F(x_k, y_k)]^2}{\bar{\sigma}_F^2} \quad (28)$$

From tabulated values of the F-ratio distribution, any value of $F_{1,\infty}$ less than 3.84 would not be considered a rare event (with

95% confidence). Thus, for values of $F_{1,\infty}$ less than 3.84, the data does not reject the hypothesis that the axial strain surface is planar. Clearly, this is a coarse estimate. The lack of sensitivity in this case is due to the fact that we have only one degree of freedom in the estimate of the total variance. (In fact, it would take seven degrees of freedom in estimating the total variance to reduce the critical value of the F-ratio to 2.01.) However, with 80% confidence, the critical value of $F_{1,\infty}$ is 1.64, and thus for values of $F_{1,\infty}$ larger than 1.64, one would begin to doubt the validity of the assumption that plane sections of the pipe remained plane after deformation.

3.0 ERROR ANALYSIS OF LOAD DETERMINATION

As discussed in the previous sections, the main purpose of the strain gages is to determine the load resultants at the various cross sections of interest in the primary coolant piping. In this section, an analysis of the errors which can be anticipated in the load determination is presented.

In any instrumentation system significant errors which are present can be categorized as being in two classes:

- (1) Dynamic Errors
- (2) Calibration Errors.

The dynamic errors are of importance when a measurement of a steady state process is being made, and are due to unavoidable changes in the conditions which exist while the measurement is being made. Examples are changes in air temperature, presence of vibration, and electrical noise. Determina-

tion of the magnitude of the dynamic errors requires the use of redundant instrumentation. Calibration errors, as their name implies, are errors due to random uncertainties in the scale factors and zero shifts in each transducer and errors in the recording and processing instruments. In a strain gage, for example, the numerical factor relating electrical output to strain is generally known only to within several percent. More precise scale factors can be obtained, in some cases, by individually calibrating each transducer, but this cannot always be done. For example, a strain gage installed on a test specimen for calibration purposes cannot generally be removed and reinstalled for use in the field. And even in cases where individual calibration is feasible, economic considerations frequently preclude this option. Thus, in many cases, one must rely on a single scale factor valid for all transducers of the same type and whose value is known within a prescribed tolerance that is specified by the transducer manufacturer.

In making a strain gage measurement, the most important dynamic errors are due to thermal effects:

- (a) Change in gage factor with temperature
- (b) Change in lead wire resistance with temperature
- (c) Thermal drift at high temperatures
- (d) Thermal lag due to insulation of strain gage from piping.
- (e) Thermal coefficients of expansion of both strain gage and body being tested being affected by temperature changes.

All of these potential sources of error must be considered when making a strain measurement over a time period during which significant temperature changes can be anticipated. The purpose of the strain

measurements on the LOFT piping, however, is to measure the loads which are imposed on the structure due to transient pressure waves which exist in the fluid inside the pipes during a Loss of Coolant Experiment. The time span during which these loads occur is on the order of 70 to 100 milliseconds. Preliminary experiments show that no significant temperature changes occur in the pipes during this period. Thus, in the measurement of subcooled decompression loads, none of the dynamic errors described above need be considered.

By contrast, however, there are several calibration errors which could be significant. In particular, one must consider:

- (1) Uncertainty in gage (scale) factor.
- (2) Errors in the data acquisition system.
- (3) Uncertainty in material properties and physical dimensions utilized in obtaining the load resultants from measured strains.

When each individual uncertainty is known, the equation of linear error propagation, equation (16) can be used to establish an error bound for each load resultant computed from the measured strains.

The uncertainty in the strain gage calibration (scale) factor is one of the larger sources of variance. The strain gages to be used on LOFT are of the weldable type manufactured by Ailtech (formerly Microdot, Inc.), City of Industry, California. According to their published catalog, Strain Gage Products, the scale factor is known to within $\pm 3\%$, and the scale factor can change by no more than $\pm 1.75\%$ per 100°F temperature change. It will be assumed that these values are known with a 99% con-

fidence level, and hence correspond to $\pm 3\sigma$ error bands. Since the strain gages will be exposed to ambient temperatures from 50°F to 650°F, the error bands are

$$3\sigma_{\text{tolerance}} = 0.03 \Gamma_{\text{nom}}$$

$$3\sigma_{\text{thermal}} = (0.0175 \Gamma_{\text{nom}} \text{ per } 100^\circ\text{F}) (650^\circ\text{F})$$

$$= 0.114 \Gamma_{\text{nom}}$$

where Γ_{nom} is the nominal scale factor specified for the strain gage.

The combined (RMS) variance in the gage factor is then

$$\sigma_{\Gamma}^2 = \left[\frac{0.03}{3} \Gamma_{\text{nom}}\right]^2 + \left[\frac{0.114}{3} \Gamma_{\text{nom}}\right]^2 = [0.0395 \Gamma_{\text{nom}}]^2.$$

The strain is obtained from the product of the electrical output E and the scale factor Γ , and hence the variance in the strain reading due to errors in the scale factor is, by equation (16),

$$\sigma_{\epsilon}^2 = (E)^2 \sigma_{\Gamma}^2 = E^2 \Gamma_{\text{nom}}^2 [0.0395]^2 = \epsilon_{\text{nom}}^2 [0.0395]^2 \quad (29)$$

where ϵ_{nom} is taken to be the indicated strain output of the strain gage.

Thus, due to uncertainties in the scale factor, there is a variance in the strain gage output which depends on the magnitude of the measured output. To this variance we must add the variance in the data acquisition system.

An analysis of the data acquisition system has been performed in Reference 3, which indicates that the accuracy of the data recording system will be no worse than 1% of the full-scale output. There may also be errors

which arise in the signal conditioning (the Wheatstone Bridge circuit and the power supply) and these generally manifest themselves as changes in the gage factor. However, by measuring the various ballast resistor and leadwire resistances, one can compute the changes which result in the gage factor. Thus, for this analysis, it is assumed that these factors have been examined, and the gage factor corrected as necessary. Hence, the data acquisition system will be taken to have a $\pm 1\%$ Full-Scale Error. Assuming that this accuracy is known with 95% certainty, this error band corresponds to a 2σ error. Since anticipated dynamic strain levels are less than $600 \mu\text{-in./in.}$ (as determined in Reference 4), the variance of the data acquisition system is

$$\sigma^2 = \left[\frac{0.01 (600 \mu\text{-in./in.})}{2} \right]^2 = 9 \times 10^{-12} (\text{in./in.})^2 \quad (30)$$

The above two values of variance are combined to give the total variance for the measured strain for each strain gage,

$$\sigma_{\epsilon}^2 = [0.0395 \epsilon]^2 + (9 \times 10^{-12}) \quad (31)$$

where

$$\sigma_{\epsilon}^2 = \text{total variance of indicated strain}$$

$$\epsilon = \text{indicated strain.}$$

If only the values of the strains themselves were of interest, then this would be the only variance required. Since, however, it is desired to compute load resultants the strains, the variances in the material properties of the pipes must also be known.

These variances are due to batch-to-batch variability in the composition of the 316 stainless steel from which the piping is fabricated. While there is little data on this subject, personal communication with ANC metallurgists, Reference 5, indicate that these material properties should vary less than 2% from their values published in the literature. This is considered to be a 99% confidence estimate, and thus corresponds to a $\pm 3\sigma$ error. Hence, the variance of the Young's Modulus E and Poisson's ratio ν are

$$\sigma_E^2 = \left[\frac{0.02 E}{3} \right]^2 \quad \sigma_\nu^2 = \left[\frac{0.02 \nu}{3} \right]^2 \quad (32)$$

where E and ν for the piping (SA-376 Grade 316 Stainless Steel) are given in Reference 6 as

$$\begin{aligned} E &= 29 \times 10^6 \text{ psi} \\ \nu &= 0.3 \end{aligned} \quad (33)$$

The allowable tolerances on the physical dimensions of the piping are specified in RDT Specification M 3-3, which in turn requires conformance to ASME Specification SA-376 contained in Reference 7. The piping is specified by a nominal outside diameter b_{nom} and a nominal thickness, t_{nom} . The tolerance on the diameter is $\pm 1\%$, and the minimum wall thickness allowed is a 12.5% reduction in the nominal wall thickness. A maximum wall thickness is not specified. Personal communication with the pipe manufacturer, Reference 8, reveals that the wall thicknesses are manufactured to a tolerance of $\pm 12.5\%$ of the nominal wall thickness. Based

on these tolerances, the variances in I/b and J/b can be calculated.

Preliminary calculations showed that a $\pm 12.5\%$ variation in wall thickness dominated all the potential errors, and gave rise to error bounds on the load resultants of $\pm 20\%$ of the value of the load resultant being measured. This large error band can be significantly reduced by measuring the pipe thickness at each location using ultrasonic techniques. Such techniques are routinely capable of measuring pipe thicknesses with a $\pm 1/32$ -inch accuracy. It will thus be necessary to make such measurements on the LOFT piping, and hence in the following analysis, the dimensional tolerances are taken as

$$\begin{aligned} b &= b_{\text{nom}} + 0.01 b_{\text{nom}} \\ t &= t_{\text{nom}} \pm 1/32 \end{aligned} \quad (34)$$

These estimates are assumed to be known with a 95% confidence level.

Using these tolerances, the variances in I/b and J/b can be computed in routine fashion using equation (16). For example, the variance in the section modulus

$$\frac{I}{b} = \frac{\pi (b^4 - a^4)}{4b}$$

is given by

$$\sigma_{I/b}^2 = \left[\frac{\partial I/b}{\partial a} \right]^2 \sigma_a^2 + \left[\frac{\partial I/b}{\partial b} \right]^2 \sigma_b^2 = \left[\frac{\pi (4a^3)}{4b} \right]^2 \sigma_a^2 + \left[\frac{\pi (3b^2 + \frac{a^4}{b^2})}{4} \right]^2 \sigma_b^2$$

where

$$\begin{aligned} \sigma_b &= \frac{0.01 b_{\text{nom}}}{2} \\ \sigma_a &= \frac{0.5 (a_{\text{max}} - a_{\text{min}})}{2} \end{aligned}$$

and in computing a_{max} and a_{min} , the tolerances in both b_{nom} and t_{nom}

must be considered. Table III lists the nominal dimensions of the LOFT coolant piping, and the computed variances in I/b , J/b , and A . In addition, let K denote the group of dimensional terms in equation (7),

$$K \triangleq \frac{a^2(b^2+a^2)}{b^2(b^2-a^2)}$$

The variance in K is computed in the same fashion as above, and is also listed in Table III.

The variances given in Table III and by equations (31) and (32) are all that are needed in order to evaluate the variances in the load resultants. The equations for the load resultants in terms of the measured strains were obtained in Section 2.3 and 2.4. Each load resultant equation has the form

$$L = L(E, I/b, \epsilon_{zz}, \dots) \quad (35)$$

Using the equation for linear error propagation, equation (16), the variance of each load resultant is obtained as

$$\sigma_L^2 = \left[\frac{\partial L}{\partial E}\right]^2 \sigma_E^2 + \left[\frac{\partial L}{\partial (I/b)}\right]^2 \sigma_{I/b}^2 + \left[\frac{\partial L}{\partial \epsilon_{zz}}\right]^2 \sigma_{\epsilon_{zz}}^2 + \dots \quad (36)$$

In using this equation, it is necessary that all the parameters E , I/b , etc., be independent, and that the errors in these parameters be random and normally distributed.

Applying this equation to the equations given in Table I which relate load resultants to strains for the six-gage, 90° configuration (Case A) gives the following expressions for the variances of the computed load resultants:

TABLE III

Size	Nom. OD	Nom. t	b max b min	a max a min	σ_A^2	$\sigma_{I/b}^2$	$\sigma_{J/b}^2$	σ_K^2
5"	5.563	0.625	2.809	2.216	0.245	0.212	0.848	.0412
			2.754	2.098				
10"	10.750	1.125	5.429	4.335	2.567	9.946	39.784	.0342
			5.321	4.165				
12"	12.750	1.312	6.439	5.158	3.626	19.944	79.776	.0336
			6.311	4.968				
14"	14.000	1.406	7.070	5.695	5.641	35.717	142.868	.0356
			6.930	5.493				
16"	16.000	1.593	8.080	6.518	9.915	90.809	363.236	.0348
			7.920	6.296				

$$\begin{aligned}\sigma_{M_x}^2 &= \left\{ \frac{I}{2b} [\epsilon_{zz}^{(2)} - \epsilon_{zz}^{(4)}] \right\}^2 \sigma_E^2 + \left\{ \frac{E}{2} [\epsilon_{zz}^{(2)} - \epsilon_{zz}^{(4)}] \right\}^2 \sigma_{I/b}^2 \\ &\quad + \left\{ \frac{EI}{2b} \right\}^2 [\sigma_{\epsilon_{zz}}^2(2) + \sigma_{\epsilon_{zz}}^2(4)].\end{aligned}\quad (37)$$

$$\begin{aligned}\sigma_{M_y}^2 &= \left\{ \frac{I}{2b} [\epsilon_{zz}^{(2)} + \epsilon_{zz}^{(4)}] - \frac{I}{b} \epsilon_{zz}^{(1)} \right\} \sigma_E^2 + \left\{ \frac{E}{2} [\epsilon_{zz}^{(2)} + \epsilon_{zz}^{(4)}] \right. \\ &\quad \left. - E \epsilon_{zz}^{(1)} \right\} \sigma_{I/b}^2 + \left\{ \frac{EI}{2b} \right\}^2 [\sigma_{\epsilon_{zz}}^2(2) + \sigma_{\epsilon_{zz}}^2(4)] + \left\{ \frac{EI}{b} \right\}^2 \sigma_{\epsilon_{zz}}^2(1).\end{aligned}\quad (38)$$

$$\begin{aligned}\sigma_{M_z}^2 &= \left\{ \frac{J}{2b(1+\nu)} [\epsilon_{45}^{(1)} + \epsilon_{45}^{(3)} - \epsilon_{\theta\theta}^{(1)} - \nu \epsilon_{zz}^{(3)}] \right. \\ &\quad \left. - \left(\frac{1-\nu}{2} \right) (\epsilon_{zz}^{(2)} + \epsilon_{zz}^{(4)}) \right\}^2 \sigma_E^2 \\ &\quad + \left\{ \frac{E}{2(1+\nu)} [\epsilon_{45}^{(1)} + \epsilon_{45}^{(3)} - \epsilon_{\theta\theta}^{(1)} - \nu \epsilon_{zz}^{(3)}] \right. \\ &\quad \left. - \left(\frac{1-\nu}{2} \right) (\epsilon_{zz}^{(2)} + \epsilon_{zz}^{(4)}) \right\}^2 \sigma_{J/b}^2 \\ &\quad + \left\{ \frac{EJ}{2b(1+\nu)} [\epsilon_{45}^{(1)} + \epsilon_{45}^{(3)} - \epsilon_{\theta\theta}^{(1)} - \nu \epsilon_{zz}^{(3)}] \right. \\ &\quad \left. - \left(\frac{1-\nu}{2} \right) (\epsilon_{zz}^{(2)} + \epsilon_{zz}^{(4)}) \right\}^2 \sigma_\nu^2 \\ &\quad + \left\{ \frac{EJ}{2(1+\nu)b} \right\}^2 \{ \sigma_{\epsilon_{45}}^2(1) + \sigma_{\epsilon_{45}}^2(3) + \sigma_{\epsilon_{\theta\theta}}^2(1) \\ &\quad + \nu^2 \sigma_{\epsilon_{zz}}^2(3) + \frac{(1-\nu)^2}{4} [\sigma_{\epsilon_{zz}}^2(2) + \sigma_{\epsilon_{zz}}^2(4)] \}\end{aligned}\quad (39)$$

$$\sigma_{M_y}^2 = \left\{ \frac{I}{2b} [\epsilon_{zz}^{(3)} - \epsilon_{zz}^{(1)}] \right\}^2 \sigma_E^2 + \left\{ \frac{E}{2} [\epsilon_{zz}^{(3)} - \epsilon_{zz}^{(1)}] \right\}^2 \sigma_{I/b}^2$$

$$+ \left\{ \frac{EI}{2b} \right\}^2 \{ \sigma_{\epsilon_{zz}}^2(3) + \sigma_{\epsilon_{zz}}^2(1) \} \quad (43)$$

$$\sigma_{M_z}^2 = \left\{ \frac{J}{2b(1+\nu)} [\epsilon_{45}^{(2)} + \epsilon_{45}^{(4)} - \epsilon_{\theta\theta}^{(1)} - \nu \epsilon_{zz}^{(1)} \right.$$

$$\left. - \frac{(1-\nu)}{2} (\epsilon_{zz}^{(1)} + \epsilon_{zz}^{(3)})] \right\}^2 \sigma_E^2$$

$$+ \left\{ \frac{E}{2(1+\nu)} [\epsilon_{45}^{(2)} + \epsilon_{45}^{(4)} - \epsilon_{\theta\theta}^{(1)} - \nu \epsilon_{zz}^{(1)} \right.$$

$$\left. - \frac{(1-\nu)}{2} (\epsilon_{zz}^{(1)} + \epsilon_{zz}^{(3)})] \right\}^2 \sigma_{J/b}^2$$

$$+ \left\{ \frac{EJ}{2b(1+\nu)^2} [\epsilon_{45}^{(2)} + \epsilon_{45}^{(4)} - \epsilon_{\theta\theta}^{(1)} - \nu \epsilon_{zz}^{(1)} \right.$$

$$\left. - \frac{(1-\nu)}{2} (\epsilon_{zz}^{(1)} + \epsilon_{zz}^{(3)})] \right\}^2 \sigma_\nu^2$$

$$+ \left\{ \frac{EJ}{2(1+\nu)b} \right\}^2 [\sigma_{\epsilon_{45}}^2(2) + \sigma_{\epsilon_{45}}^2(4) + \sigma_{\epsilon_{\theta\theta}}^2(1) + \nu^2 \sigma_{\epsilon_{zz}}^2(1)$$

$$+ \frac{(1-\nu)^2}{4} [\sigma_{\epsilon_{zz}}^2(1) + \sigma_{\epsilon_{zz}}^2(3)]] \quad (44)$$

$$\sigma_{F_z}^2 = \left\{ \frac{A}{2} [\epsilon_{zz}^{(1)} + \epsilon_{zz}^{(3)}] + \frac{Av}{1-\nu} [\epsilon_{\theta\theta}^{(1)} + \nu \epsilon_{zz}^{(1)}] \right\}^2 \sigma_E^2$$

$$+ \left\{ \frac{E}{2} [\epsilon_{zz}^{(1)} + \epsilon_{zz}^{(3)}] + \frac{Ev}{1-\nu} [\epsilon_{\theta\theta}^{(1)} + \nu \epsilon_{zz}^{(1)}] \right\}^2 \sigma_A^2$$

$$+ \left\{ \frac{EA}{2} \right\}^2 \{ \sigma_{\epsilon_{zz}}^2(1) + \sigma_{\epsilon_{zz}}^2(3) \} + \left\{ \frac{\nu EA}{1-\nu} \right\}^2 \{ \sigma_{\epsilon_{\theta\theta}}^2(1) + \nu^2 \sigma_{\epsilon_{zz}}^2(1) \}$$

$$+ \left\{ \frac{1-\nu^2}{(1-\nu^2)^2} EA \epsilon_{\theta\theta}^{(1)} + \frac{2\nu EA}{(1-\nu^2)^2} \epsilon_{zz}^{(1)} \right\}^2 \sigma_\nu^2 \quad (45)$$

$$\begin{aligned}
\sigma_{F_z}^2 = & \left\{ \frac{A}{2} [\epsilon_{zz}^{(4)} + \epsilon_{zz}^{(2)}] + \frac{Av}{1-v^2} [\epsilon_{\theta\theta}^{(1)} + v\epsilon_{zz}^{(1)}] \right\}^2 \sigma_E^2 \\
& + \frac{E}{2} [\epsilon_{zz}^{(4)} + \epsilon_{zz}^{(2)}] + \frac{Ev}{1-v^2} [\epsilon_{\theta\theta}^{(1)} + v\epsilon_{zz}^{(1)}] \right\}^2 \sigma_A^2 \\
& + \left\{ \frac{(1+v^2)EA}{(1-v^2)^2} \epsilon_{\theta\theta}^{(1)} + \frac{2vEA}{(1-v^2)^2} \epsilon_{zz}^{(1)} \right\}^2 \sigma_v^2 + \left[\frac{EA}{2} \right]^2 \sigma_{\epsilon_{zz}}^2(4) \\
& + \left[\frac{EA}{2} \right]^2 \sigma_{\epsilon_{zz}}^2(2) + \left[\frac{EA v}{1-v^2} \right]^2 \sigma_{\epsilon_{\theta\theta}}^2(1) + \left[\frac{EA v^2}{1-v^2} \right]^2 \sigma_{\epsilon_{zz}}^2(1)
\end{aligned} \tag{40}$$

$$\begin{aligned}
\sigma_P^2 = & \left\{ \frac{K}{1-v^2} [\epsilon_{\theta\theta}^{(1)} + v\epsilon_{zz}^{(1)}] \right\}^2 \sigma_E^2 + \frac{E}{1-v^2} [\epsilon_{\theta\theta}^{(1)} + \epsilon_{zz}^{(1)}] \right\}^2 \sigma_K^2 \\
& + \left\{ EK \left[\frac{-2v}{(1-v^2)^2} \right] \epsilon_{\theta\theta}^{(1)} + EK \left[\frac{1+v^2}{(1-v^2)^2} \right] \epsilon_{zz}^{(1)} \right\}^2 \sigma_v^2 \\
& + \left\{ \left[\frac{EK}{1-v^2} \right]^2 [\sigma_{\epsilon_{\theta\theta}}^2(1) + v^2 \sigma_{\epsilon_{zz}}^2(1)] \right\}
\end{aligned} \tag{41}$$

These variances can be evaluated once data from the strain gages is available.

In a similar fashion, the variances in the loads as determined by the six-gage, 90° configuration (Case B) can be determined, and these are listed below:

$$\begin{aligned}
\sigma_{M_x}^2 = & \left\{ \frac{I}{b} \epsilon_{zz}^{(2)} - \frac{I}{b} [\epsilon_{zz}^{(1)} + \epsilon_{zz}^{(3)}] \right\}^2 \sigma_E^2 \\
& + \left\{ E \epsilon_{zz}^{(2)} - \frac{E}{2} [\epsilon_{zz}^{(1)} + \epsilon_{zz}^{(3)}] \right\}^2 \sigma_{I/b}^2 \\
& + \left\{ \frac{EI}{b} \right\}^2 \left\{ \sigma_{\epsilon_{zz}}^2(2) + \frac{1}{4} [\sigma_{\epsilon_{zz}}^2(1) + \sigma_{\epsilon_{zz}}^2(3)] \right\}
\end{aligned} \tag{42}$$

$$\begin{aligned}
\sigma_P^2 = & \left\{ \frac{K}{1-\nu^2} [\epsilon_{\theta\theta}^{(1)} + \nu \epsilon_{zz}^{(1)}] \right\}^2 \sigma_E^2 + \frac{E}{1-\nu^2} [\epsilon_{\theta\theta}^{(1)} + \epsilon_{zz}^{(1)}]^2 \sigma_K^2 \\
& + \left\{ EK \left[\frac{-2\nu}{(1-\nu^2)^2} \right] \epsilon_{\theta\theta}^{(1)} + EK \left[\frac{1+\nu^2}{(1-\nu^2)^2} \right] \epsilon_{zz}^{(1)} \right\}^2 \sigma_v^2 \\
& + \left\{ \left[\frac{EK}{1-\nu^2} \right]^2 [\sigma_{\epsilon_{\theta\theta}}^2(1) + \nu^2 \sigma_{\epsilon_{zz}}^2(1)] \right\}
\end{aligned} \tag{46}$$

4.0 ILLUSTRATIVE EXAMPLE

As an illustration of the procedure for obtaining error bounds for the load resultant computations, and also to gain some insight into the order of magnitudes of the variances which could be expected in actual LOFT blowdown data, consider the following load resultants:

$$M_y = 1486 \text{ in.-kips}$$

$$M_x = 516 \text{ in.-kips}$$

$$M_z = 316 \text{ in.-kips}$$

$$F_z = 183 \text{ kips}$$

These loads were predicted in Reference 4 at the inlet nozzle of the reactor vessel (blowdown loop). The loads were predicted using the dynamic structural analysis code, SHAKER 1, based on input forcing functions (representing the fluid-induced transient loads) generated by the WHAM computer code in Reference 9. The loads listed are the maximum loads which were predicted for the subcooled blowdown, and these loads did not occur simultaneously. The loads listed correspond to a loss-of-coolant experiment in which two 8-inch, quick-opening valves are opened completely and simultaneously in 0.001 second.

These loads will be utilized as follows. Using equations (12) through (14), the strains at each gage of the Minimum Six-Gage Configuration (Case A) will be computed for each load separately. Then the variances will be computed for each load using equations (37) through (41). A pressure drop of 300 psi will be assumed. This value of pressure drop is typical near the midway point of subcooled blowdown. The reactor vessel nozzle is a 14-inch diameter pipe.

Load M_y - The Minimum Five-Gage Configuration is shown on Page 22.

From equation (12),

$$\epsilon_{zz}^{(1)} = -\frac{M_y b}{EI_{yy}} - \frac{\nu P}{EK} = 322 - 9 = 313 \text{ } \mu\text{in./in.}$$

$$\epsilon_{zz}^{(2)} = -\frac{\nu P}{EK} = -9 \text{ } \mu\text{in./in.}$$

$$\epsilon_{zz}^{(4)} = -\frac{\nu P}{EK} = -9 \text{ } \mu\text{in./in.}$$

and from equations (13) and (14)

$$\epsilon_{\theta\theta}^{(1)} = \frac{P}{EK} + \nu \frac{M_y b}{EI} = 30 - 96.68 = -66.68 \text{ } \mu\text{in./in.}$$

$$\epsilon_{45}^{(1)} = \frac{1-\nu}{2} \frac{P}{EK} - \left(\frac{1-\nu}{2}\right) \frac{M_y b}{EI} = 10.5 + 112 = 122.5 \text{ } \mu\text{in./in.}$$

The above five strains would be those indicated by a Minimum Six-Gage Configuration of strain gages due to M_y and P loads only.

If these five readings were used in the equations in Table I to solve for the load resultant M_y , then the variance $\sigma_{M_y}^2$ associated with this calculation is given by equation (37), which yields

$$\sigma_{M_y}^2 = [-0.051]^2 \sigma_E^2 + [-9338]^2 \sigma_{I/b}^2 + [2.305 \times 10^9]^2 [\sigma_{\epsilon_{zz}}^2(2) + \sigma_{\epsilon_{zz}}^2(4)] + [4.611 \times 10^9]^2 \sigma_{\epsilon_{zz}}^2(1)$$

where

$$\sigma_E^2 = 0.050 \times 10^{+12} \quad [\text{from equation (32)}]$$

$$\sigma_{I/b}^2 = 36.5 \quad [\text{from Table III}]$$

$$\sigma_{\epsilon_{zz}}^2(2) = 9 \times 10^{-12}$$

$$\sigma_{\epsilon_{zz}}^2(4) = 9 \times 10^{-12}$$

$$\sigma_{\epsilon_{zz}}^2(1) = 160 \times 10^{-12}$$

then the total variance in M_y is

$$\sigma_{M_y}^2 = [1.30 \times 10^8] \quad (\text{due to } E)$$

$$+ [3.182 \times 10^9] \quad (\text{due to } I/b)$$

$$+ [9.56 \times 10^5] \quad (\text{due to strains})$$

$$+ [3.40 \times 10^9]$$

therefore

$$\sigma_{M_y} = 81.0 \times 10^4 \text{ in.-lb}$$

$$2\sigma_{M_y} = 162 \text{ in.-kips.}$$

Thus, with 95% certainty, the measured load resultant M_y would fall in the range

$$M_y = 1486 \pm 162 \text{ in.-kips}$$

and thus there is an error band of 11% of M_y in this calculation.

It can be seen that the total variance in M_y is due primarily to the variances in I/b and in the strain gage scale factor, and that a more accurate determination of the wall thickness (which would reduce $\sigma_{I/b}^2$) would not greatly reduce the total variance. Thus measuring the pipe wall thickness to within $\pm 1/32$ -inch will be adequate.

Load M_x - Again using equations (12) through (14), the strains that would be indicated by a Minimum Five-Gage Configuration due to M_x and P are found to

be

$$\epsilon_{zz}^{(1)} = -9 \text{ } \mu\text{in./in.}$$

$$\epsilon_{zz}^{(2)} = +102 \text{ } \mu\text{in./in.}$$

$$\epsilon_{zz}^{(4)} = -120 \text{ } \mu\text{in./in.}$$

$$\epsilon_{\theta\theta}^{(1)} = 30 \text{ } \mu\text{in./in.}$$

$$\epsilon_{45}^{(1)} = 10.5 \text{ } \mu\text{in./in.}$$

Hence, from equation (38),

$$\sigma_{M_x}^2 = \{0.017\}^2 \sigma_E^2 + \{3219.0\}^2 \sigma_{I/b}^2 + \{2.305 \times 10^9\}^2 [\sigma_{\epsilon_{zz}}^2(2) + \sigma_{\epsilon_{zz}}^2(4)]$$

where

$$\sigma_E^2 = 0.05 \times 10^{12}$$

$$\sigma_{I/b}^2 = [12.2]^2$$

$$\sigma_{\epsilon_{zz}}^2(2) = 25 \times 10^{-12}$$

$$\sigma_{\epsilon_{zz}}^2(4) = 31 \times 10^{-12}$$

and thus $\sigma_{M_x}^2$ is given by

$$\sigma_{M_x}^2 = [1.44 \times 10^7] \quad (\text{due to } E)$$

$$+ [3.78 \times 10^8] \quad (\text{due to } I/b)$$

$$+ [2.97 \times 10^8] \quad (\text{due to strains})$$

$$= 6.89 \times 10^8$$

therefore

$$2\sigma_{M_x} = 52.5 \text{ in.-kips}$$

Thus, with 95% certainty, the measured load M_x would fall in the range

$$M_x = 516 \pm 52.5 \text{ in.-kips}$$

and this is an error band of $\pm 10.2\%$ of M_x in this calculation.

Load M_z - Again, using equations (12) through (14), the strains that would be indicated by a Six-Gage Configuration (Case A) due to $M_z = 316$ in.-kips and P are:

$$\epsilon_{zz}^{(1)} = -9 \text{ } \mu\text{in./in.}$$

$$\epsilon_{zz}^{(2)} = -9 \text{ } \mu\text{in./in.}$$

$$\epsilon_{zz}^{(4)} = -9 \text{ } \mu\text{in./in.}$$

$$\epsilon_{\theta\theta}^{(1)} = 30 \text{ } \mu\text{in./in.}$$

$$\epsilon_{45}^{(1)} = 55 \text{ } \mu\text{in./in.}$$

The variance in M_z , given by equation (39), becomes

$$\begin{aligned} \sigma_{M_z}^2 &= \{0.011\}^2 \sigma_E^2 \\ &+ \{992.7\}^2 \sigma_{J/b}^2 \\ &+ \{2.43 \times 10^5\}^2 \sigma_v^2 \\ &+ 2.10 \times 10^9 \left[2\sigma_{\epsilon_{45}}^2 + \sigma_{\epsilon_{zz}}^2 + \sigma_{\epsilon_{\theta\theta}}^2 \right] \end{aligned}$$

where

$$\sigma_E^2 = 0.05 \times 10^{12}$$

$$\sigma_{J/b}^2 = 146.0$$

$$\sigma_v^2 = 5.4 \times 10^{-6}$$

$$\sigma_{\epsilon_{45}}^2 = 13.8 \times 10^{-12}$$

$$\sigma_{\epsilon_{zz}}^{2(1)} = 9 \times 10^{-12}$$

$$\sigma_{\epsilon_{\theta\theta}}^{2(1)} = 10.4 \times 10^{-12}$$

Thus, the total variance in M_z becomes

$$\begin{aligned} \sigma_{M_z}^2 &= [6.05 \times 10^6] && \text{(due to E)} \\ &+ [1.33 \times 10^8] && \text{(due to J/b)} \\ &+ [3.2 \times 10^5] && \text{(due to } \nu) \\ &+ [2.07 \times 10^8] && \text{(due to strains)} \\ &= 3.51 \times 10^8 \end{aligned}$$

Therefore, $2\sigma_{M_z} = 37.5$ in.-kips, and hence, with 95% certainty, the measured value of M_z should fall in the range

$$M_z = 316 \pm 37.5 \text{ in.-kips}$$

and this corresponds to a 2σ error band of $\pm 11.9\%$ of M_z .

Load F_z - Again using equations (12) through (14), the strains that would be indicated by a Minimum Six-Gage Configuration due to

$F_z = 183$ kips and P are:

$$\epsilon_{zz}^{(1)} = 102 \text{ } \mu\text{in./in.}$$

$$\epsilon_{zz}^{(2)} = 102 \text{ } \mu\text{in./in.}$$

$$\epsilon_{zz}^{(4)} = 102 \text{ } \mu\text{in./in.}$$

$$\epsilon_{\theta\theta}^{(1)} = -4 \text{ } \mu\text{in./in.}$$

$$\epsilon_{45}^{(1)} = 50 \text{ } \mu\text{in./in.}$$

Thus, from equation (40), the variance

$$\begin{aligned} \sigma_{F_z}^2 &= \{0.0062\}^2 \sigma_E^2 + \{3264\}^2 \sigma_A^2 + \{1.11 \times 10^5\}^2 \sigma_v^2 \\ &+ \{8.09 \times 10^8\}^2 [\sigma_{\epsilon_{zz}}^2(4) + \sigma_{\epsilon_{zz}}^2(2)] \\ &+ \{5.33 \times 10^8\}^2 \sigma_{\epsilon_{\theta\theta}}^2(1) + \{1.60 \times 10^8\}^2 \sigma_{\epsilon_{zz}}^2(1) \end{aligned}$$

where

$$\sigma_E^2 = 0.05 \times 10^{12}$$

$$\sigma_A^2 = 5.641$$

$$\sigma_v^2 = 5.4 \times 10^{-6}$$

$$\sigma_{\epsilon_{zz}}^2(4) = 25 \times 10^{-12}$$

$$\sigma_{\epsilon_{zz}}^2(2) = 25 \times 10^{-12}$$

$$\sigma_{\epsilon_{zz}}^2(1) = 25 \times 10^{-12}$$

$$\sigma_{\epsilon_{\theta\theta}}^2(1) = 9 \times 10^{-12}$$

and thus the total variance $\sigma_{F_z}^2$ becomes

$$\sigma_{F_z}^2 = [1.92 \times 10^6] \quad (\text{due to E})$$

$$+ [5.76 \times 10^7] \quad (\text{due to A})$$

$$+ [6.65 \times 10^4] \quad (\text{due to } \nu)$$

$$+ [3.59 \times 10^7] \quad (\text{due to strains})$$

$$= 9.549 \times 10^7$$

Hence, $2\sigma_{F_z} = 19.5$ kips and thus, with 95% confidence, the measured load will fall within the range

$$F_z = 183 \pm 19.5 \text{ kips}$$

which corresponds to a 2σ error band of width 10.6% of F_z . Here the total variance is predominately due to the variance in the strain gage scale factor.

5.0 CONCLUSIONS AND RECOMMENDATIONS

The example problem just considered was based on the maximum (analytically predicted) loads for a Loss of Coolant Experiment. The valve opening time was 1 millisecond. This valve opening time is faster than the time anticipated for the LOFT experiments, and hence the loads that were utilized in the example are expected to be somewhat conservative. Based on these loads, it was found that all the load resultants would be measured with an uncertainty of less than $\pm 12\%$. This accuracy is certainly sufficient for making comparisons between actual measured loads and analytically predicted loads.

If the actual measured loads are smaller than the maximum loads used in the example, then the variance due to the tolerance in scale factor will be reduced. In this case, a more accurate measurement of the piping thicknesses would result in a smaller total variance for each calculated load resultant. However, if the maximum measured loads are of the same order as those used in the example, a pipe thickness determination within $\pm 1/32$ inch is adequate, and is well within the routine capabilities of ultrasonic quality control practices.

Finally, it is to be noted that the variances due to E and ν were always at least an order of magnitude smaller than the variances due to tolerances in the physical dimensions and in the scale factor of the strain gages. Hence, one can reasonably omit the variances due to E and ν in any subsequent refinements of the errors in the strain gage system.

6.0 REFERENCES

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