Inelastic Strong Interactions at High Energies

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Peter Suranyi
University of Cincinnati
Cincinnati, Ohio 45221

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ABSTRACT

The investigation of cross section generating classical solutions of field theories, sectons, was continued. Inclusive and semi-inclusive cross sections were shown to be expectation values of simple combinations of field operators in an auxiliary quantum field theory (cut field theory) that can be defined for an arbitrary field theory.

The classical solutions that may dominate inclusive cross sections were investigated. The only space-time symmetry possible for the solutions was shown to be $O(2) \otimes O(1,1)$. Solutions of this symmetry generate an inclusive cross section with Feynman scaling.

Secton solutions were compared with and shown to be distinct from other known classical solutions, instantons and solitons.

Final results in our study of a Reggeon field theory with an internal symmetry were obtained.

I. Further Developments in the Theory of Sections.

During the past one and a half years we have developed a new method for the calculation of inclusive and semi-inclusive cross sections.¹⁻⁵⁾ The method is based on the application of semi-classical methods to the cut field theory^{1,2)}, an auxiliary theory which can be defined for an arbitrary quantum field theory. This cut field theory generates the s-channel discontinuity of Green's functions and it can be used for perturbative or nonperturbative investigations of cross sections. The cut field theory is defined by the action

$$A[\psi, \bar{\psi}] = -\int d^4x d^4y D_+(g, x-y) K \psi(x) K \bar{\psi}(y) - i \int d^4x [L(\psi(x)) - L(\bar{\psi}(x))] , \quad (1)$$

where $K = \frac{m}{\sqrt{m^2 - \square}} + m$ and

$$D_+(g, x) = (2\pi)^{-3} \int d^4p \delta(p^2 - m^2) \theta(p_0) e^{-ip \cdot x} g(p) . \quad (2)$$

$L(\psi(x))$ is the Lagrangian of the original field theory.

Inclusive and semi-inclusive cross sections can be expressed in a simple way as expectation values of certain combinations of field operators in the quantum field theory defined by the action A.²⁾

The expectation values representing cross sections may be dominated by classical solutions of the field equations, derivable from the action of Eq. (1). The field equations are

$$\begin{aligned} \psi(x) &= \int D_{\text{adv}}(x-y) \psi(y) + i \int D_+(g, x-y) [V(\bar{\psi}(y)) - V(\psi(y))] , \\ \bar{\psi}(x) &= \int D_{\text{adv}}(x-y) \bar{\psi}(y) + i \int D_+^*(g, x-y) [V(\bar{\psi}(y)) - V(\psi(y))] , \end{aligned} \quad (3)$$

where $V(\psi(x)) = \partial L_{\text{int}}(\psi(x)) / \partial \psi(x)$ and $D_{\text{adv}}(x)$ is the advanced Green's function satisfying the equation $K D_{\text{adv}}(x) = -\delta^4(x)$.

The classical solutions of Eq. (3) can be put to use in various ways.⁵⁾ The solutions either provide a minimum of the action A(Eq. (1)) for physical values of the coupling constant (that may happen for some values $g = g(p)$) or they provide a minimum only for unphysical coupling constants. In the first

case the cross section can be calculated directly by semi-classical methods. In the second case the cross section can be calculated in large orders (n) of the perturbation expansion.^{6,7)} The expectation values of products of field operators factor into an n dependent, momentum independent multiplier and a multiplier dependent on momenta (or coordinates) only in leading order of n^{-1} .⁷⁾ It is conceivable that the multiplier dependent on momenta provides the appropriate momentum dependence of cross sections (up to an undetermined constant multiplier).

In our view the investigation of some questions of basic importance, such as the symmetry properties of solutions, existence, uniqueness, etc. should precede phenomenological applications. Accordingly, we investigated the existence of solutions of various symmetries. The symmetry of a solution is defined as the maximal subgroup of $O(3,1)$ that applied to the four vector x_μ (or p_μ) leaves the solution $\psi(x)$ invariant.

We could show, for general conditions on the interaction Lagrangian, that no solutions of the following symmetries exist: $O(3,1)$, $O(2,1)$, $O(3)$ and $E(2)$, corresponding to solutions dependent on x^2 , x^2 and x_3 , x^2 and x_0 , x^2 and $x_0 + x_3$, respectively. The two most important facts contributing to the proof of the non-existence theorem were

i) that in certain regions of space-time $\bar{\psi}(x) - \psi(x)$ (and also $\psi(x)$ itself) satisfy homogeneous Volterra integral equations. Volterra equations have unique - in our case, identically zero - solutions under quite general conditions.⁸⁾

ii) The solutions may not have discontinuities across the boundaries of certain regions. This condition favors solutions of symmetry $O(1,1) \otimes O(2)$ (a solution that is dependent on x^2 and $x_T^2 = x_1^2 + x_2^2$ or p^2 and $p_T^2 = p_1^2 + p_2^2$). Introducing x_{II}^2 and x_I^2 , the Casimir operators of the non-compact and compact parts of the surviving symmetry group, respectively, for an $O(k,1) \otimes O(3-k)$

symmetry group ($k = 1, 2, 3$) the field equation that has to be satisfied by the solutions of Eq. (3) reads as follows:

$$[4\lambda \partial^2 / \partial \lambda^2 + (2 + 2k) \partial / \partial \lambda - \partial^2 / \partial x_{\perp}^2 - (k/x_{\perp}) \partial / \partial x_{\perp}] \psi = -V(\psi) \quad (4)$$

where $\lambda = x_{\perp}^2$. It is easy to see that a solution that is finite at $\lambda = 0$ can have a finite jump there only if $k=1$ i.e. for a symmetry $O(1,1) \times O(2)$. Otherwise a δ function singularity appears on the left hand side of Eq. (4). This property of the solutions of Eq. (4) implies that if a solution vanishes for, say $\lambda > 0$, $x_0 > 0$, then it vanishes everywhere (except in the case of $O(1,1) \otimes O(2)$ symmetry).

Finally, we compared section solutions with soliton and instanton solutions and discovered various differences, the most significant one being the existence of a mass shell pole in the momentum representation.⁵⁾

Our future plans and research presently performed in the role of classical solutions in the calculation of cross sections will be described in the adjoining proposal.

II. Reggeon Field Theory

The Reggeon field theory which we studied has been described previously.⁴⁾

The model has an internal SU(2) symmetry, called I-spin and has three multiplets of bare reggeons of $I = 0, 1, 2$ with trajectories

$$\alpha_i(t) = 1 - \Delta_i + \alpha_i' t$$

where $i = 1, 2, 3$ and $\Delta_0, \alpha_0', \Delta_1, \alpha_1'$ are positive and most importantly Δ_2, α_2' are negative; i.e. the $I = 2$ field has negative mass and slope. The Lagrangian in zero transverse dimensions where we work is schematically

$$L = 1/2(p_0 \overleftrightarrow{\partial}_y q_0 + p_1^i \overleftrightarrow{\partial}_y q_1^i + p_2^a \overleftrightarrow{\partial}_y q_2^a) - H(p, q) \quad (5)$$

$$H = -\mu_0 p_0 q_0 - \mu_1 p_1^i q_1^i - \mu_2 p_2^a q_2^a + \lambda_{bc}^a (p_a q_b q_c + q_a p_b p_c)$$

where the triple Regge interactions are SU(2) symmetric and $\mu_1 = -\Delta_1 = \alpha_1(0) - 1$.

The classical solutions analogous to those of Ref. (9) in supercritical Pomeron theory were considered in detail. We have considered in addition the classical solutions and their consequences corresponding to the solution of Ref. (10). We have shown that such solutions exist.¹¹⁾ We studied these solutions by investigating the transition matrix element using path integral techniques and using the collective coordinate method¹²⁾ to control the dependence of the classical solutions on the variables of the internal symmetry group. This allowed for a derivation of the I dependent Hamiltonian H_I , corresponding to the overall exchange of a specific I-spin in the t channel. The total cross sections in the various I-spin channels corresponding to these solutions were calculated. Their behavior was found to be similar to that of Ref. (10) i.e. the cross sections fall with energy. We do not anticipate any further study in this area in the immediate future.

III. General Remarks

P. Suranyi, being on academic leave at the Imperial College, London, England, devoted 100% of his time, and J.J.G. Scanio devoted approximately 50% (100% during academic recesses) of his time to the investigations described in the present report; they are expected to spend the same fraction of their time in the continuation of these investigations during the rest of the current reporting period.